SIGNALS and SYSTEMS

2022-2023

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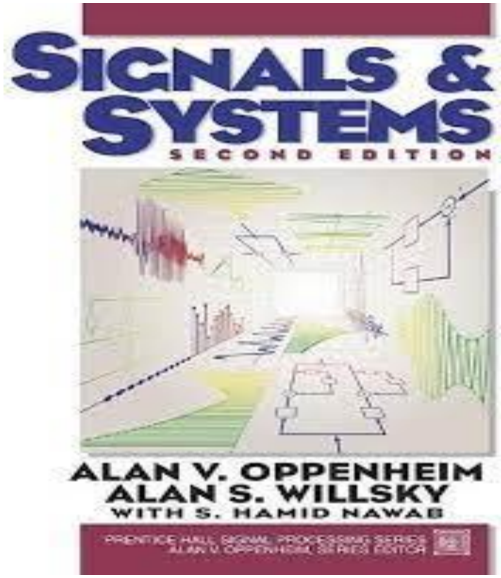
SIGNALS and SYSTEMS

Saturday: 19:30

Avarage of Quizes	15%
Mid- Term Exam	15%
Final Exam	70%

TEXTBOOK:
Signals and Systems by Alan V. Oppenheim, Alan S. Willsky with S. Hamid Nawab, Prentice Hall, Second Edition, 1997.



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TOPICS

1. Introduction (Basic signals and operation on them)
2. Linear Time Invariant (LTI) (Convolution, Correlation)
3. Fourier Series (How we represent the signal in terms of a combination of other signals)
4. Fourier Transforms
5. Laplace Transform
6. Z-Transform

1.1. SIGNAL

- **DEFINITION:** A signal is the variation of a physical, or non-physical, quantity with respect to one or more independent variable(s). Signals typically carry information that is somehow relevant for some purpose.

  - Ex: Electrical signals : voltage as a function of time
  - Ex: Acoustic signals : acoustic pressure as a function of time
  - Ex: Picture : brightness as a function of two spatial variables

- We will mostly refer to the independent variable as time (t)

- **Signals are mathematical functions.** We will represent signals by using the representation of mathematical functions like f(t), g(t),etc.

- **NOISE:** noise is a signal which carries unwanted information.

1.2 SYSTEM

❑ **DEFINITION:** System is defined as any process in which input signals are transformed to output signals.

➢ Ex: Electrical circuit with an input signal (vi(t)) and an output signal (vo(t))

