Logic

Murat Osmanoglu

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(study of the difference between valid arguments and invalid arguments)

(finding out what it is that makes an argument valid)

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1) All men are mortal
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2) John will come to the party, or Mary will come to the party
   John will not come to the party

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premises

conclusion

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most of the mathematical statements are constructed by combining one or more propositions using logical operators (connectives)

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p	~p

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p	$\sim p$
T	
F	

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Conjunction ($p \wedge q$) : "p and q" .

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- the converse of $p \rightarrow q$: $q \rightarrow p$

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Conditional Statement ($p \to q$) : "if p, then q" (p implies q).

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- the contrapositive of $p \to q$: $\sim q \to \sim p$

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T	F			
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T	T	T	T	T
T	F	T	F	F
F	T	F	T	F
F	F	T	T	T

Logical Operators

Truth Tables

$$(p \lor \sim q) \rightarrow (p \land q)$$

Logical Operators

Truth Tables

$(p \vee {\sim}q) \rightarrow (p \wedge q)$

p	q	${\sim}q$	$p \wedge q$	$p \vee {\sim}q$	$(p \vee {\sim}q) \rightarrow (p \wedge q)$

Logical Operators

Truth Tables

$(p \vee \sim q) \rightarrow (p \wedge q)$

p	q	$\sim q$	$p \wedge q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow (p \wedge q)$
1	1				
1	0				
0	1				
0	0				

Logical Operators

Truth Tables

$$(p \vee \sim q) \rightarrow (p \wedge q)$$

p	q	~q	$p \wedge q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow (p \wedge q)$
1	1	0			
1	0	1			
0	1	0			
0	0	1			

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$$(p \vee \sim q) \rightarrow (p \wedge q)$$

p	q	$\sim q$	$p \wedge q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow (p \wedge q)$
1	1	0	1		
1	0	1	0		
0	1	0	0		
0	0	1	0		

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$$(p \vee \sim q) \rightarrow (p \wedge q)$$

p	q	$\sim q$	$p \wedge q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow (p \wedge q)$
1	1	0	1	1	
1	0	1	0	1	
0	1	0	0	0	
0	0	1	0	1	

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$(p \vee \sim q) \rightarrow (p \wedge q)$

p	q	~q	$p \wedge q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow (p \wedge q)$
1	1	0	1	1	1
1	0	1	0	1	0
0	1	0	0	0	1
0	0	1	0	1	0

Logical Operators

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$q \leftrightarrow (\sim p \vee \sim q)$

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p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$q \leftrightarrow (\sim p \vee \sim q)$
1	1				
1	0				
0	1				
0	0				

Logical Operators

Truth Tables

$q \leftrightarrow (\sim p \vee \sim q)$

p	q	~p	~q	~p∨~q	q ↔ (~p∨~q)
1	1	0	0		
1	0	0	1		
0	1	1	0		
0	0	1	1		

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$q \leftrightarrow (\sim p \vee \sim q)$

p	q	~p	~q	~pV~q	$q \leftrightarrow (\sim p \vee \sim q)$
1	1	0	0	0	
1	0	0	1	1	
0	1	1	0	1	
0	0	1	1	1	

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$q \leftrightarrow (\sim p \vee \sim q)$

p	q	~p	~q	~p∨~q	q ↔ (~p∨~q)
1	1	0	0	0	0
1	0	0	1	1	0
0	1	1	0	1	1
0	0	1	1	1	0

Logical Operators

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$p \rightarrow (p \vee q)$

$p \wedge (\sim p \wedge q)$

Logical Operators

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$p \rightarrow (p \vee q)$ $p \wedge (\sim p \wedge q)$

p	q	p∨q	$p \rightarrow (p \vee q)$	~p	~p∧q	$p \wedge (\sim p \wedge q)$

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$p \rightarrow (p \vee q)$

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p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
1	1					
1	0					
0	1					
0	0					

Logical Operators

Truth Tables

$p \to (p \lor q)$ $\qquad$ $p \land (\sim p \land q)$

p	q	$p \lor q$	$p \to (p \lor q)$	$\sim p$	$\sim p \land q$	$p \land (\sim p \land q)$
1	1	1		0	0	
1	0	1		0	0	
0	1	1		1	1	
0	0	0		1	0	

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$p \rightarrow (p \vee q)$

$p \wedge (\sim p \wedge q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
1	1	1	1	0	0	
1	0	1	1	0	0	
0	1	1	1	1	1	
0	0	0	1	1	0	

