

SIGNALS and SYSTEMS

2022-2023

LECTURE 02

TOPICS

1. Operation on signals

- a) Operations performed on amplitude (scaling, addition, subtraction, multiplication)
- b) Operations on time (shifting, scaling, reversal)

2. Classification of signals

2.1. Operations on signals

❑ In general we can vary two parameters: amplitude and time

❑ Operations can be performed on amplitude are:

- Scaling
- Addition
- Subtraction
- Multiplication

❑ Operations performed on time are:

- Shifting
- Scaling
- reversal

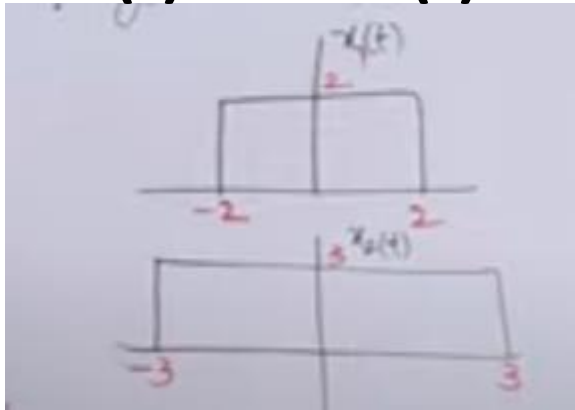
2.2 Operations on Amplitude.

□ **Scaling:** scaled version of the signal is $y(t) = Cx(t)$, where C is our scaling factor.

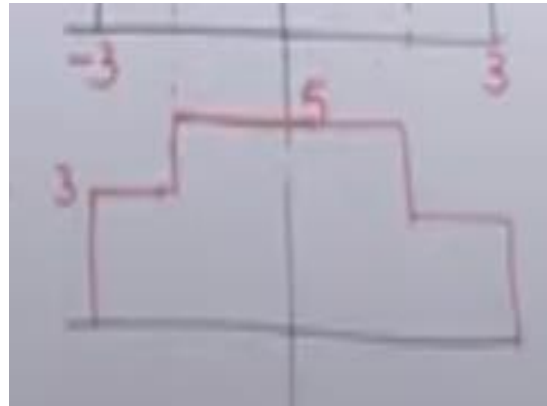
- Ex: $x(t) = 4\cos t$ $y(t) = 3x(t)$ Then $y(t) = 3(4\cos t) = 12\cos t$

□ **Addition, Subtraction, Multiplication:** Let's we have two signals here:

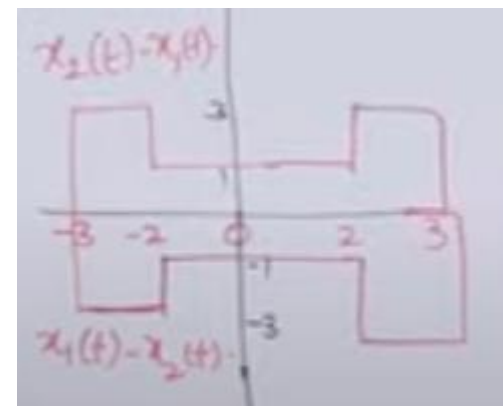
$x_1(t)$ and $x_2(t)$



addition



subtraction



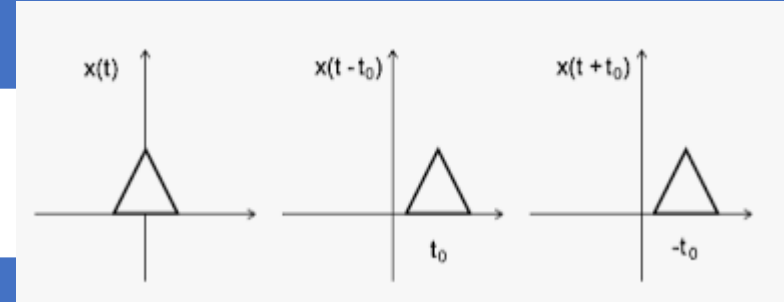
multiplication



2.3 Time related operations

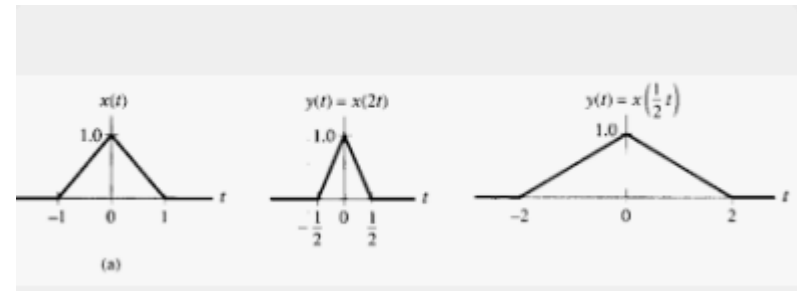
Time shifting:

