Mathematical Proof Techniques

Murat Osmanoglu

 Valid arguments that establish the truth of mathematical statements

Valid <u>arguments</u> that establish the truth of mathematical statements

<u>argument</u>: sequence of sentences (propositions); premises at the beginning and conclusion at the end

 An argument is called valid if the truthness of all its premises implies that the confusion is true

 An argument is called valid if the truthness of all its premises implies that the confusion is true

 If you have a password, then you can log onto the network.

 An argument is called valid if the truthness of all its premises implies that the confusion is true

- If you have a password, then you can log onto the network.
- · You have a password

 An argument is called valid if the truthness of all its premises implies that the confusion is true

- If you have a password, then you can log onto the network.
- · You have a password
- Therefore,
 you can log onto the network

 An argument is called valid if the truthness of all its premises implies that the confusion is true

 If you have a password, then you can log onto the network.

 $p \rightarrow q$

· You have a password

p

Therefore,
 you can log onto the network

 $p \rightarrow q$

p

Modus Ponens

 $p \rightarrow q$

p

Modus Ponens

$$p \rightarrow q$$

p

| p | q | $p \rightarrow q$ | p ∧ (p → q) | $[p \land (p \rightarrow q)] \rightarrow q$ |
|---|---|-------------------|-------------|---------------------------------------------|
| 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 |

Modus Ponens

• If $\sqrt{5} > \sqrt{3}$, then $(\sqrt{5})^2 > (\sqrt{3})^2$.

• If
$$\sqrt{5} > \sqrt{3}$$
, then $(\sqrt{5})^2 > (\sqrt{3})^2$. $p \to q$

• If
$$\sqrt{5} > \sqrt{3}$$
, then $(\sqrt{5})^2 > (\sqrt{3})^2$. $p \to q$

• We know that
$$\sqrt{5} > \sqrt{3}$$

• If
$$\sqrt{5} > \sqrt{3}$$
, then $(\sqrt{5})^2 > (\sqrt{3})^2$.

$$p \rightarrow q$$

• We know that
$$\sqrt{5} > \sqrt{3}$$

• So,
$$(\sqrt{5})^2 > (\sqrt{3})^2$$

• If
$$\sqrt{5} > \sqrt{3}$$
, then $(\sqrt{5})^2 > (\sqrt{3})^2$.

• We know that
$$\sqrt{5} > \sqrt{3}$$

• So,
$$(\sqrt{5})^2 > (\sqrt{3})^2 \rightarrow 5 > 3$$

$$p \rightarrow q$$

• To prove $\forall x \ (P(x) \to Q(x))$, show that $P(c) \to Q(c)$ is true for an arbitrary element c of the domain.

• To prove $\forall x \ (P(x) \to Q(x))$, show that $P(c) \to Q(c)$ is true for an arbitrary element c of the domain.

1) All men are mortal

Socrates is a man

Socrates is mortal

• To prove $\forall x \ (P(x) \to Q(x))$, show that $P(c) \to Q(c)$ is true for an arbitrary element c of the domain.

1) All men are mortal

Socrates is a man

Socrates is mortal

P(x) : x is a man

Q(x): x is mortal

• To prove $\forall x (P(x) \rightarrow Q(x))$, show that $P(c) \rightarrow Q(c)$ is true for an arbitrary element c of the domain.

| 1) | All | men | are | mortal |
|------------|-----|-----|------|------------|
| - / | / \ | | ul C | IIIOI I GI |

$$\forall x \ (P(x) \to Q(x))$$

Socrates is a man

P(Socrates)

Socrates is mortal

Q(Socrates)

P(x) : x is a man

Q(x): x is mortal

- To prove $\forall x \ (P(x) \to Q(x))$, show that $P(c) \to Q(c)$ is true for an arbitrary element c of the domain.
- To prove $P(c) \rightarrow Q(c)$, show that Q(c) is true if P(c) is true

- To prove $\forall x \ (P(x) \to Q(x))$, show that $P(c) \to Q(c)$ is true for an arbitrary element c of the domain.
- To prove $P(c) \rightarrow Q(c)$, show that Q(c) is true if P(c) is true (p \rightarrow q is true unless p is true but q is false)

