More on Greedy Algorithms

Murat Osmanoglu

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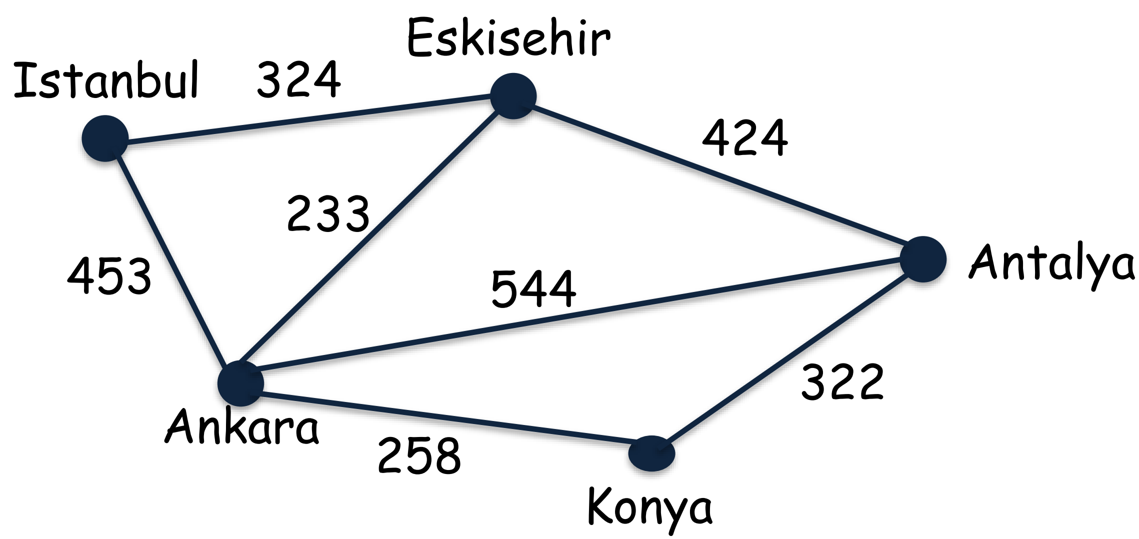

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- given a connected undirected graph $G = (V,E)$ with real-valued edge weight $w(e)$, an minimum spanning tree (MST) is a subset of the edges $T \subseteq E$ such that T is a tree that connects all vertices of the graphs, and T has minimum total weight.

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- connecting all computers in an office buildings using least amount of cables

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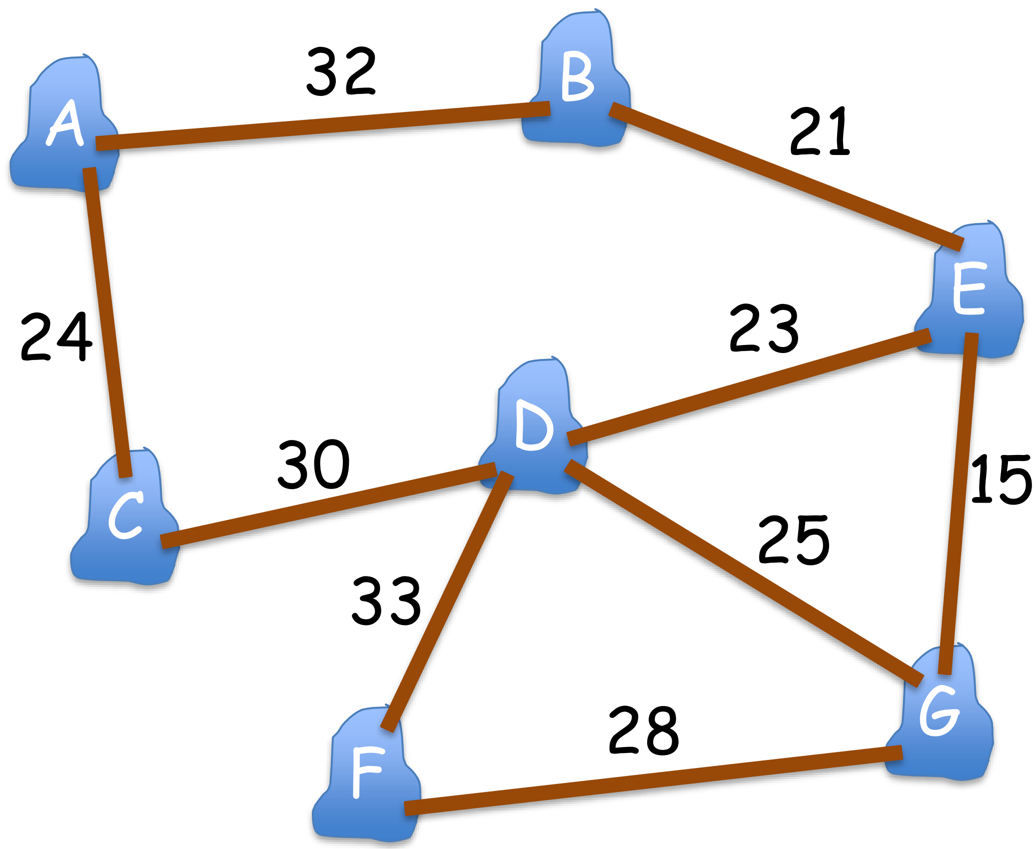

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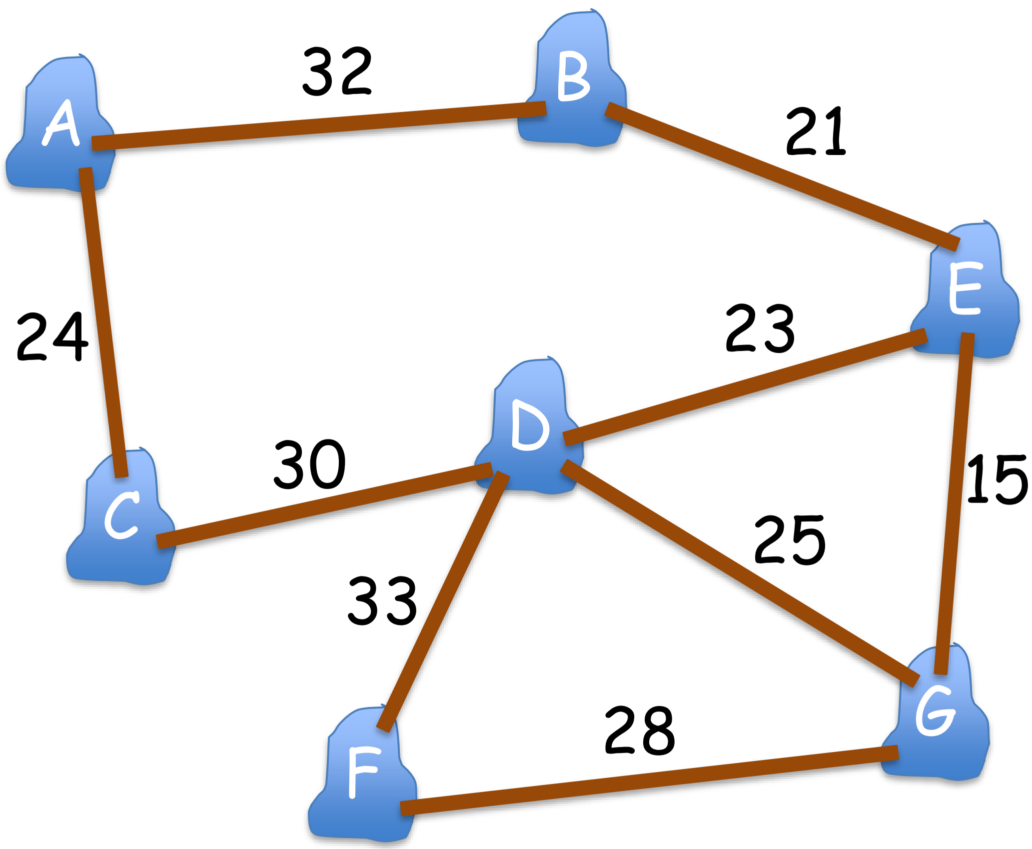

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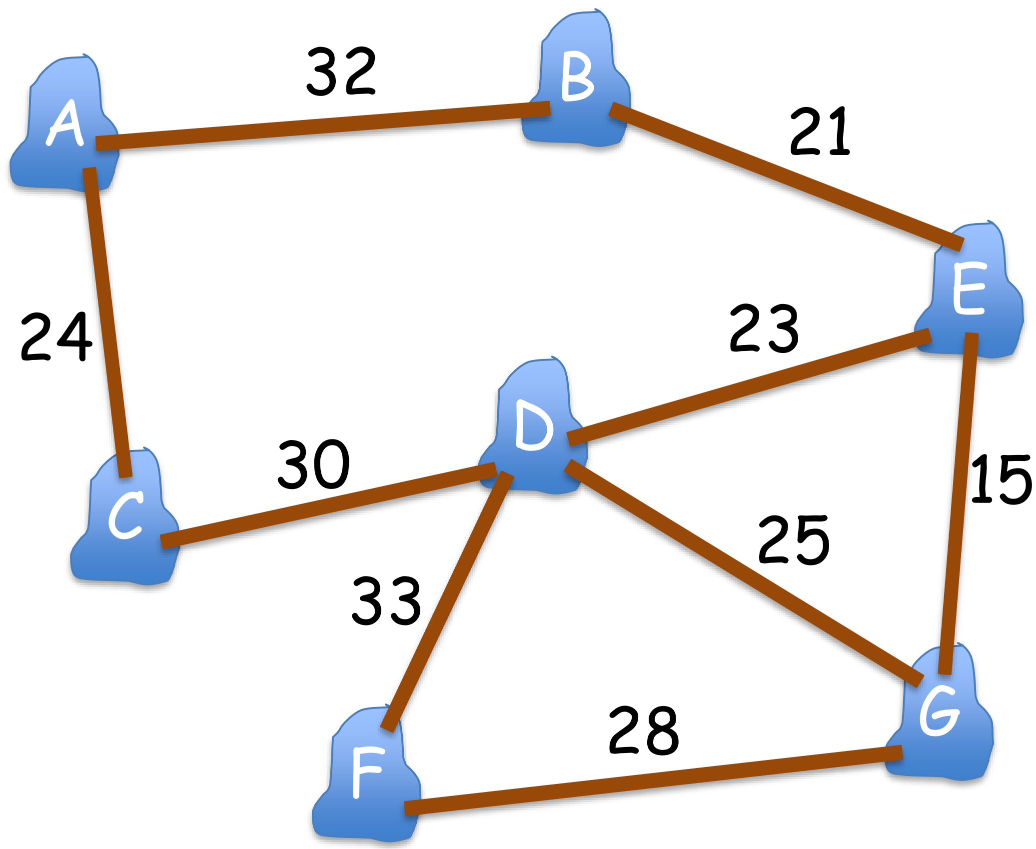

- start with an empty set of edges A
- repeat the following procedure |V| – 1 times :
  add the lightest edge that does not create a cycle to A

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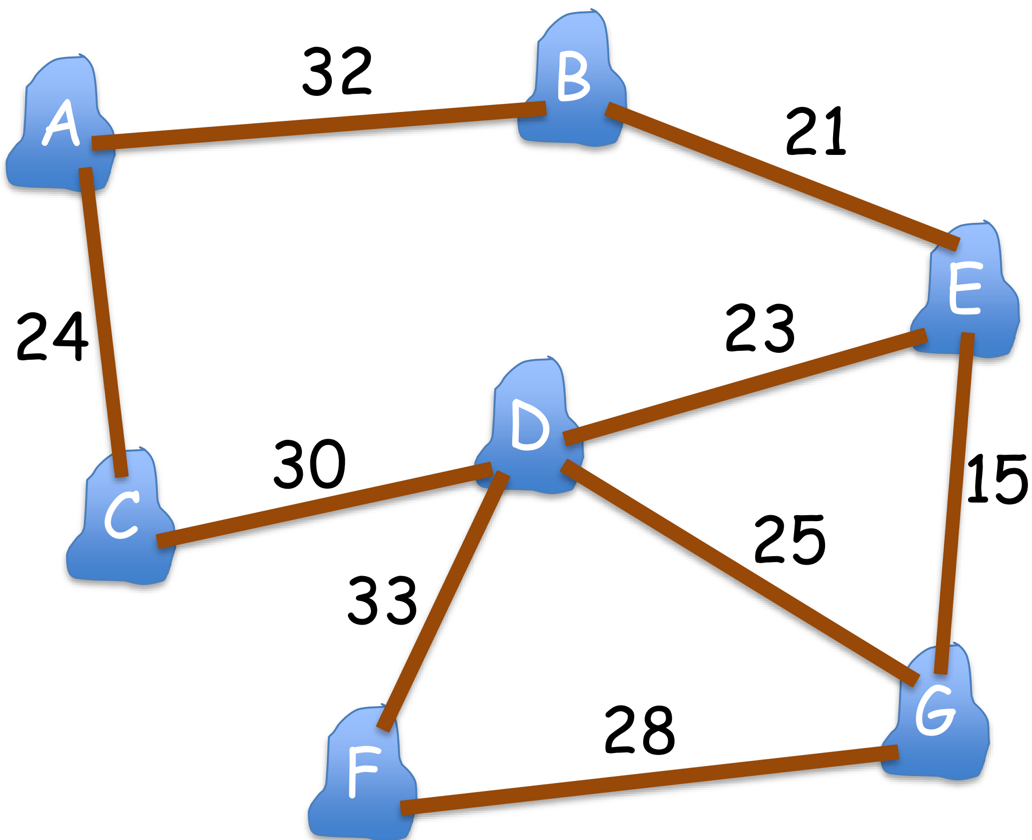

n-1 edges enough to connect n nodes, one more creates a cycle

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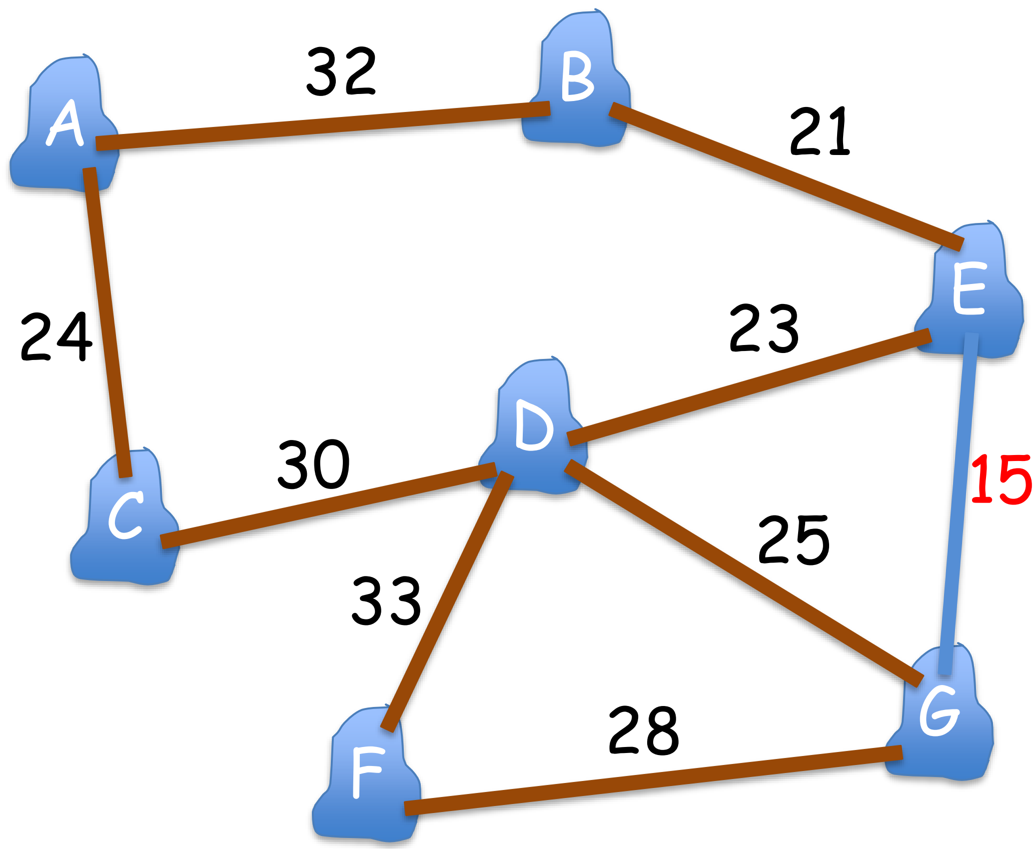

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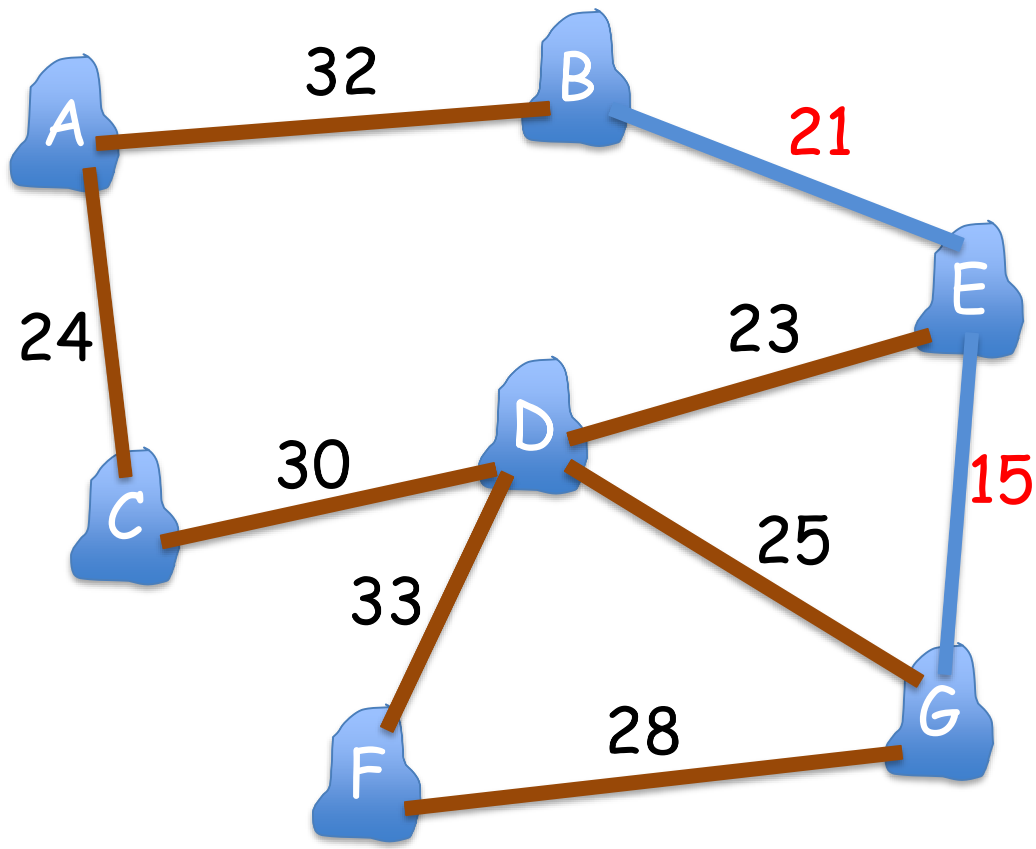

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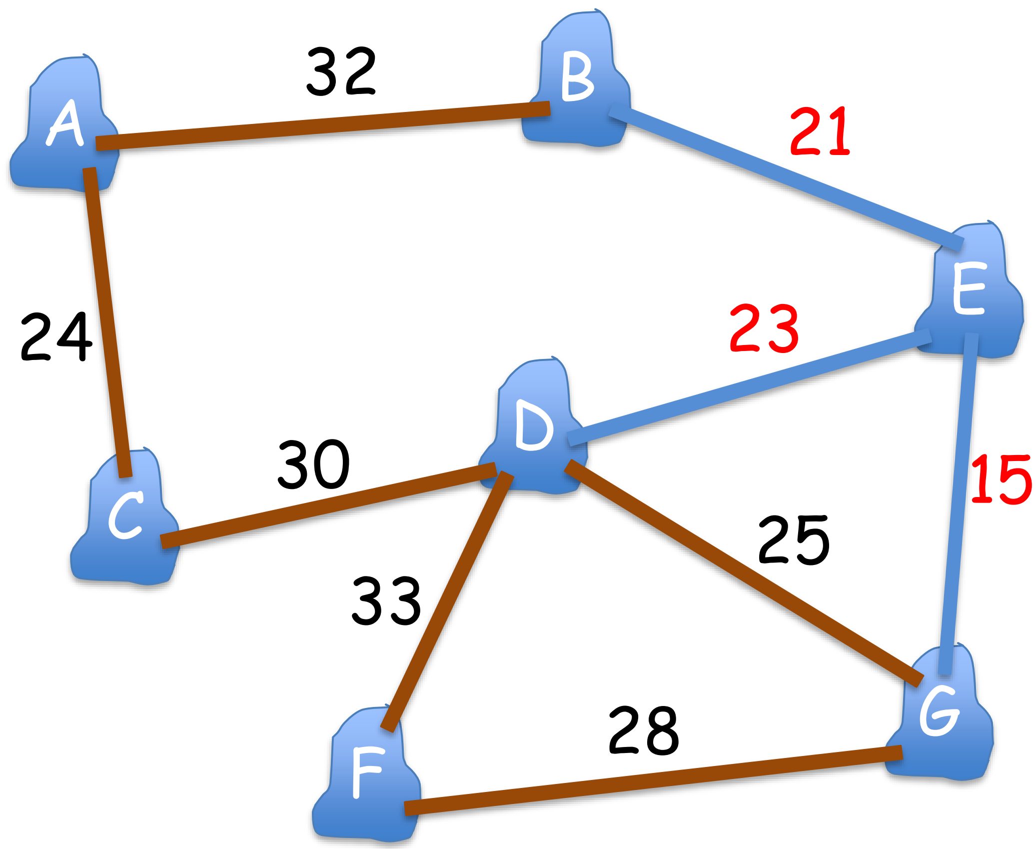

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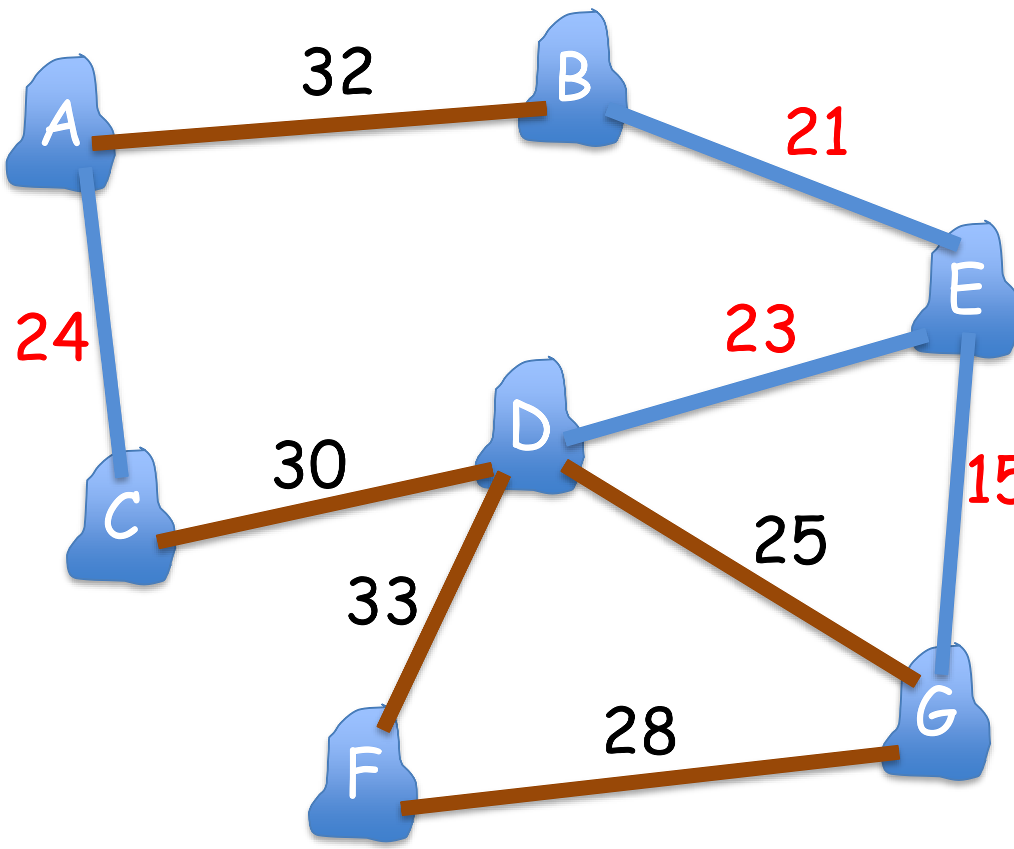

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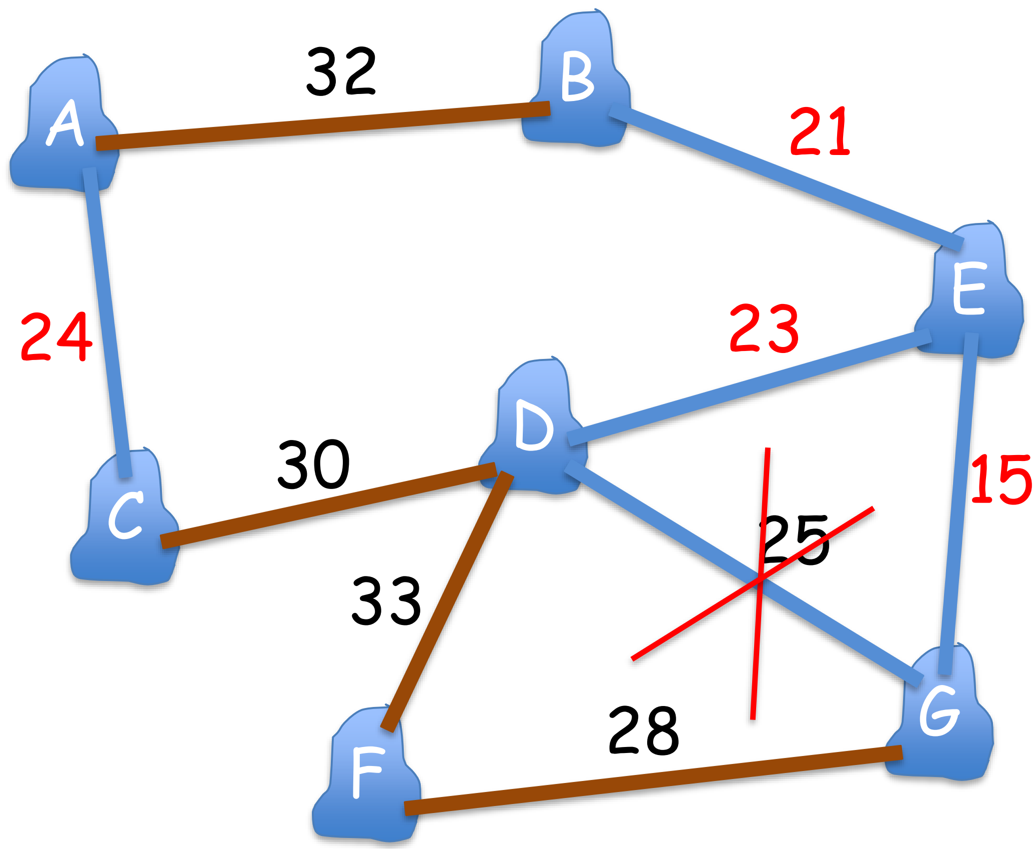