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$p \rightarrow (p \vee q)$ $\qquad$ $p \wedge (\sim p \wedge q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
1	1	1	1	0	0	0
1	0	1	1	0	0	0
0	1	1	1	1	1	0
0	0	0	1	1	0	0

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$p \rightarrow (p \vee q)$ $\qquad\qquad$ $p \wedge (\sim p \wedge q)$

p	q	p∨q	$p \rightarrow (p \vee q)$	~p	~p∧q	$p \wedge (\sim p \wedge q)$
1	1	1	1	0	0	0
1	0	1	1	0	0	0
0	1	1	1	1	1	0
0	0	0	1	1	0	0

- *A* compound proposition is called tautology if it's true for all the cases

Logical Operators

Truth Tables

$p \rightarrow (p \vee q)$ $\qquad\qquad$ $p \wedge (\sim p \wedge q)$

p	q	p∨q	p → (p∨q)	~p	~p∧q	p ∧ (~p ∧ q)
1	1	1	1	0	0	0
1	0	1	1	0	0	0
0	1	1	1	1	1	0
0	0	0	1	1	0	0

- A compound proposition is called tautology if it's true for all the cases
- A compound proposition is called contradiction if it's false for all the cases

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p	q	~p	~p∨q	$p \rightarrow q$

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p	q	~p	~p∨q	p → q
1	1			
1	0			
0	1			
0	0			

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1	1	0		
1	0	0		
0	1	1		
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1	1	0	1	
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0	1	1	1	
0	0	1	1	

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$$\sim p \lor q \equiv p \rightarrow q$$

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1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

$$\sim p \vee q \equiv p \rightarrow q$$

p	q	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$

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p	q	~p	~q	~(p ∧ q)	~p∨~q
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1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

$$\sim p \vee q \equiv p \rightarrow q$$

p	q	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
1	1	0	0	0	0
1	0	0	1	1	1
0	1	1	0	1	1
0	0	0	1	1	1

Logical Equivalences

- If the compound propositions p and q have same truth values for all the cases, they are called logically equivalent

p	q	$\sim p$	$\sim p \vee q$	$p \rightarrow q$
1	1	0	1	1
1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

$$\sim p \vee q \equiv p \rightarrow q$$

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

p	q	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
1	1	0	0	0	0
1	0	0	1	1	1
0	1	1	0	1	1
0	0	0	1	1	1

Logical Equivalences

- If the compound propositions p and q have same truth values for all the cases, they are called logically equivalent

p	q	~p	~p∨q	p → q
1	1	0	1	1
1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

$$\sim p \vee q \equiv p \rightarrow q$$

De Morgan's Low

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

p	q	~p	~q	~(p ∧ q)	~p∨~q
1	1	0	0	0	0
1	0	0	1	1	1
0	1	1	0	1	1
0	0	0	1	1	1

Logical Equivalences

- De Morgan's Low

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

- $\sim(\sim p) \equiv p$

- $p \wedge 1 \equiv p$
  $p \vee 0 \equiv p$

- $p \wedge 0 \equiv 0$
  $p \vee 1 \equiv 1$

- $p \wedge p \equiv p$
  $p \vee p \equiv p$

- $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
  $(p \vee q) \vee r \equiv p \vee (q \vee r)$

- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

- $p \wedge \sim p \equiv 0$
  $p \vee \sim p \equiv 1$

- $p \rightarrow q \equiv \sim p \vee q$
  $p \rightarrow q \equiv \sim q \rightarrow \sim p$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q) \equiv$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q) \equiv \sim p \wedge \sim(\sim p \wedge q)$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q) \equiv \sim p \wedge \sim(\sim p \wedge q)$

  $\equiv \sim p \wedge (p \vee \sim q)$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q) \equiv \sim p \wedge \sim(\sim p \wedge q)$

$$\equiv \sim p \wedge (p \vee \sim q)$$

$$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim(\sim p \wedge q)$

  $\equiv \sim p \wedge (p \vee \sim q)$

  $\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

  $\equiv 0 \vee (\sim p \wedge \sim q)$

Logical Equivalences

- $$\begin{aligned} \sim(p \vee (\sim p \wedge q)) &\equiv \sim p \wedge \sim(\sim p \wedge q) \\ &\equiv \sim p \wedge (p \vee \sim q) \\ &\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q) \\ &\equiv 0 \vee (\sim p \wedge \sim q) \\ &\equiv \sim p \wedge \sim q \end{aligned}$$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q) \equiv \sim p \wedge \sim(\sim p \wedge q)$
  $\equiv \sim p \wedge (p \vee \sim q)$
  $\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$
  $\equiv 0 \vee (\sim p \wedge \sim q)$
  $\equiv \sim p \wedge \sim q$