- To prove $\forall x \ (P(x) \to Q(x))$, show that $P(c) \to Q(c)$ is true for an arbitrary element c of the domain.
- To prove $P(c) \rightarrow Q(c)$, show that Q(c) is true if P(c) is true (p \rightarrow q is true unless p is true but q is false)

- To prove $\forall x \ (P(x) \to Q(x))$, show that $P(c) \to Q(c)$ is true for an arbitrary element c of the domain.
- To prove $P(c) \rightarrow Q(c)$, show that Q(c) is true if P(c) is true (p \rightarrow q is true unless p is true but q is false)

Direct Proof

• To prove $p \rightarrow q$ is true, first assume p is true, then show that q must also be true.

- To prove $\forall x \ (P(x) \to Q(x))$, show that $P(c) \to Q(c)$ is true for an arbitrary element c of the domain.
- To prove $P(c) \rightarrow Q(c)$, show that Q(c) is true if P(c) is true (p \rightarrow q is true unless p is true but q is false)

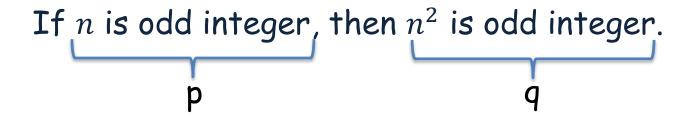
- To prove $p \rightarrow q$ is true, first assume p is true, then show that q must also be true.
- Thus, if p is true, then q must also be true, so that

- To prove $\forall x \ (P(x) \to Q(x))$, show that $P(c) \to Q(c)$ is true for an arbitrary element c of the domain.
- To prove $P(c) \rightarrow Q(c)$, show that Q(c) is true if P(c) is true (p \rightarrow q is true unless p is true but q is false)

- To prove $p \rightarrow q$ is true, first assume p is true, then show that q must also be true.
- Thus, if p is true, then q must also be true, so that the combination of p true and q false never occurs

Direct Proof

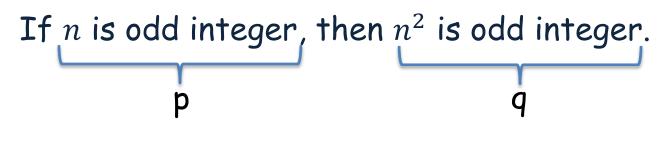
If n is odd integer, then n^2 is odd integer.