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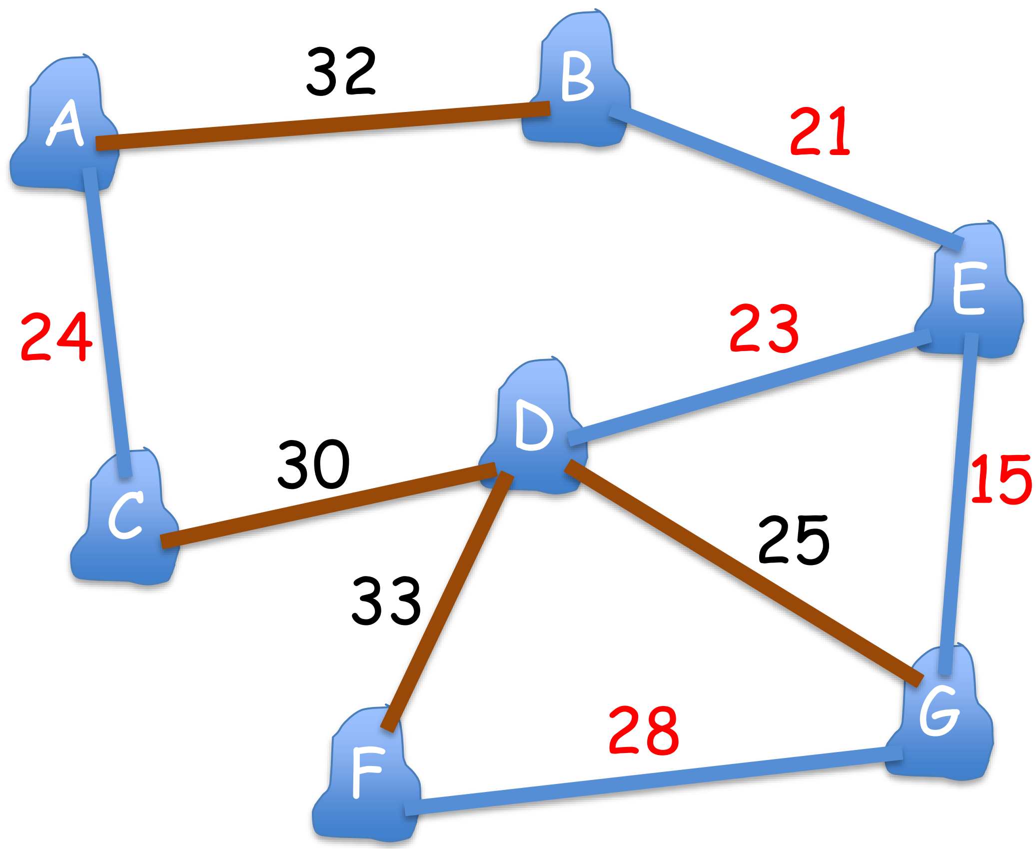

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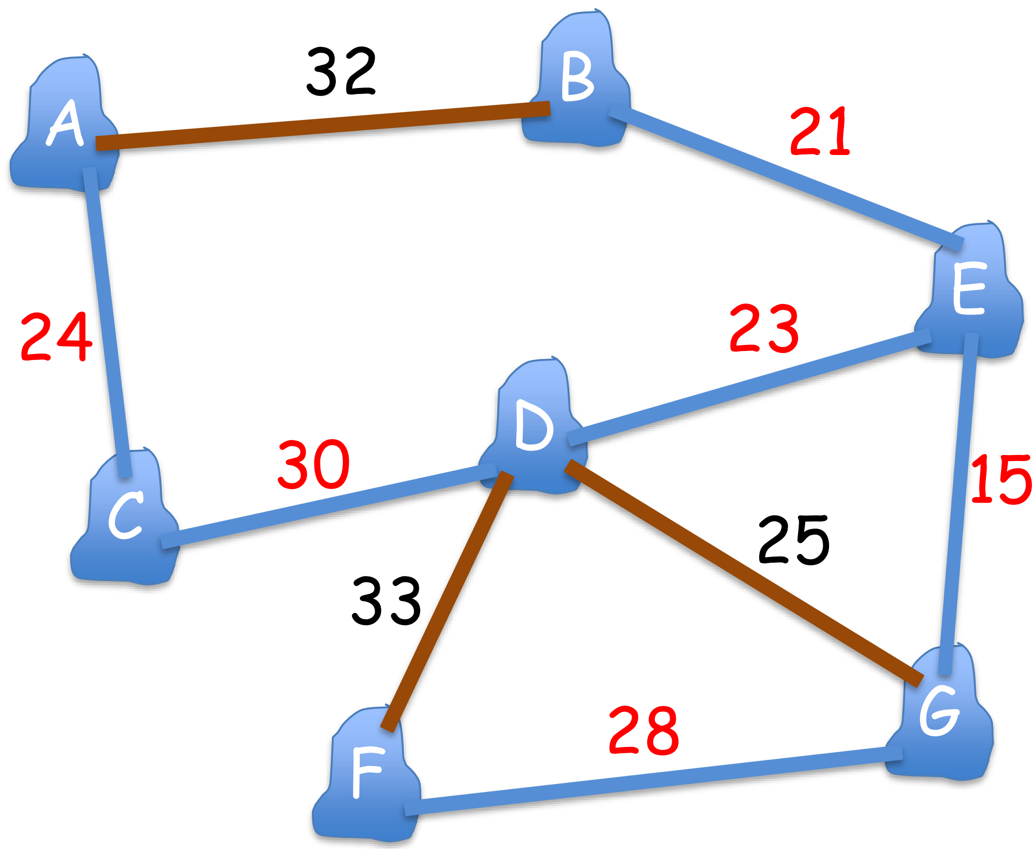

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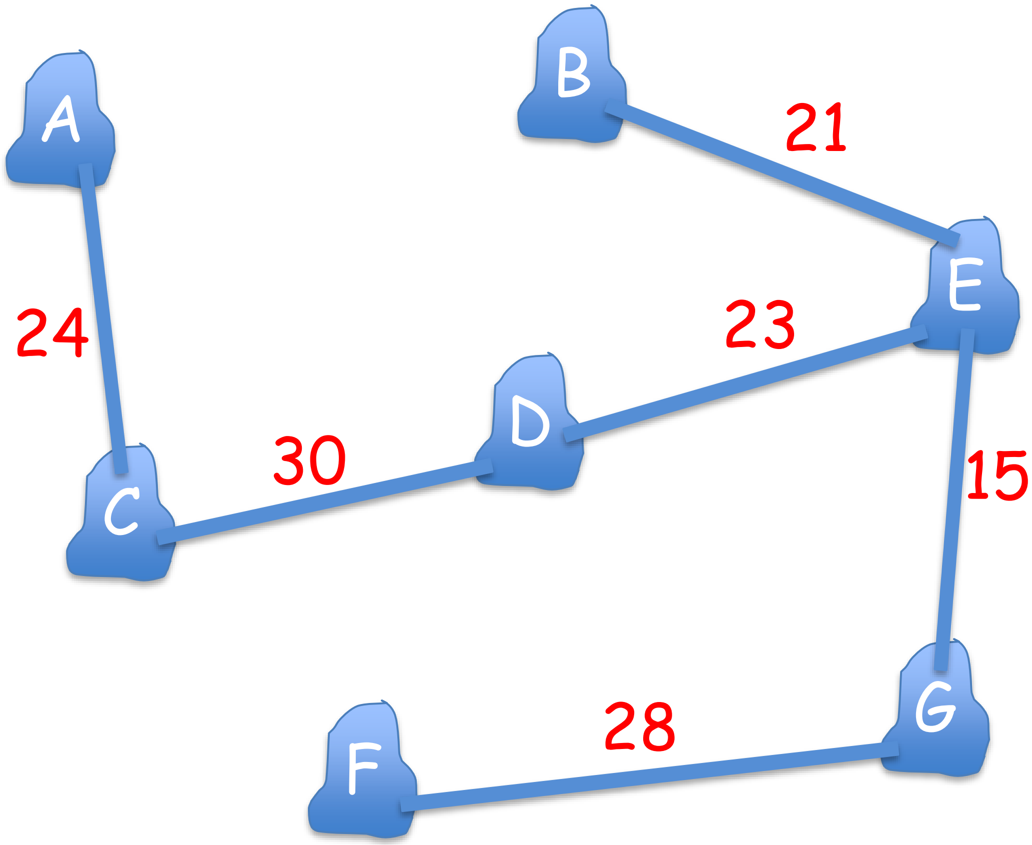

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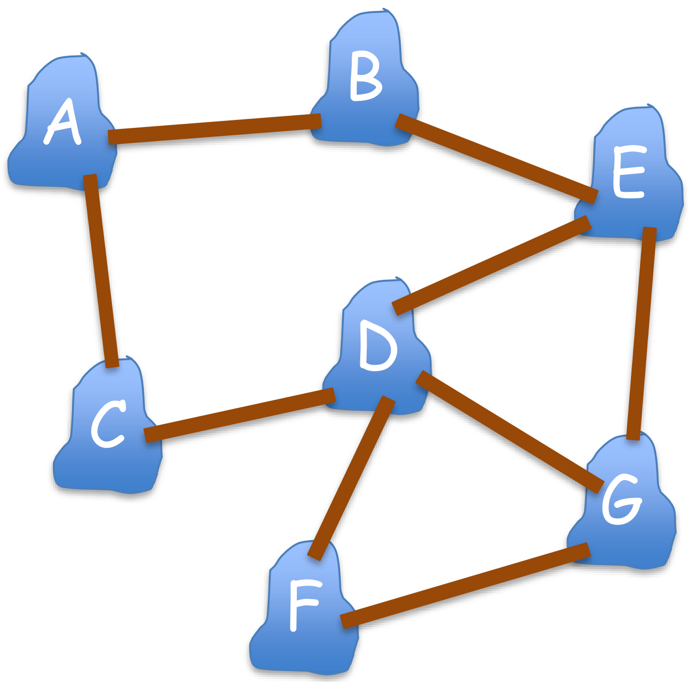

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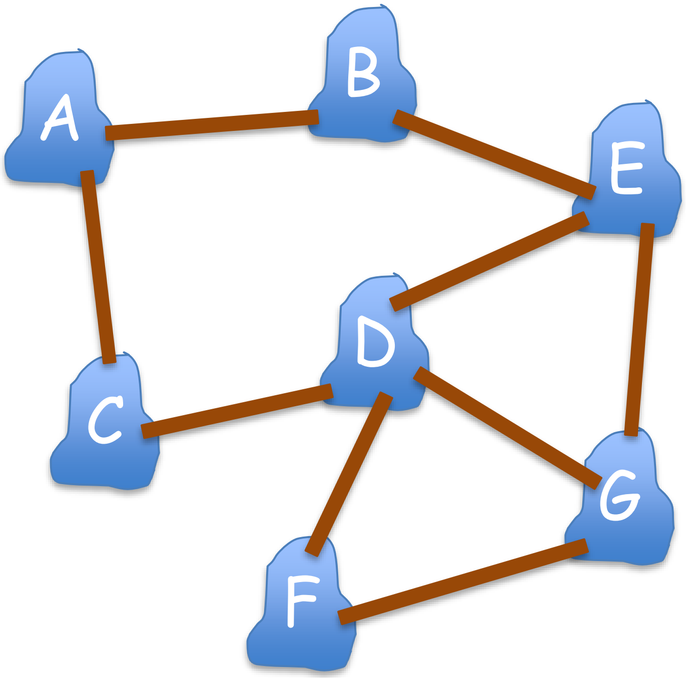

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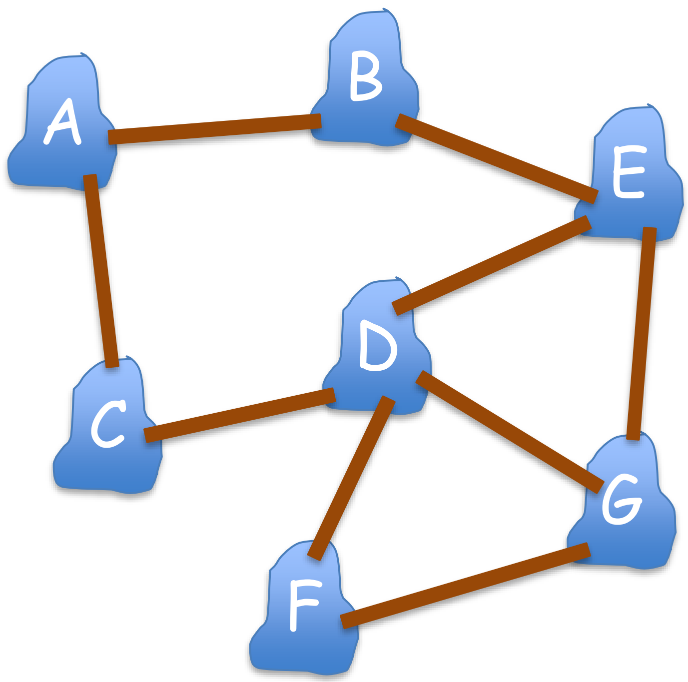

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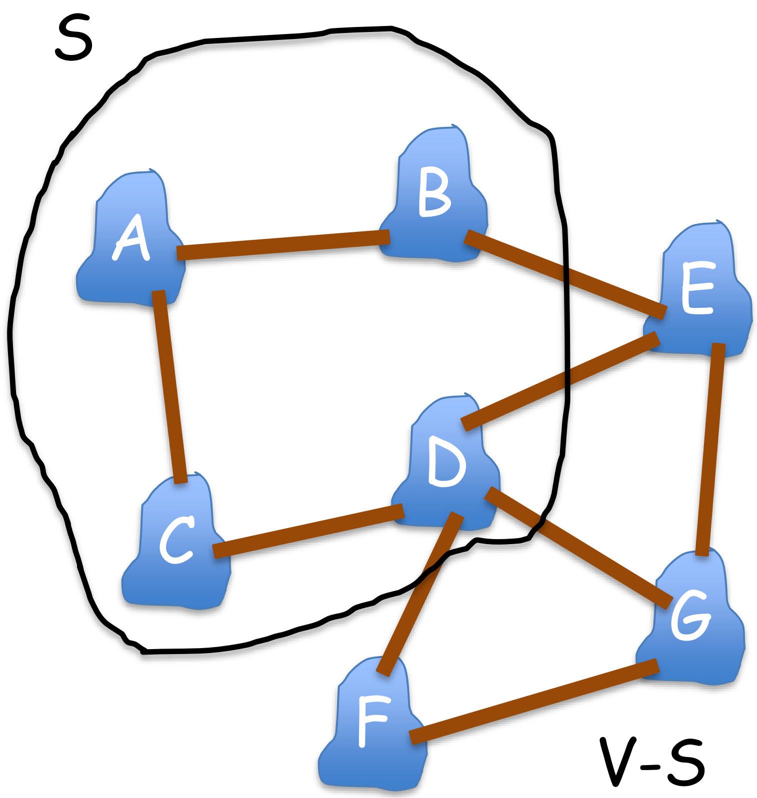

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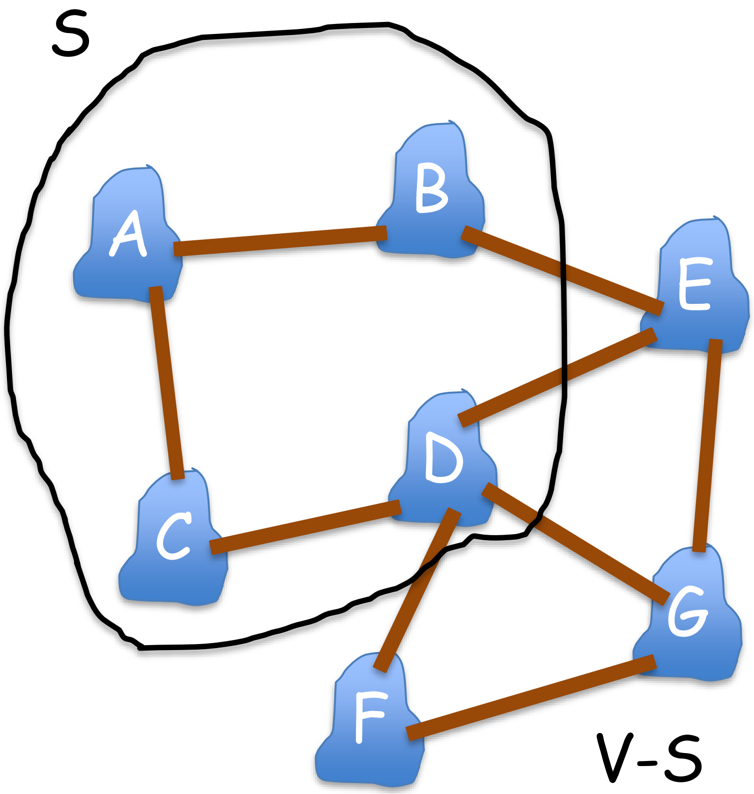

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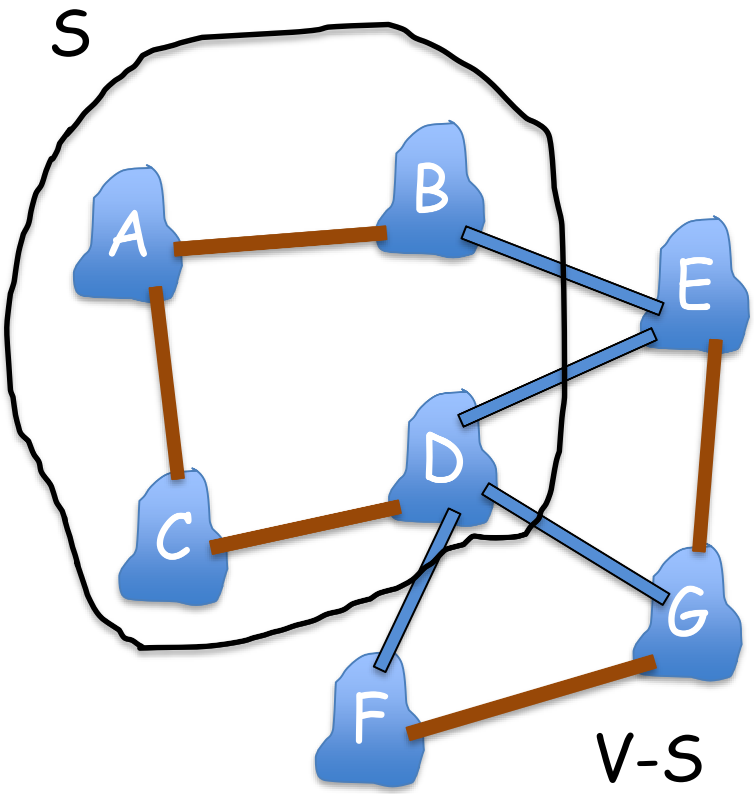

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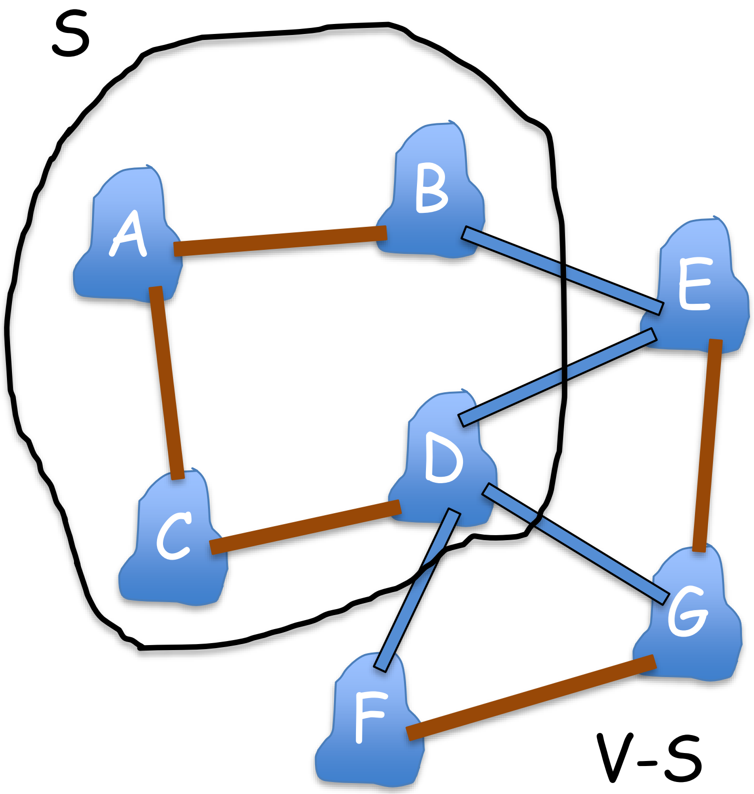

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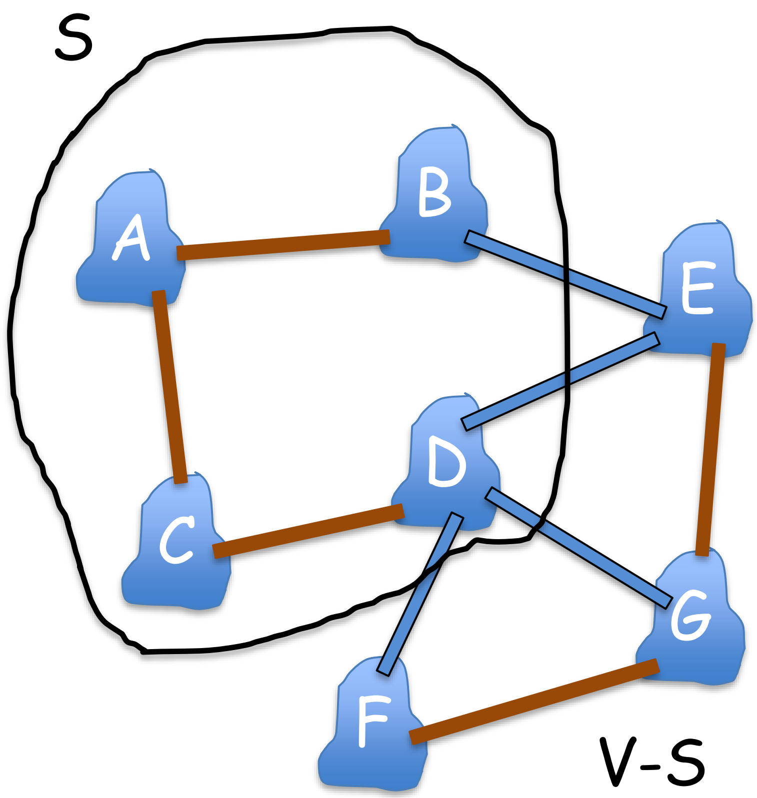

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A={(E,G),(F,G)} respects (S,V-S)

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<u>Theorem</u> : Let A be a subset of edges of some minimum spanning tree T. Let (S,V-S) be a cut that A respects and e be the lightest edge that crosses (S,V-S). Then A∪{e} is a subset of some minimum spanning tree.

