Definition [$\lambda^{1,b,\to}$ $\beta\eta$-equality]

$\Gamma \vdash e_1 \approx_{\beta\eta} e_2 : \tau$ if derivable from the following set of axioms.

$$\Gamma \vdash e \approx_{\beta\eta} e : \tau \qquad \text{(refl)}$$

$$\frac{\Gamma \vdash e \approx_{\beta\eta} e' : \tau}{\Gamma \vdash e' \approx_{\beta\eta} e : \tau} \qquad \text{(sym)}$$

$$\frac{\Gamma \vdash e \approx_{\beta\eta} e' : \tau \quad \Gamma \vdash e' \approx_{\beta\eta} e'' : \tau}{\Gamma \vdash e \approx_{\beta\eta} e'' : \tau} \qquad \text{(tran)}$$

$$\frac{\Gamma \vdash e_0 \approx_{\beta\eta} e_0' : \text{bool} \quad \Gamma \vdash e_1 \approx_{\beta\eta} e_1' : \tau \quad \Gamma \vdash e_2 \approx_{\beta\eta} e_2' : \tau}{\Gamma \vdash \text{if } e_0\, e_1\, e_2 \approx_{\beta\eta} \text{if } e_0'\, e_1'\, e_2' : \tau} \qquad \text{(cong-if)}$$

$$\frac{\Gamma, x:\tau \vdash e \approx_{\beta\eta} e' : \tau'}{\Gamma \vdash \lambda x:\tau.\, e \approx_{\beta\eta} \lambda x:\tau.\, e' : \tau \to \tau'} \qquad \text{(cong-abs)}$$

$$\frac{\Gamma \vdash e_1 \approx_{\beta\eta} e_1' : \tau \to \tau' \quad \Gamma \vdash e_2 \approx_{\beta\eta} e_2' : \tau}{\Gamma \vdash e_1 e_2 \approx_{\beta\eta} e_1' e_2' : \tau'} \qquad \text{(cong-app)}$$

$$\Gamma \vdash (\lambda x:\tau.\, e)\, e' \approx_{\beta\eta} e[e'/x] : \tau' \qquad (\beta) \quad \text{if } \Gamma \vdash \lambda x:\tau.\, e : \tau \to \tau'$$

$$\Gamma \vdash \lambda x:\tau.\, (e\, x) \approx_{\beta\eta} e : \tau \to \tau' \qquad (\eta) \quad x \notin FV(e)$$