ENERGY and POWER SIGNALS

Total Energy: $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$ (Both periodic and nonperiodic signals)

Avg. Power $P = \frac{1}{T_0} \int |x(t)|^2 dt$ (For periodic signals)

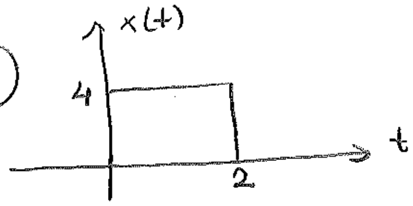
Avg Power $P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$ (For nonperiodic signals)

ENERGY SIGNALS

A signal is said to be an energy signal; if and only if its total energy is finite.

E = finite and Power = 0 in energy signal

Eg 1



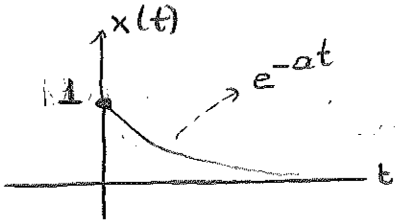
Check, if the signal is an energy signal or not.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{0} 0 \, dt + \int_{0}^{2} (4)^2 dt + \int_{2}^{\infty} 0 \, dt$$

$$\Rightarrow \int_{0}^{2} 16 \, dt = 16t \Big]_0^2 = 16.2 - 16.0 = \underline{32} \text{ J} \quad \text{(finite)}$$

$E = 32$ (finite value) $\Rightarrow$ x(t) is an energy signal.

Eg2) Calculate the total energy of the following signal : $x(t) = e^{-at} u(t) \quad a>0$



$$u(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{0} 0\, dt + \int_{0}^{\infty} |e^{-at}|^2 dt = \int_{0}^{\infty} e^{-2at} dt.$$

$$\Rightarrow -\frac{1}{2a} \left[e^{-2at}\right]_0^{\infty} \longrightarrow -\frac{1}{2a}\left[\underbrace{\cancel{e^{-2\infty a}}}_{0} - \underbrace{e^{-2.0a}}_{1}\right] = \frac{1}{2a}$$

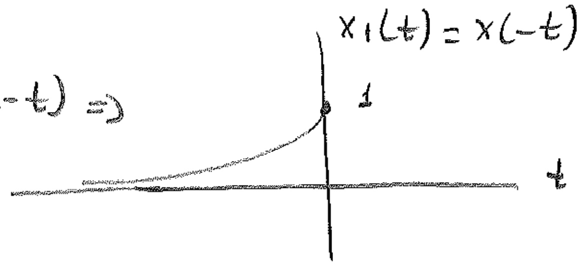
$E = \frac{1}{2a}$ (finite) $x(t)$ is an ENERGY signal

Average POWER for ENERY signal is 0

Eg3) $x(t) = e^{-at} u(t) \quad a>0$

$x_1(t) = x(-t)$ Calculate the total energy of $x_1(t)$

$x_1(t) = x(-t) \Rightarrow$

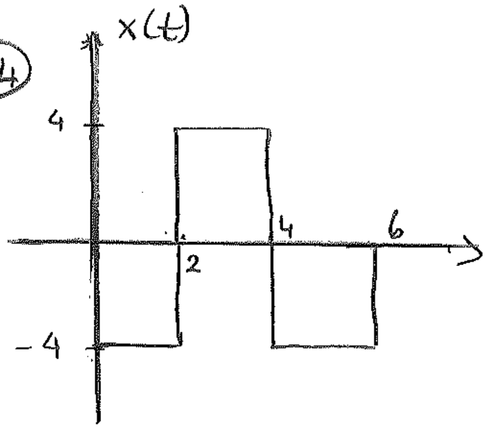


$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{0} |e^{+a(t)}|^2 dt = \int_{-\infty}^{0} e^{2at} dt.$$

$$E \Rightarrow \frac{1}{2a} \left. e^{2at} \right]_{-\infty}^{0} = \frac{1}{2a}\left(\underbrace{e^{2a.0}}_{1} - \underbrace{\cancel{e^{2a(-\infty)}}}_{0}\right) = \frac{1}{2a}.\ J$$

NOTE : Time reversal have no effect on the total Energy of the signal.

