EXPECTED VALUE/ VARIANCES:

Definition:

Let X be a random variable with pf $f(x)$, $g(x)$ be a function of X. The expected value of $g(x)$,

$$\boldsymbol{E}(\boldsymbol{g}(\boldsymbol{X})) = \begin{cases} \sum_{D_X} g(x) f_X(x), X \ \ discrete \\ \int_{-\infty}^{\infty} x\, f(x) dx\,, X \ \ continuous. \end{cases}$$

Definition:

In this case, X is a a random variable with pf $f(x)$,

a. For **discrete** random variable , the value $E(X)$,

$$E(X) = \sum_{D_X} x f_X(x)$$

is called **expected value of X random variable**.

b. For **continuous** random variable , the value $E(X)$,

$$E(X) = \int_{-\infty}^{\infty} x\, f(x) dx$$

is called **expected value of** X random variable. The value $Var(X)$,

$$Var(X) = E(X - E(X))^2 = E(X^2) - \big(E(X)\big)^2$$

is called the variance of X random variable.

the value $E(X^k)$ is called the $k.$ moment of X random variable. So where;

$E(X^2)$ is called 2. moment o X random variable.

$$E(X^2) = \sum_{D_X} x^2 f_X(x) \qquad \textbf{(for discrete random variable)}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx \qquad \textbf{(for continuous random variable)}$$

<u>Note:</u>

The expected value of X random variable is denoted by μ and the variance of X random variable is denoted by σ^2. The square root of the variance is called the standard deviation of X random variable and denoted by σ.

<u>Laws of the Expected Value:</u>

Let X be a random variable and a be a constant.

1. $E(a) = a$
2. $E(X + a) = E(X) + a$
3. $E(aX) = aE(X)$

<u>Laws of the Variance:</u>

1. $Var(a) = a$
2. $Var(X + a) = Var(X)$
3. $Var(aX) = a^2 Var(X)$

<u>Theorem:</u> Let $a, b \in IR$,

a. $E(aX + b) = aE(X) + b$

b. $Var(aX + b) = a^2 Var(X)$

Examples:

1. X is a random vaiable with pf $f(x) = \frac{1}{3}, D_X = \{-1,0,1\}$

$E(X)$ =?, $Var(X)$ =?, $E(X^3)$ =? $E(2X+3)$ =? $Var(2X+3)$=?

Solution:

$$E(X) = \sum_{x=-}^{1} x f_X(x) = \frac{1}{3}(-1+0+1) = 0$$

$$E(X^2) = \sum_{x=-}^{1} x^2 f_X(x) = \frac{1}{3}((-1)^2+0+1^2) = \frac{2}{3}$$

$Var(X) = \frac{2}{3}$ $\quad E(X^3) = \frac{1}{3}(-1^3+0+1^3)$=0

$E(2X+3)$=2 $E(X)+3 = 3$ $\quad E(2)$=2 $\quad Var(2) = 0$

$Var(2X+3) = 4Var(X) = \frac{8}{3}$

Student: X is a random vaiable with pf $f(x)$

$X = x$	2	3	5
$f(x)$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$E(X)$ =? $Var(X)$ =? $E(X^3)$ =? $E(X+X^2)$ =? $E(2X+4)$ =? $Var(2X+4)$ =?

Student: X is a random vaiable with pf $f(x)$

$f(x) = cx \quad D_X = \{1,2,3\}$

$c = ?\quad E(X) = ?\ Var(X) = ?\quad P(0 < X \leq 1) = ?\quad P(1 < X \leq 2) = ?\quad P(X > 2) = ?$

$P(X \leq 3) = ?$

1. X is a random vaiable with pdf ;

$$f(x) = \begin{cases} 3x^2, \dots 0 < \mathrm{x} < 1 \\ 0, \dots other\ wises \end{cases}$$

$E(X) = ?$ $Var(X) = ?$

Solution:

$$E(X) = \int_0^1 x\, f(x)dx = \int_0^1 x\, 3x^2 dx = \frac{3}{4}$$

$$E(X^2) = \int_0^1 x^2\, f(x)dx = \int_0^1 x^2\, 3x^2 dx = \frac{3}{5}$$

$$Var(X) = E(X^2) - \left(E(X)\right)^2 = \frac{3}{5} - \frac{9}{16} = 0.0625$$

Student: X is a random vaiable with pdf ;

$$f(x) = \begin{cases} 3x^2, \dots -1 < \mathrm{x} < 1 \\ 0, \dots other\ wises \end{cases}$$

a. $\boldsymbol{c} = ?$ $E(X) = ?$ $Var(X) = ?$ $P(0 < X \leq 1) = ?$ $P(X \leq 1.5) = ?$ $P(0 < X \leq 2) = ?$ $P(X \geq 2) = ?$ $P(X \leq 2) = ?$

Study Questions (At Home) :

1.What is the value of the c constant . Obtain the c constant; then calculate the $E(X)=?$ $Var(X)=?$. Define some probabilities and calculate them.

$$f(x)=cx^2, \quad D_X=\{-3,-2,-1,1,2,3\}$$

$$f(x)=c\binom{4}{x}, \quad D_X=\{0,1,2,3,4\}$$

2.What is the value of the c constant . Obtain the c constant; then calculate the $E(X)=?$ $Var(X)=?$. Define some probabilities and calculate them.

$$f(x)=\begin{cases} cx, \ldots 0<\mathrm{x}<2 \\ 0, \ldots other\ wises \end{cases}$$

$$f(x)=\begin{cases} c\sqrt{x}, \ldots 0<\mathrm{x}<4 \\ 0, \ldots other\ wises \end{cases}$$

Transformation of a Random Variable:

<u>Example:</u>

1.

X is a random vaiable with pf $f(x)$

$f(x) = \frac{1}{5}$ $D_X = \{-2, -1, 0, 1, 2, \}$. Y is a random variable defined as $Y = X^2$.

Obtain the pf of Y random variable.

<u>Solution:</u>

First we have to obtain the D_Y

$Y = X^2 \rightarrow D_Y = \{0,1,4\}$

$$f_Y(0) = P(Y = 0) = P(X^2 = 0) = P(X = 0) = \frac{1}{5}$$

$$f_Y(1) = P(Y = 1) = P(X^2 = 1) = P(X = 1) + P(X = -1) = \frac{2}{5}$$

$$f_Y(4) = P(Y = 4) = P(X^2 = 4) = P(X = 2) + P(X = -2) = \frac{2}{5}$$

$Y = y$	0	1	4
$P(Y = y)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

<u>Student:</u> X is a random vaiable with pf $f(x)$

$f(x) = \frac{x^2}{10}$ $D_X = \{-2, -1, , 1, 2, \}$. Y is a random variable defined as $Y = X + 2$.

Obtain the pf of Y random variable.