Final

You will have **110 minutes to prepare** your answers and **extra 10 minutes to upload** them. You write your answers on paper, scan (or photo capture through a mobile application such as CamScanner) and submit them as a single **.pdf file before 13.30 on January 13(today)**. Your answers have to be handwritten. Also, I accept the solutions of the students having some technical problems sent via email to mosmanoglu@ankara.edu.tr before 13.30. **The answer sheets sent after 13.30 will NOT be graded.**

1. Assume there is a game played by moving a small stone on a $2\times n$ chessboard such that there are numbers written in the cells of the chessboard indicating the amount of points you earn when you place the stone the corresponding cell. You initially place the stone one of the cells in the first column, and at each step, you move it to one of the cells in the right hand column. You end the game when you move the stone one of the cells in the rightmost column. Note that if you move the stone to the other row, you loose 3 points. Your task is to design a dynamic programming algorithm that takes a $2\times n$ integer matrix as input, and computes the maximum amount of points that can be earned.

In the following instance, if the shaded path is followed, (2+7 – 3+3+9 – 3) points will be earned.

5	7	3	7
2	8	2	9

a) (15p) Define a subproblem, construct the recurrence relation, and identify base case(s)

b) (15p) Use your student number as the input, and build a table for the solutions of the subproblems on your input

Assume your id is 14290519. The corresponding input matrix will be

1	4	2	9
0	5	1	9

The table for the input will be as follows :

OPT(i, j)

	5		
		6	

Note that for this example, I just filled two entries. But you must fill all the entries in the corresponding table.

2. For the question, first construct a string using your student id as follows ('14290519' will be used for the following string):

- take the square of your id

  $14290519^2 = 204218933289361$

- multiply the square with the number 123456789 (fixed for all the students)

  25212213756909716921829

- write the corresponding letters from the table in place of the numbers you obtained above in order to get your string (Note that all the students must use the following table to transform the numbers to the letters)

0	1	2	3	4	5	6	7	8	9
A	B	C	D	E	F	G	H	K	L

your string = “CFCBCCBDHFGLALHBGLCBKCL”

a) **(10p)** Find the frequencies of the characters in the string, and construct the Huffman tree corresponding to these frequencies

b) **(10p)** Based on the Huffman tree you have obtained, determine the Huffman codes of the characters, and calculate the cost of the encoding for your string

3. For the questions, first construct a weighted undirected graph using your student id as follows (‘14290519’ will be used for the following graph):

- take the square of your id

  $14290519^2 = 204218933289361$

- cut out the first 11 numbers

  20421893328

- consider each consequtive two numbers as two digit numbers (if it begins with zero, take it as one digit number)

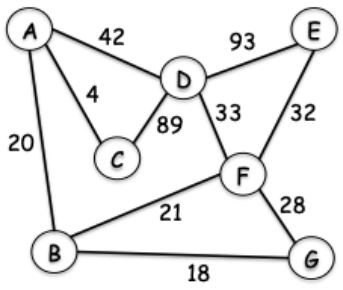
  20 – 04 – 42 – 21 – 18 – 89 – 93 – 33 – 32 – 28

- fill the empty entries in the upper triangular part of the following adjacency weighted matrix with the obtained ten numbers, with the following order ‘from the first row to the last row, and at each row, from left to right.
  Note that you are constructing an undirected graph, and there will be symmetry in the adjacency matrix. Thus, the upper triangular part will be enough to construct the graph.

	A	B	C	D	E	F	G
A	0				∞	∞	∞
B		0	∞	∞	∞		
C		∞	0		∞	∞	∞
D		∞		0			∞
E	∞	∞	∞		0		∞
F	∞		∞			0	
G	∞		∞	∞	∞		0

	A	B	C	D	E	F	G
A	0	20	4	42	∞	∞	∞
B		0	∞	∞	∞	21	18
C		∞	0	89	∞	∞	∞
D		∞		0	93	33	∞
E	∞	∞	∞		0	32	∞
F	∞		∞			0	28
G	∞		∞	∞	∞		0

- construct the corresponding graph

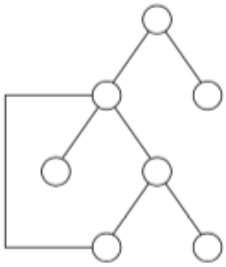


a) (15p) Apply Prim's algorithm to your graph using the vertex 'A' as the source vertex. For the answer, just draw the MST tree that you obtain after the algorithm terminates (to get full credit, you should reflect the final keys of the vertices to the graph).

b) (15p) Apply Dijkstra's algorithm to your graph using the vertex 'A' as the source vertex. For the answer, just draw the shortest path tree that you obtain after the algorithm terminates (to get full credit, you should reflect the final keys of the vertices to the graph).

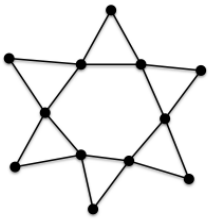
4.(20p) For the following two questions, you just pick one of them to answer.

Option 1. Consider an undirected graph which is formed by taking a binary tree and adding an edge from exactly one of the leaves to another node in the tree. We call such a graph a loop graph. An example of a loop graph could be the following one:



Assume you are given a weighted loop graph (each edge has a positive integer weight). Design an efficient algorithm (better than Kruskal and Prim) for finding the minimum spanning tree of the given graph. To get full credit, you need to argue the running time.

Option 2. A star graph is an undirected graph of the following form, i.e. let n be the number of vertices, a star graph consists of a center cycle with n/2 vertices where each pair of connected vertices are also connected to a third vertex:



Assume you are given a weighted star graph (each edge has a positive integer weight). Design an efficient algorithm (better than Kruskal and Prim) for finding the minimum spanning tree of the given graph. To get full credit, you need to argue the running time.

Note that first employ your student id to calculate your specific number, then create your own question set. Don't use the numbers provided here.