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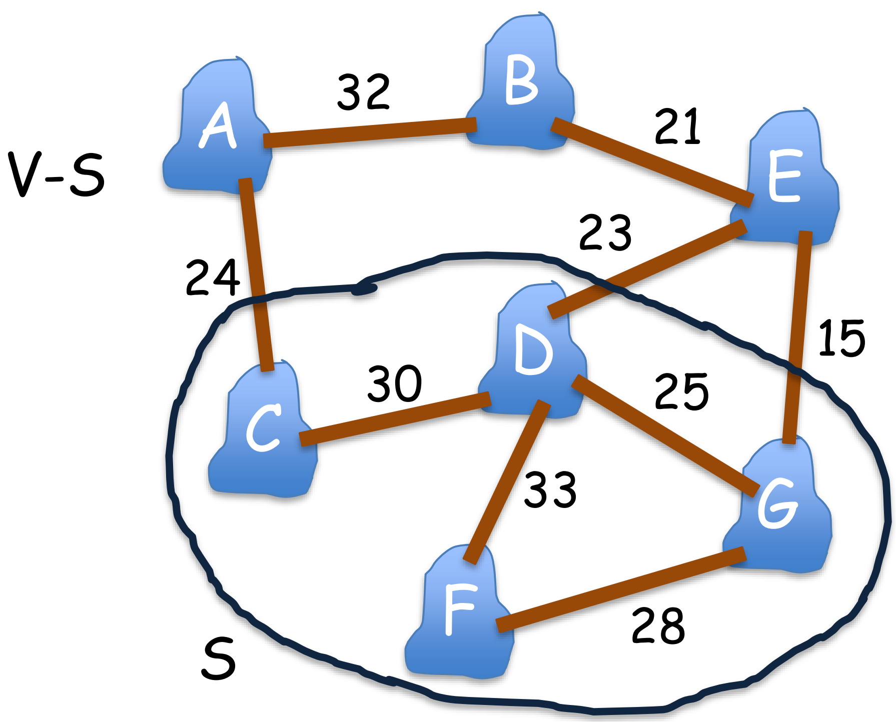

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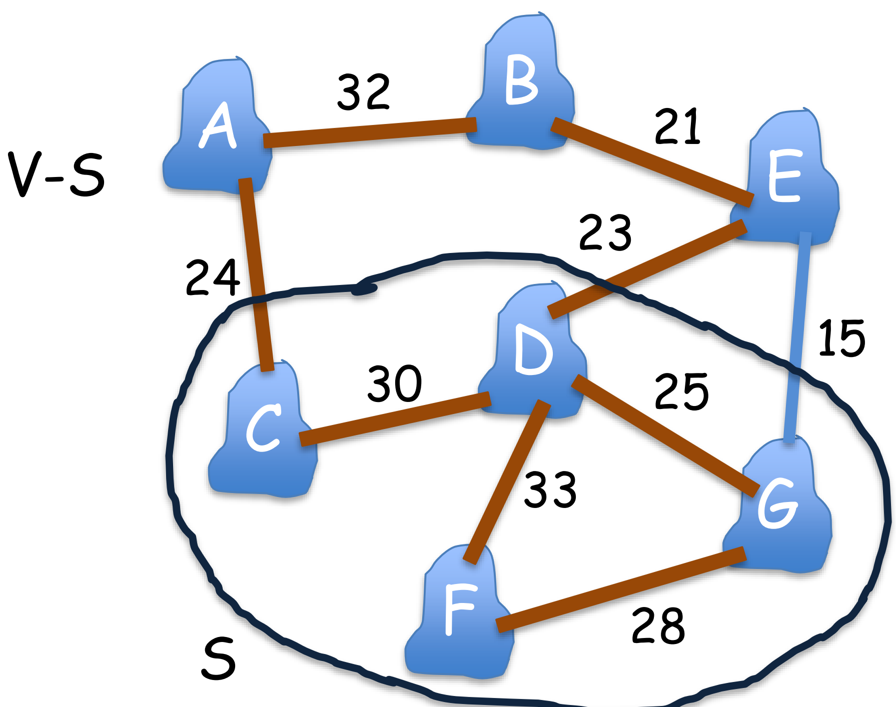

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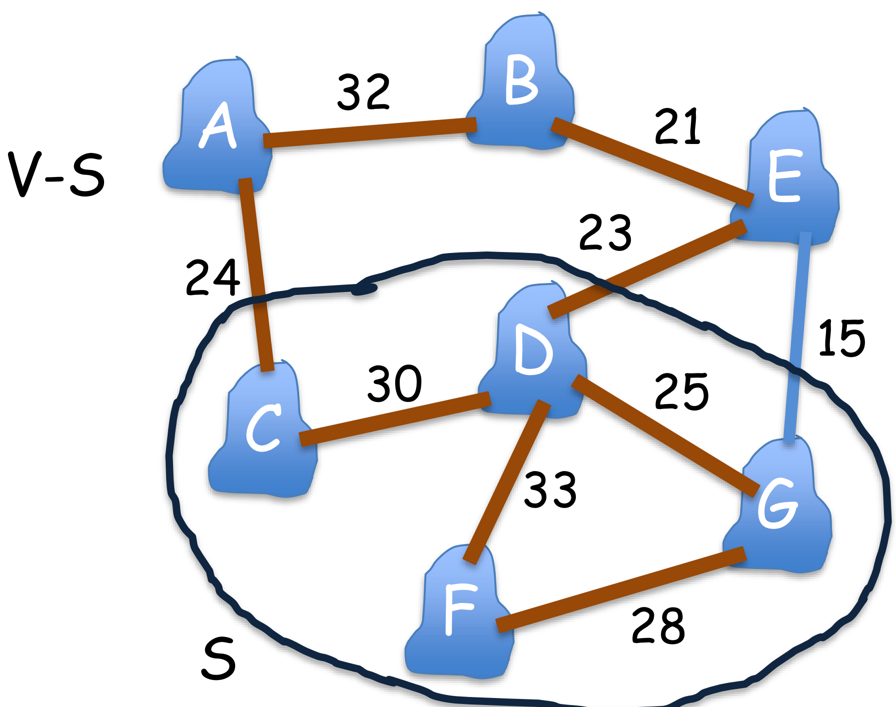

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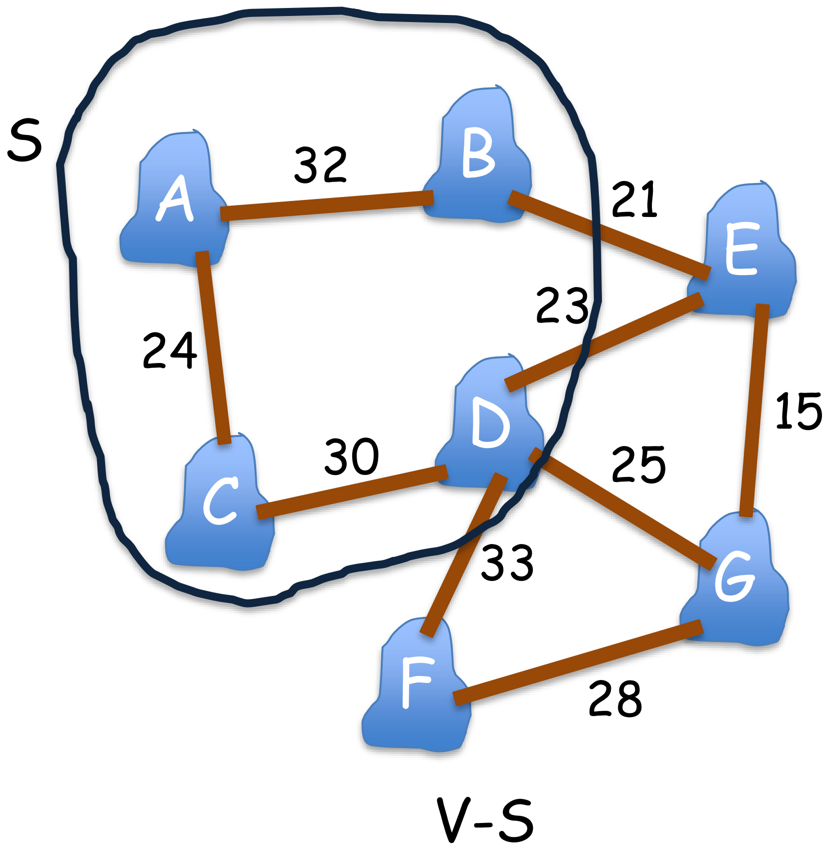

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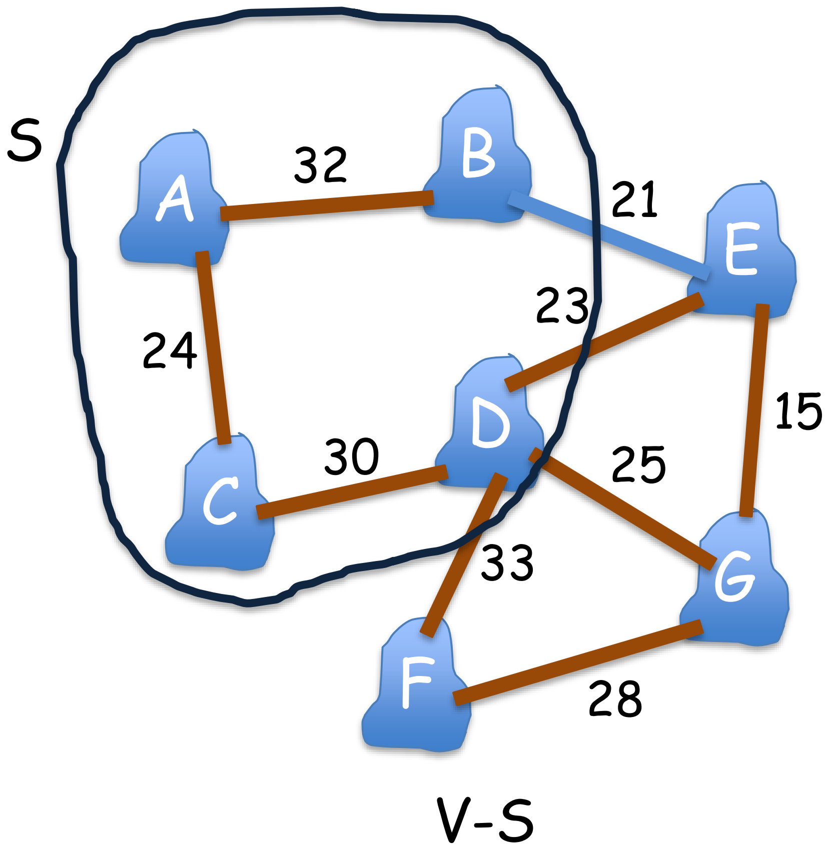

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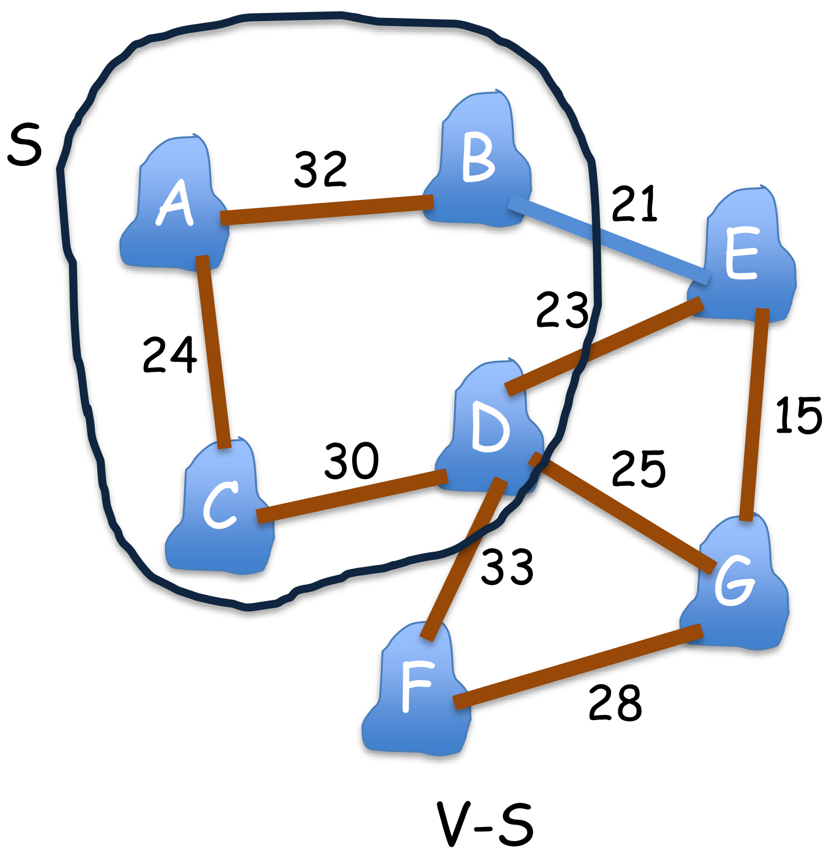

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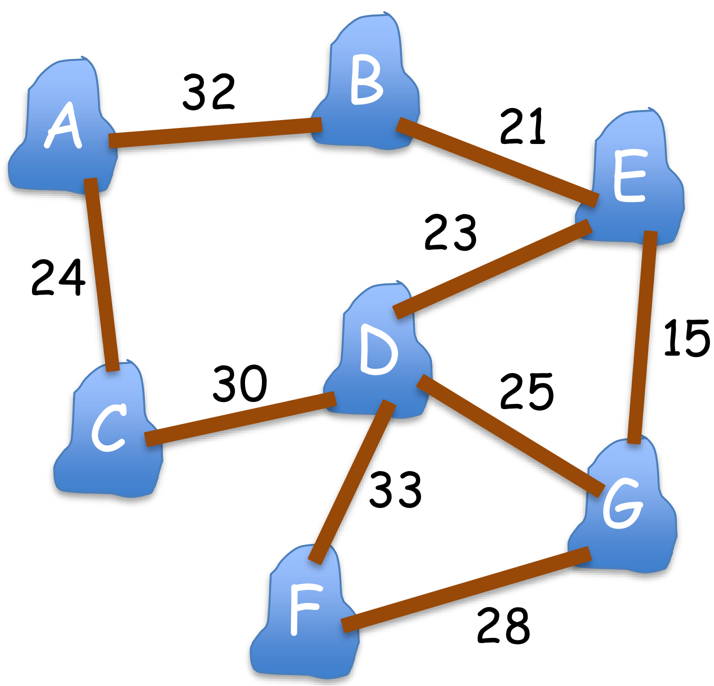

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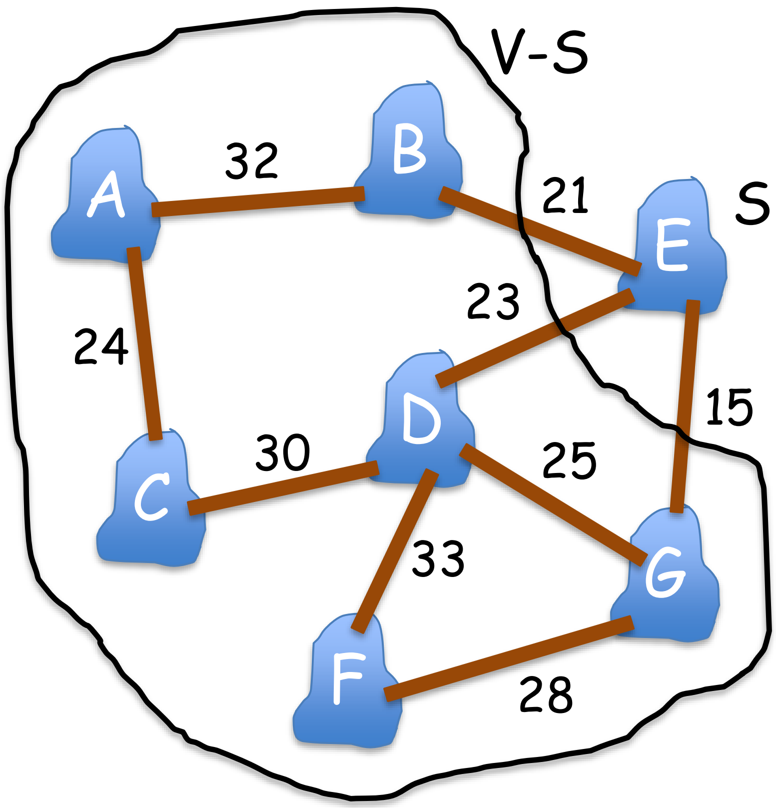

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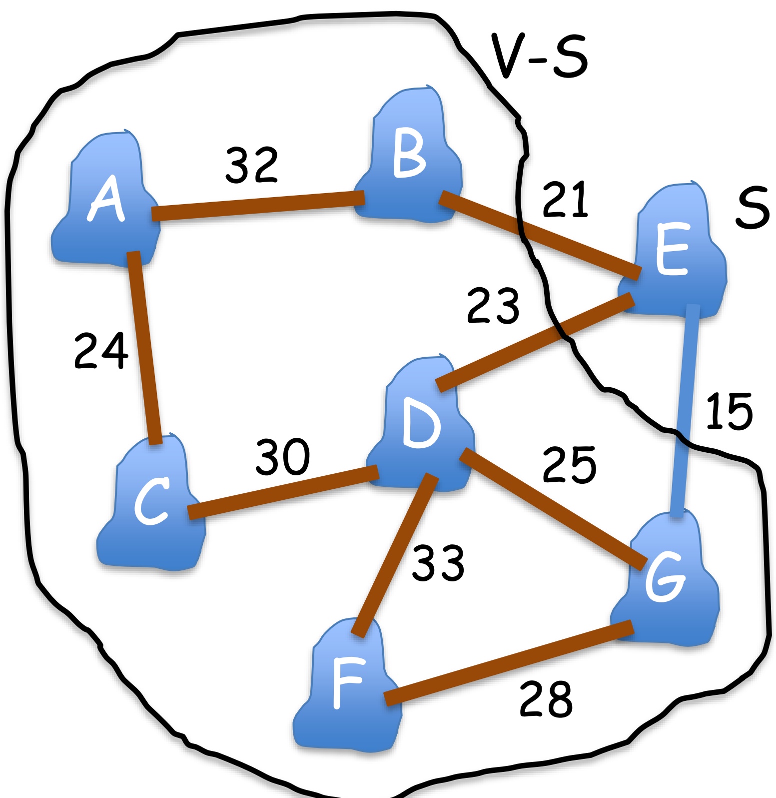

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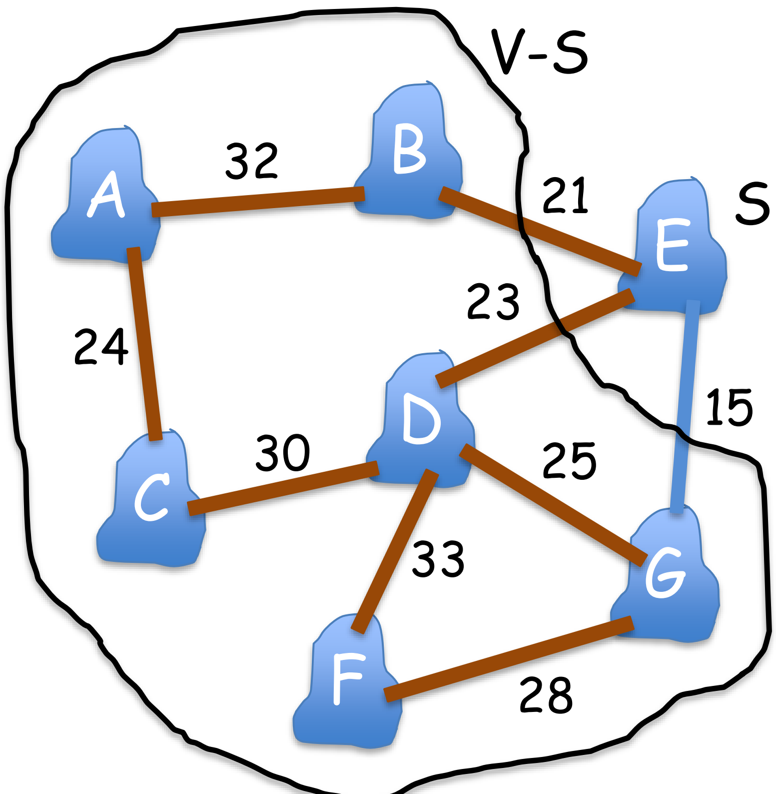

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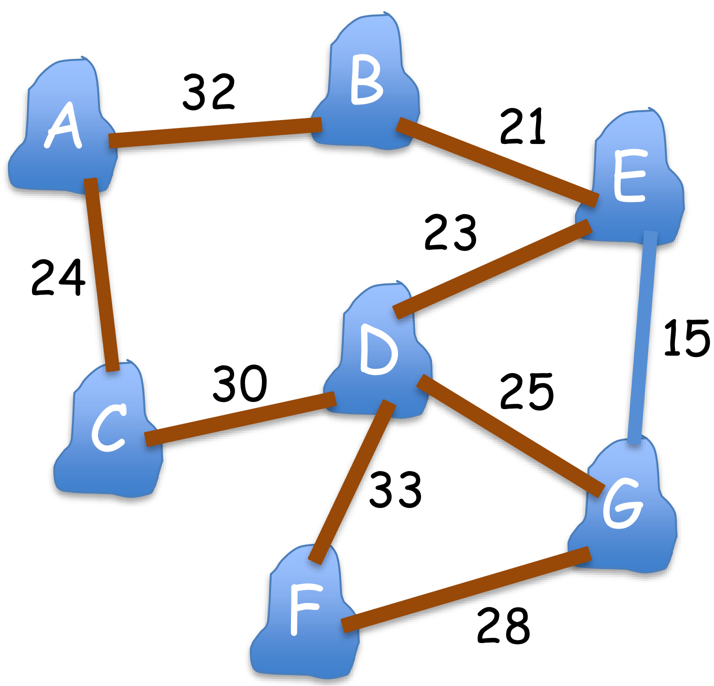

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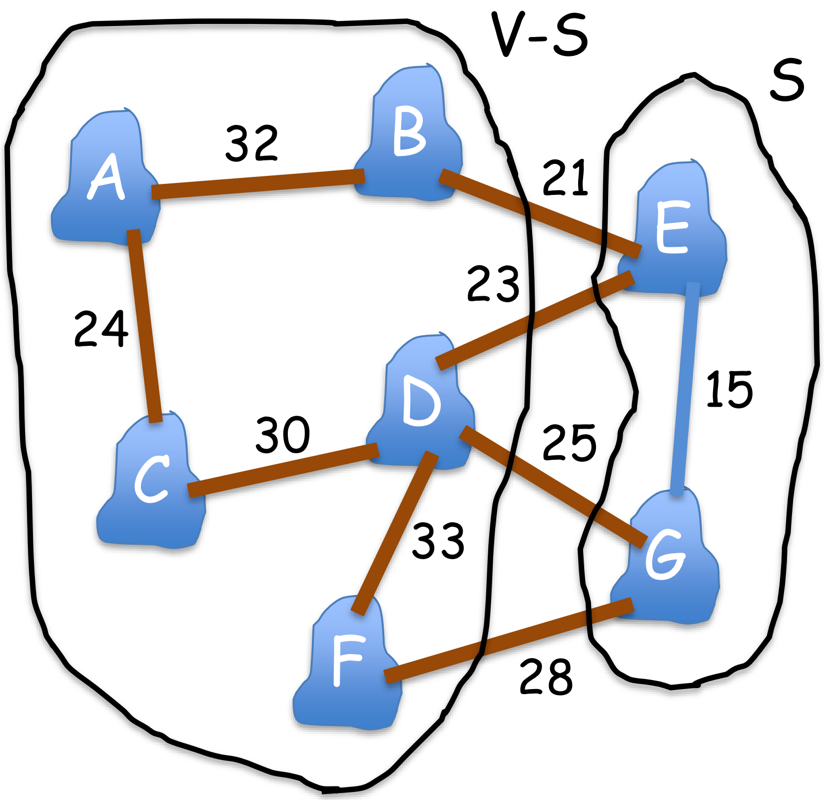

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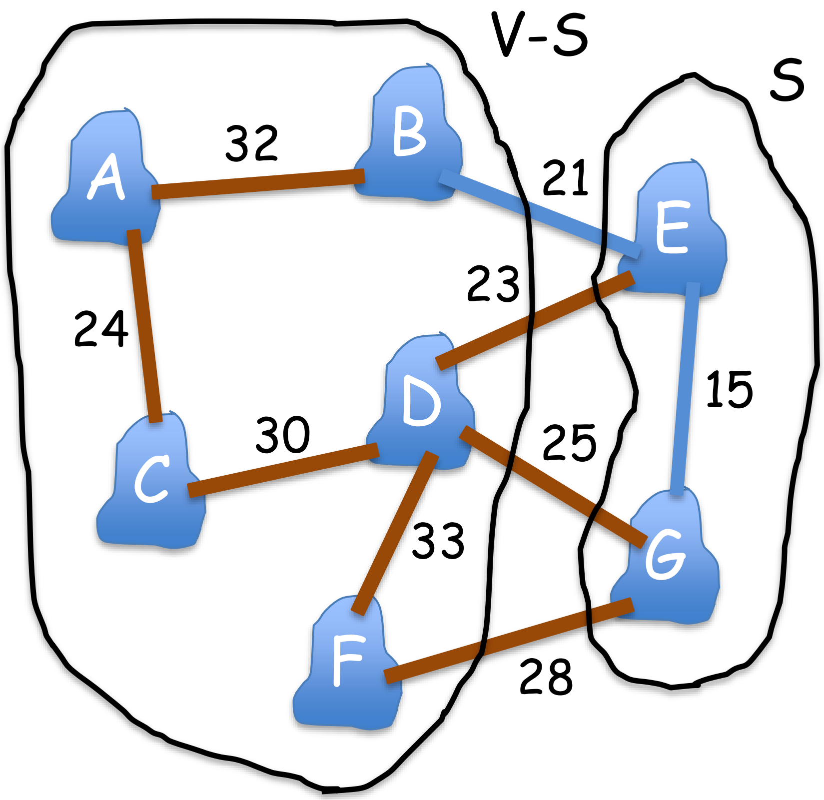

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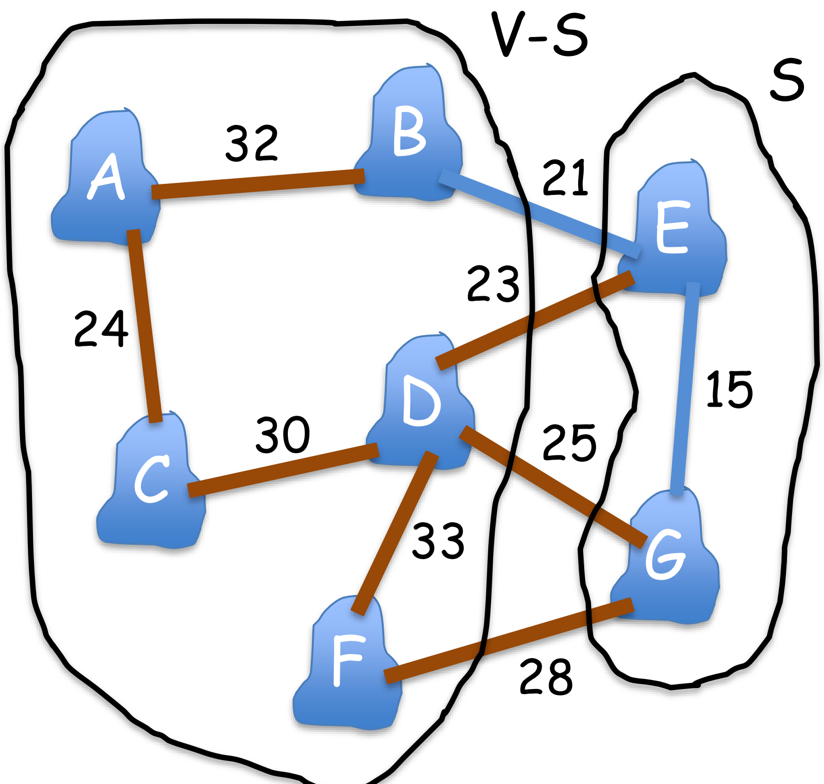

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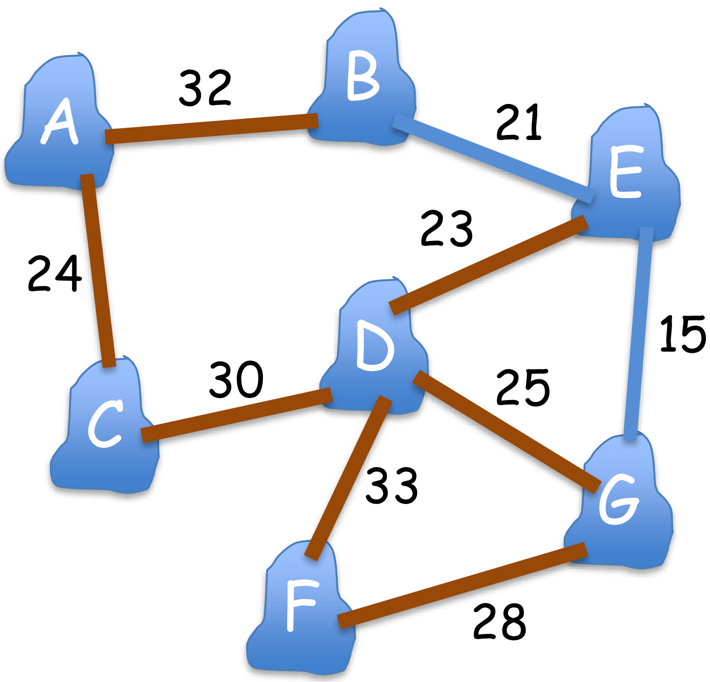

