

Bilateral Filters

Digital Visual Effects

Yung-Yu Chuang

with slides by Fredo Durand, Ramesh Raskar, Sylvain Paris, Soonmin Bae

Bilateral filtering



[Ben Weiss, Siggraph 2006]

Image Denoising

DigiVFX



noisy image



naïve denoising
Gaussian blur



better denoising
edge-preserving filter

Smoothing an image without blurring its edges.

A Wide Range of Options

DigiVFX

- Diffusion, Bayesian, Wavelets...

- All have their pros and cons.

- Bilateral filter

- not always the best result [Buades 05] but often good
 - easy to understand, adapt and set up

Basic denoising

DigiVFX

Noisy input



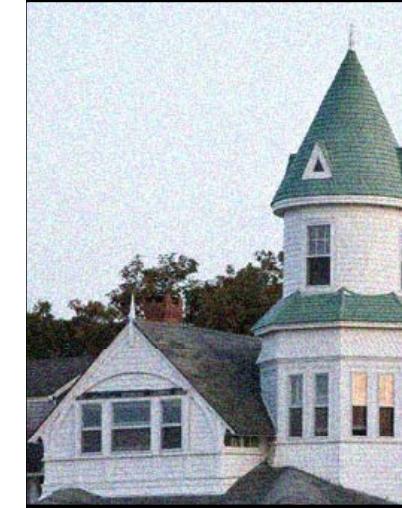
Median 5x5



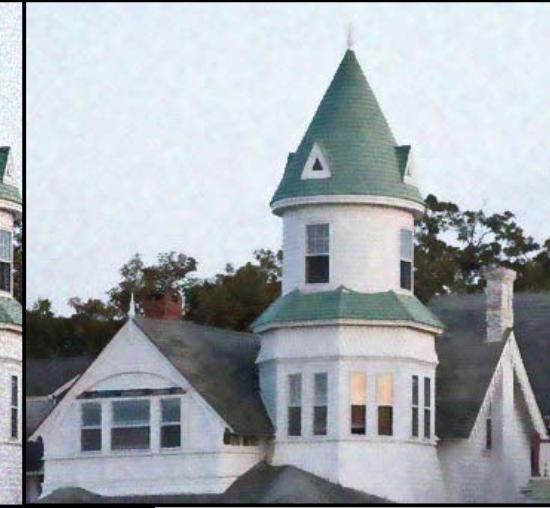
Basic denoising

DigiVFX

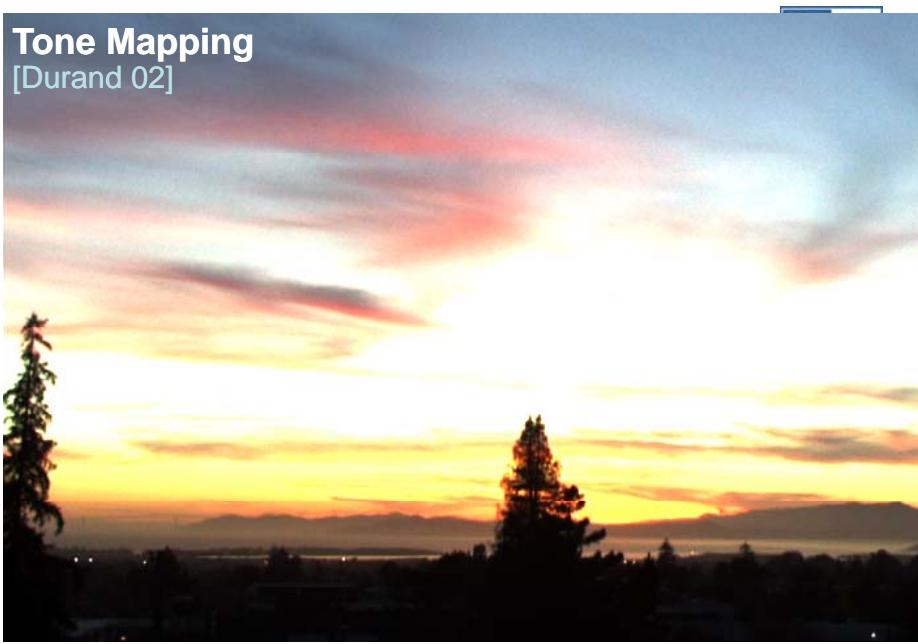
Noisy input



Bilateral filter 7x7 window

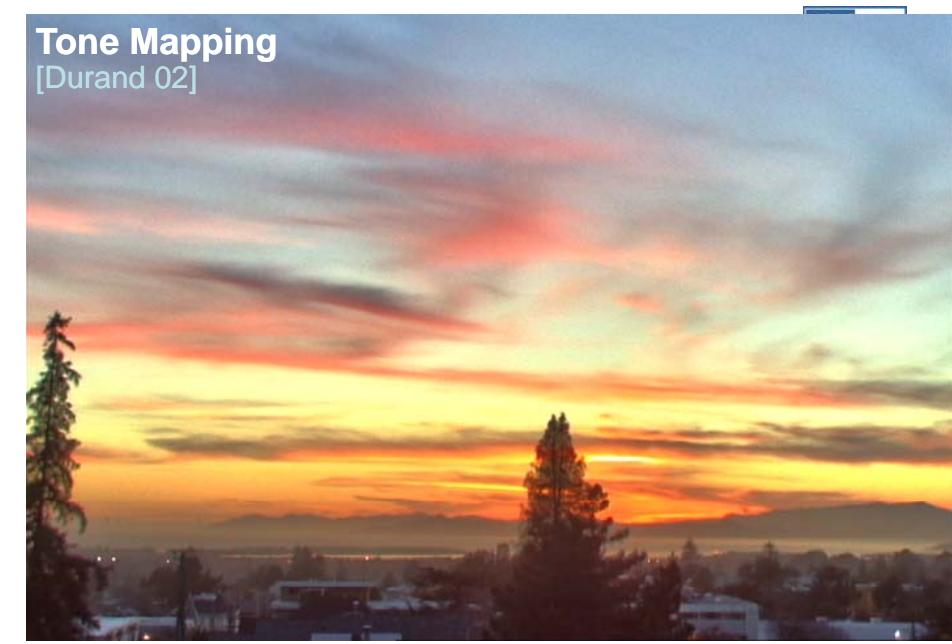


Tone Mapping [Durand 02]



HDR input

Tone Mapping [Durand 02]



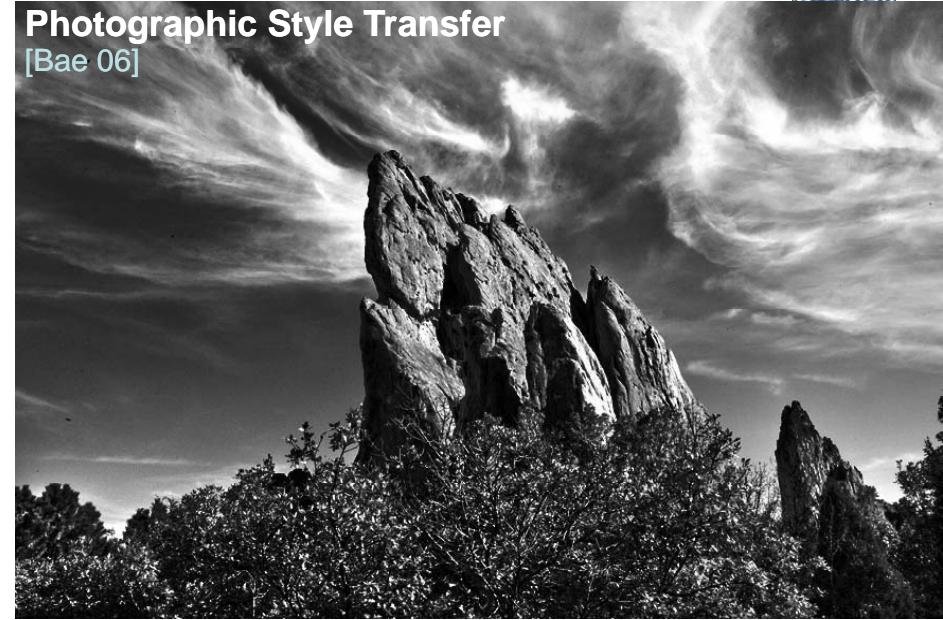
output

Photographic Style Transfer
[Bae 06]



input

Photographic Style Transfer
[Bae 06]



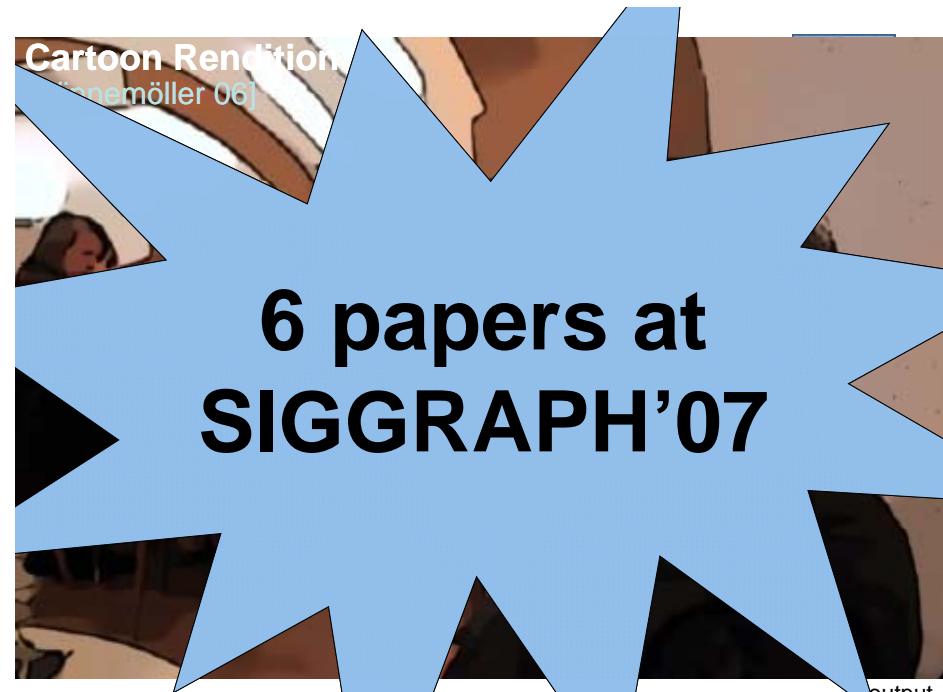
output

Cartoon Rendition
[Winnemöller 06]



input

Cartoon Rendition
[Winnemöller 06]

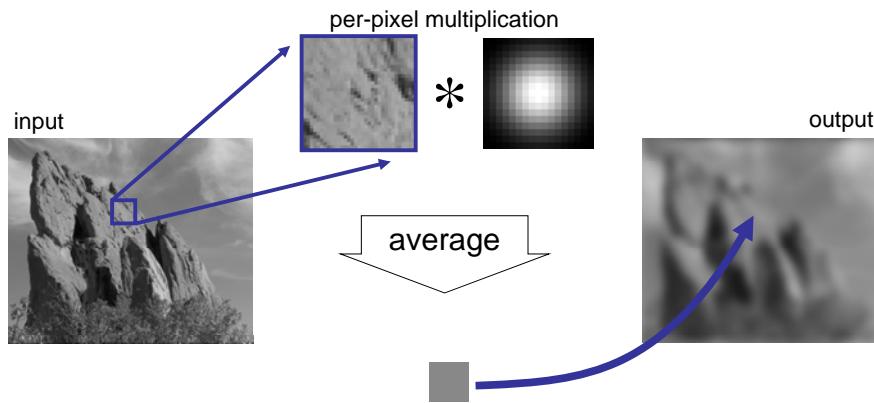


**6 papers at
SIGGRAPH'07**

output

Gaussian Blur

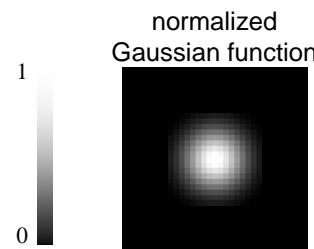
DigiVFX



Equation of Gaussian Blur

Same idea: **weighted average of pixels.**

$$GB[I]_p = \sum_{q \in S} G_\sigma(\|p-q\|) I_q$$

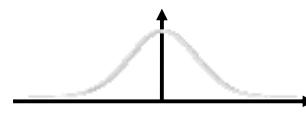
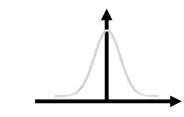


Spatial Parameter



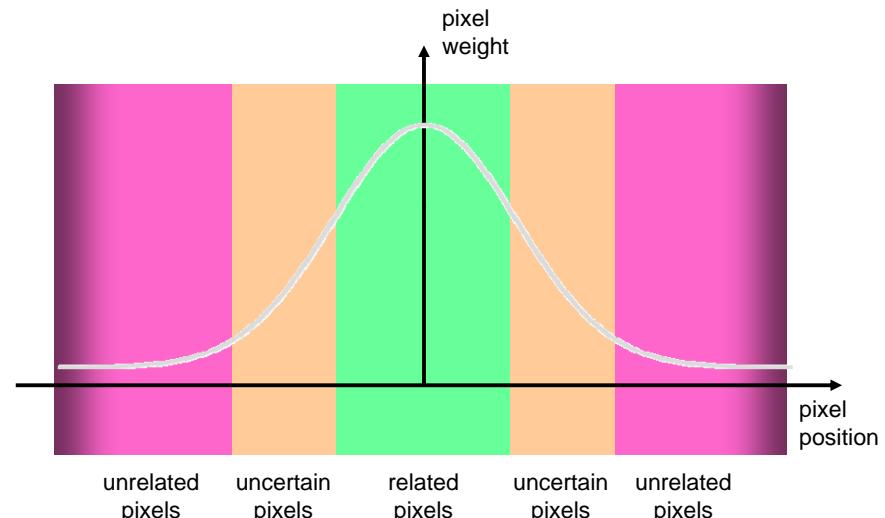
$$GB[I]_p = \sum_{q \in S} G_\sigma(\|p-q\|) I_q$$

σ size of the window



Gaussian Profile

$$G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$



How to set σ

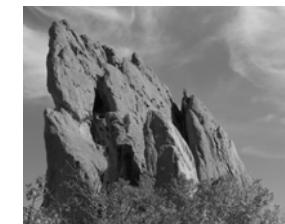
- Depends on the application.
- Common strategy: proportional to image size
 - e.g. 2% of the image diagonal
 - property: independent of image resolution

Properties of Gaussian Blur

- Weights independent of spatial location
 - linear convolution
 - well-known operation
 - efficient computation (recursive algorithm, FFT...)

