Tobias Dahlberg
Hyejeong Cheon
Robert Cordts

Department of Physics
Umeå University

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GAS FLOW IN A VENTURI TUBE –LIQUID FLOW IN A CAPILLARY TUBE

Tobias Dahlberg
toda0006@gmail.com

Hyejeong Cheon
hyejungchj@gmail.com

Robert Cordts
Robert.Cordts@posteo.de

ABSTRACT

Two experiments which were conducted – the first one with the aim of measuring the flow velocity in a Venturi tube, the second one looking at the flow of water in a thin capillary with the aim of determining both the water's viscosity and Reynolds number and characterizing the transition from turbulent to laminar flow inside the capillary.

In order to determine the flow velocities in the Venturi tube, the pressure inside the tube was measured. Both a manometer making use of the Venturi-structure and a Pitot-tube where used. In the experiment examining liquid flow, a vertical column of water was used, which provided a means of knowing the pressure difference inside the capillary as a function of time.

As opposed to what we had assumed, the gas flow in the Venturi tube showed to be rather turbulent than strictly viscously laminar. The viscosity of water could be determined to be μ_{water} = 1.61 mPa s at 19.5°C, although hardly explicable results in the last part of the experiment call this result into question.

Fluid Mechanics

Supervisors: Narges Mortezaei, Dmitry Kobyakov

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Hyejeong Cheon
Robert Cordts

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1. Introduction

Having an understanding of the characteristics of a flow can be important in many industrial applications, such as injection molding. In this report we analyze the properties of the velocity profile, at various flow speeds, in a gas flow confined to a pipe. In addition, the impact of viscosity and turbulence on a liquid flow through a thin capillary was explored.

2. Theory

In this section a theoretical background to the phenomena discussed is given.

2.1 Gas flow in pipe

In this section we discuss the properties of the velocity profile of a gas flow in a cylindrical pipe with pressure P_1 in one end and P_2 in the opposite; a schematic view of this system is shown below in fig. 1.

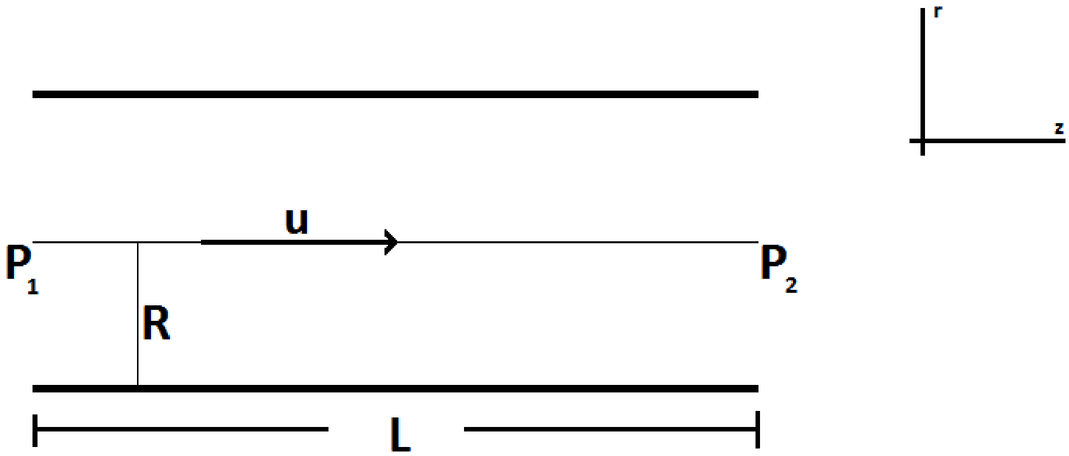

Figure 1 Schematic depiction of a pipe (radius R , length L) with pressure P_1 and P_2 applied to its ends with the resulting flow indicated by u

In the following sections we derive the flow profile for this system for two different cases, laminar and non-viscous flow.

2.1.1 Laminar flow

For a laminar flow, we assume the flow to be a shear flow and the geometry to be cylindrical, this gives us a velocity described by Eq. (1)

$$\bar{u} = u_z(r)\hat{a}_z$$
$$P(z, r) \qquad (1)$$

Where, P is the pressure, u_z is the z-component of the velocity distribution, $\bar{u}$ and $\hat{a}_z$ is the unit vector pointing in z-direction. The flow is also assumed stationary, which implies that

$$\frac{d\bar{u}}{dt} = 0 \qquad (2)$$

To describe the flow we make use of the Navier-Stokes equation, Eq.(3)

$$\rho\frac{d\bar{u}}{dt} = -\nabla P + \eta\nabla^2\bar{u} \tag{3}$$

where η is the viscosity and ρ is the density. If we now expand the terms ∇P and $\nabla^2 u$ in conjunction with the assumption in Eq.(2), Eq.(3) becomes the following.

$$\frac{dP}{dz}\hat{a}_z + \frac{dP}{dr}\hat{a}_r + \frac{dP}{d\phi}\hat{a}_\phi = \eta\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d\bar{u}}{dr}\right) + \frac{1}{r^2}\frac{d\bar{u}}{d\phi^2} + \frac{x^2\bar{u}}{dz^2}\right] \tag{4}$$

