Analysis

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Contents

1 Curl

Theorem 1.0.0.1.

$$\nabla \times \vec{F} = 0 \Rightarrow \exists\, f : X \to \mathbb{R},\, \nabla f = \vec{F},$$

given that $\vec{F}$ is defined on X, an open, simply connected subset of $\mathbb{R}^3$.

2 Integrals

We define the Riemann-Stieltjis integral. Let μ be our non-decreasing, bounded "measure function", and f bounded over the interval $[a, b]$. Then, we define the lower and upper sums of a partition P of that interval:

$$U(f, \mu, P) := \sum_{p \in P} \sup_p(f) \Delta_p \mu \,;$$

$$L(f, \mu, P) := \sum_{p \in P} \inf_p(f) \Delta_p \mu \,.$$

Where $\Delta_p \mu = \mu(\text{endpoint}) - \mu(\text{startpoint})$. In turn we define the upper and lower Riemann-Stieltjis integrals:

$$\overline{\int_a^b} f \, d\mu := \inf_P U(f, \mu, P) \,;$$

$$\underline{\int_a^b} f \, d\mu := \sup_P L(f, \mu, P) \,.$$

(Is it okay to take the inf/sup over partitions? Aren't there "more" partitions than there are real numbers?) The crucial property is that refining the partition non-strictly increases the lower sum and non-strictly decreases the upper sum. Also, we note that

A Appendix