关系的基本运算

Operations on Relations（I）

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关系的基本运算

- 假设 A、B 是两个集合，$a\in A$，$b\in B$，R、S 为 A 到 B 的两个关系，
- 于是 R 和 S 都是 $A\times B$ 的子集。
- 于是集合的运算也可以应用于 R 和 S 。

关系的基本运算

- R 与 S 的**交**（**intersection**）关系 $R\cap S$ 定义为：
  $(a, b)\in R\cap S$ 当且仅当 $(a, b)\in R$ 且 $(a, b)\in S$；

- R 与 S 的**并**（**union**）关系 $R\cup S$ 定义为：
  $(a, b)\in R\cup S$ 当且仅当 $(a, b)\in R$ 或 $(a, b)\in S$；

- R 的**补**（**complement**）关系 $\overline{R}$ 定义为；
  $a\overline{R}b$ 当且仅当 $a\not{R}b$

关系的基本运算

- {张, 白, 宋, 方} × {离散数学, 数据结构, 计算机网络} =

  ~~{(张, 离散数学),~~ ~~(张, 数据结构),~~ (张, 计算机网络)

  (白, 离散数学), ~~(白, 数据结构),~~ (白, 计算机网络),

  (宋, 离散数学), (宋, 数据结构), (宋, 计算机网络),

  (方, 离散数学), (方, 数据结构), ~~(方, 计算机网络)~~ }

- ***R*** = { (张, 数据结构), (张, 离散数学),

  (白, 数据结构), (方, 计算机网络)}

- $\overline{R}$ = { (张,计算机网络), (白, 离散数学), (白,计算机网络),

  (宋, 离散数学), (宋, 数据结构), (宋, 计算机网络),

  (方, 离散数学), (方, 数据结构), }

关系的基本运算

- 例：
  - $A = \{ 1, 2, 3, 4 \}$
  - $R = \{ (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4) \}$
  - $S = \{ (1, 2), (1, 3), (2, 1), (2, 2), (2, 4), (4, 4) \}$
  - $R \cap S = \{ (1, 2), (2, 1) \}$
  - $R \cup S = \{ (1, 1), (1, 2), (1, 3), (1, 4), (2, 1),$
    $(2, 2), (2, 4), (3, 2), (3, 4), (4, 4) \}$

关系的基本运算

- 假设 A、B 是两个集合，R 为 A 到 B 的关系，则 R 的**逆（inverse）关系**定义为

$$R^{-1}=\{(b, a)|b\in B, a\in A , (a, b)\in R\}\subseteq B\times A。$$

- 简言之，R 的逆关系是由将 R 中每个有序对的元素顺序交换构成的。
- 例：
  - $A = \{1, 2, 3, 4\}$
  - $R = \{ (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4) \}$
  - $R^{-1} = \{ (1, 1), (2,1), (4, 1), (1, 2), (2, 3), (4, 3) \}$

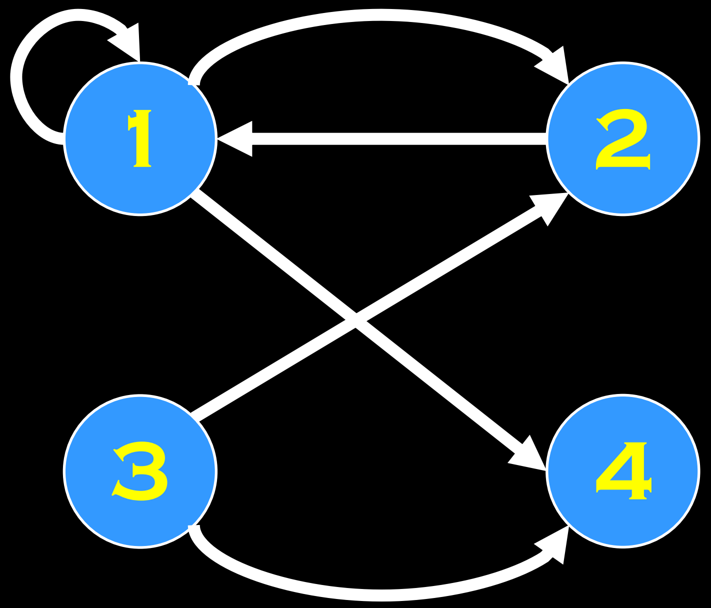
关系的基本运算

- 例
  - R = { (张, 数据结构), (张, 离散数学),
    (白, 数据结构), (方, 计算机网络)}
  - R^{-1} = { (数据结构, 张), (离散数学, 张),
    (数据结构, 白), (计算机网络, 方)}

关系的基本运算

- $A = \{1, 2, 3, 4\}$
- $R = \{ (1,1), (1,2), (1,4), (2,1), (3,2), (3,4) \}$
- $S = \{ (1,2), (1,3), (2,1), (2,2), (2,4), (4,4) \}$

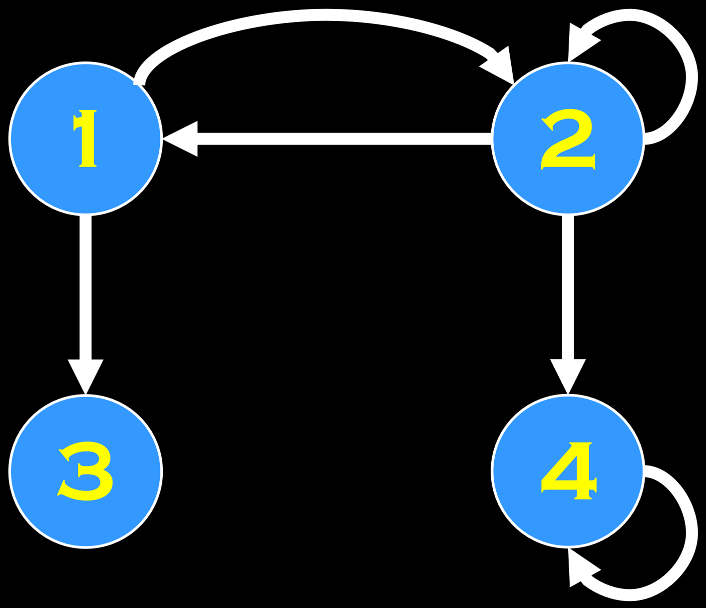
R	1	2	3	4
1	1	1	0	1
2	1	0	0	0
3	0	1	0	1
4	0	0	0	0



关系的基本运算

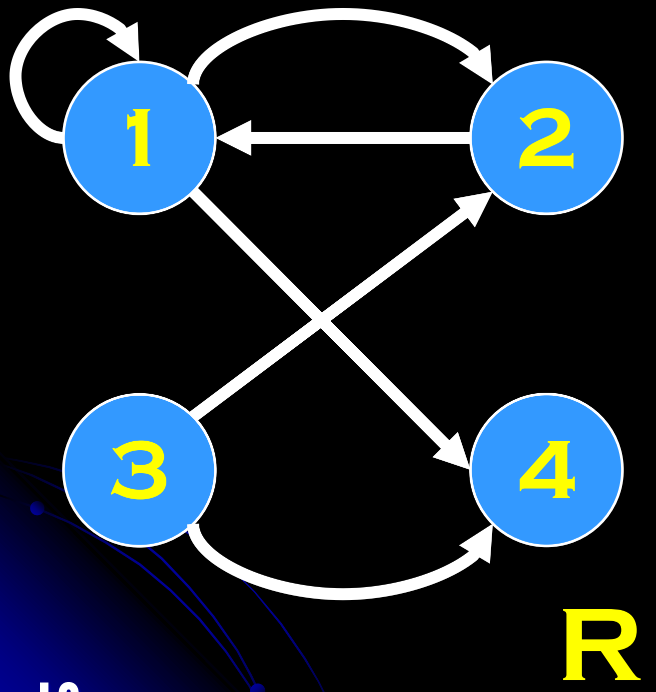
- $A = \{1, 2, 3, 4\}$
- $R = \{ (1,1), (1,2), (1,4), (2,1), (3,2), (3,4) \}$
- $S = \{ (1,2), (1,3), (2,1), (2,2), (2,4), (4,4) \}$

