Mathematical Induction

Murat Osmanoglu

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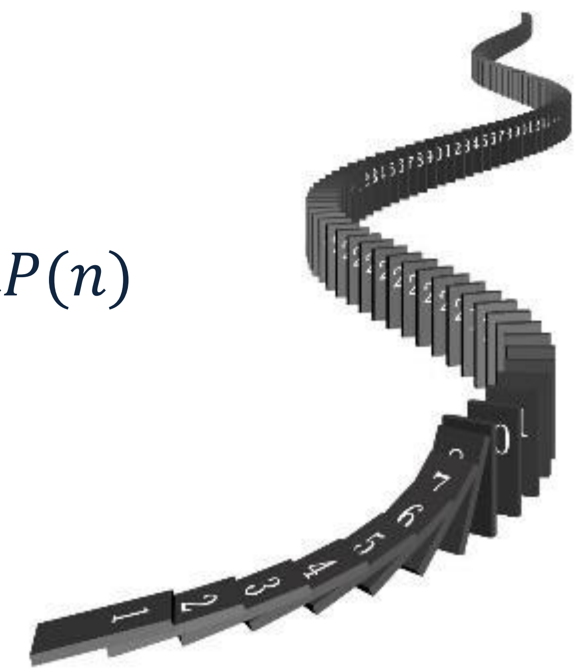

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- Prove that for every integer $n \geq 4,\ 2^n < n!$

<u>Basic Step</u> P(4) : $2^4 = 16 < 4! = 24$

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    for all $k \in Z^{+}$ (Inductive Step)

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Thus, $k+1 = a.b$ can also be written as the product of primes.

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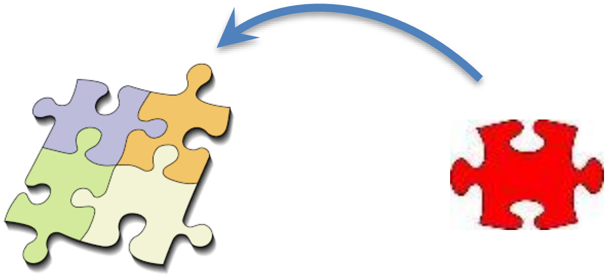

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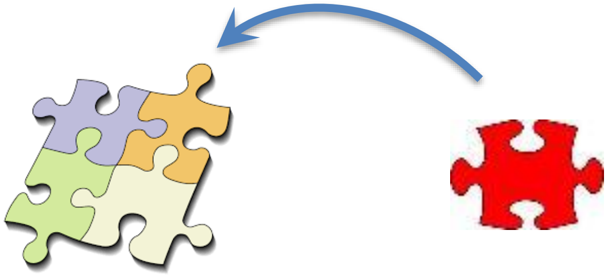

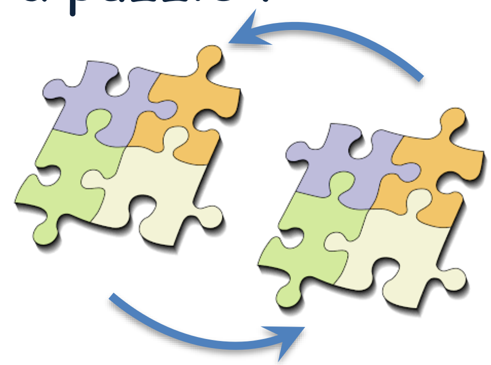

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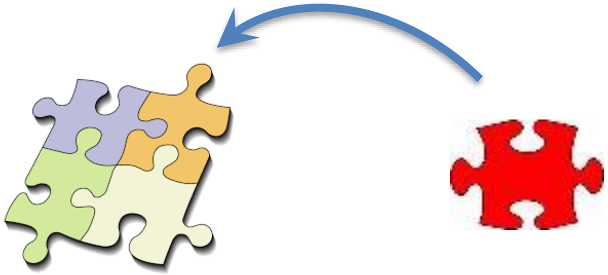

