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Project 1

MATH345 MATH MODELING

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Abstract

Gas prices are known to change aggressively and unpredictably, oftentimes changing on a day to day basis. People are very sensitive to changes in gas prices, especially when they are rising. The average American driver bought approximately 570 gallons of gas in 2021, adding up to an estimated 2,000 dollars [7]. The cost of gas oftentimes leads people to driving out of their way to a farther gas station to buy the cheapest gas.

We created a mathematical model to help understand which conditions would make it be worth a farther drive to a cheaper gas station. Our model took into account price, distance, fuel efficiency, and how people value their time to determine which gas station would be the best option. Analysis of our model concluded that the model is rather sensitive to how people value their time.

Using the gas station locations and prices in Fulton County, GA and Gwinnett County, GA, we used our model to determine which gas station a family with a certain car in Sandy Springs, GA should use in order to minimize cost. In this example, the closer but more expensive gas station would be preferred, however, this decision might change based on how this family value their time.

Keywords: gas cost, fuel efficiency, gas mathematical model, gas station problem

Contents

1 Introduction

1.1 Background

How do you save cost on gas? The most obvious answer is to find the cheapest gas possible. However, gas prices constantly change and different people value costs aside from monetary costs, such as the value of their time. A buyer may consider driving further to a gas station that sells less expensive gas. However, another buyer may value their time more and decide that they should not drive further to save a few dollars. Moreover, the amount of gas spent in order to drive to the further gas station is also significant. Ultimately, a buyer needs to determine how they value their time and their money, and then they can determine when it would be more or less costly to drive further for gas.

Studying gas prices and where one should buy gas is extremely useful. For instance, it could help someone on a tight budget meet their goals. It can help someone make a difficult decision and genuinely consider their potential options. Also, studying gas prices and modeling where to buy gas is important as gas prices continually rise throughout the country. Gas prices are rising due to a number of factors, and the demand for gas rarely decreases. Therefore, finding less expensive gas is often a subject of interest for a wide array of consumers.

Moreover, this type of problem setup is useful in many different circumstances. There are many examples in everyday life where people are forced to choose between a more monetarily costly option and a more time consuming option that costs less money. For example, one article discusses the options one faces in a parking situation [10]. In this article, a driver wants to find parking. Parking on the curb is free and there is a lot with parking, but it costs money. At a given moment in time, there are no available spots on the street, so the driver would either need to drive around looking for a spot or pay for parking in the lot. This is similar to the gas station problem as there are two options, and one of the options has a greater monetary cost while the other option has a greater time cost. Therefore, the model from this problem is similar to the model in this paper, as it takes into consideration the different costs of each option and determines which would end up being less costly. Also, this model incorporates a time-value variable that is essential as it enables the model to personal preferences regarding time into consideration, which we take into account in our model.

1.2 Assumptions

There are several assumptions that must be made in order to form a model:

1. The driver is buying gas from a specific gas station.

   The driver knows exactly which gas stations have the cheap and the expensive gas. They do not drive aimlessly and there exists a clear route to get to the gas station.

2. Traffic is constant.

   Our model takes into consideration the time it takes to get to the further gas station and the time it takes to get to the closer gas station. However, it assumes that the amount of traffic does not change between successive trips to the two gas stations, as traffic congestion would increase fuel consumption. The time it takes to get to the further gas station will always take x minutes and the time it takes to get to the closer gas station takes y minutes.

3. People tend to save money.

   This assumption indicates that people favor cheaper alternatives, in this case, cheaper gas. When given the opportunity to lower the cost of gas, the buyer will try to pay less money for gas.

4. People tend to save time.

   This assumption indicates that people do not like to waste time and favor time efficiency. Even though cheap gas may exist, people do not want to spend hours every week driving to get their gas. Therefore, time will limit their ability to get cheap gas.

5. The car conditions are consistent.

   We assume the car conditions such as the state of the car tires and engines are consistent. For example, there is no "wear and tear" cost per mile on the car.

6. Speed is consistent.

   We assume that the speed at which the car is going is consistent. For instance, the driver either decides to take highways or decides to take back roads or a combination of the two. However, the driver cannot decide to change the route they take to the gas station on different trips, as this would alter the amount of gas it would take to get to the gas station. We assume that speed is consistent whether the driver goes to the closer gas station or goes to a further gas station. Moreover, the routes to both of the gas stations should be similar. For example, they both should be normal roads or both on highways.

7. Road condition is consistent.

   We assume that the road condition at which the car is going is consistent. MIT researchers pointed out that both the roughness and deflection have significant impact on the fuel consumption [4].