S	1	2	3	4
1	0	1	1	0
2	1	1	0	1
3	0	0	0	0
4	0	0	0	1



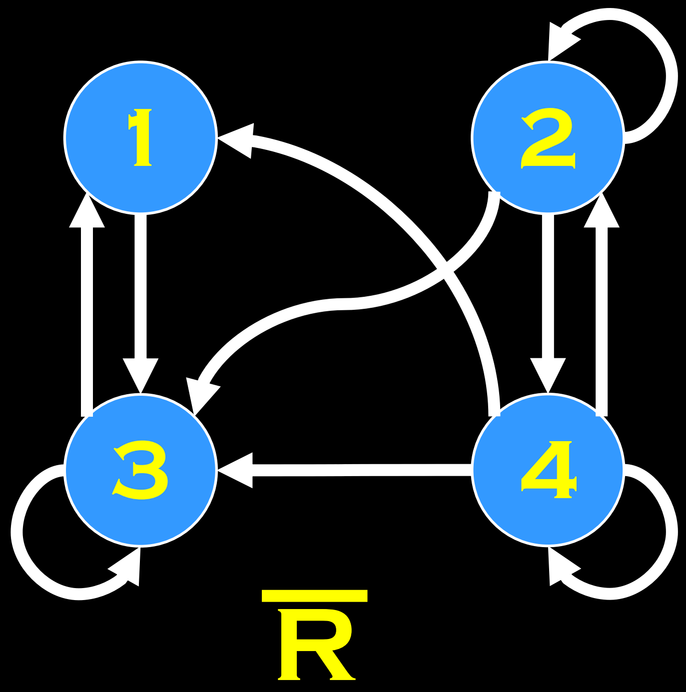
关系的基本运算

- $A = \{1,2,3,4\}$
- $R = \{ (1,1), (1,2), (1,4), (2,1), (3,2), (3,4) \}$
- $\overline{R} = \{ (1,3), (2,2), (2,3), (2,4), (3,1),$
  $(3,3), (4,1), (4,2), (4,3), (4,4) \}$



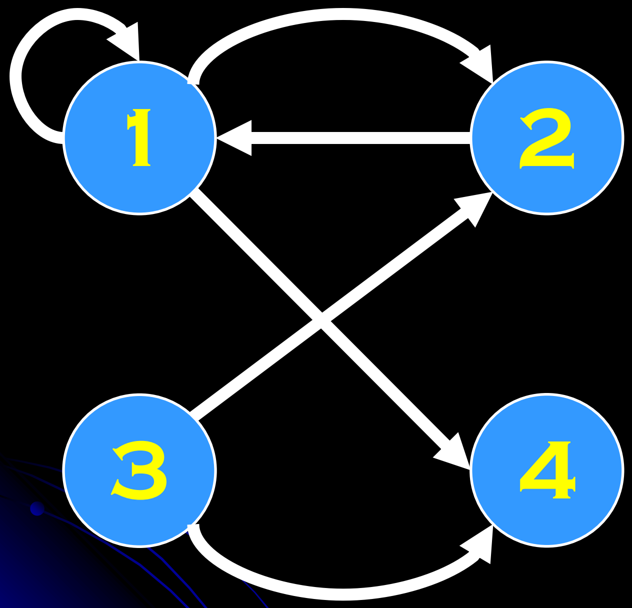
R	1	2	3	4
1	1	1	0	1
2	1	0	0	0
3	0	1	0	1
4	0	0	0	0

$\overline{R}$	1	2	3	4
1	0	0	1	0
2	0	1	1	1
3	1	0	1	0
4	1	1	1	1



关系的基本运算

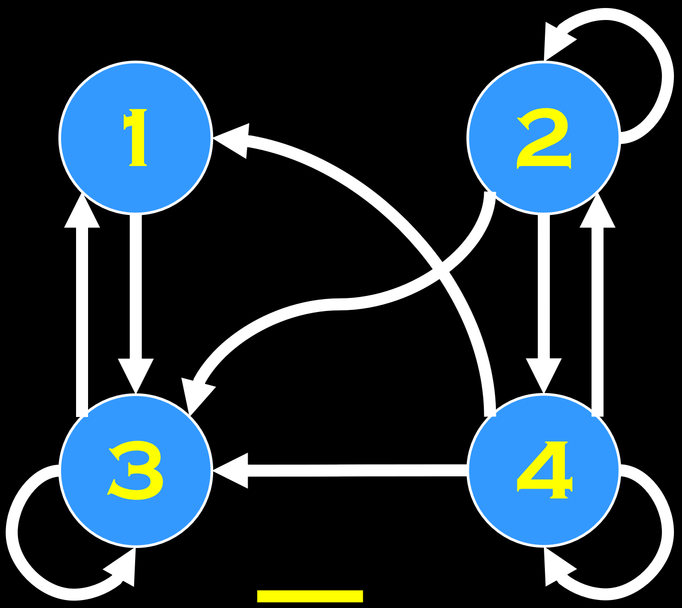
- $A = \{1,2,3,4\}$
- $R = \{\ (1,1),\ (1,2),\ (1,4),\ (2,1),\ (3,2),\ (3,4)\ \}$
- $\overline{R} = \{\ (1,3),\ (2,2),\ (2,3),\ (2,4),\ (3,1),$
  $(3,3),\ (4,1),\ (4,2),\ (4,3),\ (4,4)\ \}$



