

Problem 1

Implement the method of bisection , Newton's, secant, regular falsi.

1. Optimizing Method Class

```
class Method {
public:
    Method(std::function<float(const float&)> f):function(f){};

    // optimization methods
    float bisection(float start, float end);
    float newtons(float x);
    float secant(float x1, float x0);
    float regular_falsi(float start, float end);

protected:
    // target function as member
    std::function<float(const float&)> function;
};
```

2. Bisection method

method 1: bisection

```
float Method::bisection(float start, float end) {
    assert( function(start)*function(end)<0 );

    auto midpoint = (start + end)/2.f;

    if(function(midpoint)==0 || end-start<MIN)
        return midpoint;

    if(function(midpoint)*function(start)<0)
        midpoint = bisection(start, midpoint);
    else
        midpoint = bisection(midpoint, end);

    return midpoint;
}
```

3. Newton's method

method 2: Newton's

```
float Method::newtons(float x0) {  
    // approximation of derivative lambda function  
    auto d =  
    [](std::function<float(const float&)> func, float x, float eps=1e-6)  
    {  
        return (func(x+eps) - func(x))/eps;  
    };  
  
    float x1 = x0;  
    while(function(x1)>0.f) {  
        float t = x1;  
        x1 = t - function(t)/d(function, t);  
    }  
    return x1;  
}
```

4. Secant method

method 3: secant

```
// Two point approximation method  
float Method::secant(float x1, float x0) {  
    // no matter which is bigger  
    x1 = std::min(x1, x0);  
    x0 = std::max(x1, x0);  
  
    float x2 = MAX; // initial next point  
    while(function(x2)>0.f) {  
        x2 = x1 - ((x1-x0)/(function(x1)-function(x0))) * function(x1);  
  
        x0 = x1;  
        x1 = x2;  
    }  
    return x2;  
}
```

5. Regular-falsi method

method 4: regular false

```
float Method::regular_falsi(float start, float end) {  
    // secant method lambda  
    auto sec = [](std::function<float(const float&)> func, float x1,  
        float x0)  
    {  
        return x1 - ((x1-x0)/(func(x1)-func(x0))) * func(x1);  
    };  
  
    assert( function(start)*function(end)<0 );  
  
    // new x-axis intersection point  
    float x = sec(function, start, end);  
  
    if(function(x)==0 || end-start<MIN)  
        return x;  
  
    // do recursively until the end  
    if(function(start)*function(x) < 0)  
        x = regular_falsi(start, x);  
    else  
        x = regular_falsi(x, end);  
  
    return x;  
}
```

Problem 2

Discuss their comparative performance for at least four different problems you generate.

1. Target functions

$$X(m, n) = \left\{ \begin{array}{ll} x(n), & \text{for } 0 \leq n \leq 1 \\ \frac{x(n-1)}{2}, & \text{for } 0 \leq n \leq 1 \\ \log_2 \lceil n \rceil & \text{for } 0 \leq n \leq 1 \end{array} \right\} = xy$$

2. The derivative of target functions

3. Performance comparison