# V19 exam notes

## MNF130V2020

May 24, 2020

# 1 Prop. logic

### c)

What we want to show:

$$(p \wedge \neg q) \to \neg r \equiv (p \wedge r) \to q$$

Using identities:

$$(p \wedge \neg q) \to \neg r \equiv \neg(p \wedge \neg q) \vee \neg r \equiv (\neg p \vee q) \vee \neg r$$

$$(\neg p \vee q) \vee \neg r \equiv (\neg p \vee \neg r) \vee q \equiv (p \wedge r) \to q$$

### d)

Truth value of:

$$\exists n \forall m P(m,n)$$

F, just set $m = n + 1$.

Truth value of:

$$\forall n \exists m P(n,m)$$

T, take n and set $m = n$.

# 2 2 Set theory and functions

### a)

$$A - (B \cap C) = (A - B) \cap (A - C)$$

Counterexample:

$$A = \{1,2,3\}, B = \{2,3\}, C = \{3\}$$

$$A - (B \cap C) = \{1,2,3\} - (\{2,3\} \cap \{3\}) = \{1,2,3\} - \{3\} = \{1,2\}$$

$$(A - B) \cap (A - C) = (\{1,2,3\} - \{2,3\}) \cap (\{1,2,3\} - \{3\}) = \{1\} \cap \{1,2\} = \{1\}$$

**b)**

$$A-(B\cap C)=(A-B)\cup(A-C)$$

Either do Venn diagram (this one is easy to visualize), or:

$$\begin{aligned} x\in(A-(B\cap C)) &\equiv x\in A\wedge\neg x\in(B\cap C)\\ &\equiv x\in A\wedge\neg(x\in B\wedge x\in C)\\ &\equiv x\in A\wedge(x\notin B\wedge x\notin C)\end{aligned}$$

$$\begin{aligned} &\equiv((x\in A\wedge(x\notin B))\vee(x\in A)\wedge(x\notin C))\\ &\equiv(x\in(A-B))\vee(x\in(A-C))\\ &\equiv x\in((A-B)\cup(A-C))\end{aligned}$$

**c)**

Injective: Yes. For all integers $n$ and $m$, $m\neq n$, then $3n\neq 3m$
Surjective: No. No instance, where $3n=2$
Bijective: No.

## 3 3 Number theory

Let $a=bq+r$, where $a,b,q,r$ are integers. Prove that $gcd(a,b)=gcd(b,r)$.
Let $S$ be the set of common divisors of $a$ and $b$, $S=\{c|c|a\wedge c|b\}$, and let $T$ be the set of common divisors of $b$ and $r$, $T=\{c|c|b\wedge c|r\}$. We prove that $S=T$ by showing that $S\subseteq T$ and $T\subseteq S$.
Let $c\in S$. Then there exist integers $k,l$ such that $a=ck$ and $b=cl$. Hence $r=a-bq=c(k-lq)$. Because $k-lq$ is an integer scaled by c and $c|r$, $c\in T$.

Let $c\in T$. Then there exist integers $k,l$ such that $b=ck$ and $r=cl$. Hence $a=bq+r=c(kq+l)$. Because $kq+l$ is an integer, $c|a$ and $c\in S$. Because $gcd(a,b)$ is the maximal element of $S$ and $gcd(b,r)$ is the maximal element of $T$ and $S=T$, it will follow that $gcd(a,b)=gcd(b,r)$.

## 4 4 is not worth wasting more time on.

## 5 5 Counting

**a)**

In each group, there will be $\binom{5}{2}$ games. With 4 total groups, this will be:

$$\binom{5}{2}+\binom{5}{2}+\binom{5}{2}+\binom{5}{2}=4\cdot\binom{5}{2}=4\frac{5!}{2!3!}=4\frac{5\cdot 4}{2}=40$$

**b)**

In each group, there can be 5 possible winners. With 4 groups and by the product rule, this becomes:

$$5\cdot 5\cdot 5\cdot 5=5^4$$

**c)**

For the first group, there is a total of $\binom{20}{5}$ combinations. Next group: $\binom{15}{5}$, then $\binom{10}{5}$ and only one configuration for the last group:

$$\binom{20}{5} \cdot \binom{15}{5} \cdot \binom{10}{5} = \frac{20}{5!15!} \cdot \frac{15!}{5!10!} \cdot \frac{10!}{5!5!} = \frac{20!}{(5!)^4}$$

## 6 6 Relations

**a)**

**Reflexive:** for every integer $x$, $x \equiv x(\mod 3)$, so $(x,x) \in R$.
**Symmetric:** for all intergers $x,y,x \equiv y(\mod 3) \leftrightarrow x \mod 3 = y \mod 3$, hence $(x,y) \in R$ which implies $(y,x) \in R$.
**Transitive:** for all integers $x,y,z$, assume that $(x,y) \in R$ and $(y,z) \in R$, then $x \mod 3 = y \mod 3$ and $y \mod 3 = z$ mod 3. Hence $x \mod 3$ and therefore $(x,z) \in R$

$$[2]_R = \{x \in \mathbb{Z} | x \equiv 2(\mod 3)\} = \{x \in Z | x \mod 3 = 2\} = \{x \in \mathbb{Z} | x = 3k + 2 \ for \ some \ integer \ k\}$$

Hence $[2]_R$ is the set of integers which have 2 as a remainder after division by 3, that is the set of integers which can be written as 2 plus an integer multiple of 3.

**b)**

The $\vee$ statement breaks transitivity for the relation. To show with a counterexample:

$$x = 2, y = 0, z = 3 \Rightarrow x \equiv y(\mod 2) \wedge y \equiv z(\mod 3) \therefore (x,y) \in R \wedge (y,z) \in R$$

But

$$x \mod 2 = 0 \neq 1 = z \mod 2 \wedge x \mod 3 = 2 \neq 0 \Rightarrow (x,z) \notin R$$