Astron 5205 Project 4 Writeup

Group 5

April 2025

1 Introduction

In this paper, we seek to simulate the internal structure and composition of Trappist-1f using ExoPlex models given the measured mass and radius. Trappist-1f is a confirmed exoplanet that is thought to be both a rocky planet and in the habitable zone around its star. It is in a multi-planetary system, and it has well-confined radius and mass measurements, which makes it ideal for internal modeling. Furthermore, internal modeling may recontextualize Trappist-1f's habitability potential, and provide a reference for how composition affects the overall mass-radius relationship for all planets and exoplanets.

2 Methodology

There were four steps to our work in this project: composition analysis, contextualization, structure modeling and tuning, and mantle mineralogy comparison. First, we utilized the obtained host star composition of iron, magnesium, and silicon to find the silicon-magnesium ratio and the iron-magnesium ratio. This was accomplished using the definition of a dex composition value and working back to find the molar ratios. The planet ratios were taken to be equivalent to the host star.

Next, we found the stellar flux at the distance Trappist-1f orbits, and used this to find the total incident luminosity of the planet from its host star. We used the following equation to find the stellar flux:

$$f_d = \frac{L_*}{4\pi d^2} \tag{1}$$

where L_* is the luminosity of Trappist-1f, and d is its semi-major axis. Equation 1 dictates how the power given off by a star is distributed at a given given distance from that star. This flux is then used to find the total incident luminosity of the planet via:

$$I = \pi r^2 f_d \tag{2}$$

where r is the radius of Trappist-1f. This incident flux informs the temperature of the planet (and thus its habitability) and puts it into context of other planets

and exoplanets.
Then, we calculated the expected radius and density of the planet given the measured mass. These qualities will be used to model the planet's interior structure and composition by introducing the constraints specific to Trappist-1f. These are calculated by a naive density calculation of mass over volume, and carrying uncertainty through as necessary to obtain reasonable bounds. We continued by feeding these values into ExoPlex to determine a range of possible solutions to the interior structure of the planet. We explored several 2-layer structures that worked. Most of the work was done by manually adjusting the inputs into ExoPlex, the full details of which is detailed in the results section.
Finally, we compared the modeled minerology of Trappist-1f with Earth's. These are given directly as a result from ExoPlex, and merely needed to be interpreted.

3 Results

The values for the abundance of the Trappist-1 system were given. The results of the stellar composition analysis, taken to be planet's composition, is summarized by the following table:

Ratio	Value (dex)
Fe/H	0.05350
Mg/H	0.1635
Si/H	0.0785
Si/Mg	0.6167
Fe/Mg	0.7762

The stellar flux was calculated to be 506.44 $\frac{\mathrm{W}}{\mathrm{m}^2}$, and the total incident luminosity as $7.068 \cdot 10^{16}$ W. For context, the figure 1 shows the incident luminosity and stellar flux of the solar system planets in relation to Trappist-1f.

This result puts Trappist-1f in the habitable zone, despite having less total incident luminosity and less stellar flux than Earth, although it is quite close to the edge.
Next, we calculated a base structure for the model, including the expected radius and core fractions, including their uncertainties. The density was calculated as a naive bulk density, while the core fractions were given by a starting ExoPlex model. The results can be summarized in the following table, where CMF is the core mass fraction and CRF is the core radius fraction. The mass and radius are given in terms of Earth. The error on the CMF was not significant to the figures given by the model.

Mass	$1.039^{+0.031}_{-0.031}$
Radius	$1.011^{+0.009}_{-0.008}$
CMF	0.3368
CRF	$0.5332^{+0.0004}_{-0.0004}$
Density	$5525^{+15.5}_{-35.6}\ \frac{\mathrm{kg}}{\mathrm{m}^3}$

