Homework 2
Due Date: February 5th, 2021

1. Describe the following prepreg laminate fabrication processes and the equipment involved for each: cutting, tooling, layup, bagging and curing. (**20 points**)

2. Describe the types of distortion that occur in a laminate due to 1) a mismatch in thermal expansion and 2) a mismatch in elastic constants during testing. Why and when do they arise? What might the resulting distorted shapes look like for a [0/90] non-symmetric laminate? (**20 points**)

3. A unidirectional carbon fiber composite plate is rigidly bonded to a tool to cure. The fiber properties are $E_f = 320\ GPa$, $\nu_f = 0.21$, $\alpha_f = 0.3 \cdot 10^{-6}\ C^{-1}$, and the matrix properties are $E_m = 2.4\ GPa$, $\nu_m = 0.42$, $\alpha_f = 90 \cdot 10^{-6}\ C^{-1}$. The fibers have a $V_f = 65\%$ volume fraction. After the part has cured in an autoclave at $T = 80°C$, it is brought back to room temperature ($T = 20°C$). What is the stress in the plate after it has been brought back to room temperature while it is still attached to the tool? Assume the tool has a negligible thermal expansion. (**20 points**)

4. A glass fiber reinforced epoxy composite with stiffness $\underline{C} = \begin{bmatrix} 46.5 & 2.1 & 0 \\ 2.1 & 6.0 & 0 \\ 0 & 0 & 1.6 \end{bmatrix} GPa$ (in the fiber direction for a single ply) is laminated in a [0/90]$_s$ configuration. Each laminate layer has the same thickness. The laminate has a global stress applied of $\underline{\sigma} = \begin{bmatrix} 20 \\ -5 \\ 5 \end{bmatrix} MPa$ with σ_x aligned with the 0° fiber direction. What is the stress in the outer 0° layer? (**20 points**)

5. Two CFRP laminates have stackings of 1) [0°/45°/90°/-45°] and 2) [0°/90°]$_s$. The composite global stiffness tensors are $\underline{C}_1 = \begin{bmatrix} 91.05 & 30.59 & 0 \\ 30.59 & 91.05 & 0 \\ 0 & 0 & 30.23 \end{bmatrix}$ and $\underline{C}_2 = \begin{bmatrix} 115.85 & 5.79 & 0 \\ 5.79 & 115.85 & 0 \\ 0 & 0 & 5.42 \end{bmatrix}$ respectively. Which, if either, laminate is (approximately) balanced? Show the work you did to arrive at your answer. (*Hint: you can use the rotation function from the previous homework*). (**20 points**)

6. Using the modified shear lag model, plot the stiffness of a SiC ($E_f = 192\ GPa$, $\nu_f = 0.16$) reinforced Al ($E_m = 72\ GPa$, $\nu_m = 0.33$) composite with a $V_f = 35\%$ for varying reinforcement aspect ratios $s = 1 - 10$. Assume all the reinforcements are aligned. Compare this against the corresponding stiffness prediction for an equivalent long-fiber composite using a rule of mixtures estimate. (**20 points**)

Coding

7. a) Write a function to generate the stiffness tensor for an arbitrary laminate stacking. The inputs should be the stiffness tensor for a single type of laminate ($\underline{C}$), a list of ply angles ($[\theta_1, \theta_2, \dots, \theta_n]$) and a list of ply thicknesses ($[t_1, t_2, \dots, t_n]$). It should output a global laminate stiffness tensor $\underline{\bar{C}}$. (**15 points**)

   b) Use your laminate stiffness tensor function to generate a stiffness tensor for a carbon fiber reinforced epoxy composite laminate. The stiffness of each ply is $\underline{C} = \begin{bmatrix} 151 & 3.4 & 0 \\ 3.4 & 16.7 & 0 \\ 0 & 0 & 4.7 \end{bmatrix}$, the composite layup is $[0°/90°/60°/-60°/30°/-30°]$, and the thicknesses are $t_{0°} = t_{90°} = 1\ mm$ and $t_{\pm 30°} = t_{\pm 60°} = 0.5\ mm$. (**15 points**)

   As a reference to check your function, a [0°/45°/90°/-45°]$_s$ laminate with the same stiffness tensor and a thickness of $t = 0.5\ mm$ for each ply should give a stiffness tensor of $\underline{\bar{C}} = \begin{bmatrix} 66.09 & 21.16 & 0 \\ 21.16 & 66.09 & 0 \\ 0 & 0 & 22.46 \end{bmatrix}$. If the thicknesses of the same laminate are changed to $t_{0°} = t_{90°} = 0.5\ mm$ and $t_{\pm 45°} = 1\ mm$, the resulting stiffness tensor will be $\underline{\bar{C}} = \begin{bmatrix} 60.17 & 27.08 & 0 \\ 27.08 & 60.17 & 0 \\ 0 & 0 & 28.38 \end{bmatrix}$.

8. The laminate in the previous problem has an applied strain of $\underline{\varepsilon} = \begin{bmatrix} 600\ \mu m/m \\ 300\ \mu m/m \\ -120\ \mu rad \end{bmatrix}$ with ε_x aligned with the 0° fiber direction. Find the stress in the 90° ply and plot the axial stress component σ_{xk} at angles from $\theta = 0° \rightarrow 90°$ in 10° increments. (*Note: you should be rotating the stress σ_{1k} that you obtain in the 90° ply to obtain σ_{xk} at various angles*). (*Hint: you should be able to use your rotation functions from the previous homework*). (**20 points**).

Total value of this homework = 12.5% of the course grade (150 points)

Note: The total adds up to 170 points. You can choose 5/6 of the first 6 problems to solve or solve all of them for potential extra credit.