$\lambda^{1,b,+,0,\bullet}$ # Contexts and Contextual Equivalence.

$C^v ::= [\cdot]^v \mid \lambda x{:}\tau.\, C \mid \mathrm{fold}\ C^v$

$C ::= [\cdot] \mid C^v \mid \mathrm{if}\ C\ e_1\ e_2 \mid \mathrm{if}\ e_0\ C\ e_2 \mid \mathrm{if}\ e_0\ e_1\ C \mid$

$C\ e \mid e\ C \mid \mathrm{fold}\ C \mid \mathrm{unfold}\ C$

$\boxed{\Gamma' \vdash C^v : (\Gamma \triangleright \rho) \rightsquigarrow \tau}$

$$\frac{}{\Gamma' \vdash [\cdot]^v : (\Gamma \triangleright \tau) \rightsquigarrow \tau}$$

$$\frac{\Gamma' \vdash C^v : (\Gamma \triangleright \tau) \rightsquigarrow \tau'[\mu\alpha.\tau'/\alpha]}{\Gamma' \vdash \mathrm{fold}\ C^v : (\Gamma \triangleright \tau) \rightsquigarrow \mu\alpha.\tau'}$$

$$\frac{\Gamma', x{:}\tau_0 \vdash C : (\Gamma, x{:}\tau_0 \triangleright E^{\delta}_{\tau}) \rightsquigarrow E^{\delta_0}_{\tau'}}{\Gamma' \vdash \lambda x{:}\tau_0.\, C : (\Gamma, x{:}\tau_0 \triangleright E^{\delta}_{\tau}) \rightsquigarrow (\tau_0 \to E^{\delta_0}_{\tau'})} \quad \text{where } \delta <: \delta_0$$

$$\boxed{\Gamma' \vdash C : (\Gamma \triangleright \rho) \rightsquigarrow \sigma}$$

$$\frac{}{\Gamma' \vdash [\cdot] : (\Gamma \triangleright \sigma) \rightsquigarrow \sigma}
\qquad
\frac{\Gamma' \vdash_v C_v : (\Gamma \triangleright \rho) \rightsquigarrow \tau}{\Gamma' \vdash_e C_v : (\Gamma \triangleright \rho) \rightsquigarrow E^{0} \tau}$$

$$\frac{\Gamma' \vdash C : (\Gamma \triangleright E^{\delta} \tau) \rightsquigarrow E^{\delta_0} bool \quad \Gamma' \vdash e_1 : E^{\delta_1} \tau' \quad \Gamma' \vdash e_2 : E^{\delta_2} \tau'}{\Gamma' \vdash \text{if } C\ e_1\ e_2 : (\Gamma \triangleright E^{\delta} \tau) \rightsquigarrow E^{\delta_0 \wedge \delta_1 \wedge \delta_2} \tau'}$$

$$\frac{\Gamma' \vdash e_0 : E^{\delta} bool \quad \Gamma' \vdash C : (\Gamma \triangleright E^{\delta} \tau) \rightsquigarrow E^{\delta_1} \tau' \quad \Gamma' \vdash e_2 : E^{\delta_2} \tau'}{\Gamma' \vdash \text{if } e_0\ C\ e_2 : (\Gamma \triangleright E^{\delta} \tau) \rightsquigarrow E^{\delta_0 \wedge \delta_1 \wedge \delta_2} \tau'}$$

$$\frac{\Gamma' \vdash e_0 : E^{\delta_0} bool \quad \Gamma' \vdash e_1 : E^{\delta_1} \tau' \quad \Gamma' \vdash C : (\Gamma \triangleright E^{\delta} \tau) \rightsquigarrow E^{\delta_2} \tau'}{\Gamma' \vdash \text{if } e_0\ e_1\ C : (\Gamma \triangleright E^{\delta} \tau) \rightsquigarrow E^{\delta_0 \wedge \delta_1 \wedge \delta_2} \tau'}$$

$$\frac{\Gamma' \vdash C : (\Gamma \triangleright E^{\delta} \tau) \rightsquigarrow E^{\delta_0} (\tau_1 \to E^{\delta_2} \tau_2) \quad \Gamma' \vdash e : E^{\delta_1} \tau_1}{\Gamma' \vdash C\, e : (\Gamma \triangleright E^{\delta} \tau) \rightsquigarrow E^{\delta_0 \wedge \delta_1 \wedge \delta_2} \tau_2}$$

$$\frac{\Gamma' \vdash e : E^{\delta_0}(\tau_1 \to E^{\delta_2}\tau_2) \qquad \Gamma' \vdash C : (\Gamma \triangleright E^{\delta}\tau) \rightsquigarrow E^{\delta_1}\tau_1}{\Gamma' \vdash e\,C : (\Gamma \triangleright E^{\delta}\tau) \rightsquigarrow E^{\delta_0 \wedge \delta_1 \wedge \delta_2}\tau_2}$$

$$\frac{\Gamma' \vdash C : (\Gamma \triangleright E^{\delta}\tau) \rightsquigarrow E^{\delta_0}\tau'[\mu\alpha.\tau'/\alpha]}{\Gamma' \vdash \text{fold}\, C : (\Gamma \triangleright E^{\delta}\tau) \rightsquigarrow E^{\delta_0}\mu\alpha.\tau'}$$

$$\frac{\Gamma' \vdash C : (\Gamma \triangleright E^{\delta}\tau) \rightsquigarrow E^{\delta_0}\mu\alpha.\tau'}{\Gamma' \vdash \text{unfold}\, C : (\Gamma \triangleright E^{\delta}\tau) \rightsquigarrow E^{\bullet}\tau'[\mu\alpha.\tau'/\alpha]}$$

$$\frac{\Gamma' \vdash C : (\Gamma_1 \triangleright E^{\delta_1}\tau_1) \rightsquigarrow E^{\delta_2}\tau' \qquad \Gamma_1 \vdash C_1 : (\Gamma \triangleright E^{\delta}\tau) \rightsquigarrow E^{\delta_1}\tau_1}{\Gamma' \vdash C[C_1[\cdot]] : (\Gamma \triangleright E^{\delta}\tau) \rightsquigarrow E^{\delta_2}\tau'}$$

where $\delta <: \delta_1$ & $\delta_1 <: \delta_2$

$$\boxed{\Gamma' \vdash C[e] : \sigma}$$

$$\frac{\Gamma' \vdash C : (\Gamma \triangleright \rho) \rightsquigarrow \sigma \qquad \Gamma \vdash e : \rho}{\Gamma' \vdash C[e] : \sigma}$$

$$\boxed{\Gamma' \vdash C^{v}[v]^{v} : \tau}$$

$$\frac{\Gamma' \vdash C^{v} : (\Gamma \triangleright \rho) \rightsquigarrow \tau \qquad \Gamma \vdash e : \rho}{\Gamma' \vdash C^{v}[e]^{v} : \tau}$$