ME 450 HW 2

1. Before cutting the prepreg, the manufacturer usually creates templates of the features of the part or parts that they wish to create. These templates are laid on to the sheet of prepreg and the manufacturer may trace the template outlines on to the prepreg. After this, the manufacturer cuts the prepreg into the desired shapes. There is a variety of methods which the manufacturer may employ to cut the prepreg. Simple and cheap, yet less precise methods include cutting the prepreg with scissors or an x-acto knife. CNC cutting machines offer more precision and greater mass-production capability at a drastically higher price point. Such machines may use either a laser or a scalpel to cut the material.

   The tooling is the mold that the prepreg will be shaped to in order to create the desired form factor. Though the prepreg tooling can be made from a wide variety of materials, the choice of tool material is dependent upon the production quantity and desired shape. For tools that will be used in long production runs, harder materials such as Invar and steel are suitable as they will maintain surface finish and dimensional accuracy better than softer materials. Soft tools may be constructed easier than hard tools and generally are less expensive to acquire and shape.

   The layup is when the manufacturer shapes the prepreg to the tool. Generally, the manufacturer first lays down a release film on the tool that will allow the cured prepreg to decouple from the tool. This may also be accomplished with a spray-on release agent. Then the manufacturer lays down the prepreg layers in their desired orientations. A second layer of release film is laid on top of the prepreg. Then, the manufacturer may add a layer of bleeder, which absorbs excess resin from the layup. The manufacturer may also add a layer of breather, which creates air pathways for air to evacuate when a vacuum is applied.

   The layup is then bagged inside of a film and a vacuum is applied to the contents inside. The application of the vacuum creates a pressure on the layup inside and may squeeze any excess resin out, which is then absorbed in the bleeder. A vacuum pump and vacuum fitting are required to create the vacuum inside the bagging.

   Once bagged, the layup may be inserted into either an oven or autoclave, where the layup is heated and the resin in the prepreg cures.

2. As a lamina usually has a much larger expansion coefficient in the transverse direction than in the axial direction, heating of a crossply laminate will lead to the transverse expansion of each ply being strongly inhibited by the presence of the other ply. This leads to internal stresses developing in the laminate. These internal stresses vary in the transverse direction of the laminate and cause a compression/tension differential which causes the laminate to bend. In the case of a [0/90] non-symmetric laminate, this thermal expansion leads to a saddle-shaped distortion.

   When a well-bonded laminate is subjected to uniaxial tension the poisson's ratios of the different plies may be different in the loading orientation. This difference in poisson's ratios may cause the different plies to try to contract at different magnitudes in the in-plane direction perpendicular to the loading, and to try to expand at different magnitudes in the in-plane direction parallel to the loading. This difference in contraction and expansion tendency causes a stress differential to form in the transverse out-of-plane direction, which in a [0/90] non-symmetric laminate leads to a bent U-shape.

3. $E_f = 320 \cdot 10^9\ Pa$ $E_m = 2.4 \cdot 10^9\ Pa$

$\nu_f = 0.21$ $\nu_m = 0.42$ $V_f = 0.65$

$\alpha_f = 0.3 \cdot 10^{-6}\ C^{-1}$ $\alpha_m = 90 \cdot 10^{-6}\ C^{-1}$ $T_1 = 80^\circ C \longrightarrow T_2 = 20^\circ C$

$$\alpha_{11} = \frac{\alpha_f V_f E_f + \alpha_m (1 - V_f) E_m}{V_f E_f + (1 - V_f) E_m} = 6.61 \cdot 10^{-7}$$

$$\alpha_{22} = \alpha_f V_f (1 + \nu_f) + \alpha_m (1 - V_f)(1 + \nu_m) - \alpha_{11} \nu_{12} = 4.5 \cdot 10^{-5}$$

$$\nu_{12} = \nu_f V_f + \nu_m (1 - V_f)$$

$$\varepsilon_{11} = \alpha_{11} \Delta T = 0.00004$$

$$\varepsilon_{22} = \alpha_{22} \Delta T = 0.0027$$

$$E_1 = E_f V_f + E_m (1 - V_f) = 2.09 \cdot 10^{11}\ Pa$$

$$E_2 = \left(\frac{V_f}{E_f} + \frac{1 - V_f}{E_m} \right)^{-1} = 6.76 \cdot 10^9\ Pa$$