Properties of Gaussian Blur

- Does smooth images
- But smoothes too much:
edges are blurred.
 - Only spatial distance matters
 - No edge term

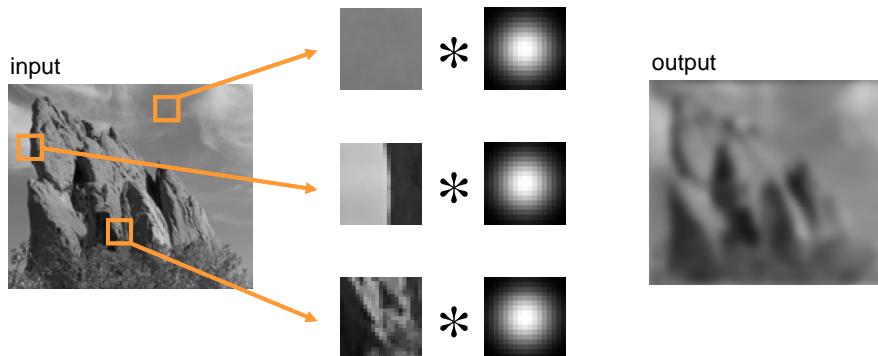


↓
output

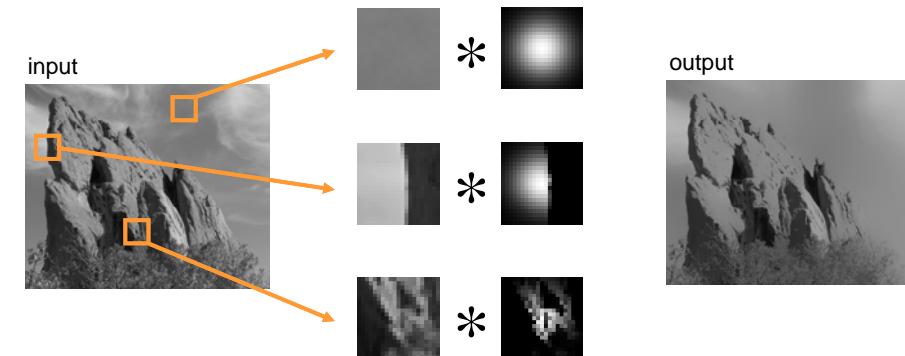


$$GB[I]_p = \sum_{q \in S} G_\sigma(\|p - q\|) I_q$$

Blur Comes from Averaging across Edges



Bilateral Filter No Averaging across Edges



Bilateral Filter Definition

Same idea: weighted average of pixels.

$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_s}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|) I_q$$

new
not new
new

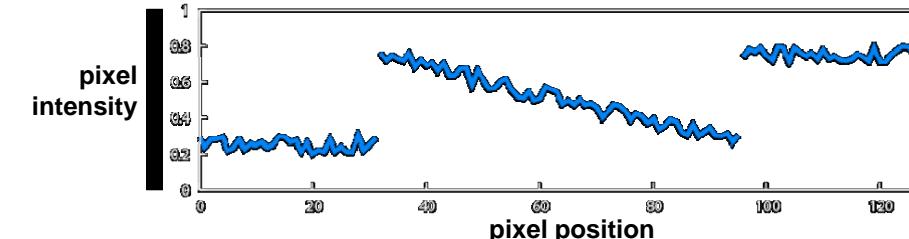
normalization factor
space weight
range weight

Illustration a 1D Image

- 1D image = line of pixels

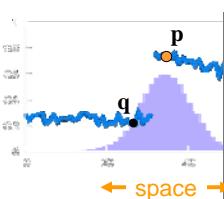


- Better visualized as a plot



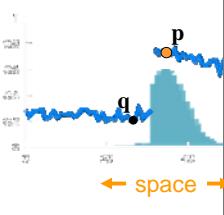
Gaussian Blur and Bilateral Filter

Gaussian blur



Bilateral filter

[Aurich 95, Smith 97, Tomasi 98]



$$GB[I]_p = \sum_{q \in S} G_\sigma(\|p - q\|) I_q$$

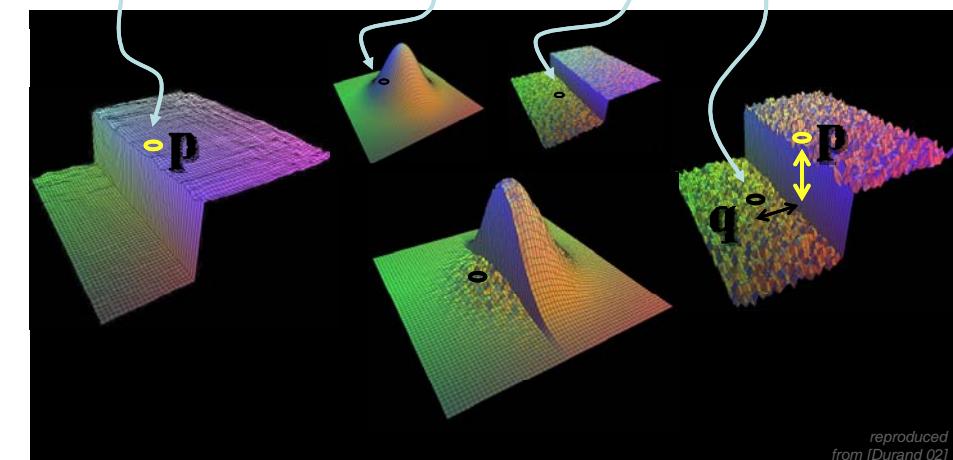
space

$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_s}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|) I_q$$

normalization

Bilateral Filter on a Height Field

$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_s}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|) I_q$$



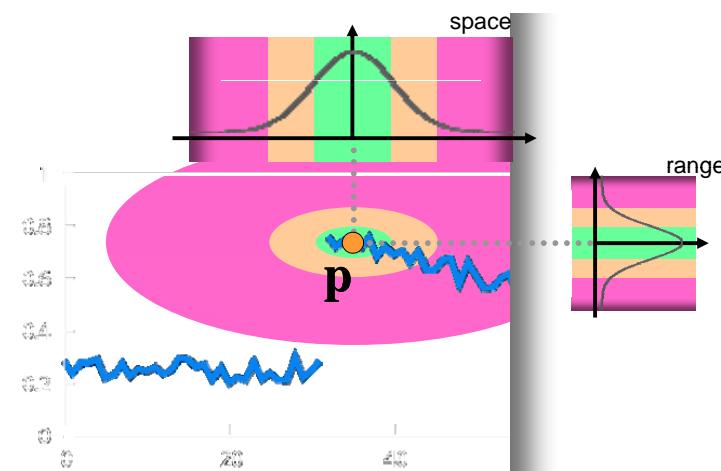
Space and Range Parameters

$$BF [I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_s}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|) I_q$$

- space σ_s : spatial extent of the kernel, size of the considered neighborhood.
- range σ_r : “minimum” amplitude of an edge

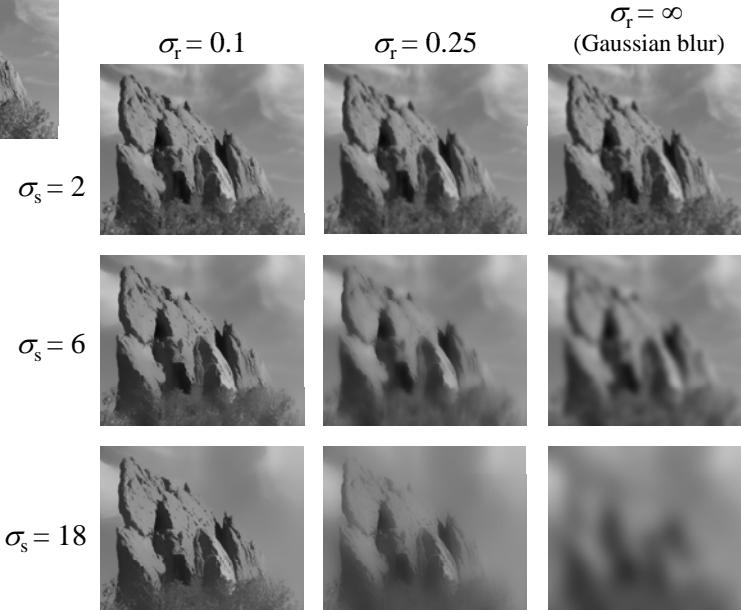
Influence of Pixels

Only pixels close in space and in range are considered.



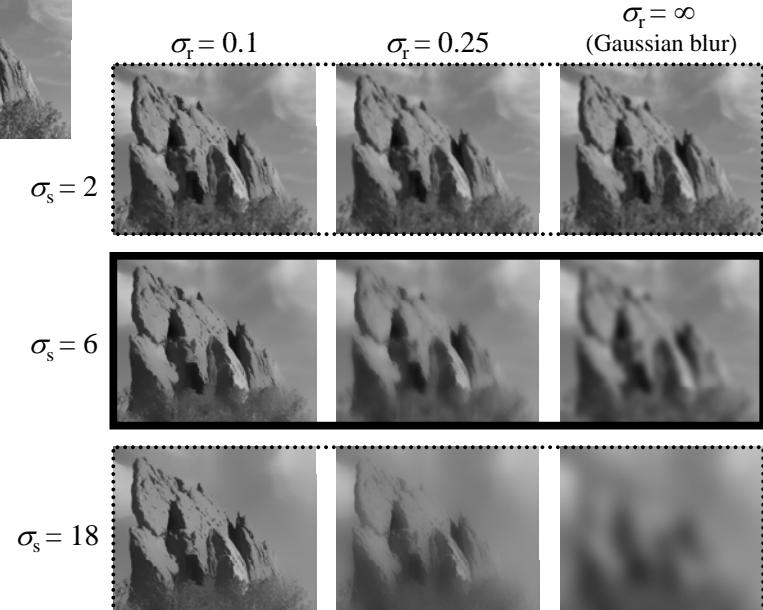
input

Exploring the Parameter Space



input

Varying the Range Parameter

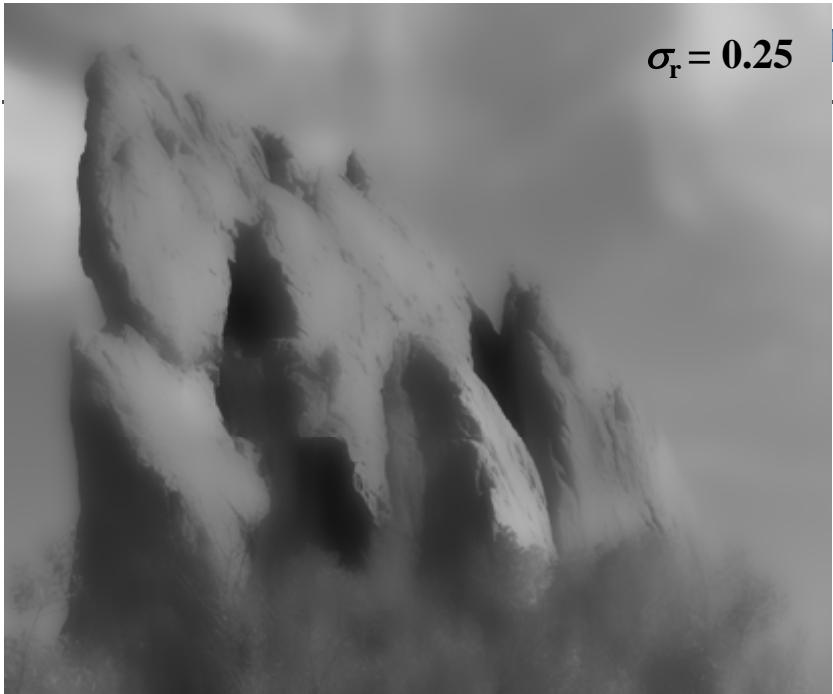




input



$\sigma_r = 0.1$



$\sigma_r = 0.25$



$\sigma_r = \infty$
(Gaussian blur)



input

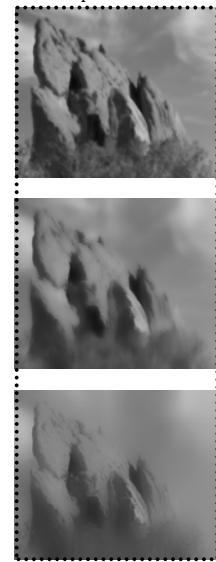
Varying the Space Parameter

$\sigma_r = 0.1$



$\sigma_s = 2$

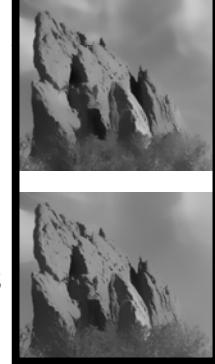
$\sigma_r = 0.25$



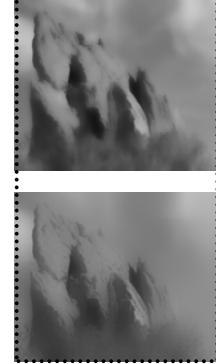
$\sigma_r = \infty$
(Gaussian blur)



$\sigma_s = 6$



$\sigma_s = 18$



input



$\sigma_s = 2$



$\sigma_s = 6$



$\sigma_s = 18$

Iterating the Bilateral Filter

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$$I_{(n+1)} = BF[I_{(n)}]$$

- Generate more piecewise-flat images
- Often not needed in computational photo, but could be useful for applications such as NPR.

How to Set the Parameters

DigiVFX

Depends on the application. For instance:

- space parameter: proportional to image size
 - e.g., 2% of image diagonal
- range parameter: proportional to edge amplitude
 - e.g., mean or median of image gradients
- independent of resolution and exposure

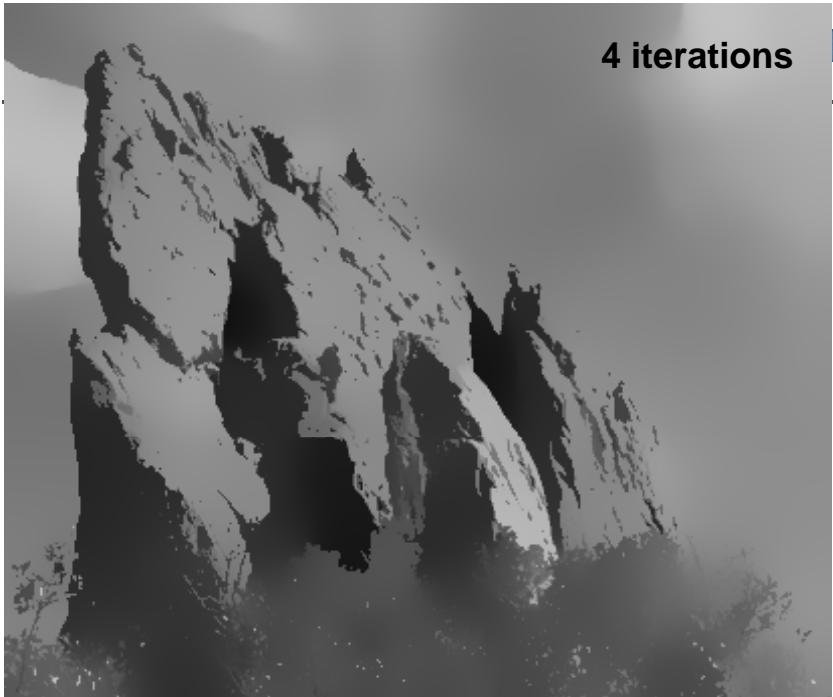




1 iteration



2 iterations



4 iterations

Advantages of Bilateral Filter

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- Easy to understand
 - Weighted mean of nearby pixels
- Easy to adapt
 - Distance between pixel values
- Easy to set up
 - Non-iterative

Hard to Compute

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- Nonlinear $BF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_p - I_q|) I_q$
- Complex, spatially varying kernels
 - Cannot be precomputed, no FFT...