Eg 4



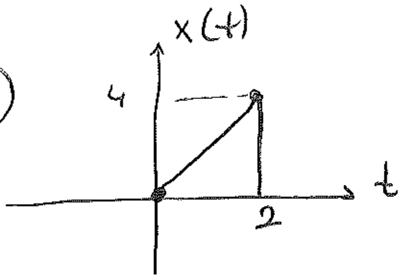
Calculate the total energy of x(t)

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \Rightarrow \int_{-\infty}^{0} 0\,dt + \int_0^2 (-4)^2 dt + \int_2^4 |4|^2 dt + \int_4^6 |-4|^2 dt + \int_6^{\infty} 0\,dt$$

$$E = \int_0^2 16 dt + \int_2^4 16\,dt + \int_4^6 16 dt \Rightarrow 16t\Big]_0^2 + 16t\Big]_2^4 + 16t\Big]_4^6$$

$$\Rightarrow 16(2-0) + 16(4-2) + 16(6-4) = 32.3 = 96 \text{ joule}$$

Eg5

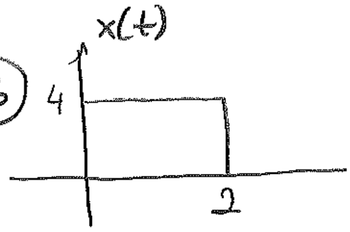


Total energy of x(t)?

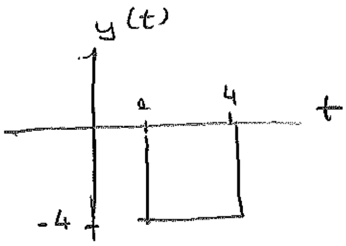
- The standard equation of the straight line : $y = mx + c$.
- Our example line is passing through origin. So $c = 0$
- m is slope of line : $\frac{y_2 - y_1}{t_2 - t_1} = \frac{4-0}{2-0} = 2$
- Equation of line is : $y = 2t$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \Rightarrow \int_0^2 |2t|^2 dt = \frac{4t^3}{3}\Big]_0^2$$

$$E = \frac{4}{3} \cdot (2^3 - 0) = \frac{4 \cdot 8}{3} = \frac{32}{3} \text{ J}$$

Eg6 

Calculate the energy of following signal: $y(t) = -x(t-2)$



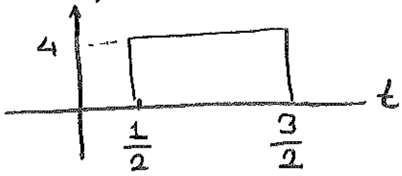
$$\int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_2^4 |-4|^2\,dt = 16t\Big]_2^4.$$

$E = 16.(4-2) = 32$

NOTE: No effect of time scaling, time reversal and amplitude shifting on total energy of signal

Eg7 For the signal in Eg6; calculate the total energy of signal $y(t) = 2j\,x(2t-1)$

The signal $x(2t-1)$ is :



$$\int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_{1/2}^{3/2} |2j\,4|^2\,dt = \int_{1/2}^{3/2} |-8|^2\,dt = \int_{1/2}^{3/2} 64\,dt.$$

$j^2 = -1$

$$E = 64t\Big]_{1/2}^{3/2} = 64.\left(\frac{3}{2} - \frac{1}{2}\right) = 64 \text{ joule.}$$

POWER SIGNALS

$P \Rightarrow$ finite $\quad E \Rightarrow$ infinite.

NOTE : For an energy signal E = finite $P = 0$

For a power signal P = finite $E \Rightarrow \infty$

* Periodic signals are power signals, but vice-versa is not true. If there is a power signal, we can't say that signal is periodic.

Eg 8) $x_1(t) = A\sin\omega_0 t$ $\quad$ Calculate average power.

$\hookrightarrow$ periodic.

For periodic signal $P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x_1(t)|^2$

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A^2 \underbrace{\sin^2\omega_0 t}_{\frac{1-\cos 2\omega_0 t}{2}}\, dt \Rightarrow \frac{A^2}{2T_0}\int_{-T_0/2}^{T_0/2} 1-\cos 2\omega_0 t\, dt$$

$$\Rightarrow \frac{A^2}{2T_0}\left[\, t\Big]_{-T_0/2}^{T_0/2} + \underbrace{\cancel{\frac{\sin 2\omega_0 t}{2}\Big]_{-T_0/2}^{T_0/2}}}_{0}\,\right] = \frac{A^2}{2}$$