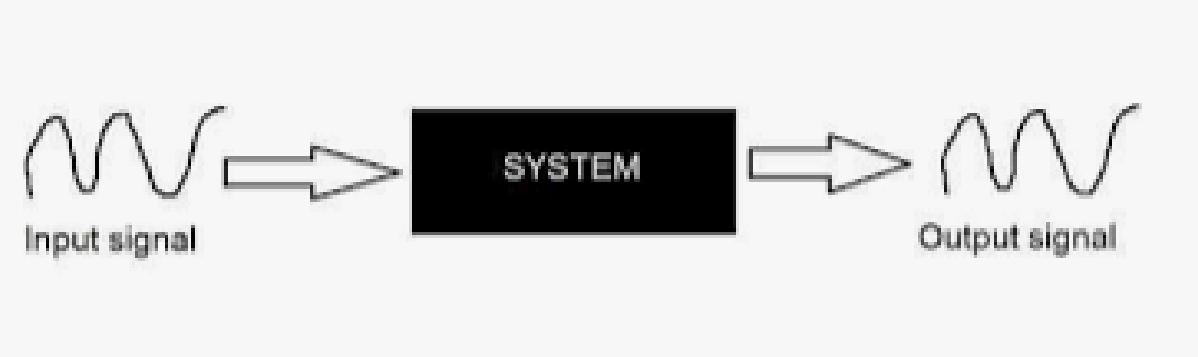


Fig 1.1

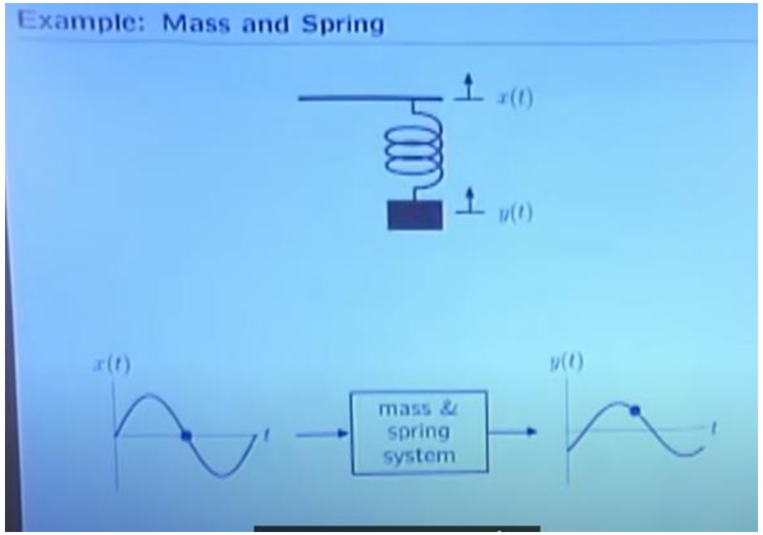
❑ **INPUT/OUTPUT:**

❑ A system has input and output. The input is sometimes called as excitation and the output is generally called as response.

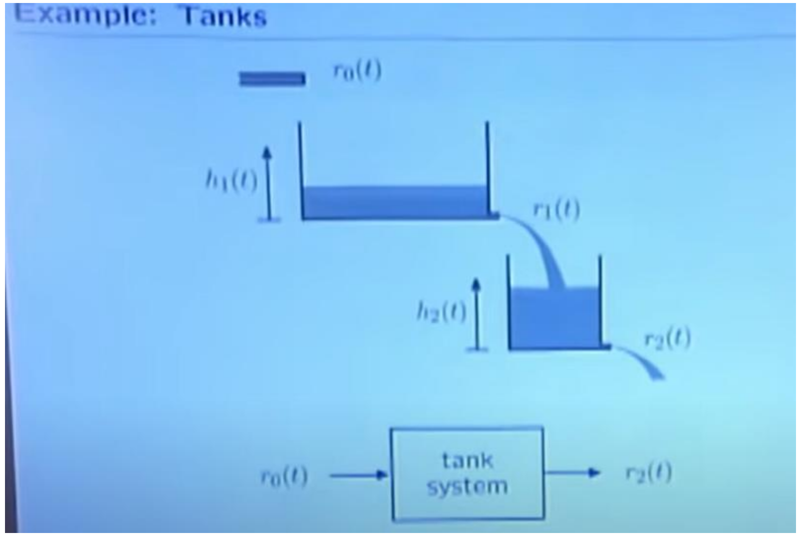
❑ In many textbooks you can read the statements as "A system excited by unit step signal". This means the unit step signal is the input of the given system.

❑ Similarly, "Response of the system g" means the output of that system.

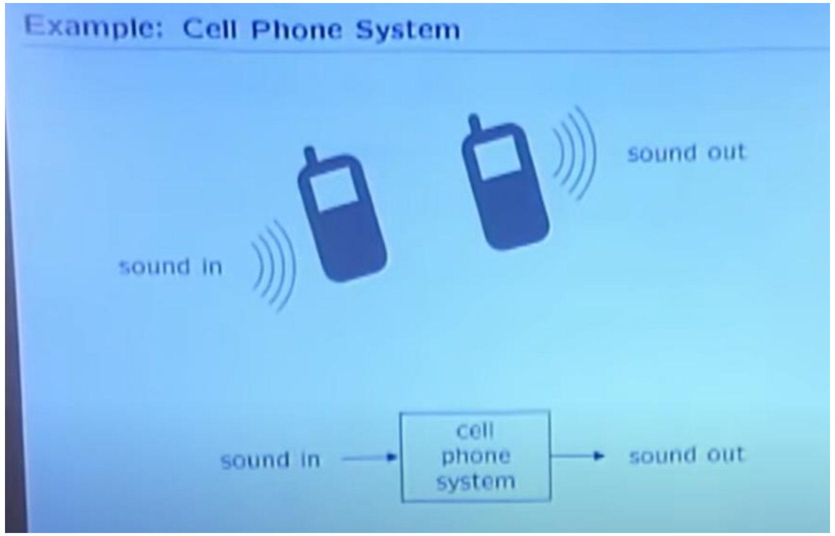
System is an abstraction of real world.