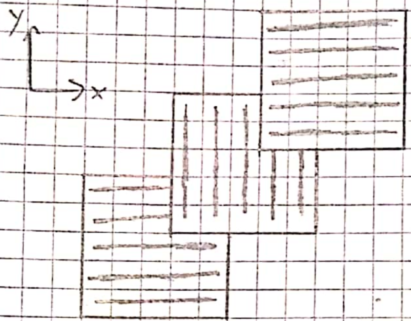
$$\sigma_{11} = \varepsilon_{11} E_1 = 8.36 \cdot 10^6\ Pa$$

$$\sigma_{22} = \varepsilon_{22} E_2 = 1.825 \cdot 10^7\ Pa$$

$$C_{0^\circ} = \begin{bmatrix} 46.5 & 2.1 & 0 \\ 2.1 & 6.6 & 0 \\ 0 & 0 & 1.6 \end{bmatrix} GPa \quad \text{in } [0/90]_s \quad \text{applies } \sigma = \begin{bmatrix} 20 \\ -5 \\ 5 \end{bmatrix} MPa$$

aligns with fibers

σ_x aligns with fibers



$$C_{0,9} = \frac{t_k C_{0^\circ} + t_k C_{90^\circ} + t_k C_{0^\circ}}{t_k + t_k + t_k}$$

$$C_{90^\circ} = T^{-1} C_{0^\circ} T' = \begin{bmatrix} \cos 90^2 & \sin 90^2 & 2\cos 90 \sin 90 \\ \sin 90^2 & \cos 90^2 & -2\cos 90 \sin 90 \\ -\cos 90 \sin 90 & \cos 90 \sin 90 & \cos 90^2 - \sin 90^2 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} 46.5 & 2.1 & 0 \\ 2.1 & 6.6 & 0 \\ 0 & 0 & 1.6 \end{bmatrix} \cdot 10^9$$

$$\begin{bmatrix} \cos 90^2 & \sin 90^2 & \cos 90 \sin 90 \\ \sin 90^2 & \cos 90^2 & -\cos 90 \sin 90 \\ -2\cos 90 \sin 90 & 2\cos 90 \sin 90 & \cos 90^2 - \sin 90^2 \end{bmatrix}$$

$$C_{90^\circ} = \begin{bmatrix} 6 & 2.1 & 0 \\ 2.1 & 46.5 & 0 \\ 0 & 0 & 1.6 \end{bmatrix} GPa$$

$$C_{0,9} = \frac{1}{3}(C_{0^\circ} + C_{90^\circ} + C_{0^\circ}) = \begin{bmatrix} 33 & 2.1 & 0 \\ 2.1 & 19.5 & 0 \\ 0 & 0 & 1.6 \end{bmatrix} GPa$$

$$S_{0,9} = C_{0,9}^{-1} = \begin{bmatrix} 3.05 \cdot 10^{-11} & -3.28 \cdot 10^{-12} & 0 \\ -3.29 \cdot 10^{-12} & 5.16 \cdot 10^{-11} & 0 \\ 0 & 0 & 6.25 \cdot 10^{-10} \end{bmatrix}$$

$$\sigma_{0^\circ} = C_{0^\circ} T' S_{0,9} \sigma_9 = \begin{bmatrix} 3.53 \cdot 10^7 \\ -3.17 \cdot 10^5 \\ 5 \cdot 10^6 \end{bmatrix} Pa$$

Scanned with CamScanner

5.

1) $[0°/45°/90°/-45°]$

$$C_{1,g} = \begin{bmatrix} 91.05 & 30.59 & 0 \\ 30.59 & 91.05 & 0 \\ 0 & 0 & 30.23 \end{bmatrix}$$

2) $[0°/90°]_s$

$$C_{2,g} = \begin{bmatrix} 115.85 & 5.79 & 0 \\ 5.79 & 115.85 & 0 \\ 0 & 0 & 5.42 \end{bmatrix}$$

Rotate 45°:

$$C_{1,45°} = T^{-1}_{45°} C_{1,g} T'_{45°} = \begin{bmatrix} 91.05 & 30.59 & 0 \\ 30.59 & 91.05 & 0 \\ 0 & 0 & 30.23 \end{bmatrix}$$

Both are balanced?

Rotate 90°:

$$C_{1,90°} = T^{-1}_{90°} C_{1,g} T'_{90°} = \begin{bmatrix} 91.05 & 30.59 & 0 \\ 30.59 & 91.05 & 0 \\ 0 & 0 & 30.23 \end{bmatrix}$$

Rotate 45°:

$$C_{1,-45°} = T^{-1}_{-45°} C_{1,g} T'_{-45°} = \begin{bmatrix} 91.05 & 30.59 & 0 \\ 30.59 & 91.05 & 0 \\ 0 & 0 & 30.23 \end{bmatrix}$$

6.

