Oscillating Magnetic Dipole In An Inhomogeneous Magnetic Field

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Abstract

A magnetic dipole was placed on a air track inside a Helmholtzcoil. The oscillations of the dipole was measured per time unit and the frequency of the oscillating dipole was calculated. The experimentally determined frequency was compared with a theoretically determined frequency calculated by a point-like dipole moment and a dipole moment of finite size. The frequency that came closer to the experimentally one was the frequency calculated by a point-like dipole moment.

Foundations of Electromagnetics
Supervisors: Avazeh Hashemloo, Dmitry Kobyakov

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1 Introduction

A magnetic dipole is an object that gives magnetic field consisting two magnetic poles with opposite signs by a small distance. A magnetic field of a magnetic dipole is depicted as a closed loop of field and it has the constant magnetic moment. A Helmholtz coil is a device for generating steady magnetic field, which consists of a circular coil. In region of a Helmholtz coil, there is regular magnetic field which is perpendicular to the plane of coil.

In this experiment, a oscillator which consists of a magnetic dipole (a permanent magnet that has north and sputh poles) and glider is located on an air track. The air track passes through the coil perpendicularly and reduces friction between the air track and the glider when the magnetic dipole moves. When the electric current flows on the Helmholtz coil, a magnetic field from coils passes through the magnetic dipole and it moves back and forth. This oscillating magnetic dipole moves with a frequency because a magnetic field generated from the magnetic dipole is on same axis with magnetic field generated from the Helmholtz coil. The velocity of magnetic dipole increases when directions of two fields that were from dipole and coils are same, the velocity of magnetic dipole becomes zero and the oscillator stops when directions of two fields are opposite and cancelled.

The frequency of oscillating magnetic dipole could be obtained with counting oscillating numbers in unit time. In this experiment, we count 10 seconds for measuring how many numbers the oscillator passes the central point of air track - executes oscillation. By multiply 2π to the frequency obtained above, the frequency of the oscillation could be calculated. The first frequency by theoretical approach where the finite size of the magnet and thickness of coil are ignored is obtained and the frequency of the oscillation for a point-like dipole moment and a finite size dipole moment are also obtained by measuring the magnetic moment and it is measured by a magnetometer.

2 Theory

The frequency of the harmonic oscillations of a magnet that travels along the axis of a circular coil can be determined theoretically. To calculate the frequency of the oscillations for a point-like dipole moment, the following equation can be used

$$\omega_1^2 = \frac{3\mu_0 ImN}{2MR^3} \tag{1}$$

where m is the dipole moment, ω_1^2 is the frequency, μ_0 is the permeability constant, M is the mass of the oscillating particle and R is the radius of the coil where N is the turns where each turn carry a current I.

The magnetic field, B, can be determined by following equation

$$\begin{aligned} B = {} & \frac{\mu_0 m}{8\pi Lab}\left[tan^{-1}\left(\frac{a}{b}\frac{z+2L}{[d^2+(z+2L)^2]^{1/2}}\right) - tan^{-1}\left(\frac{a}{b}\frac{z}{[d^2+z^2]^{1/2}}\right)\right] \\ & + \frac{\mu_0 m}{8\pi Lab}\left[tan^{-1}\left(\frac{b}{a}\frac{z+2L}{[d^2+(z+2L)^2]^{1/2}}\right) - tan^{-1}\left(\frac{b}{a}\frac{z}{[d^2+z^2]^{1/2}}\right)\right] \end{aligned} \tag{2}$$

where L is the length of the dipole, a is the height of the dipole, b is the width of the dipole and d are given by

$$d = \sqrt{a^2+b^2} \tag{3}$$

To calculate the frequency for a finite-size dipole moment the equation below can be used

$$\omega_1^2 = \frac{\mu_0 NIm}{8Ll_1l_2M}\left(ln\left(\frac{K_+}{K_-}\right) + A_- - A_+\right) \tag{4}$$

where l_1 is the half thickness of the coil in radius direction and l_2 is the half width of the coil. K_+, K_-, A_+ and A_- are constants that are defined as