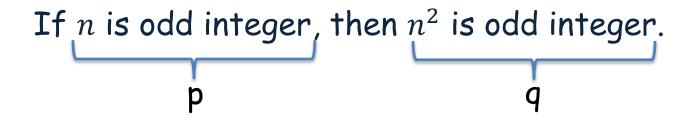


$$p \rightarrow q$$

Direct Proof

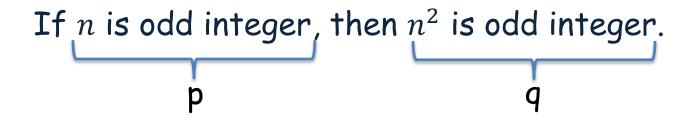


 $p \rightarrow q$ assume p is true



$$p \rightarrow q$$
 assume p is true

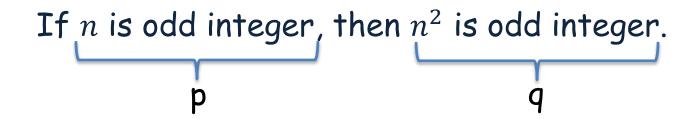
$$n = 2k + 1, \exists k \in Z$$



$$p \rightarrow q$$
 assume p is true

$$n = 2k + 1, \exists k \in Z$$

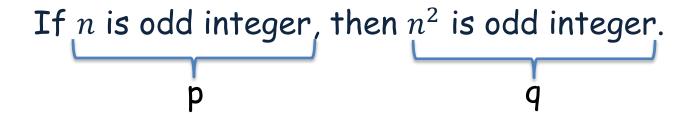
 $n^2 = (2k + 1)^2$



$$p \rightarrow q$$
 assume p is true

$$n = 2k + 1, \exists k \in Z$$

 $n^2 = (2k + 1)^2$
 $n^2 = 4k^2 + 4k + 1$



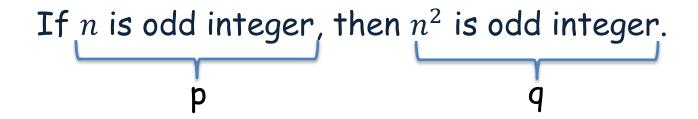
$$p \rightarrow q$$
 assume p is true

$$n = 2k + 1, \exists k \in \mathbb{Z}$$

$$n^{2} = (2k + 1)^{2}$$

$$n^{2} = 4k^{2} + 4k + 1$$

$$n^{2} = 2(2k^{2} + 2k) + 1$$



$$p \rightarrow q$$
 assume p is true

$$n = 2k + 1, \exists k \in Z$$

 $n^2 = (2k + 1)^2$
 $n^2 = 4k^2 + 4k + 1$
 $n^2 = 2(2k^2 + 2k) + 1$
 $n^2 = 2m + 1, \exists m \in Z$

Direct Proof

If
$$n$$
 is odd integer, then n^2 is odd integer.

$$p \rightarrow q$$
 assume p is true

$$n = 2k + 1, \exists k \in Z$$

$$n^{2} = (2k + 1)^{2}$$

$$n^{2} = 4k^{2} + 4k + 1$$

$$n^{2} = 2(2k^{2} + 2k) + 1$$

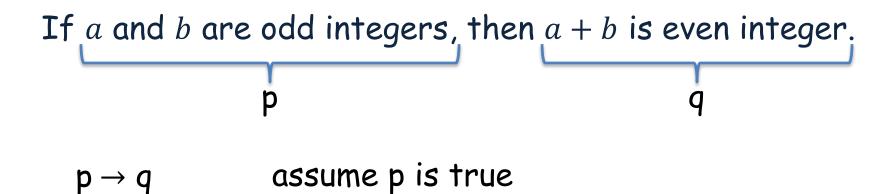
$$n^{2} = 2m + 1, \exists m \in Z$$

q is also true

Direct Proof

Direct Proof

Direct Proof



Direct Proof

$$p \rightarrow q$$
 assume p is true

$$a = 2x + 1$$
 and $b = 2y + 1 \exists x, y \in Z$

Direct Proof

$$p \rightarrow q$$
 assume p is true

$$a = 2x + 1$$
 and $b = 2y + 1$ $\exists x, y \in Z$
 $a + b = 2x + 1 + 2y + 1$

Direct Proof

$$p \rightarrow q$$
 assume p is true

$$a = 2x + 1$$
 and $b = 2y + 1$ $\exists x, y \in Z$
 $a + b = 2x + 1 + 2y + 1$
 $a + b = 2x + 2y + 2$

Direct Proof

$$p \rightarrow q$$
 assume p is true

$$a = 2x + 1$$
 and $b = 2y + 1$ $\exists x, y \in Z$
 $a + b = 2x + 1 + 2y + 1$
 $a + b = 2x + 2y + 2$
 $a + b = 2(x + y + 1)$

Direct Proof

$$p \rightarrow q$$
 assume p is true

$$a = 2x + 1$$
 and $b = 2y + 1$ $\exists x, y \in Z$
 $a + b = 2x + 1 + 2y + 1$
 $a + b = 2x + 2y + 2$
 $a + b = 2(x + y + 1)$
 $a + b = 2m$, $\exists m \in Z$

Direct Proof

$$p \rightarrow q$$
 assume p is true

$$a = 2x + 1$$
 and $b = 2y + 1$ $\exists x, y \in Z$
 $a + b = 2x + 1 + 2y + 1$
 $a + b = 2x + 2y + 2$
 $a + b = 2(x + y + 1)$
 $a + b = 2m$, $\exists m \in Z$
q is also true

Direct Proof

If m and n are perfect squares, then m.n is also a perfect square.