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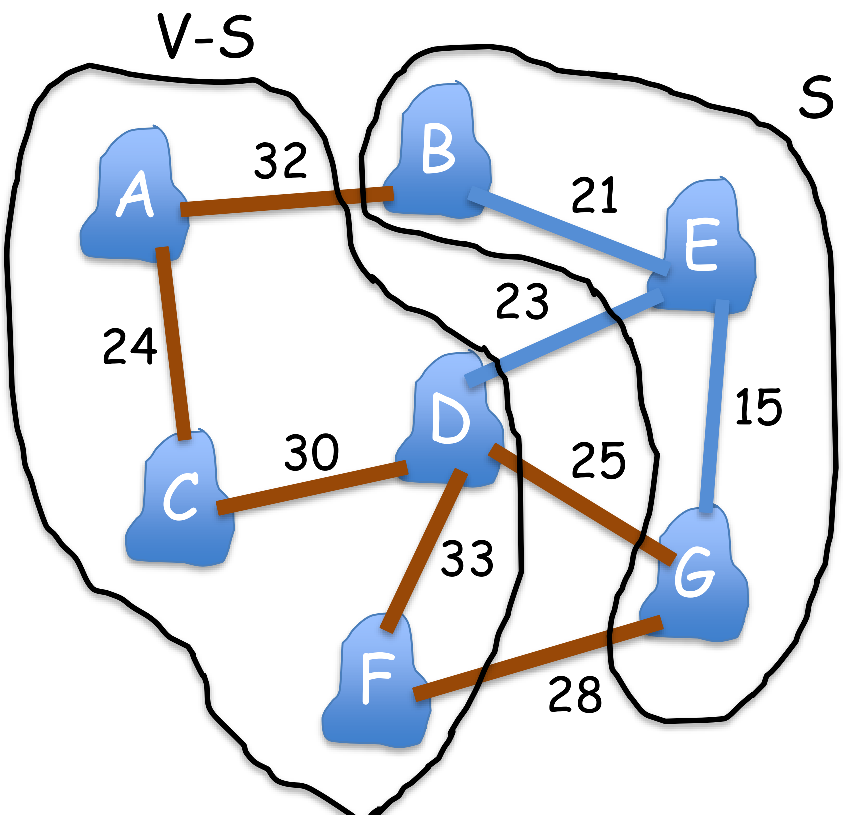

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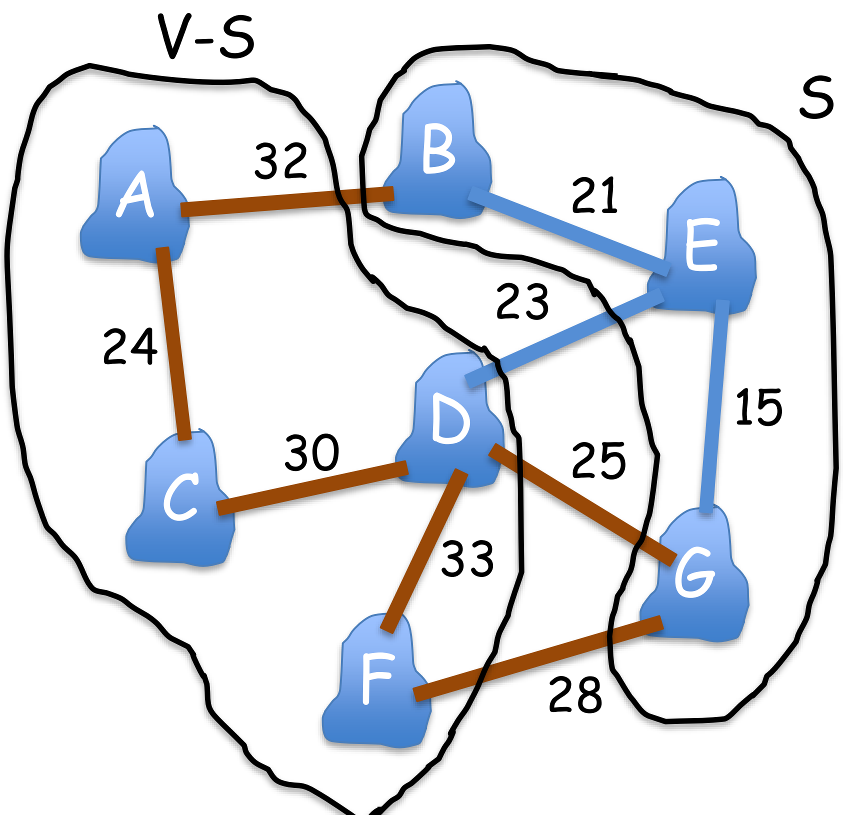

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Proof : Assume T is an MST and $\{e\}$ is contained in T.

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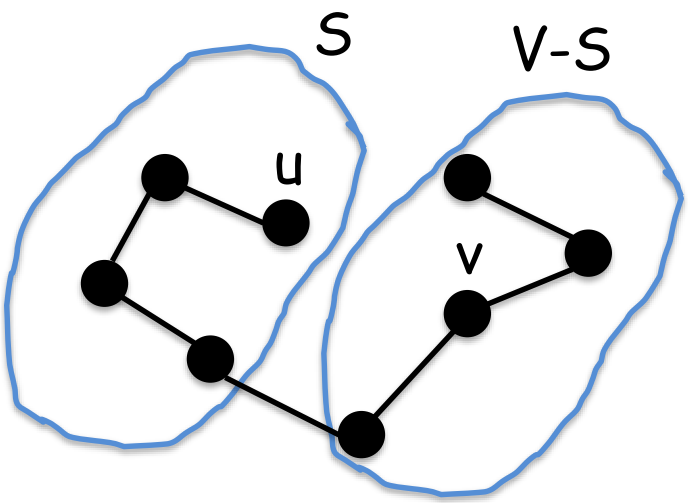

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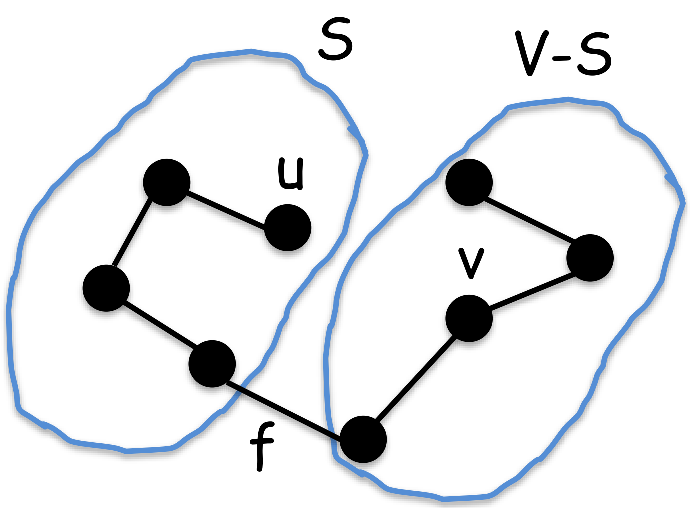

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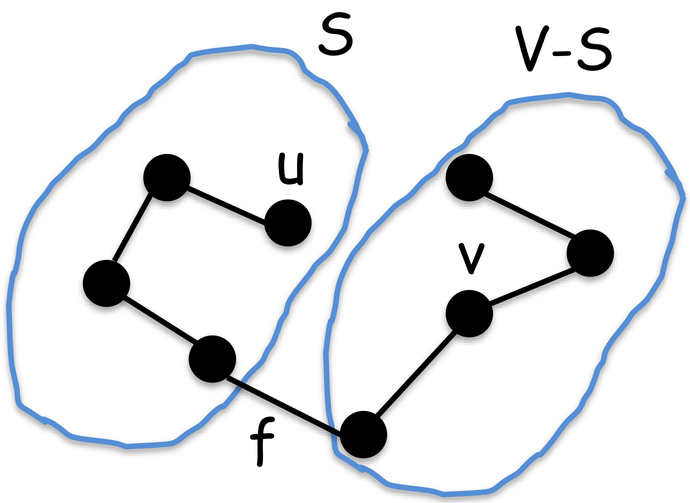
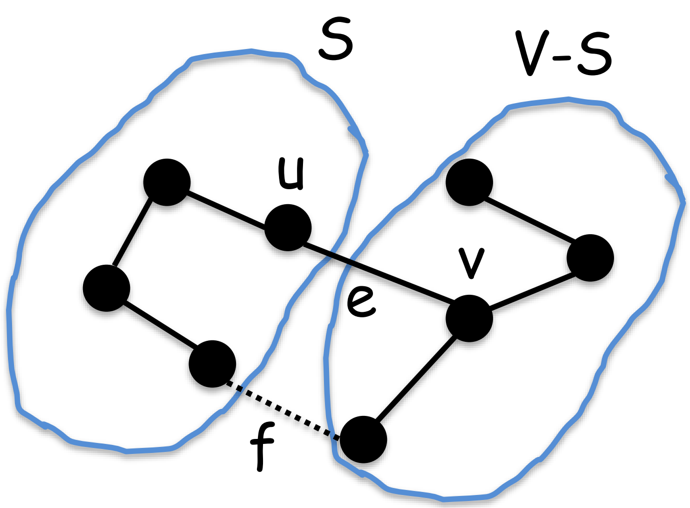


- Let f be the edge on this path that crosses (S,V-S). Such edge must exist since there should be an edge that connects this cut

Greedy Choice Property

Theorem : Let A be a subset of edges of some minimum spanning tree T. Let (S,V-S) be a cut that A respects and e be the lightest edge that crosses (S,V-S). Then A∪{e} is a subset of some minimum spanning tree.

Proof : Assume T is an MST and {e} is contained in T. Let e=(u,v) . Since T is an MST, T must contain a path from v to u.



- Let f be the edge on this path that crosses (S,V-S). Such edge must exist since there should be an edge that connects this cut
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$$T' = T \cup \{e\} - \{f\}$$

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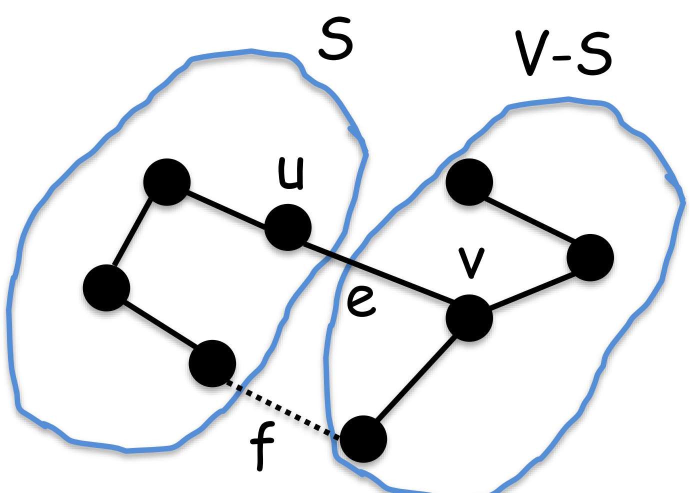

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  w(e) ≤ w(f)

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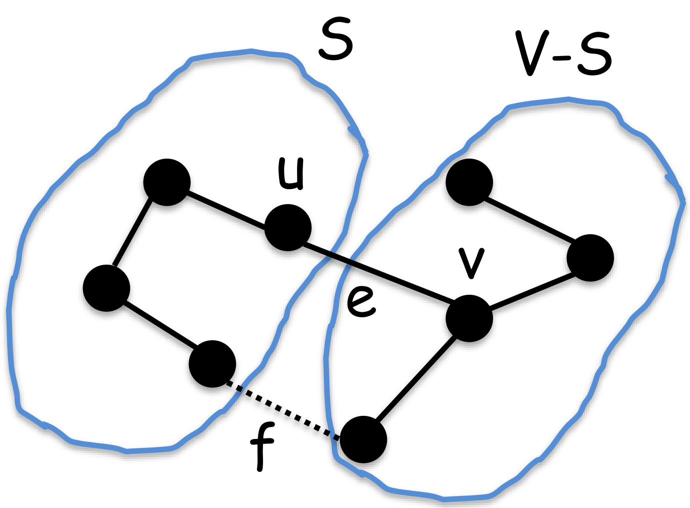

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- Thus, $w(T') = w(T) - w(f) + w(e) \leq w(T)$

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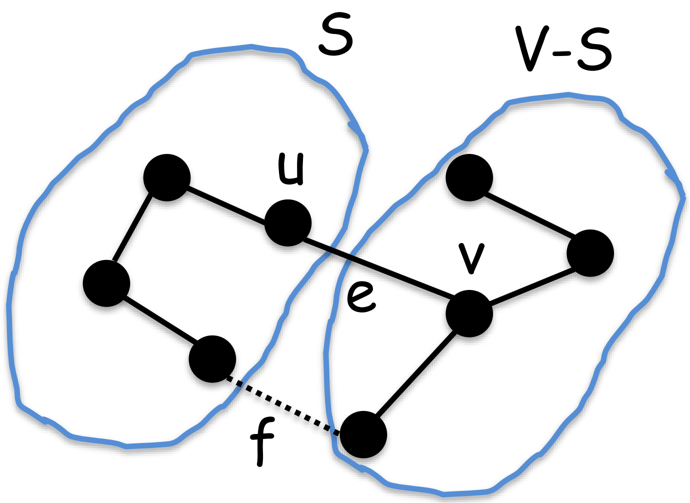

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this is a contradiction !

Kruskal's Algorithm

- start with an empty tree T
- consider edges in increasing order of weights
- add the lightest edge to T if it does not create a cycle
- If It creates a cycle, discard it

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- use union-find data structure

Union-Find

- represent each set as a tree
- each node has a pointer to its parent
- the root has a parent pointer to itself
- each tree has a height

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Find-Set(x)
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      x = x.parent
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<u>Union(x,y)</u>

```
a = Find-Set(x)
b = Find-Set(y)
if (a.height ≤ b.height)
    if (a.height = b.height)
        b.height = b.height + 1
    a.parent = b
else
    b.parent = a
```

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Always make the root of the taller one the parent of the shorter

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MST-Kruskal(G)

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A = { }
for each v of V
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sort all edges in increasing
order by weight
for each edge (u,v)
      if Find-Set(u) ≠ Find-Set(v)
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O(|V|)

O(|E|.log|E|)

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order by weight
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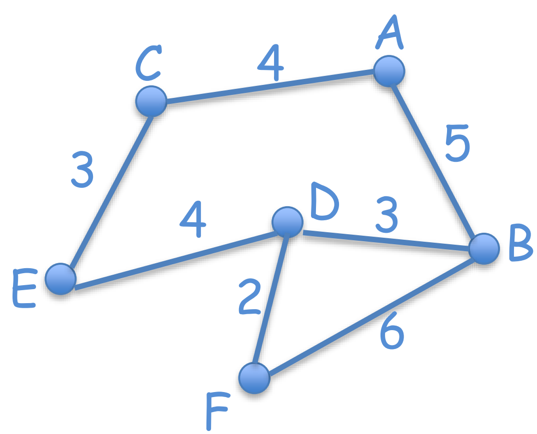

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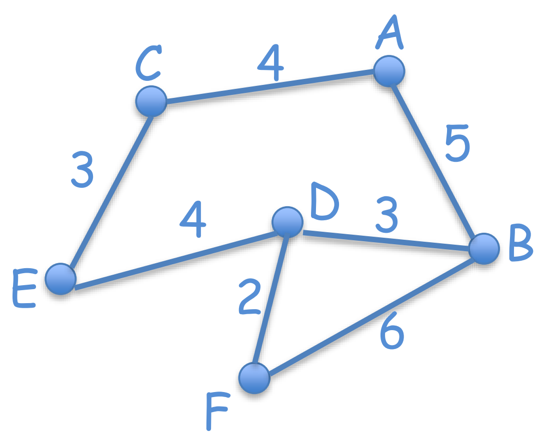

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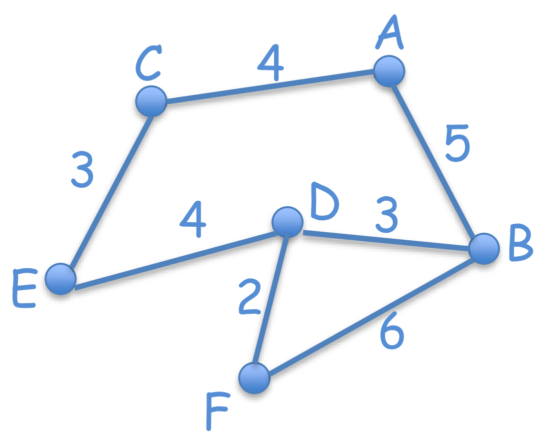

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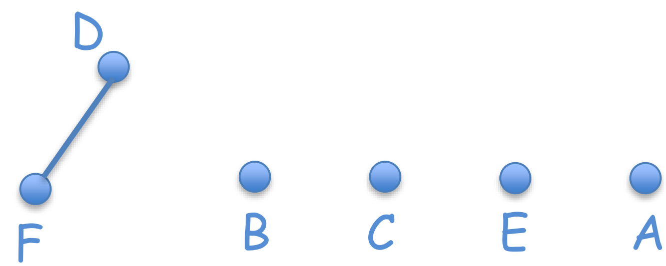

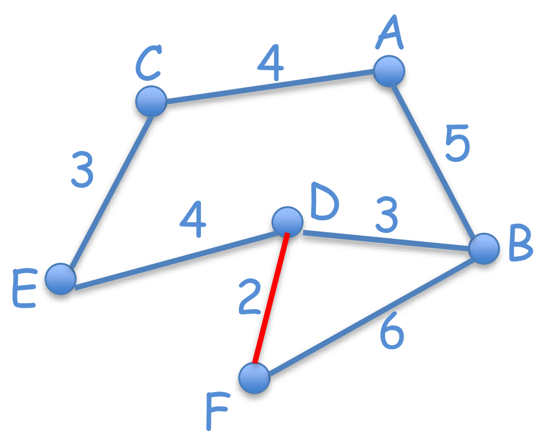

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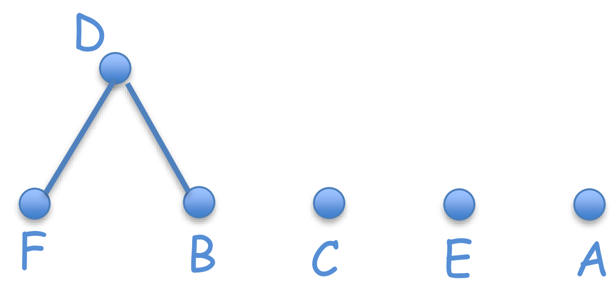

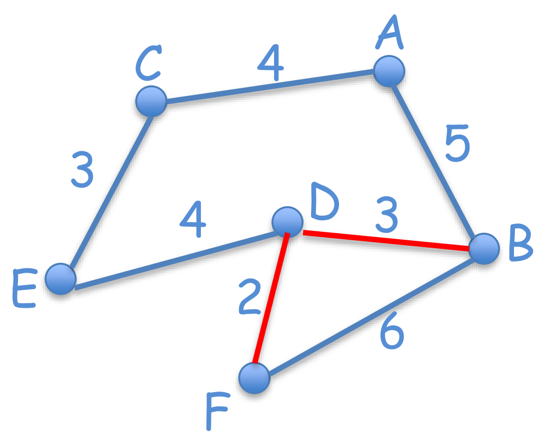

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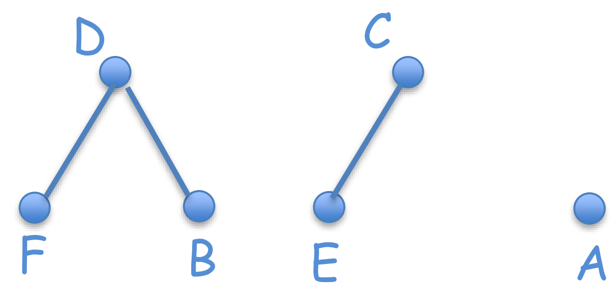

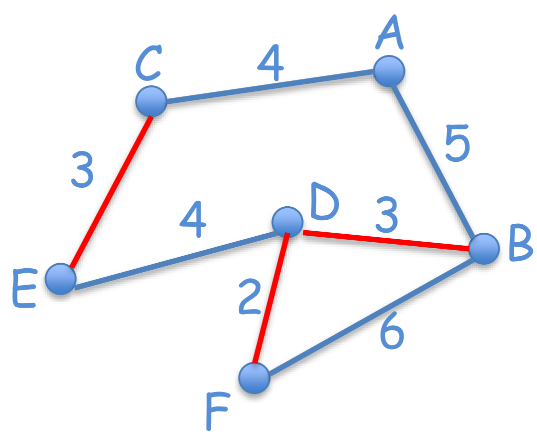

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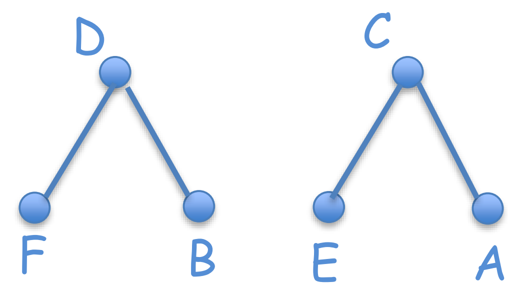

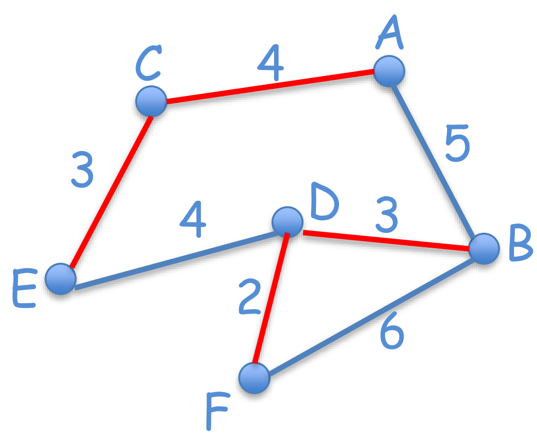

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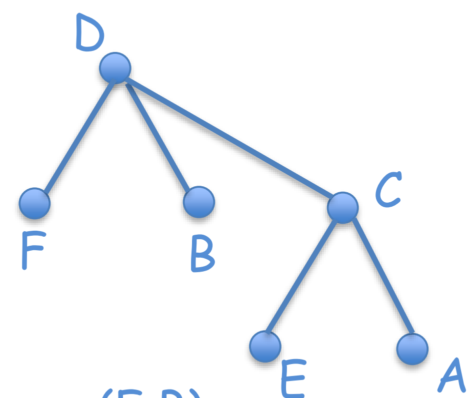

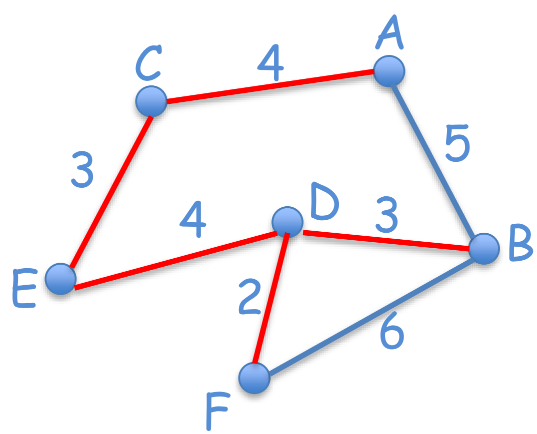

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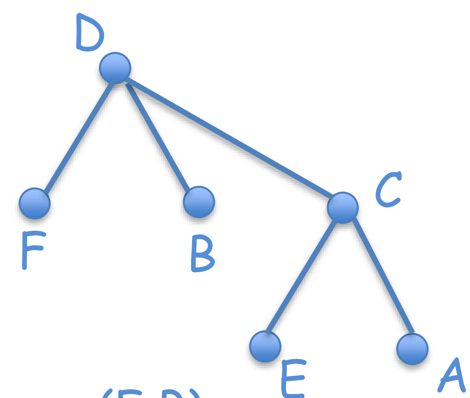

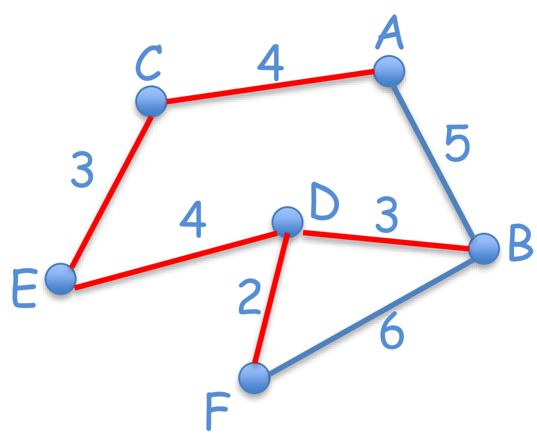

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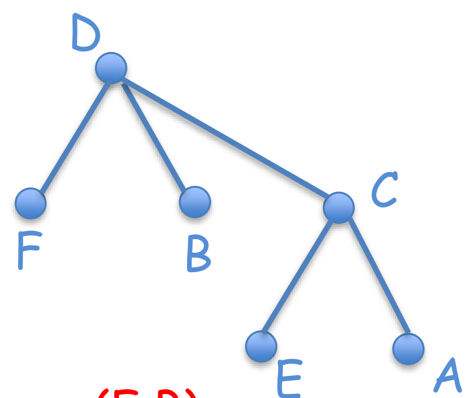