NOTE: Time scaling: $x_2(t) = x_1(2t)$

$= A_0 \sin 2\omega_0 t$

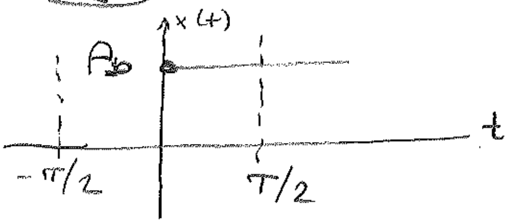
Phase shift: $x_3(t) = A \sin(\omega_0 t + \phi)$

Time Reversal: $x_4(t) = x_1(-t) = A_0 \sin(-\omega_0 t)$

Time Shifting: $x_5(t) = A_0 \sin(\omega_0 (t+2))$

* All time operations listed above have no effect on average power.

Eg 9 Calculate the average power of step signal below



$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

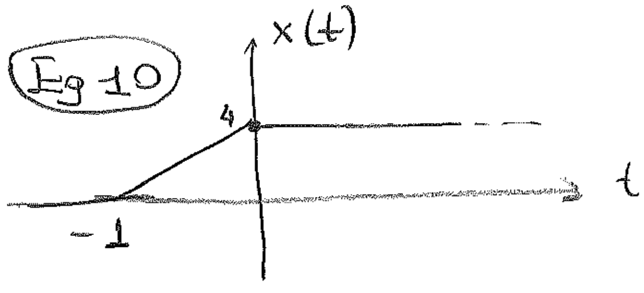
for nonperiodic signals

$$P = \lim_{T\to\infty} \frac{1}{T} \left[\int_{-T/2}^{0} 0 \, dt + \int_{0}^{T/2} |A_0|^2 \, dt \right]$$

$$P = \lim_{T\to\infty} \frac{1}{T} \left[A_0^2 t \Big|_0^{T/2} \right] = \lim_{T\to\infty} \frac{1}{T} \left[A_0^2 \left[\frac{T}{2} - 0 \right] \right]$$

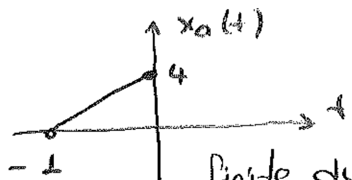
$$P = A_0^2 / 2$$

Eg 10

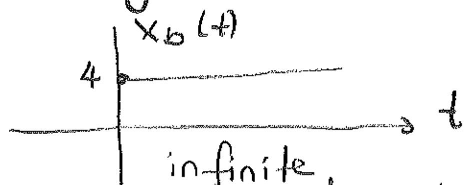


Calculate the average power?

Solution: We can divide this signal into two subsignals



finite duration signal



infinite duration signal.

$x_a(t) = at + b$

$t = 0 \rightarrow x_a(t) = 4 = b$

$$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{0 - (-1)} = 4$$

$x_a(t) = 4t + 4$

$$P = \lim_{T \to \infty} \frac{1}{T} \left[\int_{-T/2}^{0} (4t+4)^2 dt + \cancel{\int_{0}^{T/2} 0 \, dt} \right]$$

$$\lim_{T \to \infty} \frac{1}{T} \left[\int_{-1}^{0} 16t^2 + 16t + 16 \; dt \right]$$

$$\lim_{T \to \infty} \frac{1}{T} \left[\left. 16\frac{t^3}{3} + 16\frac{t^2}{2} + 16t \right]_{-1}^{0} \right]$$

$$\lim_{T \to \infty} \frac{1}{T} \left[\left(\frac{16.0^3}{3} + \frac{16.0^2}{2} + 16.0 \right) - \left(\frac{16.(-1)^3}{3} + \frac{16(-1)^2}{2} + 16.(-1) \right) \right]$$

$$\lim_{T \to \infty} \frac{1}{T} \left[-\frac{16}{3} + \frac{16}{2} - 16 \right] = 0$$

infinity constant

P of $x_a(t) = 0$

HINT: $x_a(t)$ is a finite duration signal. All finite duration signals are energy signals. So in fact E of $x_a(t)$ = finite and average power of an Energy signal $P(x_a(t)) = 0$

