

2022 Enterprise Boost Program

AI/ML Basic 05

08. 이미지를 위한 신경망

CNN으로 패션상품을 분류해보자!

전현상

Solutions Architect

2022.06.28

AI/ML Basic Tracks



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Convolutional Neural Networks

How images are stored in a Computer?

Grayscale image



| | | | | | | | | | | | | | | | |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|
| 0 | 2 | 15 | 0 | 0 | 11 | 10 | 0 | 0 | 0 | 0 | 9 | 9 | 0 | 0 | 0 |
| 0 | 0 | 0 | 4 | 60 | 157 | 236 | 255 | 255 | 177 | 95 | 61 | 32 | 0 | 0 | 29 |
| 0 | 10 | 16 | 119 | 238 | 255 | 244 | 245 | 243 | 250 | 249 | 255 | 222 | 103 | 10 | 0 |
| 0 | 14 | 170 | 255 | 255 | 244 | 254 | 255 | 253 | 245 | 255 | 249 | 253 | 251 | 124 | 1 |
| 2 | 98 | 255 | 228 | 255 | 251 | 254 | 211 | 141 | 116 | 122 | 215 | 251 | 238 | 255 | 49 |
| 13 | 217 | 243 | 255 | 155 | 33 | 226 | 52 | 2 | 0 | 10 | 13 | 232 | 255 | 255 | 36 |
| 16 | 229 | 252 | 254 | 49 | 12 | 0 | 0 | 7 | 7 | 0 | 70 | 237 | 252 | 235 | 62 |
| 6 | 141 | 245 | 255 | 212 | 25 | 11 | 9 | 3 | 0 | 115 | 236 | 243 | 255 | 137 | 0 |
| 0 | 87 | 252 | 250 | 248 | 215 | 60 | 0 | 1 | 121 | 252 | 255 | 248 | 144 | 6 | 0 |
| 0 | 13 | 113 | 255 | 255 | 245 | 255 | 182 | 181 | 248 | 252 | 242 | 208 | 36 | 0 | 19 |
| 1 | 0 | 5 | 117 | 251 | 255 | 241 | 255 | 247 | 255 | 241 | 162 | 17 | 0 | 7 | 0 |
| 0 | 0 | 0 | 4 | 58 | 251 | 255 | 246 | 254 | 253 | 255 | 120 | 11 | 0 | 1 | 0 |
| 0 | 0 | 4 | 97 | 255 | 255 | 255 | 248 | 252 | 255 | 244 | 255 | 182 | 10 | 0 | 4 |
| 0 | 22 | 206 | 252 | 246 | 251 | 241 | 100 | 24 | 113 | 255 | 245 | 255 | 194 | 9 | 0 |
| 0 | 111 | 255 | 242 | 255 | 158 | 24 | 0 | 0 | 6 | 39 | 255 | 232 | 230 | 56 | 0 |
| 6 | 218 | 251 | 250 | 137 | 7 | 11 | 0 | 0 | 0 | 2 | 62 | 255 | 250 | 125 | 3 |
| 0 | 173 | 255 | 255 | 101 | 9 | 20 | 0 | 13 | 3 | 13 | 182 | 251 | 245 | 61 | 0 |
| 0 | 107 | 251 | 241 | 255 | 230 | 98 | 55 | 19 | 118 | 217 | 248 | 253 | 255 | 52 | 4 |
| 0 | 18 | 146 | 250 | 255 | 247 | 255 | 255 | 255 | 249 | 255 | 240 | 255 | 129 | 0 | 5 |
| 0 | 0 | 23 | 113 | 215 | 255 | 250 | 248 | 255 | 255 | 248 | 248 | 118 | 14 | 12 | 0 |
| 0 | 0 | 6 | 1 | 0 | 52 | 153 | 233 | 255 | 252 | 147 | 37 | 0 | 0 | 4 | 1 |
| 0 | 0 | 5 | 5 | 0 | 0 | 0 | 0 | 0 | 14 | 1 | 0 | 6 | 6 | 0 | 0 |

(height x width) pixels

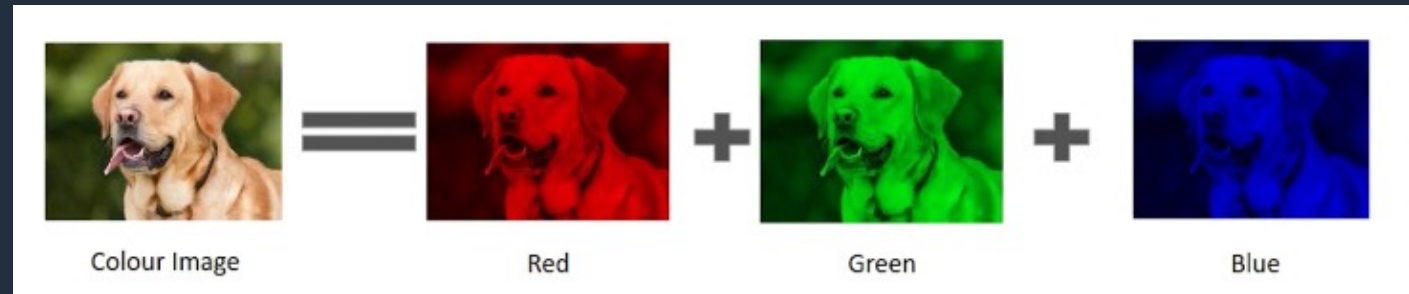
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How images are stored in a Computer?

Colored image



| | | | | | | | | | | | |
|--|--|--|----|----|-----|-----|-----|-----|-----|-----|-----|
| | | | | | 141 | 142 | 143 | 144 | 145 | | |
| | | | | | 151 | 152 | 153 | 154 | 155 | | |
| | | | | | 161 | 162 | 163 | 164 | 165 | | |
| | | | | 35 | 36 | 37 | 38 | 39 | 173 | 174 | 175 |
| | | | | 45 | 46 | 47 | 48 | 49 | 183 | 184 | 185 |
| | | | | 55 | 56 | 57 | 58 | 59 | 193 | 194 | 195 |
| | | | | 65 | 66 | 67 | 68 | 69 | | | |
| | | | 31 | 32 | 33 | 34 | 35 | 6 | 77 | 78 | 79 |
| | | | 41 | 42 | 43 | 44 | 45 | 16 | 87 | 88 | 89 |
| | | | 51 | 52 | 53 | 54 | 55 | | | | |
| | | | 61 | 62 | 63 | 64 | 65 | | | | |
| | | | 71 | 72 | 73 | 74 | 75 | | | | |
| | | | 81 | 82 | 83 | 84 | 85 | | | | |