R

R	1	2	3	4
1	√	√	0	√
2	√	0	0	0
3	0	√	0	√
4	0	0	0	0

$\overline{R}$	1	2	3	4
1	0	0	√	0
2	0	√	√	√
3	√	0	√	0
4	√	√	√	√



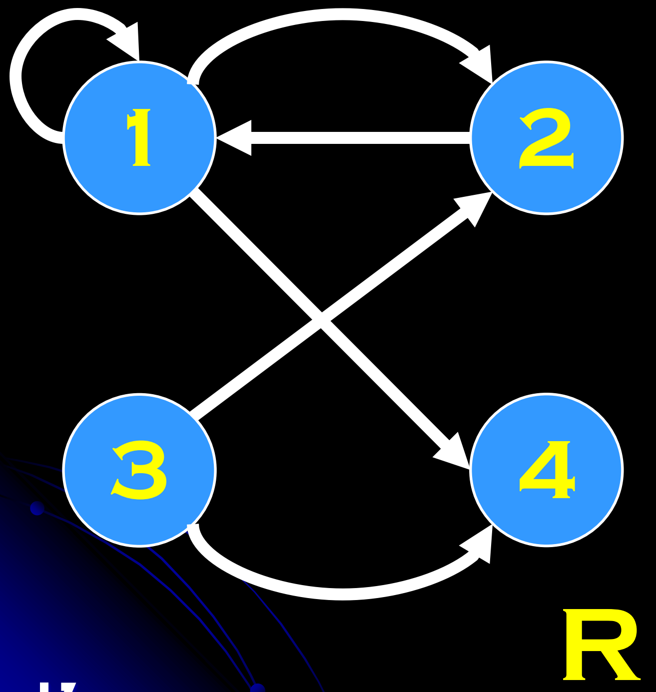
$\overline{R}$

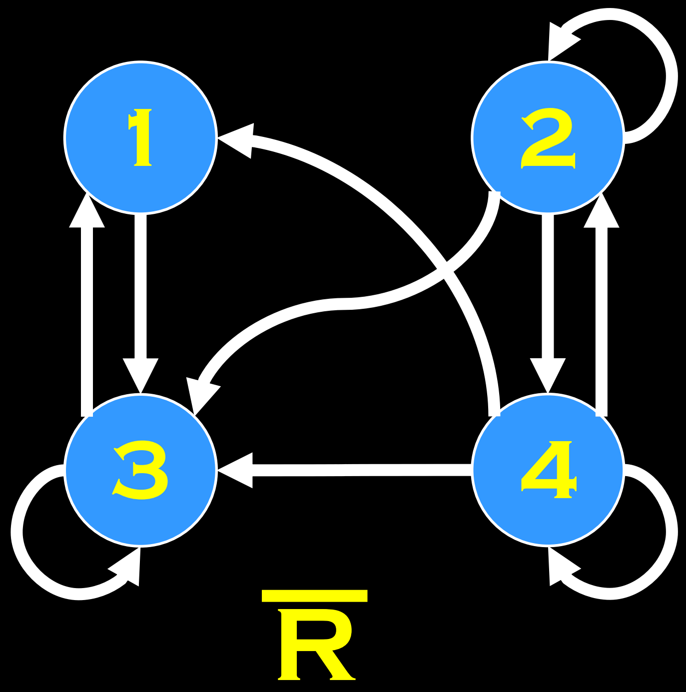
关系的基本运算

R	1	2	3	4
1	√	√		√
2	√			
3		√		√
4				

$\overline{R}$	1	2	3	4
1			√	
2		√	√	√
3	√		√	
4	√	√	√	√

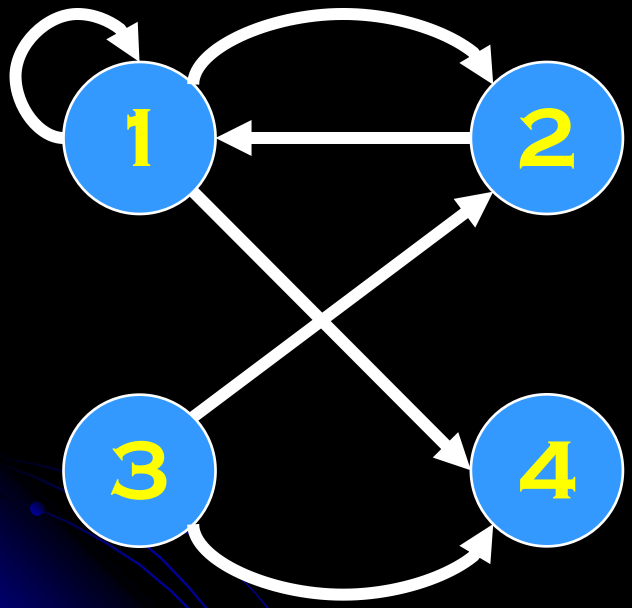
关系的基本运算





关系的基本运算

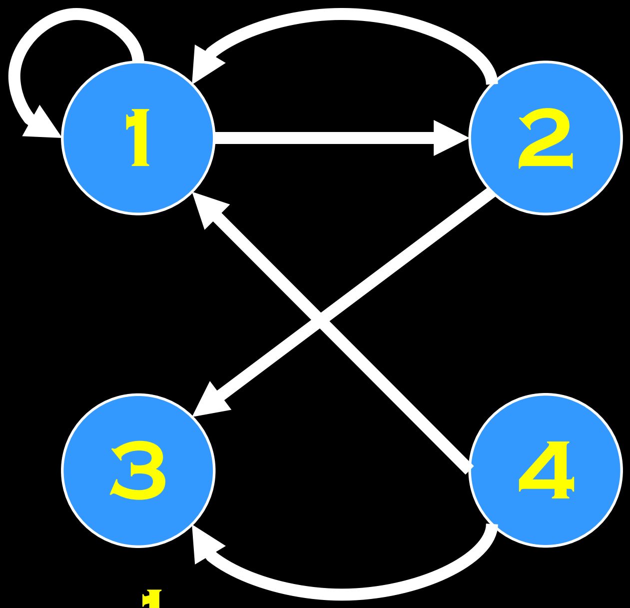
- $A = \{1, 2, 3, 4\}$
- $R = \{\ (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4)\ \}$
- $R^{-1} = \{\ (1, 1), (2, 1), (4, 1), (1, 2), (2, 3), (4, 3)\ \}$



R

R	1	2	3	4
1	1	1	0	1
2	1	0	0	0
3	0	1	0	1
4	0	0	0	0

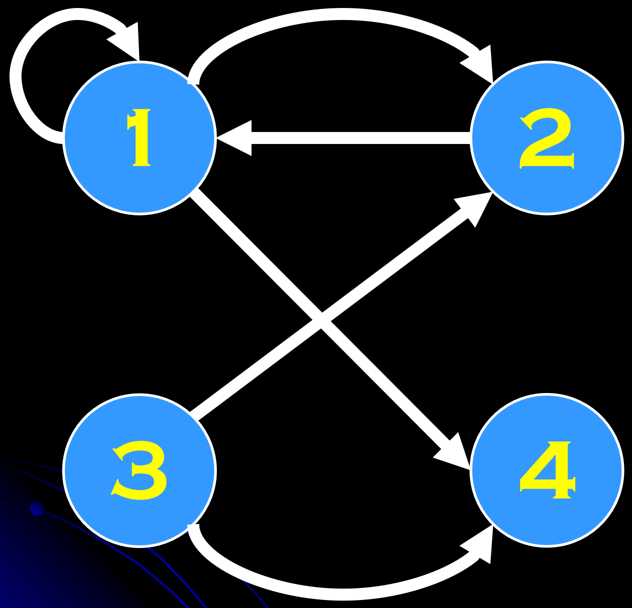
R^{-1}	1	2	3	4
1	1	1	0	0
2	1	0	1	0
3	0	0	0	0
4	1	0	1	0



R^{-1}

关系的基本运算

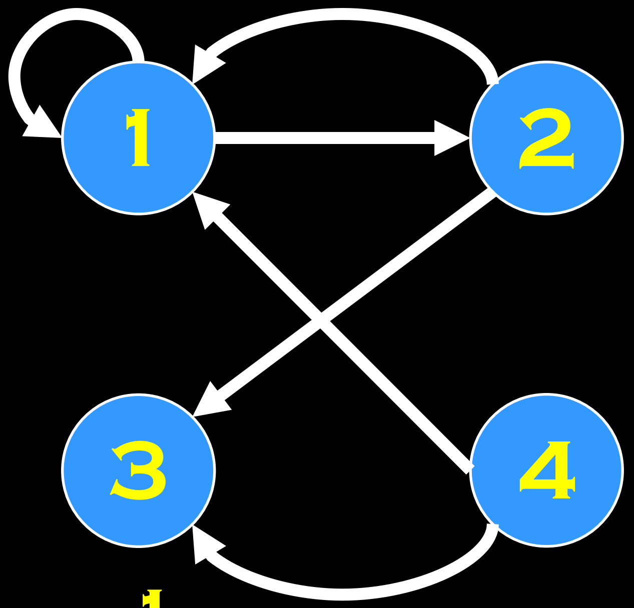
- $A = \{1, 2, 3, 4\}$
- $R = \{ (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4) \}$
- $R^{-1} = \{ (1, 1), (2, 1), (4, 1), (1, 2), (2, 3), (4, 3) \}$