$x_b(t) = 4u(t)$ where $u(t)$ is a step signal

$$P = \lim_{T \to \infty} \frac{1}{T}\left(\cancel{\int_{-T/2}^{0} 0^2 \, dt} + \int_{0}^{T/2} 4^2 \, dt\right)$$

$$P = \lim_{T \to \infty} \frac{1}{T}\left(\int_{0}^{T/2} 4^2 \, dt\right)$$

$$P = \lim_{T \to \infty} \frac{1}{T} \; 16t \Big]_0^{T/2} = \frac{16}{T} \cdot (T/2) = \frac{16}{2} = 8$$

So; the total average power of signal $x(t)$ is

$$P = P_a + P_b = 0 + 8 = 8$$

Eg 11) $x(t) = 5\cos(10t + \phi) + 10\sin(5t + \phi)$

Average power? (Hint: There is no effect of phase shift on average power)

$$x_b(t) = 10\sin(5t + \phi)$$

For sin and cos signal: $A_0 \sin w_0 t$; $P = A_0^2/2$.

So power of $P(x_b(t)) = 10^2/2 = 50$

$x_a = 5\cos(10t + \phi)$ $P(x_a(t)) = 5^2/2 = 25/2$

$$P = 50 + \frac{25}{2} = 62{,}5 \text{ watt}$$

NOTE: Neither Energy Nor Power Signals (NENP)

If magnitude of signal is infinite at any instant of time than the signal will be neither energy nor power signal.

Eg 12 $x(t) = e^{-at}u(t)$

* In example 3; we calculated energy of the signal x(t). Now we will calculate the average power of signal x(t) [Hint: This was an energy signal and value of E = 1/2a joule]

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x^2(t)|\, dt$$

NOTE: In other examples we use $\frac{1}{T}$ and $\int_{-T/2}^{T/2}$. Both representations are correct.

$$P(x(t)) = \lim_{T\to\infty} \frac{1}{2T} \left[\int_{-T}^{0} 0\, dt + \int_{0}^{T} e^{-at}\, dt \right]$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\left[-\frac{e^{-at}}{a} \right]_0^T \right]$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[-\frac{e^{-aT}}{a} - \frac{e^{-a(0)}}{a} \right] = 0$$

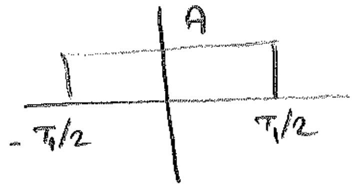
$$-\frac{1}{e^{aT}} \quad - \quad 1$$

$T \to \infty$

NOTE: Average Power of an Energy Signal is always ZERO

Eg 13 $x(t) = A\,\text{rect}(t/T_1)$ Energy ? Power ?

a) Energy $= \int_{-\infty}^{\infty} |x(t)|^2 \, dt$



$$\Rightarrow \int_{-T/2}^{T/2} |A|^2 \, dt \Rightarrow A^2 t \Big]_{-T_1/2}^{+T_1/2} = A^2 \frac{T_1}{2} - A^2 \frac{(-T_1)}{2} = \underline{A^2 T_1} \text{ J}$$

b) Power $= \lim_{T \to \infty} \frac{1}{2T} \left(\int_{-T}^{T} x^2(t) \, dt \right)$

We have calculated this part in (a). This shows the value of total energy $= A^2 T_1$

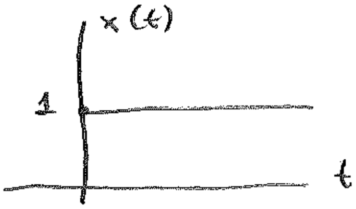
$$= \lim_{T \to \infty} \frac{1}{2T} \cdot A^2 T_1 \Big\} = 0$$

Note: $A^2 T_1$ is a finite quantity. A finite quantity divided by T is always ZERO.

Power $= \underline{\underline{0}}$

Rectangular signal $x(t) = A\,\text{rect}\left(\frac{t}{T_1}\right)$ is an energy signal. $E = A^2 T_1$ $P = 0$

Eg 14 Average power of $x(t) = u(t)$



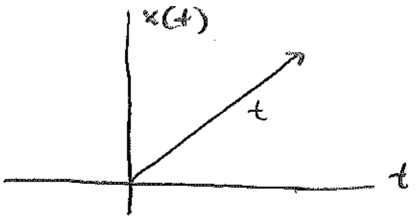
$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T \to 0}^{T} (1)^2 \, dt .$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \left(t \Big]_{+0}^{T} \right) \Rightarrow \lim_{T \to \infty} \frac{1}{2T} \cdot (T) = \underline{\underline{1/2}}$$

$P = \frac{1}{2}$ watt. (finite) $\downarrow$ power signal

NOTE: For power signals ($E \to \infty$)

Eg 15 $x(t) = r(t)$ (r = ramp signal) (Power signal or Energy signal ?)



