Analysis

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Contents

1. Recursive Functions

The comment on the footnote of §1.2, that intuitionists reject the claim that the function

$$h(x) = \begin{cases} 1, & \text{if the Goldbach conjecture is true;} \\ 0, & \text{otherwise;} \end{cases}$$

is primitive recursive from the reasoning that it must either be true or false, is actually trivial. One just needs to understand that intuitionists have a different notion of "true" and "false". If one asks me in the midst of a chess game whether I've won, I would answer no, because the game is still going, but that does not mean I have lost; similarly, a statement to an intuitionist is true, false, or contingent/neither, with the latter being due to a non-existence of an explicit construction. The idea behind intuitionism is to define truth/existence/reality to hinge on (somewhat circularly) existence/reality/explicit construction. (Perhaps this is saying even consistent constructions are not "real" if not realised in the universe?)

2. Unsolvable Problems

Problem 2.1. Show that the function

$$g(x) = \begin{cases} 1, & \text{if a consecutive run of at least } x \text{ 5's occurs in the decimal expansion of } \pi; \\ 0, & \text{otherwise;} \end{cases}$$

is primitive recursive.

Solution. We just need to find an algorithm for π that is primitive recursive; the BBP-formula trivially solves this. (Actually, I think the question wants you to explicitly develop a primitve parsing algorithm, supposing that π is already given. But whatever.) □

Problem 2.2. Define f by

$$f(x) = \begin{cases} 1, & \text{if } \varphi_x(x) = 1\ \pi; \\ 0, & \text{otherwise.} \end{cases}$$

Is f recursive?

Solution. Immediately this reeks of being unrecursive. Suppose that f is recursive. Then $g = \neg f$ is also recursive. Let $g = \varphi_y$. Is $g(y) = 0$? Then $\varphi_y(y) = 0$ and $\varphi_y(y) = 1$; if $g(y) = 1$ we get a similar contradiction. □

Problem 2.3. Consider the list of primitive recursive derivations decribed in §1.4. Let f_x be the primitive recursive function determined by the $(x+1)$st derivation in this list $x = 0, 1, 2, \ldots$. Define $g = \lambda xy[f_x(y)]$. Is g recursive? Is g primitive recursive?

Solution. g is primitive recursive, as it is effectively the concatenation of two primitive recursive operations; finding f_x and computing f_x. f_x may be found easily by iterating through the method by which the enumeration was generated (e.g iterating through symbols). □