R

G

B

(height x width) x 3 pixels

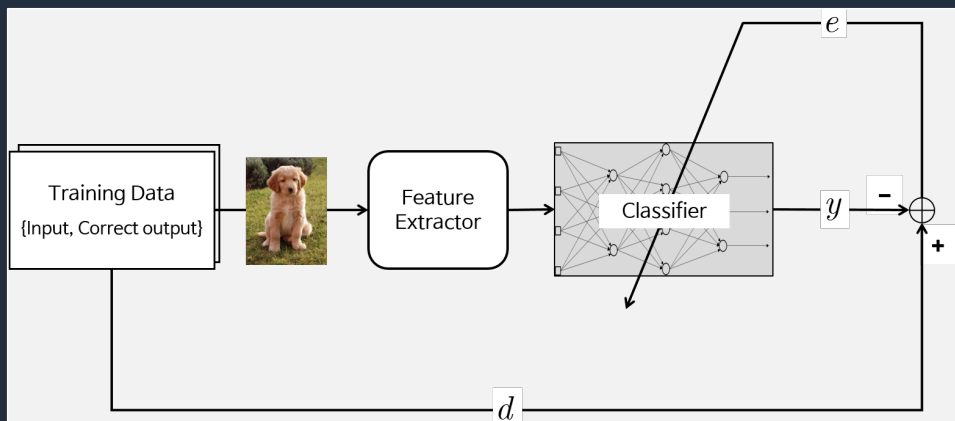
Figures copyright , Himanshi, <https://www.analyticsvidhya.com/blog/2021/03/grayscale-and-rgb-format-for-storing-images/>

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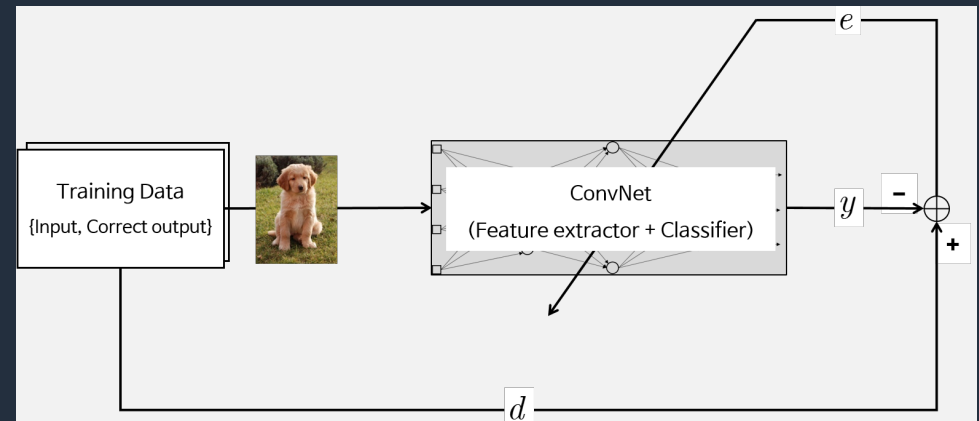
Image recognition

Perceptron neural network



Feature extraction + Classification network

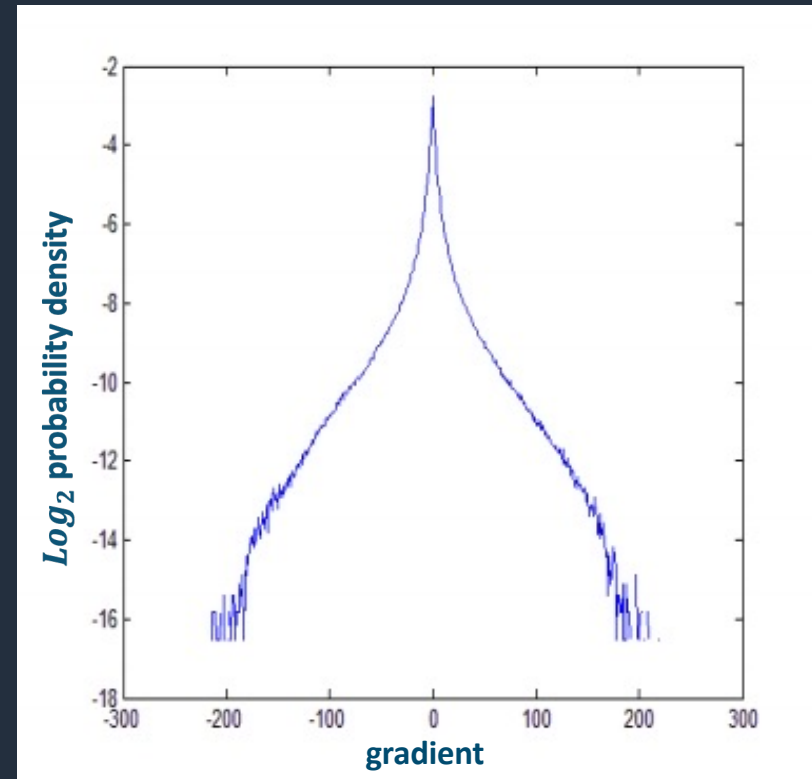
Convolutional neural network



Feature extraction network + Classification network

Figures copyright , 김성필, 머신러닝에서 컨벌루션 신경망까지, 딥러닝 첫걸음

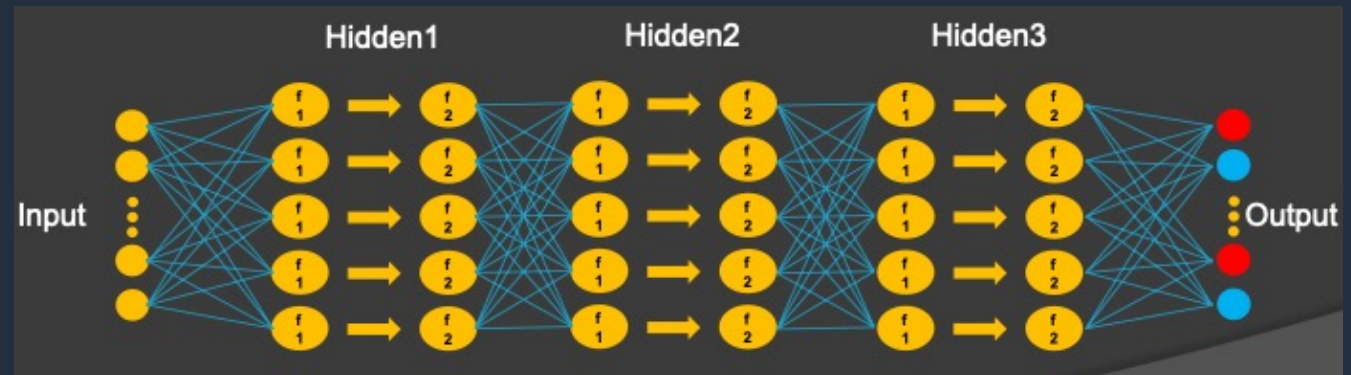
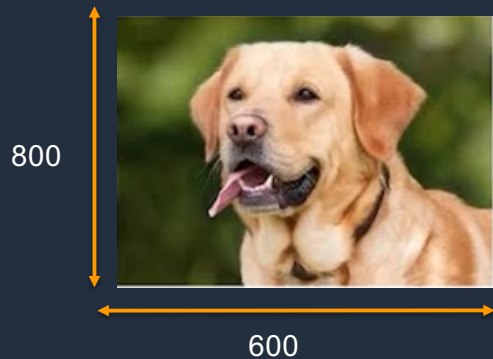
Statistics of image gradients



Logarithmic density of gradients from 10 natural images.