Using the assumptions made in Eq.(1) we find that Eq.(4) reduces to the following equations

$$\frac{dP}{dz} = \frac{1}{r}\frac{d}{dr}\left(r\frac{du_z}{dr}\right) \tag{5}$$

$$\frac{dP}{dz}=0 \tag{6}$$

From Eq.(6) we see that this implies that P is a function of z which in turn means that Eq.(5) is separable as the both the sides of the equation are independent of each other. This tells also tell us that the equations are constant, giving us Eq.(7) and (8)

$$\frac{dP}{dz} = const = C_s \tag{7}$$

$$\frac{\eta}{r}\left(r\frac{du_z}{dr}\right) = \text{const} = C_s \tag{8}$$

Where is C_s the separation constant.

We start with solving Eq.(7). Integration gives us that

$$P(z) = C_s z + C_1 \tag{9}$$

where is C_1 an integration constant. Using the boundary conditions that $P(0)=P_1$ and $P(L)=P_2$ we see that Eq.(9 becomes)

$$P(z)=\frac{P_2-P_1}{L}z + P_1 \tag{10}$$

We continue to solve Eq.(7) which after integration becomes

$$u_z(r) = \frac{C_s r^2}{4\eta} + C_3 ln(\text{r})+C_4 \tag{11}$$

As Eq.(11) should be limited, we see that the integration constant C_3 should be zero as $\ln(r) \rightarrow -\infty$ when $r \rightarrow 0$. Then C_4 can be determined by use of the boundary condition $u_z(R)=0$ and the value of C_s from Eq.(10) which gives us $C_4 = -\frac{(P_2 - P_2)r^2}{4\eta L}$. Knowing these constants Eq.(11) can then be expressed as

$$u_z = -\frac{(P_2-P_1)R^2}{4\eta L}\left(1 - \frac{r^2}{R^2}\right) = u_m\left(1 - \frac{r^2}{R^2}\right) \tag{12}$$

which is a parabolic function with its maximum value at r=0. it also implies that for $P_2 < P_1$ we have a flow in the positive z-direction which is plausible and a maximum velocity $u_m = -\frac{(P_2-P_1)R^2}{4\eta L}$ From this the average velocity can then be calculated with use of Eq.(13)

$$u_{avg} = \frac{1}{R^2}\int_0^R u_z(r)rdr \tag{13}$$

Inserting Eq.(12) into Eq.(13) results in

$$u_{avg} = -\frac{(P_2-P_1)R^2}{8\eta L} \tag{14}$$

Which we can rewrite with Eq.(12) as

$$u_{avg} = \frac{u_m}{2} \tag{15}$$

2.1.2 Non-viscous flow

For a inviscid flow no internal friction will hinder the movement of the laminas in the flow resulting in a plane velocity profile. The equation describing this state I called the Bernoulli equation and I given by

$$P_1 + \frac{\rho u_z^2}{2} = P_2 + \frac{\rho u_z^2}{2} \tag{16}$$

Eq.(15) can then be solved for u_z giving us

$$u_z = \sqrt{\frac{2(P_2 - P_1)}{\rho}} \tag{17}$$

2.2 Flow in pipe connected to column

In this section we look at the viscosity of a flow through a thin capillary connected to a large column of water, the system can be seen below in fig. 2..

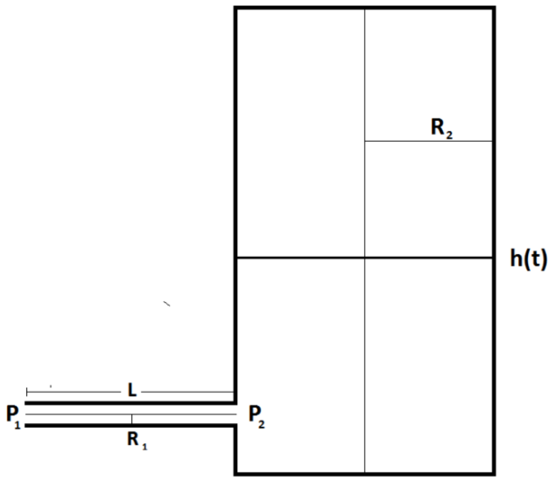

Figure 2 capillary of length L and radius R_1 connected to a column of radius R_2 filled with water to a height of h(t)