- $(p \rightarrow r) \wedge (p \rightarrow q) \equiv$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q)$ $\equiv \sim p \wedge \sim(\sim p \wedge q)$
  $\equiv \sim p \wedge (p \vee \sim q)$
  $\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$
  $\equiv 0 \vee (\sim p \wedge \sim q)$
  $\equiv \sim p \wedge \sim q$

- $(p \rightarrow r) \wedge (p \rightarrow q) \equiv (\sim p \vee r) \wedge$

Logical Equivalences

- $$\begin{aligned} \sim(p \vee (\sim p \wedge q)) &\equiv \sim p \wedge \sim(\sim p \wedge q) \\ &\equiv \sim p \wedge (p \vee \sim q) \\ &\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q) \\ &\equiv 0 \vee (\sim p \wedge \sim q) \\ &\equiv \sim p \wedge \sim q \end{aligned}$$

- $(p \rightarrow r) \wedge (p \rightarrow q) \equiv (\sim p \vee r) \wedge (\sim p \vee q)$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim(\sim p \wedge q)$
  $\equiv \sim p \wedge (p \vee \sim q)$
  $\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$
  $\equiv 0 \vee (\sim p \wedge \sim q)$
  $\equiv \sim p \wedge \sim q$

- $(p \rightarrow r) \wedge (p \rightarrow q) \equiv (\sim p \vee r) \wedge (\sim p \vee q)$
  $\equiv \sim p \vee (r \wedge q)$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q) \equiv \sim p \wedge \sim(\sim p \wedge q)$
  $\equiv \sim p \wedge (p \vee \sim q)$
  $\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$
  $\equiv 0 \vee (\sim p \wedge \sim q)$
  $\equiv \sim p \wedge \sim q$

- $(p \rightarrow r) \wedge (p \rightarrow q) \equiv (\sim p \vee r) \wedge (\sim p \vee q)$
  $\equiv \sim p \vee (r \wedge q)$
  $\equiv p \rightarrow (r \wedge q)$

Propositional Logic

- deals with propositions and logical relationship between propositions

(has only propositions and logical operators to represent, to interpret, to infer the knowledge)

(utilizes only propositions and operators to establish valid arguments)

- If Hasan does not die, then Zeynep will not get any money and Hasan's family will be happy

p : Hasan dies
q : Zenep will get money
r : Hasan's family will be happy

$$\sim p \rightarrow (\sim q \wedge r)$$

Propositional Logic

- deals with propositions and logical relationship between propositions

(has only propositions and logical operators to represent, to interpret, to infer the knowledge)

(utilizes only propositions and operators to establish valid arguments)

- All men are mortal
  Socrates is a man

  Socrates is mortal

How do we represent such argument ?

- propositional logic does not work for 'all' everyone'
- propositional logic does not have quantifiers

Predicate Logic

Predicate Logic

- p : '2 + 3 = 5'

  q : 'my computer is vulnerable to side channel attacks'

Predicate Logic

- p : '2 + 3 = 5'

  q : 'my computer is vulnerable to side channel attacks'

- 'x + 3 = 5'

  'computer x is vulnerable to side channel attacks'

Predicate Logic

- p : '2 + 3 = 5'

  q : 'my computer is vulnerable to side channel attacks'

- 'x + 3 = 5'

  'computer x is vulnerable to side channel attacks'

Definition Propositions (or statements) that contains variables

Predicate Logic

- p : '2 + 3 = 5'

  q : 'my computer is vulnerable to side channel attacks'

- P(x) : 'x + 3 = 5'

  Q(x) : 'computer x is vulnerable to side channel attacks'

Definition Propositions (or statements) that contains variables

Predicate Logic

- p : '2 + 3 = 5'

  q : 'my computer is vulnerable to side channel attacks'

- P(x) : 'x + 3 = 5'

  Q(x) : 'computer x is vulnerable to side channel attacks'

Definition Propositions (or statements) that contains variables

- When a value is assigned to the variable x, then P(x) becomes a proposition and has a truth value.