(a)



(b)



(c)

- Once, you model the real world and then only focus on input and output

1.3 Important Points About Signals

- A signal f1(t) can be represented in terms of another signal f2(t):

$f1(t) = C_{12} * f2(t)$ where C_{12} = coefficient of approximation

$$\mathbf{C_{12}} = \frac{\int_{t1}^{t2} f1(t) f2(t) dt}{\int_{t1}^{t2} f2(t)^2 \, dt}$$

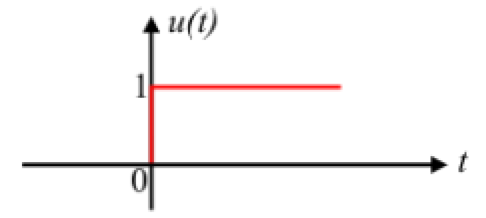
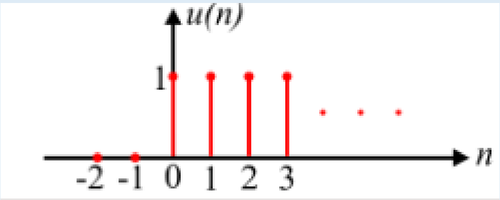
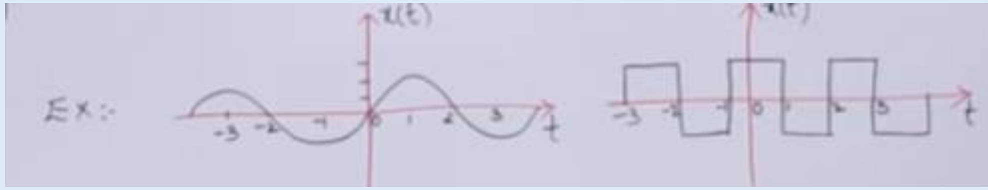
- If two signals are orthogonal the coefficient of approximation is zero

$\mathbf{C_{12} = 0}$

So the condition for orthogonality is: $\int_{t1}^{t2} f1(t) f2(t) dt = 0$

- Sin and cos functions are orthogonal to each other

1.4. Continous and Discrete Time SIGNALS

Continous time signals (CTS)	Discrete-time signals (CTS)
1-A signal which id defined for all values of t is called as continous time signal	1- A signal which is defined only at discrete intervals of time is called discrete time signals.
	
2-We represent the functions with x(t)	2-We represent the functions with x[n] (n is not directly time. It discrete time refers time intervals)
	

- For discrete time signals only time is discrete but the amplitude is continous.
- For digital signals both amplitude and time are discrete
- We convert a CTS to DTS. For this we multiply the CTS x(t) with a pulse train. This process is sometimes called as sampling proces.
- We can convert DTS to CTS by continous steps.

1.4. BASIC SIGNALS

•

1- Unit step signal	6- Parabolic signal
2- Impulse signal	7-Rectangular pulse
3-Signum function	8-Triangular signal
4-Exponential signal	9-Sinusoidal signal
5-Unit ramp signal	10 Sinc function and sampling function

1.4.1 UNIT STEP SIGNAL

- The step signal or step function is that type of standard signal which exists only for positive time and it is zero for negative time. It is denoted **with u(t) or u[n]**
- In other words, a signal **u(t) or u[n]** is said to be step signal if and only if it exists for t > 0 and zero for t < 0. The step signal is an important signal used for analysis of many systems.
- If a step signal has unity magnitude, then it is known as unit step signal or unit step function. It is denoted by u(t).
- In practice, the unit step signal is used as a test signal because the response of a system for the unit step signal gives the information about how quickly the system responds to a sudden change in the input signal.
- This generally accepted as best test signal to observe any systems response.
- Properties of Unit step function
  - $[u(t)]^n = u(t)$
  - $[u(t-to)]^n = u(t-to)$
  - Time scaling does not applicable: $u(at) = u(t)$
  - $u(at-to) = u(t-to/a)$

i) The unit-step function, defined by

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

CTS

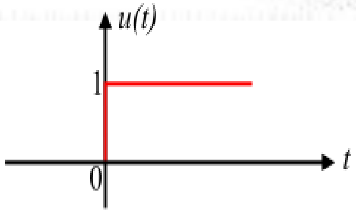


Figure-1

ii) The unit-step function, defined by

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

DTS

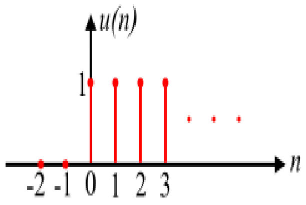


Figure-2

1.4.2 IMPULSE SIGNAL

- It is denoted **with δ(t) or δ[n]**
- Hence, by the definition, the unit impulse signal has zero amplitude every where except at t = 0. At the origin (t = 0) The continuous-time impulse signal is also called Dirac Delta Signal.
- The time integral of unit impulse signal is a unit step signal. In other words, the time derivative of a unit step signal is a unit impulse signal,