- Brute-force implementation is slow > 10min

A Fast Approximation of the Bilateral Filter using a Signal Processing Approach

Sylvain Paris and Frédéric Durand

Computer Science and Artificial Intelligence Laboratory
Massachusetts Institute of Technology



But Bilateral Filter is Nonlinear

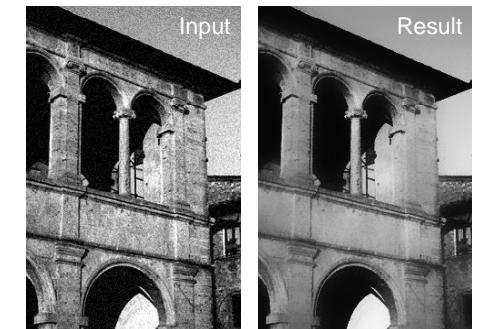
DigiVFX

- Slow but some accelerations exist:
 - [Elad 02]: Gauss-Seidel iterations
 - Only for many iterations
 - [Durand 02, Weiss 06]: fast approximation
 - No formal understanding of accuracy versus speed
 - [Weiss 06]: Only box function as spatial kernel

Definition of Bilateral Filter

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- [Smith 97, Tomasi 98]
- Smoothes an image and preserves edges
- Weighted average of neighbors
- Weights
 - Gaussian on *space* distance
 - Gaussian on *range* distance
 - sum to 1



$$I_p^{bf} = \frac{1}{W_p^{bf}} \sum_{q \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) \underset{\text{space}}{G_{\sigma_r}(|I_p - I_q|)} I_q \underset{\text{range}}{}$$

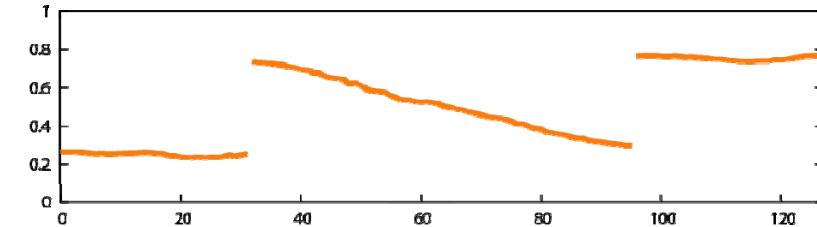
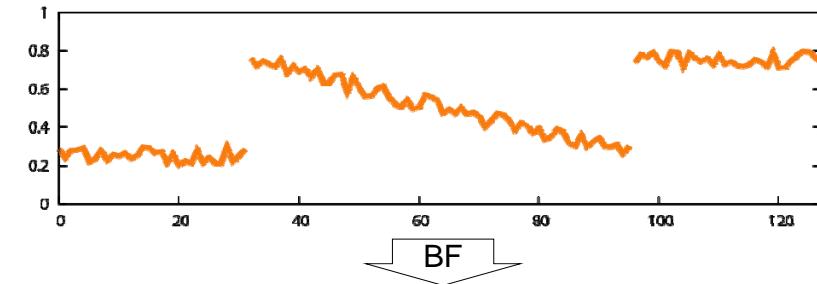
Contributions

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- Link with linear filtering
- Fast and accurate approximation

Intuition on 1D Signal

DigiVFX



Basic idea

DigiVFX

1D Gaussians

$$BF [I]_p = \frac{1}{W_p} \sum_{q \in S} G(q; p, \sigma_s) G(I_q; I_p, \sigma_r) I_q$$

Basic idea

DigiVFX

1D Gaussians

$$BF [I]_p = \frac{1}{W_p} \sum_{q \in S} G(q; p, \sigma_s) G(I_q; I_p, \sigma_r) I_q$$

2D Gaussians

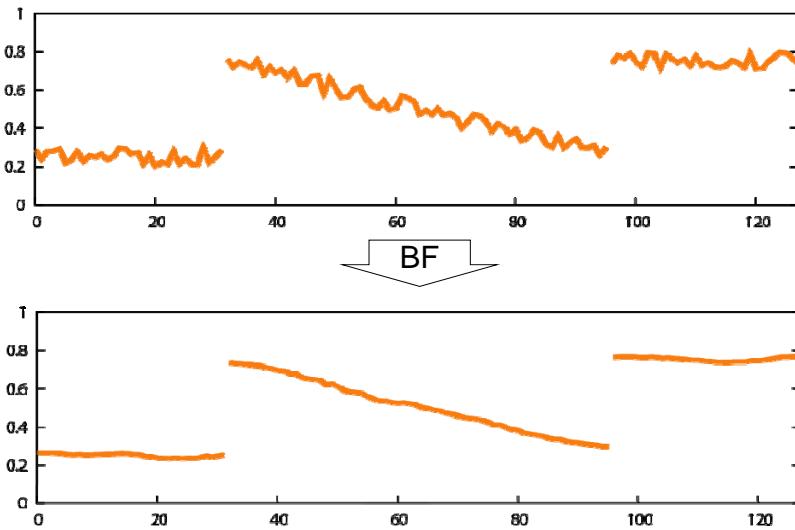
$$BF [I]_p = \frac{1}{W_p} \sum_{\langle q, I_q \rangle \in S'} G(q, I_q; p, I_p, \sigma_s, \sigma_r) I_{\langle q, I_q \rangle}$$

a special
2D image



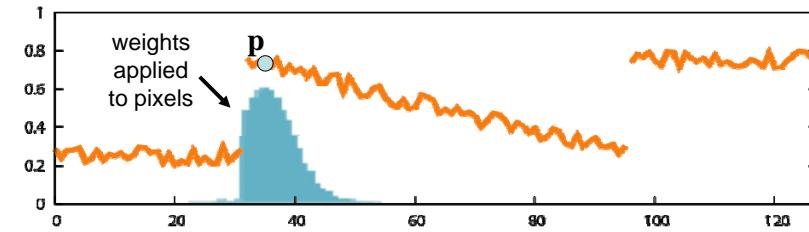
Intuition on 1D Signal

DigiVFX



Intuition on 1D Signal Weighted Average of Neighbors

DigiVFX

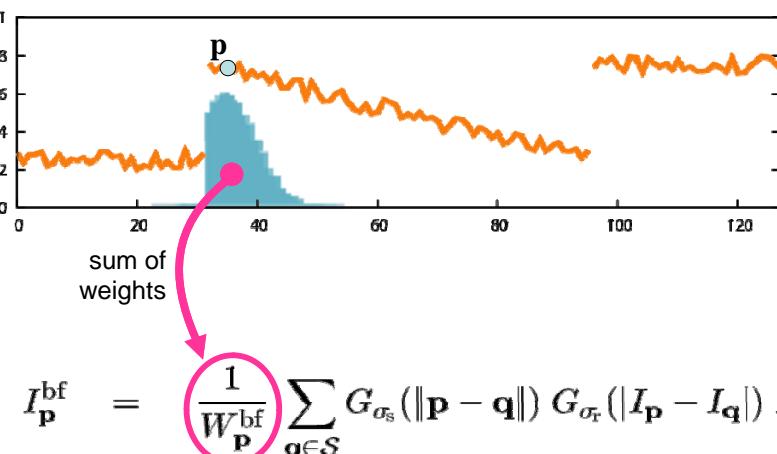


- Near and similar pixels have influence.
- Far pixels have no influence.
- Pixels with different value have no influence.

Link with Linear Filtering

1. Handling the Division

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Handling the division with a projective space.

Formalization: Handling the Division

DigiVFX

$$\begin{aligned} I_p^{\text{bf}} &= \frac{1}{W_p^{\text{bf}}} \sum_{q \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_p - I_q|) I_q \\ W_p^{\text{bf}} &= \sum_{q \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_p - I_q|) \end{aligned}$$

- Normalizing factor as homogeneous coordinate
 - Multiply both sides by W_p^{bf}

$$\begin{pmatrix} W_p^{\text{bf}} & I_p^{\text{bf}} \\ W_p^{\text{bf}} & \end{pmatrix} = \sum_{q \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_p - I_q|) \begin{pmatrix} I_q \\ 1 \end{pmatrix}$$

Formalization: Handling the Division

DigiVFX

$$\begin{pmatrix} W_p^{bf} I_p^{bf} \\ W_p^{bf} \end{pmatrix} = \sum_{q \in S} G_{\sigma_s}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|) \begin{pmatrix} W_q I_q \\ W_q \end{pmatrix} \text{ with } W_q = 1$$

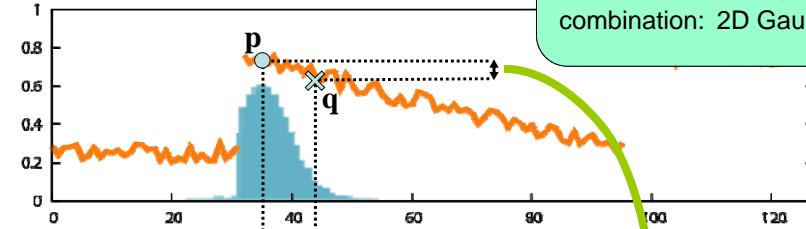
- Similar to homogeneous coordinates in projective space
- Division delayed until the end
- Next step: Adding a dimension to make a convolution appear

Link with Linear Filtering

2. Introducing a Convolution

space: 1D Gaussian
 \times range: 1D Gaussian

combination: 2D Gaussian



$$\begin{pmatrix} W_p^{bf} I_p^{bf} \\ W_p^{bf} \end{pmatrix} = \sum_{q \in S} \underbrace{G_{\sigma_s}(\|p - q\|)}_{\text{space}} \underbrace{G_{\sigma_r}(|I_p - I_q|)}_{\text{range}} \begin{pmatrix} W_q I_q \\ W_q \end{pmatrix}$$

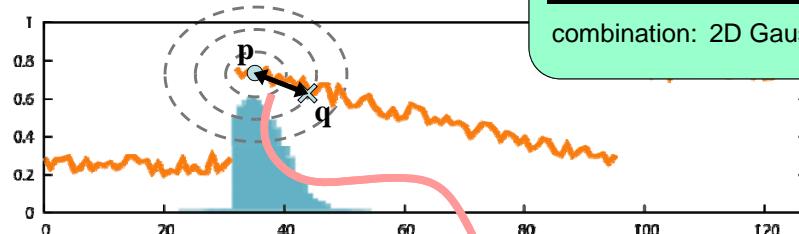
Link with Linear Filtering

2. Introducing a Convolution

DigiVFX

space: 1D Gaussian
 \times range: 1D Gaussian

combination: 2D Gaussian

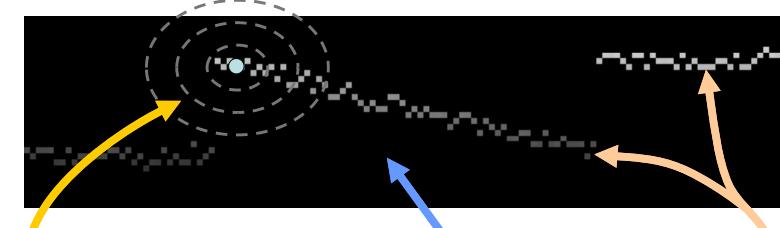


$$\begin{pmatrix} W_p^{bf} I_p^{bf} \\ W_p^{bf} \end{pmatrix} = \sum_{q \in S} \underbrace{G_{\sigma_s}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|)}_{\text{space} \times \text{range}} \begin{pmatrix} W_q I_q \\ W_q \end{pmatrix}$$

Corresponds to a 3D Gaussian on a 2D image.

Link with Linear Filtering

2. Introducing a Convolution

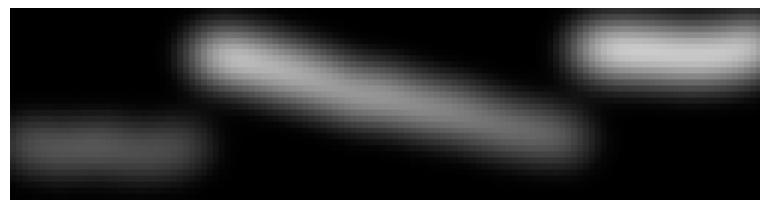


$$\begin{pmatrix} W_p^{bf} I_p^{bf} \\ W_p^{bf} \end{pmatrix} = \boxed{\sum_{(q,\zeta) \in S \times R} \underbrace{\quad}_{\text{space-range Gaussian}} \begin{pmatrix} W_q I_q \\ W_q \end{pmatrix}}$$

sum all values multiplied by kernel \Rightarrow convolution

Link with Linear Filtering 2. Introducing a Convolution

DigiVFX

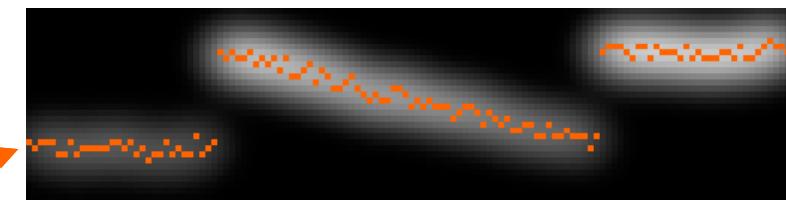


result of the convolution

$$\begin{pmatrix} W_p^{bf} & I_p^{bf} \\ W_p^{bf} \end{pmatrix} = \sum_{(\mathbf{q}, \zeta) \in \mathcal{S} \times \mathcal{R}} \text{space-range Gaussian} \begin{pmatrix} W_q & I_q \\ W_q \end{pmatrix}$$

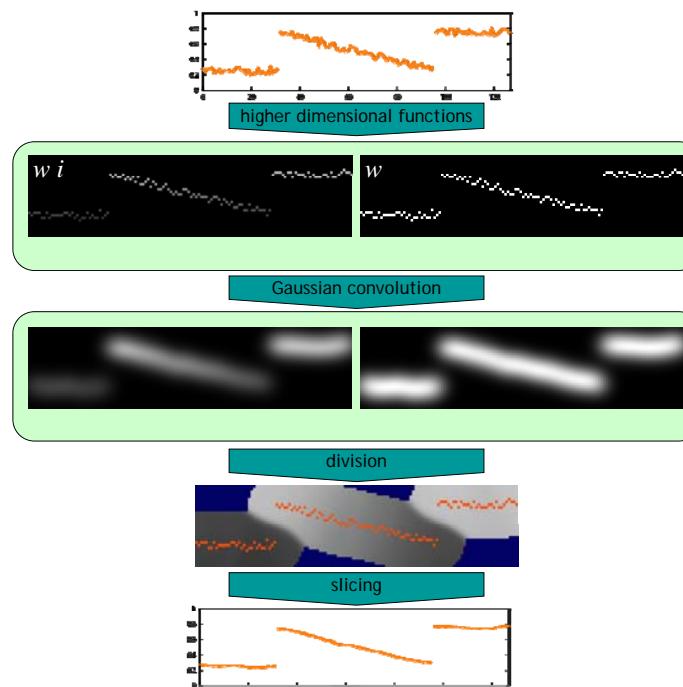
Link with Linear Filtering 2. Introducing a Convolution

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result of the convolution

$$\begin{pmatrix} W_p^{bf} & I_p^{bf} \\ W_p^{bf} \end{pmatrix} = \sum_{(\mathbf{q}, \zeta) \in \mathcal{S} \times \mathcal{R}} \text{space-range Gaussian} \begin{pmatrix} W_q & I_q \\ W_q \end{pmatrix}$$



