

HW #2

Homework exercises should be done individually (You should write the solution by yourself). Solutions must be prepared in the Python programming language and submitted electronically as .py file before **11.59 pm on Sunday, November 14**. No credit will be given to solutions obtained verbatim from the Internet or other sources. **To get full credit for each question, you need to provide a brief explanation of your codes and the efficiency analysis with comments.**

5. Consider the matrices A_0, A_1, A_2, \dots that are recursively formed as follows:

- $A_0 = [1]_{1 \times 1}$, and $A_1 = \begin{bmatrix} A_0 & A_0 & A_0 & -A_0 \end{bmatrix}_{2 \times 2} = \begin{bmatrix} 1 & 1 & 1 & -1 \end{bmatrix}_{2 \times 2}$
- $A_k = \begin{bmatrix} A_{k-1} & A_{k-1} & A_{k-1} & -A_{k-1} \end{bmatrix}_{2^k \times 2^k}$ where A_{k-1} is the $2^{k-1} \times 2^{k-1}$ matrix.

Devise a divide-and-conquer algorithm that takes a column integer vector v whose length is $n = 2^k$, and computes the matrix-vector product $A_k * v$. The running of your algorithm should be at most $O(n \log n)$.