DNN Image recognition

Colored image



- 2차원 input data
- Pixel array : (800, 600, 3)
- Bytes : $800 \times 600 \times 3 = 1,440,000$ bytes
- Dense Layer에서는 모든 입력이 Hidden Layer에 연결(fully connected layer)
- Hidden Layer가 늘어날수록 back propagation에서 weight, bias update 계산량이 기하급수로 늘어남
- Image의 평탄한 영역은 학습에 불필요한 영역, 특징이 잘 드러나지 않음
- 특징 추출기는 사람이 직접 설계
- 학습에서 모든 data(pixels)를 각각 node에 연결하는 것보다 효율적인 방법을 찾아보자.

Figures copyright , Deep Neural Net by Daebang Kim.

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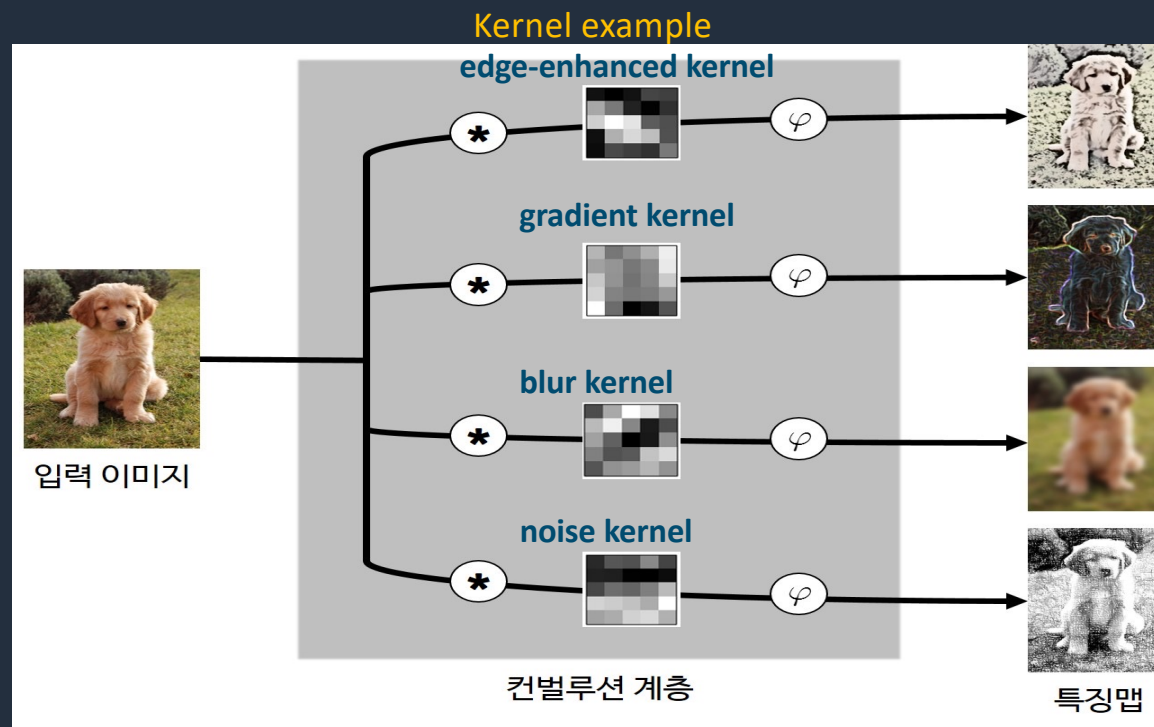
Figures copyright , Himanshi, <https://www.analyticsvidhya.com/blog/2021/03/grayscale-and-rgb-format-for-storing-images/>



CNN Image recognition

Convolution Layer

- 특징점(feature point) 추출filter(kernel)을 사용하여 입력데이터를 처리
- kernel의 개수가 특징맵(feature map)개수



CNN operation

Convolution?

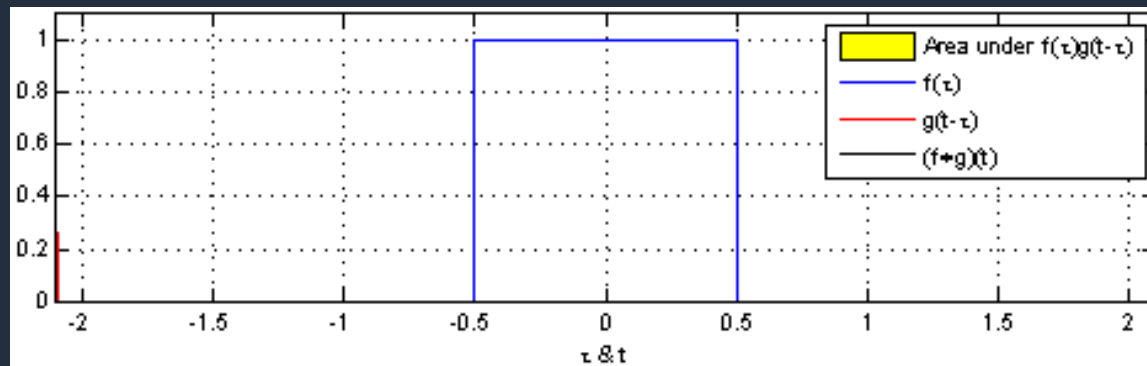
두 신호함수의 적분(중첩)

Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t - \tau)d\tau$$

Cross-Correlation

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t + \tau)d\tau$$



Figures copyright , Convolution, Wikipedia.

2D convolution for image filtering

Convolution

$$f(x, y) * k(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tau_1, \tau_2) k(x - \tau_1, y - \tau_2) d\tau_1 d\tau_2$$

$$g = f * k \quad g[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k k[u, v] f[i - u, j - v]$$