Reformulation: Summary

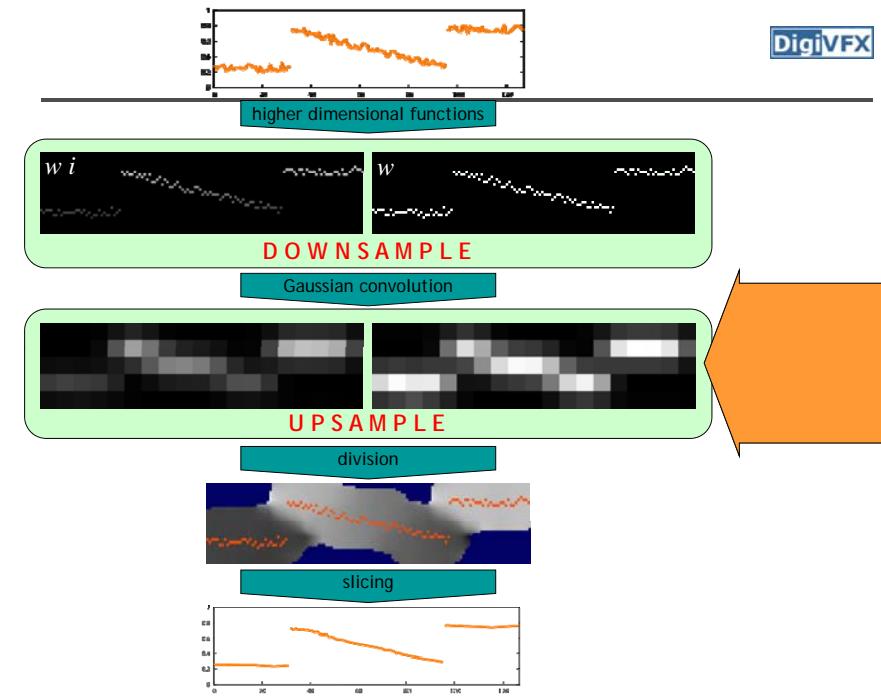
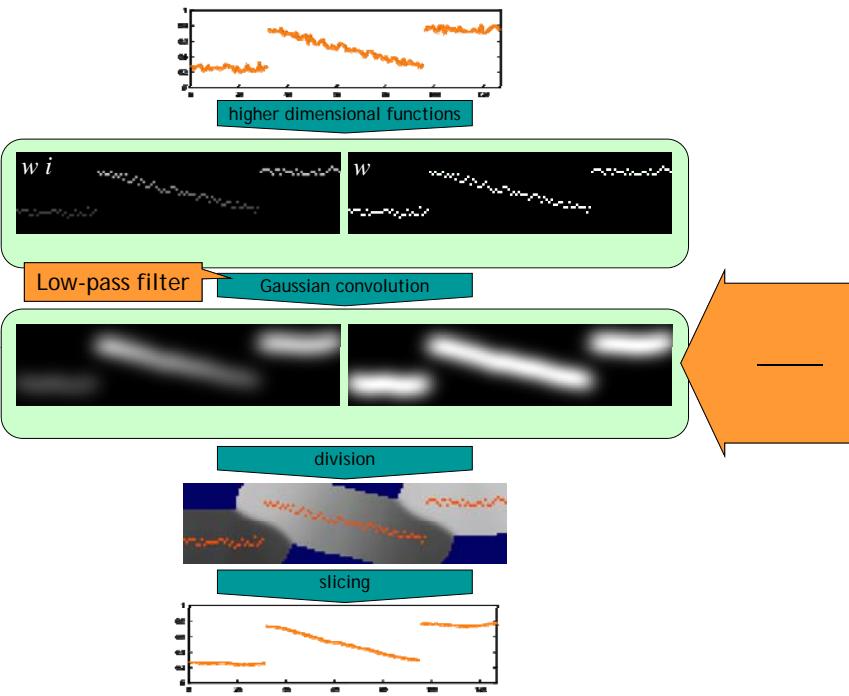
DigiVFX

linear: $(w^{bf}, i^{bf}, w^{bf}) = g_{\sigma_s, \sigma_r} \otimes (wi, w)$

nonlinear: $I_p^{bf} = \frac{w^{bf}(\mathbf{p}, I_p) i^{bf}(\mathbf{p}, I_p)}{w^{bf}(\mathbf{p}, I_p)}$

1. Convolution in higher dimension
 - expensive but well understood (linear, FFT, etc)
2. Division and slicing
 - nonlinear but simple and pixel-wise

Exact reformulation



Fast Convolution by Downsampling

- Downsampling cuts frequencies above Nyquist limit
 - Less data to process
 - But induces error
- Evaluation of the approximation
 - Precision versus running time
 - Visual accuracy

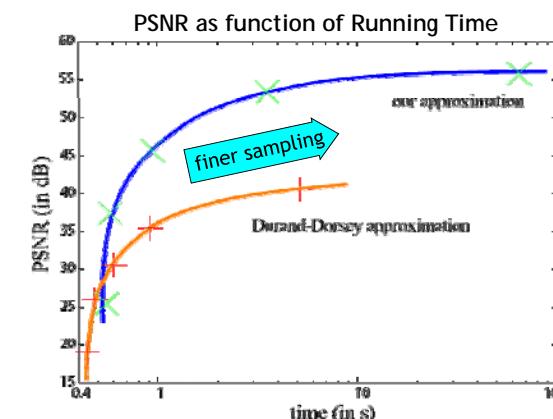
DigiVFX

Accuracy versus Running Time

- Finer sampling increases accuracy.
- More precise than previous work.



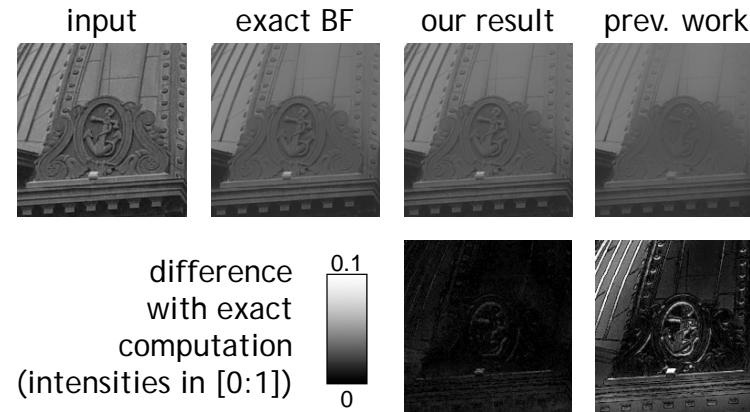
Digital photograph
1200 × 1600



Straightforward implementation is over 10 minutes.

Visual Results

- Comparison with previous work [Durand 02]
 - running time = 1s for both techniques



Conclusions

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higher dimension \Rightarrow “better” computation

Practical gain

- Interactive running time
- Visually similar results
- Simple to code (100 lines)

Theoretical gain

- Link with linear filters
- Separation linear/nonlinear
- Signal processing framework

DigiVFX

DigiVFX

Two-scale Tone Management for Photographic Look

Soonmin Bae, Sylvain Paris, and Frédo Durand

MIT CSAIL

SIGGRAPH2006

Ansel Adams



Ansel Adams, *Clearing Winter Storm*

An Amateur Photographer

DigiVFX



Goals

DigiVFX

- Control over photographic look
- Transfer “look” from a model photo

For example,

we want

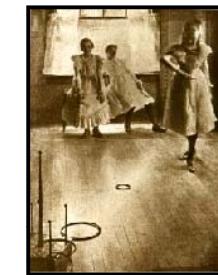


with the look of



A Variety of Looks

DigiVFX



Aspects of Photographic Look

DigiVFX

- Subject choice
- Framing and composition
 - ➔ Specified by input photos



Input

- Tone distribution and contrast
 - ➔ Modified based on model photos



Model

Tonal Aspects of Look

DigiVFX



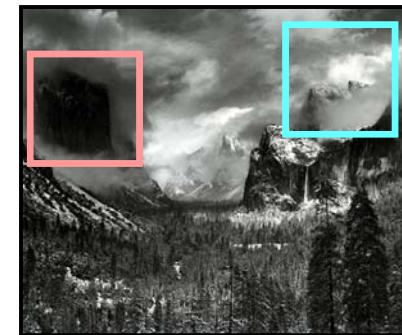
Ansel Adams



Kenro Izu

Tonal aspects of Look - Global Contrast

DigiVFX



Ansel Adams



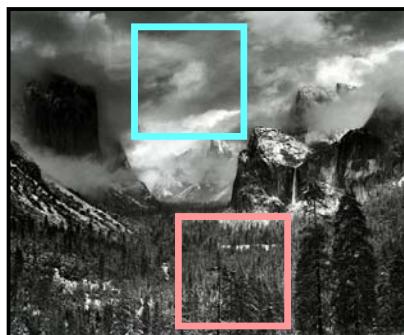
Kenro Izu

High Global Contrast

Low Global Contrast

Tonal aspects of Look - Local Contrast

DigiVFX



Ansel Adams



Kenro Izu

Variable amount of texture

Texture everywhere

Overview

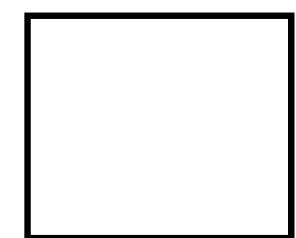
DigiVFX



Input Image



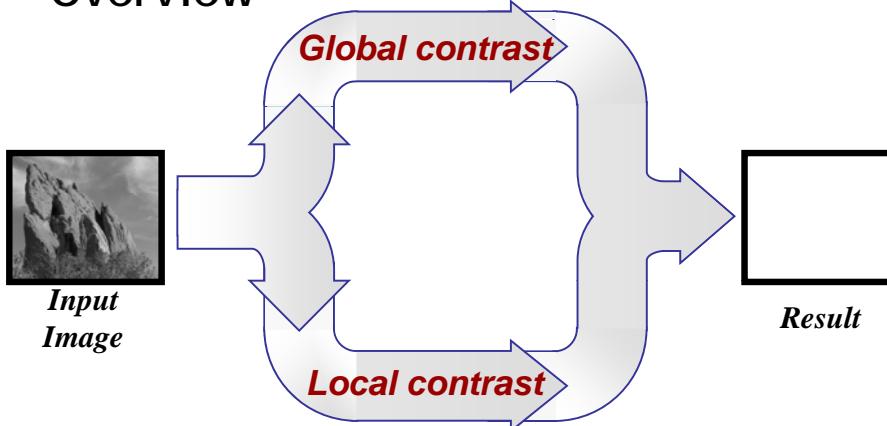
Model



Result

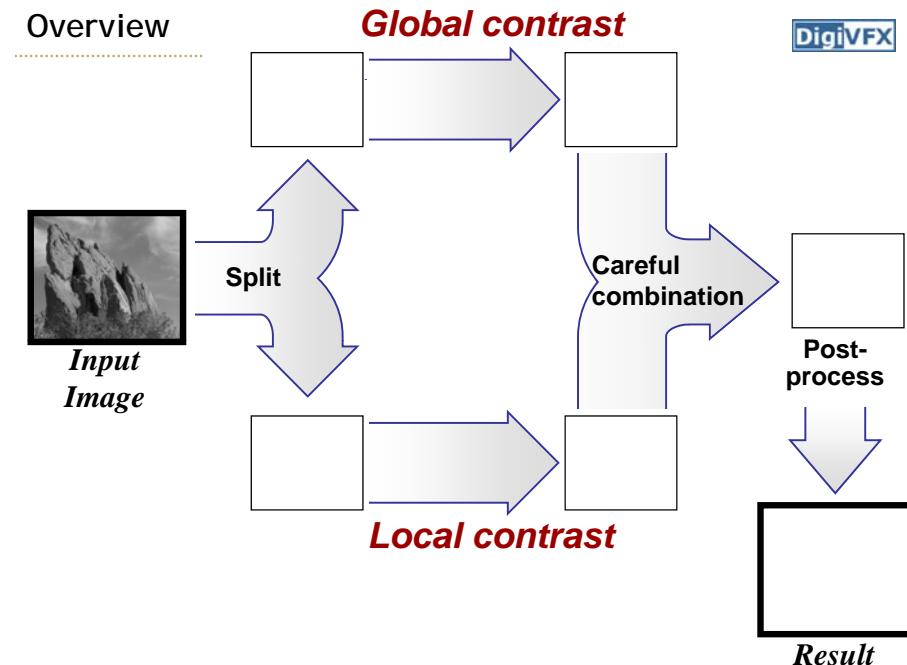
- Transfer look between photographs
 - Tonal aspects

Overview

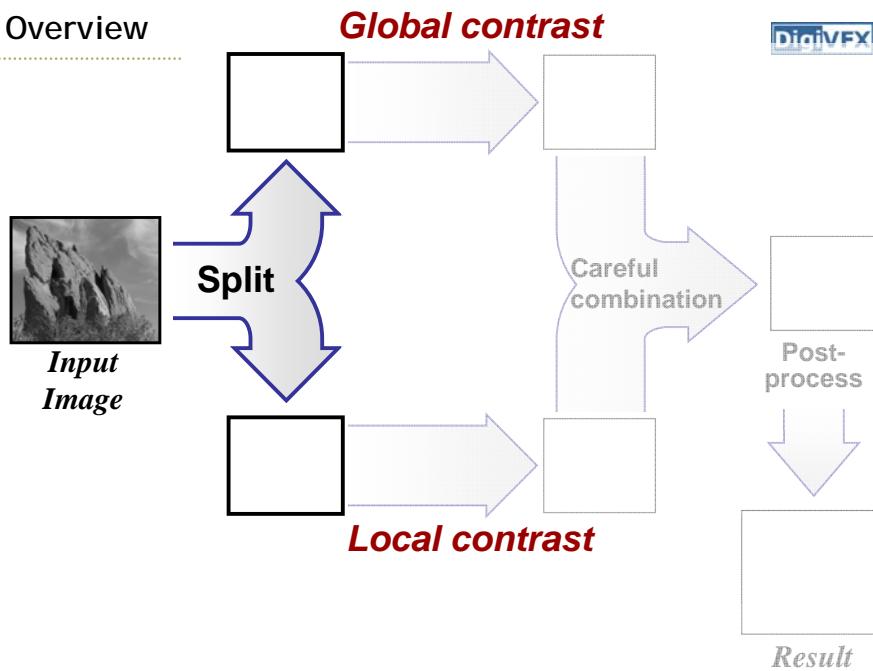


- Separate global and local contrast

Overview

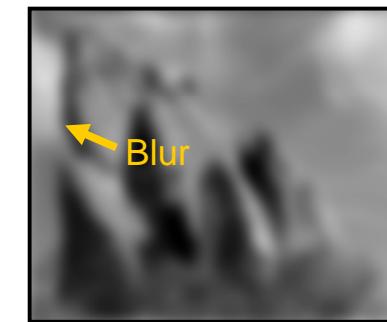


Overview

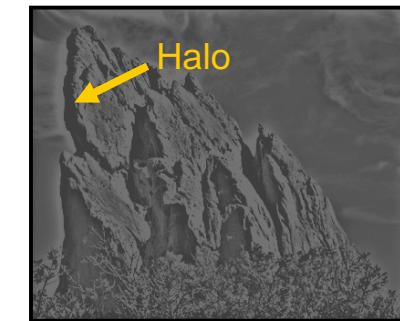


Split Global vs. Local Contrast

- Naïve decomposition: low vs. high frequency
 - Problem: introduce blur & halos



Low frequency
Global contrast



High frequency
Local contrast

Bilateral Filter

- Edge-preserving smoothing [Tomasi 98]
- We build upon tone mapping [Durand 02]