R

R	1	2	3	4
1	1	1	0	1
2	1	0	0	0
3	0	1	0	1
4	0	0	0	0

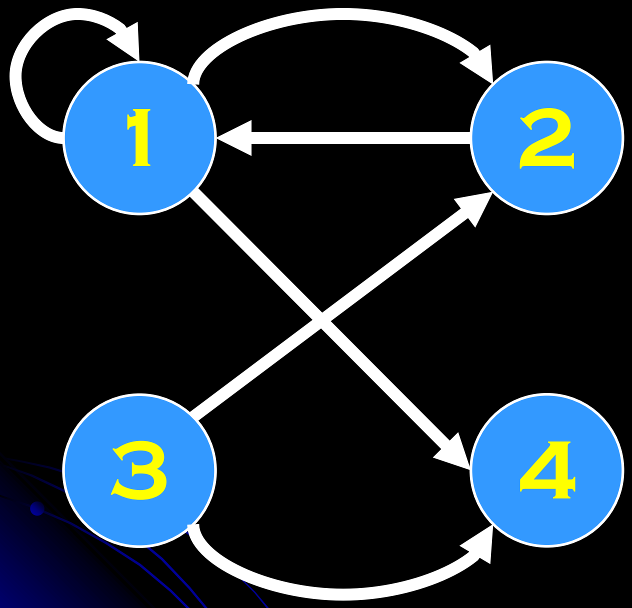
R^{-1}	1	2	3	4
1	1	1	0	0
2	1	0	1	0
3	0	0	0	0
4	1	0	1	0



R^{-1}

关系的基本运算

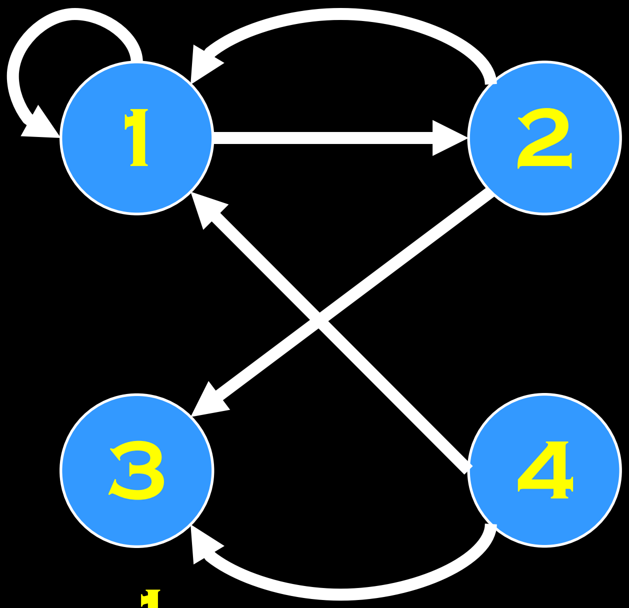
- $A = \{1, 2, 3, 4\}$
- $R = \{ (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4) \}$
- $R^{-1} = \{ (1, 1), (2, 1), (4, 1), (1, 2), (2, 3), (4, 3) \}$



R

R	1	2	3	4
1	1	1	0	1
2	1	0	0	0
3	0	1	0	1
4	0	0	0	0

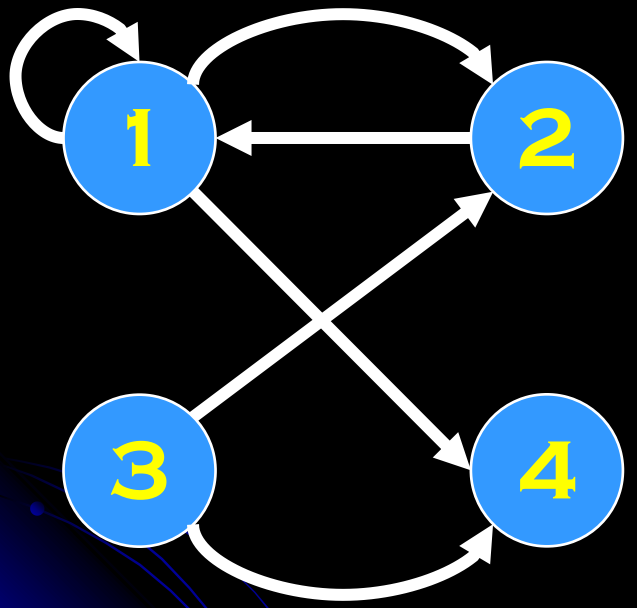
R^{-1}	1	2	3	4
1	1	1	0	0
2	1	0	1	0
3	0	0	0	0
4	1	0	1	0



R^{-1}

关系的基本运算

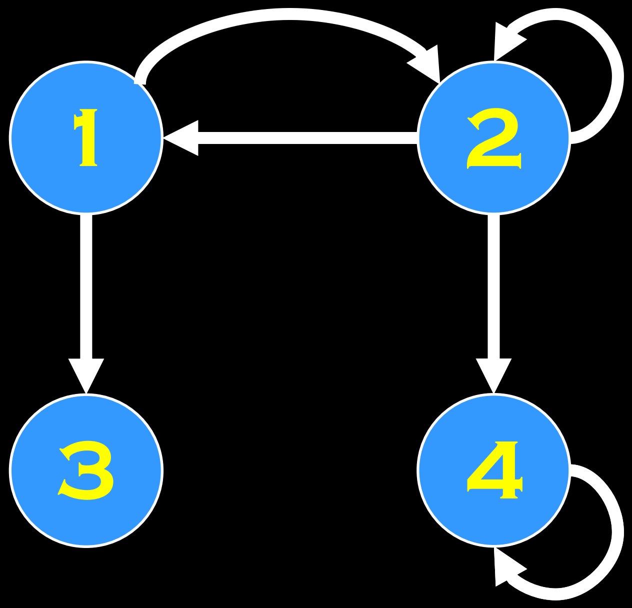
- R = { (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4) }
- S = { (1, 2), (1, 3), (2, 1), (2, 2), (2, 4), (4, 4) }
- $R \cap S$ = { (1, 2), (2, 1) }