Predicate Logic

- $P(x)$: '$x > 3$'
- $Q(x,y)$: '$x + 3 = y$'
- $R(x,y,z)$: '$x + y = z$'

Predicate Logic

- $P(x)$: '$x > 3$'
  $P(4)$ is true
- $Q(x,y)$: '$x + 3 = y$'
- $R(x,y,z)$: '$x + y = z$'

Predicate Logic

- $P(x)$: '$x > 3$'
  $P(4)$ is true, but $P(2)$ is false
- $Q(x,y)$: '$x + 3 = y$'
- $R(x,y,z)$: '$x + y = z$'

Predicate Logic

- $P(x)$: '$x > 3$'
  $P(4)$ is true, but $P(2)$ is false

- $Q(x,y)$: '$x + 3 = y$'
  $Q(4,7)$ is true, but $Q(4,2)$ is false

- $R(x,y,z)$: '$x + y = z$'

Predicate Logic

- $P(x)$: '$x > 3$'
  $P(4)$ is true, but $P(2)$ is false

- $Q(x,y)$: '$x + 3 = y$'
  $Q(4,7)$ is true, but $Q(4,2)$ is false

- $R(x,y,z)$: '$x + y = z$'
  $R(2,1,3)$ is true, but $R(3,2,2)$ is false

Quantifiers

- Another way of creating a proposition from a propositional function

Quantifiers

- Another way of creating a proposition from a propositional function

Universal Quantifier

Quantifiers

- Another way of creating a proposition from a propositional function

Universal Quantifier

$Q : \forall x \, P(x)$

Quantifiers

- Another way of creating a proposition from a propositional function

Universal Quantifier

$Q : \forall x\, P(x)$ If $P(x)$ is true for all x in the domain, then Q is true

Quantifiers

- Another way of creating a proposition from a propositional function

Universal Quantifier

Q : $\forall$x P(x) If P(x) is true for all x in the domain, then Q is true
If there is an x_0 such that P(x_0) is not true, then Q is false

Quantifiers

- Another way of creating a proposition from a propositional function

Universal Quantifier

Q : ∀x P(x)

If P(x) is true for all x in the domain, then Q is true
If there is an x_0 such that P(x_0) is not true, then Q is false

Existential Quantifier

Quantifiers

- Another way of creating a proposition from a propositional function

Universal Quantifier

Q : $\forall x$ P(x) If P(x) is true for all x in the domain, then Q is true
If there is an x_0 such that P(x_0) is not true, then Q is false

Existential Quantifier

R : $\exists x$ P(x)

Quantifiers

- Another way of creating a proposition from a propositional function

Universal Quantifier

Q : ∀x P(x) — If P(x) is true **for all x** in the domain, then Q is true. If there is an x_0 such that P(x_0) is not true, then Q is false

Existential Quantifier

R : ∃x P(x) — If there **exists an x_0** such that P(x_0) is true, then R is true

Quantifiers

- Another way of creating a proposition from a propositional function

Universal Quantifier

Q : ∀x P(x)

If P(x) is true for all x in the domain, then Q is true

If there is an x_0 such that P(x_0) is not true, then Q is false

Existential Quantifier

R : ∃x P(x)

If there exists an x_0 such that P(x_0) is true, then R is true

If P(x) is false for all x in the domain, then R is false

Quantifiers

- P(x) : $x^2 \geq x$

  What is the truth value of $\forall$x P(x) if the domain is Z^+ ?

Quantifiers

- P(x) : $x^2 \geq x$

  What is the truth value of ∀x P(x) if the domain is Z^+ ?

  For all $x \in Z^+$ $x^2 \geq x$.

Quantifiers

- P(x) : $x^2 \geq x$

  What is the truth value of $\forall$x P(x) if the domain is Z^+ ?

  For all $x \in Z^+$ $x^2 \geq x$. So $\forall$x P(x) is true for Z^+ .

Quantifiers

- P(x) : $x^2 \geq x$

  What is the truth value of ∀x P(x) if the domain is Z^+ ?

  For all $x \in Z^+$ $x^2 \geq x$. So ∀x P(x) is true for Z^+ .

- Q(x) : x = x + 1

  What is the truth value of ∃x Q(x) if the domain is R?

Quantifiers

- P(x) : $x^2 \geq x$

  What is the truth value of ∀x P(x) if the domain is Z^+ ?