$$K_+ = \frac{R_+ + \sqrt{R_+^2 + L_+^2}}{R_- + \sqrt{R_-^2 + L_+^2}} \tag{5}$$

$$K_- = \frac{R_+ + \sqrt{R_+^2 + L_-^2}}{R_- + \sqrt{R_-^2 + L_-^2}} \tag{6}$$

$$A_+ = \frac{L_+^2 \left(R_- \sqrt{R_-^2 + L_+^2} - R_+ \sqrt{R_+^2 + L_+^2} - 4Rl_2\right)}{\left(R_+ \sqrt{R_+^2 + L_+^2} + R_+^2 + L_+^2\right)\left(R_- \sqrt{R_-^2 + L_+^2} + R_-^2 + L_+^2\right)} \tag{7}$$

$$A_- = \frac{L_-^2 \left(R_- \sqrt{R_-^2 + L_-^2} - R_+ \sqrt{R_+^2 + L_-^2} - 4Rl_2\right)}{\left(R_+ \sqrt{R_+^2 + L_-^2} + R_+^2 + L_-^2\right)\left(R_- \sqrt{R_-^2 + L_-^2} + R_-^2 + L_-^2\right)} \tag{8}$$

where R_+ is the outer radius of the coil defined as

$$R_+ = R + l_2 \tag{9}$$

, R_- is the inner radius of the coil defined as

$$R_- = R - l_2 \tag{10}$$

, L_+ is the outer length of the finite-size oscillating dipole defined as

$$L_+ = L + l_1 \tag{11}$$

and L_- is the inner length of the finite-size oscillating dipole defined as

$$L_- = L - l_1 \tag{12}$$

3 Method

3.1 Experimental Setup

In order to implement this experiment an air track(1), a glider(2), an air generator(3), a Helmholtz coil(4), an magnetic dipole(5), a current source(6), an ammeter(7), a magnetometer(8) and a stopwatch is needed. In fig. 2 the dimensions of the Helmotz coil(4) is shown.

The air generator was used to create an airflow throughout the track, thus reducing the friction on the glider with the attatched magnet. When the Hemlholtz coil is given an initial current, I, near the centre of the coil, a homogenous magnetic field is created. This cause the glider to oscillate around the equilibirum point. An ammeter were used to measure the current. With the help of an stopwatch, the period time could be measured.

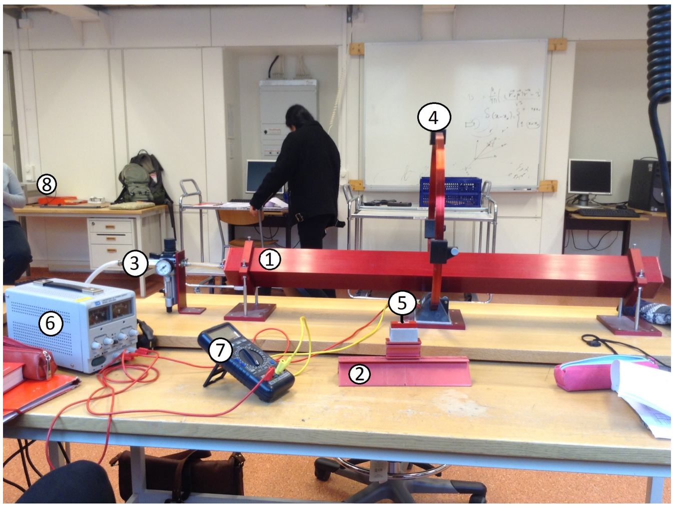

Figure 1 – A picture of the setup for the experiment

3.2 Experiment

To conduct the experiment, the air generator was turned on so that the friction would significally decrease. Then the air track were calibrated so that it stood still in the middle of the coil. When the current was applied to the coil, a magnetic field parallell to the air track was created. This magnetic field made the dipole oscillate around the middle of the coil. The period time were measured one period at time, for ten periods. Then calculating the mean value of the period time to remove some measuring errors. Then the measuring were done for five different values of the current I. With the use of a scale, ruler and a magnetometer, the dimensions, mass and magnetic field were measured.
When the measuring was done, the values were analyzed using MATLAB.

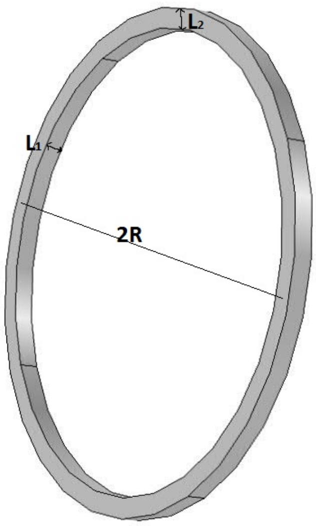

Figure 2 – A schematic figure made in Comsol of the coli, where $L_2 = 2 \cdot l_2$, $L_1 = 2 \cdot l_1$, $l_1 = 0.008[m]$, $l_2 = 0.0085[m]$ and $R = 0.2[m]$

4 Result and calculations

The measured frequency of the oscillation ω_{exp} for 5 different values of current I in the Helmholtz coil were calculated by

$$\omega_{exp} = \frac{2\pi I}{t} \tag{13}$$

where the values of the current, I, and the time, t, it takes for the dipole to oscillate ten times in the Helmholtz coil are taken from table 2. The calculated experimental frequency can be found in the same table.
To calculate the magnetic dipole moment, m, equation 14 was used so this equation was obtained

$$\begin{aligned}\frac{B}{m} = \frac{\mu_0}{8\pi Lab}\left[tan^{-1}\left(\frac{a}{b}\frac{z+2L}{[d^2+(z+2L)^2]^{1/2}}\right) - tan^{-1}\left(\frac{a}{b}\frac{z}{[d^2+z^2]^{1/2}}\right)\right] \\ + \frac{\mu_0 m}{8\pi Lab}\left[tan^{-1}\left(\frac{b}{a}\frac{z+2L}{[d^2+(z+2L)^2]^{1/2}}\right) - tan^{-1}\left(\frac{b}{a}\frac{z}{[d^2+z^2]^{1/2}}\right)\right]\end{aligned} \tag{14}$$

where the length of the dipole L was equal to $0.025m$, the height a was $0.0051m$, the width b was $0.0071m$, z was the distance that can be found in table 1 and d was equal to $0.0087m$. $\frac{B}{m}$ was plotted against the values for the magnetic field, B, for different distances, z, that was measured by the magnetometer for the dipole. This can be found in table 1. The value for μ_0 are equal to $4\pi \cdot 10^{-7}$ [1].
When plotting these against each other, the result in figure 3 was obtained.

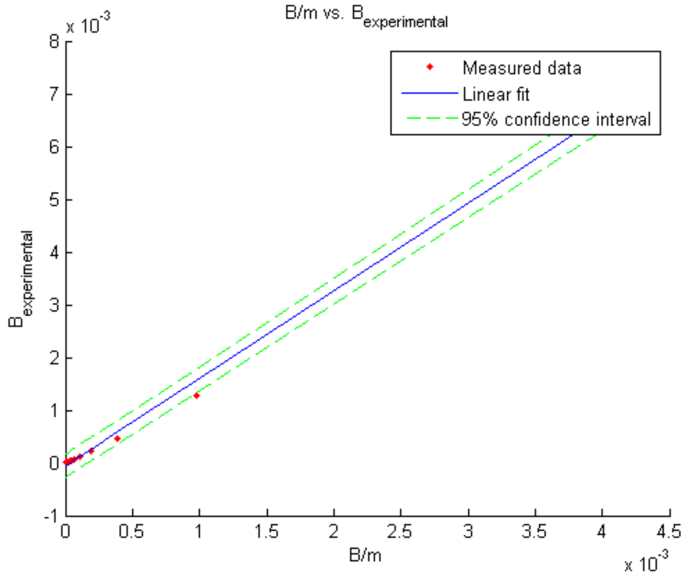

Figure 3 – A plot of the experimentally measured magnetic field versus the theoretically magnetic field divided by the dipole moment.

By taking the incline of this plot, the dipole moment was

$$m = 1.6632 \tag{15}$$

To calculate the frequency theoretically for a point-like dipole moment equation 1 was used. The coil radius, R, was equal to $0.199m$ and the turns, N, in this coil was equal to 154. The current flowing in each turn of the coil was gradually increased (table 2). The mass M of the oscillating dipole with the glider was equal to $0.3659kg$,

When using equation 1 and the dipole moment m to calculate the frequency, ω_{point}, for a point-like dipole moment, the values in table 3 was obtained.

