1 Notations

What is given by the user is:
$\boldsymbol{l_{y_{out}}}$, the least significant bits desired for all outputs y

$\boldsymbol{l_u}$, the least significant bits desired for all inputs u

What we want is the error $\boldsymbol{e_y}$ introduced by the whole filter, given all intermediates errors, in order to compute back those errors:

$$\boldsymbol{e_y} < \frac{1}{2}\boldsymbol{l_{y_{out}}}$$

Intermediate errors are defined as follows:
$\boldsymbol{\varepsilon_{y^*}}$, the error introduced computing y from t, x, and u

$\boldsymbol{\varepsilon_x}$, the error introduced computing x from t, x, and u

$\boldsymbol{\varepsilon_t}$, the error introduced computing t from t, x, and u

What we search are:
$\boldsymbol{l_t}$, the least significant bits desired for all intermediate variables t

$\boldsymbol{l_x}$, the least significant bits desired for all state variables x

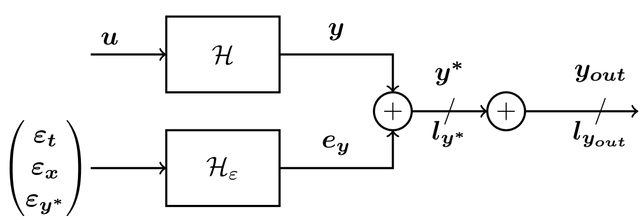

Figure 1: A signal view of the error propagation with respect to the ideal filter

Further definitions:

$$\boldsymbol{l_{y^*}} = log_2\lfloor\boldsymbol{\varepsilon_{y^*}}\rfloor$$
$$\boldsymbol{l_x} = log_2\lfloor\boldsymbol{\varepsilon_x}\rfloor$$
$$\boldsymbol{l_t} = log_2\lfloor\boldsymbol{\varepsilon_t}\rfloor$$

2 Error Analysis

What Lopez claims:

$$e_y = \|\mathcal{H}_\varepsilon\| \cdot \begin{pmatrix} \boldsymbol{\varepsilon_t} \\ \boldsymbol{\varepsilon_x} \\ \boldsymbol{\varepsilon_{y^*}} \end{pmatrix}$$

Resuming, we want:

$$\boldsymbol{e_y} < \frac{1}{2} l_{y_{out}}$$

$$\Leftrightarrow$$

$$\frac{1}{2} l_{y_{out}} > \|\mathcal{H}_\varepsilon\| \cdot \begin{pmatrix} \boldsymbol{\varepsilon_t} \\ \boldsymbol{\varepsilon_x} \\ \boldsymbol{\varepsilon_{y^*}} \end{pmatrix}$$

$$\Leftrightarrow$$

$$\frac{1}{2} l_{y_{out}\, i} > \sum_{j=1}^{n} \|\mathcal{H}_\varepsilon\|_{i,j} \cdot \boldsymbol{\varepsilon}_j, \quad \forall 1 \leq i \leq n_y$$

Idea: find the solution using PNL, trying to maximize:

$$\sum \boldsymbol{\varepsilon}_i$$