The flow through the capillary is as in the previous section a stationary laminar flow driven by a pressure difference. The difference in this case is that the pressure on one side is time dependent as it originates from the height of the water column, which is continually drained through the capillary. We begin by defining the discharge Q , which is the flow of volume going out of the system, Eq.(18)

$$Q = u_{avg}A \tag{18}$$

where A is the cross-sectional area of the channel through which the flow is passing. Using Eq.(14) Eq.(18) can be rewritten as

$$Q = u_{avg}\pi R_1^2 = -\frac{\pi(P_2 - P_1)R_1^4}{8\eta L} \tag{19}$$

where is R_1 the radius of the capillary. By Gauss law, $Q = u_{avg}A = \frac{dV}{dt} = \frac{d(\pi R_2^2\, h(t))}{dt}$, we can directly relate it to the height of the water within the column, h(t). This is done below in Eq.(20)

$$h'(t)\pi R_2^2 = -\pi\frac{(P_2 - P_1)R_1^4}{8\eta L} \tag{20}$$

where R_2 is the radius of the column. As the pressure P_2 is also dependent on the height as

$$P_2 = \rho g h(t) \tag{21}$$

Where g is the gravitational acceleration. We insert Eq.(21) into Eq.(20) and solve for η resulting in

$$\eta = -\frac{(\rho g h(t) - P_1) R_1^4}{8 L h'(t) R_2^2} \tag{22}$$

3. Experimental

In order to implement this experiment the following equipment was used: A Venturi tube with a regular diameter of 10cm and a 5cm diameter at the constriction (1 in fig. 3), a U-type manometer (2 in fig. 3), a Pitot tube (3 in fig. 3), a differential pressure gauge (4 in fig. 3), a multimeter (*HP34410*) (5 in fig. 3), a current and voltage source (6 in fig. 3) and a scaled mount (7 in fig. 3).

3.1 *Measuring gas flow*

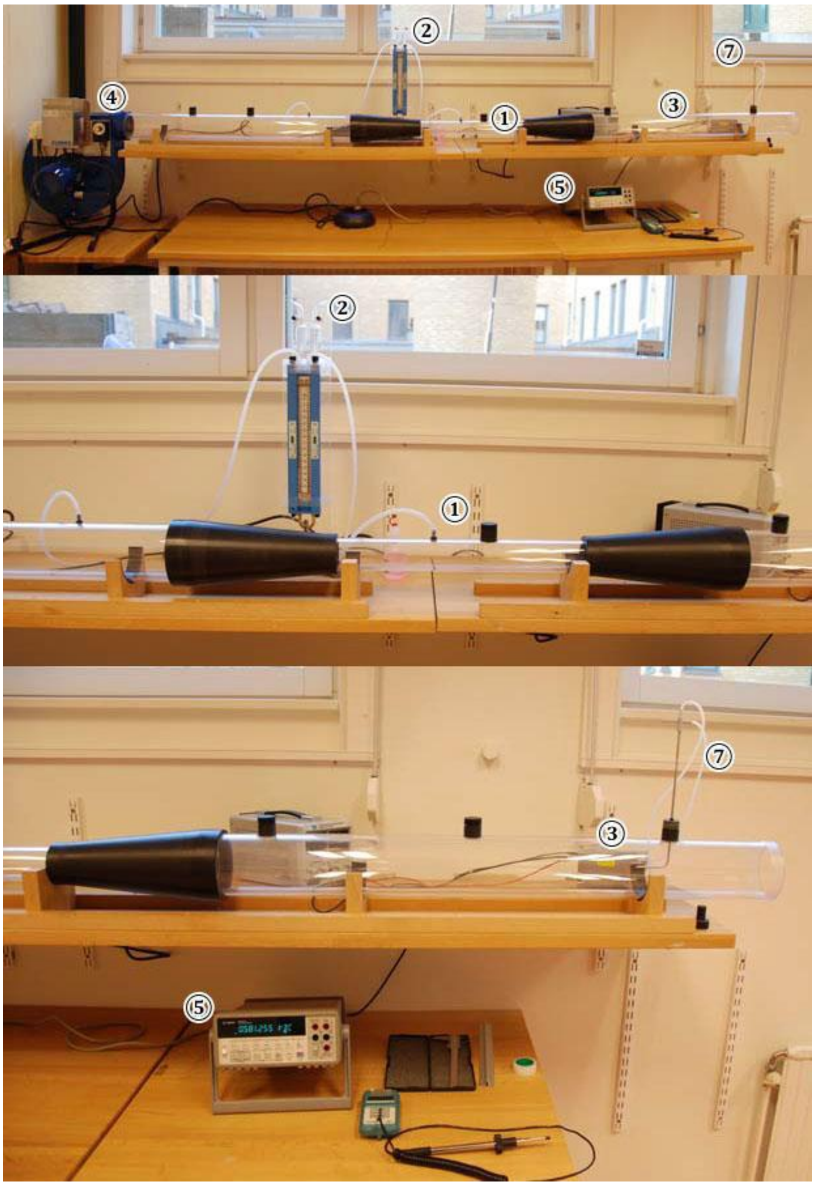

Figure 3 picture of setup for this experiment

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Hyejeong Cheon
Robert Cordts

3.1.1 Venturi tube and Pitot tube

A Venturi tube is a tube that has a constriction placed where the cross-sectional area decreases and velocity of a fluid passing through increases according to Bernoulli's theorem Eq. (15).
A Pitot tube is an apparatus for measuring velocity of fluid flow. The Pitot tube includes two tubes and a pressure sensor, which measure pressure difference between the dynamic and stagnation pressure in a flow.