SiC:

$E_f = 192$ GPa

$\nu_f = 0.16$

Al:

$E_m = 72$ GPa

$\nu_m = 0.33$

$V_f = 0.35$

$$E_{1-mod} = V_f E_f \left(1 - \frac{(E_f - E_m')\tanh(ns)}{E_f n s}\right) + (1 - V_f)E_m$$

$$n = \sqrt{\frac{2E_m}{E_f(1+\nu_m)\ln\left(\frac{1}{V_f}\right)}} = 0.7349$$

$$E_m' = E_f(1 - \operatorname{sech}(ns)) + E_m$$

$$E_{1,mod} = V_f E_f \left(1 - \frac{\left[E_f - E_f(1-\operatorname{sech}(ns)) - E_m\right]\tanh(ns)}{E_f\, ns}\right) + (1 - V_f)E_m$$

↓ plug in n, s values

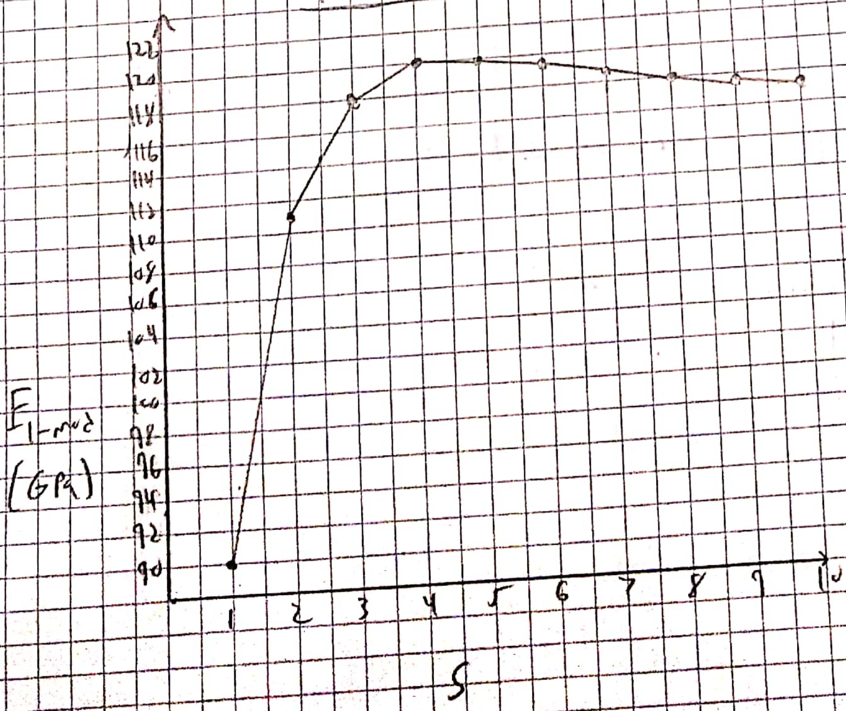
$$E_{1,long} = E_f V_f + E_m(1 - V_f)$$

$E_{1,long} = 1.14 \cdot 10^{11}$ Pa

$\boxed{E_{1,long} = 114 \text{ GPa}}$

s	E_{1-mod}
1	9.075 E10
2	1.114 E11
3	1.186 E11
4	1.201 E11
5	1.199 E11
6	1.193 E11
7	1.1875 $\cdot 10^{11}$
8	1.18 E11
9	1.177 E11
10	1.174 E11

E_{1-mod} vs. s



Untitled

February 7, 2021

[2]:
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###Problem 7

import numpy as np
import math as m

def LaminateC(C,angles,thicknesses):
    i = 0
    angles_rads = []
    for angle in angles:
        angles_rads.append(m.radians(angle))

    Cg_t_sum = np.array([[0,0,0],[0,0,0],[0,0,0]])
    for angle in angles_rads:
        T = np.array([[(m.cos(angle))**2,(m.sin(angle))**2,2*(m.cos(angle))*(m.
↪sin(angle))],[(m.sin(angle))**2,(m.cos(angle))**2,-2*(m.cos(angle))*(m.
↪sin(angle))],[-(m.cos(angle))*(m.sin(angle)),(m.cos(angle))*(m.
↪sin(angle)),(m.cos(angle))**2-(m.sin(angle))**2]])
        T_prime = np.array([[(m.cos(angle))**2,(m.sin(angle))**2,(m.
↪cos(angle))*(m.sin(angle))],[(m.sin(angle))**2,(m.cos(angle))**2,-(m.
↪cos(angle))*(m.sin(angle))],[-2*(m.cos(angle))*(m.sin(angle)),2*(m.
↪cos(angle))*(m.sin(angle)),(m.cos(angle))**2-(m.sin(angle))**2]])
        Cg = np.linalg.inv(T)*C*T_prime
        Cg_t = Cg*thicknesses[i]
        Cg_t_sum = Cg_t_sum + Cg_t
        i+=1

    Cg_total = Cg_t_sum / np.sum(thicknesses)

    return Cg_total




angles = [0,45,90,-45,-45,90,45,0]
thicknesses = [0.0005,0.0005,0.0005,0.0005,0.0005,0.0005,0.0005,0.0005]
C = np.array([[151,3.4,0],[3.4,16.7,0],[0,0,4.7]])
LaminateC(C,angles,thicknesses)
```

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[2]:
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array([[56.625 ,  1.275 ,  0.    ],
       [ 1.275 ,  6.2625,  0.    ],
       [ 0.    ,  0.    ,  2.35  ]])
```

[ ]:
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###Problem 8
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