Matrices

Rosen Section 2.6

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Matrices

An $m \times n$ **matrix** is a rectangular array of numbers with m rows and n columns. If $m = n$, the matrix is called *square*. Two matrices are *equal* if they have the same number of rows and columns and the same entries in every position.

We write $\mathbf{A} = (a_{ij})$ as a shorthand for

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

The **sum** of two $m \times n$ matrices $\mathbf{A} = (a_{ij})$ and $\mathbf{B} = (b_{ij})$ is the matrix $\mathbf{A} + \mathbf{B} = (a_{ij} + b_{ij})$.

The **product** of an $m \times r$ matrix $\mathbf{A} = (a_{ij})$ and $r \times n$ matrix $\mathbf{B} = (b_{ij})$ is the $m \times n$ matrix $\mathbf{AB} = \left(\sum_{k=1}^{r} a_{ik} b_{kj}\right)$.

This should all be known from linear algebra course ...

Boolean/logical/zero-one matrices

Boolean, **logical** or **zero-one matrices** whose entries are either 0 (false) or 1 (true) are often used to represent *discrete structures*, such as trees or graphs.

Recall **logical operators on bits**

$$b_1 \wedge b_2 = \begin{cases} 1 & \text{if } b_1 = b_2 = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$b_1 \vee b_2 = \begin{cases} 1 & \text{if } b_1 = 1 \text{ or } b_2 = 1 \\ 0 & \text{otherwise} \end{cases}$$

Boolean/logical/zero-one matrices

The **join** of two Boolean $m \times n$ matrices $\mathbf{A} = (a_{ij})$ and $\mathbf{B} = (b_{ij})$ is the Boolean matrix $\mathbf{A} \vee \mathbf{B} = (a_{ij} \vee b_{ij})$.

The **meet** of two Boolean $m \times n$ matrices $\mathbf{A} = (a_{ij})$ and $\mathbf{B} = (b_{ij})$ is the Boolean matrix $\mathbf{A} \wedge \mathbf{B} = (a_{ij} \wedge b_{ij})$.

The **Boolean product** of a Boolean $m \times r$ matrix $\mathbf{A} = (a_{ij})$ and Boolean $r \times n$ matrix $\mathbf{B} = (b_{ij})$ is the Boolean $m \times n$ matrix $\mathbf{A} \odot \mathbf{B} = \left(\bigvee_{k=1}^{r}(a_{ik} \wedge b_{kj})\right)$.