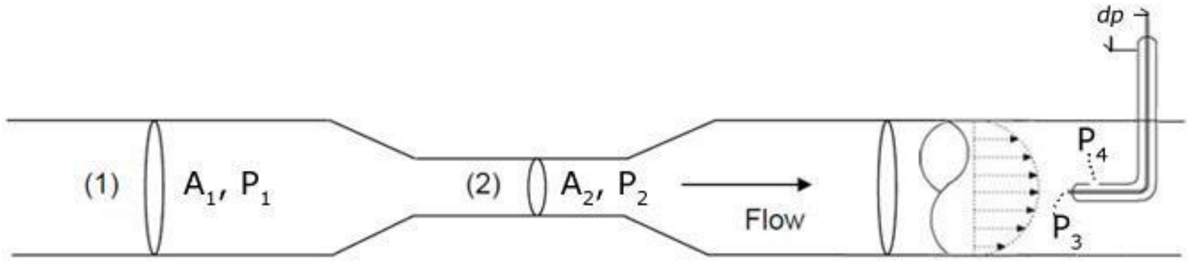

Figure 4 Conceptual picture of Venturi tube and Pitot tube

3.1.2 Procedure

First, in order to find a function for relationship between voltage and pressure, we measured the voltage from pressure sensor in Pitot tube at gauge setting for the flow source of 50. After calibrating the voltage from this value, we get a linear function to use when converting voltage to pressure. Our measurements started with an air compressor setting of 50 and measuring in increments of 5 up until it reached 100. The measurement data was collected by a LabVIEW program, which was connected, to sensor of Pitot tube. By this method, we measured the pressure profile in the center of the tube.
Secondly, in order to investigate the velocity profile in the vertical z-axis, we used the scaled mount for the Pitot tube to move the device in this axis. Using this we measured the Pitot pressure in the range +5cm to -5cm (increments of 0.5 cm) with respect to the center point of the Venturi tube. The same data collection method as in the previous experiment was used.

3.2 Measuring liquid flow

In order to implement this experiment the following equipment was used: A pump (1 in fig. 5), an integrated differential pressure sensor MPX5010 (2 in fig. 5), a 95cm tall hollow cylinder (3 in fig. 5), two capillaries with diameters 3.00mm and 1.60mm (4, 5 in fig. 5), a tube for refilling the water (6 in fig. 5) and a water container (7 in fig. 5).

Figure 5 The experimental setup for investigating liquid flow

3.2.1 Calibration

In order to determine the viscosity of the fluid, we measured how the pressure sensor signal changes as the water level decreases as a calibration. First, we fill water in the cylinder as much as possible and release the water in height increments of 5 cm, and record the corresponding change in pressure. From this data, we can get a relationship between signal and the water height.

We use two capillaries in this experiment and the capillary we used for calibrating is one that has 1.60mm diameter and 303.5mm length. The water cylinder has a 647mm diameter and a 95cm height. In addition, the other capillary, which is for investigating flow velocity when the flow transition from a laminar to a turbulent flow has 3.00mm diameter and 304.6 mm length, fig. 6 shows this.

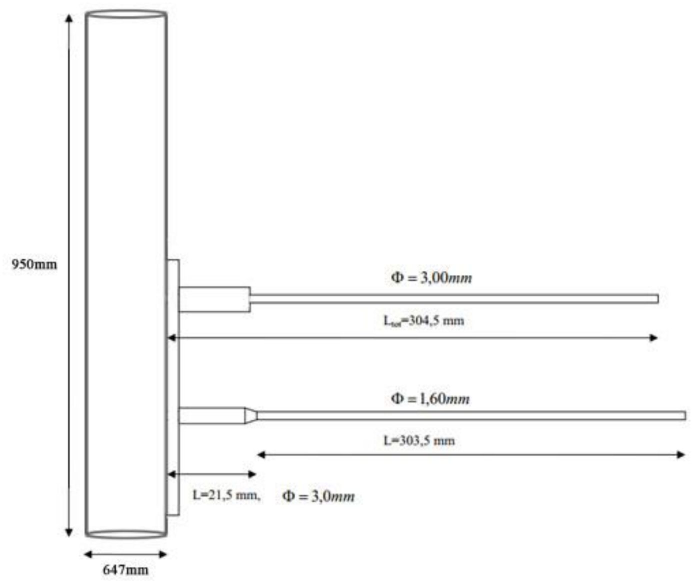

Figure 6 A water cylinder and two capillaries

3.2.2 Procedure

We flow water after filling the water cylinder by 95cm high level and obtain signal data. The signal data could be transformed to water level and time by the data we got in the calibration. Both capillaries were used to get signal data for water level and time. When calculating the Reynold's number in the transition from laminar to turbulent flow, we use the capillary with 3.00mm diameter. Additionally we measured the temperature of water to calculate the density and viscosity. All signal data are collected by National instruments LabVIEW program.

Tobias Dahlberg
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4. RESULTS

4.1 Gas flow in a venturi tube

4.1.1 Average and maximum velocity in the Venturi tube

As laid out above both the average velocity $v_{venturi}$ of the flow and its maximum velocity v_{pitot} where measured via the pressure difference in the Venturi and the Pitot rube respectively. In fig. 7 the average velocity is depicted over the maximum velocity. Besides to the measured values the velocities of non-viscous laminar, viscous laminar and different turbulent flows, based on the theory described in chapter 2, are shown in the illustration. As we assumed a viscous laminar flow, one would expect the blue line to coincide with the red line of viscous laminar flow. It does however coincide more with the line for a turbulent flow of a factor n = 8.