R

R	1	2	3	4
1	√	√	0	√
2	√	0	0	0
3	0	√	0	√
4	0	0	0	0

S	1	2	3	4
1	0	√	√	0
2	√	√	0	√
3	0	0	0	0
4	0	0	0	√



S

关系的基本运算

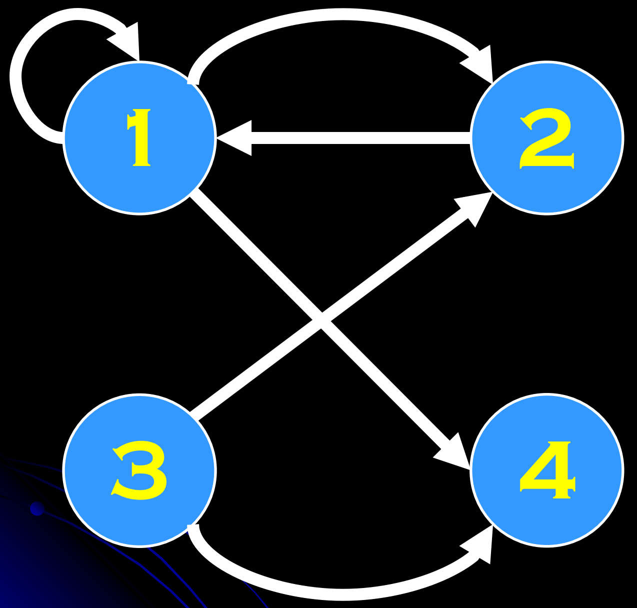
- $R = \{ (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4) \}$
- $S = \{ (1, 2), (1, 3), (2, 1), (2, 2), (2, 4), (4, 4) \}$
- $R \cap S = \{ (1, 2), (2, 1) \}$

R	1	2	3	4
1	√	√		√
2	√			
3		√		√
4				

S	1	2	3	4
1		√	√	
2	√	√		√
3				
4				√

关系的基本运算

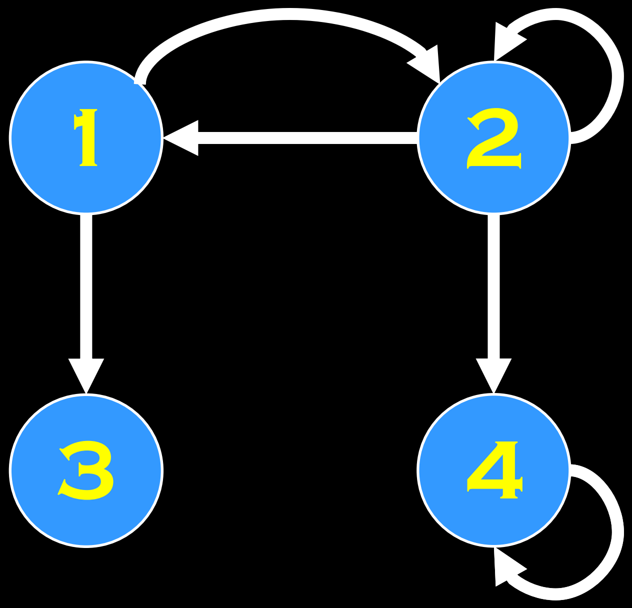
- $R = \{ (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4) \}$
- $S = \{ (1, 2), (1, 3), (2, 1), (2, 2), (2, 4), (4, 4) \}$
- $R \cap S = \{ (1, 2), (2, 1) \}$



R

R	1	2	3	4
1	1	1	0	1
2	1	0	0	0
3	0	1	0	1
4	0	0	0	0

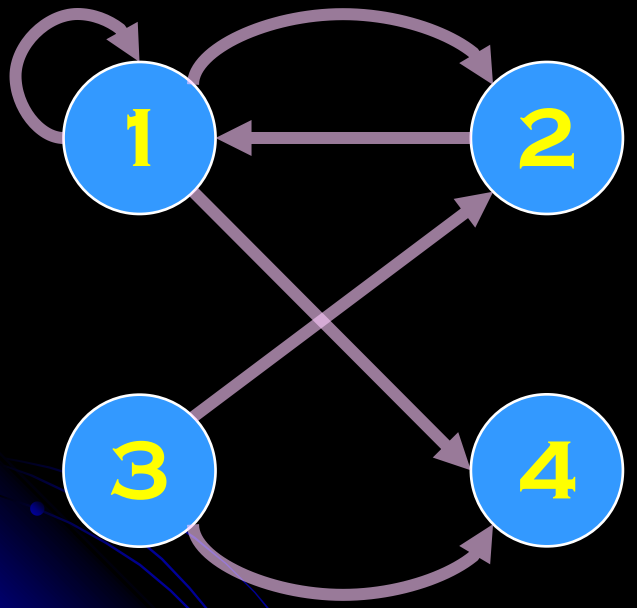
S	1	2	3	4
1	0	1	1	0
2	1	1	0	1
3	0	0	0	0
4	0	0	0	1



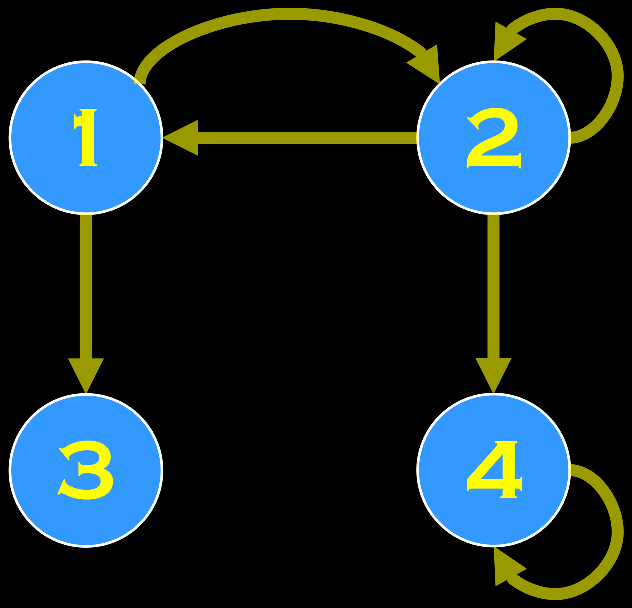
S

关系的基本运算

- $R = \{ (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4) \}$
- $S = \{ (1, 2), (1, 3), (2, 1), (2, 2), (2, 4), (4, 4) \}$
- $R \cap S = \{ (1, 2), (2, 1) \}$