8. Gas stations were visited on the same day of the week.

   We assume that the individual visited both gas stations on the same day of the week because the gas prices are different depending on the day of the week. According to Gasbuddy.com, the best day to fill up the tank is on Monday, whereas the worst day is on Sunday[8].

9. The number of gallons of gas you need to fill up is constant

   We assume that no matter where you choose to fill up your gas tank, you always want to fill up with the same amount of gas.

2 Methods

The primary hypothesis, that the threshold difference in distance between the two gas stations would approximately be 10 miles, will be tested in the mathematical model we constructed. While we were constructing the mathematical model, we made assumptions to model the real world case, took into consideration what factors would affect the cost and incorporate them as variables into the model. We used a linear model to identify the cost difference of two different gas stations.

Data

The locations of the two gas stations are found from the Google Maps website. The Google Maps also automatically displays the approximate time that would be spent traveling to another gas station. The price of gas at both stations are from the Gasbuddy website. This website shows the current gas price for different gas stations.

3 Our Model

3.1 Variables and Model

Determining whether to buy gas from a closer or further gas station. First, we will declare the variables necessary for our model.

- p_1 = price of gas at the further gas station $(\frac{\$}{gal})$
- p_2 = price of gas at the closer gas station $(\frac{\$}{gal})$
- g = number of gallons you need to fill up your car's tank (gal)
- c_1 = time spent going to the further gas station $(mins)$
- c_2 = time spent going to the closer gas station $(mins)$
- f_1 = fuel cost of going to the further gas station (\$)
- f_2 = fuel cost of going to the closer gas station (\$)
- v = value of time spent driving to the gas station $(\frac{\$}{mins})$
- d_1 = miles to get to the further gas station $(miles)$
- d_2 = miles to get to the closer gas station $(miles)$
- k = the fuel efficiency of the car; miles per gallon for a certain car

Then, we form some basic equations in order to put together our model:

1. $v * c_1$ = monetized cost of time driving to the further gas station
2. $v * c_2$ = monetized cost of time driving to the closer gas station
3. $g * p_1$ = the cost of getting gas at the further gas station
4. $g * p_2$ = the cost of getting gas at the closer gas station
5. $\frac{d_1 * p_1}{k} = f_1$ = cost of getting to the further gas station
6. $\frac{d_2 * p_2}{k} = f_2$ = cost of getting to the closer gas station

Let us also define a variable ω.

- If $\omega > 0$, then go to the closer gas station.
- If $\omega < 0$, then go to the further gas station.
- If $\omega = 0$, then go to either gas station.

Our model:

$$\omega = (\frac{d_2 p_2}{k} + vc_2 + gp_2) - (\frac{d_1 p_1}{k} + vc_1 + gp_1). \quad (1)$$

3.2 Model explained

Our model takes in a number of variables and outputs a positive or negative number, and the sign of that number helps to determine where to buy gas.

We chose the variables above based on the assumptions we made. Overall, each of the variables is an integral part of determining the total cost of visiting each gas station. It takes price of gas, distance from station, and how much the person values their time [9]. This time value variable, v, was very important to us as it will change depending on who is driving to the different gas stations. If a person greatly values their time, they may have a higher value of v, making it less likely that our model will tell them to go to the further gas station. However, if the value of v is rather small or negligible, then the other variables will have greater weight in determining where the person should buy gas.

The variable ω denotes the value that our model outputs, and its value will determine where to buy gas. If ω is positive, and in our model we put the further gas station in the first half of the equation, this indicates that the total cost of going to the further gas station is greater than the total cost of going to the closer gas station, and therefore, the buyer should buy gas from the closer gas station. On the other hand, when the value of ω is negative, this indicates that the total cost of going to the closer gas station is greater than the total cost of going to the further gas station, so the buyer should buy gas at the further gas station. If the value of ω is zero, then it does not matter where the buyer buys gas because the total costs are equivalent [6].

4 Solution

We implemented our model in python (see **Appendix 6.1** for codes) for calculating ω in our model, and simulated some real world cases to show how the model works in **Section 4.1**. With the model we solved the real world problem in Fulton County in **Section 4.2** and based on, this case, did sensitivity analysis on parameter v (time value) in **Section 4.3.2**.

4.1 Example

With example data we experimented how our model helps making decisions. With a close (0.5 miles away) but expensive (\$3.9 per gallon) gas station and some far way (2 to 20 miles away) but cheap (\$3.2 per gallon) gas stations, for a person who values per minute \$0.3 (equivalent to \$18 hourly salary), our model shows the threshold distance difference to decide where to gas up is about 9 miles as show in **Figure 1**. That is to say, if the cheap gas station is within that far away from the expensive one, it's worth the drive for less expensive gas.