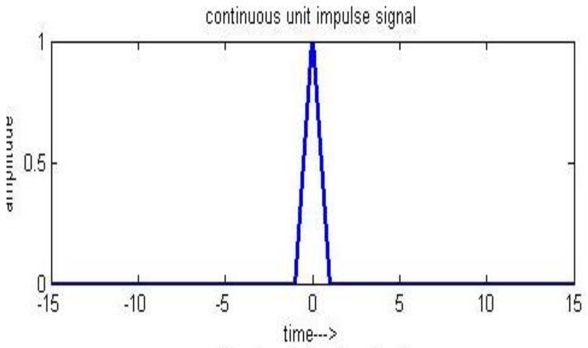
$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

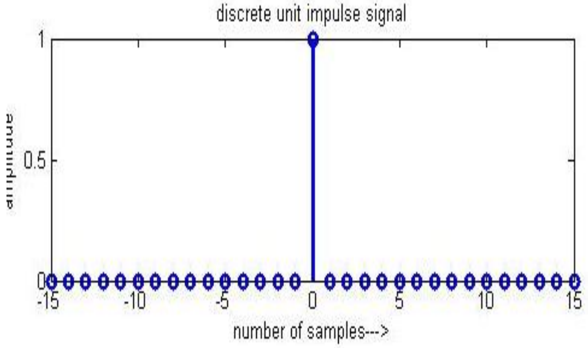
$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0. \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = u(t)$$

And

$$\delta(t) = \frac{d}{dt} u(t)$$





* Properties :-

(i) $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

(ii) $\delta(n-k) = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$

(iii) $\delta(n) = u(n) - u(n-1)$

(iv) $f(t)\,\delta(t) = f(0)$

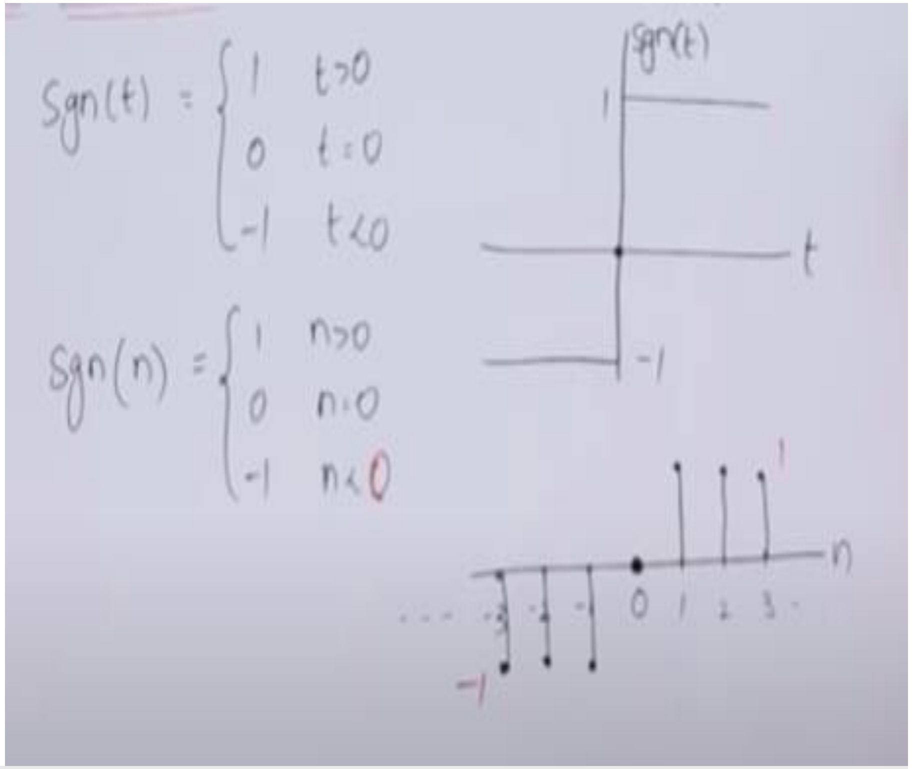
(v) $\delta(t-t_0)\, f(t) = f(t_0)$

(vi) $\delta(kt) = \frac{1}{|k|}\delta(t)$

(vii) $\delta(t) - \delta(t)$

1.4.3 SIGNUM FUNCTION

- It is denoted **with sgn(t) or sgn[n]**
- **Signum stands for sign in Latin. We call this function as sign because it gives the sign of the number.**
- **If the sign of a number is negative (Eg; -2, -2.5, …) then result of signum function is -1**
- **If the sign of a number is positive (Eg; 2, 2.5, …) then result of signum function is 1**
- **The sign function is not continuous at t=0 and n=0**