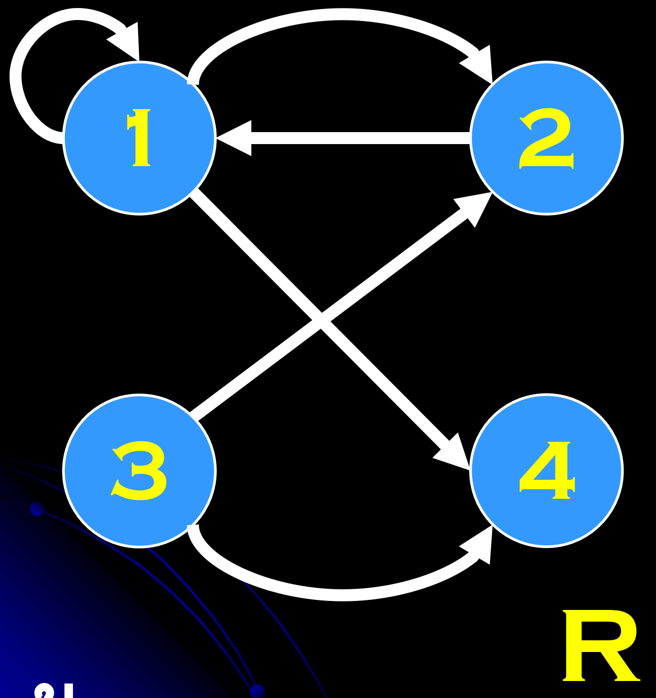
R



S

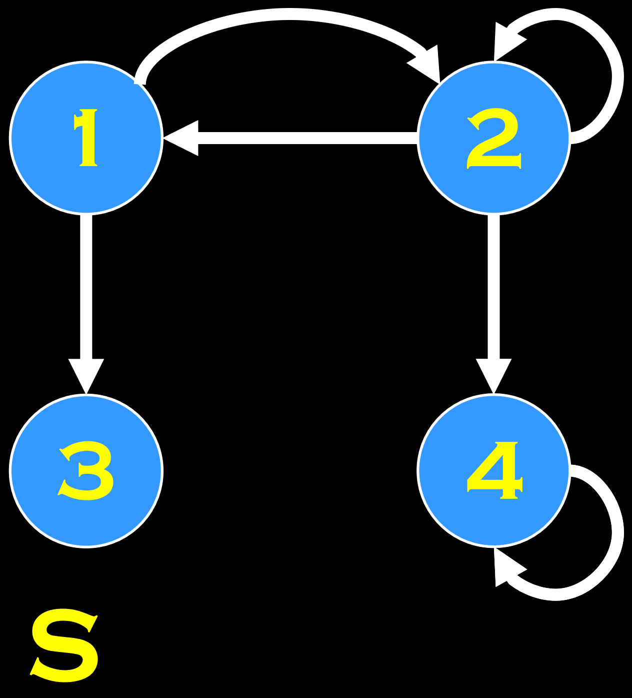
关系的基本运算

- R = { (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4) }
- S = { (1, 2), (1, 3), (2, 1), (2, 2), (2, 4), (4, 4) }
- $R \cup S$ = { (1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 4), (3, 2), (3, 4), (4, 4) }



R	1	2	3	4
1	1	1	0	1
2	1	0	0	0
3	0	1	0	1
4	0	0	0	0

S	1	2	3	4
1	0	1	1	0
2	1	1	0	1
3	0	0	0	0
4	0	0	0	1



关系的基本运算

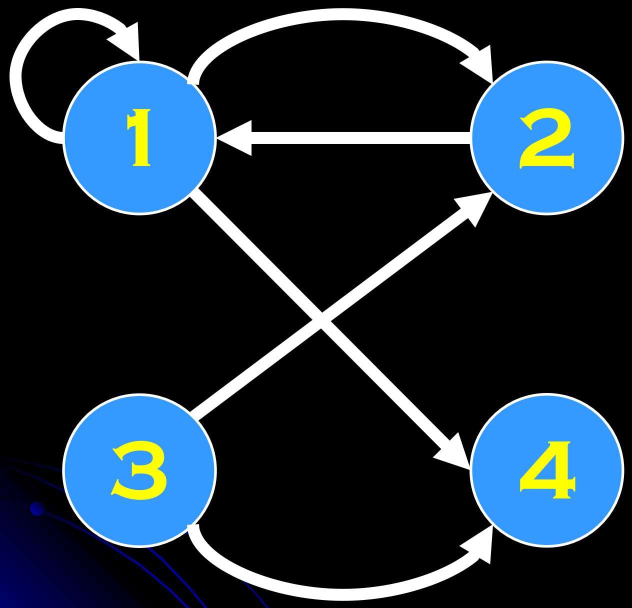
- $R = \{ (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4) \}$
- $S = \{ (1, 2), (1, 3), (2, 1), (2, 2), (2, 4), (4, 4) \}$
- $R \cup S = \{ (1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 4),$
  $(3, 2), (3, 4), (4, 4) \}$

R	1	2	3	4
1	√	√		√
2	√			
3		√		√
4				

S	1	2	3	4
1		√	√	
2	√	√		√
3				
4				√

关系的基本运算

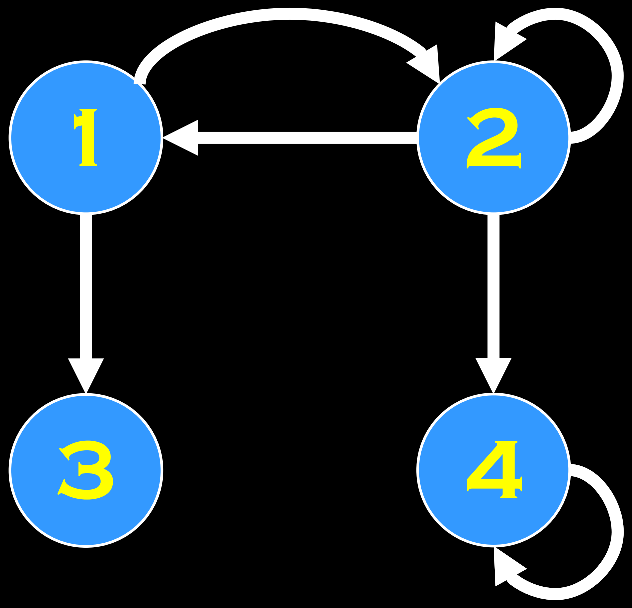
- $R = \{ (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4) \}$
- $S = \{ (1, 2), (1, 3), (2, 1), (2, 2), (2, 4), (4, 4) \}$
- $R \cup S = \{ (1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 4),$
  $(3, 2), (3, 4), (4, 4) \}$



R

R	1	2	3	4
1	1	1	0	1
2	1	0	0	0
3	0	1	0	1
4	0	0	0	0

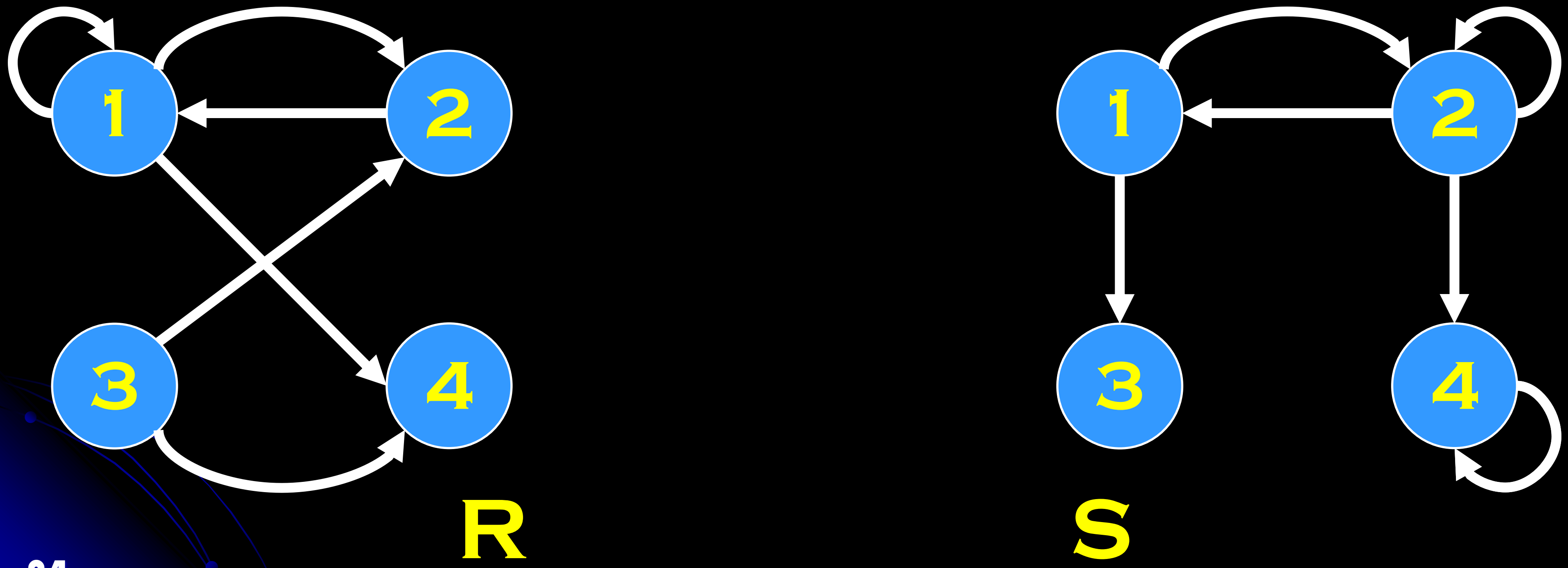
S	1	2	3	4
1	0	1	1	0
2	1	1	0	1
3	0	0	0	0
4	0	0	0	1