2D Convolution ? Cross-Correlation

$$Y = I - K + 1$$

| | | |
|---|---|---|
| 1 | 6 | 2 |
| 5 | 3 | 1 |
| 7 | 0 | 4 |

 \star

| | |
|----|---|
| 1 | 2 |
| -1 | 0 |

 =

| | |
|---|--|
| 8 | |
| | |

input kernel output

$$1 \cdot 1 + 2 \cdot 6 + -1 \cdot 5 + 0 \cdot 3 = 8$$

| | | |
|---|---|---|
| 1 | 6 | 2 |
| 5 | 3 | 1 |
| 7 | 0 | 4 |

 \star

| | |
|----|---|
| 1 | 2 |
| -1 | 0 |

 =

| | |
|---|---|
| 8 | 7 |
| | |

input kernel output

$$1 \cdot 6 + 2 \cdot 2 + -1 \cdot 3 + 0 \cdot 1 = 7$$

| | | |
|---|---|---|
| 1 | 6 | 2 |
| 5 | 3 | 1 |
| 7 | 0 | 4 |

 \star

| | |
|----|---|
| 1 | 2 |
| -1 | 0 |

 =

| | |
|---|---|
| 8 | 7 |
| 4 | |

input kernel output

$$1 \cdot 5 + 2 \cdot 3 + -1 \cdot 7 + 0 \cdot 0 = 4$$

| | | |
|---|---|---|
| 1 | 6 | 2 |
| 5 | 3 | 1 |
| 7 | 0 | 4 |

 \star

| | |
|----|---|
| 1 | 2 |
| -1 | 0 |

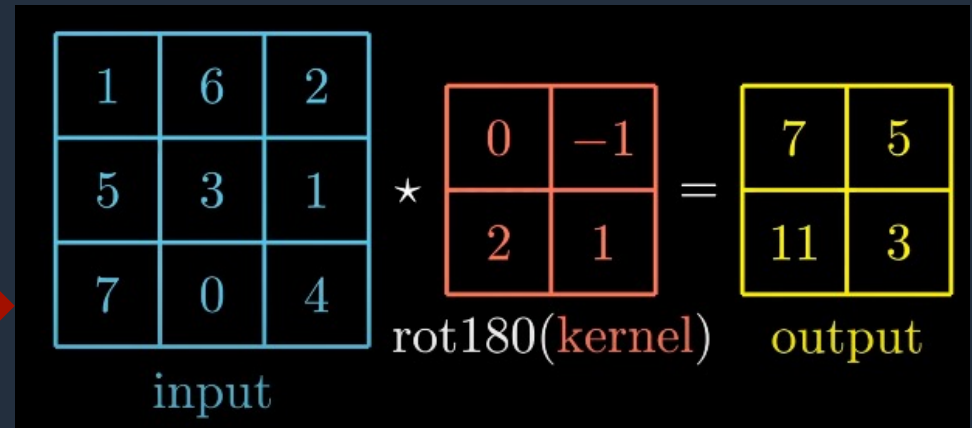
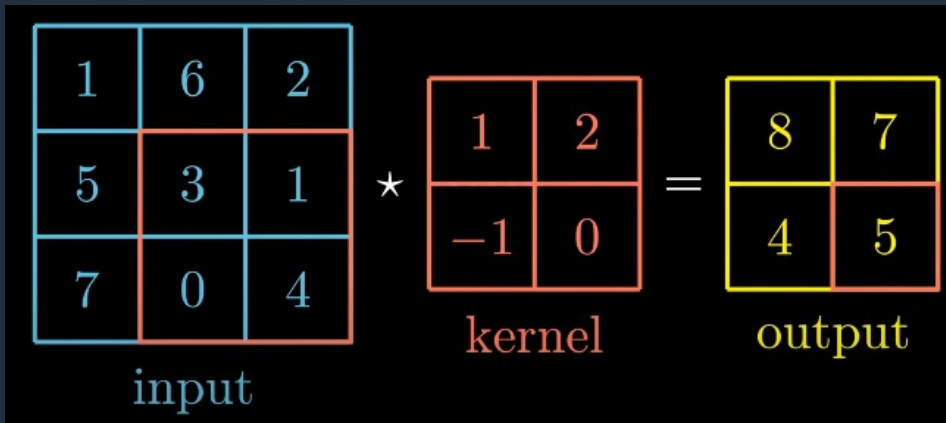
 =

| | |
|---|---|
| 8 | 7 |
| 4 | 5 |

input kernel output

$$1 \cdot 3 + 2 \cdot 1 + -1 \cdot 0 + 0 \cdot 4 = 5$$

Convolution & Cross Correlation



Cross-Correlation

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t + \tau)d\tau$$

Convolution

$$I * K = I \star \text{rot180}(K)$$

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t - \tau)d\tau$$

Padding

- valid

| | | |
|---|---|---|
| 1 | 6 | 2 |
| 5 | 3 | 1 |
| 7 | 0 | 4 |

 \star

| | |
|----|---|
| 1 | 2 |
| -1 | 0 |

 $=$

| | |
|---|---|
| 8 | 7 |
| 4 | 5 |

input kernel output

- full cross correlation(same)

| | | |
|---|---|---|
| 1 | 6 | 2 |
| 5 | 3 | 1 |
| 7 | 0 | 4 |

 $\overset{\star}{full}$

| | |
|----|---|
| 1 | 2 |
| -1 | 0 |

 $=$

| | | | |
|----|----|----|----|
| 0 | -1 | -6 | -2 |
| 2 | 8 | 7 | 1 |
| 10 | 4 | 5 | -3 |
| 14 | 7 | 8 | 4 |

Keras functional parameter : padding (default = 'valid')

```
1 model.add(keras.layers.Conv2D(32, kernel_size=3, activation='relu',  
2 padding='same', input_shape=(28,28,1)))
```

- valid : kernel에 매칭되지 않는 부분 skip, (0,0)시작
- same : kernel에 매칭 되지 않는 부분 0 padding

Convolution Layer

CNN for Image

Tensorflow keras CNN example

```
1 model.add(keras.layers.Conv2D(32, kernel_size=3, activation='relu', padding='same', input_shape=(28,28,1)))
```