- Relation between u(t) and sgn(t) are:
  - sgn(t)= 2u(t) – 1
  - It can also be written as :$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

x < 0

x	−1	−2	−3
y	−1	−1	−1

x > 0

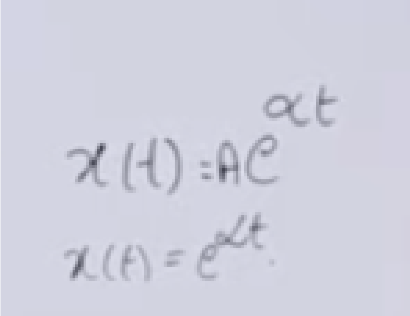
x	1	2	3
y	1	1	1

1.4.4 EXPONENTIAL SIGNALS

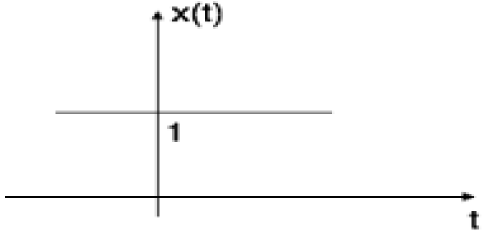
Exponential Signal

Exponential signal is in the form of x(t) = $e^{\alpha t}$.

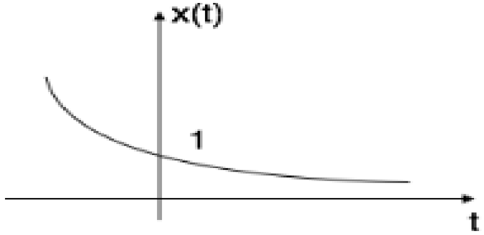
The shape of exponential can be defined by α .



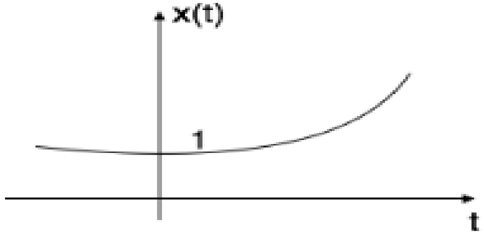
Case i: if α = 0 $\rightarrow$ x(t) = e^{0} = 1



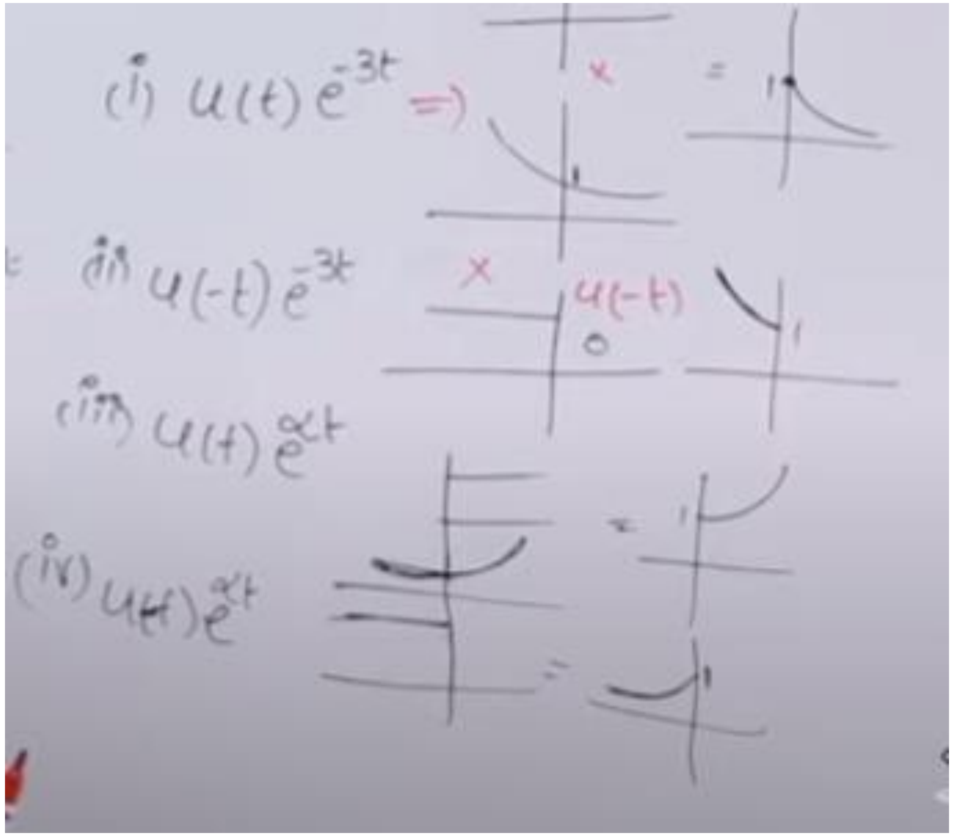
Case ii: if α < 0 i.e. -ve then x(t) = $e^{-\alpha t}$. The shape is called decaying exponential.