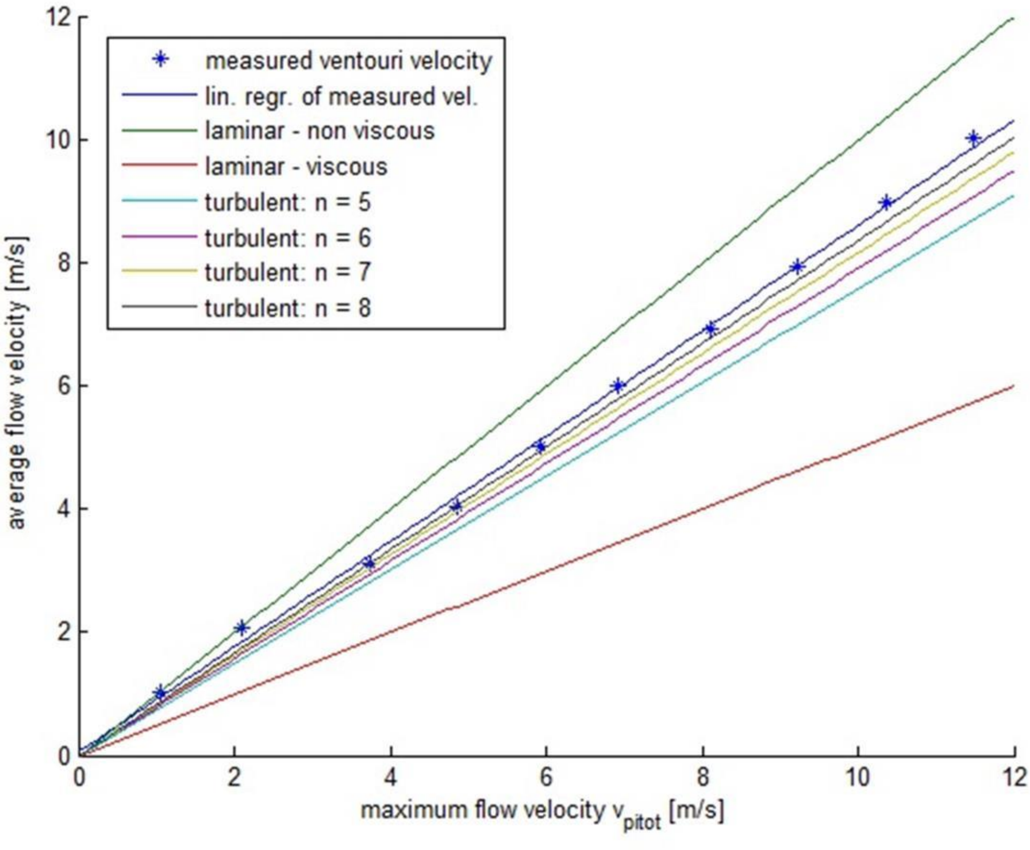

Figure 7 depiction of the average flow velocity over the maximum velocity; theoretical velocity distributions based on non-viscous laminar, viscous laminar and turbulent flow

4.1.2 Radial distribution of velocity

Using the Pitot tube, we then measured a velocity profile for the flow. In fig. 8, the velocity inside the Venturi tube is depicted over the distance from the center of the tube. As can be seen the measured velocity profile lies far from the expected distribution of a viscous laminar flow. It is again best approximated by a turbulent flow of n = 8.