After bilateral filtering
Global contrast



Residual after filtering
Local contrast

Bilateral Filter

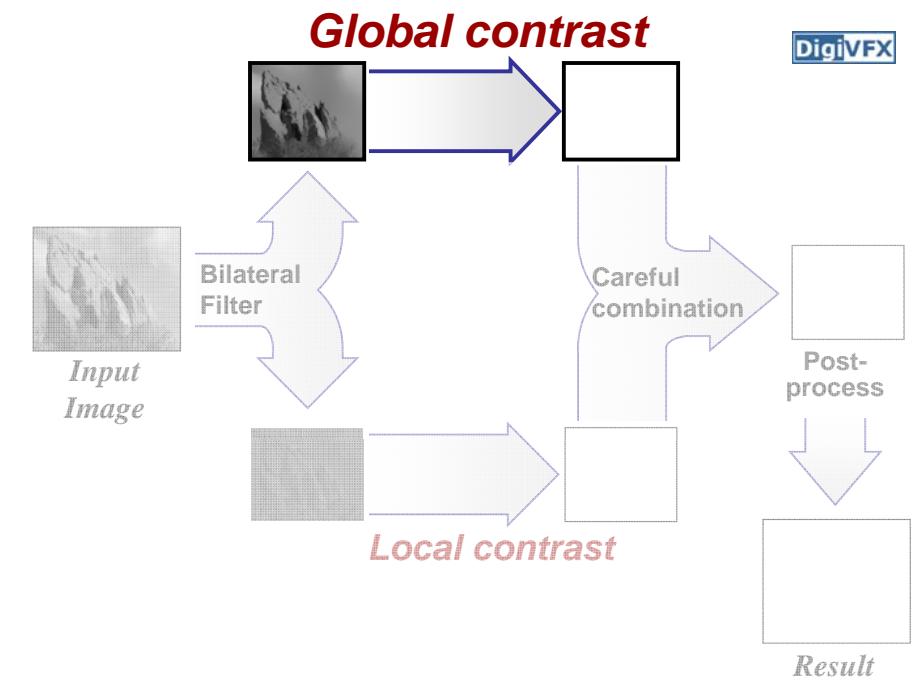
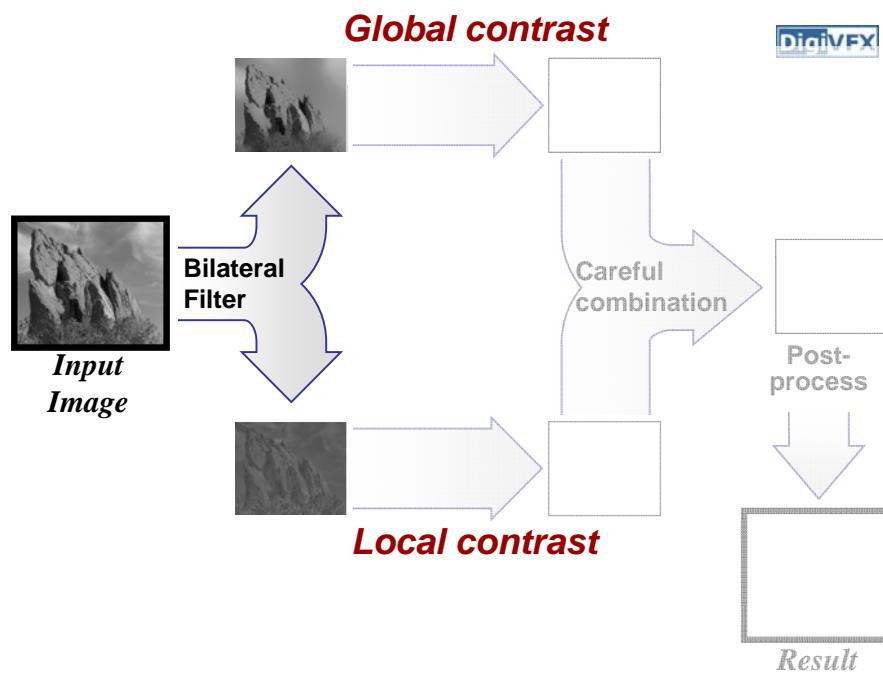
- Edge-preserving smoothing [Tomasi 98]
- We build upon tone mapping [Durand 02]



After bilateral filtering
Global contrast



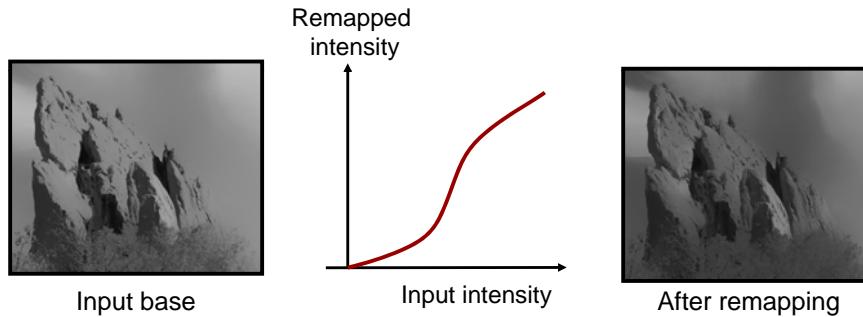
Residual after filtering
Local contrast



Global Contrast

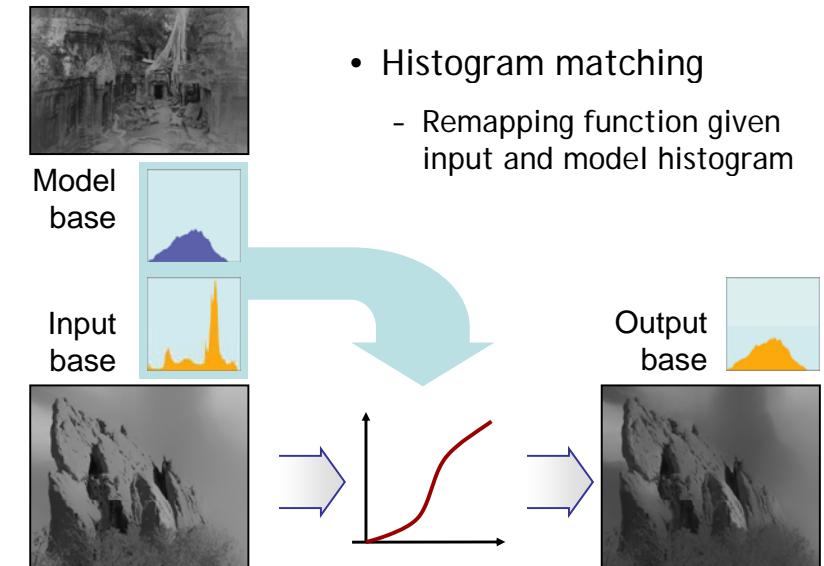
DigiVFX

- Intensity remapping of base layer



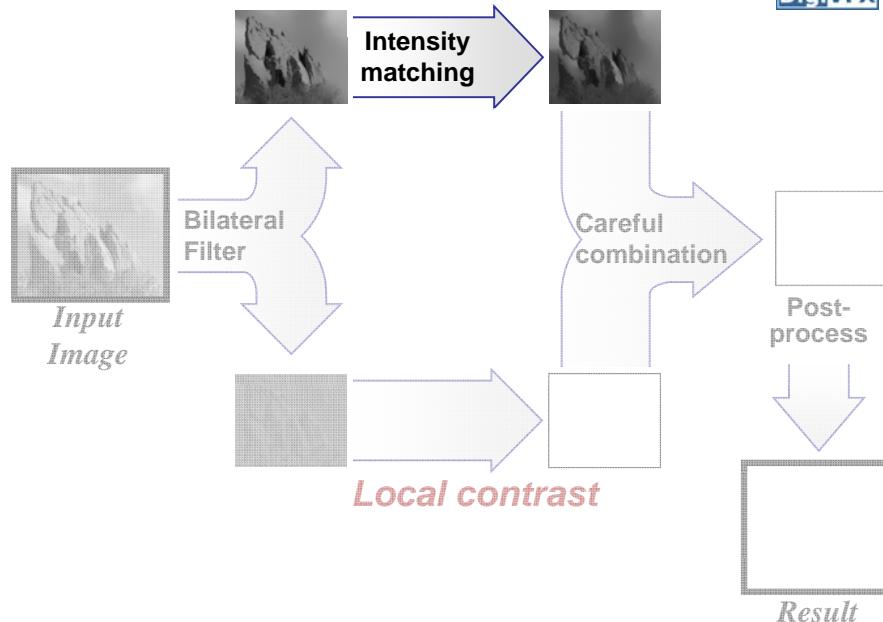
Global Contrast (Model Transfer)

DigiVFX



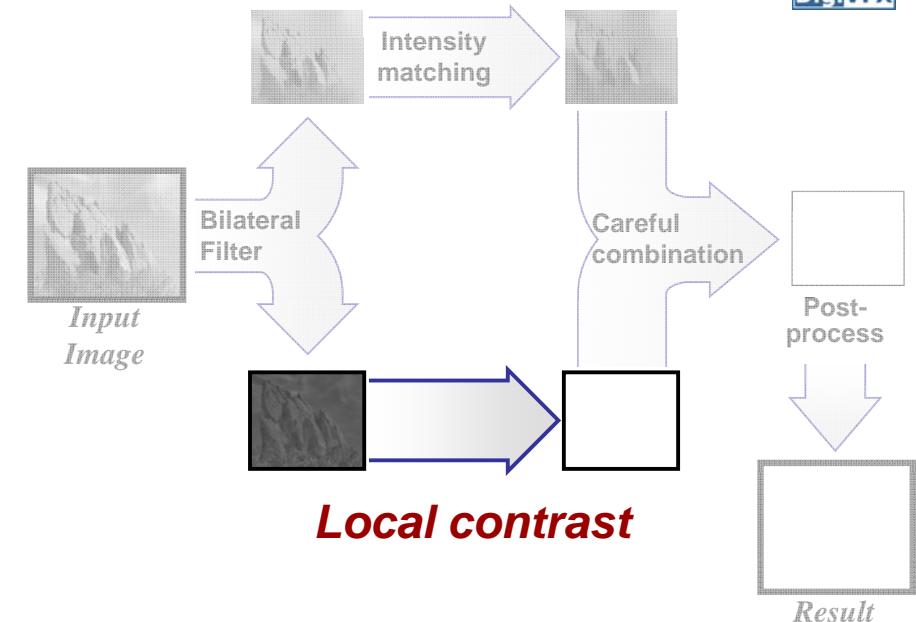
Global contrast

DigiVFX



Global contrast

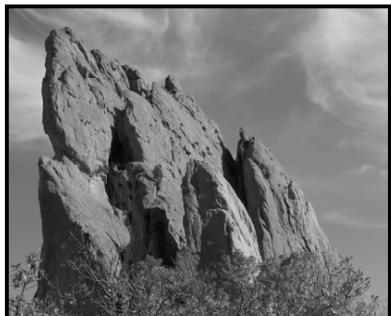
DigiVFX



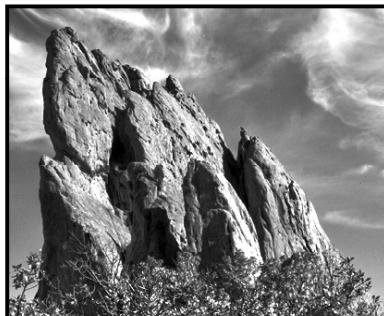
Local Contrast: Detail Layer

DigiVFX

- Uniform control:
 - Multiply all values in the detail layer



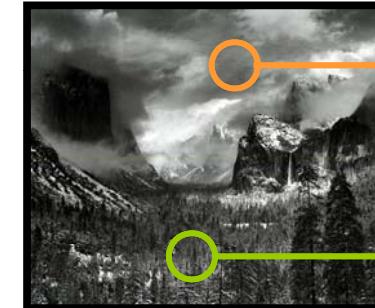
Input



Base + 3 × Detail

The amount of local contrast is not uniform

DigiVFX



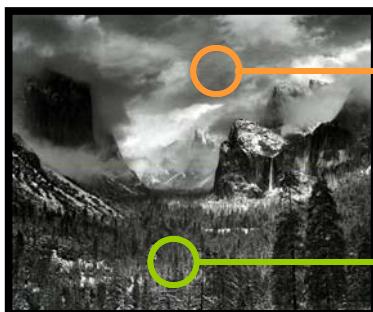
Smooth region

Textured region

Local Contrast Variation

DigiVFX

- We define “textureness”: amount of local contrast
 - at each pixel based on surrounding region