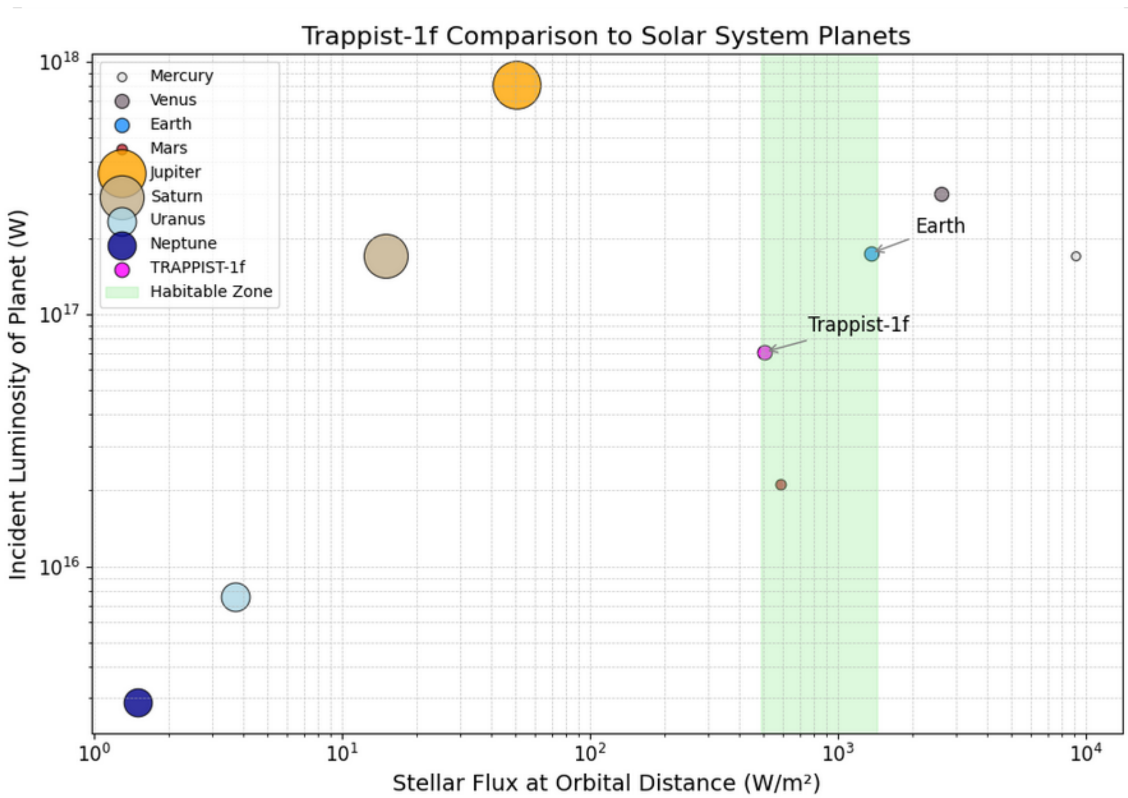

Figure 1: A plot relating stellar flux to incident luminosity of the solar system planets and Trappist-1f.

Using these values as a base, we then ran multiple different ExoPlex models that explained the observed mass and expected radius. Figure 2 overviews the 8 different trials we made. Trials 1 and 2 were obtained by simulating with no core impurities, and adjusting Fe/Mg and FeO as necessary. Trials 3-6 were made by using Trappist-1 Fe/Mg, assuming no FeO in the mantle, and adjusting the core impurities as needed. Trial 7 utilized Trappist-1 Fe/Mg and adjusted FeO in the mantle and impurities in the core. Trial 8 was the last trial, and put together everything learned from the other trials to create the most realistic interior structure. Figure 3 shows Trial 8's interior structure up close. This gives us a good idea as to what the most realistic interior structure of Trappist-1f may be. The final values are notably all fairly close to Earth.

Finally, using ExoPlex, we are able to compare the modeled mineralogy of Trial 8 to Earth. The values are summarized in the following table, and each is quite close to terran values.