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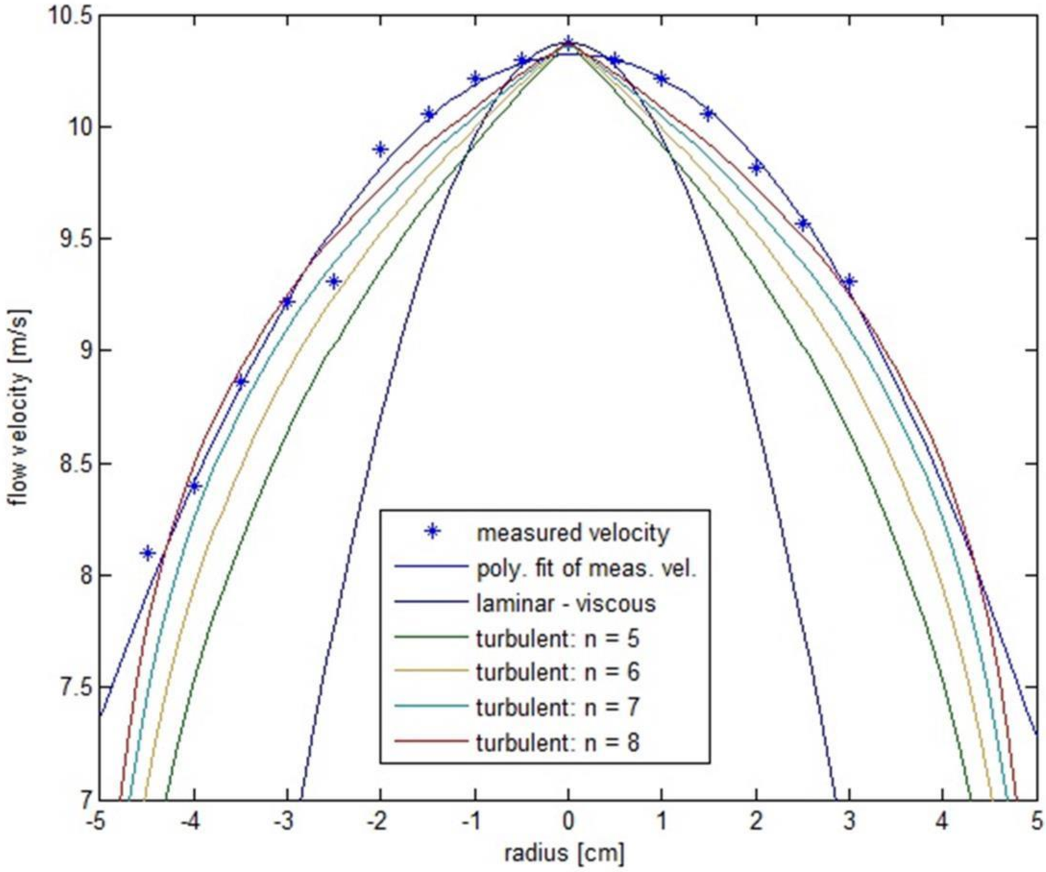

Figure 8 Dependance of the flow velocity in the Ventouri tube on the radial positon. Theoretical prediction of the velocity distribution for viscous laminar and turbulent flows.

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4.2 Liquid flow in a capillary tube

4.2.1 Calibration of the height measurement

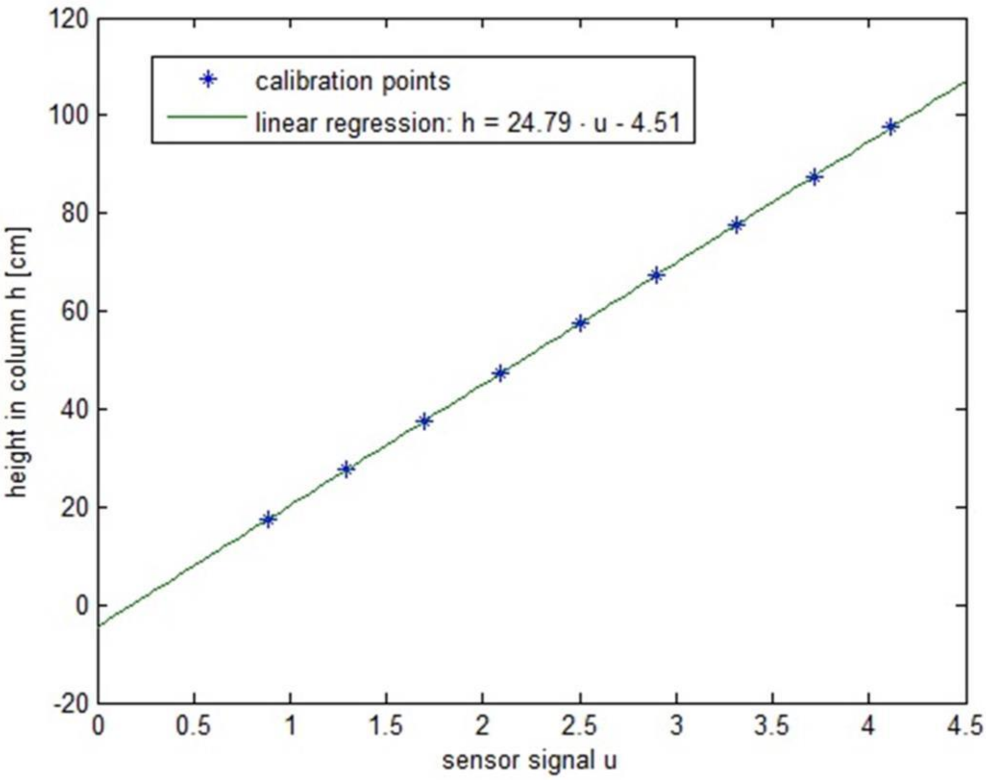

Figure 9 Calibration of the sensor signal: dependence of the water level h in the column on the sensor signal u.

