Some Special Discrete Random Variables:

1. Bernoulli Random Variable

2. Binom Random Variable

3.Geometric Random Variable

4.Poisson Random Variable

Bernoulli Random Variable:

Bernoulli Experiment: The experiment consists of one trial. There are two outcomes called failure and success. It can result in one of 2 outcomes.

X: The number of successes in one trial.

$D_X = \{0,1\}$ dir.   The probability function of X Bernoulli Random Variable is

$$f(x) = P(X = x) = p^x(1-p)^{1-x},\ \ x = 0,1$$

The probability table is given,

x		0	1
$P(X = x)$		$q = 1 - p$	p

where;

p: The probability of success

$q = 1 - p$: The probability of failure.

Examples (Bernoulli Experiments)

1.Tossing a coin

Sucess: head (or tail randomly; it means : absent or present)

2. There are black and white balls in a box. Select and record the color of the ball.

Only For one trial.

Success: Black (or white randomly; it means : absent or present)

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The Expected Value and the Variance of X

$$E(X) = \sum_x x\, f(x) = 0(1-p) + 1p = p$$

$$E(X^2) = \sum_x x^2\, f(x) = 0^2(1-p) + 1^2 p = p$$

$$Var(X) = E(X^2) - [E(X)]^2 = p - p^2 = p(1-p) = pq$$

dır. **Notation:**

$X \sim Bernoulli(1, p)$.

2. Binomial Random Variable:

Conditions of Binomial Experiment:

1.Experiment consists of a series of n identical trails.

2.Each trial is called a Bernoulli Trial.

3.Experiment consists of n repetad trails.

3. Two possible outcomes: Failure and Success.

4. p: The probability of success.

5.Each trail is independent.

Under these conditions ;

Experiment is called: Binomial Experiment,

X is called : Binomial Random Variable,

where;

$D_X = \{0,1,2,\dots,n\}$; the probability function of X ;

$$f(x) = P(X = x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0,1,\dots,n$$

<u>The Expected Value and the Variance of</u> X

$E(X) = np$

$Var(X) = npq$

dır. **<u>Notation:</u>**

$X \sim Binom(n,p)$

<u>Examples:</u>

1.There are black and white balls in a box. Select and record the color of the ball. Put it back and re-pick (sampling *with* replacement).

•n: number of independent and identical trials

•p: probability of success (e.g. probability of picking a black ball)

•X: number of successes in n trials

2.Tossing a coin (Success: Head) in two times.

Examples:

1.

For an exam have 20 questions with 5 multiple choices.

X: The number of the correctly marked questions.

a. $P(X \geq 10) =?$
b. $E(X) =?$

Solution:

a.

$$X \sim Binom(n = 20, p = \frac{1}{5})$$

$$f(x) = P(X = x) = \binom{20}{x} \frac{1}{5}^{x} \frac{4}{5}^{20-x} \quad , \quad x = 0,1, \ldots ,20$$

$$P(X \geq 10) = \sum_{x=10}^{20} \binom{20}{x} \frac{1}{5}^{x} \frac{4}{5}^{20-x}$$

b.

$$E(X) = np = 4$$

Student:

The special machine produces a 5 pieces per day.

$p = \frac{4}{5}$: The probability to produce the piece perfectly

X: The number of pieces produced perfectly in one day.

a. Obtain the probability function.
b. $E(X) =?$

Homework:

Experiment: Tossing a coin three times .

X: The number of tails. (Success is defined As tail.it means absent or present)

X is a Binomial Random Variable.

a. Show the notation.
b. Obtain the probability function.
c. $E(X), Var(X) =$?

Stat 250

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Geometric Random Variable:

Let a Bernoulli experiment has the probability of success p repeates until the first success achieved (under the same conditions, independently). Where X is defined as;

X : The number of trials until the first success is achieved.