S

关系的基本运算

- R = { (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4) }
- S = { (1, 2), (1, 3), (2, 1), (2, 2), (2, 4), (4, 4) }
- $R \cup S$ = { (1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 4), (3, 2), (3, 4), (4, 4) }



关系的基本运算

- 假设 A、B、C 是集合，R 为 A 到 B 的关系，S 为 B 到 C 的关系，则 $S\circ R$ 表示 A 到 C 的一个关系

$S\circ R=\{(a, c) \mid$

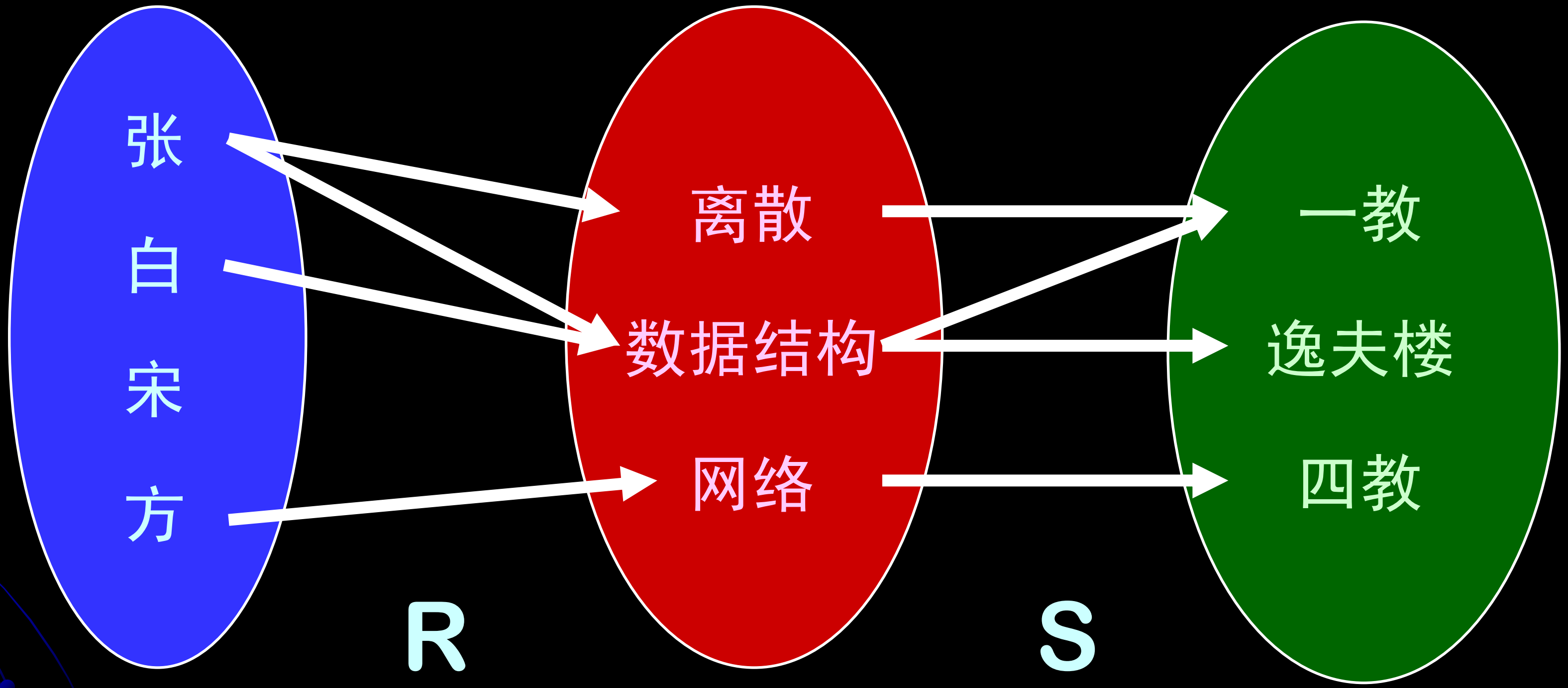
$a\in A, c\in C$, 存在 $b\in B$ 使得 $(a, b)\in R$ 且 $(b, c)\in S\}$，