Direct Proof

If m and n are perfect squares, then m.n is also a perfect square.

Direct Proof

If m and n are perfect squares, then m.n is also a perfect square.

$$p \rightarrow q$$

Direct Proof

If m and n are perfect squares, then m.n is also a perfect square.

 $p \rightarrow q$ assume p is true

Direct Proof

If m and n are perfect squares, then m.n is also a perfect square.

þ

q

$$p \rightarrow q$$
 assume p is true

$$m = x^2$$
 and $n = y^2$, $\exists x, y \in Z$

Direct Proof

If m and n are perfect squares, then m.n is also a perfect square.

þ

q

$$p \rightarrow q$$

$$m = x^2$$
 and $n = y^2$, $\exists x, y \in Z$
 $m.n = x^2y^2$

Direct Proof

 $p \rightarrow q$

If m and n are perfect squares, then m.n is also a perfect square.

þ

$$m = x^2$$
 and $n = y^2$, $\exists x, y \in Z$
 $m.n = x^2y^2$
 $m.n = (x.y)^2$

Direct Proof

If m and n are perfect squares, then m.n is also a perfect square.

p

 $p \rightarrow q$ assume p is true

$$m = x^2$$
 and $n = y^2$, $\exists x, y \in Z$
 $m.n = x^2y^2$
 $m.n = (x.y)^2$
 $m.n = k^2$, $\exists k \in Z$

Direct Proof

If m and n are perfect squares, then m.n is also a perfect square.

p

 $p \rightarrow q$

$$m=x^2$$
 and $n=y^2$, $\exists x,y \in Z$
 $m.n=x^2y^2$
 $m.n=(x.y)^2$
 $m.n=k^2$, $\exists k \in Z$
q is also true

Proof by Contraposition

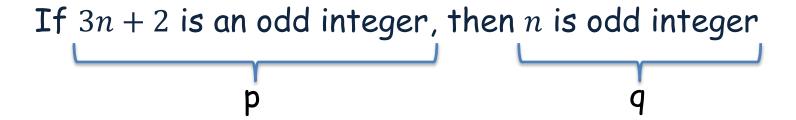
• Instead of proving $p \rightarrow q$, prove logically equivalent proposition $\sim q \rightarrow \sim p$

Proof by Contraposition

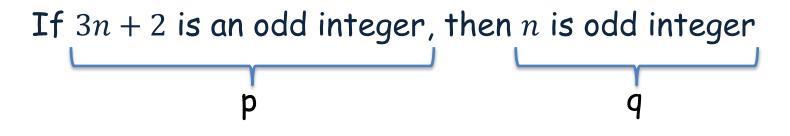
• Instead of proving $p \rightarrow q$, prove logically equivalent proposition $\sim q \rightarrow \sim p$ --WHY?

Proof by Contraposition

If 3n + 2 is an odd integer, then n is odd integer



Proof by Contraposition



 $p \rightarrow q$ assume p is true



$$p \rightarrow q$$
 assume p is true

$$3n + 2 = 2k + 1, \exists k \in Z$$



$$p \rightarrow q$$
 assume p is true

$$3n + 2 = 2k + 1, \exists k \in \mathbb{Z}$$

 $3n = 2k - 1$

If
$$3n + 2$$
 is an odd integer, then n is odd integer

$$p \rightarrow q$$
 assume p is true

$$3n + 2 = 2k + 1, \exists k \in \mathbb{Z}$$
$$3n = 2k - 1$$
$$n = \frac{2k - 1}{3}$$