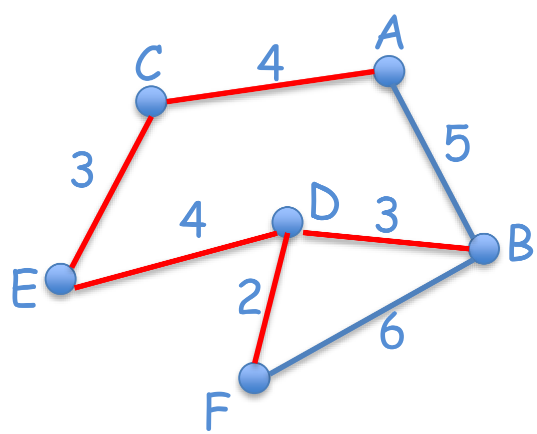

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Prim's Algorithm

MST-Prim(G,s)

```
for each u of V
     u.key = ∞
     u.par = nil
s.key = 0
create a minimum priority Q on V
while Q ≠ { }
     u = ExtractMin(Q)
     for each v of Adj(u)
          if v in Q and w(u,v) < v.key
               v.par = u
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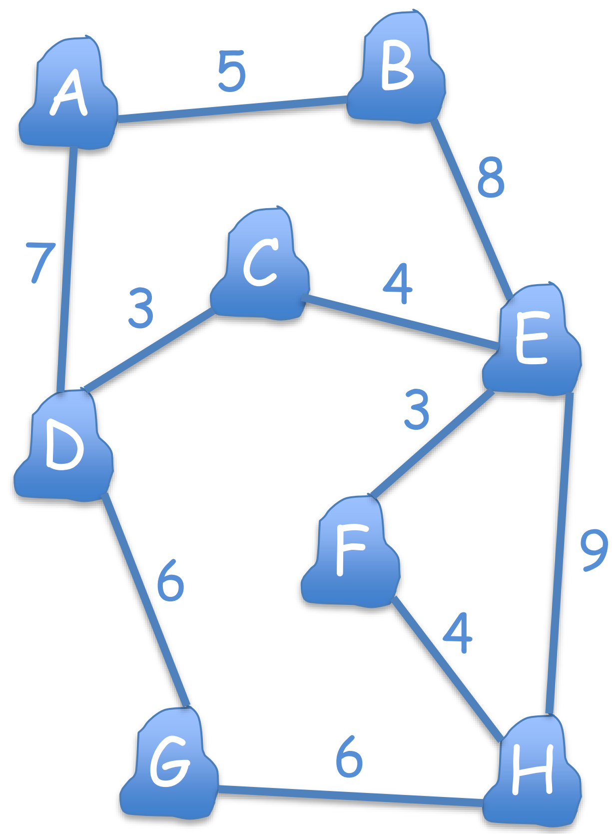

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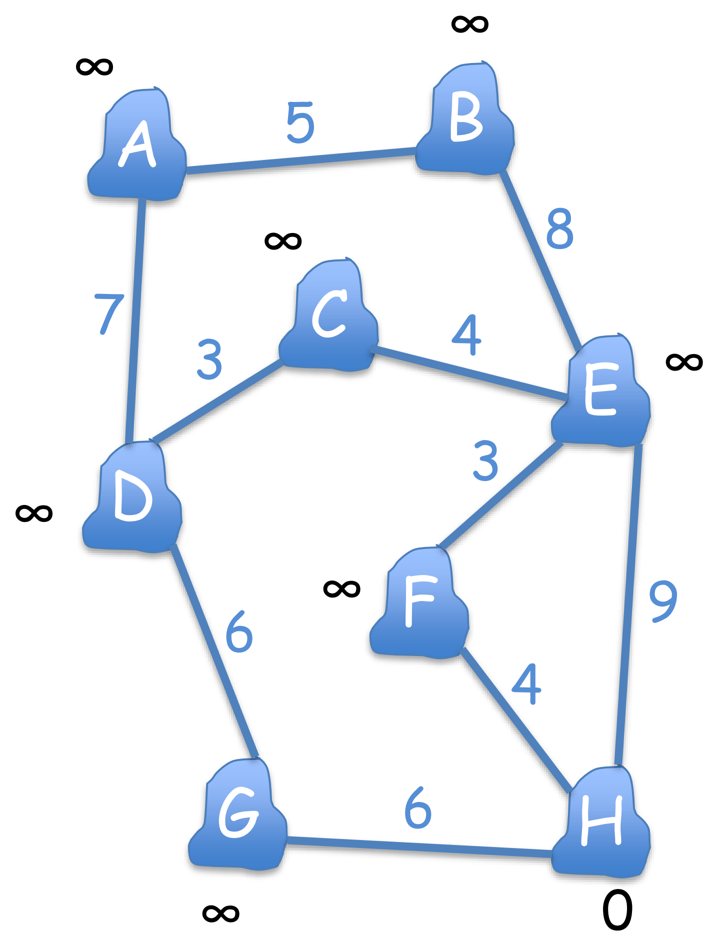

H F G E D C B A

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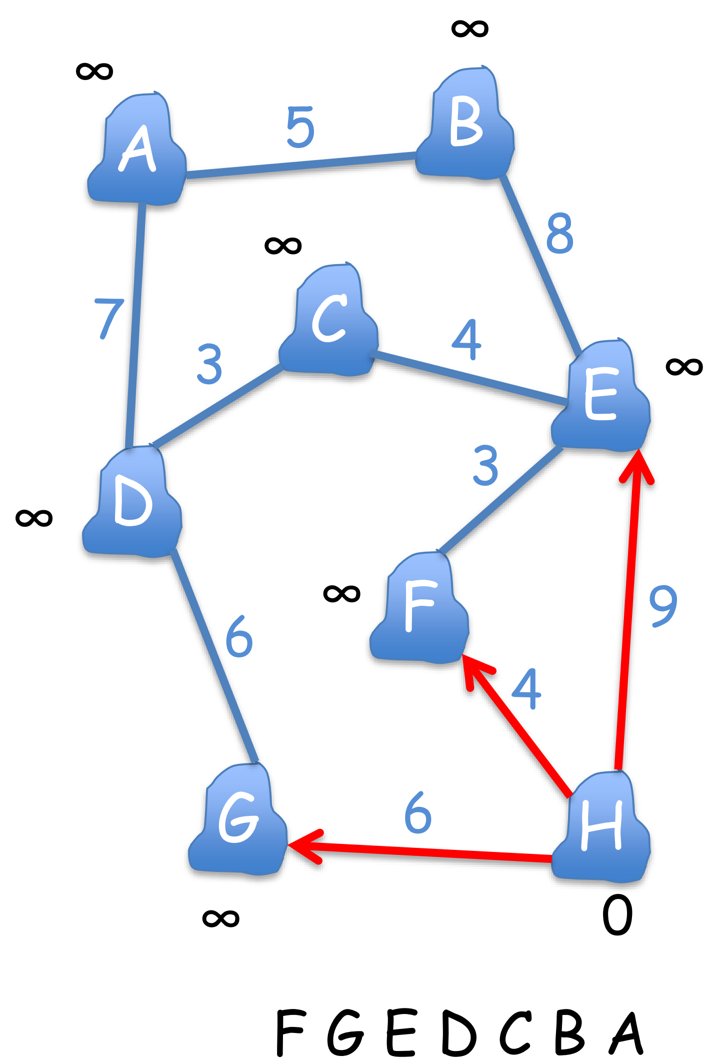

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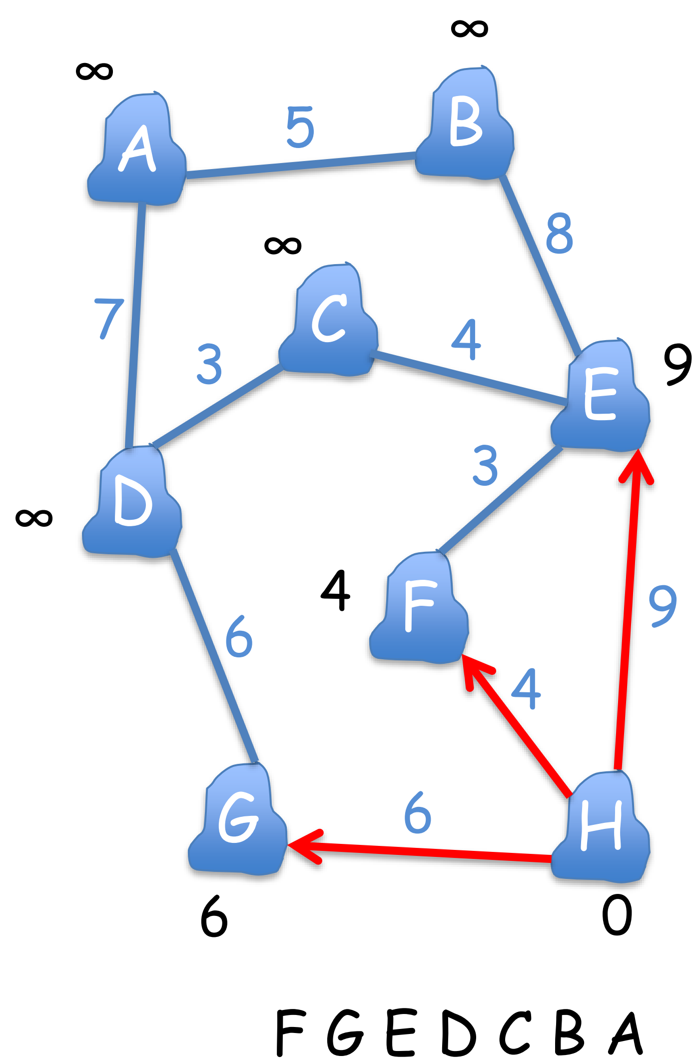

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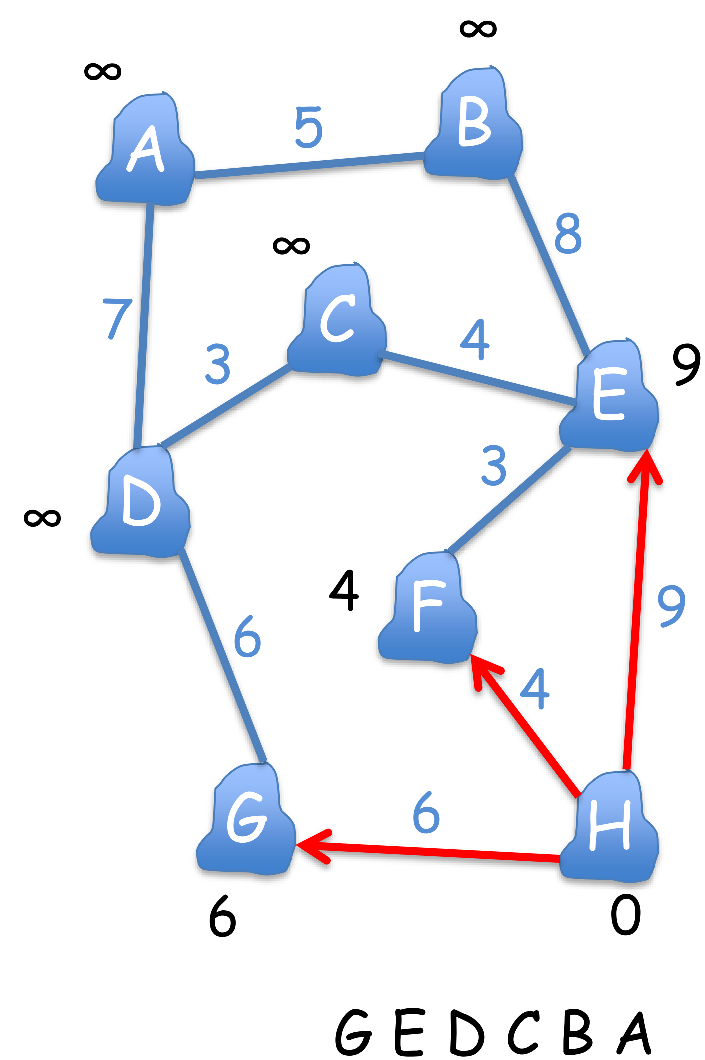

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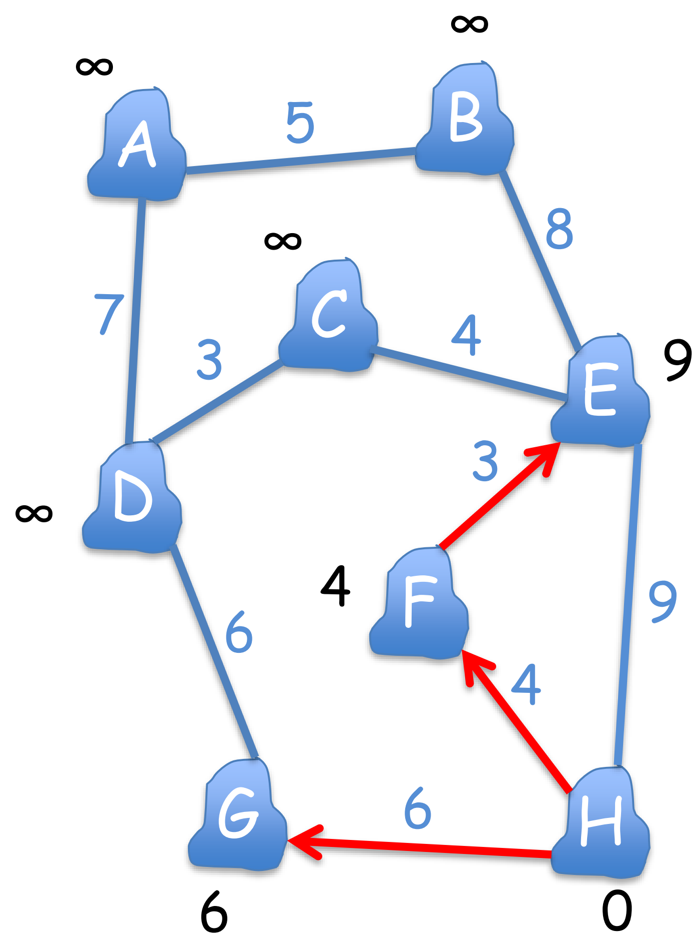

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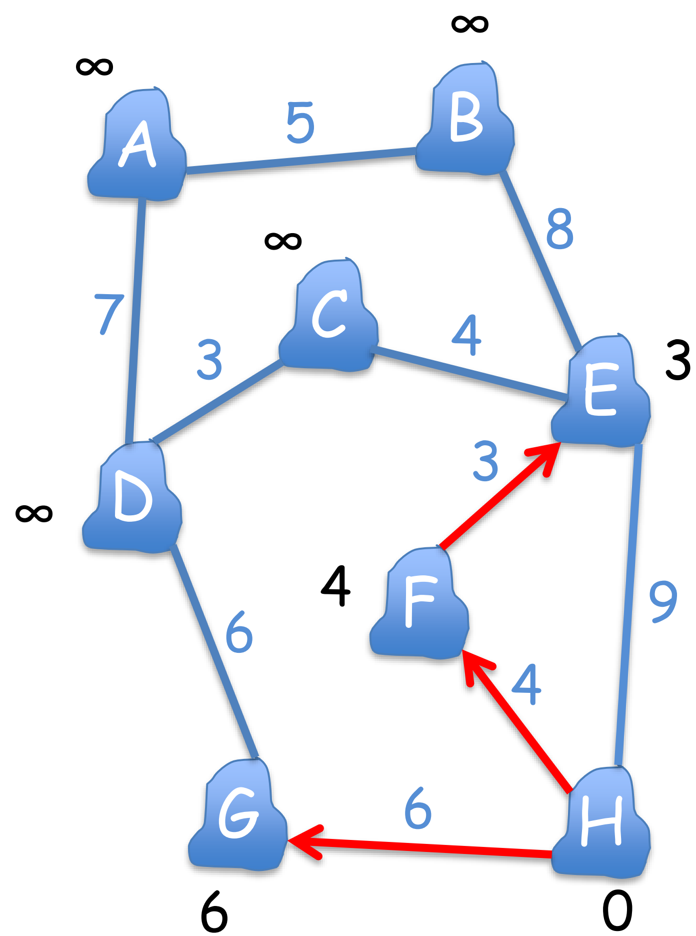

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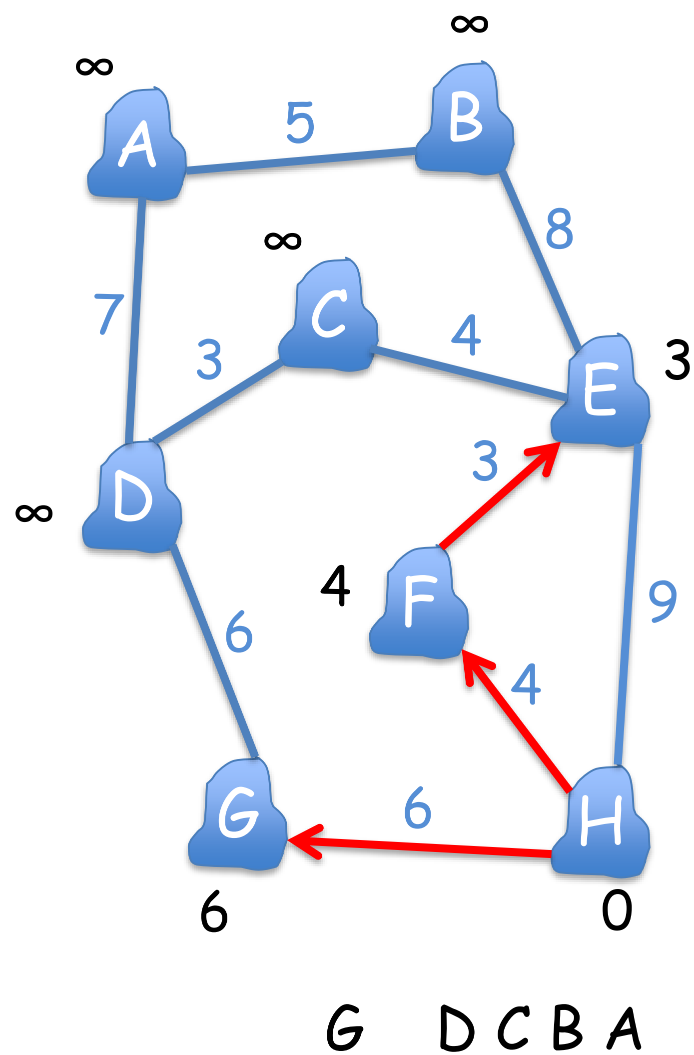

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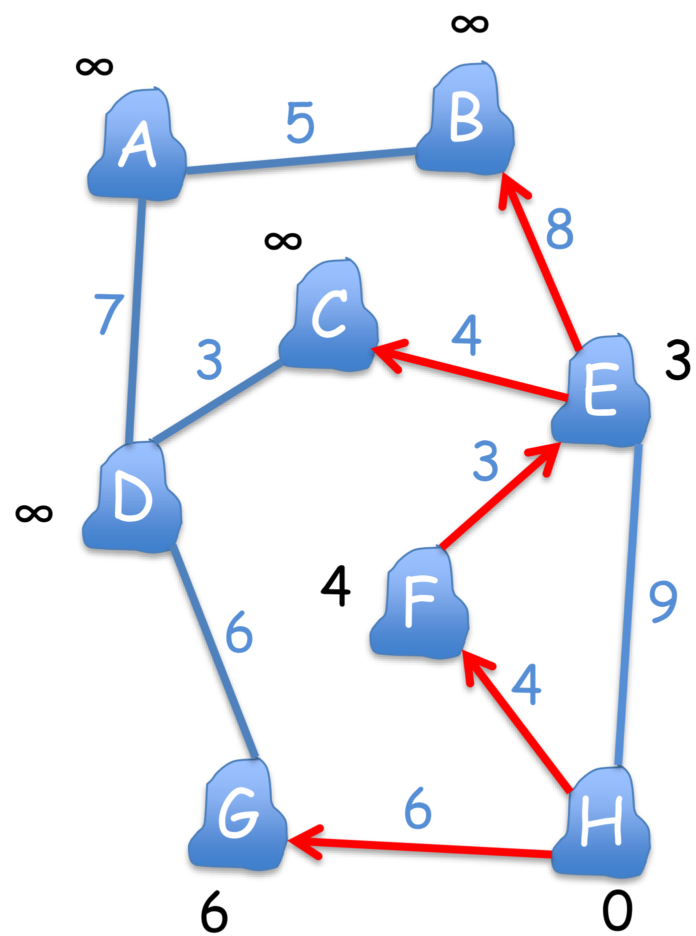

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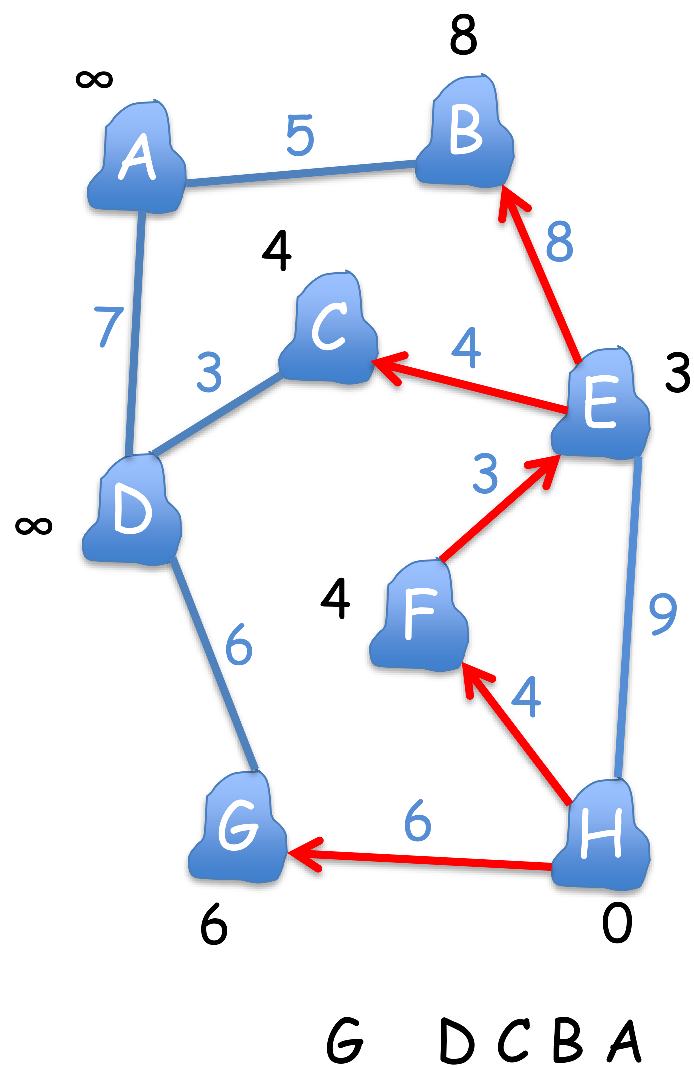

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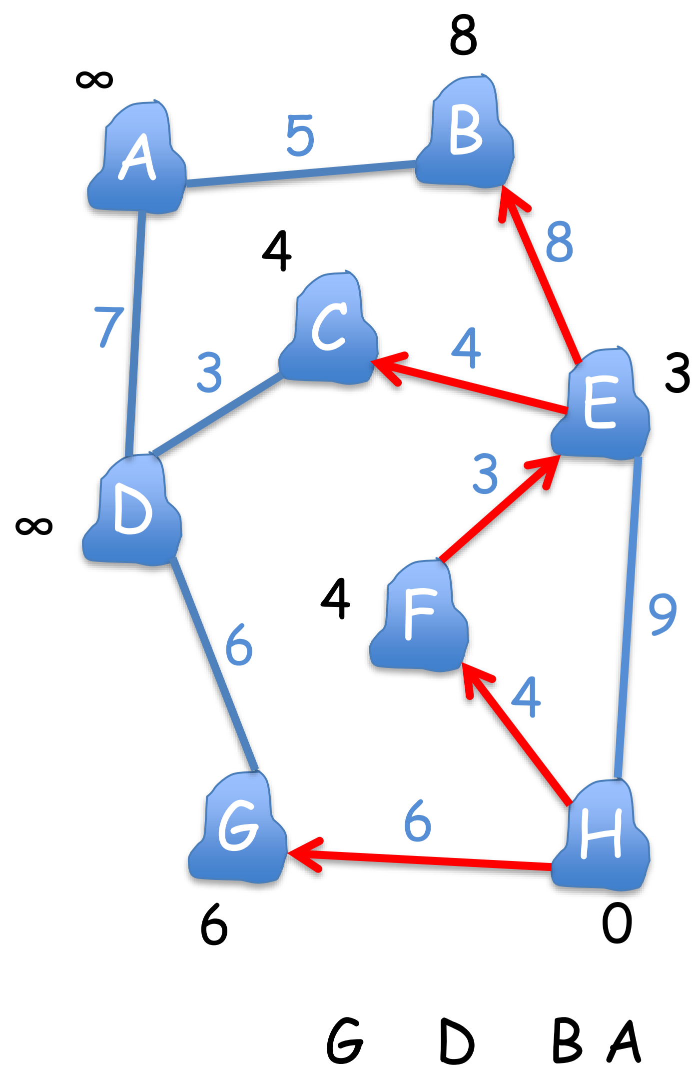

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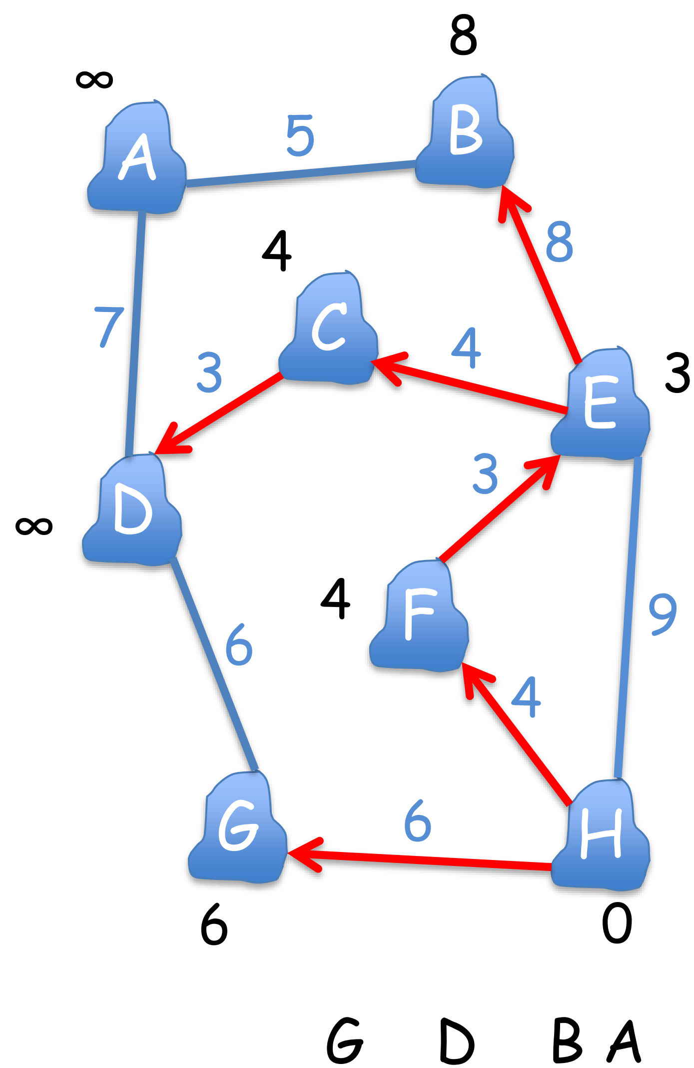

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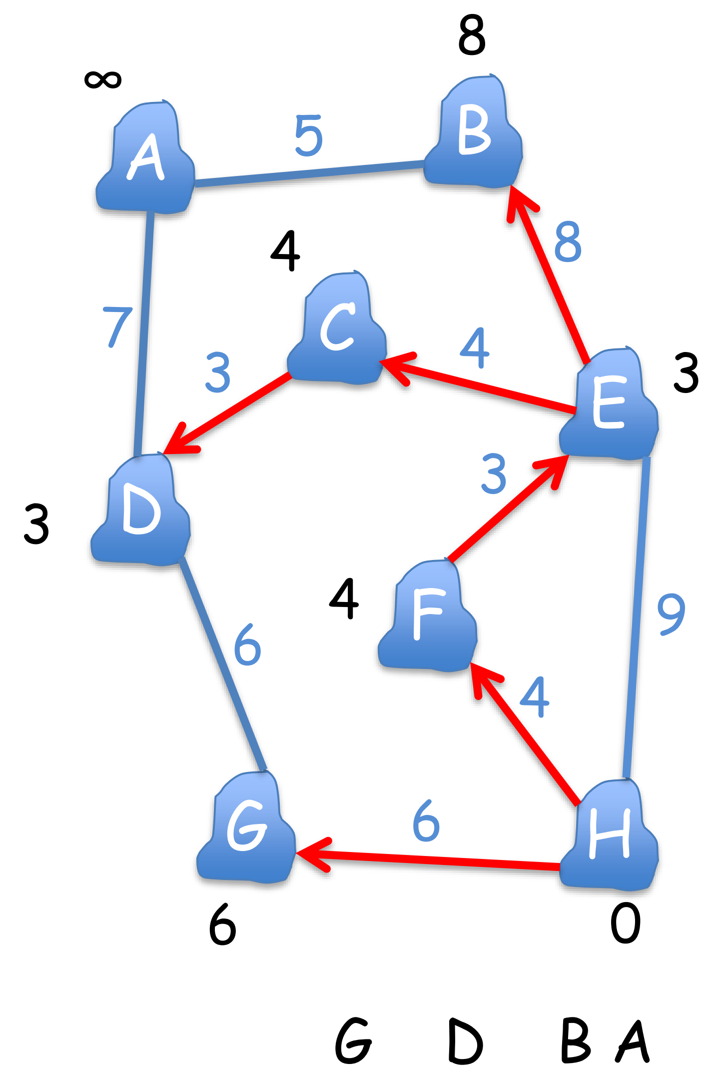

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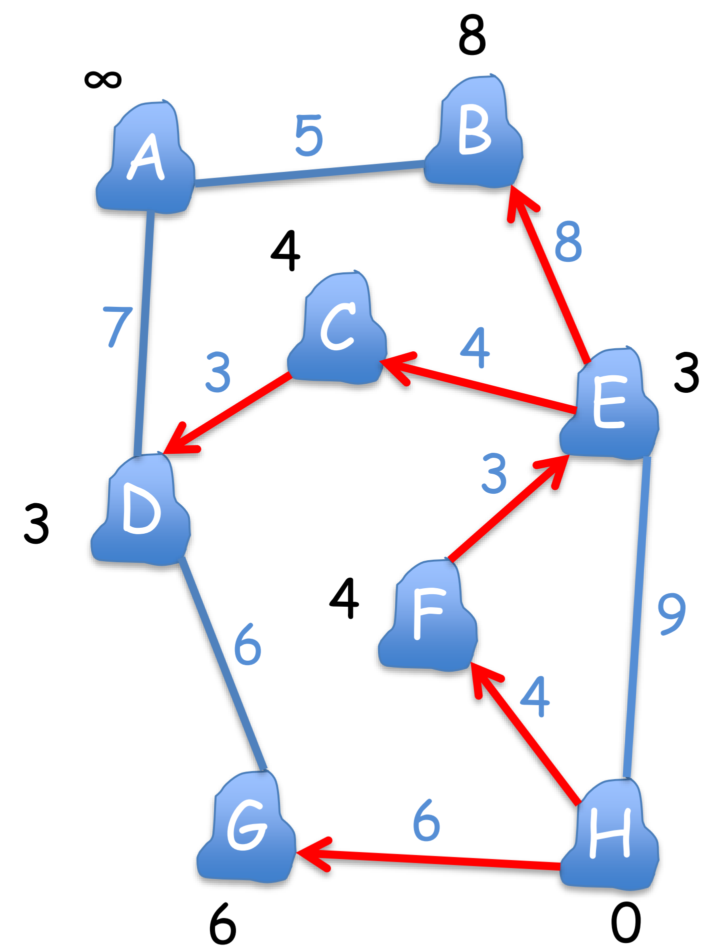

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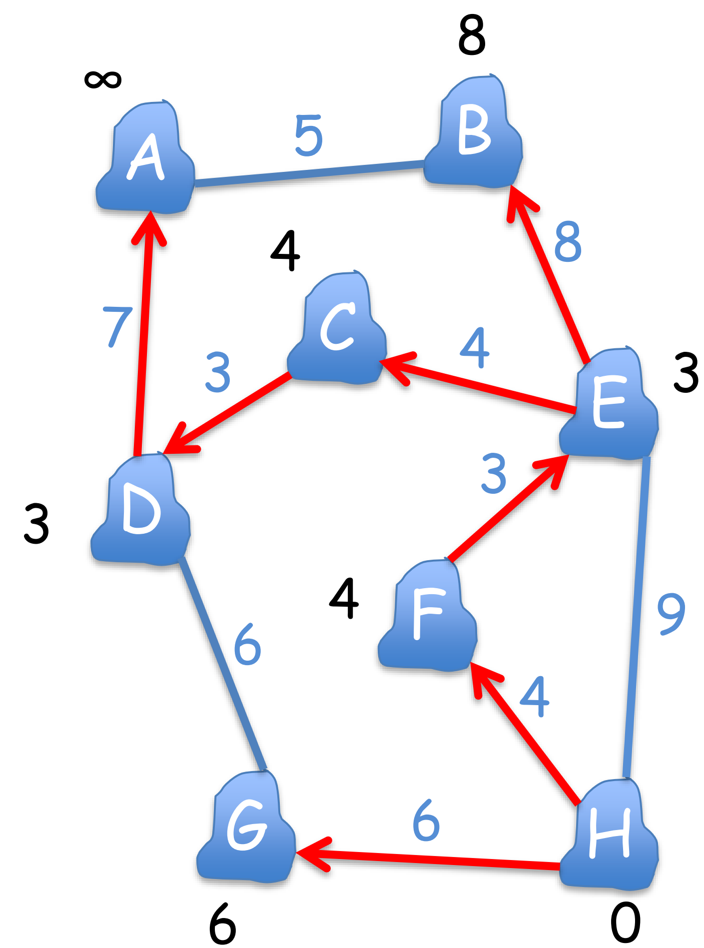

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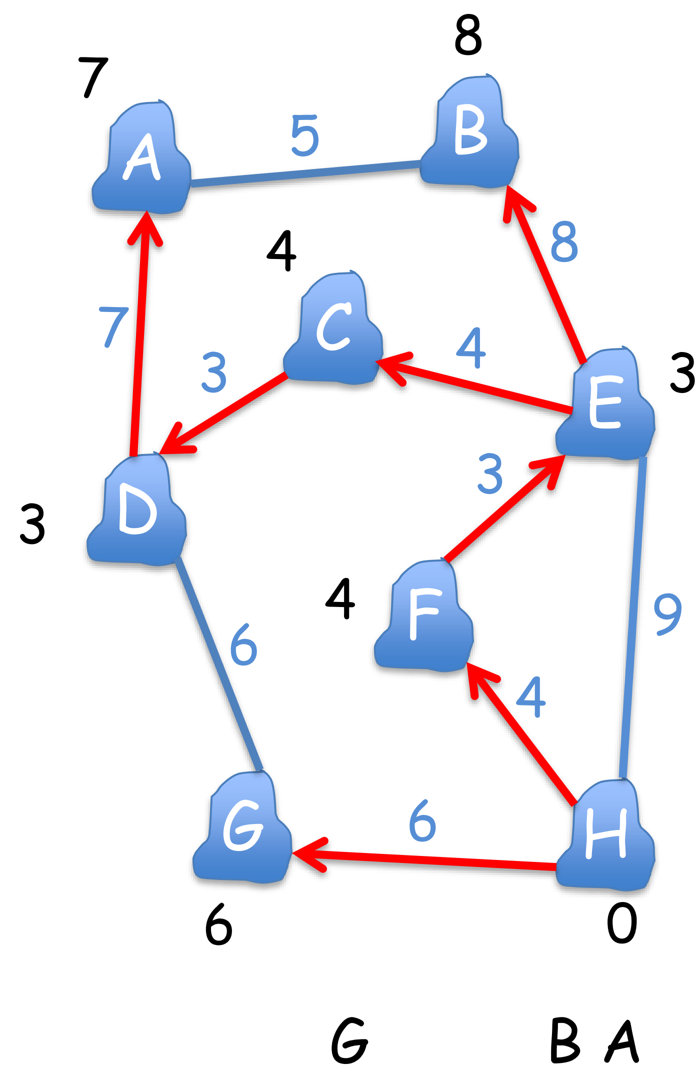

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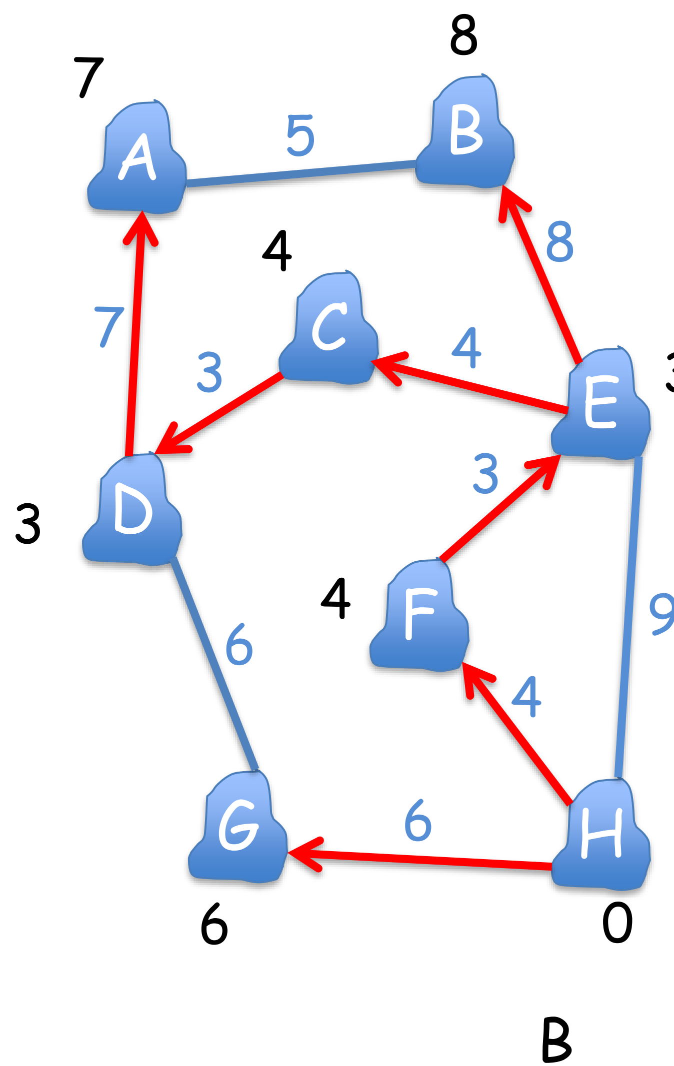

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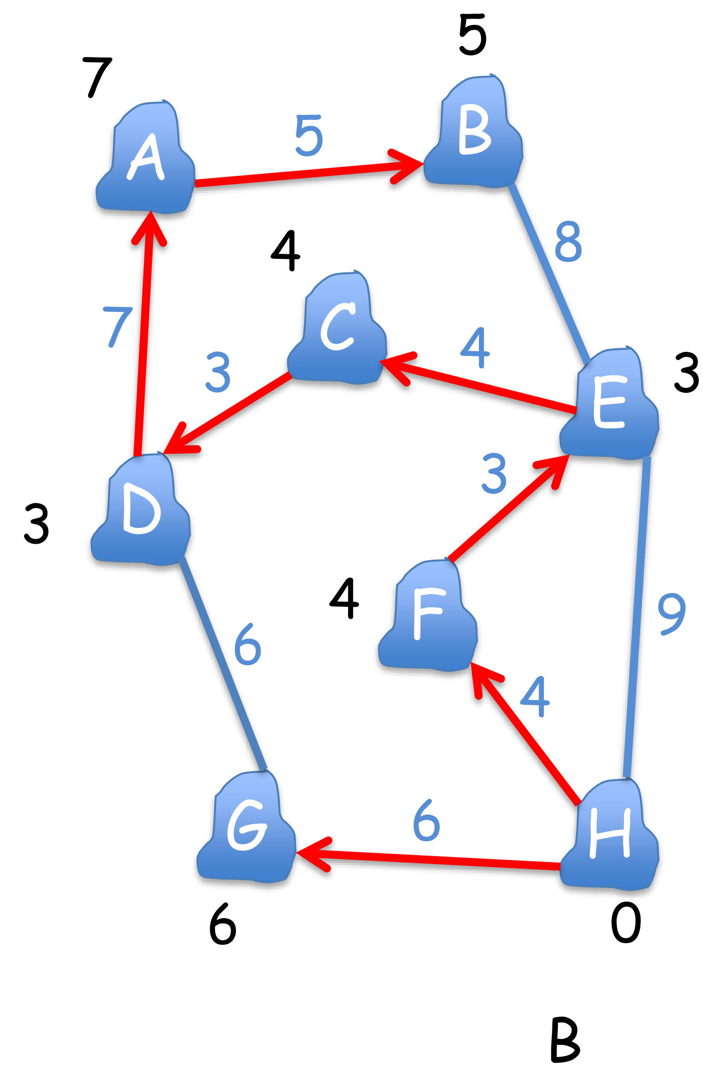

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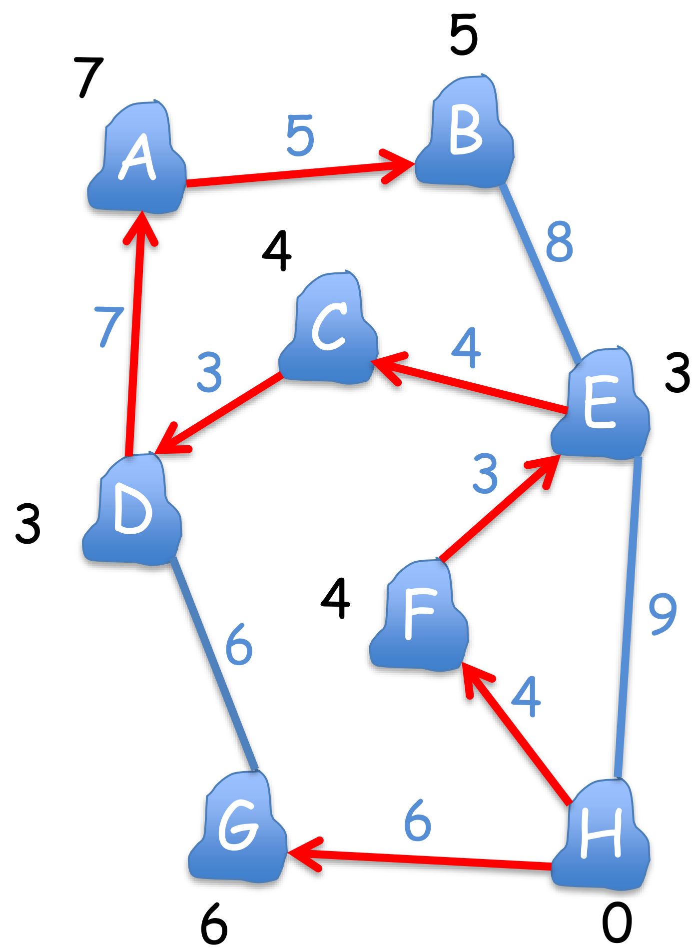

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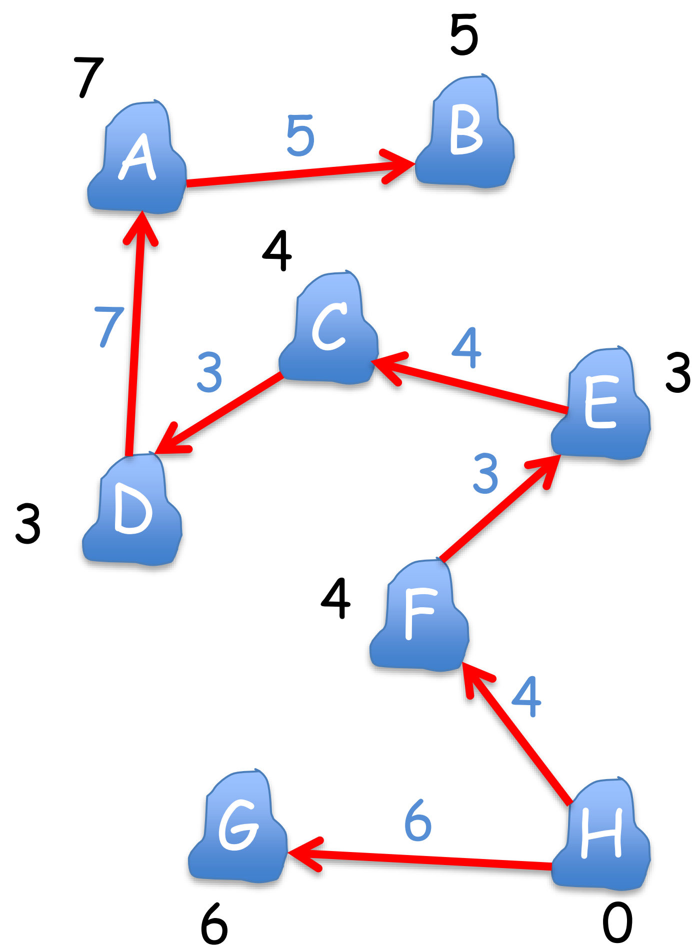

SSSP

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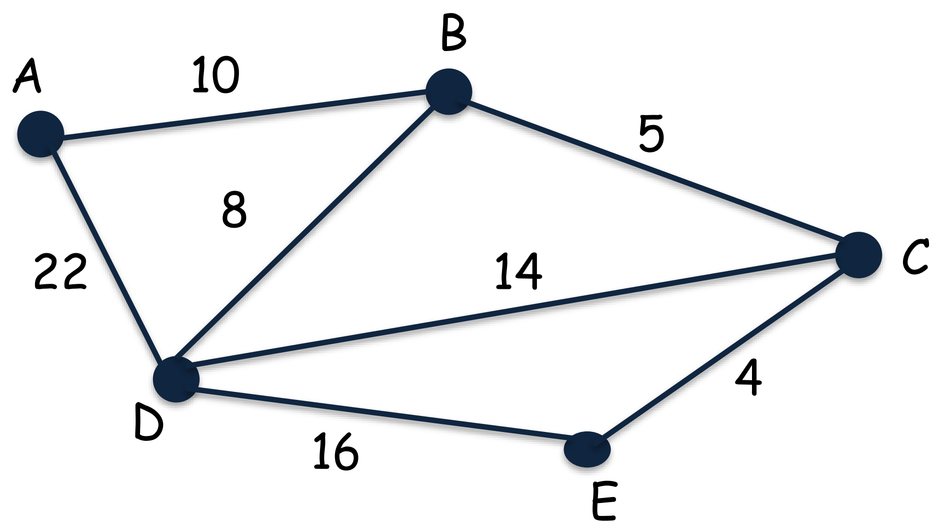

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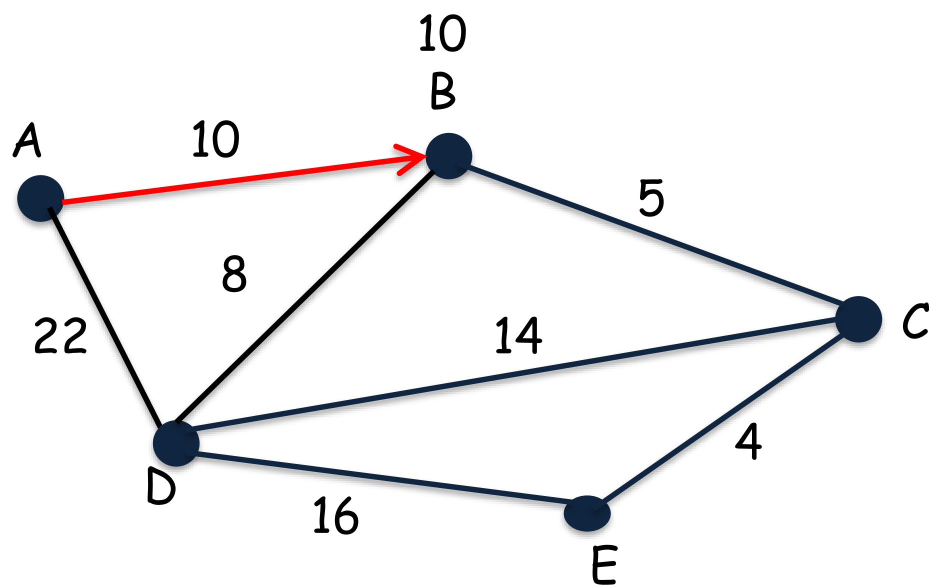