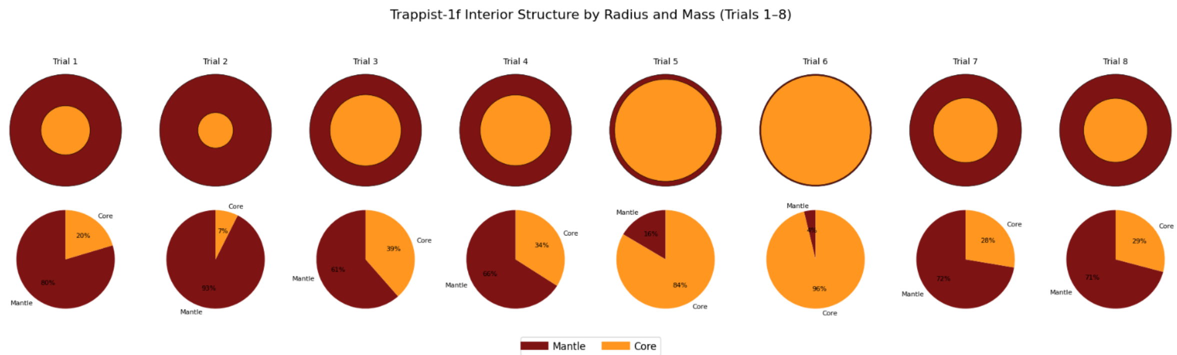

Figure 2: An overview of every ExoPlex model we generated with the proper radius and mass.

Mineral	Trial 8	Earth
FeO	8.52%	8.18%
SiO_2	36.5%	44.71%
MgO	45.4%	38.73%
CaO	4.42%	3.17%
Al_2O_3	5.17%	3.98%

4 Conclusion

We have shown that Trappist-1f is a planet lying within its star's habitable zone that likely has a composition and structure similar to Earth. Trappist-1f receives a moderate stellar flux, placing it near the outside edge of the habitable zone. Our models suggest that Trappist-1f has a core mass fraction of around 29% to 34%, a radius fraction of around 57%, and a density of 5525 $\frac{\text{kg}}{\text{m}^3}$, which is comparable to Earth. Mineralogy analysis suggests a similar mantle composition to that of Earth, with slightly elevated MgO and slightly lowered SiO_2. Overall, our results highlight the importance of stellar refractory abundances in constraining planetary interior models and suggest that Trappist-1f is a differentiated rocky planet, which is similar in many ways to Earth.

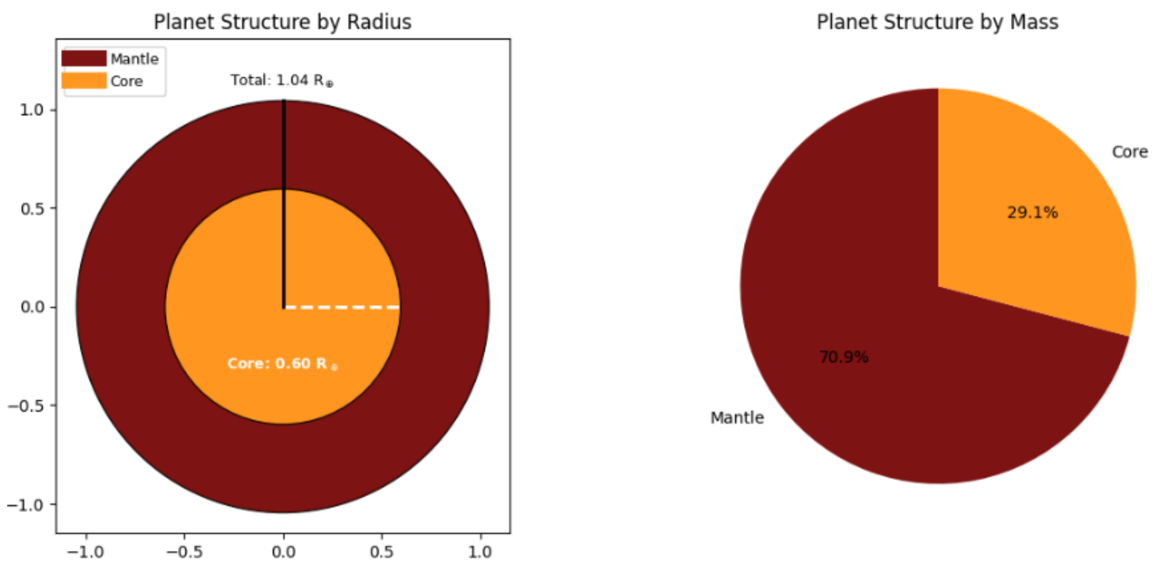

Figure 3: A close up of the final trial (Trial 8) mass radius results from ExoPlex.