In order to measure the change in height of water in the column over time, the sensor signal function was calibrated first. To do so, the sensor signal was measured over several seconds for 9 different heights and then averaged. The water level can then be determined as a function of the sensor signal via a linear regression. Both the calibration points and the linear regression are given in fig. 9

4.2.2 Measurement of the viscosity

First, the viscosity of water was measured using a thin capilar. Corresponding to the theory laid out in section 2 the logarithmic height of water in the column depends linearly on the time. The slope of this function will in turn be inversely related to the viscosity of the medium. In fig. 10 the height is depicted logarithmically over the time. A linear regression yields the slope m = -1.12 10^{-3}. The viscosity μ of water at a temperature of 19.5℃ then follows to be:

$$\mu_{water} = 1.61 \text{ mPa s}$$

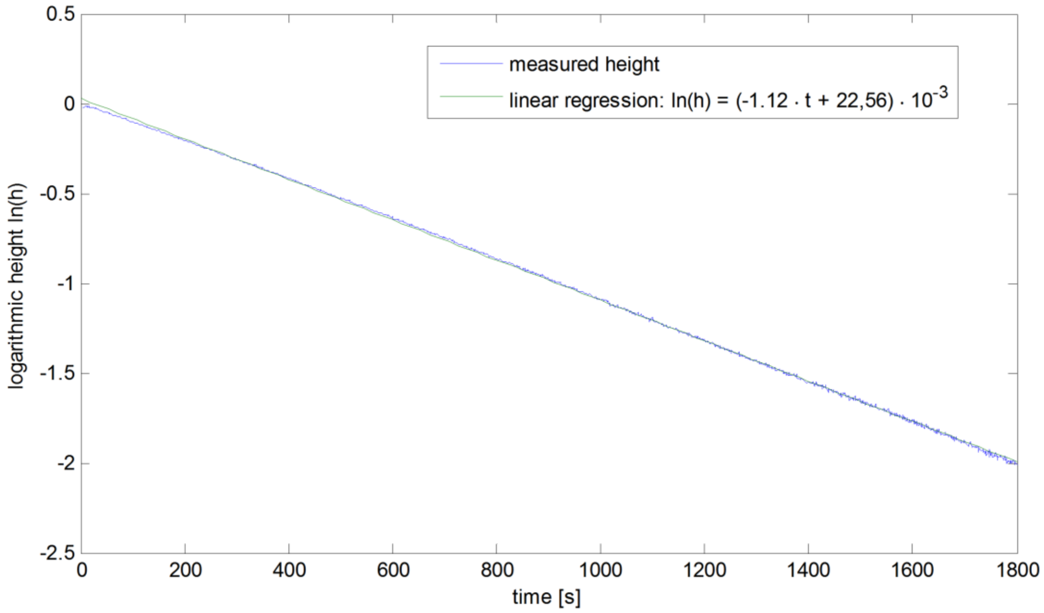

Figure 10 Logarithmic depiction of the height in the column over time. Linear regression of ln(h) over t.

4.2.3 Transition between turbulent and laminar flow

In order to observe the transition from a turbulent to a laminar flow inside the capillary the same experimental procedure was repeated for a thicker capillary. In fig. 11 the same plot as in section 4.2.2 is presented for the wider capillary.

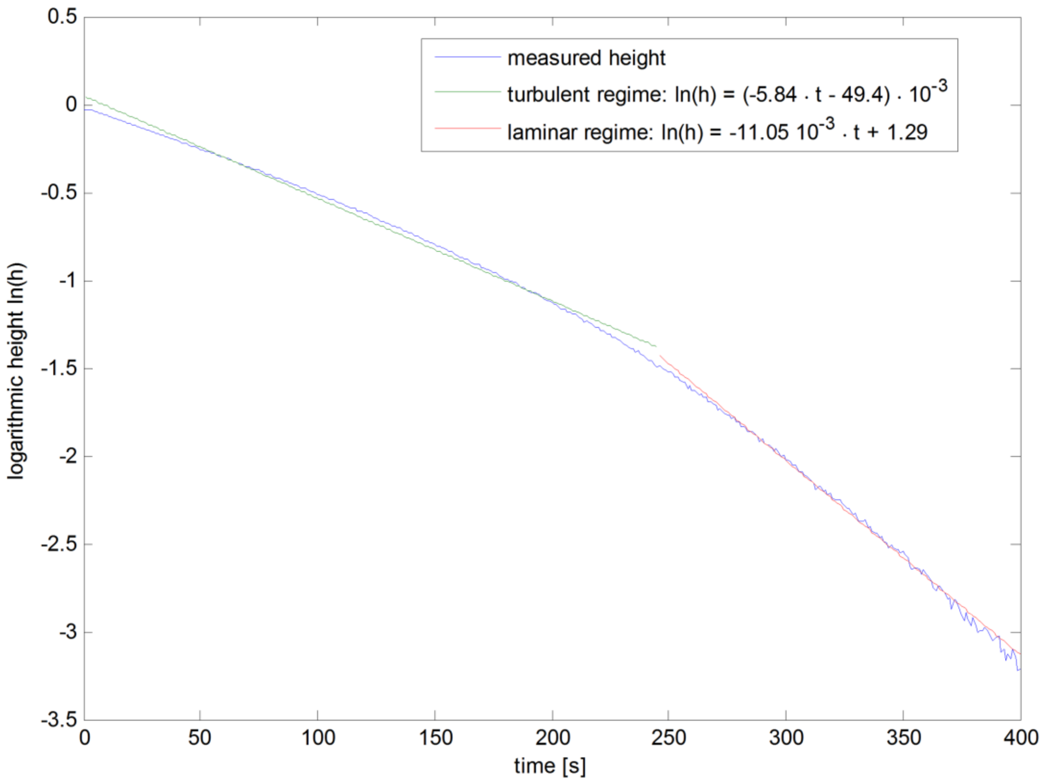

Figure 11 Depiction of the logarithmic height over time.