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

If 3n + 2 is an odd integer, then n is odd integer

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

If 3n + 2 is an odd integer, then n is odd integer

If n is not odd integer, then 3n + 2 is not odd integer

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

If 3n + 2 is an odd integer, then n is odd integer

If n is not odd integer, then 3n + 2 is not odd integer $\sim q$

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

If 3n + 2 is an odd integer, then n is odd integer

If n is not odd integer, then 3n + 2 is not odd integer $\sim a$

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

If 3n + 2 is an odd integer, then n is odd integer

If n is not odd integer, then 3n + 2 is not odd integer $\sim q$

assume $\sim q$ is true $n = 2k, \exists k \in Z$

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

$$p \rightarrow q \equiv \sim q \rightarrow \sim p$$

If 3n + 2 is an odd integer, then n is odd integer

If n is not odd integer, then 3n + 2 is not odd integer

$$n = 2k, \exists k \in Z$$

 $3n + 2 = 6k + 2$

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

$$p \rightarrow q \equiv \sim q \rightarrow \sim p$$

If 3n + 2 is an odd integer, then n is odd integer

If n is not odd integer, then 3n + 2 is not odd integer



$$n = 2k$$
, $\exists k \in Z$
 $3n + 2 = 6k + 2$
 $3n + 2 = 2(3k + 1)$

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

$$p \rightarrow q \equiv \sim q \rightarrow \sim p$$

If 3n + 2 is an odd integer, then n is odd integer

If n is not odd integer, then 3n + 2 is not odd integer



$$n = 2k, \exists k \in \mathbb{Z}$$

 $3n + 2 = 6k + 2$
 $3n + 2 = 2(3k + 1)$
 $3n + 2 = 2m, \exists m \in \mathbb{Z}$

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

$$p \rightarrow q \equiv \sim q \rightarrow \sim p$$

If 3n + 2 is an odd integer, then n is odd integer

If n is not odd integer, then 3n + 2 is not odd integer

~q ~p

$$n = 2k$$
, $\exists k \in Z$
 $3n + 2 = 6k + 2$
 $3n + 2 = 2(3k + 1)$
 $3n + 2 = 2m$, $\exists m \in Z$
~p is also true

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

Prove that for all real numbers x and y, if $x + y \ge 100$, then $x \ge 50$ or $y \ge 50$.

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

Prove that for all real numbers x and y, if $x + y \ge 100$, then $x \ge 50$ or $y \ge 50$.

If
$$x < 50$$
 and $y < 50$, then $x + y < 100$

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

Prove that for all real numbers x and y, if $x + y \ge 100$, then $x \ge 50$ or $y \ge 50$.

If
$$x < 50$$
 and $y < 50$, then $x + y < 100$

assume ~q is true

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

Prove that for all real numbers x and y, if $x + y \ge 100$, then $x \ge 50$ or $y \ge 50$.

If
$$x < 50$$
 and $y < 50$, then $x + y < 100$

assume
$$\sim q$$
 is true $x < 50$ and $y < 50$

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

$$p \rightarrow q \equiv \sim q \rightarrow \sim p$$

Prove that for all real numbers x and y, if $x + y \ge 100$, then $x \ge 50$ or $y \ge 50$.

If
$$x < 50$$
 and $y < 50$, then $x + y < 100$

assume
$$\sim q$$
 is true $x < 50$ and $y < 50$

$$x + y < 100$$

<u>Proof by Contraposition</u> $p \rightarrow q \equiv \sim q \rightarrow \sim p$

Prove that for all real numbers x and y, if $x + y \ge 100$, then $x \ge 50$ or $y \ge 50$.

If
$$x < 50$$
 and $y < 50$, then $x + y < 100$

assume $\sim q$ is true x < 50 and y < 50

$$x < 50$$
 and $y < 50$

$$x + y < 100$$

~p is also true

Proof by Contradiction

Proof by Contradiction

Proof by Contradiction

$$\sim p \rightarrow q$$

Proof by Contradiction

$$\sim p \rightarrow q$$
 $\rightarrow F \equiv T$

Proof by Contradiction

$$\sim p \rightarrow q$$

$$F \rightarrow F \equiv T$$

Proof by Contradiction

$$\sim p \rightarrow q$$

$$F \rightarrow F \equiv T$$

$$q \equiv r \wedge \sim r$$

Proof by Contradiction

$$\sim p \rightarrow q$$

$$F \rightarrow F \equiv T$$

$$q \equiv r \wedge \sim r \equiv 0$$

Proof by Contradiction

• To prove that 'p is true', find a contradiction q such that $\sim p \rightarrow q$ is true.

$$\sim p \rightarrow q$$

$$F \rightarrow F \equiv T$$

 assuming '~p is true' leads us a contradiction

$$q \equiv r \wedge \sim r \equiv 0$$

Proof by Contradiction

 Prove that the sum of an irrational number and rational number is irrational.

