

Homework 6: Due on Mar. 24

Guideline

- Homework should be submitted via Gradescope by Friday midnight (11:59 pm. CDT).
 - Homework answers to Simulations and data analysis should be written in R Markdown. Please use `\newpage` in R Markdown to separate answers to different questions. For simulations, please use `set.seed(4211)` to make your result reproducible.
 - Please also upload your executable “xxx.Rmd” file to the corresponding homework assignment on Canvas.
 - Please find the following exercises from the book ”Extending the Linear Model with R” by Faraway, 2016.
 - The total score for the homework is 50 points, with the first two questions each worth 20 points and the last one worth 10 points.
1. Logistic regression. Ex. 2.1, on page 46.
 2. Logistic regression. Ex. 2.2, on page 47.
 3. Check each of the following distributions to see whether it belongs to the exponential family.

(a) Gamma:

$$f(y; \mu, \nu) = \frac{1}{\Gamma(\nu)} \frac{1}{y} \left(\frac{\nu y}{\mu}\right)^\nu \exp\left\{-\frac{\nu y}{\mu}\right\},$$

for $y \geq 0, \nu > 0, \mu > 0$.

(Please compare the above pdf with the classical parameterization: $\text{Gamma}(\alpha, \beta)$:

$$f(y; \alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} y^{\alpha-1} \exp\left\{-\frac{y}{\beta}\right\}, \quad .)$$

(b) Weibull:

$$f(y; \theta, k) = \theta k y^{k-1} \exp\{-\theta y^k\},$$

for $y > 0, \theta > 0, k > 0$.

(Note that the Weibull distributin with $k = 1$ is the exponential distribution.)

(c) Inverse Gaussian:

$$f(y; \mu, \sigma) = (2\pi\sigma^2 y^3)^{-1/2} \exp\left\{-\frac{(\mu - y)^2}{2\sigma^2 \mu^2 y}\right\},$$

for some $\sigma^2 > 0$.

(d) Negative Binomial:

$$f(y; \theta, n) = \binom{n+y-1}{y} (1-\theta)^y \theta^n,$$

for $y = 0, 1, 2, \dots, \theta > 0$.