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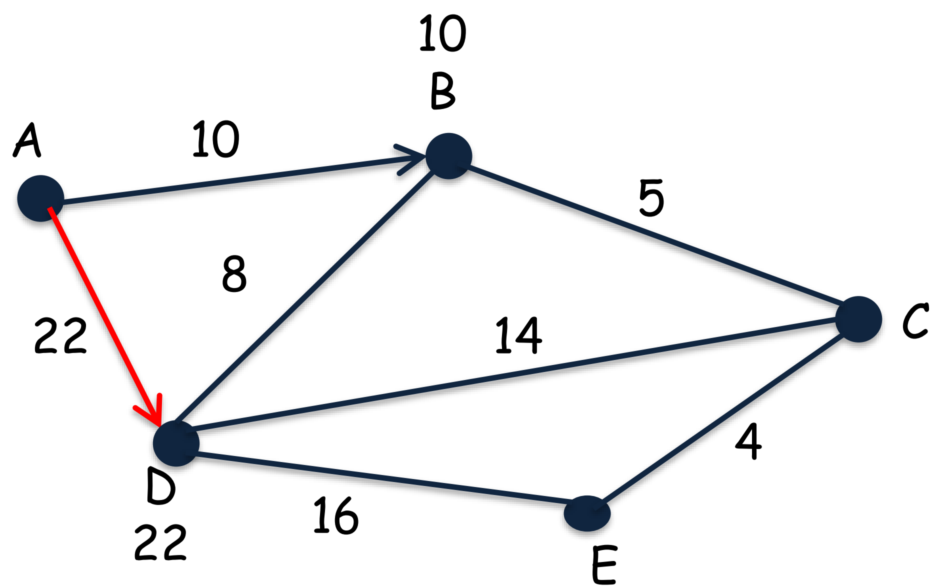

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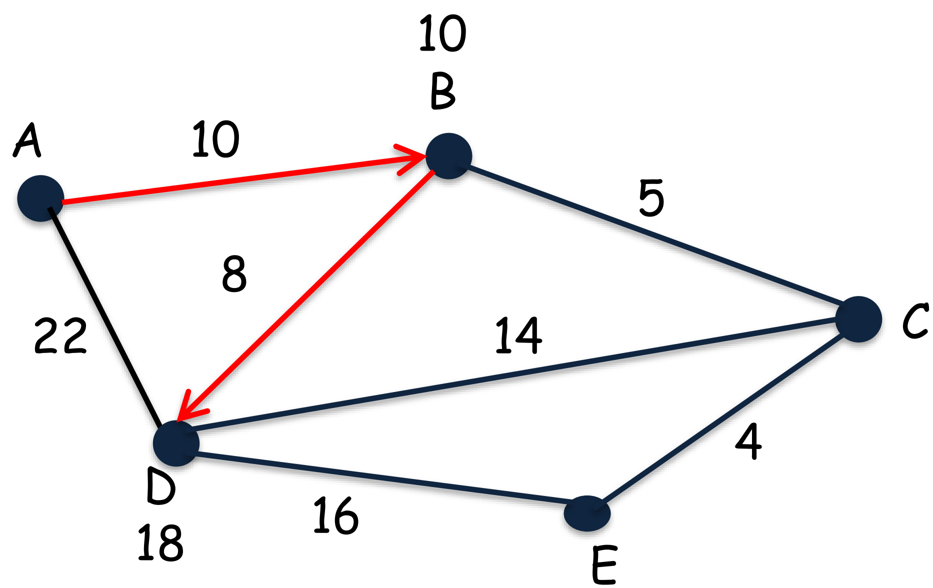

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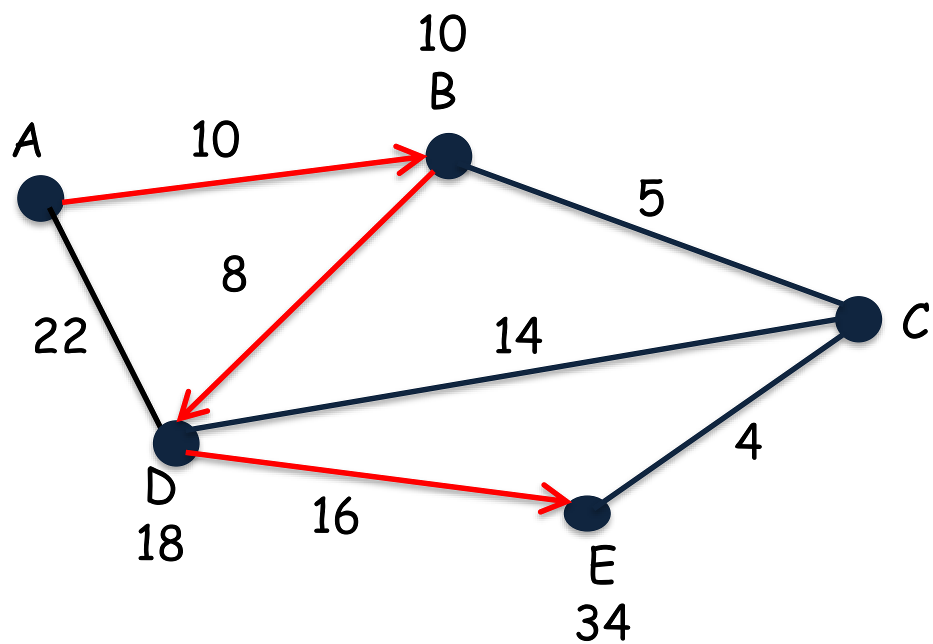

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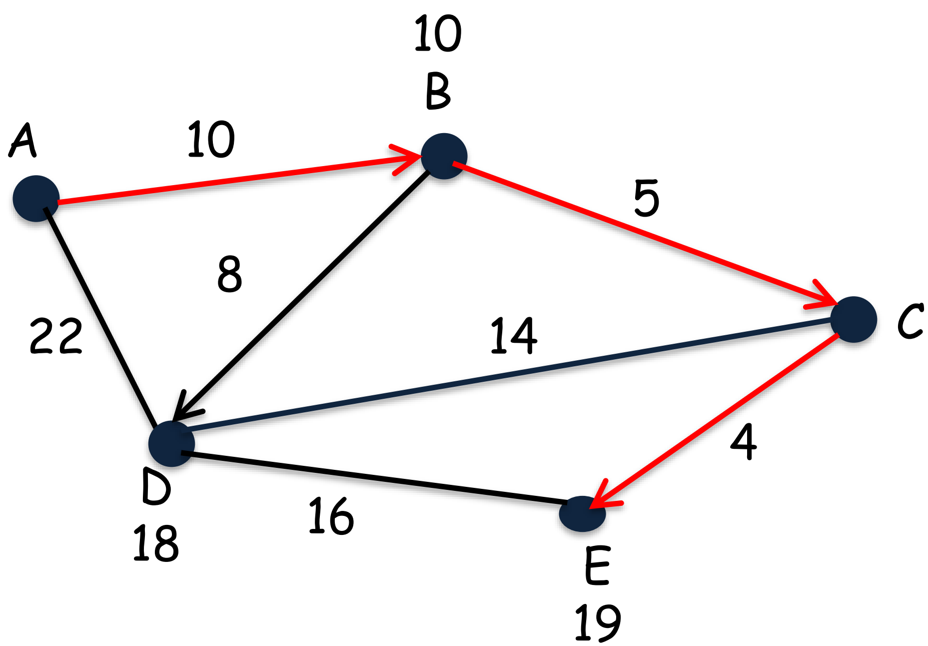

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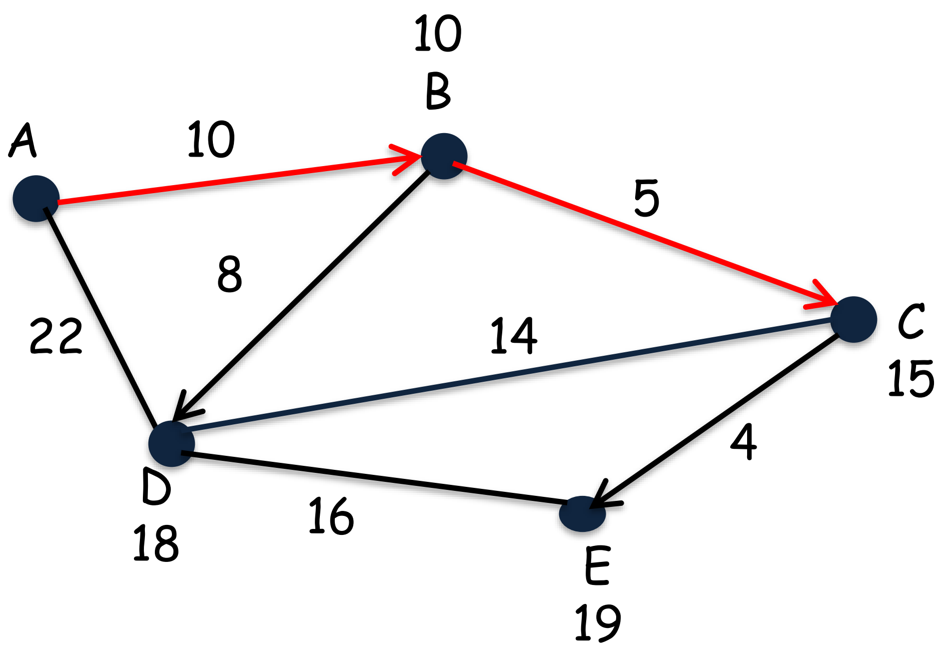

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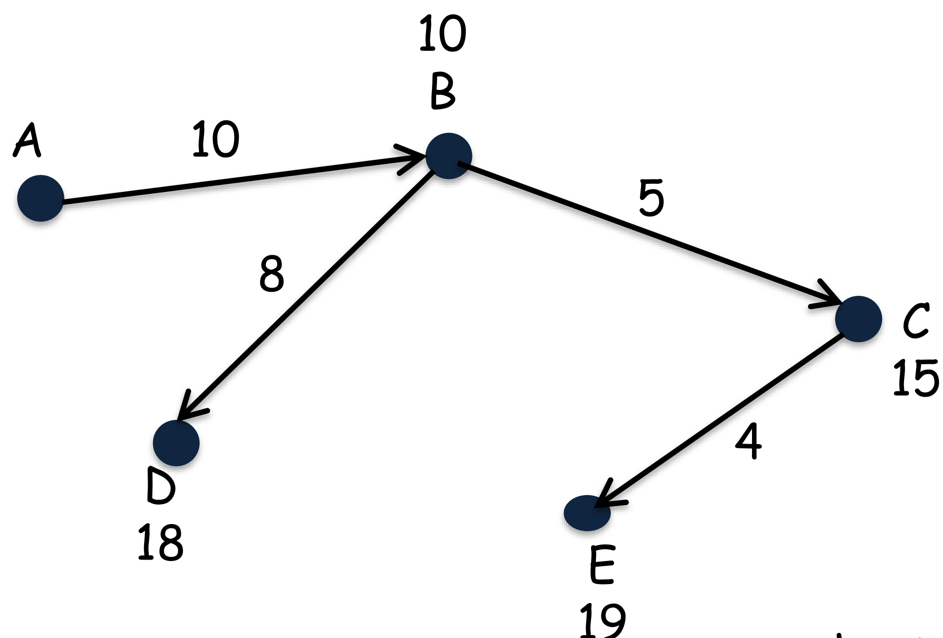

shortest-paths tree

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--Dijkstra—

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Initialize (G, s)

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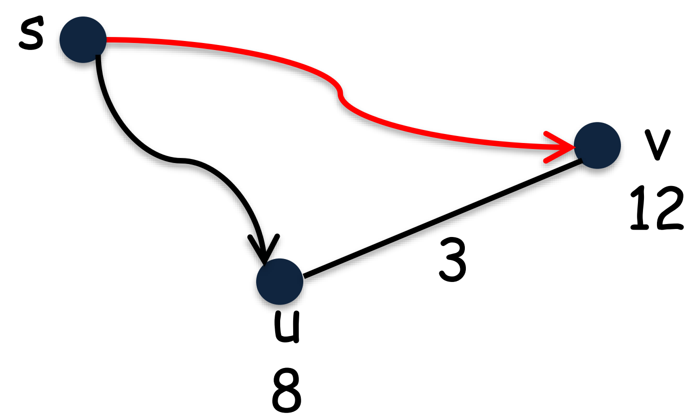

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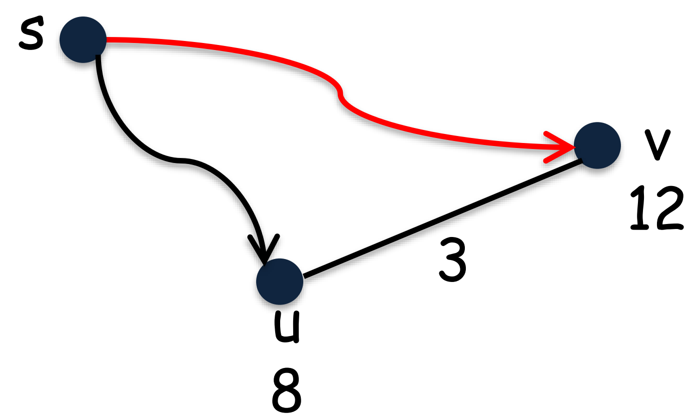

v.dis > u.dis + w(u,v)
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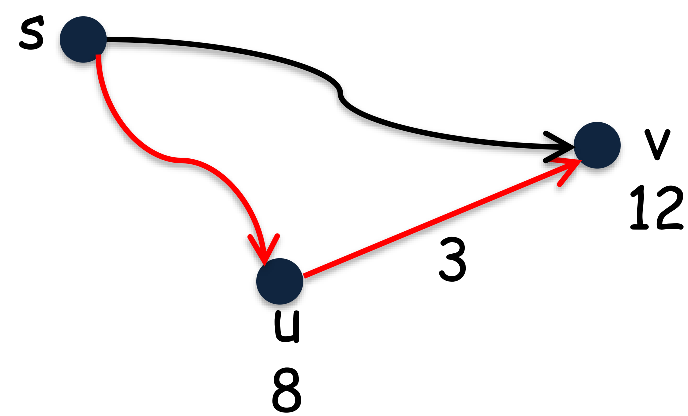

v.dis > u.dis + w(u,v) → v.dis = u.dis + w(u,v)
12 > 8 + 3
v.dis = 11
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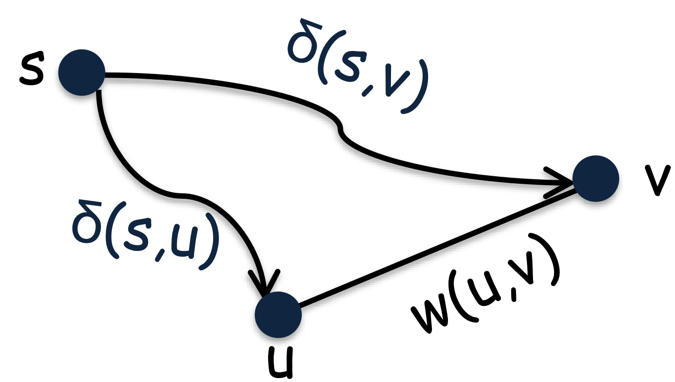
Relaxation

- Let δ(s,v) be the weight of the shortest path from source to the vertex v (after the termination of the program)



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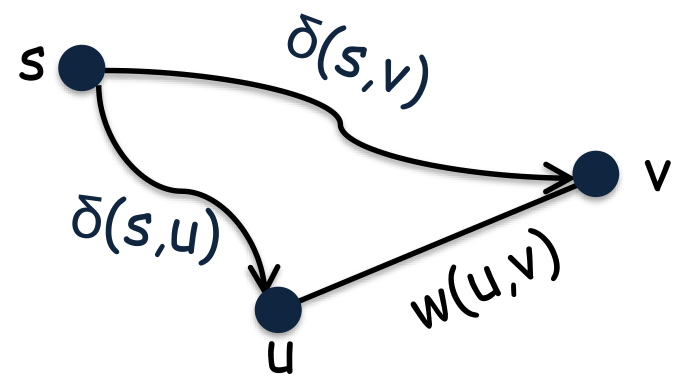

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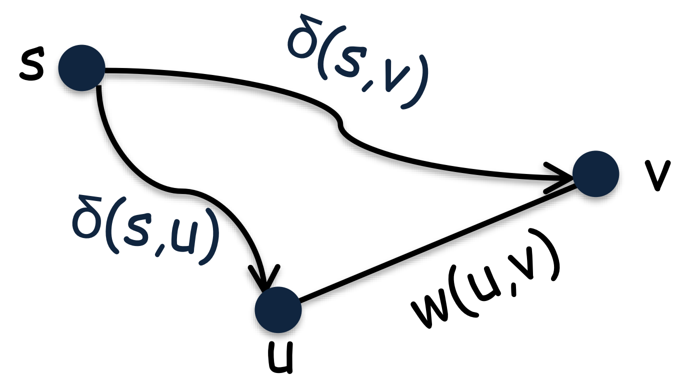

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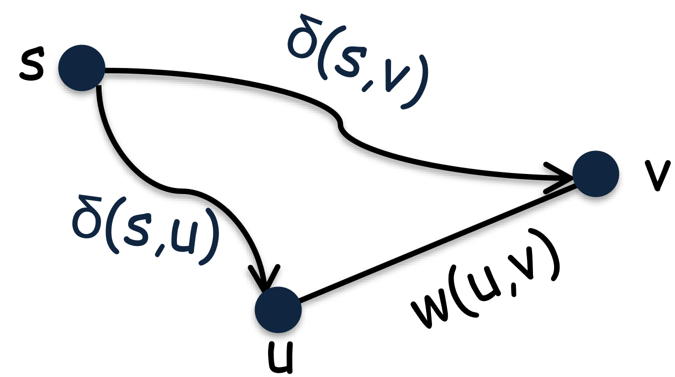

- For all vertices v in V,

$$v.dis \geq \delta(s,v)$$

- If there is no path from s to v, then

$$v.dis = \delta(s,v) = \infty$$

Dijkstra's Algorithm

Dijkstra(G,s)

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for each u of V
    u.key = ∞
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initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
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create a minimum priority Q on V         O(|V|)
while Q ≠ { }
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Relax(u,v)
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      for each v of Adj(u)
            if  v.dis > u.dis + w(u,v)
                 v.dis = u.dis + w(u,v)
                 v.par = u
            update Q
```
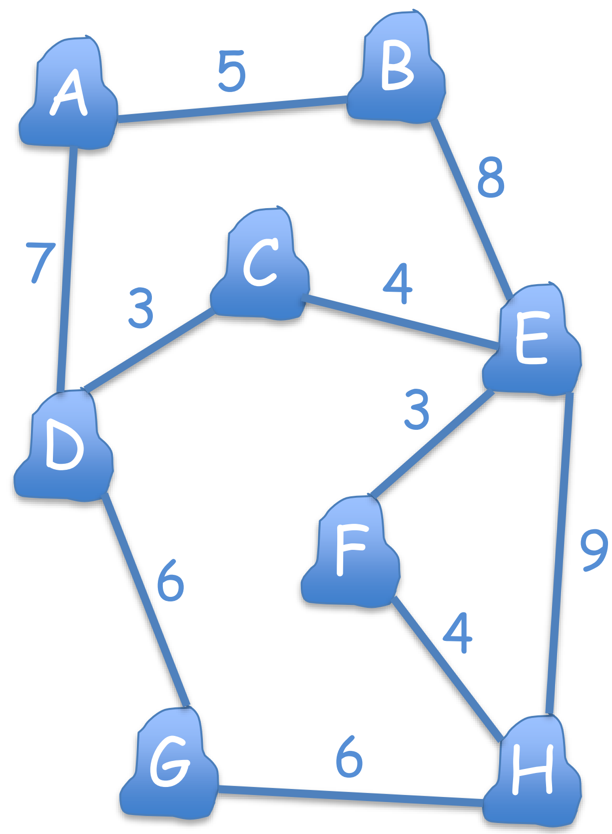
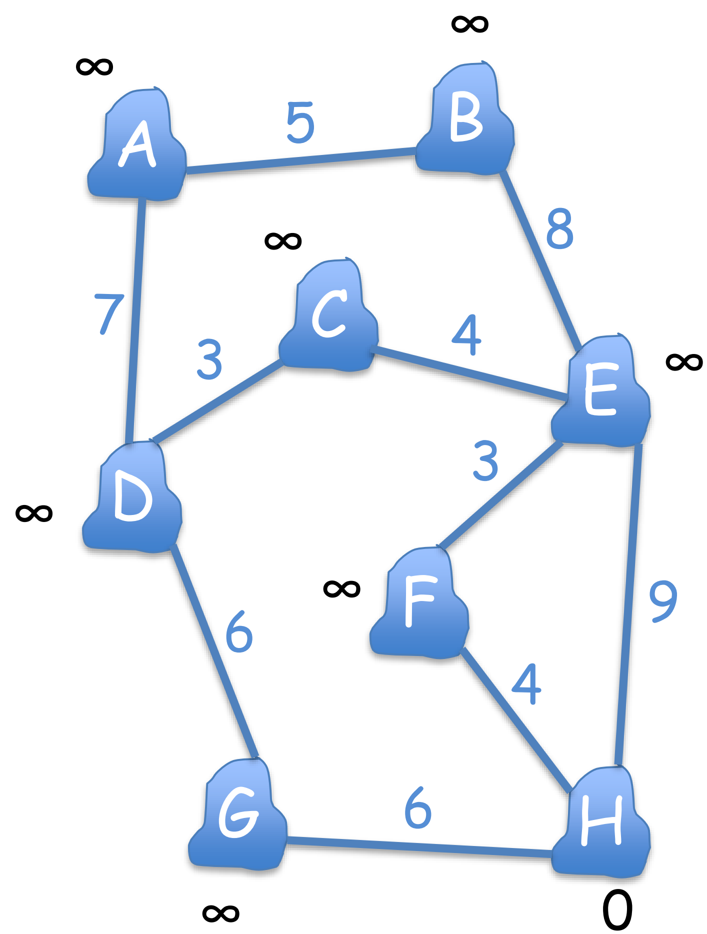