  For all $x \in Z^+$ $x^2 \geq x$. So ∀x P(x) is true for Z^+ .

- Q(x) : x = x + 1

  What is the truth value of ∃x Q(x) if the domain is R?

  There is no real number x such that x = x + 1.

Quantifiers

- P(x) : $x^2 \geq x$

  What is the truth value of ∀x P(x) if the domain is Z^+ ?

  For all $x \in Z^+$ $x^2 \geq x$. So ∀x P(x) is true for Z^+ .

- Q(x) : x = x + 1

  What is the truth value of ∃x Q(x) if the domain is R?

  There is no real number x such that x = x + 1. So ∃x Q(x) is false for R.

Quantifiers

- P(x) : $x^2 + 1 < 10$, D = { 1, 2, 3}

  What is the truth value of ∀x P(x) if the domain is D?

Quantifiers

- P(x) : $x^2 + 1 < 10$, D = { 1, 2, 3}

  What is the truth value of ∀x P(x) if the domain is D?

  If the domain consists of n elements,
  then ∀x P(x) ≡ P(x_1) ∧ P(x_2) ∧ . . . ∧ P(x_n)

Quantifiers

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  then ∀x P(x) ≡ P(x_1) ∧ P(x_2) ∧ . . . ∧ P(x_n)

  P(1) : $2 < 10$, true

Quantifiers

- P(x) : $x^2 + 1 < 10$, D = { 1, 2, 3}

  What is the truth value of ∀x P(x) if the domain is D?

  If the domain consists of n elements,
  then ∀x P(x) ≡ $P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$

  P(1) : $2 < 10$, true
  P(2) : $5 < 10$, true

Quantifiers

- P(x) : $x^2 + 1 < 10$, D = { 1, 2, 3}

  What is the truth value of ∀x P(x) if the domain is D?

  If the domain consists of n elements,
  then ∀x P(x) ≡ $P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$

  P(1) : $2 < 10$, true
  P(2) : $5 < 10$, true
  P(3) : $10 < 10$, false

Quantifiers

- P(x) : $x^2 + 1 < 10$, D = { 1, 2, 3}

  What is the truth value of ∀x P(x) if the domain is D?

  If the domain consists of n elements,
  then ∀x P(x) ≡ P(x_1) ∧ P(x_2) ∧ . . . ∧ P(x_n)

  P(1) : $2 < 10$, true
  P(2) : $5 < 10$, true
  P(3) : $10 < 10$, false

  Since 1 ∧ 1 ∧ 0 ≡ 0, then ∀x P(x) is false for D.

Quantifiers

- Q(x) : $x^2 < 3$, D = {1, 2, 3}

  What is the truth value of ∃x Q(x) if the domain is D?

Quantifiers

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  What is the truth value of ∃x Q(x) if the domain is D?

  If the domain consists of n elements,
  then ∃x P(x) ≡ P(x_1) ∨ P(x_2) ∨ . . . ∨ P(x_n)

Quantifiers

- Q(x) : $x^2 < 3$, D = {1, 2, 3}

  What is the truth value of ∃x Q(x) if the domain is D?

  If the domain consists of n elements,
  then ∃x P(x) ≡ P(x_1) ∨ P(x_2) ∨ . . . ∨ P(x_n)

  P(1) : $1 < 3$, true

Quantifiers

- Q(x) : $x^2 < 3$, D = {1, 2, 3}

  What is the truth value of ∃x Q(x) if the domain is D?

  If the domain consists of n elements,
  then ∃x P(x) ≡ P(x_1) ∨ P(x_2) ∨ . . . ∨ P(x_n)

  P(1) : $1 < 3$, true
  P(2) : $4 < 3$, false

Quantifiers

- Q(x) : $x^2 < 3$, D = {1, 2, 3}

  What is the truth value of ∃x Q(x) if the domain is D?

  If the domain consists of n elements,
  then ∃x P(x) ≡ P(x_1) ∨ P(x_2) ∨ . . . ∨ P(x_n)

  P(1) : $1 < 3$, true
  P(2) : $4 < 3$, false
  P(3) : $9 < 3$, false

Quantifiers

- Q(x) : $x^2 < 3$, D = {1, 2, 3}

  What is the truth value of ∃x Q(x) if the domain is D?

