Stats for DS HW 10

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7.3.10

(a)

```
phat = 985/1516
n = 1516
SE = sqrt(phat*(1-phat)/n)
alpha = qnorm(0.975)
upper = phat + SE*alpha
lower = phat - SE*alpha
upper
```

```
## [1] 0.6737502
```

```
lower
```

```
## [1] 0.6257221
```

(b)

False. The true proportion value and the interval obtained in part a are both non-random constants, so it doesn't make sense to talk about probability with regards to them.

7.4.4

(a)

```r
phatpr = 75/193
alpha = qnorm(0.975)
n = (4*(alpha^2)*phatpr*(1-phatpr))/(0.06^2)
ceiling(n)
```

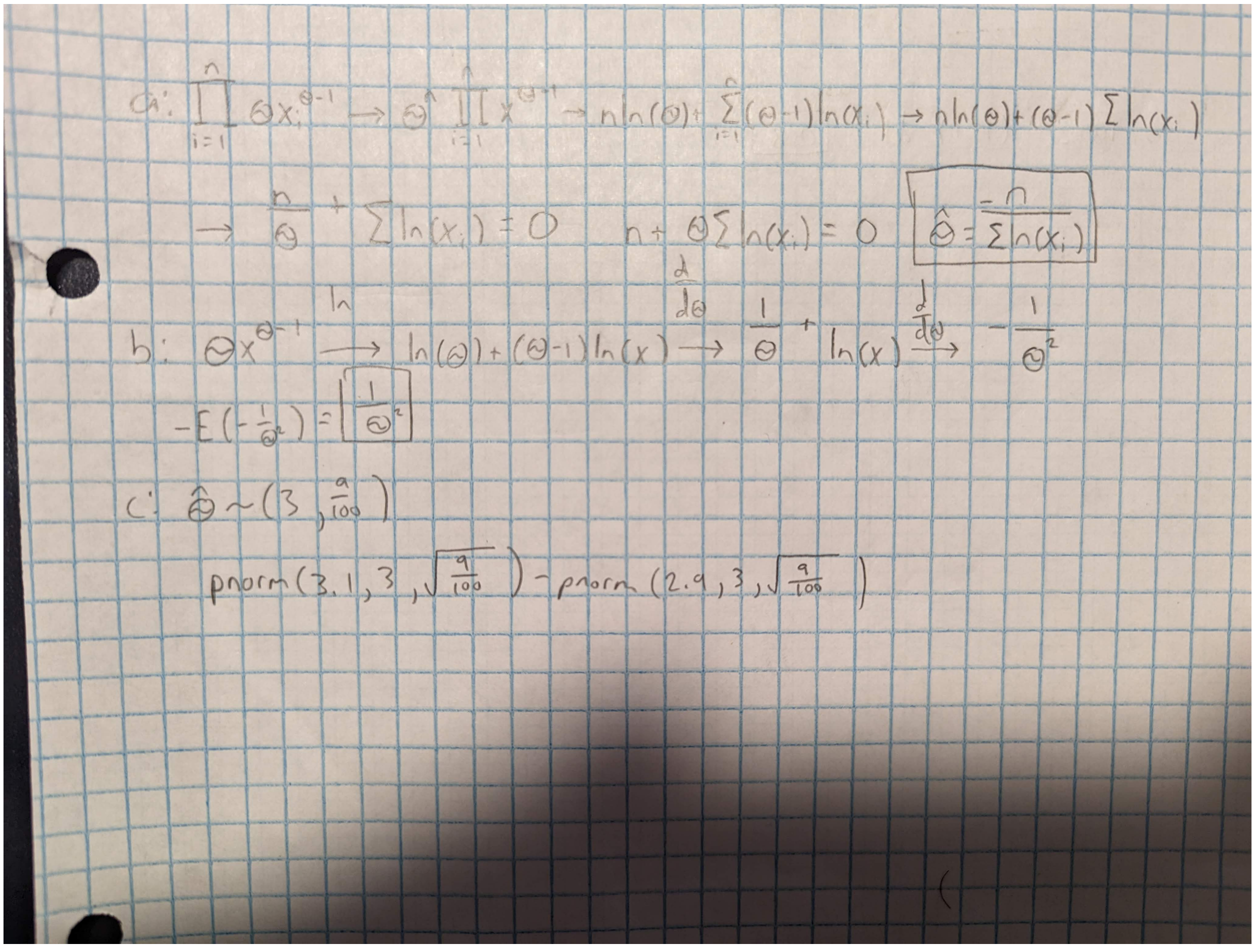
```
## [1] 1015
```

(b)

```r
phatpr = 0.5
alpha = qnorm(0.975)
n = (4*(alpha^2)*phatpr*(1-phatpr))/(0.06^2)
ceiling(n)
```

```
## [1] 1068
```

Additional Problem 1



a: $\prod_{i=1}^{n} \theta x_i^{\theta-1} \to \theta^n \prod_{i=1}^{n} x_i^{\theta-1} \to n\ln(\theta) + \sum_{i=1}^{n} (\theta-1)\ln(x_i) \to n\ln(\theta) + (\theta-1)\sum \ln(x_i)$

$\to \frac{n}{\theta} + \sum \ln(x_i) = 0 \quad n + \theta \sum \ln(x_i) = 0 \quad \boxed{\hat{\theta} = \frac{-n}{\sum \ln(x_i)}}$

b: $\theta x^{\theta-1} \xrightarrow{\ln} \ln(\theta) + (\theta-1)\ln(x) \xrightarrow{\frac{d}{d\theta}} \frac{1}{\theta} + \ln(x) \xrightarrow{\frac{d}{d\theta}} -\frac{1}{\theta^2}$

$-E\left(-\frac{1}{\theta^2}\right) = \boxed{\frac{1}{\theta^2}}$

c: $\hat{\theta} \sim \left(3, \frac{9}{100}\right)$

pnorm$\left(3.1, 3, \sqrt{\frac{9}{100}}\right)$ - pnorm$\left(2.9, 3, \sqrt{\frac{9}{100}}\right)$

Figure 1: Work and answers for 1

```
s = sqrt(9/100)
pnorm(3.1,3,s) - pnorm(2.9,3,s)
```

```
## [1] 0.2611173
```

Additional Problem 2

(2) a: prior distr : $h_\Theta(t) = 4t^3$ for $0<t<1$

$$P_{X|\Theta=t}(x_1,\dots,x_{10}) = t^{x_1}(1-t)^{1-x_1}\cdots t^{x_{10}}(1-t)^{1-x_{10}} = t^{(x_1+\dots+x_{10})}(1-t)^{(10-(x_1+\dots+x_{10}))}$$

$$g_{\Theta|x}(t) = \frac{1}{k}\, t^{\Sigma x_i}(1-t)^{(10-\Sigma x_i)}\, 4t^3 = \frac{4}{k}\, t^{\Sigma x_i+3}(1-t)^{(10-\Sigma x_i)} = \frac{4}{k}\, t^{11}(1-t)^2$$

$\frac{1}{k}\int_0^1 4t^{11}(1-t)^2 = 1$, using software, $k = \frac{1}{273}$

$$g_{\Theta|x}(t) = 1092\, t^{11}(1-t)^2$$

b: using software

$$\int_0^1 1092\, t^{12}(1-t)^2\, dt = \boxed{0.8}$$

Figure 2: Work and answers for 2