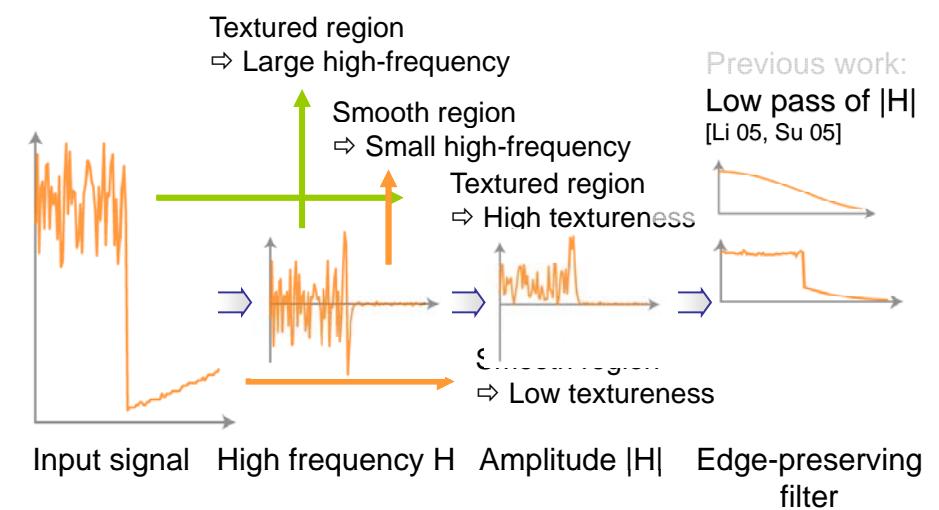


Smooth region
⇒ Low textureness

Textured region
⇒ High textureness

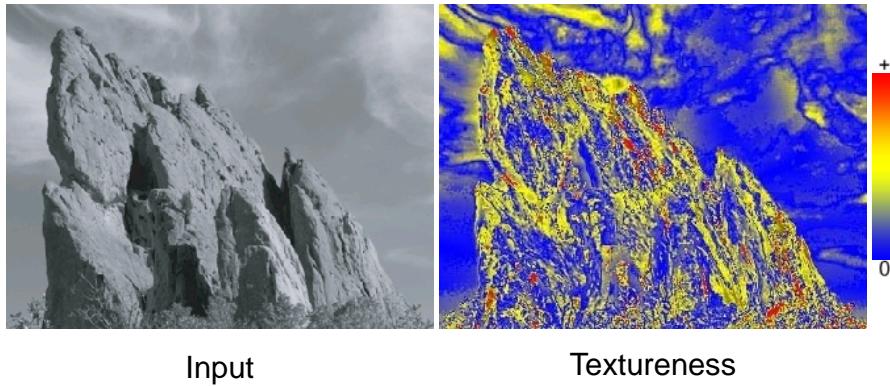
“Textureness”: 1D Example

DigiVFX



Textureness

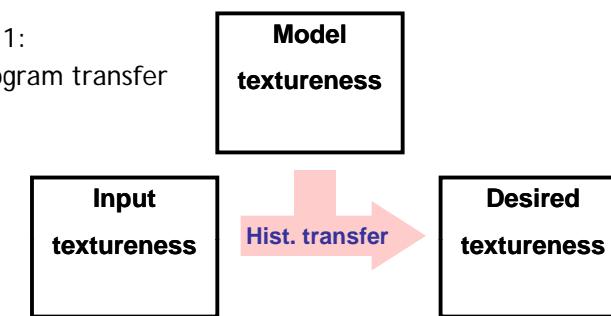
DigiVFX



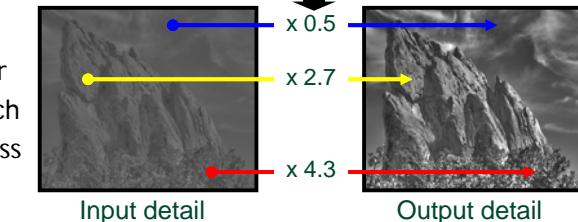
Textureness Transfer

DigiVFX

Step 1:
Histogram transfer

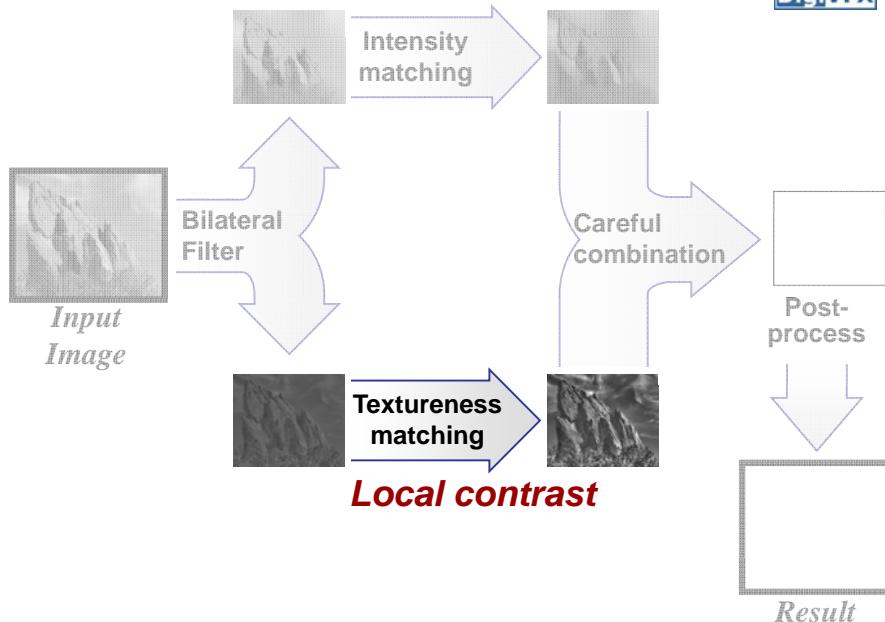


Step 2:
Scaling detail layer
(per pixel) to match
desired textureness



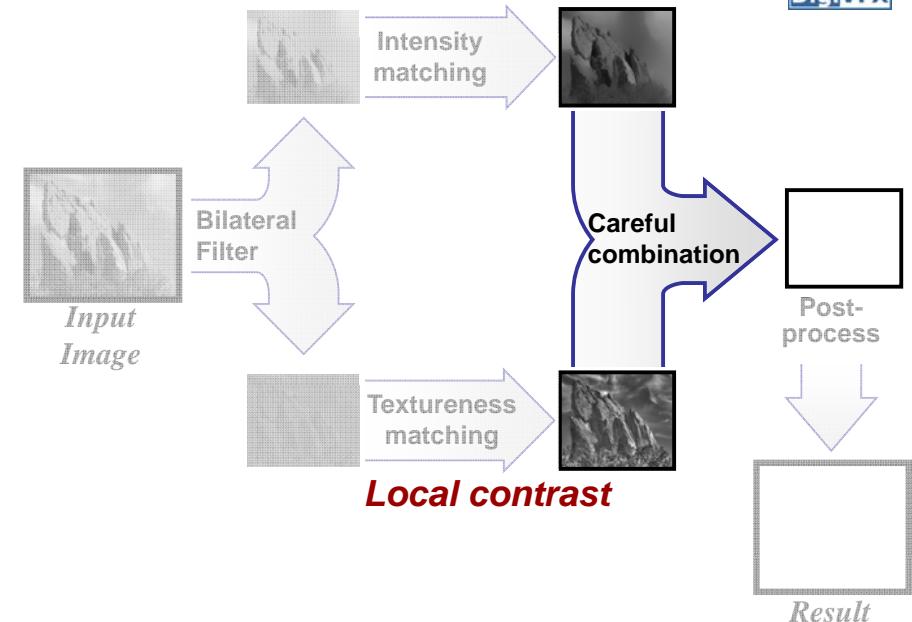
Global contrast

DigiVFX



Global contrast

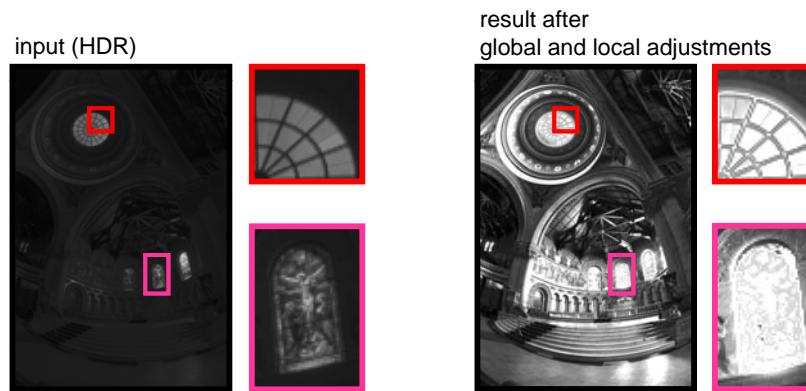
DigiVFX



A Non Perfect Result

DigiVFX

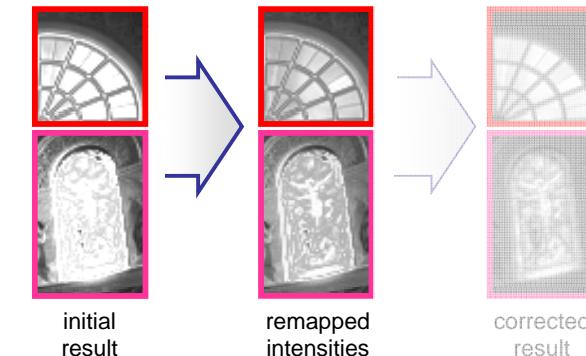
- Decoupled and large modifications (up to 6x)
→ Limited defects may appear



Intensity Remapping

DigiVFX

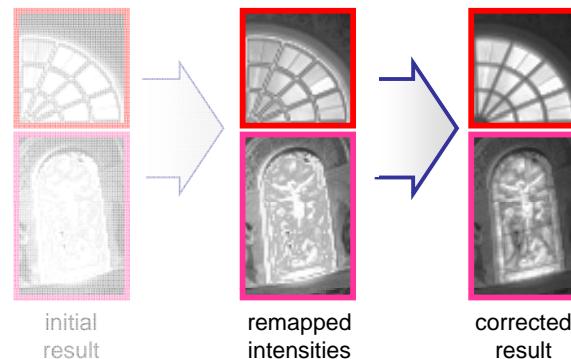
- Some intensities may be outside displayable range.
→ Compress histogram to fit visible range.



Preserving Details

DigiVFX

1. In the gradient domain:
 - Compare gradient amplitudes of input and current
 - Prevent extreme reduction & extreme increase
2. Solve the Poisson equation.



Effect of Detail Preservation

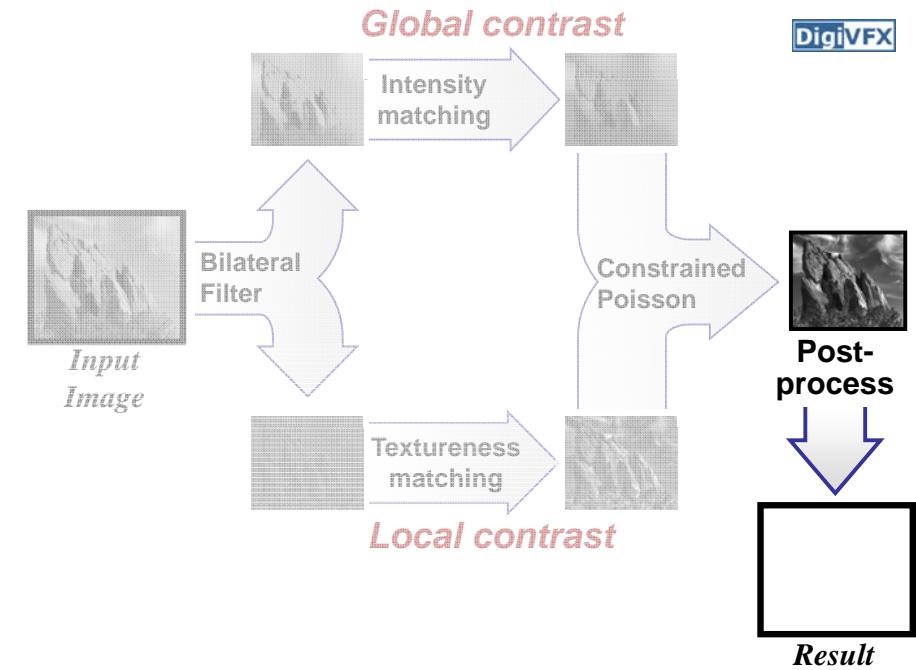
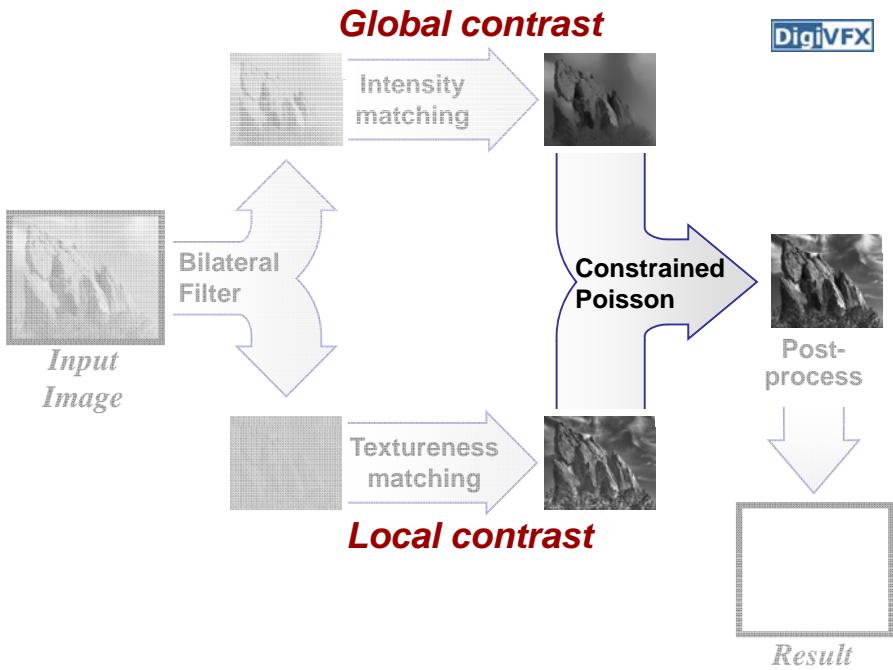
DigiVFX

uncorrected result



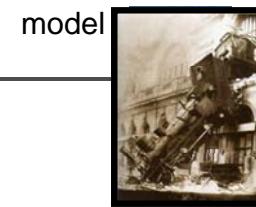
corrected result



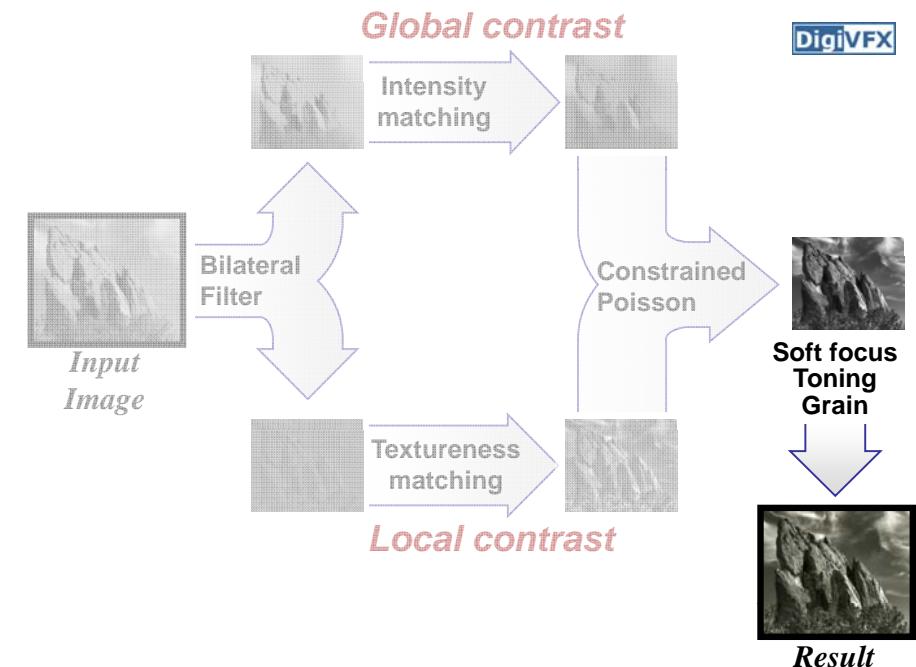
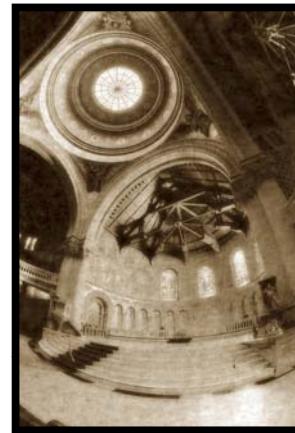


Additional Effects

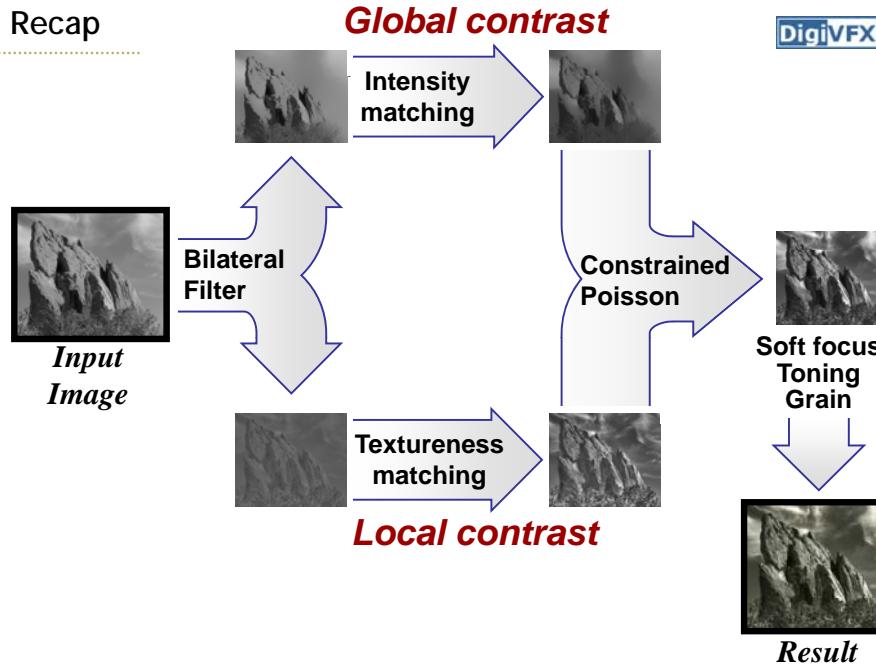
- Soft focus (high frequency manipulation)
- Film grain (texture synthesis [Heeger 95])
- Color toning (chrominance = f (luminance))



before effects



Recap



Results

DigiVFX

User provides input and model photographs.

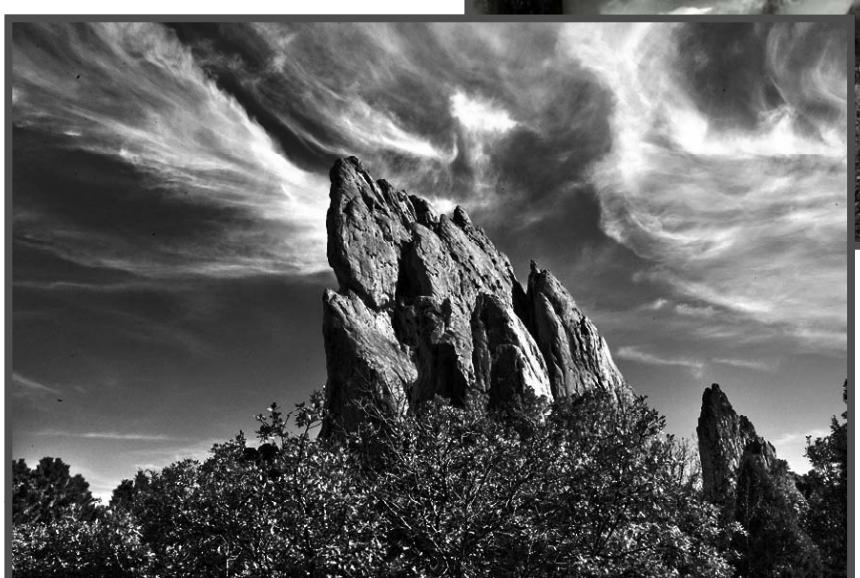
→ Our system automatically produces the result.