Case iii: if α > 0 i.e. +ve then x(t) = $e^{\alpha t}$. The shape is called raising exponential.



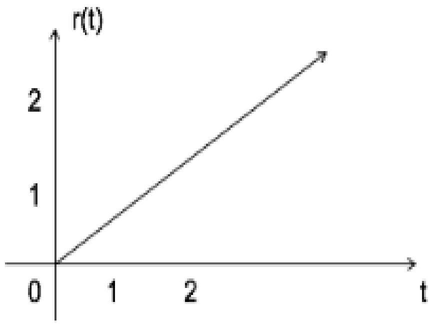
- The shape of exponential signal depends upon α



1.4.5 UNIT RAMP SIGNALS

Ramp Signal

Ramp signal is denoted by r(t), and it is defined as r(t) = $\begin{cases} t & t \geqslant 0 \\ 0 & t < 0 \end{cases}$

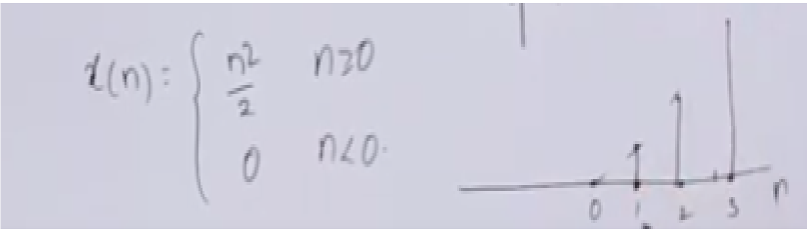


$$\int u(t) = \int 1 = t = r(t)$$

$$u(t) = \frac{dr(t)}{dt}$$

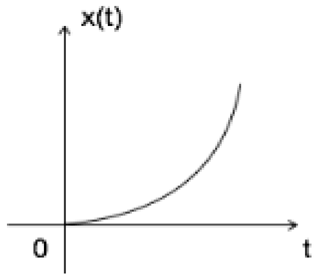
Area under unit ramp is unity.

1.4.6 UNIT PARABOLIC SIGNALS



Parabolic Signal

Parabolic signal can be defined as x(t) = $\begin{cases} t^2/2 & t \geqslant 0 \\ 0 & t < 0 \end{cases}$



