Stats for DS HW 6

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4.3.7

(a)

```
expecteda = 0.66*1 + 0.34*2
vara = (0.66*1 + 0.34*4) - expecteda**2
expecteda
```

```
## [1] 1.34
```

```
vara
```

```
## [1] 0.2244
```

(b)

```
joint = data.frame(
  X_equals_one = c(0.132, 0.068),
  X_equals_two = c(0.24, 0.06),
  X_equals_three = c(0.33, 0.17)
)

rownames(joint) = c("y = one", "y = two")
joint
```

```
##         X_equals_one X_equals_two X_equals_three
## y = one        0.132         0.24           0.33
## y = two        0.068         0.06           0.17
```

(c)

0.702

(d)

```r
ans = 0.132/0.702
ans
```

```
## [1] 0.1880342
```

4.3.11

(a)

marginal PDF of X is the integral from 0 to 1 of $x + y$ dy, which is equal to $x+1/2$. Therefore, the conditional PDF is the joint PDF over $(x+1/2)$, so $(x+y)/(x+1/2)$ for $0<x<1$, $0<y<1$.

Taking the integral of the conditional PDF from 0.3 to 0.5, we get $(1/(x+1/2))(0.2x+0.08)$

(b)

By 4.3.16, this is equal to the integral from 0 to 1 of the result we calculated in part (a). This is

```
f = function(x) {(0.2*x+0.08)/(x+0.5)}
integrate(f, lower = 0, upper = 1)
```

```
## 0.1780278 with absolute error < 2e-15
```

4.3.15

The conditional pdf is (1/y)*eˆ(-x/y). This clearly depends on Y, so X and Y are not independent.

4.4.4

(a)

X1+X2+X3+X4+X5+Y1+Y2+Y3

(b)

I am assuming all waiting times are independent. With that in mind, the expected value and variance of the linear combination is just the sums of the expected values and variances of each individual waiting time, so expected value = 33, and the variance = 22.

4.4.8

The marginal PDF of X is 12x(1-x)^2. The expected value of X is the integral from 0 to 1 of x times the marginal PDF of X, which comes out to

```
f2 = function(x) {x*(12*x)*((1-x)**2)}
integrate(f2, lower = 0, upper = 1)
```

```
## 0.4 with absolute error < 4.4e-15
```

0.4. By the symmetry between x and y in the joint pdf, we can see that the expected value of y is also 0.4. The expected value of XY would be xy times the joint pdf of (X,Y), which would be 2/15 (work in photo). Therefore, the covariance is 2/15-0.16 = -2/75.

$$\int\int 24x^2y^2 \qquad x \le 1-y$$

$$24\int_0^1\int_0^{1-y} x^2y^2\,dx\,dy$$

$$\Big|_0^{1-y} \frac{y^2x^3}{3} \to 24\int_0^1 \frac{y^2(1-y)^3}{3}\,dy$$

$$u=1-y \qquad du=-1 \qquad y=1-u$$

$$8\int_0^1 y^2(1-y)^3\,dy \qquad \int -(1-u)^2u^3$$

$$\int -(1-2u+u^2)u^3 \to \int -u^3+2u^4-u^5$$

$$8\Big|_0^1 -\frac{u^4}{4}+\frac{2u^5}{5}-\frac{u^6}{6} \to 8\Big|_0^1 \frac{-(1-y)^4}{4}+\frac{2(1-y)^5}{5}+\frac{-(1-y)^6}{6}$$

$$\boxed{\frac{2}{15}}$$

Figure 1: Work for 4.4.8