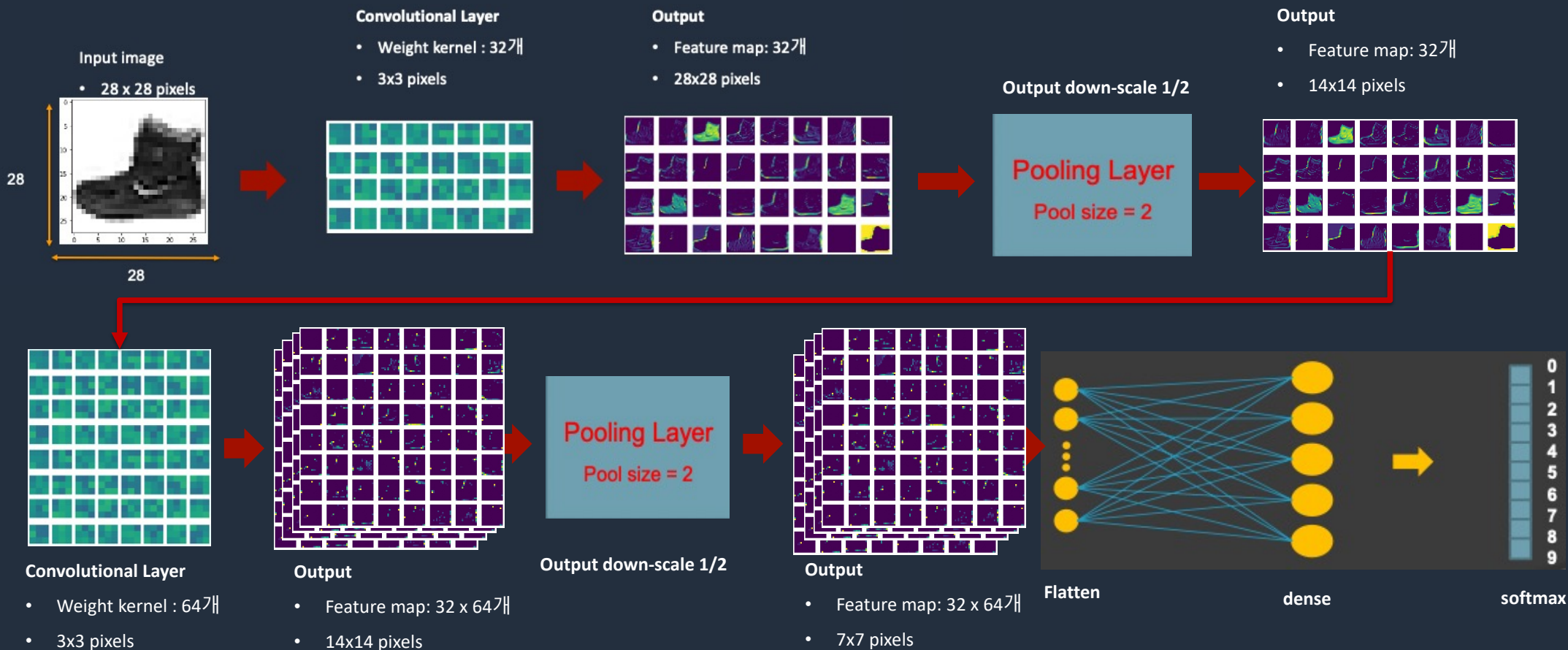


Weight Parameters :

- $\text{Input_channel} * w_kernel * h_kernel + \text{biases}$
- $1 * 3 * 3 * 32 + 32 = 320$

CNN for Image

Tensorflow keras CNN example

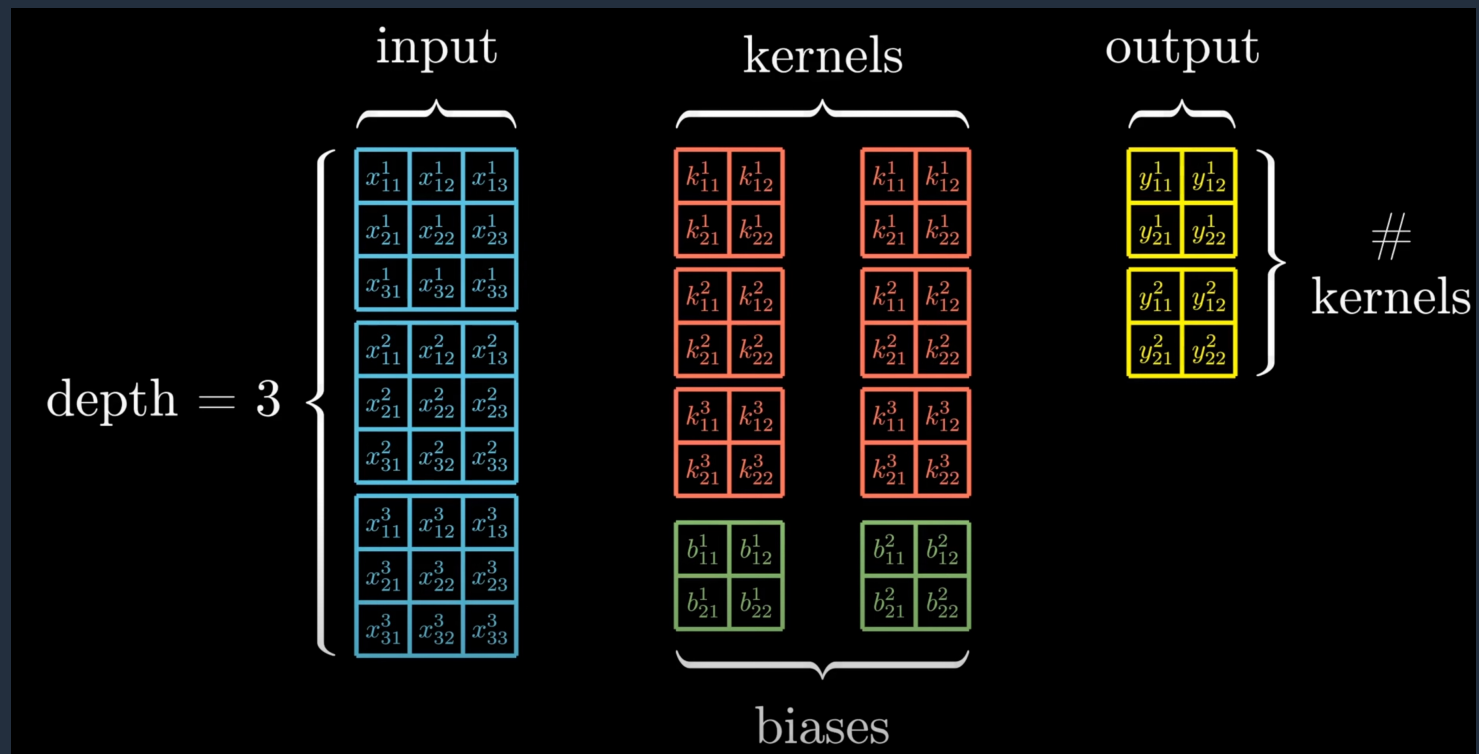


Visualization of Multi-Layer Perceptron & CNN



Figures copyright , Denis Dmitriev, Neural Network 3D Simulation, <https://www.youtube.com/watch?v=3JQ3hYko51Y&t=98s>

Forward propagation

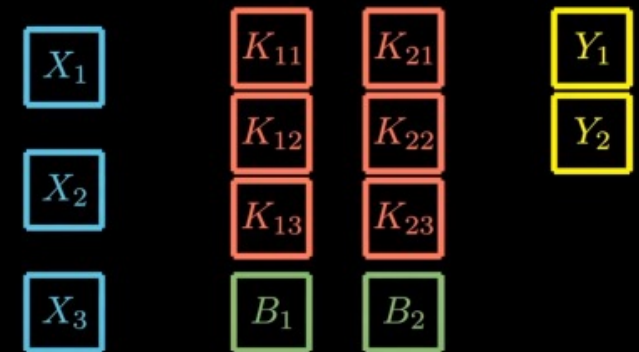


Forward propagation



Forward propagation

$$\begin{aligned} Y_1 &= B_1 + X_1 \star K_{11} + X_2 \star K_{12} + X_3 \star K_{13} \\ Y_2 &= B_2 + X_1 \star K_{21} + X_2 \star K_{22} + X_3 \star K_{23} \\ &\vdots \\ Y_d &= B_d + X_1 \star K_{d1} + X_2 \star K_{d2} + X_3 \star K_{d3} \end{aligned}$$



