Analysis

Edwin Park

2023

Contents

1 Taylor Polynomials

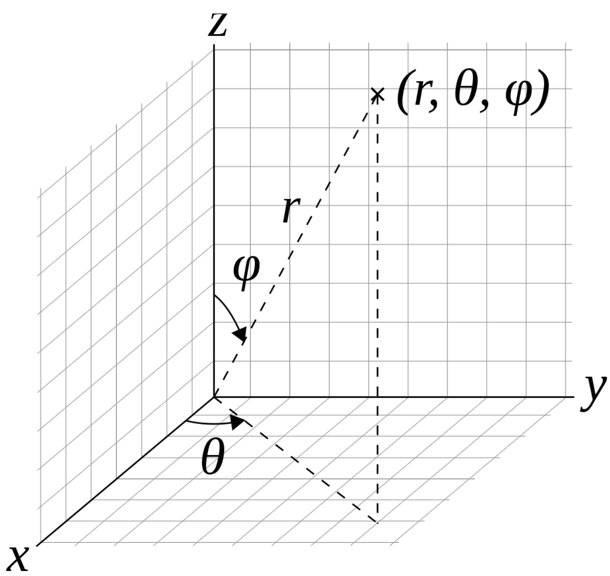
$$
\begin{aligned}
f(\vec{x}) &= f(\vec{x_0} + (\vec{x} - \vec{x_0})) \\
&\approx f(\vec{x_0}) + D_f(\vec{x_0})(\vec{x} - \vec{x_0}) + \frac{1}{2}(\vec{x} - \vec{x_0})^\top H_f(\vec{x_0})(\vec{x} - \vec{x_0}) \\
&= f(\vec{x_0}) + \begin{bmatrix} f_x(\vec{x_0}) & f_y(\vec{x_0}) \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} x - x_0 & y - y_0 \end{bmatrix} \begin{bmatrix} f_{xx}(\vec{x_0}) & f_{xy}(\vec{x_0}) \\ f_{yx}(\vec{x_0}) & f_{yy}(\vec{x_0}) \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}
\end{aligned}
$$

Say we wanted to find an upper bound for the error of a first-order approximation at $\vec{x}$. The error then is equal to the quadratic term evaluated at some point in between the central point, $\vec{x_0}$, and the point of interest, $\vec{x}$. In particular, it is, for some $t \in [0, 1]$,

$$
\frac{1}{2}\left[D^2_{\vec{x}-\vec{x_0}} f\right](t\vec{x} + (1-t)\vec{x_0}) = \frac{1}{2} \begin{bmatrix} x - x_0 & y - y_0 \end{bmatrix} \begin{bmatrix} f_{xx}(t\vec{x} + (1-t)\vec{x_0}) & f_{xy}(t\vec{x} + (1-t)\vec{x_0}) \\ f_{yx}(t\vec{x} + (1-t)\vec{x_0}) & f_{yy}(t\vec{x} + (1-t)\vec{x_0}) \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}.
$$

2 Classification of Points

3 Spherical



$$
\begin{aligned}
x &= r\sin(\phi)\cos(\theta) \\
y &= r\sin(\phi)\sin(\theta) \\
z &= r\cos(\phi) \\
\det D\Phi &= r^2\sin(\phi) \\
\theta &\in [0, 2\pi) \\
\phi &\in [0, \pi)
\end{aligned}
$$