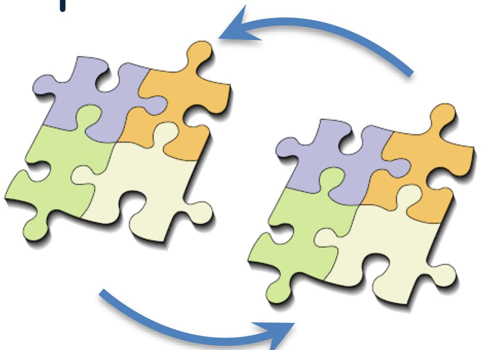

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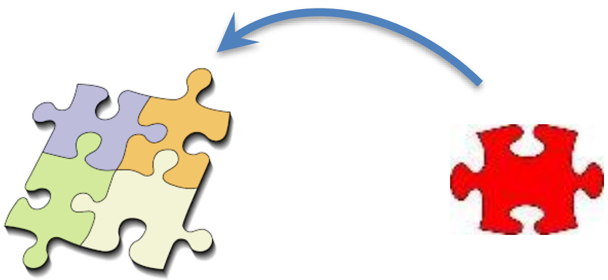

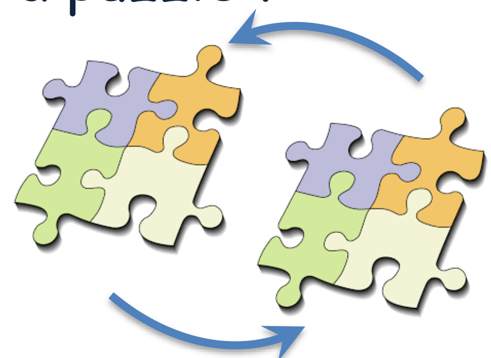

- Show that no matter which move we make, n-1 noves required to assemble a puzzle with n pieces.

<u>Basic Step</u> P(1) is true, i.e. no move required for just 1 piece

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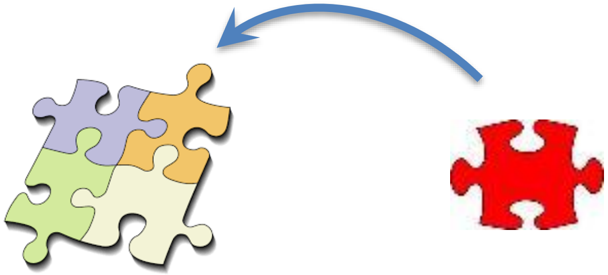

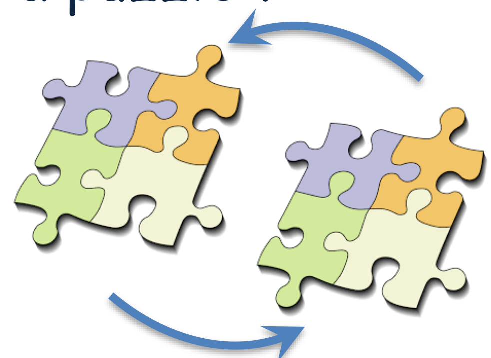

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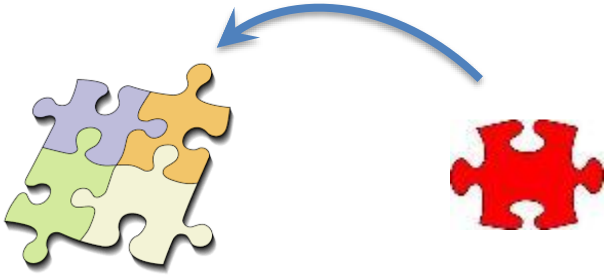

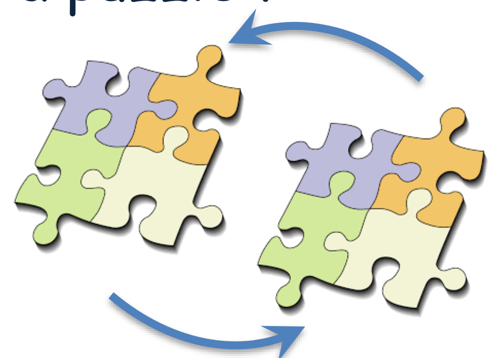

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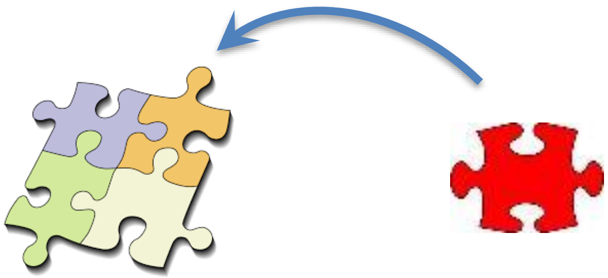