Forward propagation

$$Y_i = B_i + \sum_{j=1}^n X_j \star K_{ij}, \quad i = 1 \dots d$$

$$\begin{aligned} Y_1 &= B_1 + X_1 \star K_{11} + \dots + X_n \star K_{1n} \\ Y_2 &= B_2 + X_1 \star K_{21} + \dots + X_n \star K_{2n} \\ &\vdots \\ Y_d &= B_d + X_1 \star K_{d1} + \dots + X_n \star K_{dn} \end{aligned}$$

Forward propagation

$$Y_i = B_i + \sum_{j=1}^n X_j \star K_{ij}, \quad i = 1 \dots d$$

$$Y_1 = B_1 + X_1 \star K_{11} + \dots + X_n \star K_{1n}$$

$$Y_2 = B_2 + X_1 \star K_{21} + \dots + X_n \star K_{2n}$$

$$\vdots$$

$$Y_d = B_d + X_1 \star K_{d1} + \dots + X_n \star K_{dn}$$

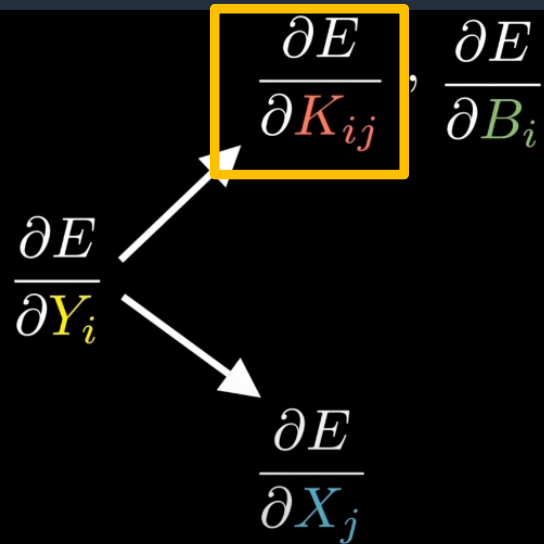
$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_d \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_d \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1n} \\ K_{21} & K_{22} & \dots & K_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ K_{d1} & K_{d2} & \dots & K_{dn} \end{bmatrix} \cdot \star \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$$

$$Y = B + K \cdot \star X$$

Convolution Layer

Backward Operation

Backward Kernel



$$Y_i = B_i + \sum_{j=1}^n X_j \star K_{ij}$$

Forward propagation

$$Y_i = B_i + X_1 \star K_{i1} + \dots + X_n \star K_{in}$$

Simplified example

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \star \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

$$\begin{cases} y_{11} = b_{11} + k_{11}x_{11} + k_{12}x_{12} + k_{21}x_{21} + k_{22}x_{22} \\ y_{12} = b_{12} + k_{11}x_{12} + k_{12}x_{13} + k_{21}x_{22} + k_{22}x_{23} \\ y_{21} = b_{21} + k_{11}x_{21} + k_{12}x_{22} + k_{21}x_{31} + k_{22}x_{32} \\ y_{22} = b_{22} + k_{11}x_{22} + k_{12}x_{23} + k_{21}x_{32} + k_{22}x_{33} \end{cases}$$

Backward Kernel

$$\begin{cases} y_{11} = b_{11} + k_{11}x_{11} + k_{12}x_{12} + k_{21}x_{21} + k_{22}x_{22} \\ y_{12} = b_{12} + k_{11}x_{12} + k_{12}x_{13} + k_{21}x_{22} + k_{22}x_{23} \\ y_{21} = b_{21} + k_{11}x_{21} + k_{12}x_{22} + k_{21}x_{31} + k_{22}x_{32} \\ y_{22} = b_{22} + k_{11}x_{22} + k_{12}x_{23} + k_{21}x_{32} + k_{22}x_{33} \end{cases}$$

$$\frac{\partial E}{\partial k_{11}} = \frac{\partial E}{\partial y_{11}} \frac{\partial y_{11}}{\partial k_{11}} + \frac{\partial E}{\partial y_{12}} \frac{\partial y_{12}}{\partial k_{11}} + \frac{\partial E}{\partial y_{21}} \frac{\partial y_{21}}{\partial k_{11}} + \frac{\partial E}{\partial y_{22}} \frac{\partial y_{22}}{\partial k_{11}}$$

$$\frac{\partial E}{\partial k_{11}} = \underbrace{\frac{\partial E}{\partial y_{11}} \frac{\partial y_{11}}{\partial k_{11}}}_{x_{11}} + \underbrace{\frac{\partial E}{\partial y_{12}} \frac{\partial y_{12}}{\partial k_{11}}}_{x_{12}} + \underbrace{\frac{\partial E}{\partial y_{21}} \frac{\partial y_{21}}{\partial k_{11}}}_{x_{21}} + \underbrace{\frac{\partial E}{\partial y_{22}} \frac{\partial y_{22}}{\partial k_{11}}}_{x_{22}}$$

$$\frac{\partial E}{\partial k_{11}} = \frac{\partial E}{\partial y_{11}} x_{11} + \frac{\partial E}{\partial y_{12}} x_{12} + \frac{\partial E}{\partial y_{21}} x_{21} + \frac{\partial E}{\partial y_{22}} x_{22}$$

$$\frac{\partial E}{\partial Y} = \begin{bmatrix} \frac{\partial E}{\partial y_{11}} & \frac{\partial E}{\partial y_{12}} \\ \frac{\partial E}{\partial y_{21}} & \frac{\partial E}{\partial y_{22}} \end{bmatrix} \rightarrow \begin{bmatrix} \frac{\partial E}{\partial k_{11}} & \frac{\partial E}{\partial k_{12}} \\ \frac{\partial E}{\partial k_{21}} & \frac{\partial E}{\partial k_{22}} \end{bmatrix}$$

$$\begin{aligned} \frac{\partial E}{\partial k_{11}} &= \frac{\partial E}{\partial y_{11}} x_{11} + \frac{\partial E}{\partial y_{12}} x_{12} + \frac{\partial E}{\partial y_{21}} x_{21} + \frac{\partial E}{\partial y_{22}} x_{22} \\ \frac{\partial E}{\partial k_{12}} &= \frac{\partial E}{\partial y_{11}} x_{12} + \frac{\partial E}{\partial y_{12}} x_{13} + \frac{\partial E}{\partial y_{21}} x_{22} + \frac{\partial E}{\partial y_{22}} x_{23} \\ \frac{\partial E}{\partial k_{21}} &= \frac{\partial E}{\partial y_{11}} x_{21} + \frac{\partial E}{\partial y_{12}} x_{22} + \frac{\partial E}{\partial y_{21}} x_{31} + \frac{\partial E}{\partial y_{22}} x_{32} \\ \frac{\partial E}{\partial k_{22}} &= \frac{\partial E}{\partial y_{11}} x_{22} + \frac{\partial E}{\partial y_{12}} x_{23} + \frac{\partial E}{\partial y_{21}} x_{32} + \frac{\partial E}{\partial y_{22}} x_{33} \end{aligned}$$