$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |t|^2 \, dt$$

$$\Rightarrow \lim_{T \to \infty} \frac{1}{2T} \left(\int_0^T t^2 \, dt \right) \Rightarrow \lim_{T \to \infty} \frac{1}{2T} \left(\frac{t^3}{3} \Big]_0^T \right) =$$

$$\Rightarrow \lim_{T \to \infty} \frac{1}{2T} \cdot \frac{T^{3-2}}{3} \Rightarrow \infty$$

$P = \infty$ (This is not a power signal)

$$\Rightarrow E = \int_{-\infty}^{\infty} |t|^2 \, dt \Rightarrow \int_0^{\infty} t^2 \, dt \Rightarrow \frac{t^3}{3} \Big]_0^{\infty} = \infty$$

$E = \infty$ (Not an energy signal)

$x(t) = r(t)$ is neither energy nor power signal.

Eg 16 $x(t) = Ae^{j\left(2t+\frac{\pi}{4}\right)}$ Check Energy signal or Power signal.

Solution: $x(t)$ is a complex exponential signal. For a complex exponential signal; everything depends on amplitudes.

$e^{j\omega t} = \cos\omega t + j\sin\omega t$ So this is also a periodic signal.

For a periodic signal $P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 \, dt$.

First; we have to find $|x(t)| = \left|Ae^{j\left(2+\frac{\pi}{4}\right)}\right|$

For a standard complex exponential signal:

$$z(t) = e^{j\omega t} = \underbrace{\cos\omega t}_{a} + \underbrace{j\sin\omega t}_{jb}$$

$$z = a + jb \qquad |z| = \sqrt{a^2+b^2}$$

$$|z| = \underbrace{\sqrt{\sin^2\omega t + \cos^2\omega t}}_{1} = \underline{1}$$

So for our example signal

$$x(t) = A\underline{e^{j(2t+\pi/4)}} \qquad |x(t)| = A \cdot \underbrace{1}_{|z|} = A$$

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (A)^2 \, dt = \frac{A^2}{T_0} t \Big]_{-T_0/2}^{T_0/2} = \underline{\underline{A^2}}$$

(P = finite. Power signal)

Eg 17 $x(t) = A\cos(\omega t + \theta)$

$$E = \int_{-\infty}^{\infty} |x^2(t)|\, dt = \int_{-\infty}^{\infty} A^2 \underbrace{\cos^2(\omega t + \theta)}\, dt$$

$$\cos^2(\omega t + \theta) = \frac{1 + \cos 2(\omega t + \theta)}{2}$$

$$E \Rightarrow A^2 \int_{-\infty}^{\infty} 1 + \cos 2(\omega t + \phi)\, dt$$

$$E = A^2 \left(\int_{-\infty}^{\infty} \frac{1}{2}\, dt + \cancel{\int_{-\infty}^{\infty} \frac{\cos 2(\omega t + \phi)}{2}\, dt} \right)$$

$$= 0$$

$$E = A^2 \left. t \right]_{-\infty}^{\infty} \qquad E = \infty \text{ (Not an energy signal}$$

$$P = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A^2 \cos^2(\omega t + \theta)\, dt$$

$$P = \frac{A^2}{T_0} \left[\int_{-T_0/2}^{T_0/2} \frac{1}{2} + \cancel{\int_{-T_0/2}^{T_0/2} \frac{\cos 2(\omega t + \theta)}{2}\, dt} \right]$$

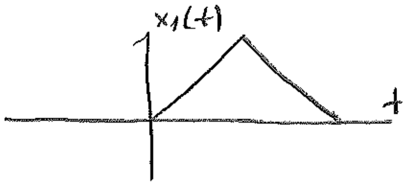
$$0$$

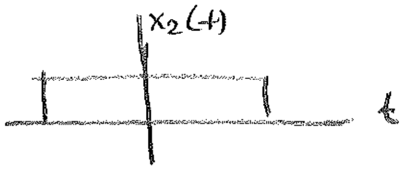
$$P = \frac{A^2}{T_0} \left(\left. \frac{1}{2} \right]_{-T_0/2}^{T_0/2} \right) = \frac{A^2}{T_0} \left(\frac{T_0/2}{2} - \left(-\frac{T_0/2}{2} \right) \right)$$