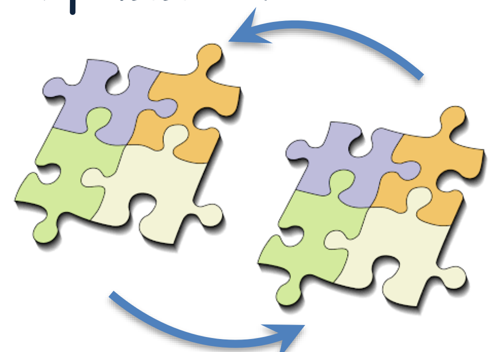

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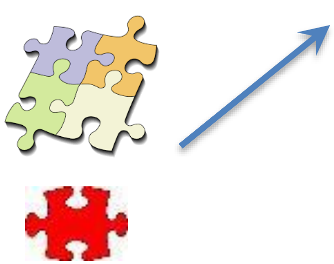

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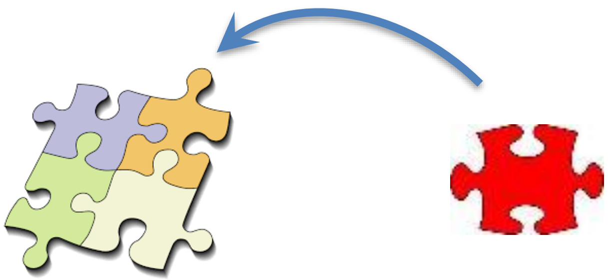

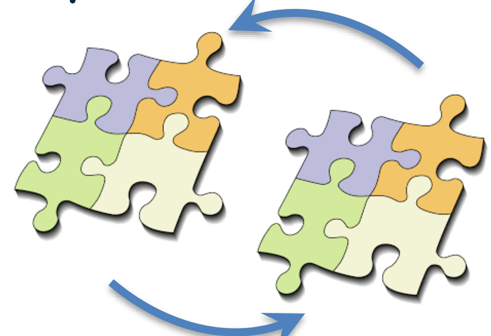

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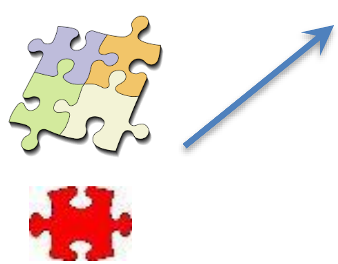

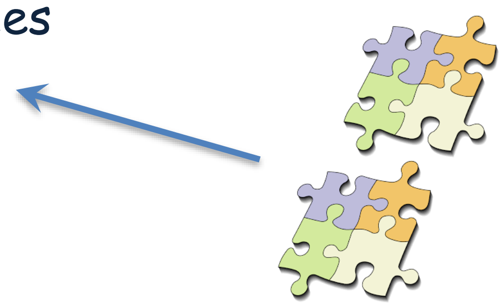

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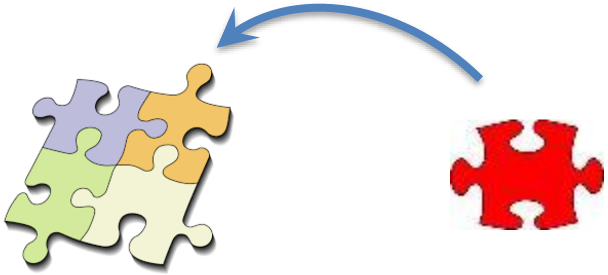

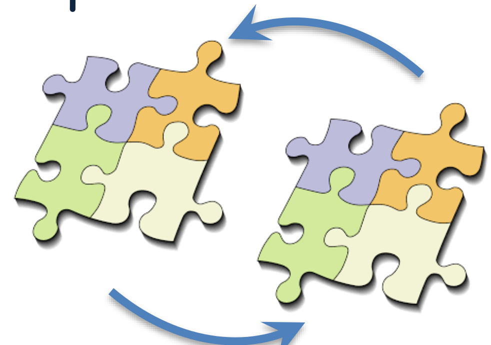

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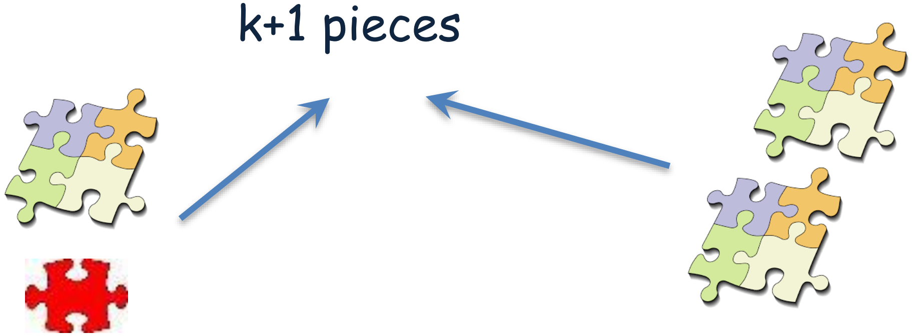