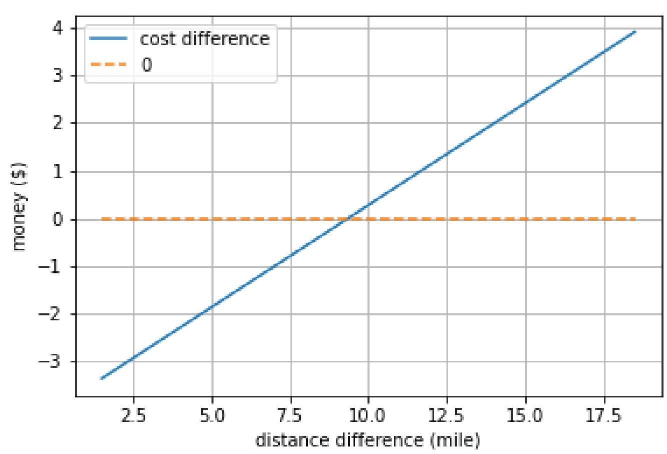

Figure 1: Example Cost Difference vs. Distance Difference

However for the same person and for the threshold distance 0.5 and 9.5 miles, we experimented different fuel efficiencies of cars and results in **Figure 2** show the influence of car's fuel efficiency on choosing gas station:

the higher the fuel efficiency, more likely the less expensive gas station would be preferred.

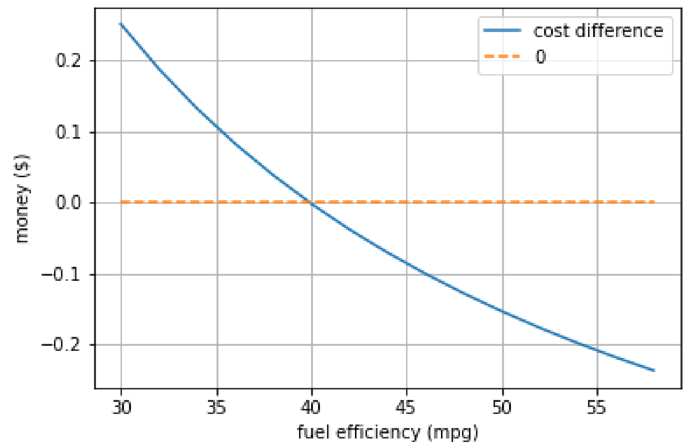

Figure 2: Example Cost Difference vs. Car Fuel Efficiency

See **Appendix 6.2** for codes we implemented.

4.2 Real World Problem

As of February 8, 2022, the average gas price in Fulton County, Georgia was \$3.386. In Gwinnett County, the average gas price was \$3.225 [1]. Let us say that a family lives in Sandy Springs, Georgia, which is in Fulton County. They have the option to either get gas in Sandy Springs or drive to Norcross, Georgia, which is located in Gwinnett County and has cheaper gas. The family lives 2 miles away from their local gas station and they live 9 miles away from the gas station in Norcross.

Our variables:

- $p_1 = \$3.225$
- $p_2 = \$3.386$
- $g = 10$ gal
- $c_1 = 12$ mins

- $c_2 = 5$ mins
- $v = 0.4 \frac{\$}{\text{mins}}$
- $d_1 = 9$ miles
- $d_2 = 2$ miles
- $k = 50$ mph

Given data above, for a person who values each minute of their time at \$0.40, the cost difference of going to 2 different gas stations is \$1.64. Therefore, since this value is positive, the total cost of getting gas at the closer gas station is less expensive than getting gas at the gas station that is further away. The driver should purchase their gas at the more expensive gas station nearby, rather than driving farther for less expensive gas.

4.3 Sensitivity Analysis

4.3.1 Analytical Analysis

Variable	Partial Derivative
p_1 (price of gas at farther gas station)	$\frac{d_2}{k}$
p_2 (price of gas at closer gas station)	$-\frac{d_1}{k}$
g (number of gallons you need to fill up)	$p_2 - p_1$
c_1 (time spent going to the further gas station)	$-v$
c_2 (time spent going to the closer gas station)	v
v (value of time spent driving to the gas station)	$c_2 - c_1$
d_1 (miles to get to the further gas station)	$\frac{p_1}{k}$
d_2 (miles to get to the closer gas station)	$\frac{p_2}{k}$
k (the fuel efficiency of the car)	$\frac{d_1 p_1 - d_2 p_2}{k^2}$

4.3.2 Numerical Analysis

Based on the real world problem we discussed in **Section 4.2**, we did sensitivity analysis on time value v to see how sensitive the model decision is over how much a person values time. As shown in **Figure 3**, the modeling result is rather sensitive on v: in this case a person with $v = 0.15\$/min$ (equivalent to hourly salary \$9) and a person with $v = 0.2\$/min$ (equivalent to hourly salary \$12) would make different choice to minimize cost.