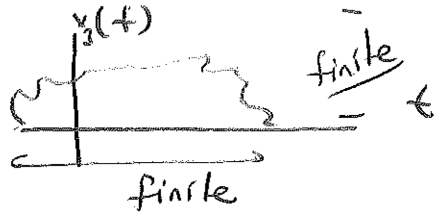
$$= \frac{A^2}{\cancel{T_0}} \cdot \frac{\cancel{T_0}}{2} = \frac{A^2}{2}$$

NOTES

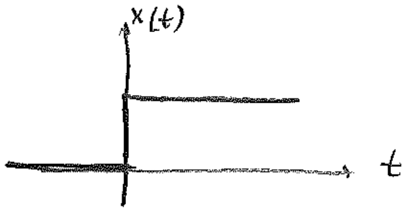
① All signals with finite duration and finite amplitude are ENERGY Signals







② Signals which have finite amplitude and infinite duration are POWER signals.



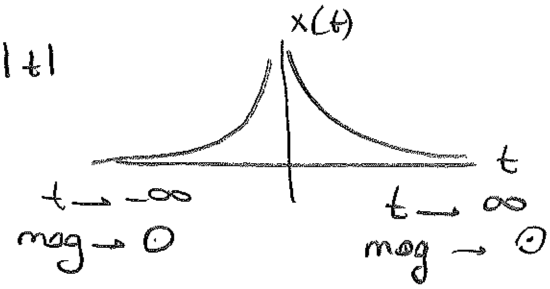
③ $A\cos(\omega t+\phi)$ or $Ae^{j\omega t}$ (Periodic signals)

All periodic signals are POWER signals

But

All power signals are Not periodic.

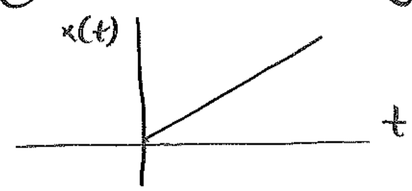
④ $e^{-a|t|}$ $(a>0)$



This exponential is always ENERGY signal

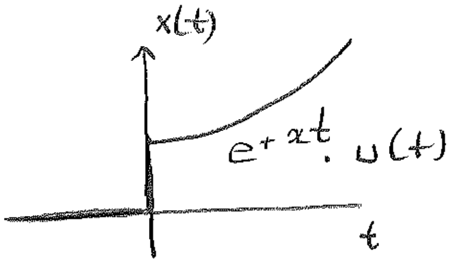
No matter it is one sided or not

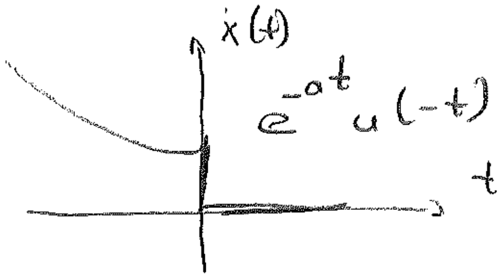
⑤ Unit ramp signal

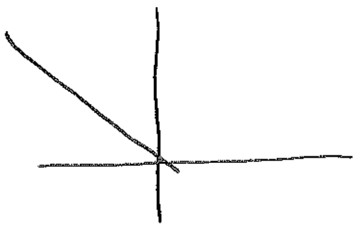


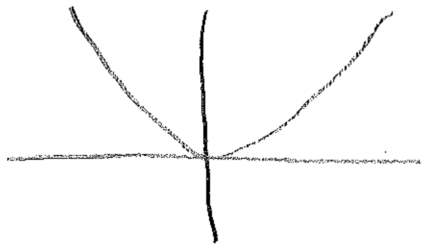
Both infinite duration and amplitude diverges to infinity by $t \to \infty$

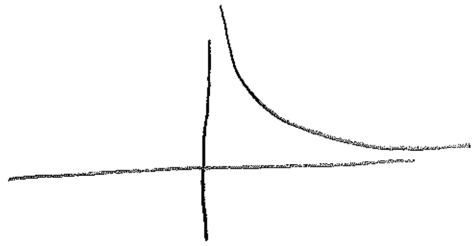
Neither Energy Nor Power











These signals have no finite amplitude and have no finite duration.

NEITHER ENERGY NOR POWER SIGNALS.