  If the domain consists of n elements,
  then ∃x P(x) ≡ P(x_1) ∨ P(x_2) ∨ . . . ∨ P(x_n)

  P(1) : $1 < 3$, true
  P(2) : $4 < 3$, false
  P(3) : $9 < 3$, false

  Since 1 ∨ 0 ∨ 0 ≡ 1, then ∃x P(x) is true for D.

Quantifiers

Negation

Quantifiers

- Every student in this class has entered the entrance exam

Negation

Quantifiers

- Every student in this class has entered the entrance exam

$\forall x\, P(x)$, ' x has taken the entrance exam'

Negation

Quantifiers

- Every student in this class has entered the entrance exam

$\forall x\, P(x)$, ' x has taken the entrance exam'

Negation

- It's not the case that every student in this class has entered the entrance exam.

Quantifiers

- Every student in this class has entered the entrance exam

$\forall x\, P(x)$, ' x has taken the entrance exam'

Negation

- It's not the case that every student in this class has entered the entrance exam.

  There is a student in this class who has not taken the entrance exam.

Quantifiers

- Every student in this class has entered the entrance exam

$\forall x\, P(x)$, ' x has taken the entrance exam'

Negation

- It's not the case that every student in this class has entered the entrance exam.

  There is a student in this class who has not taken the entrance exam.

$\sim(\forall x\, P(x)) \equiv$

Quantifiers

- Every student in this class has entered the entrance exam

$\forall x\ P(x)$, ' x has taken the entrance exam'

Negation

- It's not the case that every student in this class has entered the entrance exam.

  There is a student in this class who has not taken the entrance exam.

$\sim(\forall x\ P(x)) \equiv \sim(P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n))$

Quantifiers

- Every student in this class has entered the entrance exam

$\forall x\ P(x)$, ' x has taken the entrance exam'

Negation

- It's not the case that every student in this class has entered the entrance exam.

  There is a student in this class who has not taken the entrance exam.

$$\begin{aligned}\sim(\forall x\ P(x)) &\equiv \sim(P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)) \\ &\equiv \sim P(x_1) \vee \sim P(x_2) \vee \ldots \vee \sim P(x_n)\end{aligned}$$

Quantifiers

- Every student in this class has entered the entrance exam

$\forall x\ P(x)$, ' x has taken the entrance exam'

Negation

- It's not the case that every student in this class has entered the entrance exam.

  There is a student in this class who has not taken the entrance exam.

$$\begin{aligned} \sim(\forall x\ P(x)) &\equiv \sim(P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)) \\ &\equiv \sim P(x_1) \vee \sim P(x_2) \vee \ldots \vee \sim P(x_n) \\ &\equiv \exists x \sim P(x) \end{aligned}$$

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Quantifiers

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$\exists x\, P(x)$, ' x has taken the entrance exam'

Negation

Quantifiers

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  $\exists x\, P(x)$,   ' x has taken the entrance exam'

Negation

- It's not the case that There is a student in this class who has taken the entrance exam

Quantifiers

- There is a student in this class who has taken the entrance exam.

$\exists x\, P(x)$, ' x has taken the entrance exam'

Negation

- It's not the case that There is a student in this class who has taken the entrance exam

None of the students in this class has taken the entrance exam.

Quantifiers

- There is a student in this class who has taken the entrance exam.

  $\exists x\, P(x)$, ' x has taken the entrance exam'

Negation

- It's not the case that There is a student in this class who has taken the entrance exam

  None of the students in this class has taken the entrance exam.

$\sim(\exists x\, P(x)) \equiv$

Quantifiers

- There is a student in this class who has taken the entrance exam.

$\exists x\, P(x)$, ' x has taken the entrance exam'

Negation

- It's not the case that There is a student in this class who has taken the entrance exam

  None of the students in this class has taken the entrance exam.

$$\sim(\exists x\, P(x)) \equiv \sim(P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n))$$

Quantifiers

- There is a student in this class who has taken the entrance exam.

  $\exists x\, P(x)$, ' x has taken the entrance exam'

Negation

- It's not the case that There is a student in this class who has taken the entrance exam

  None of the students in this class has taken the entrance exam.

$$\sim(\exists x\, P(x)) \equiv \sim(P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n))$$
$$\equiv \sim P(x_1) \wedge \sim P(x_2) \wedge \ldots \wedge \sim P(x_n)$$

Quantifiers

- There is a student in this class who has taken the entrance exam.