Proof by Contradiction

 Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational.

Proof by Contradiction

 Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

Proof by Contradiction

 Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$$y = \frac{a}{b}$$
 and $x + y = \frac{c}{d}$, $\exists a, b, c, d \in Z$

Proof by Contradiction

 Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$$y = \frac{a}{b}$$
 and $x + y = \frac{c}{d}$, $\exists a, b, c, d \in Z$

There is no integers e,f such that $x = \frac{e}{f}$

Proof by Contradiction

 Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. ($\sim p$ is true)

$$y = \frac{a}{b}$$
 and $x + y = \frac{c}{d}$, $\exists a, b, c, d \in Z$

Proof by Contradiction

 Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. ($\sim p$ is true)

$$y = \frac{a}{b}$$
 and $x + y = \frac{c}{d}$, $\exists a, b, c, d \in Z$

$$x + y = \frac{c}{d}$$

Proof by Contradiction

 Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$$y = \frac{a}{b}$$
 and $x + y = \frac{c}{d}$, $\exists a, b, c, d \in Z$

$$x + y = \frac{c}{d} \rightarrow x + \frac{a}{b} = \frac{c}{d}$$

Proof by Contradiction

 Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. ($\sim p$ is true)

$$y = \frac{a}{b}$$
 and $x + y = \frac{c}{d}$, $\exists a, b, c, d \in Z$

$$x + y = \frac{c}{d} \rightarrow x + \frac{a}{b} = \frac{c}{d} \rightarrow x = \frac{c}{d} - \frac{a}{b}$$

Proof by Contradiction

 Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$$y = \frac{a}{b}$$
 and $x + y = \frac{c}{d}$, $\exists a, b, c, d \in Z$

$$x + y = \frac{c}{d} \rightarrow x + \frac{a}{b} = \frac{c}{d} \rightarrow x = \frac{c}{d} - \frac{a}{b} \rightarrow x = \frac{e}{f}$$
, $\exists e, f \in Z$

Proof by Contradiction

 Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$$y = \frac{a}{b}$$
 and $x + y = \frac{c}{d}$, $\exists a, b, c, d \in Z$

$$x + y = \frac{c}{d} \rightarrow x + \frac{a}{b} = \frac{c}{d} \rightarrow x = \frac{c}{d} - \frac{a}{b} \rightarrow x = \frac{e}{f}$$
, $\exists e, f \in Z \text{ (~r)}$

Proof by Contradiction

 Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$$y = \frac{a}{b}$$
 and $x + y = \frac{c}{d}$, $\exists a, b, c, d \in Z$

There is no integers e,f such that $x = \frac{e}{f}$ (the proposition r)

$$x + y = \frac{c}{d} \rightarrow x + \frac{a}{b} = \frac{c}{d} \rightarrow x = \frac{c}{d} - \frac{a}{b} \rightarrow x = \frac{e}{f}$$
, $\exists e, f \in Z \text{ (~r)}$

 $\sim p \rightarrow (r \land \sim r)$: assuming ' $\sim p$ is true' leads us a contradiction.

Proof by Contradiction

• Prove that if 3n + 2 is an odd integer, then n is odd integer

Proof by Contradiction

Prove that if 3n + 2 is an odd integer, then n is odd integer

Proof by Contradiction

• Prove that if 3n + 2 is an odd integer, then n is odd integer

Proof by Contradiction

• Prove that if 3n + 2 is an odd integer, then n is odd integer

$$\sim (p \rightarrow q)$$

Proof by Contradiction

• Prove that if 3n + 2 is an odd integer, then n is odd integer p

$$\sim (p \rightarrow q) \equiv \sim (\sim p \lor q)$$

Proof by Contradiction

• Prove that if 3n + 2 is an odd integer, then n is odd integer

$$\sim (p \rightarrow q) \equiv \sim (\sim p \lor q) \equiv p \land \sim q$$

Proof by Contradiction

• Prove that if 3n + 2 is an odd integer, then n is odd integer

Assuming 'p \rightarrow q is not true' leads us a contradiction.

$$\sim (p \rightarrow q) \equiv \sim (\sim p \lor q) \equiv p \land \sim q$$

Assuming 'p $\land \neg q$ is not true' leads us a contradiction.