As opposed to the graph in section 4.2.2, the gradient of the curve is not a constant value but rather changes over time. This is well represented by the slope $m_{laminar} = -11.05\ 10^{-3}$ of the laminar regime and the significantly smaller slope $m_{turbulent} = -5.84\ 10^{-3}$ of the turbulent regimes, both of which can be seen in the regression lines in fig. 11. The change in slope is due to a change of the velocity distribution inside the capillary when the fluid goes from the turbulent into the laminar regime.

Conducting the same mathematical method as before, the viscosity of the fluid within the laminar regime can be determined to be

$$\mu_{water} = 0.793 \text{ mPa s}.$$

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As can be seen in the graph, the transition from turbulent to laminar flow occurs after about 245 s. In order calculate the Reynolds number *Re of the flow* at the point of transition the average velocity needs to be determined. This can be done by relating the momentary change of height in the column to the flow velocity inside the capillary. In fig. 12 the height is depicted over the time. By fitting the distribution of height over time with a 3rd order polynomial function the momentary change can be determined, represented by the tangential to the curve in fig. 12.

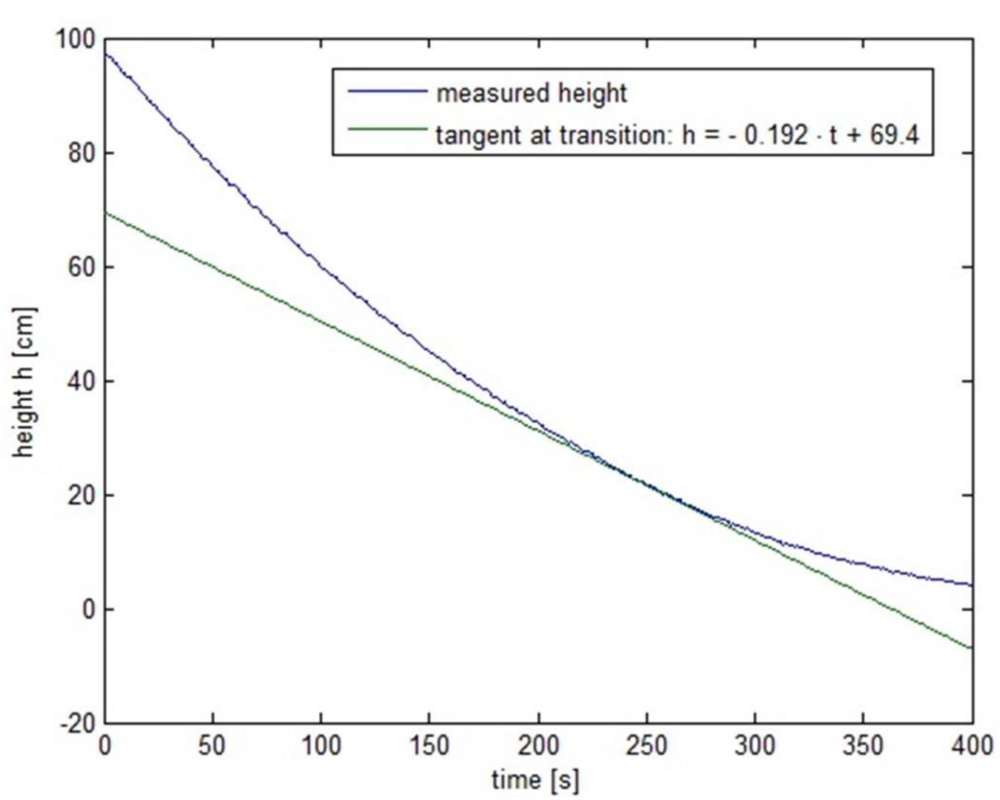

Figure 12 Depiction of the height of water in the column over time.

Following this procedure, the Reynolds number as the point of transition is determined to be

$$Re = 3318$$

Tobias Dahlberg
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5. DISCUSSION AND CONCLUSIONS

5.1 Gas flow in the venturi tube

The flow inside the Venturi tube was assumed to be a viscous laminar flow. The experiment showed however, that of the models' compared a turbulent flow with a power of n = 8 is the best approximation for the flow observed. The fact that both series of experiments yielded this result suggests strongly that the velocity was high enough for friction with the tube wall to become less relevant and the flow being a rather turbulent one.

5.2 Liquid flow in a capillary

The viscosity measured in the first part of this experiment is in the range of small mPa s, which is according to literature values. In the second part however, conducting the same experiment with a wider capillary the viscosity calculated from the measured velocities is about 2 times smaller, which is a deviation that can be assumed be beyond what is caused by the inaccuracy of the height measurement. When we conduct height measurement, we used narrower capillary but second experiment is conducted by wider one. The transition from turbulent to laminar could be confirmed to happen as expected qualitatively, the Reynolds number calculated is in range of 2300~4000 and the result would be a little small but in the range of that.

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