- $x(t) \rightarrow x(t-t_0)$ (+shifting or right shifting)
- $x(t) \rightarrow x(t+t_0)$ (- shifting or left shifting)



Time Scaling:

- $x(t) \rightarrow x(at)$
- If $|a| > 1$ compression of the signal
- If $0 < |a| < 1$ expansion of the signal
- Does not work for unit step function



Time reversal: folding about y axis

- $X(t) \rightarrow x(-t)$
- Called as mirror



2.4. Classification of signals.

.Basically signals are classified into 6 different types

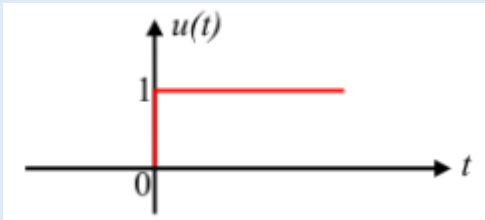
1- Continuous time and discrete time signals:
2- Deterministic and non-deterministic (random) signals
3-Even and Odd Signals
4-Periodic and aperiodic signals
5-Energy Signals and power signals
6-Real and Imaginary signals.

2.4.1 Continuous and Discrete Time SIGNALS

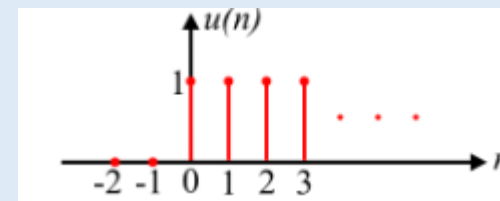
Continuous time signals (CTS)

Discrete-time signals (CTS)

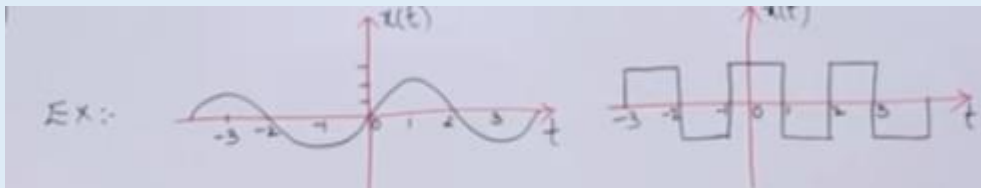
1-A signal which is defined for all values of t is called as continuous time signal



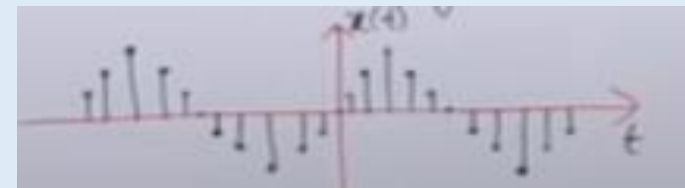
1- A signal which is defined only at discrete intervals of time is called discrete time signals.



2-We represent the functions with $x(t)$



2-We represent the functions with $x[n]$ (n is not directly time. It discrete time refers time intervals)



- ☐ For discrete time signals only time is discrete but the amplitude is continuous.
- ☐ For digital signals both amplitude and time are discrete
- ☐ We convert a CTS to DTS. For this we multiply the CTS $x(t)$ with a pulse train. This process is sometimes called as sampling process.
- ☐ We can convert DTS to CTS by continuous steps.

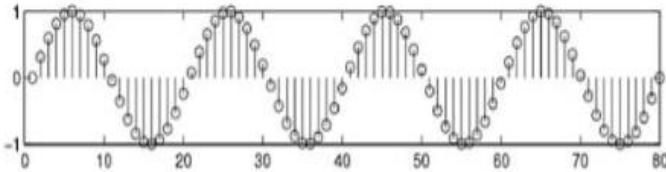
2.4.2 Deterministic and non-deterministic (random) signals

Deterministic signals

A signal is said to be deterministic if there is no uncertainty with respect to its value at any instant of time.

Deterministic signals

- There is no uncertainty with respect to its value at any time. (ex) $\sin(3t)$



Signals which can be defined exactly by a mathematical formula are known as deterministic signals.

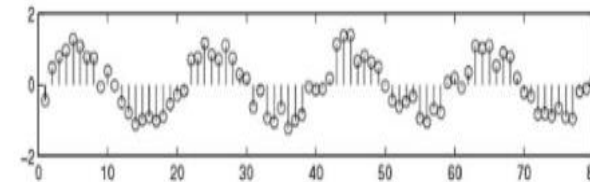
For example, the signal $x(n) = \sin(\pi n/4)$ is deterministic.

Non-deterministic signals (random)

A signal is said to be non-deterministic if there is uncertainty at any particular instant of time.

