*Permutations.java lab1.java

```java
     * characters are fixed. For example, if str corresponds to "abCDEfgh",
     * low is 2, and high is 5, then the following output is produced:
     *    abCDEfgh
     *    abCEDfgh
     *    abDCEfgh
     *    abDECfgh
     *    abEDCfgh
     *    abECDfgh
     *
     * @param str is an array representation of the string we wish to permute.
     * @param low is the index of the leftmost character in the fluid range.
     * @param high is one beyond the index of the rightmost character in the
     * fluid range.
     */
    public static void permute(char[] str, int low, int high) {
        if (low == high)
          System.out.println(new String(str));
        else {
            for (int i = low; i < high; i++) {
                change(str, low, i);
                permute(str, low + 1, high);
                change(str, low, i);
            }
        }
    }

    private static void change (char[] str, int i, int j) {
        char temp;
        temp = str[i];
        str[i] = str[j];
        str[j] = temp;
    }

  public static void main(String[] args) {
    permute("abcd");
    /* Expected output:
       abcd
```

2. Problem 1.11

a) $\sum_{i=1}^{N-2} F_i = F_N - 2$

$F_i = F_1 + F_2 + F_3 + \cdots + F_{N-2} = F_N - 2$

Base Case :

$F_1 = F_{3-2}$ $\therefore N=3$

$F_{3-2} = F_3 - 2$, so $3-2 = 1$ ✓

$1 = 1$ $\therefore$ Base case verified

Inductive Hypothesis

Assume $F_k = F_1 + F_2 + F_3 + \cdots + F_{k-2} = F_k - 2$

For some $k>3$,

$F_k = F_1 + F_2 + F_3 + \cdots + F_{k-2} + F_{k+1-2} = F_{k+1} - 2$

$\therefore F_k - 2 + F_{k-1} = F_k + F_{k-1} - 2$

$= F_{k+1} - 2$ By definition of Fibonacci

By the induction, $\sum_{i=1}^{N-2} F_i = F_N - 2$ is the case for all $N \geq 3$

3. Problem 1.12

a) $\sum_{i=1}^{N} (2i-1) = N^2$

<u>Base Case</u>

$(2-1) = 1^2 \Rightarrow 1 = 1$, $\therefore$ base case is verified.

<u>Inductive Hypothesis</u>

Assume $(2\times 1-1) + (2\times 2-1) + \dots + (2k-1) = k^2$

$$1 + 3 + 5 + \dots + 2k-1 = k^2$$

$$1 + 3 + 5 + \dots + 2k-1 + 2k+1 = (k+1)^2$$

$$k^2 + 2k+1 = (k+1)^2$$

By mathematical Induction, $\sum_{i=1}^{N} (2i-1) = N^2$ for all $N \geq 1$

4.

Problem 5.1

b) {4371, 1323, 6173, 4199, 4344, 9679, 1989}

$h(x) = x \bmod 10$

0	9679
1	4371
2	1989
3	1323
4	6173
5	4344
6	
7	
8	
9	4199

Linear probing

c) {4371, 1323, 6173, 4199, 4344, 9679, 1989}

$h(x) = x \bmod 10$

0	9679
1	4371
2	
3	1323
4	6173
5	4344
6	
7	
8	1989
9	4199

Quadratic Probing

5. Problem 5.2

b)

0	
1	4371
2	
3	
4	
5	
6	
7	
8	
9	6173
10	
11	1989
12	1323
13	4199
14	
15	
16	
17	
18	
19	9679
20	4344
21	
22	

$\{4371, 1323, 6173, 4199, 4344, 9679, 1989\}$

$h(x) = x \bmod 23$

$h(4371) = 4371 \bmod 23 = 1$

$h(1323) = 1323 \bmod 23 = 12$

$h(6173) = 6173 \bmod 23 = 9$

$h(4199) = 4199 \bmod 23 = 13$

$h(4344) = 4344 \bmod 23 = 20$

$h(9679) = 9679 \bmod 23 = 19$

$h(1989) = 1989 \bmod 23 = 11$

Linear probing

c)

{4371, 1323, 6173, 4199, 4344, 9679, 1989}

0	
1	4371
2	
3	
4	
5	
6	
7	
8	
9	6173
10	
11	1989
12	1323
13	4199
14	
15	
16	
17	
18	
19	9679
20	4344
21	
22	

Quadratic Probing

6.

Problem 5.1

b) {4371, 1323, 6173, 4199, 4344, 9679, 1989}

$h(x) = 7 - (x \bmod 7)$

0	
1	6173
2	4199
3	4344
4	4371
5	9679
6	1989
7	1323
8	
9	

$h(4371) = 7 - (4371 \bmod 7) = 4$

$h(1323) = 7 - (1323 \bmod 7) = 7$

$h(6173) = 7 - (6173 \bmod 7) = 1$

$h(4199) = 7 - (4199 \bmod 7) = 1$

$h(4344) = 7 - (4344 \bmod 7) = 3$

$h(9679) = 7 - (9679 \bmod 7) = 2$

$h(1989) = 7 - (1989 \bmod 7) = 6$

$\rightarrow h(4199) = (7 - (4199 \bmod 7)) + 1 = 2$

$\rightarrow h(9679) = (7 - (9679 \bmod 7)) + 1 = 3 + 1$

$= 4 + 1$

$= 5$

c) {4371, 1323, 6173, 4199, 4344, 9679, 1989}

$h(x) = 7 - (x \bmod 7)$

0	1989
1	6173
2	4199
3	4344
4	4371
5	
6	9679
7	1323
8	
9	

$h(4371) = 7 - (4371 \bmod 7) = 4$

$h(1323) = 7 - (1323 \bmod 7) = 7$

$h(6173) = 7 - (6173 \bmod 7) = 1$

$h(4199) = 7 - (4199 \bmod 7) = 1$

$h(4344) = 7 - (4344 \bmod 7) = 3$

$h(9679) = 7 - (9679 \bmod 7) = 2$

$h(1989) = 7 - (1989 \bmod 7) = 6$

$\rightarrow h(4199) = (7 - (4199 \bmod 7)) + 1^2 = 2$

$\rightarrow h(9679) = (7 - (4199 \bmod 7)) + 1^2 = 3$

$+ 2^2 = 6$

$\rightarrow h(1989) = (7 - (1989 \bmod 7)) + 1^2 = 7$

$+ 2^2 = 0$