Dijkstra's Algorithm

Dijkstra(G,s)

```
for each u of V
      u.key = ∞
      u.par = nil
s.key = 0
initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
      u = ExtractMin(Q)
      S = S ∪ {u}
      for each v of Adj(u)
            if  v.dis > u.dis + w(u,v)
                 v.dis = u.dis + w(u,v)
                 v.par = u
           update Q
```



H F G E D C B A

S = { }

Dijkstra's Algorithm

Dijkstra(G,s)

```
for each u of V
      u.key = ∞
      u.par = nil
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initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
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                v.dis = u.dis + w(u,v)
                v.par = u
          update Q
```

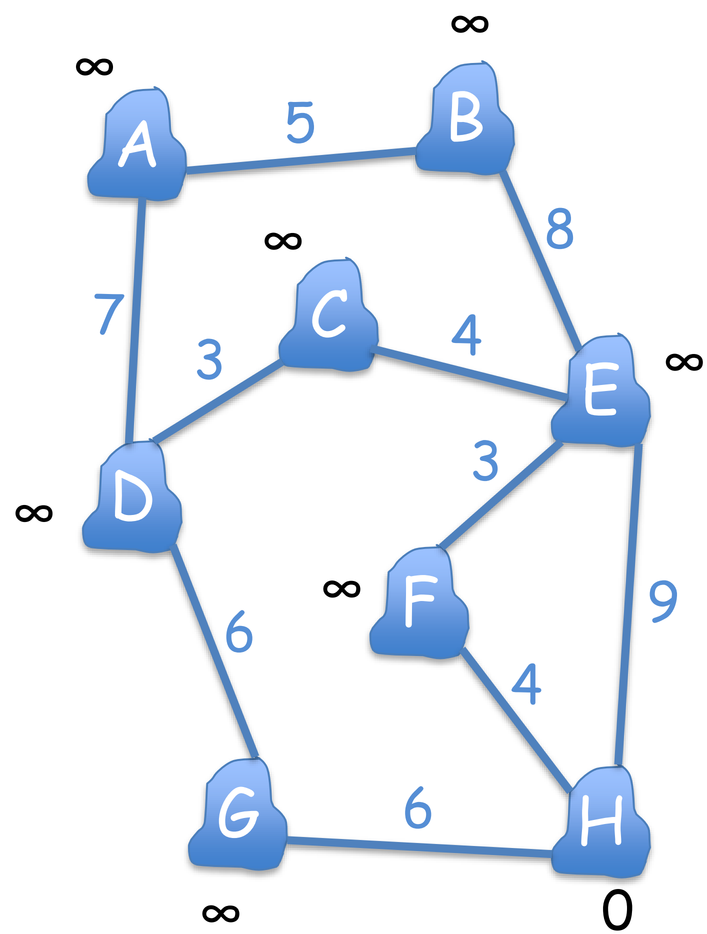

F G E D C B A

S = {H}

Dijkstra's Algorithm

Dijkstra(G,s)

```
for each u of V
      u.key = ∞
      u.par = nil
s.key = 0
initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
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      S = S ∪ {u}
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            if  v.dis > u.dis + w(u,v)
                 v.dis = u.dis + w(u,v)
                 v.par = u
           update Q
```
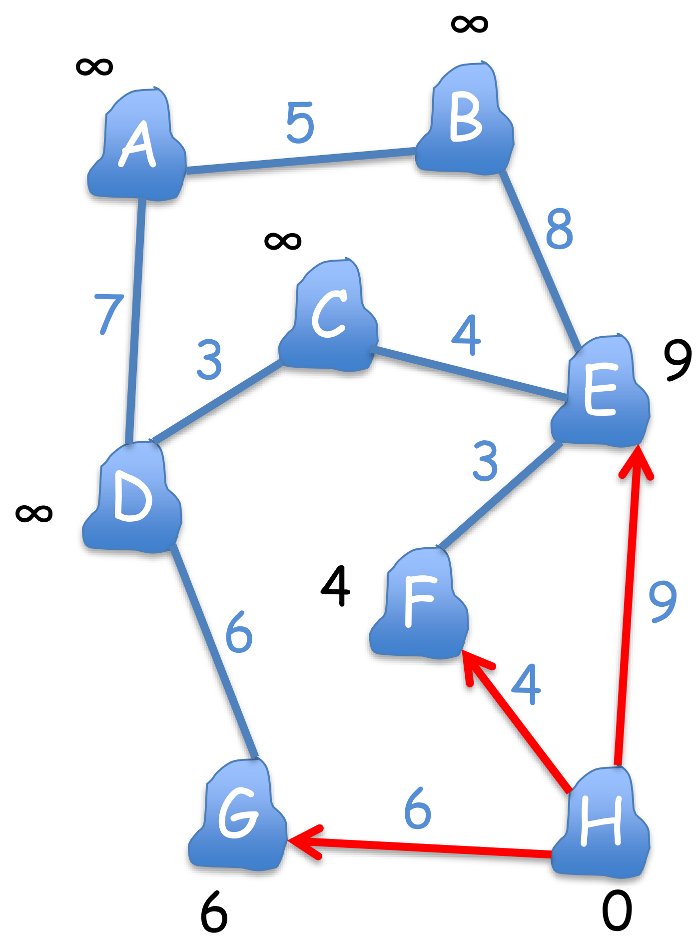



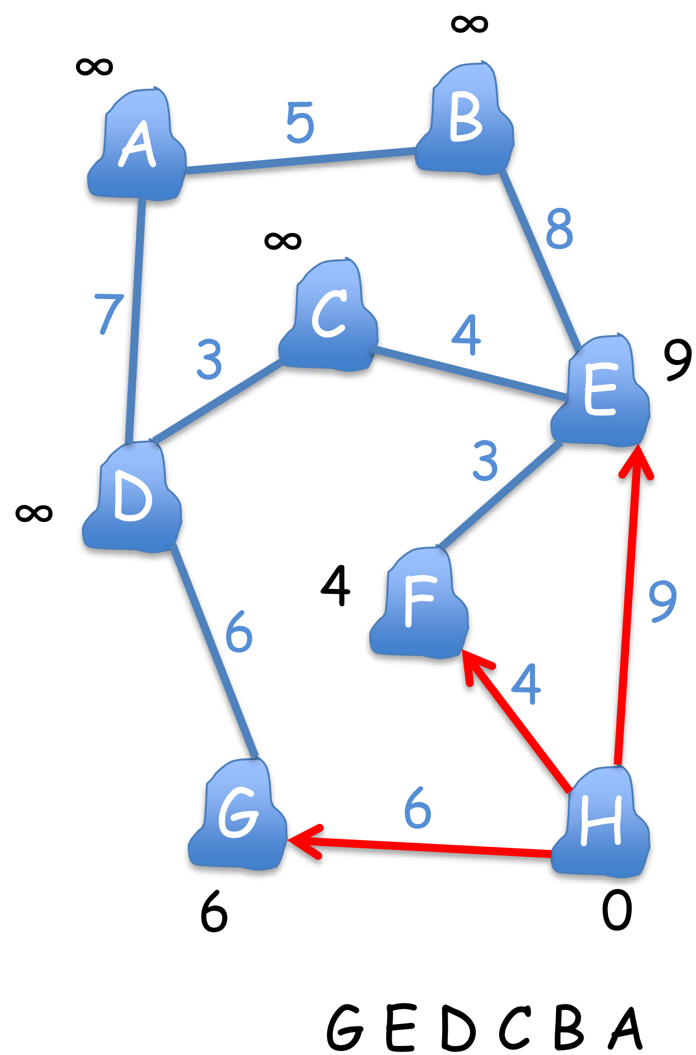
F G E D C B A

S = {H}

Dijkstra's Algorithm

<u>Dijkstra(G,s)</u>

```
for each u of V
    u.key = ∞
    u.par = nil
s.key = 0
initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
    u = ExtractMin(Q)
    S = S ∪ {u}
    for each v of Adj(u)
        if  v.dis > u.dis + w(u,v)
            v.dis = u.dis + w(u,v)
            v.par = u
        update Q
```



S = {H,F}

Dijkstra's Algorithm

Dijkstra(G,s)

```
for each u of V
      u.key = ∞
      u.par = nil
s.key = 0
initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
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                  v.dis = u.dis + w(u,v)
                  v.par = u
            update Q
```

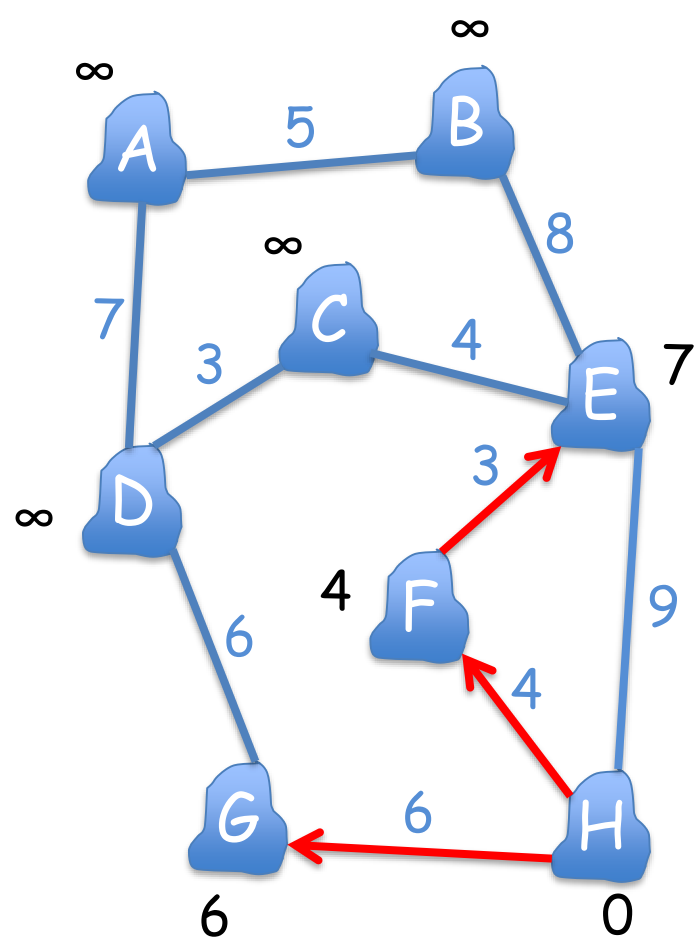

G E D C B A

S = {H,F}

Dijkstra's Algorithm

Dijkstra(G,s)

```
for each u of V
    u.key = ∞
    u.par = nil
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initialize an empty set S
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while Q ≠ { }
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            v.dis = u.dis + w(u,v)
            v.par = u
        update Q
```