Backward Kernel

$$\begin{aligned}\frac{\partial E}{\partial k_{11}} &= \frac{\partial E}{\partial y_{11}} x_{11} + \frac{\partial E}{\partial y_{12}} x_{12} + \frac{\partial E}{\partial y_{21}} x_{21} + \frac{\partial E}{\partial y_{22}} x_{22} \\ \frac{\partial E}{\partial k_{12}} &= \frac{\partial E}{\partial y_{11}} x_{12} + \frac{\partial E}{\partial y_{12}} x_{13} + \frac{\partial E}{\partial y_{21}} x_{22} + \frac{\partial E}{\partial y_{22}} x_{23} \\ \frac{\partial E}{\partial k_{21}} &= \frac{\partial E}{\partial y_{11}} x_{21} + \frac{\partial E}{\partial y_{12}} x_{22} + \frac{\partial E}{\partial y_{21}} x_{31} + \frac{\partial E}{\partial y_{22}} x_{32} \\ \frac{\partial E}{\partial k_{22}} &= \frac{\partial E}{\partial y_{11}} x_{22} + \frac{\partial E}{\partial y_{12}} x_{23} + \frac{\partial E}{\partial y_{21}} x_{32} + \frac{\partial E}{\partial y_{22}} x_{33}\end{aligned}$$



$$\begin{bmatrix} \frac{\partial E}{\partial k_{11}} & \frac{\partial E}{\partial k_{12}} \\ \frac{\partial E}{\partial k_{21}} & \frac{\partial E}{\partial k_{22}} \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \star \begin{bmatrix} \frac{\partial E}{\partial y_{11}} & \frac{\partial E}{\partial y_{12}} \\ \frac{\partial E}{\partial y_{21}} & \frac{\partial E}{\partial y_{22}} \end{bmatrix}$$



$$Y = B + X \star K \Rightarrow \frac{\partial E}{\partial K} = X \star \frac{\partial E}{\partial Y}$$

Simplified version

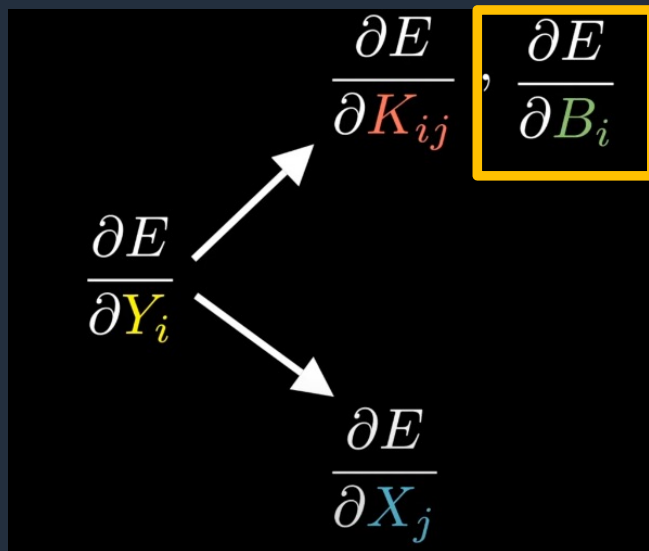
$$\frac{\partial E}{\partial K} = X \star \frac{\partial E}{\partial Y}$$

Actual version

$$Y_i = B_i + \sum_{j=1}^n X_j \star K_{ij}, \quad i = 1 \dots d$$

$$\frac{\partial E}{\partial K_{ij}} = X_j \star \frac{\partial E}{\partial Y_i}$$

Backward bias



$$Y_i = B_i + \sum_{j=1}^n X_j \star K_{ij}$$

Forward propagation

$$Y_i = B_i + X_1 \star K_{i1} + \dots + X_n \star K_{in}$$

Simplified example

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \star \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

$$\begin{cases} y_{11} = b_{11} + k_{11}x_{11} + k_{12}x_{12} + k_{21}x_{21} + k_{22}x_{22} \\ y_{12} = b_{12} + k_{11}x_{12} + k_{12}x_{13} + k_{21}x_{22} + k_{22}x_{23} \\ y_{21} = b_{21} + k_{11}x_{21} + k_{12}x_{22} + k_{21}x_{31} + k_{22}x_{32} \\ y_{22} = b_{22} + k_{11}x_{22} + k_{12}x_{23} + k_{21}x_{32} + k_{22}x_{33} \end{cases}$$

Backward bias

$$\begin{cases} y_{11} = b_{11} + k_{11}x_{11} + k_{12}x_{12} + k_{21}x_{21} + k_{22}x_{22} \\ y_{12} = b_{12} + k_{11}x_{12} + k_{12}x_{13} + k_{21}x_{22} + k_{22}x_{23} \\ y_{21} = b_{21} + k_{11}x_{21} + k_{12}x_{22} + k_{21}x_{31} + k_{22}x_{32} \\ y_{22} = b_{22} + k_{11}x_{22} + k_{12}x_{23} + k_{21}x_{32} + k_{22}x_{33} \end{cases}$$

$$\frac{\partial E}{\partial b_{11}} = \frac{\partial E}{\partial y_{11}} \underbrace{\frac{\partial y_{11}}{\partial b_{11}}}_1 + \frac{\partial E}{\partial y_{12}} \underbrace{\frac{\partial y_{12}}{\partial b_{11}}}_0 + \frac{\partial E}{\partial y_{21}} \underbrace{\frac{\partial y_{21}}{\partial b_{11}}}_0 + \frac{\partial E}{\partial y_{22}} \underbrace{\frac{\partial y_{22}}{\partial b_{11}}}_0$$

$$\frac{\partial E}{\partial b_{11}} = \frac{\partial E}{\partial y_{11}}$$

$$\frac{\partial E}{\partial b_{12}} = \frac{\partial E}{\partial y_{12}}$$

$$\frac{\partial E}{\partial b_{21}} = \frac{\partial E}{\partial y_{21}}$$

$$\frac{\partial E}{\partial b_{22}} = \frac{\partial E}{\partial y_{22}}$$

$$\frac{\partial E}{\partial B} = \frac{\partial E}{\partial Y}$$

Simplified version

$$Y = B + X \star K \Rightarrow \frac{\partial E}{\partial B} = \frac{\partial E}{\partial Y}$$