  $\exists x\, P(x)$, ' x has taken the entrance exam'

Negation

- It's not the case that There is a student in this class who has taken the entrance exam

  None of the students in this class has taken the entrance exam.

$$
\begin{aligned}
\sim(\exists x\, P(x)) &\equiv \sim(P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)) \\
&\equiv \sim P(x_1) \wedge \sim P(x_2) \wedge \ldots \wedge \sim P(x_n) \\
&\equiv \forall x \sim P(x)
\end{aligned}
$$

Quantifiers

- $\sim(\forall x(x^2 > x)) \equiv$

- $\sim(\exists x(x^2 = 7)) \equiv$

Quantifiers

- $\sim(\forall x(x^2 > x)) \equiv \exists x \sim(x^2 > x)$

- $\sim(\exists x(x^2 = 7)) \equiv$

Quantifiers

- $\sim(\forall x(x^2 > x)) \equiv \exists \mathrm{x} \sim(x^2 > x)$
  $\equiv \exists \mathrm{x}\ x^2 \leq x$

- $\sim(\exists x(x^2 = 7)) \equiv$

Quantifiers

- $\sim(\forall x(x^2 > x)) \equiv \exists x \sim(x^2 > x)$
  $\equiv \exists x\ x^2 \leq x$

- $\sim(\exists x(x^2 = 7)) \equiv \forall x \sim(x^2 = 7)$

Quantifiers

- $\sim(\forall x(x^2 > x)) \equiv \exists \mathrm{x} \sim(x^2 > x)$
  $\equiv \exists \mathrm{x}\ x^2 \leq x$

- $\sim(\exists x(x^2 = 7)) \equiv \forall \mathrm{x} \sim(x^2 = 7)$
  $\equiv \forall \mathrm{x}\ x^2 \neq 7$

Quantifiers

- $\forall x \, \forall y \, ((x > 0) \land (y < 0) \rightarrow (xy < 0))$ D = R

Quantifiers

- $\forall x\ \forall y\ \overbrace{((x > 0) \land (y < 0) \rightarrow (xy < 0))}^{\text{If x is positive and y is negative, then xy is negative}}$ D = R

Quantifiers

- $\underbrace{\forall x\ \forall y\ \overbrace{((x > 0) \wedge (y < 0) \rightarrow (xy < 0))}^{\text{If x is positive and y is negative, then xy is negative}}}_{\text{For every real numbers x and y, if x is positive and y is negative, then xy is negative}}$ D = R

Quantifiers

- For every two integers, if these integers are both positive, then the sum of these integers is also positive

Quantifiers

- For every two integers, if these integers are both positive, then the sum of these integers is also positive

- For two integers x and y, if $x > 0$ and $y > 0$, then $\mathrm{x} + y > 0$

Quantifiers

- For every two integers, if these integers are both positive, then the sum of these integers is also positive

- For two integers x and y, if $\underbrace{x > 0 \text{ and } y > 0, \text{ then } x + y > 0}_{(x > 0) \wedge (y > 0) \rightarrow (x + y > 0)}$

Quantifiers

- For every two integers, if these integers are both positive, then the sum of these integers is also positive

- For two integers x and y, if $x > 0$ and $y > 0$, then $x + y > 0$

$$(x > 0) \wedge (y > 0) \to (x + y > 0)$$

$$\forall x\, \forall y\, ((x > 0) \wedge (y > 0) \to (x + y > 0))$$

Quantifiers

- There exist integers x and y such that $x + y = 6$

Quantifiers

- There exist integers x and y such that $x + y = 6$

$$\exists x\ \exists y\ (x + y = 6)$$

Quantifiers

- There exist integers x and y such that $x + y = 6$

$$\exists x\ \exists y\ (x + y = 6)$$

or

$$\exists y\ \exists x\ (x + y = 6)$$

Quantifiers

- There exist integers x and y such that $x + y = 6$

$$\exists x\ \exists y\ (x + y = 6)$$

or

$$\exists y\ \exists x\ (x + y = 6)$$

- $\forall x\ \exists y\ (x + y = 6)$

Quantifiers

- There exist integers x and y such that $x + y = 6$

$$\exists x \, \exists y \, (x + y = 6)$$

or

$$\exists y \, \exists x \, (x + y = 6)$$

- $\forall x \, \exists y \, (x + y = 6)$

For every integer x, there exists an integer y such that $x + y = 6$

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- $\exists \mathrm{y}\, \forall x\, (x + y = 6)$

  There exists an integer y so that for all integers x, $x + y = 6$ (It's false)