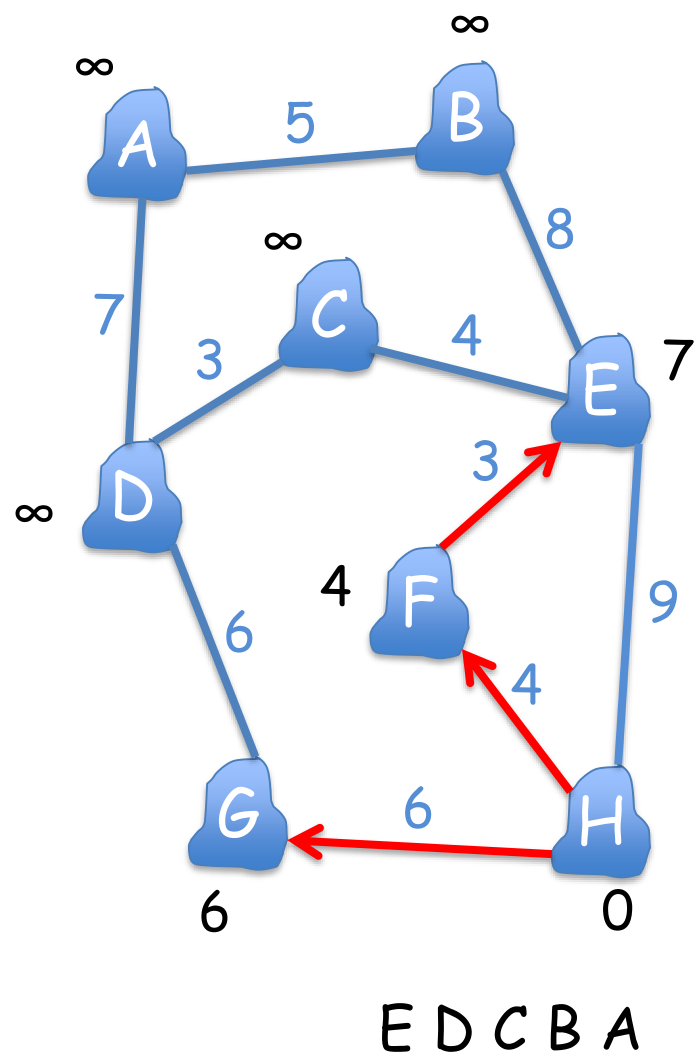

S = {H,F,G}

Dijkstra's Algorithm

Dijkstra(G,s)

```
for each u of V
      u.key = ∞
      u.par = nil
s.key = 0
initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
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                  v.dis = u.dis + w(u,v)
                  v.par = u
            update Q
```

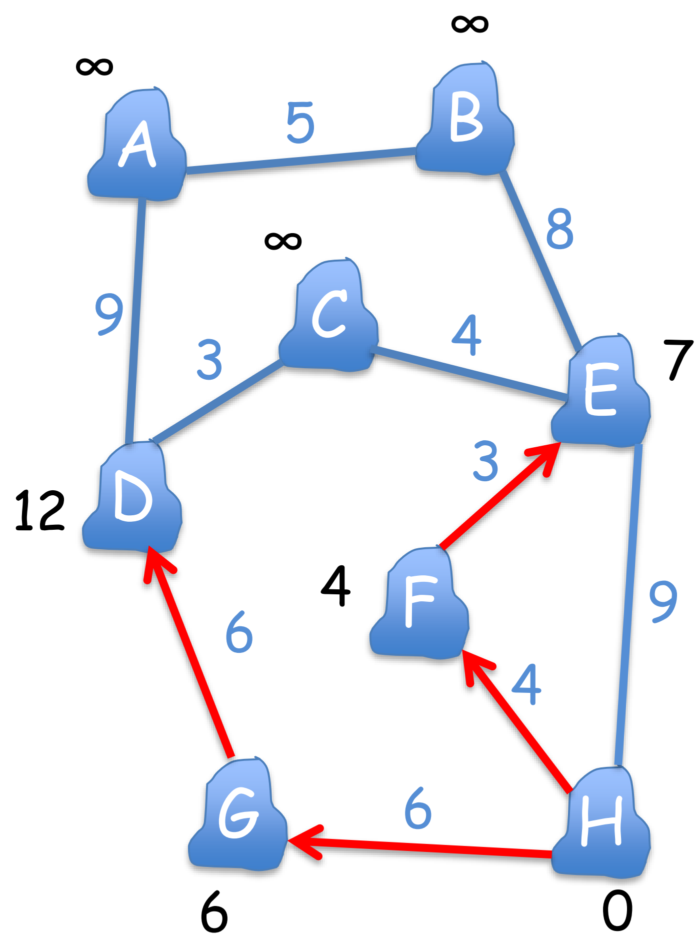

E D C B A

S = {H,F,G}

Dijkstra's Algorithm

```
Dijkstra(G,s)

for each u of V
      u.key = ∞
      u.par = nil
s.key = 0
initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
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            if  v.dis > u.dis + w(u,v)
                 v.dis = u.dis + w(u,v)
                 v.par = u
            update Q
```

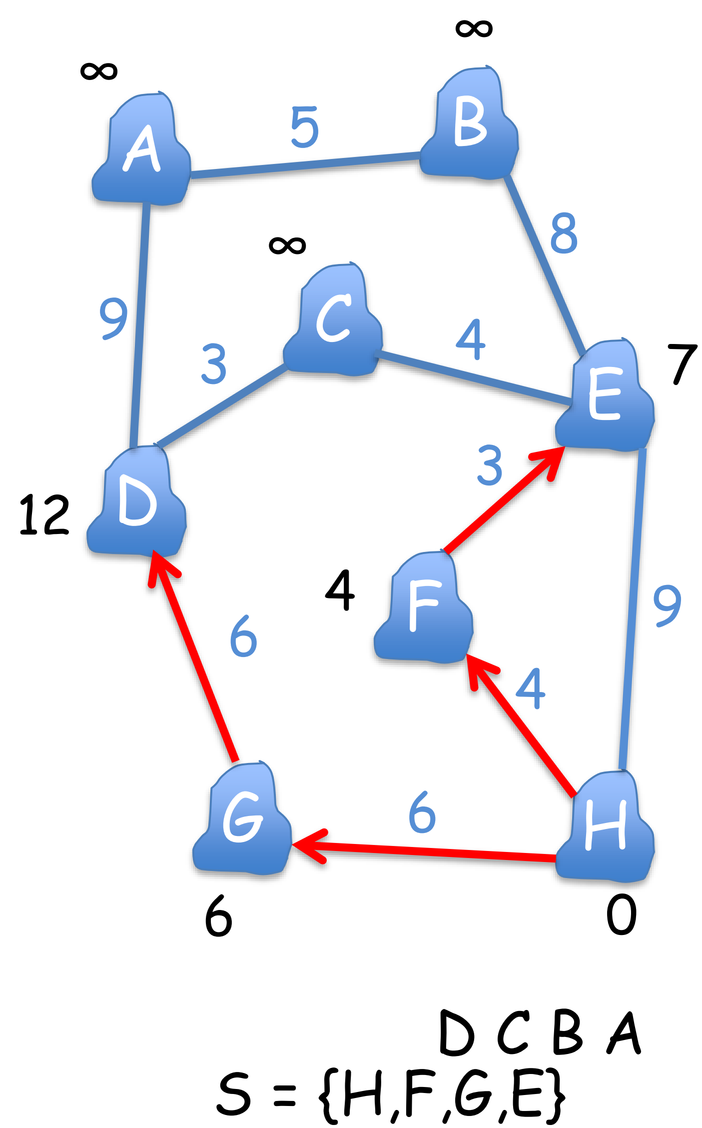

Dijkstra's Algorithm

Dijkstra(G,s)

```
for each u of V
      u.key = ∞
      u.par = nil
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initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
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            if  v.dis > u.dis + w(u,v)
                 v.dis = u.dis + w(u,v)
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            update Q
```

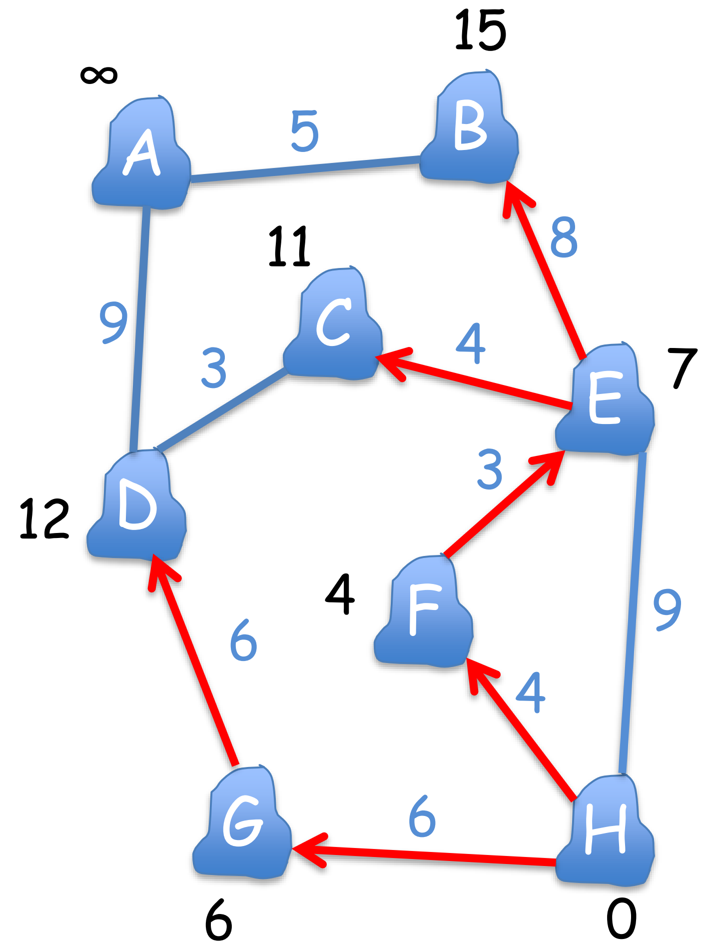

S = {H,F,G,E}

Dijkstra's Algorithm

Dijkstra(G,s)

for each u of V
    u.key = ∞
    u.par = nil
s.key = 0
initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
    u = ExtractMin(Q)
    S = S ∪ {u}
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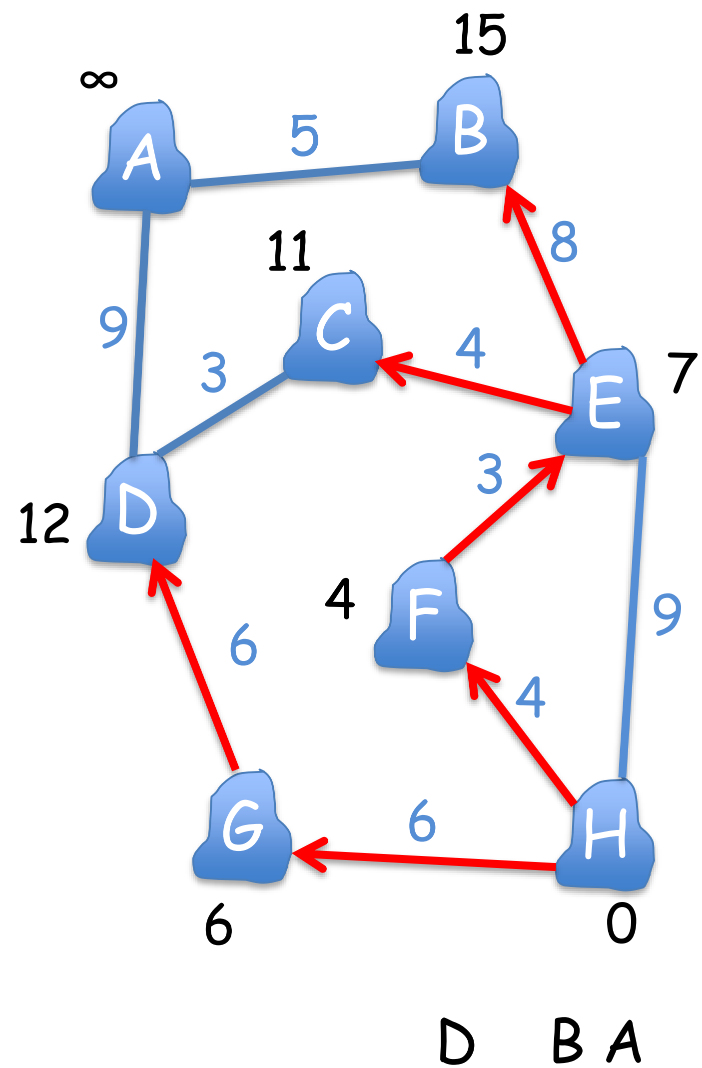
        if v.dis > u.dis + w(u,v)
            v.dis = u.dis + w(u,v)
            v.par = u
        update Q

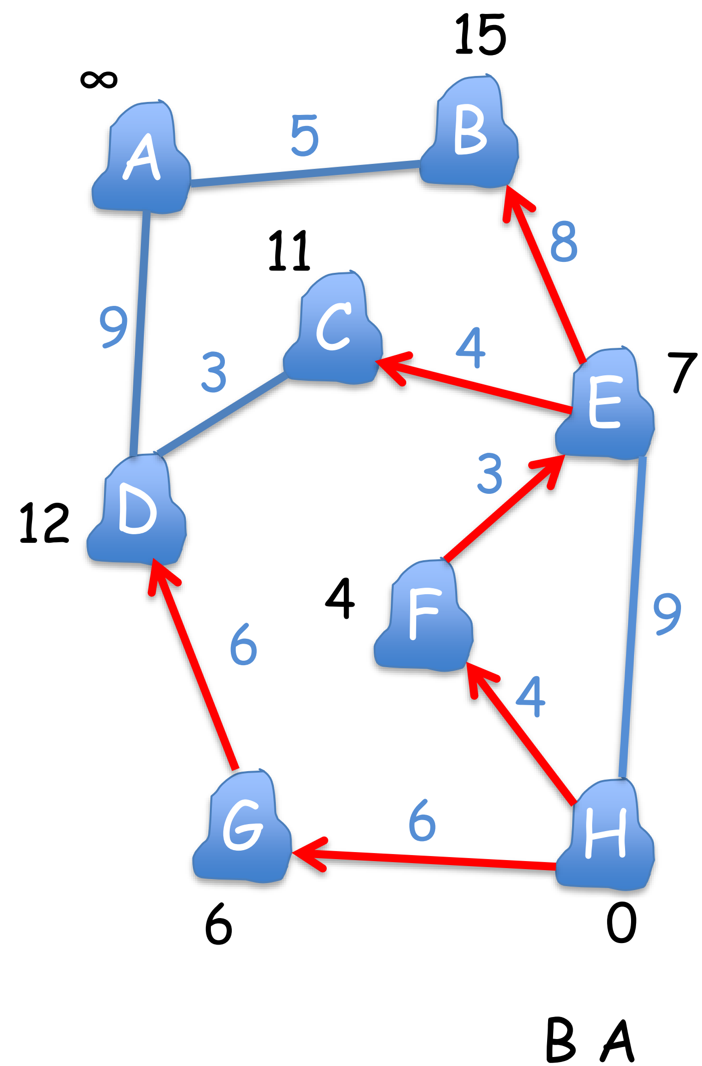


S = {H,F,G,E,C}

Dijkstra's Algorithm

Dijkstra(G,s)

```
for each u of V
      u.key = ∞
      u.par = nil
s.key = 0
initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
      u = ExtractMin(Q)
      S = S ∪ {u}
      for each v of Adj(u)
            if  v.dis > u.dis + w(u,v)
                 v.dis = u.dis + w(u,v)
                 v.par = u
           update Q
```



S = {H,F,G,E,C}

Dijkstra's Algorithm

Dijkstra(G,s)

```
for each u of V
      u.key = ∞
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initialize an empty set S
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while Q ≠ { }
      u = ExtractMin(Q)
      S = S ∪ {u}
      for each v of Adj(u)
            if  v.dis > u.dis + w(u,v)
                 v.dis = u.dis + w(u,v)
                 v.par = u
           update Q
```
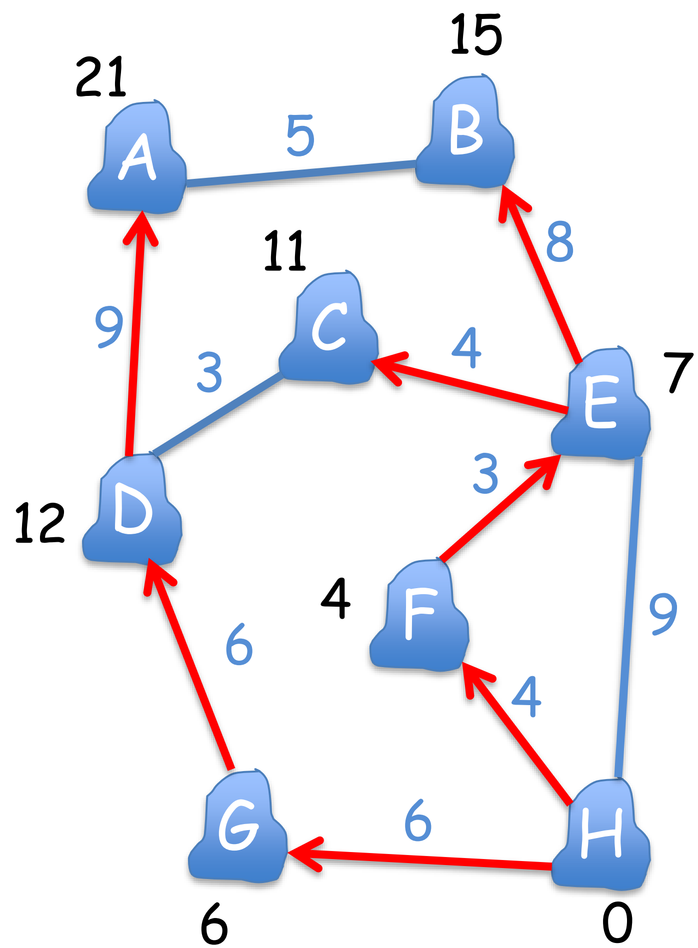



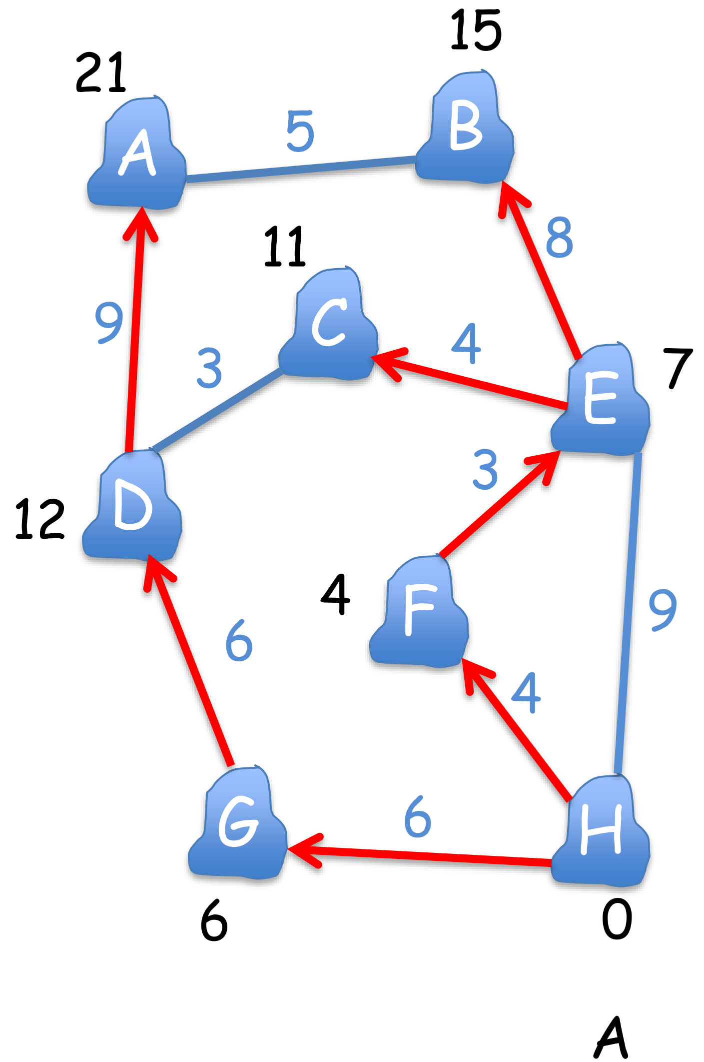
B A

S = {H,F,G,E,C,D}

Dijkstra's Algorithm

Dijkstra(G,s)

for each u of V
    u.key = ∞
    u.par = nil
s.key = 0
initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
    u = ExtractMin(Q)
    S = S ∪ {u}
    for each v of Adj(u)
        if v.dis > u.dis + w(u,v)
            v.dis = u.dis + w(u,v)
            v.par = u
        update Q

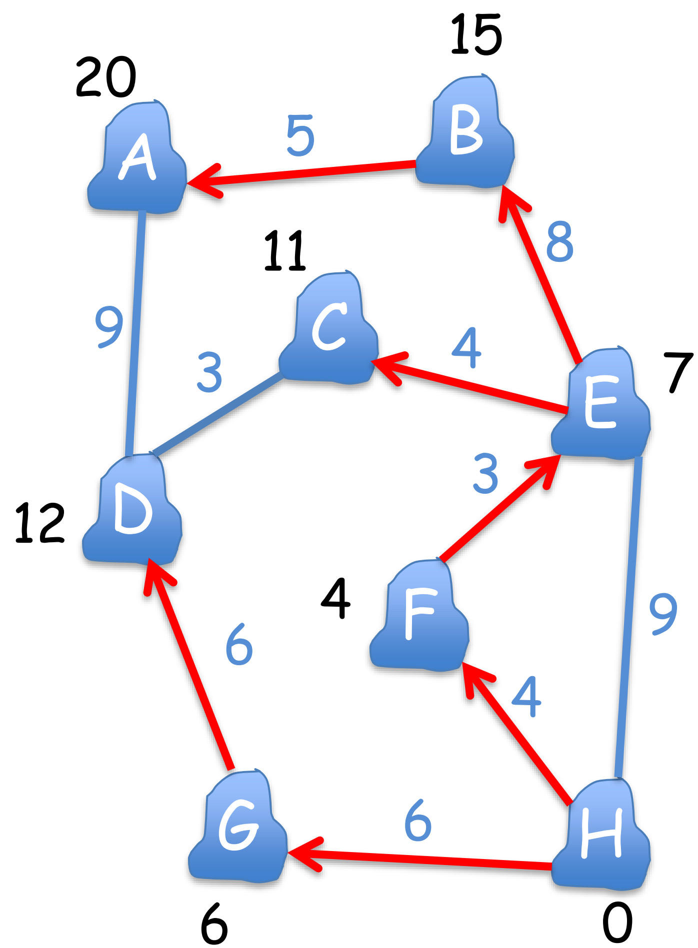


S = {H,F,G,E,C,D,B}

Dijkstra's Algorithm

Dijkstra(G,s)

```
for each u of V
      u.key = ∞
      u.par = nil
s.key = 0
initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
      u = ExtractMin(Q)
      S = S ∪ {u}
      for each v of Adj(u)
            if  v.dis > u.dis + w(u,v)
                  v.dis = u.dis + w(u,v)
                  v.par = u
            update Q
```



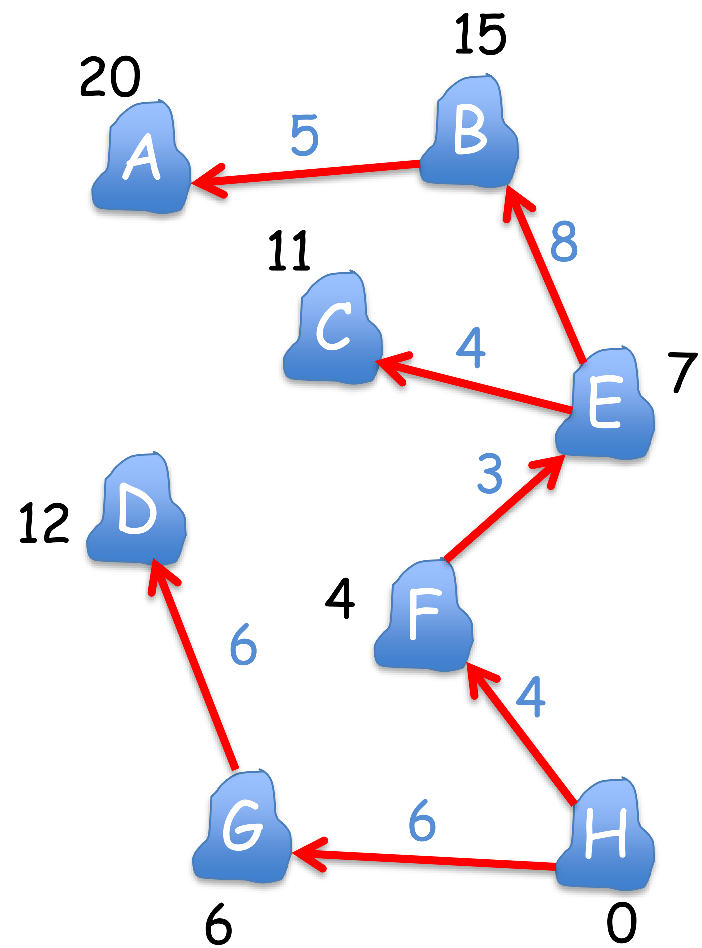
A

S = {H,F,G,E,C,D,B}

Dijkstra's Algorithm

Dijkstra(G,s)

```
for each u of V
      u.key = ∞
      u.par = nil
s.key = 0
initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
      u = ExtractMin(Q)
      S = S ∪ {u}
      for each v of Adj(u)
            if  v.dis > u.dis + w(u,v)
                  v.dis = u.dis + w(u,v)
                  v.par = u
            update Q
```

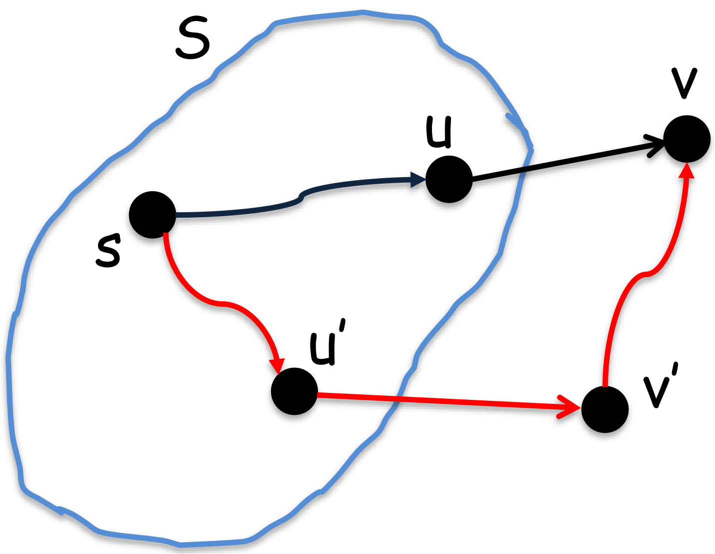


S = {H,F,G,E,C,D,B,A}

Greedy Choice Property

Theorem : For each vertex v in V, v.dis = $\delta(s,v)$ at the time when v is added to S. (Dijkstra's Algorithm computes all shortest path distances correctly)

Proof : Let v be the first vertex that v.dis $\neq \delta(s,v)$ at the time it's added to S. Let's check the true shortest path from s to v.



- v.dis ≤ u.dis + w(u,v)
  = δ(s,u)+w(u,v), since u.dis = δ(s,u)
- v'.dis ≤ u'.dis + w(u',v')
  = δ(s,u')+w(u',v'), since u'.dis = δ(s,u')
- Since v' is in the shortest path from s to v, v'.dis < δ(s,v). From upper bound property, δ(s,v) ≤ v.dis. So v'.dis < v.dis
- Since the vertices extracted from priority queue Q in the order of (…, v, … , v', …), v.dis < v'.dis. Thus, it's a contradiction!