Proof by Contradiction

• Prove that if 3n + 2 is an odd integer, then n is odd integer

Assuming 'p $\land \neg q$ is not true' leads us a contradiction.

Proof by Contradiction

• Prove that if 3n + 2 is an odd integer, then n is odd integer p

Assuming 'p $\land \neg q$ is not true' leads us a contradiction.

3n + 2 is an odd integer and n is even integer. (p $\land \neg q$)

Proof by Contradiction

• Prove that if 3n + 2 is an odd integer, then n is odd integer

Assuming 'p $\land \neg q$ is not true' leads us a contradiction.

3n + 2 is an odd integer and n is even integer. (p $\land \neg q$)

$$n = 2k$$
, $\exists k \in Z$.

<u>Proofs</u>

Proof by Contradiction

• Prove that if 3n + 2 is an odd integer, then n is odd integer

Assuming 'p $\land \neg q$ is not true' leads us a contradiction.

3n+2 is an odd integer and n is even integer. (p $\land \neg q$)

$$n = 2k$$
, $\exists k \in \mathbb{Z}$. So $3n + 2 = 6k + 2$

<u>Proofs</u>

Proof by Contradiction

• Prove that if 3n + 2 is an odd integer, then n is odd integer

Assuming 'p $\land \neg q$ is not true' leads us a contradiction.

3n + 2 is an odd integer and n is even integer. (p $\land \neg q$)

$$n = 2k$$
, $\exists k \in \mathbb{Z}$. So $3n + 2 = 6k + 2 = 2(3k + 1)$

Proof by Contradiction

• Prove that if 3n + 2 is an odd integer, then n is odd integer p

Assuming 'p $\land \neg q$ is not true' leads us a contradiction.

3n + 2 is an odd integer and n is even integer. (p $\land \neg q$)

$$n = 2k$$
, $\exists k \in \mathbb{Z}$. So $3n + 2 = 6k + 2 = 2(3k + 1) = 2m$, $\exists m \in \mathbb{Z}$

Proof by Contradiction

• Prove that if 3n + 2 is an odd integer, then n is odd integer

Assuming 'p $\land \neg q$ is not true' leads us a contradiction.

3n + 2 is an odd integer and n is even integer. (p $\land \neg q$)

$$n = 2k$$
, $\exists k \in \mathbb{Z}$. So $3n + 2 = 6k + 2 = 2(3k + 1) = 2m$, $\exists m \in \mathbb{Z}$

3n + 2 is an even integer. (Contradiction!)

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \to q) \land (q \to p)$$

n is odd integer if and only if 5n + 4 is odd integer

 $p \rightarrow q$ (direct proof)

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

p

 $p \rightarrow q$ (direct proof)

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

p

 $p \rightarrow q$ (direct proof)

$$n = 2k + 1, \exists k \in Z$$

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

p

 $p \rightarrow q$ (direct proof)

$$n = 2k + 1, \exists k \in Z$$

 $5n + 4 = 10k + 9$

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

þ

 $p \rightarrow q$ (direct proof)

$$n = 2k + 1, \exists k \in Z$$

$$5n + 4 = 10k + 9$$

$$5n + 4 = 2(5k + 4) + 1$$

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

þ

 $p \rightarrow q$ (direct proof)

$$n = 2k + 1, \exists k \in Z$$

$$5n + 4 = 10k + 9$$

$$5n + 4 = 2(5k + 4) + 1$$

$$5n + 4 = 2m + 1$$
, $\exists m \in Z$

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

þ

 $p \rightarrow q$ (direct proof)

assume p is true

$$n = 2k + 1, \exists k \in \mathbb{Z}$$

 $5n + 4 = 10k + 9$

$$5n + 4 = 2(5k + 4) + 1$$

 $5n + 4 = 2m + 1, \exists m \in Z$

q is true

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

p

 $p \rightarrow q$ (direct proof)