Random signals

- There is *uncertainty* before its actual occurrence.



On the other hand, random signals have uncertain values. They can not be described by mathematical equations. They are described using their statistics, or we can say they are modelled in probabilistic times.

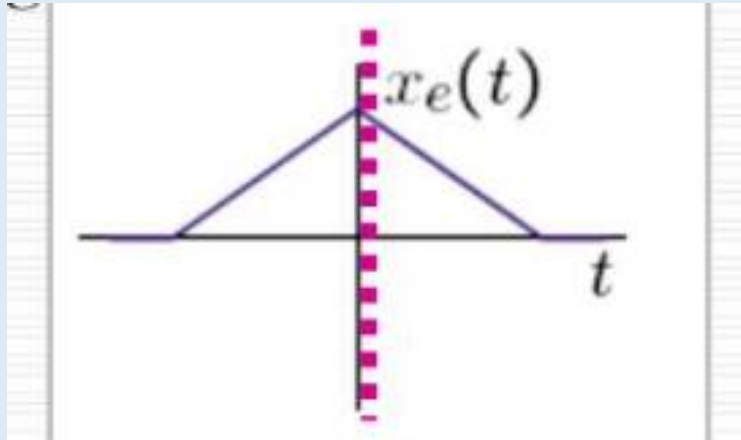
Non-deterministic signals are random in nature hence are called random signals.

2.4.3 Even and Odd Signals

Even signals

A signal is said to be even when it satisfies the condition:

$$x(t) = x(-t) \quad \text{Ex: } \cos(t), t^2$$

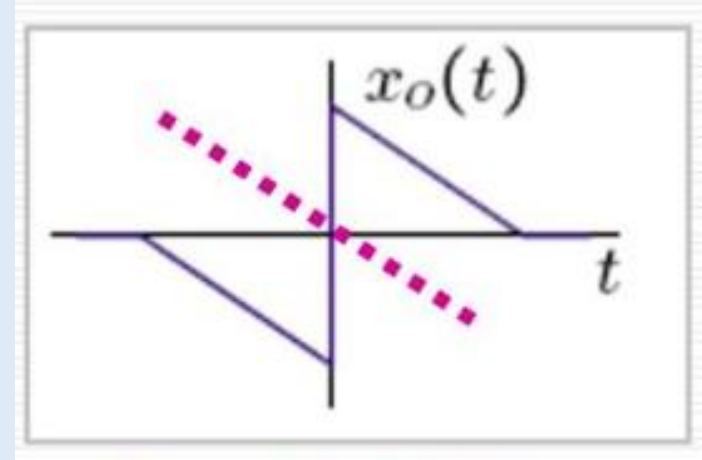


Even signals are symmetric around the vertical axis.

Odd signals

A signal is said to be odd when it satisfies the condition:

$$x(-t) = -x(t) \quad \text{Ex: } \sin(t), t^3$$



Odd signals are symmetric about origin.

*Sum of two or more even functions, product of two or more even functions, product of even number of odd functions results even function.

*Sum of two or more odd functions, product of odd number of odd functions results odd function.

2.4.4 Even and Odd Component of a Signal

- Each signal have its odd and even components:
 - $x(t) = x_e(t) + x_o(t)$ (x_e : even component / x_o : odd component)
 - $x(-t) = x_e(t) - x_o(t)$
- $x_e(t) = (x(t) + x(-t)) / 2$
- $x_o(t) = (x(t) - x(-t)) / 2$

2.4.4 Periodic and Aperiodic signals

•

A signal is said to be a periodic signal if it has a definite pattern and repeats itself at a regular interval of time.

Whereas, the signal which does not at the regular interval of time is known as an aperiodic signal or non-periodic signal

A signal is said to be periodic if it satisfies the condition: $x(t) = x(t+T)$ / $x(n) = x(n+N)$

The smallest value of T which satisfies the above condition is called « fundamental time period »

$$F = 1/T \quad \omega = 2\pi f = 2\pi/T \quad \text{in } \sin \omega t$$

When two signals of **same** frequency are added the resultant sinusoidal signal will be periodic.

When two signals of **different** frequency are added the resultant sinusoidal signal can be periodic or non-periodic.

2.4.5 – Checking Periodicity

- Ratio: Consider two signals $x_1(t)$ and $x_2(t)$ with periods T_1 and T_2 respectively. When they are summed the resultant signal is periodic if
 - T_1/T_2 is a rational number
- GCD (Greatest common divisor) : If GCD is possible then the signal will be periodic, otherwise aperiodic.

- ❖ Next week, we will discuss
 - ❖ Energy and power signals.
 - ❖ Real and imaginary signals
- ❖ Linear Time Invariant systems.
- ❖ Please check our lessons from ekampus.ankara.edu.tr

Thank You