Homework 2: Due on Feb. 10

Guideline

- Homework should be submitted via Gradescope by Friday midnight (11:59 pm. CDT).

- Homework answers to Simulations and data analysis should be written in R Markdown.

- Please find the following exercise question in [HMC] (Hogg, Mckean, and Craig 2018).

- Each question is worth of 10 points and the total is 50 points.

1. Recall the average weight of 200 newborns were 3500 grams with a standard deviation of 300 grams. It was believed that baby boys and baby girls may weight differently. Hence the weight of a newborn may not follow a normal distribution but rather a mixture of normal distributions.

   (a) Please construct a test $H_0 : \mu \geq 3400$, v.s $H_1 : \mu > 3400$ with $\alpha = 0.05$. Why t-test is no better than z-test here?

   (b) What is the power of the above test? How can we increase the power to 80%?

2. Ex 4.6.6

3. Ex 4.6.8 (Critical region is the rejection region of the test.)

4. A researcher used a simulation study to compare the coverage probabilities of two types of confidence intervals. For the type A, he found that 476 out of 500 constructed confidence intervals contained the true parameter; while for the type B, only 463 out of 500 contained the true parameter.

   (a) Is there a difference between the two types of confidence intervals? Find the p-value of the test and draw a conclusion. Please use $\alpha = 0.05$.

   (b) Now use "prop.test" function in R to check your result in part (a). If the researcher increased the number of simulations from 500 to 1000, would your conclusion in part (a) still hold? Suppose he found 952 out of the 1000 type A confidence intervals contained the true parameter, but only 926 of the 1000 type B CIs contained the true parameter.

5. Continue from the previous question. Although nominal confidence level is 95% for both type of confidence intervals, actually coverage probability for type B is only 93%.

   (a) Consider the test $H_0 : p = 0.95$ v.s $H_1 : p = 0.93$. What is the power of the test with 500 simulations? Manually calculate it first and then R to check it.

   (b) How many simulations should the researcher run to detect the problem of the type B confidence interval with 80% of power? Please use R function "pwr.p.test" from the library "pwr".
   Hint: Note that this is a one-sided test. You can use "ES.h(p1,p2)" in R to calculate the effect size.