 $q \rightarrow p$ (proof by contraposition)

assume p is true

$$n = 2k + 1, \exists k \in \mathbb{Z}$$

 $5n + 4 = 10k + 9$
 $5n + 4 = 2(5k + 4) + 1$

$$5n+4=2m+1, \exists m \in Z$$

q is true

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

p

 $p \rightarrow q$ (direct proof)

 $q \rightarrow p$ (proof by contraposition)

$$n = 2k + 1, \exists k \in Z$$

 $5n + 4 = 10k + 9$
 $5n + 4 = 2(5k + 4) + 1$
 $5n + 4 = 2m + 1, \exists m \in Z$
q is true

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

þ

 $p \rightarrow q$ (direct proof)

assume p is true

$$n = 2k + 1, \exists k \in Z$$

$$5n + 4 = 10k + 9$$

$$5n + 4 = 2(5k + 4) + 1$$

$$5n + 4 = 2m + 1, \exists m \in Z$$

q is true

 $q \rightarrow p$ (proof by contraposition)

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

p

 $p \rightarrow q$ (direct proof)

assume p is true

$$n = 2k + 1, \exists k \in Z$$

$$5n + 4 = 10k + 9$$

$$5n + 4 = 2(5k + 4) + 1$$

$$5n + 4 = 2m + 1, \exists m \in Z$$

q is true

$$q \rightarrow p$$
 (proof by contraposition)

$$n=2k$$
, $\exists k \in Z$

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

p

 $p \rightarrow q$ (direct proof)

assume p is true

$$n = 2k + 1$$
, $\exists k \in Z$

$$5n + 4 = 10k + 9$$

$$5n + 4 = 2(5k + 4) + 1$$

$$5n + 4 = 2m + 1, \exists m \in Z$$

q is true

 $q \rightarrow p$ (proof by contraposition)

$$n = 2k$$
, $\exists k \in Z$

$$5n + 4 = 10k + 4$$

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

þ

 $p \rightarrow q$ (direct proof)

assume p is true

$$n = 2k + 1$$
, $\exists k \in Z$

$$5n + 4 = 10k + 9$$

$$5n + 4 = 2(5k + 4) + 1$$

$$5n + 4 = 2m + 1, \exists m \in Z$$

q is true

 $q \rightarrow p$ (proof by contraposition)

$$n = 2k$$
, $\exists k \in Z$

$$5n + 4 = 10k + 4$$

$$5n + 4 = 2(5k + 2)$$

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

p

 $p \rightarrow q$ (direct proof)

assume p is true

$$n = 2k + 1, \exists k \in Z$$

 $5n + 4 = 10k + 9$

$$5n + 4 = 2(5k + 4) + 1$$

$$5n + 4 = 2m + 1, \exists m \in Z$$

q is true

 $q \rightarrow p$ (proof by contraposition)

$$n=2k$$
, $\exists k \in Z$

$$5n + 4 = 10k + 4$$

$$5n + 4 = 2(5k + 2)$$

$$5n + 4 = 2m$$
, $\exists m \in Z$

<u>Proof of Equivalence</u> (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

$$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$$

n is odd integer if and only if 5n + 4 is odd integer

þ

 $p \rightarrow q$ (direct proof)

assume p is true

$$n = 2k + 1, \exists k \in \mathbb{Z}$$

 $5n + 4 = 10k + 9$
 $5n + 4 = 2(5k + 4) + 1$
 $5n + 4 = 2m + 1, \exists m \in \mathbb{Z}$

q is true

 $q \rightarrow p$ (proof by contraposition)

assume ~p is true

$$n = 2k, \exists k \in \mathbb{Z}$$

 $5n + 4 = 10k + 4$
 $5n + 4 = 2(5k + 2)$

$$5n+4=2m, \exists m \in Z$$

~q is true