$$\iint u(t)dt = \int r(t)dt = \int tdt = \frac{t^2}{2} = parabolicsignal$$

$$\Rightarrow u(t) = \frac{d^2x(t)}{dt^2}$$

$$\Rightarrow r(t) = \frac{dx(t)}{dt}$$

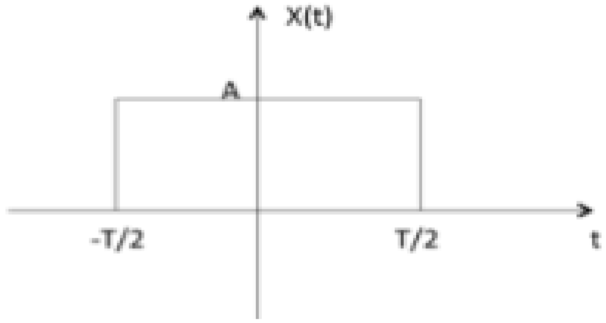
1.4.7 UNIT RECTANGLE PULSE

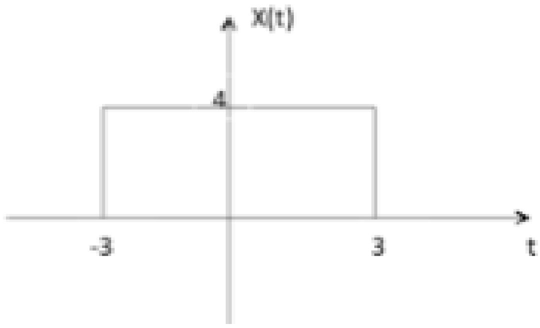
Rectangular Signal

Let it be denoted as x(t) and it is defined as

$$x(t) = A\,rect\left[\frac{r}{T}\right]$$

$$\text{ex: } 4\,rect\left[\frac{r}{6}\right]$$



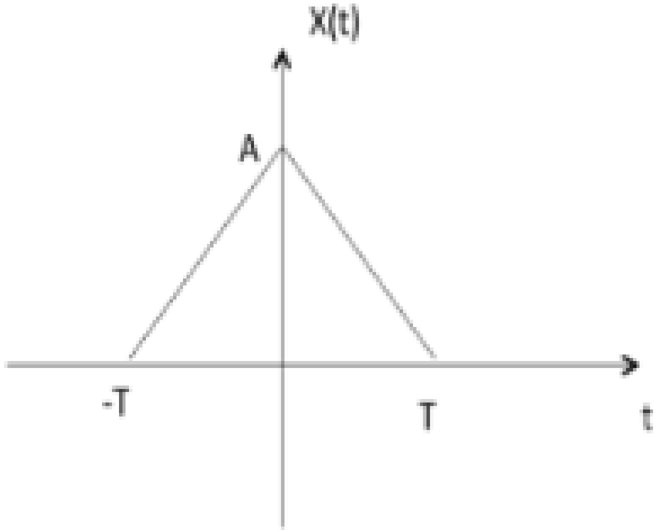


1.4.8 Triangular Signal

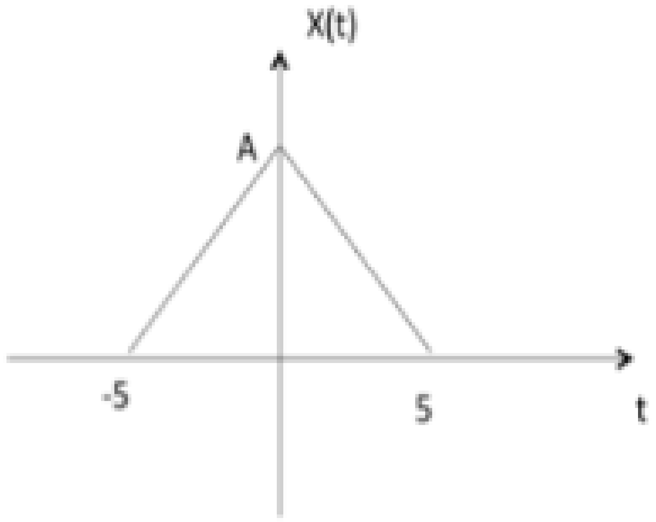
Triangular Signal

Let it be denoted as x(t)

$$x(t) = A\left[1 - \frac{|t|}{T}\right]$$



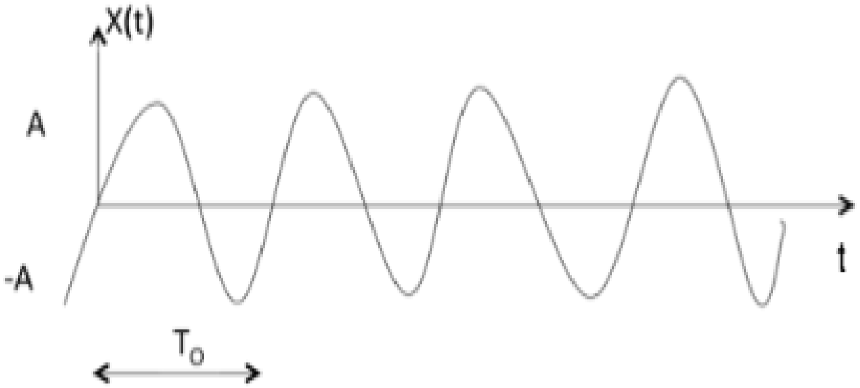
$$\text{ex}: x(t) = A\left[1 - \frac{|t|}{5}\right]$$