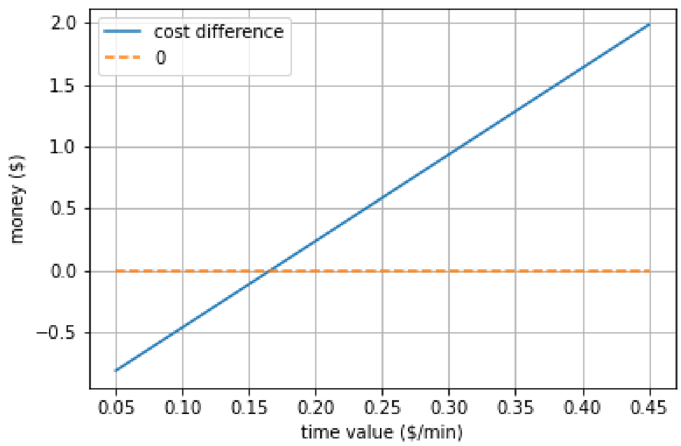

Figure 3: Time Value Sensitivity

See **Appendix 6.2** for codes.

5 Discussion

Our findings may imply that the threshold to make the decision on which gas station to fill up would be at 9 miles distance difference. This is when the gas price difference is 0.5 dollars/gallon, fuel efficiency is 50 miles/gallon, time value is 0.3 dollars/minute, and the gas tank holds 8 gallons. Based on **Figure 3**, we can see that the threshold for the time value sensitivity would be approximately at 0.17 dollars/minute. This can be interpreted to be 10.2 dollars per hour, which is lower than the average hourly rate salary in Georgia (18.59 dollars/hour) according to the U.S. Bureau of Labor Statistics [3]. Moreover, **Figure 2** shows the influence of car's fuel efficiency on choosing gas station. Higher the fuel efficiency, more likely the less expensive gas station would be preferred.

5.1 Strengths and Weaknesses

Some potential problems in this model are that even though we take it into account of how the time is monetized, it might be ambiguous and vague for the individuals to determine how valuable time is for them. Moreover, there's uncertainty about these approximations of variables like time, cost, and quality etc. which may lead to a biased model. The list of assumptions that we have, may also lead to a biased model. This could be resolved using a multi-objective programming model as future research.

Moreover, even though many assumptions were made in order to form this mathematical model, it still gives a strong understanding of the apparent costs that occur when driving to two different gas stations for gas. Another potential weakness for this model is that it does not take into consideration the changes in gas station prices due to outside influences. For instance, as more people begin to drive further for cheaper gas, the demand for gas from the further may increase, such that the price for gas at that gas station also increases. Then, the price at that gas station may no longer be better than the price at the closer gas station. Another weakness of this model is that it does not take any other factors of the gas station into consideration. For example, a gas station's food mart, a gas station's determination to help climate change, or a gas station's bathrooms may all be alternative reasons to visit that gas station over a different one. It could be interesting to implement a variable containing these alternative factors that would minimize the cost of visiting a certain gas station.

Some strengths for this model were that the model includes not only the distance, gas cost, and the time quality but also the fuel efficiency. The fuel efficiency variable takes the car sizes into consideration so that it includes all sizes like compact cars, mid-sized cars, as well as large cars [5]. Since our model would work regardless the car sizes, it would lead to a more accurate model. This model also includes the variable for the value of time, which is incredibly important. Different people may be more or less willing to drive further away because of how they value time. For instance, an extremely busy person may be more inclined to save time and buy more expensive gas and another person may have free time to spare and would rather save extra money. This factor plays a significant role in who would drive further for gas.

5.2 Potential Future Research

A possibility for future research is comparing car sizes to road conditions and determine which factor would have a greater impact on gas prices. Another idea would be how the rush hour and connecting that to traffic would impact the gas price as well. Moreover, it would be interesting to investigate on how the problem would be formulated in a different way if it takes the increasing number of electric vehicles into account. There were approximately 10 million electric cars on the world's roads in 2020, and this may affect the variables that we have in our model, eventually affecting the overall problem [2]. Moreover, as awareness increase regarding climate change and greenhouse gas emissions, this may lead to more electric cars, and possibly less gas stations, and increase in gas costs. Therefore, another future research idea could be to investigate deeply into how the public perception of climate change can shift the gas cost, and eventually how it would affect this overall problem long-term. Given this model, it is easier now to take more factors into consideration that simply distance, and instead save money and minimize unnecessary cost.