Running times:

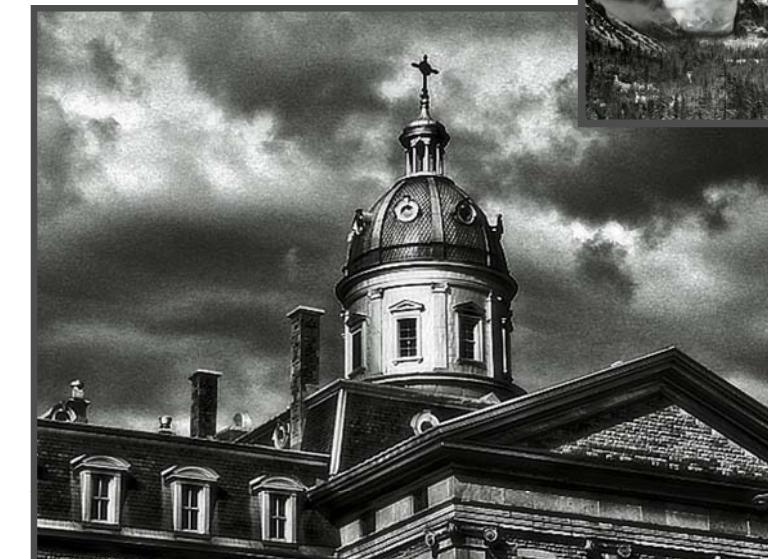
- 6 seconds for 1 MPixel or less
- 23 seconds for 4 MPixels
- multi-grid Poisson solver and fast bilateral filter [Paris 06]

Result

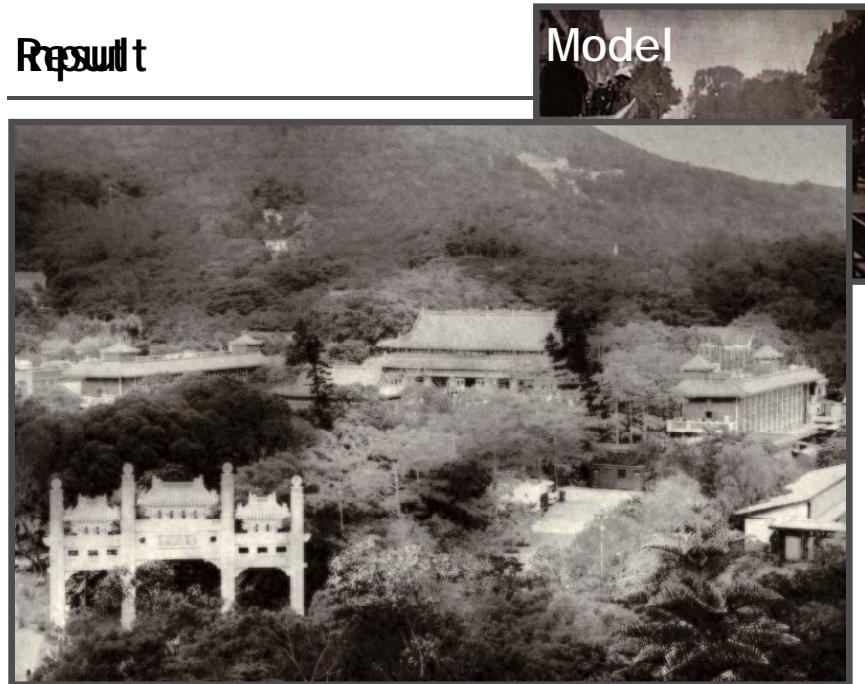
Model



Result



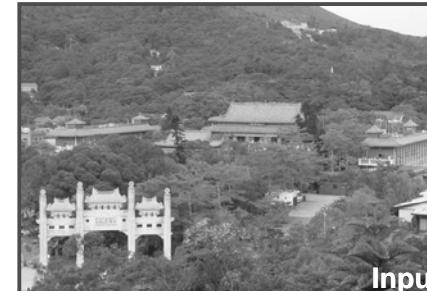
Result



Model

Comparison with Naïve Histogram Matching

DigiVFX

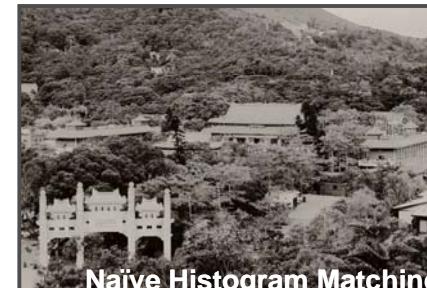


Input



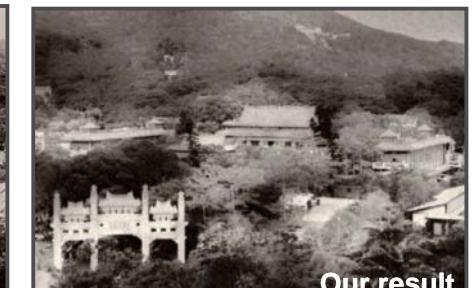
Model

Snapshot, Alfred Stieglitz



Naïve Histogram Matching

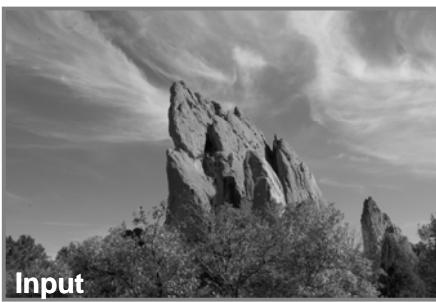
Local contrast, sharpness unfaithful



Our result

Comparison with Naïve Histogram Matching

DigiVFX



Input



Model

Clearing Winter Storm, Ansel Adams



Histogram Matching

Local contrast too low

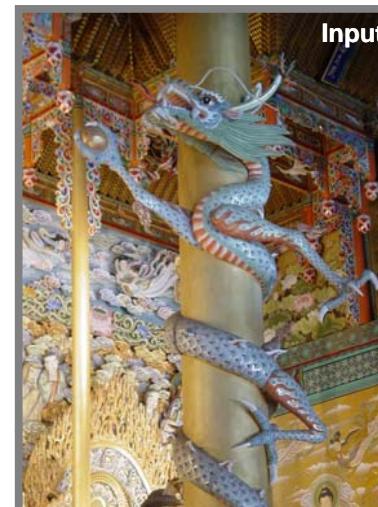


Our Result

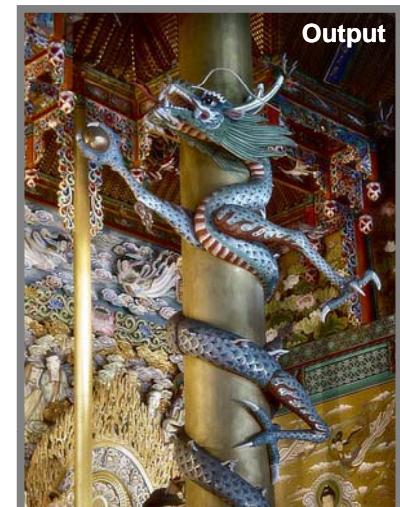
Color Images

DigiVFX

- Lab color space: modify only luminance



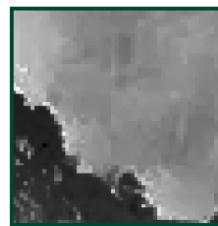
Input



Output

Limitations

- Noise and JPEG artifacts
 - amplified defects



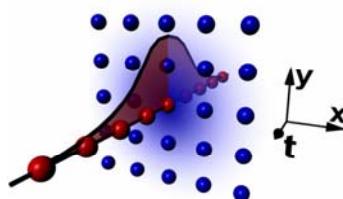
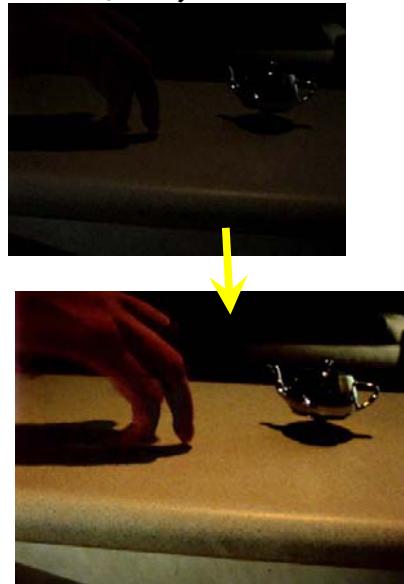
- Can lead to unexpected results if the image content is too different from the model
 - Portraits, in particular, can suffer



Video Enhancement Using Per Pixel Exposures (Bennett, 06)

From this video:

ASTA: Adaptive Spatio-Temporal Accumulation Filter



Conclusions

- Transfer “look” from a model photo
- Two-scale tone management
 - Global and local contrast
 - New edge-preserving textureness
 - Constrained Poisson reconstruction
 - Additional effects

Joint bilateral filtering

$$J_p = \frac{1}{k_p} \sum_{q \in \Omega} I_q f(||p - q||) g(||I_p - I_q||)$$

$$J_p = \frac{1}{k_p} \sum_{q \in \Omega} I_q f(||p - q||) g(||\tilde{I}_p - \tilde{I}_q||)$$

Flash / No-Flash Photo Improvement DigiVFX (Petschnigg04) (Eisemann04)

Merge best features: warm, cozy candle light (no-flash)
low-noise, detailed flash image



Petschnigg:

- Flash



Overview DigiVFX

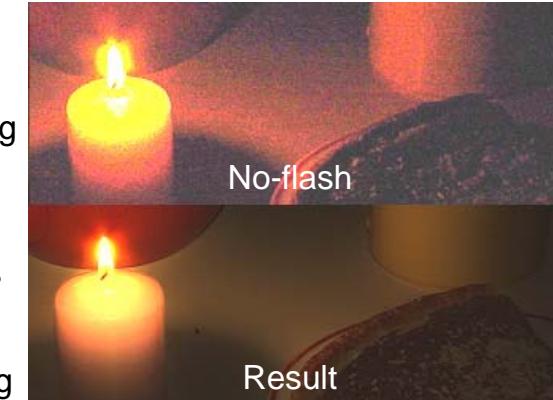
Basic approach of both flash/noflash papers

Remove noise + details
from image A,

Keep as image A Lighting

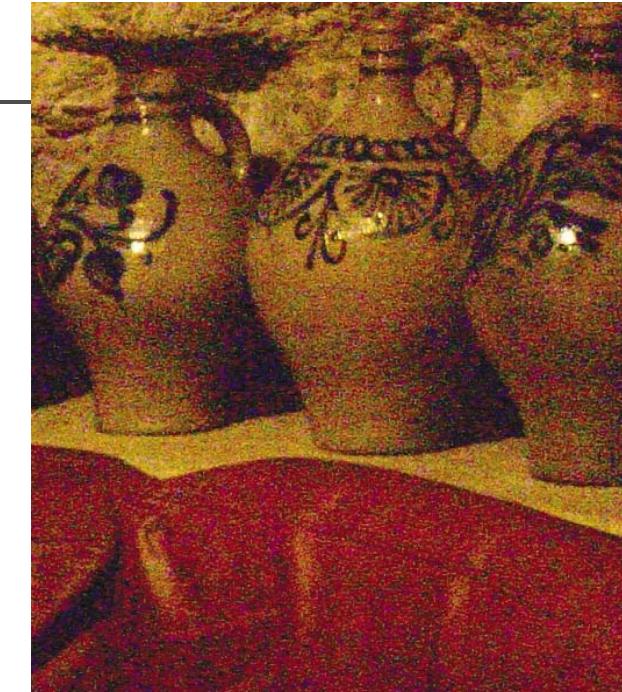
Obtain noise-free details
from image B,

Discard Image B Lighting



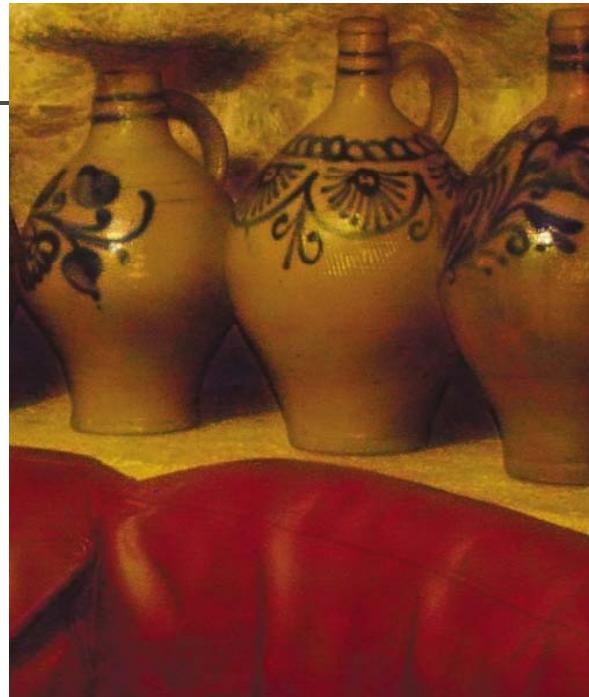
Petschnigg:

- No Flash,



Petschnigg:

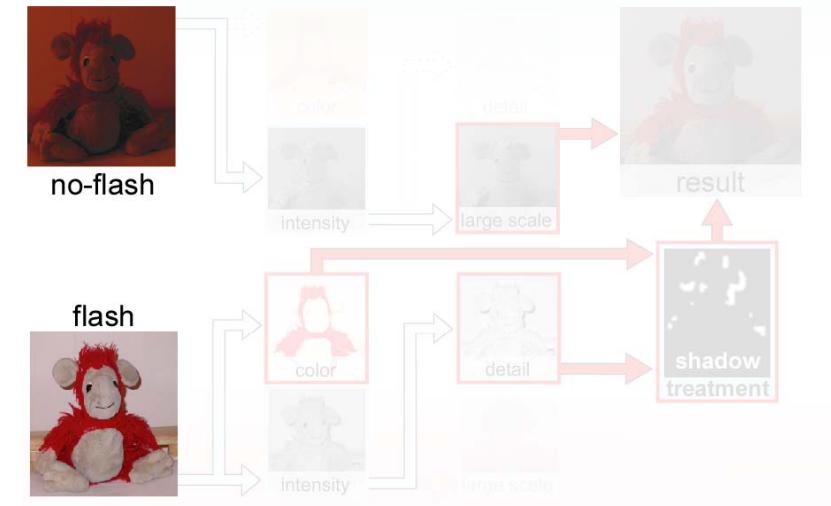
- Result



DigiVFX

Our Approach

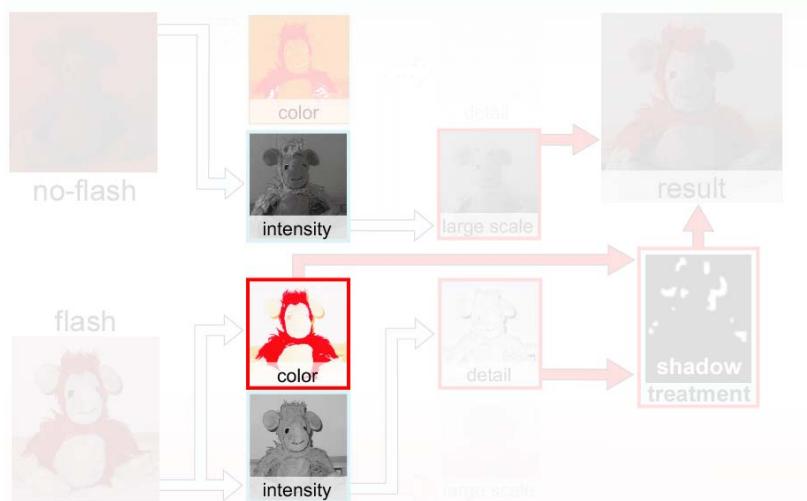
Registration



Our Approach

DigiVFX

Decomposition



Decomposition

DigiVFX

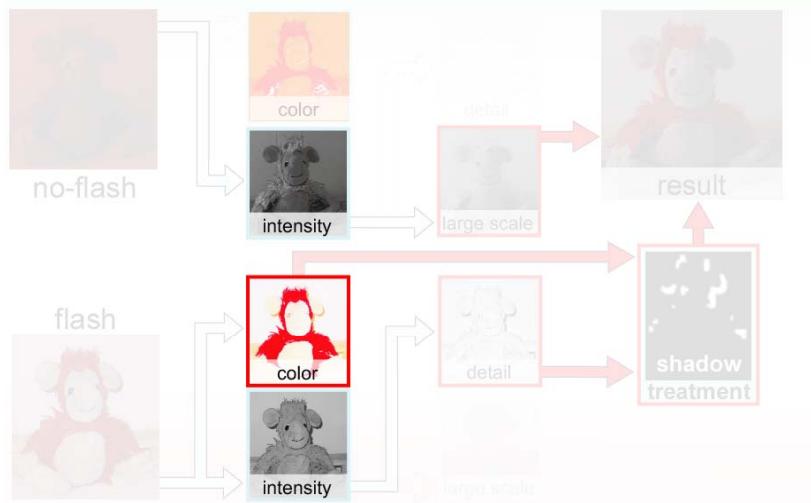
Color / Intensity:

$$\text{original} = \text{intensity} * \text{color}$$

Our Approach

DigiVFX

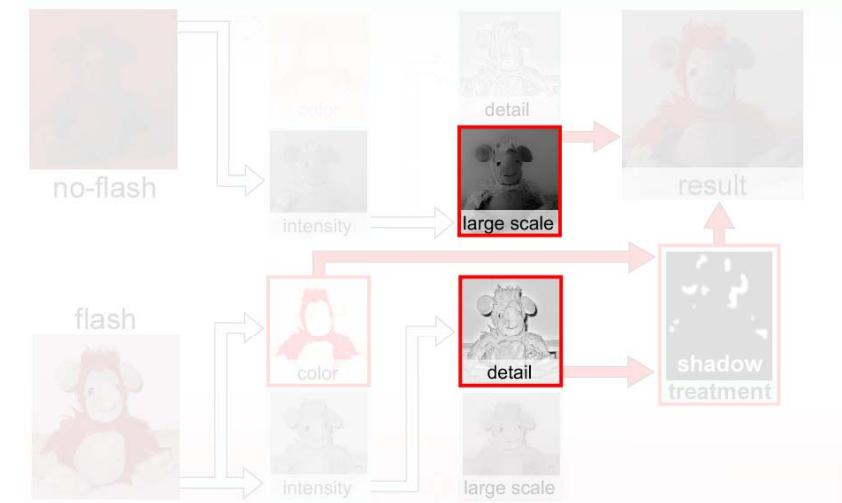
Decomposition



Our Approach

DigiVFX

Decoupling



Decoupling

DigiVFX

- Lighting : Large-scale variation
- Texture : Small-scale variation



Lighting



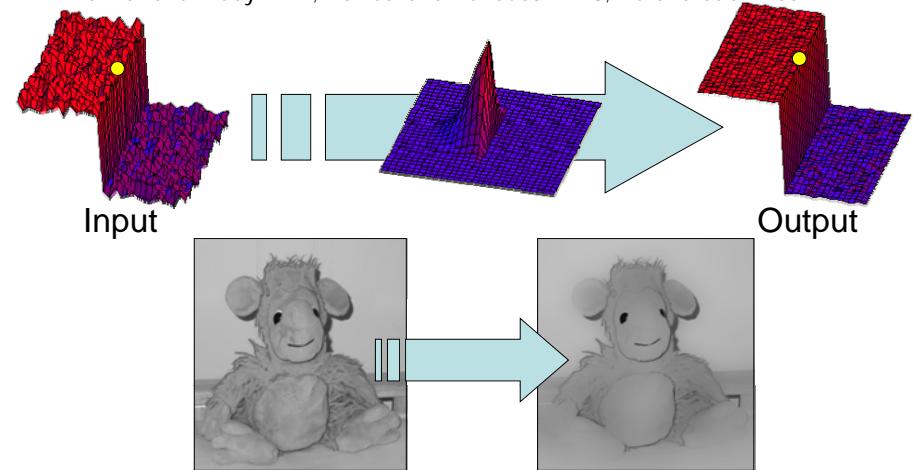
Texture

Large-scale Layer

DigiVFX

- **Bilateral filter – edge preserving filter**

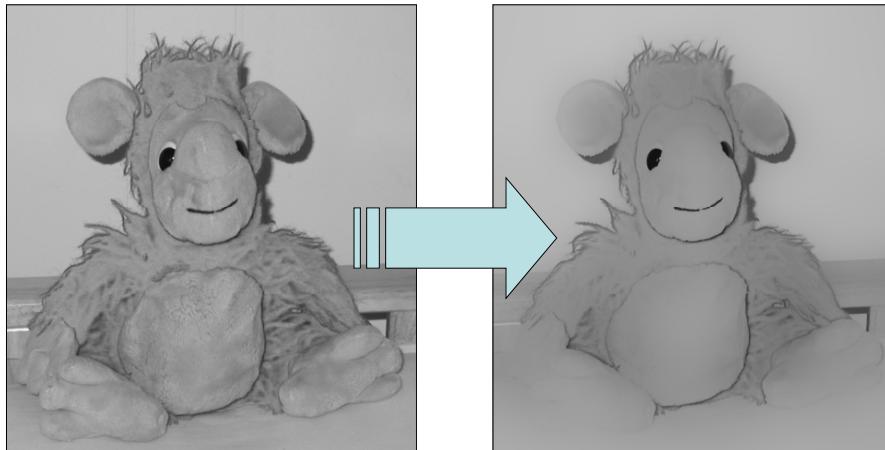
Smith and Brady 1997; Tomasi and Manducci 1998; Durand et al. 2002



Large-scale Layer

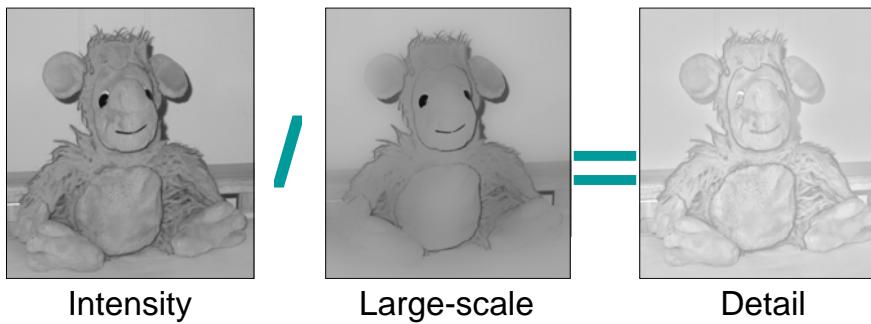
DigiVFX

- Bilateral filter



Detail Layer

DigiVFX



Recombination: Large scale * Detail = Intensity

Cross Bilateral Filter

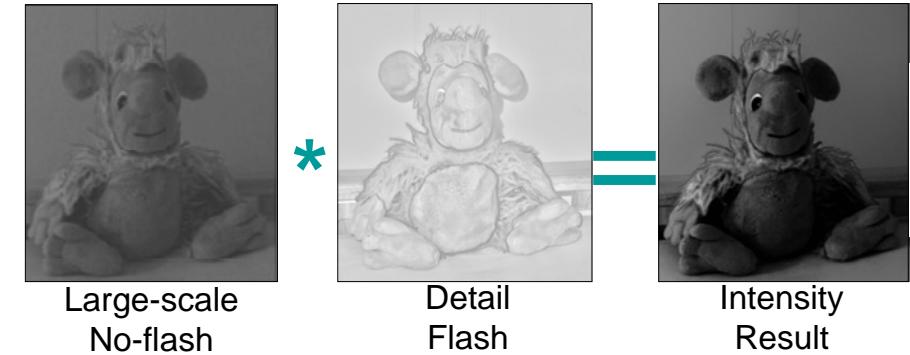
DigiVFX

- Similar to joint bilateral filter by Petschnigg et al.
- When no-flash image is too noisy
- Borrow similarity from flash image
 - edge stopping from flash image



Recombination

DigiVFX



Recombination: Large scale * Detail = Intensity

Recombination

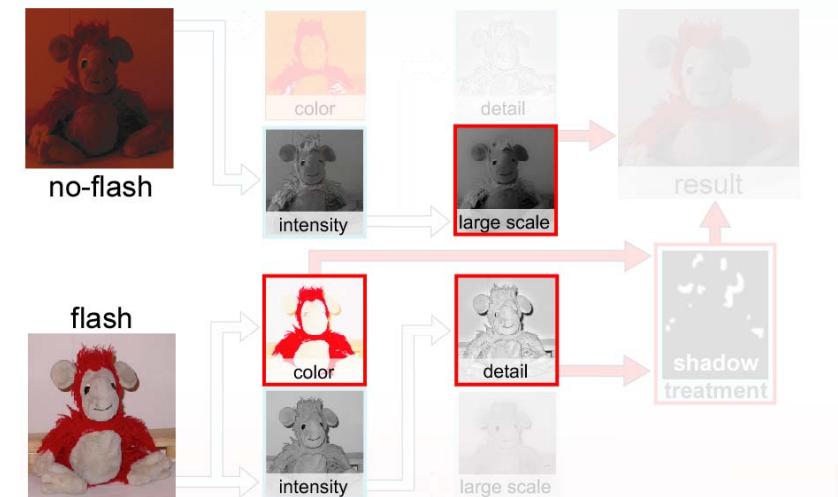
DigiVFX



Recombination: Intensity * Color = Original

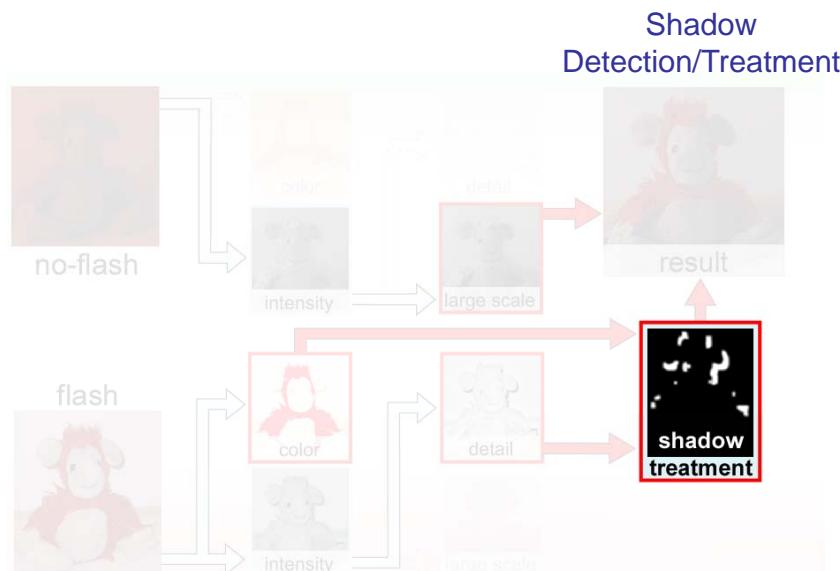
Our Approach

DigiVFX



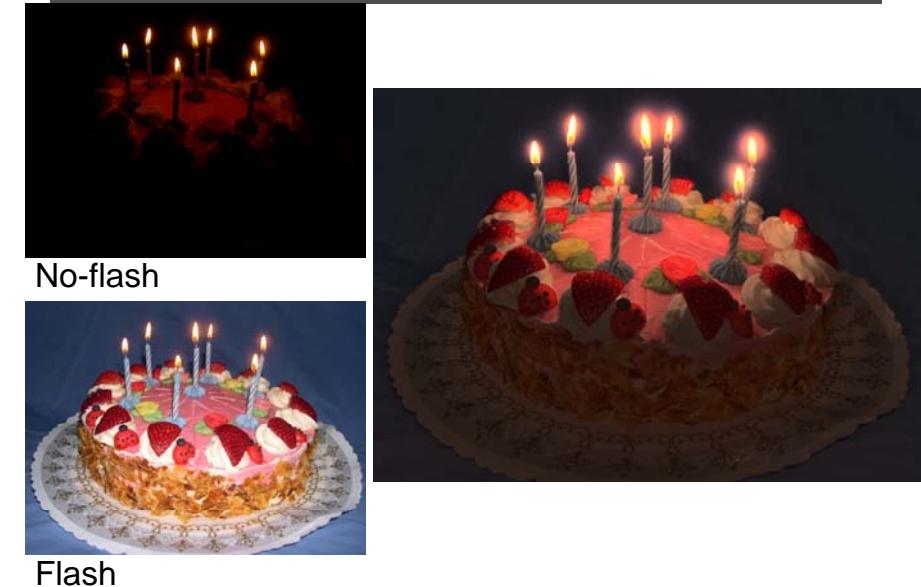
Our Approach

DigiVFX



Results

DigiVFX



Joint bilateral upsampling

DigiVFX

$$J_p = \frac{1}{k_p} \sum_{q \in \Omega} I_q f(||p - q||) g(||I_p - I_q||)$$

$$J_p = \frac{1}{k_p} \sum_{q \in \Omega} I_q f(||p - q||) g(||\tilde{I}_p - \tilde{I}_q||)$$

$$\tilde{S}_p = \frac{1}{k_p} \sum_{q_\downarrow \in \Omega} S_{q_\downarrow} f(||p_\downarrow - q_\downarrow||) g(||\tilde{I}_p - \tilde{I}_q||)$$

Joint bilateral upsampling

DigiVFX



Nearest Neighbor

Bicubic

Gaussian

Joint Bilateral

Ground Truth

Joint bilateral upsampling

DigiVFX



Upsampled Result

Joint bilateral upsampling

DigiVFX



Upsampled Result

Joint bilateral upsampling

DigiVFX



Nearest Neighbor Upsampling



Bicubic Upsampling



Gaussian Upsampling



Joint Bilateral Upsampling

Joint bilateral upsampling

DigiVFX



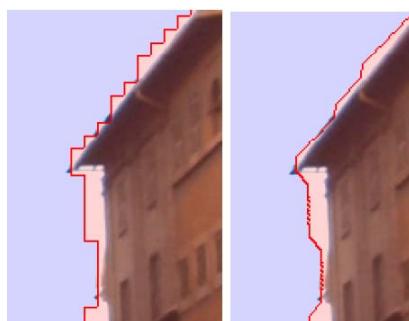
Downsampled
Input Solution



Input Images

Joint bilateral upsampling

DigiVFX



Nearest Neighbor



Bicubic



Gaussian



Joint Bilateral

Joint bilateral upsampling

DigiVFX



Upsampled Result