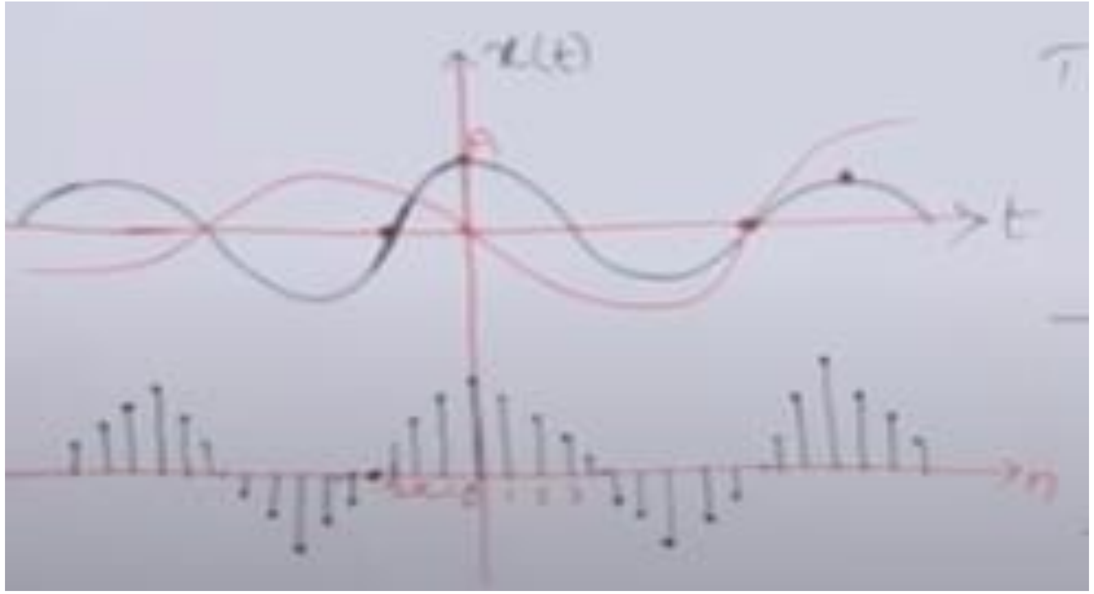
1.4.9 Sinusoidal Signal

Sinusoidal Signal

Sinusoidal signal is in the form of $x(t) = A\cos(\ w_0 \pm \phi\ )$ or $A\sin(\ w_0 \pm \phi\ )$



Where $T_0 = \frac{2\pi}{w_0}$



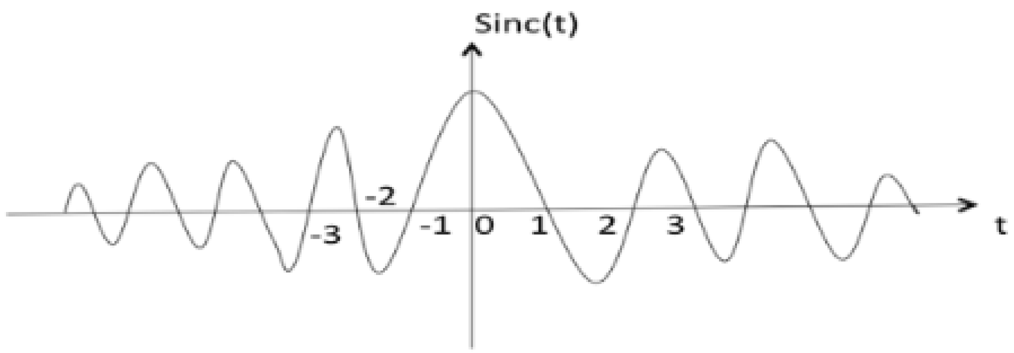
1.4.10 SINC and SAMPLING FUNCTIONS

Sinc Function

It is denoted as sinc(t) and it is defined as sinc

$$(t) = \frac{sin\pi t}{\pi t}$$

$$= 0 \text{ for t} = \pm 1, \pm 2, \pm 3...$$

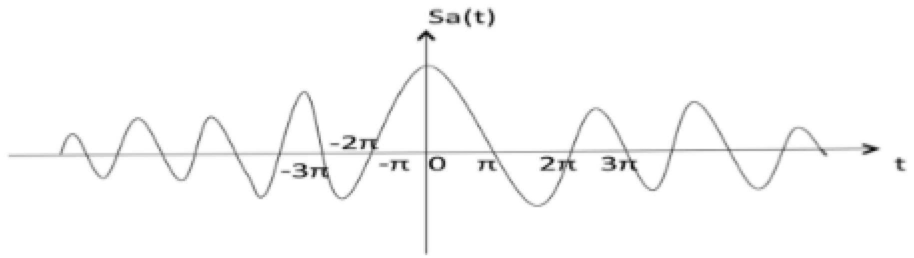


Sampling Function

It is denoted as sa(t) and it is defined as

$$sa(t) = \frac{sint}{t}$$

$$= 0 \text{ for t} = \pm\pi,\ \pm 2\pi,\ \pm 3\pi...$$



- Next week, we will discuss
  - operations on signals.
  - classificiations of signals

- I will upload some examples also.
- Please check our lessons from ekampus.ankara.edu.tr

Thank You