6 Appendix

6.1 Model Program

Python

```
def cost_difference(station1, station2, time_value, fuel_eff, gas):
    """
    Calculating cost of going to further gas station2
    - cost of going to closer gas station1;
    time is moneitized in this model.
    """
    distance1, price1, time_cost1 = station1
    distance2, price2, time_cost2 = station2
    cost_difference = (distance2 * price2 - distance1 * price1) / fuel_eff \
        + (time_cost2 - time_cost1) * time_value + gas * (price2 - price1)
    return cost_difference
```

R

```
cost_difference <- function(distance1, price1, timecost1, distance2, price2,
                            timecost2, time_value, fuel_eff, gas){
  cost_diff = (distance2 * price2 - distance1 * price1) / fuel_eff +
    (timecost2 - timecost1) * time_value + gas * (price2 - price1)
  return(cost_diff)
}
```

6.2 Example Program

```
# on distance
sdistances = .5
ldistances = np.arange(start=2, stop=20, step=1)
hprice = 3.9
lprice = 3.4
s_time_cost = sdistances * 1.2
l_time_cost = ldistances * 1.2
fuel_eff = 50
gas = 8
time_value = .3
station1 = []
station2 = []
diff = []
for i in range(len(ldistances)):
    station1.append((sdistances, hprice, s_time_cost))
    station2.append((ldistances[i], lprice, l_time_cost[i]))
    diff.append(cost_difference(station1[i], station2[i], \
        time_value, fuel_eff, gas))

plt.plot(ldistances - sdistances, diff, linestyle="-", label='cost difference')
plt.plot(ldistances - sdistances, np.zeros(len(diff)), linestyle="--", label='0')
plt.xlabel('distance difference (mile)')
plt.ylabel('money ($)')
plt.grid()
plt.legend()
plt.savefig('pic1.png')

# on fuel efficiency
sdistances = .5
```

```
ldistances = 9.5
hprice = 3.9
lprice = 3.4
s_time_cost = sdistances * 1.2
l_time_cost = ldistances * 1.2
fuel_eff = np.arange(start=30, stop=60, step=2)
gas = 8
time_value = .3
station1 = []
station2 = []
diff = []
for i in range(len(fuel_eff)):
    station1.append((sdistances, hprice, s_time_cost))
    station2.append((ldistances, lprice, l_time_cost))
    diff.append(cost_difference(station1[i], station2[i], \
        time_value, fuel_eff[i], gas))

plt.plot(fuel_eff, diff, linestyle="-", label='cost difference')
plt.plot(fuel_eff, np.zeros(len(diff)), linestyle="--", label='0')
plt.xlabel('fuel efficiency (mpg)')
plt.ylabel('money ($)')
plt.grid()
plt.legend()
plt.savefig('pic3.png')
```

6.3 Real World Problem and Sensitivity Analysis

```
# real world case
station1 = [2, 3.386, 5]
station2 = [9, 3.225, 12]
g = 10
v = .4
k = 50
print(cost_difference(station1, station2, v, k, g))
v = np.arange(start=.05, stop=.5, step=.05)
diff = cost_difference(station1, station2, v, k, g)
plt.plot(v, diff, linestyle="-", label='cost difference')
plt.plot(v, np.zeros(len(diff)), linestyle="--", label='0')
```

```
plt.xlabel('time value ($/min)')
plt.ylabel('money ($)')
plt.grid()
plt.legend()
plt.savefig('pic2.png')
```

References

[1] Georgia average gas prices.

[2] Trends and developments in electric vehicle markets – global ev outlook 2021 – analysis.

[3] Georgia - may 2020 state occupational employment and wage estimates, 2021.

[4] Matt Ball. Mit study: Traffic jams magnify how roads affect fuel consumption, Aug 2015.

[5] Christine Belledin. Teaching mathematical modeling with the driving for gas problem - ppt video online download, 2015.

[6] Kimberly Crandell. Which costs you more... nearby expensive gas, or cheaper gas across town?, Aug 2014.

[7] Drew DeSilver. Gasoline costs more these days, but price spikes have a long history and happen for a host of reasons, Dec 2021.

[8] GasBuddy and Nicole Petersen. The best and worst days of the week to buy gas, Jan 2020.

[9] Samir Khuller, Azarakhsh Malekian, and Julián Mestre. To fill or not to fill: The gas station problem - umd.

[10] Donald C. Shoup. Cruising for parking. *Transport Policy*, 13(6):479–486, 2006. Parking.