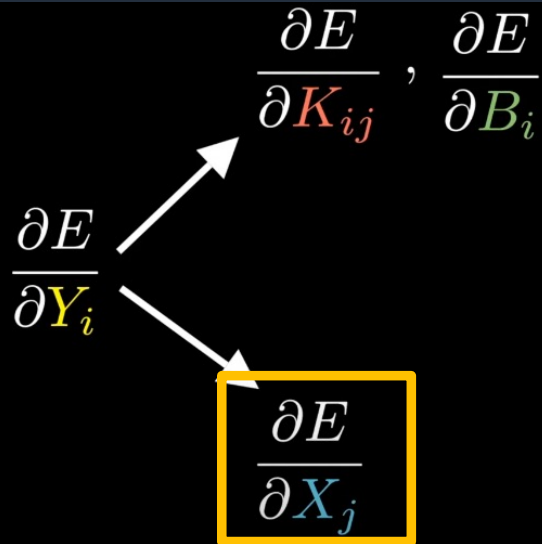
$$\frac{\partial E}{\partial Y} = \begin{bmatrix} \frac{\partial E}{\partial y_{11}} & \frac{\partial E}{\partial y_{12}} \\ \frac{\partial E}{\partial y_{21}} & \frac{\partial E}{\partial y_{22}} \end{bmatrix} \rightarrow \begin{bmatrix} \frac{\partial E}{\partial b_{11}} & \frac{\partial E}{\partial b_{12}} \\ \frac{\partial E}{\partial b_{21}} & \frac{\partial E}{\partial b_{22}} \end{bmatrix}$$

Actual version

$$Y_i = B_i + \sum_{j=1}^n X_j \star K_{ij}, \quad i = 1 \dots d$$

$$\frac{\partial E}{\partial B_i} = \frac{\partial E}{\partial Y_i}$$

Backward input



$$Y_i = B_i + \sum_{j=1}^n X_j \star K_{ij}$$

Forward propagation

$$Y_i = B_i + X_1 \star K_{i1} + \cdots + X_n \star K_{in}$$

Simplified example

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \star \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

$$\begin{cases} y_{11} = b_{11} + k_{11}x_{11} + k_{12}x_{12} + k_{21}x_{21} + k_{22}x_{22} \\ y_{12} = b_{12} + k_{11}x_{12} + k_{12}x_{13} + k_{21}x_{22} + k_{22}x_{23} \\ y_{21} = b_{21} + k_{11}x_{21} + k_{12}x_{22} + k_{21}x_{31} + k_{22}x_{32} \\ y_{22} = b_{22} + k_{11}x_{22} + k_{12}x_{23} + k_{21}x_{32} + k_{22}x_{33} \end{cases}$$

Backward bias

$$\begin{cases} y_{11} = b_{11} + k_{11}x_{11} + k_{12}x_{12} + k_{21}x_{21} + k_{22}x_{22} \\ y_{12} = b_{12} + k_{11}x_{12} + k_{12}x_{13} + k_{21}x_{22} + k_{22}x_{23} \\ y_{21} = b_{21} + k_{11}x_{21} + k_{12}x_{22} + k_{21}x_{31} + k_{22}x_{32} \\ y_{22} = b_{22} + k_{11}x_{22} + k_{12}x_{23} + k_{21}x_{32} + k_{22}x_{33} \end{cases}$$

$$\frac{\partial E}{\partial x_{11}} = \frac{\partial E}{\partial y_{11}} \underbrace{\frac{\partial y_{11}}{\partial x_{11}}}_{k_{11}} + \frac{\partial E}{\partial y_{12}} \underbrace{\frac{\partial y_{12}}{\partial x_{11}}}_0 + \frac{\partial E}{\partial y_{21}} \underbrace{\frac{\partial y_{21}}{\partial x_{11}}}_0 + \frac{\partial E}{\partial y_{22}} \underbrace{\frac{\partial y_{22}}{\partial x_{11}}}_0$$

$$\begin{bmatrix} \frac{\partial E}{\partial x_{11}} & \frac{\partial E}{\partial x_{12}} & \frac{\partial E}{\partial x_{13}} \\ \frac{\partial E}{\partial x_{21}} & \frac{\partial E}{\partial x_{22}} & \frac{\partial E}{\partial x_{23}} \\ \frac{\partial E}{\partial x_{31}} & \frac{\partial E}{\partial x_{32}} & \frac{\partial E}{\partial x_{33}} \end{bmatrix} - \begin{bmatrix} \frac{\partial E}{\partial y_{11}} & \frac{\partial E}{\partial y_{12}} \\ \frac{\partial E}{\partial y_{21}} & \frac{\partial E}{\partial y_{22}} \end{bmatrix} \star_{full} \begin{bmatrix} k_{22} & k_{21} \\ k_{12} & k_{11} \end{bmatrix}$$

$$Y = B + X \star K \Rightarrow \frac{\partial E}{\partial X} = \frac{\partial E}{\partial Y} \star_{full} K$$

$$\frac{\partial E}{\partial Y} = \begin{bmatrix} \frac{\partial E}{\partial y_{11}} & \frac{\partial E}{\partial y_{12}} \\ \frac{\partial E}{\partial y_{21}} & \frac{\partial E}{\partial y_{22}} \end{bmatrix} \rightarrow \begin{bmatrix} \frac{\partial E}{\partial x_{11}} & \frac{\partial E}{\partial x_{12}} & \frac{\partial E}{\partial x_{13}} \\ \frac{\partial E}{\partial x_{21}} & \frac{\partial E}{\partial x_{22}} & \frac{\partial E}{\partial x_{23}} \\ \frac{\partial E}{\partial x_{31}} & \frac{\partial E}{\partial x_{32}} & \frac{\partial E}{\partial x_{33}} \end{bmatrix}$$

Actual version

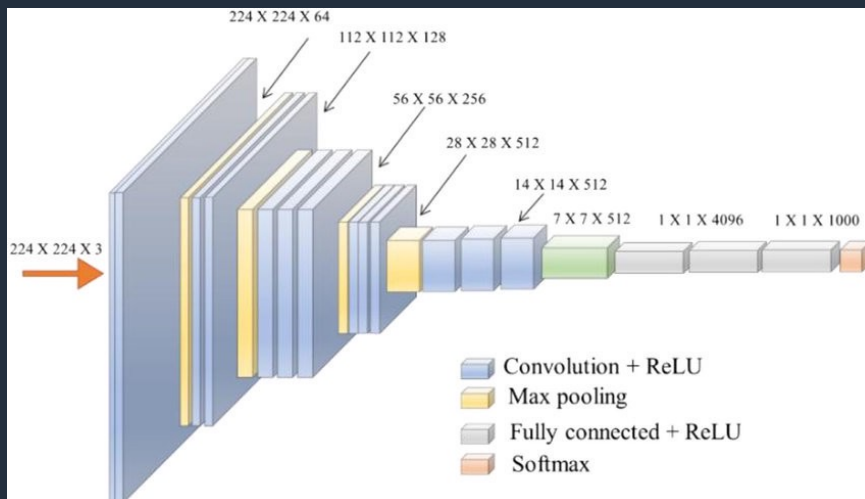
$$\frac{\partial E}{\partial X_j} = \sum_{i=1}^n \frac{\partial E}{\partial Y_i} \star_{full} K_{ij}$$



CNN Visualization

Deep CNN Layer Visualization

VGG16



Figures copyright, Denis Dmitriev, VGG16 Neural Network Visualization, <https://www.youtube.com/watch?v=RNnKtNrsmg&t=2s>

Q&A



감사합니다