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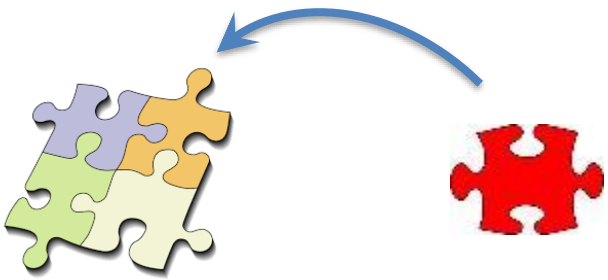

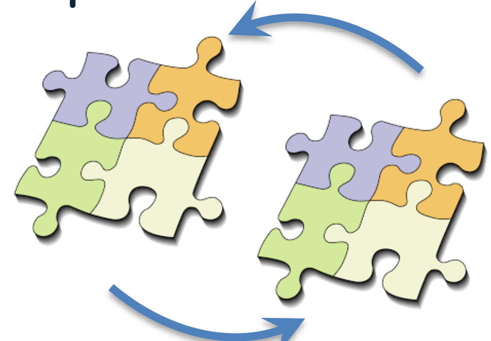

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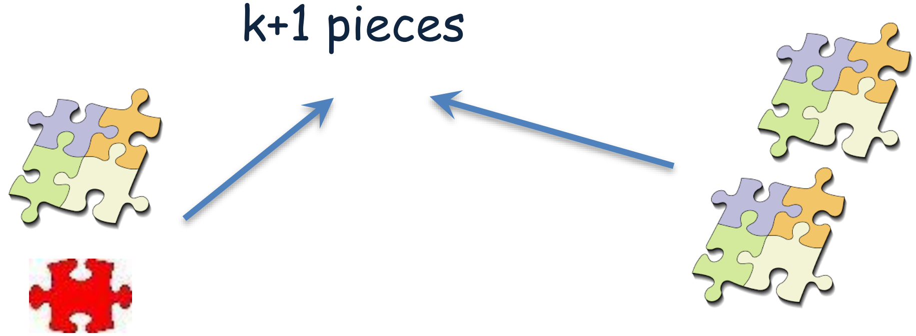

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k + 1 = a + b

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- $\sum_{j=1}^{n} F(j)^2 = F(n).F(n+1)$, where $n \geq 2$

  Basic Step P(2) :$F(1)^2 + F(2)^2 = 2 = \mathrm{F}(2).\mathrm{F}(3)$

  Inductive Step $P(k) \rightarrow P(k+1)$

assume that P(i) is true for all i such that $2 \leq i \leq k$, i.e. $\sum_{j=1}^{i} F(j)^2 = F(i).F(i+1)$

for $P(k+1)$: $F(1)^2 + \cdots + F(k)^2 + F(k+1)^2 = F(k).F(k+1) + F(k+1)^2$

$$= F(k+1)(F(k) + F(k+1))$$

Proofs

Conjecture a formula for the sum of the squares of the first n terms in Fibonacci sequence, then prove your conjecture using mathematical induction

- $F(1)^2 = 1, F(1)^2 + F(2)^2 = 2, F(1)^2 + F(2)^2 + F(3)^2 = 6,$
  $F(1)^2 + F(2)^2 + F(3)^2 + F(4)^2 = 15,$
  $F(1)^2 + F(2)^2 + F(3)^2 + F(4)^2 + F(5)^2 = 40,$

  $1.1 \quad 1.2 \quad 2.3 \quad 3.5 \quad 5.8$

- $\sum_{j=1}^{n} F(j)^2 = F(n).F(n+1)$, where $n \geq 2$

  Basic Step P(2) :$F(1)^2 + F(2)^2 = 2 = \mathrm{F}(2).\mathrm{F}(3)$

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$$= F(k+1)(F(k) + F(k+1))$$

$$= F(k+1)F(k+2)$$