称为 R 和 S 的**复合（composition）关系**或**合成关系**。

关系的基本运算

R={ (张, 数据结构), (张, 离散数学),
(白, 数据结构), (方, 计算机网络)},

S={ (数据结构, 逸夫楼), (数据结构, 一教),
(离散数学, 一教), (计算机网络, 四教)}

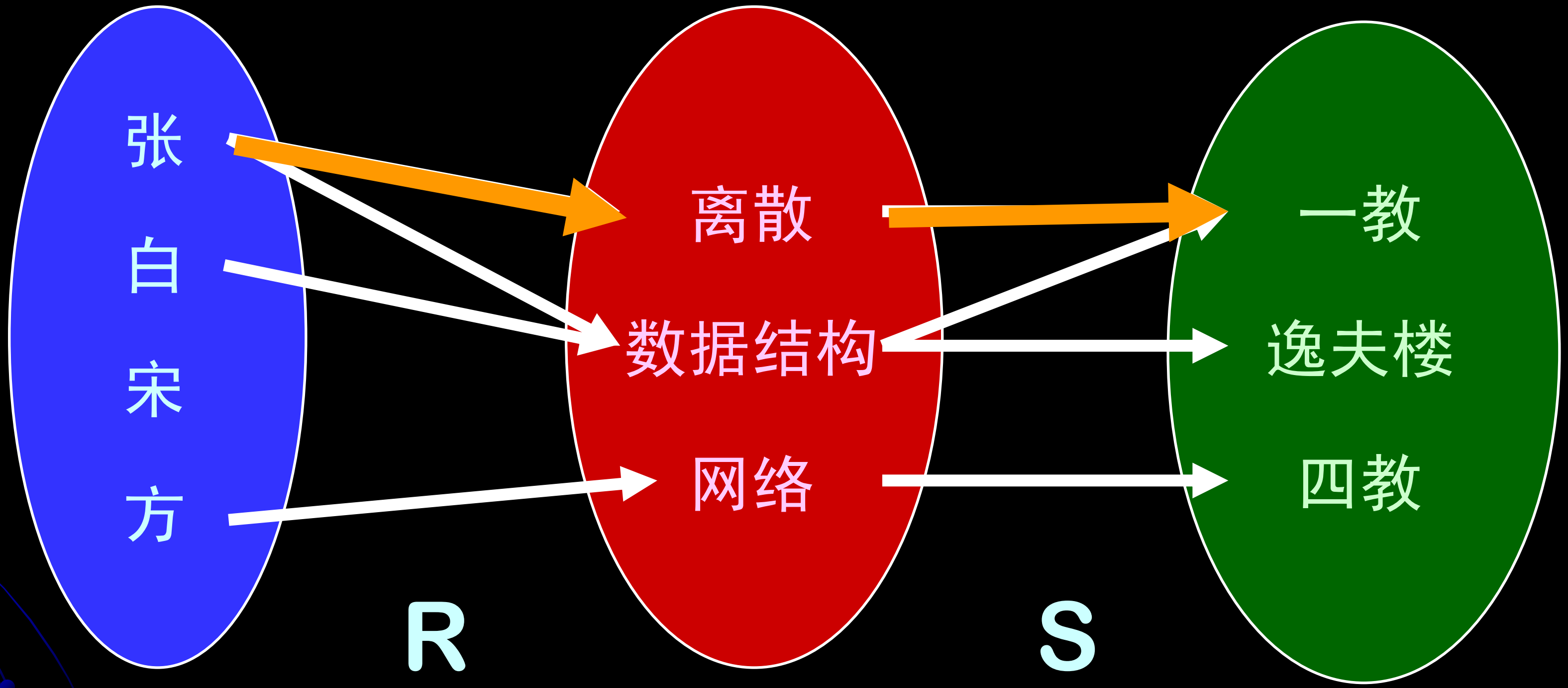


关系的基本运算

$S \circ R = ?$

$= \{$ (张,一教),(张,逸夫楼),(白,逸夫楼),

(白,一教), (方,四教) $\}$



关系的基本运算

- R={ (1, 1), (1, 2), (1, 4), (2, 1), (3, 2), (3, 4) }
- S={ (1, 2), (1, 3), (2, 1), (2, 2), (2, 4), (4, 4) }
- $R \circ S$ = ?
- $R \circ S$={ (1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (2, 4) }

- $S \circ R$ = ?
- $S \circ R$ ={ (1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (3, 1), (3, 2), (3, 4) }

关系的基本运算

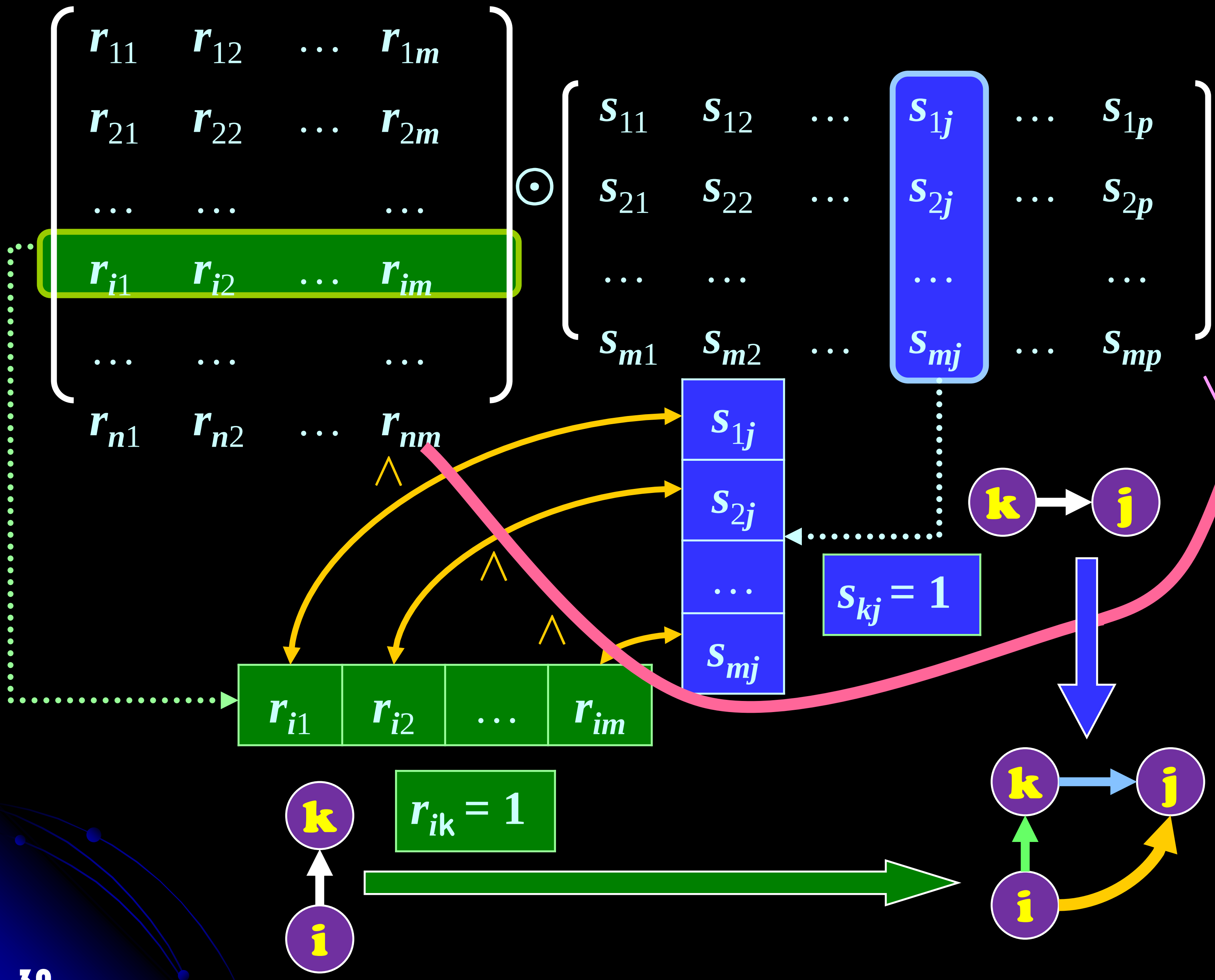
- 假设 *A*、*B*、*C* 为有限集合，*R* 为 *A* 到 *B* 的关系，*S* 为 *B* 到 *C* 的关系，则有

$$M_{S\circ R} = M_R \odot M_S$$

$$M_R=\begin{bmatrix}1&1&0&1\\1&0&0&0\\0&1&0&1\\0&0&0&0\end{bmatrix}\quad M_S=\begin{bmatrix}0&1&1&0\\1&1&0&1\\0&0&0&0\\0&0&0&1\end{bmatrix}\quad M_R\odot M_S=\begin{bmatrix}1&1&1&1\\0&1&1&0\\1&1&0&1\\0&0&0&0\end{bmatrix}$$

V.S.

$$M_{S\circ R}=\begin{bmatrix}1&1&1&1\\0&1&1&0\\1&1&0&1\\0&0&0&0\end{bmatrix}$$



r_{11} r_{12} … r_{1m}
r_{21} r_{22} … r_{2m}
r_{i1} r_{i2} … r_{im}
r_{n1} r_{n2} … r_{nm}
$\odot$
s_{11} s_{12} … s_{1j} … s_{1p}
s_{21} s_{22} … s_{2j} … s_{2p}
s_{m1} s_{m2} … s_{mj} … s_{mp}
c_{ij}
$\vee$
$\wedge$
s_{1j}
s_{2j}
…
s_{mj}
k
j
$s_{kj} = 1$
r_{i1} r_{i2} … r_{im}
k
i
$r_{ik} = 1$
k
j
i

关系的基本运算

	关系矩阵形式	关系图形式
$\overline{R}$	$M_{\overline{R}} = \overline{M_R}$	图的“补”
$R \cap S$	$M_{R\cap S} = M_R \wedge M_S$	图的“交”
$R \cup S$	$M_{R\cup S} = M_R \vee M_S$	图的“并”
R^{-1}	$M_{R^{-1}} = (M_R)^T$	每条边做“反向”
$S \circ R$	$M_{S\circ R} = M_R \odot M_S$	

End