To calculate the frequency theoretically for a finite-size dipole moment equation 4 was used. The outer radius, R_+, of the coil, given by equation 9, was equal to $0.2075m$ and the inner radius,R_-, was given by 10, which was equal to $0.1905m$. The radius R was equal to $0.199m$, which means that l_2 was equal to $0.0085m$. The length of the dipole, L, was equal to $0.05m$, and the width of the coil,l_1, was equal to 0.008. The constant K_+ calculated by equation 5 was approximately 1.0802 and the constant K_- calculated by equation 6 was approximately 1.0819. The constant A_+ calculated by 7 was approximately -0.062 and the constant A_- calculated by equation 8 was approximately -0.0033.

So, by using above with m and equation 4 the frequency, ω_{finite}, for the dipole was found for different currents, see table 4. By plotting the different frequencies against eachother, the following figure was obtained.

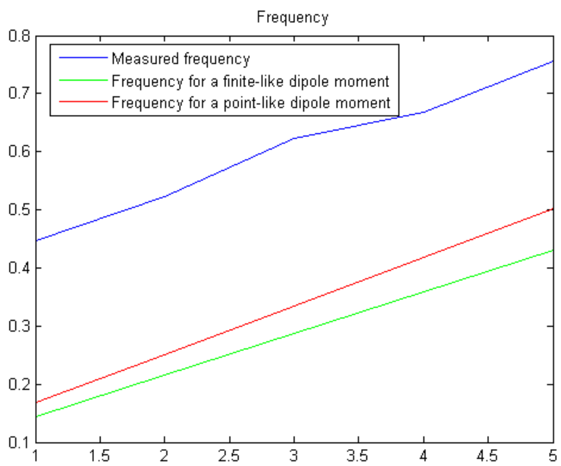

Figure 4 – The different frequencies

5 Conclusions

When plotting the angular frequency against the current, see figure 4, one can clearly see that the slope of the measured values are almost the same as the theoretical ones. One can also see thatthe frequency determined by the point-like dipole moment are slightly closer than the finite sized one. This is strange since the dipole used in the experiment are of a finite size and its frequency should therefore be better approximated with the frequency calculated of the finite-sized dipole moment. The reason for this unexpected result could be that we didn't take all factors in this experiment in consideration. For example was the friction of the dipole not considerd in any calculations eventhough it was not completely eliminated by the airglider. Also, the setup was pretty sensitive and if the airtrack was slightly unbalanced, the dipole would no longer oscillate around the coils centre and therefore it would not "feel" the magnetic field as strong as it should and the whole experiment would be affected. When changing the current in the Helmholtz coil, the dipole needed time to adjust in the new magnetic field which we didn't consider. To get a more accurate and vaild result we should have done more measuring series than just one and also have the dipole oscillate more times than ten.

References

[1] $\mu_0 = 4\pi \cdot 10^{-7}[Vs/Am]$, Nordling, Österman. Physics Handbook Eight edition,ISBN: 978-91-44-04453-8.

A Appendix

A.1 Tables

Table 1 – Tables with values of the distance of the magnetometer and the dipole and the magnetic field that was measured in the distances.

Distance [m]	Magnetic field [$\cdot 10^{-3}$ T]
0.02	7
0.04	1.36
0.06	0.53
0.08	0.29
0.10	0.193
0.12	0.146
0.14	0.120
0.16	0.105
0.18	0.096
0.20	0.090
0.22	0.086
0.24	0.083
0.26	0.080
0.28	0.078

Table 2 – Table with values of the current, time it takes for the dipole to oscillate ten times in the Helmholtz coil and the frequency of the oscillating.

Current [A]	Time [s]	Frequency
1	141	0.4456
1.5	120	0.5236
2	101	0.6221
2.5	94	0.6684
3	83	0.7570

Table 3 – Table with values of the current and the theoretically calculated frequency for a point-like dipole moment using table 1.

Current [A]	Frequency
1	0.1674
1.5	0.2512
2	0.3349
2.5	0.4186
3	0.5023

Table 4 – Table with values of the current and the theoretically calculated frequency for a finite-size dipole moment using table 1.

Current [A]	Frequency
1	0.1435
1.5	0.2153
2	0.2870
2.5	0.3588
3	0.4305