$D_X = \{1, \dots\}$

X is called as Geometric Random Variable. The pf of X random varaiable is given as;

$$f(x) = P(X = x) = p(1-p)^{x-1}, \quad x = 1,2,3, \dots \quad 0 < p < 1$$

dır. Burada;

p:The probability of success (parameter)

$q = 1 - p$: The probability of failure.

The expected value and the varaince of the Geometric Random Variable:

$$\mu = E(X) = \frac{1}{p}$$

$$\sigma^2 = Var(X) = \frac{1-p}{p^2}$$

Notation:

$X \sim Geo(p)$

Examples:

1. For a shooter, the probability of hitting a certain target p is $0{,}75$.

The shooter repeats the trials until the first shot achieved ;

a) Define the X.

b) $E(X)$ =? $Var(X)$ =?

c) What is the proability that; the number of trials is less than 4?

d) What is the proability that; at least the number of trials is 3?

Solution:

a)

X : The number of trials.

$$f(x) = P(X = x) = 0{,}75\ (0{,}25)^{x-1} \quad x = 1,2,3, \ldots$$

$$E(X) = \frac{1}{p} = \frac{4}{3}$$

$$Var(X) = \frac{1-p}{p^2} = \frac{4}{9}$$

b)

$$P(X < 4) = f(1) + f(2) + f(3) = \frac{63}{64}$$

c)

$$P(X \geq 3) = 1 - P(X < 3) = 1 - \big(f(1) + f(2)\big) = \frac{1}{16}$$

2.

There are 7 white and 5 black balls in a box. Select a ball and record the color of the ball. Put it back and re-pick (sampling with replacement). Where;

X: The number of trials to get the first black ball.

a) What is the probability that the first black ball is achieved at the 5.trials.
b) $E(X) = ? \quad Var(X) = ?$
c) What is the probability that : the number of trials is more than 4

Solution:

<u>Poisson Random Variable :</u>

This Distribution is used in the modeling of experiments that give discrete results in continuous environments (time, area,)

X : The number of occurrences in a given $(0, t]$ time interval.

<u>Examples:</u>

1.The number of accidents in a certain way.

2.The number of customers in the shop.

$D_X = \{0,1,\dots\}$. The pf of X random varaiable is given as;

$$f(x) = P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0,1,2,\dots$$

dır.

λ: average number of occurences.

<u>Notation:</u>

$X \sim Poisson(\lambda)$

<u>The expected value and the varaince of the Geometric Random Variable:</u>

$\mu = E(X) = \lambda$

$\sigma^2 = Var(X) = \lambda$

Examples:

The average number of the patients coming to a hospital's emergency servise over a 15 minute interval is 4. In this period of time;

a) What is the probability that; no patient came to service?
b) What is the probability; that the number of patients is 1?
c) What is the probability that ; the number of patients is **least** 2?
d) What is the probability that; the number of patients is at **most** 3 ?

Solution:

a)

X:

The number of patients over a 15 minutes interval.

$f(x) = P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \;\; x = 0,1,2,\ldots, \qquad \lambda = 4$

$$P(X = 0) = \frac{e^{-4}4^{0}}{0!} = 0.0183$$

b)

$$P(X = 1) = \frac{e^{-4}4^{1}}{1!} = 4 * 0.0183 = \mathbf{0.0732}$$

c) $P(X \geq 2)$ =1- $P(X < 2) =$

=$\mathbf{1} -\; (f(0) + f(1))$

= $\mathbf{1} -\; (e^{-4} + 4e^{-4})$

= $\mathbf{1} -\; (5 * e^{-4})$

= $\mathbf{1} -\; (5 * 0.0183)$

d)

$P(X \leq 3) = \left(f(0) + f(1) + f(2) + f(3)\right)$

=$\left(e^{-4} + 4e^{-4} + 8e^{-4} + \frac{32}{3}e^{-4}\right)$

=$\frac{71}{3}e^{-4}$

Student:

In a city; there an average 5 traffic accidents in a day. For a certain day what is the probability that; the number of accidents

a) is 6?

b) is less than 6?

c) is more than 6?

d) is zero?

e) for a certain month, is 100?

Solution: