COUNTING I

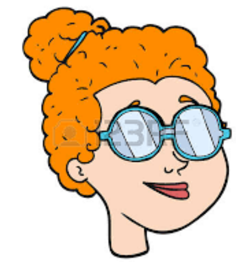
Murat Osmanoglu

Counting

- Ali and Buse eat lunch together in a specific restaurant regularly.

Counting

'soup' XOR 'sandwich'



'soup' AND 'sandwich'

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Counting

'soup' XOR 'sandwich'

'soup' AND 'sandwich'

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- restaurant has 9 choices for soup
  and 16 choices for sandwich

Counting

'soup' XOR 'sandwich'

'soup' AND 'sandwich'

- Ali and Buse eat lunch together in a specific restaurant regularly.
- restaurant has 9 choices for soup
  and 16 choices for sandwich
- How many different meals can Ali order ?
- How many different meals can Buse order ?

Counting

- Ali can order either one soup among 9 different soups or one sandwich among 16 different sandwiches.

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- apply the Sum Rule here : 9 + 16 = 25 different choices

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- If $A_1, A_2, \ldots, A_n$ are mutually disjoint sets ($A_i \cap A_j = \emptyset$), then the number of ways of choosing a single element from A_1 or A_2 or … A_n is

$$|A_1 \cup \ldots \cup A_n| = |A_1| + \ldots + |A_n|$$

Counting

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$$|A \times B| = |A| \cdot |B|$$

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Product Rule

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- Suppose that a task can be broken into a sequence of two small tasks. If there are n_1 ways to do the first task, and for each one there are n_2 ways to do the second task, then there are $n_1 \, . \, n_2$ different ways to finish the task

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Product Rule

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- If $A_1, A_2, \ldots, A_n$ are finite sets, then the number of ways of choosing an element from $A_1, \ldots,$ an element from A_n is

$$|A_1 \times \ldots \times A_n| = |A_1| \cdot |A_2| \cdot \ldots \cdot |A_n|$$

Counting

- How many bit-strings can you create with 3-digits ?

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  101, 001, 110, . . .

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___ ___ ___

Counting

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- What is the number of subsets of the set {a, b, c} ?

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  {a, b, c}

  {a, b}

  {c}

Counting

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$$\underline{\;2\;} \times \underline{\;2\;} \times \underline{\;2\;} = 8$$

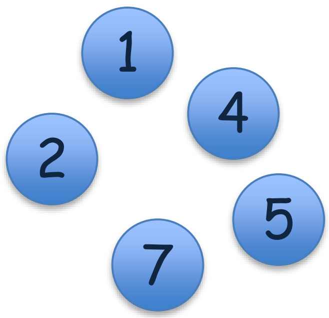
- What is the number of subsets of the set {a, b, c} ?

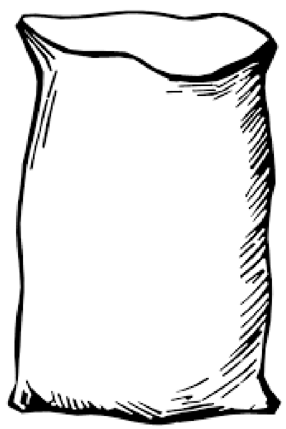
$\{a, b, c\} \longrightarrow 1\ 1\ 1$

$\{a, b\} \longrightarrow 1\ 1\ 0$

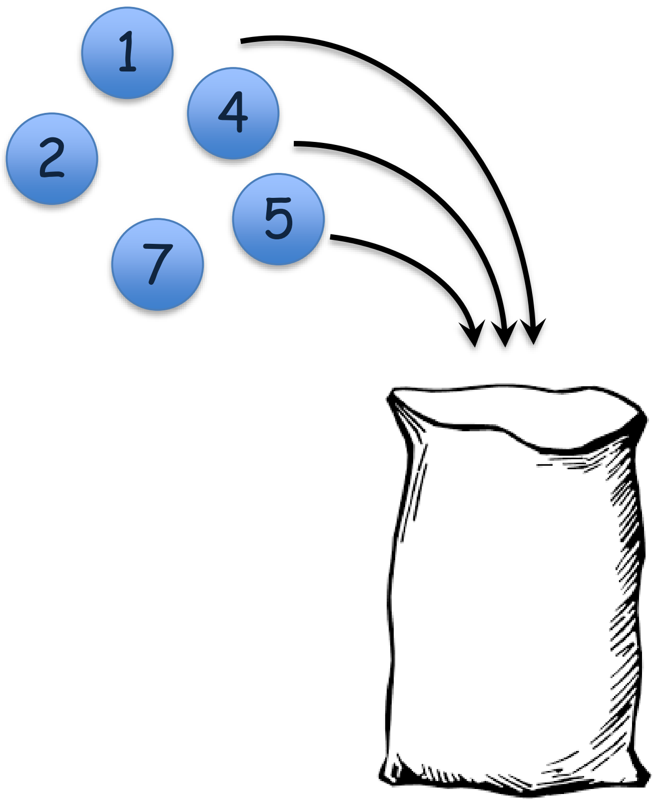
$\{c\} \longrightarrow 0\ 0\ 1$

Counting

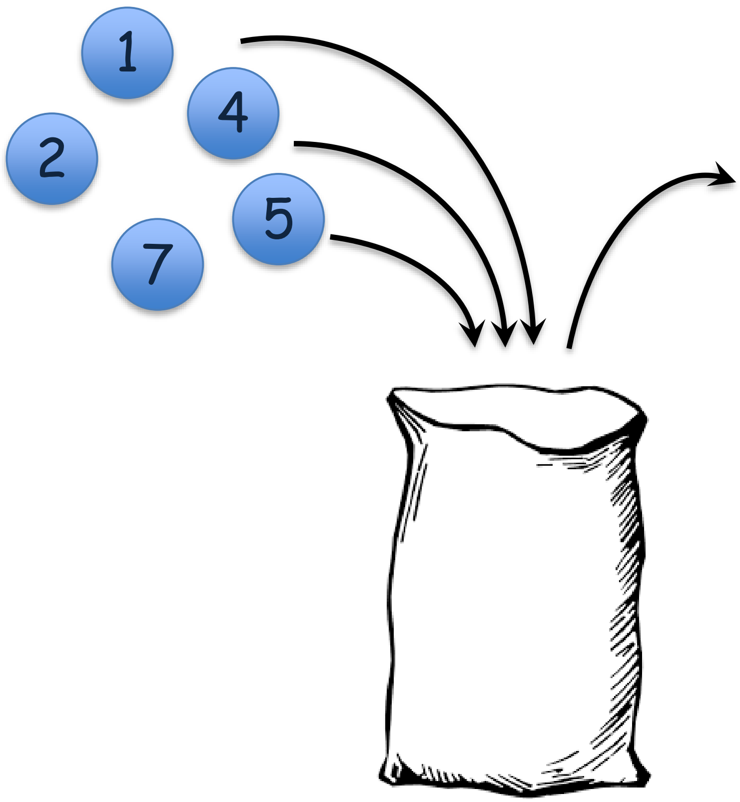




Counting

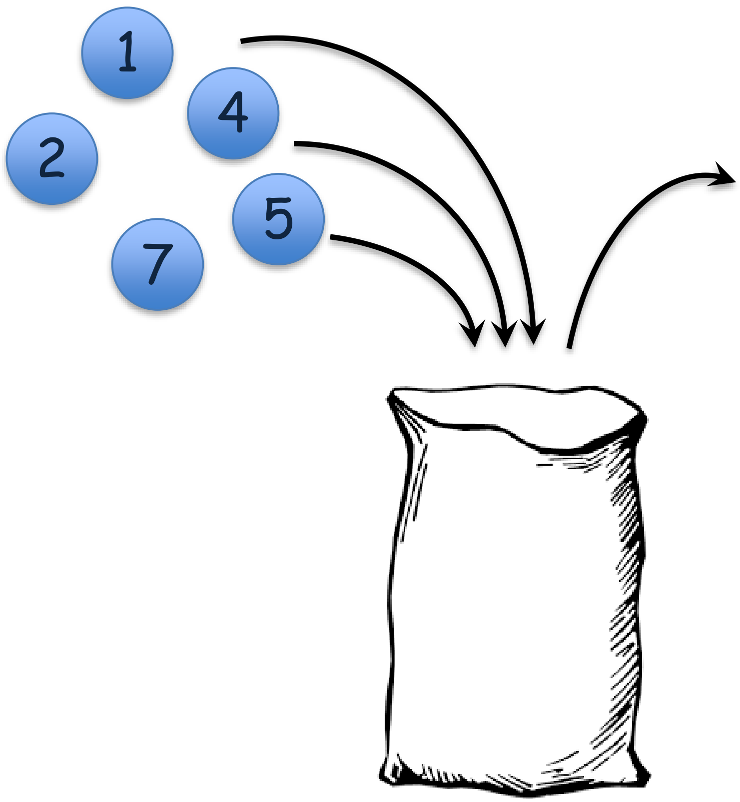


Counting



you pick one ball at a time

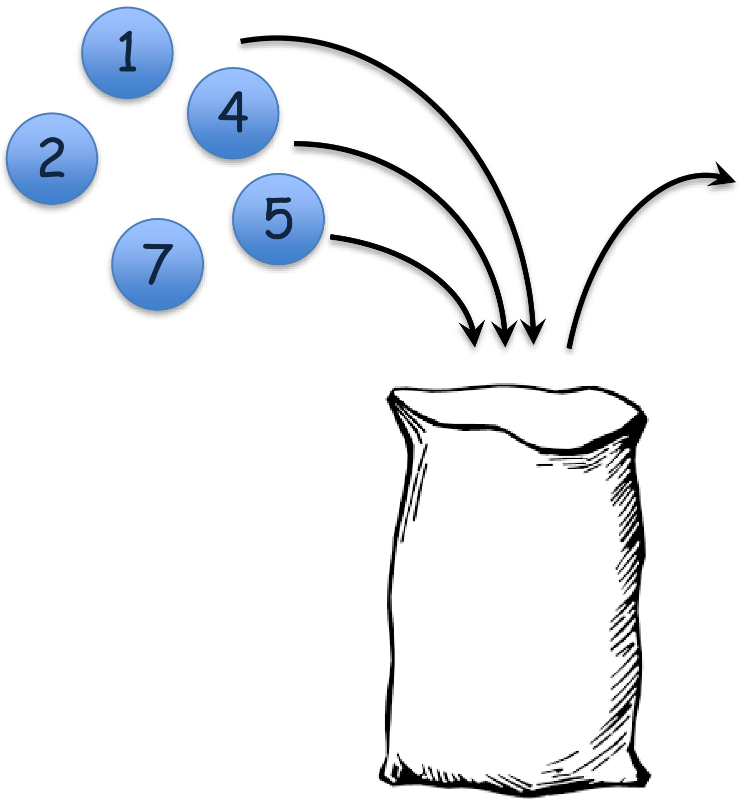
Counting



you pick one ball at a time

How many 3-digits numbers can you create with the picked numbers ?

Counting

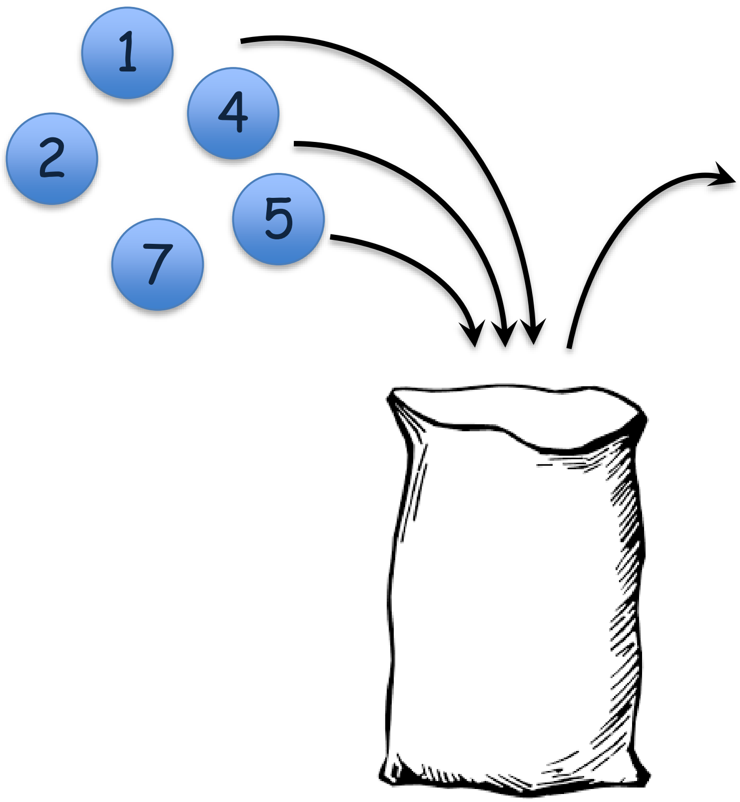


you pick one ball at a time

How many 3-digits numbers can you create with the picked numbers ?

- If you leave them to the bag after you pick

Counting



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How many 3-digits numbers can you create with the picked numbers ?

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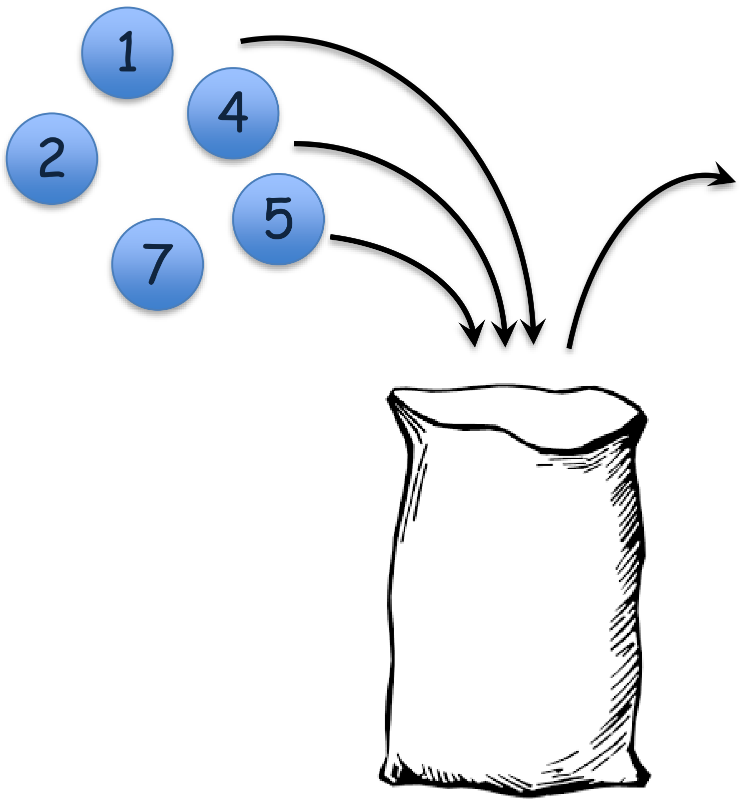

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___ ___ ___

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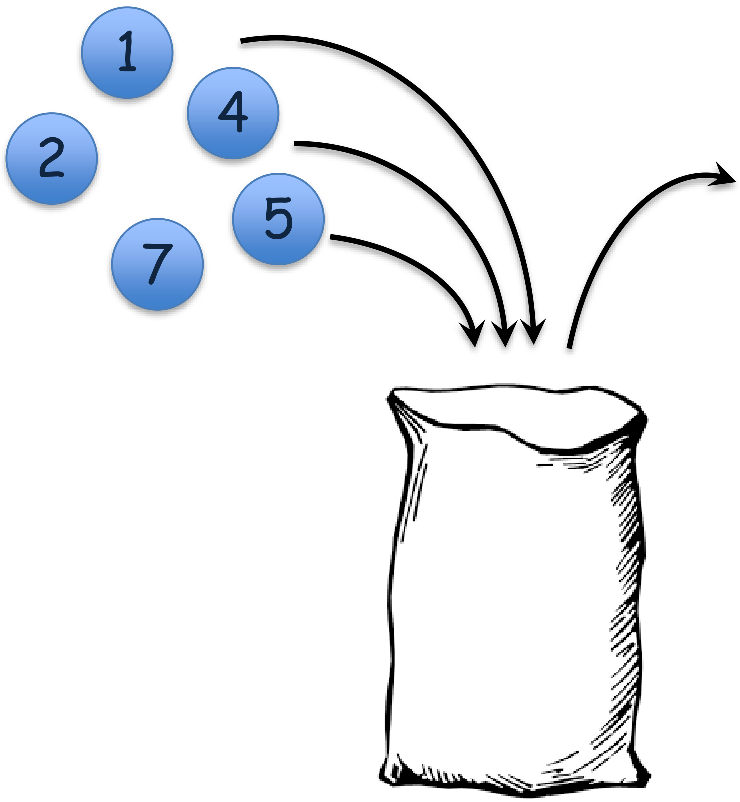
___ ___ ___

Counting



you pick one ball at a time

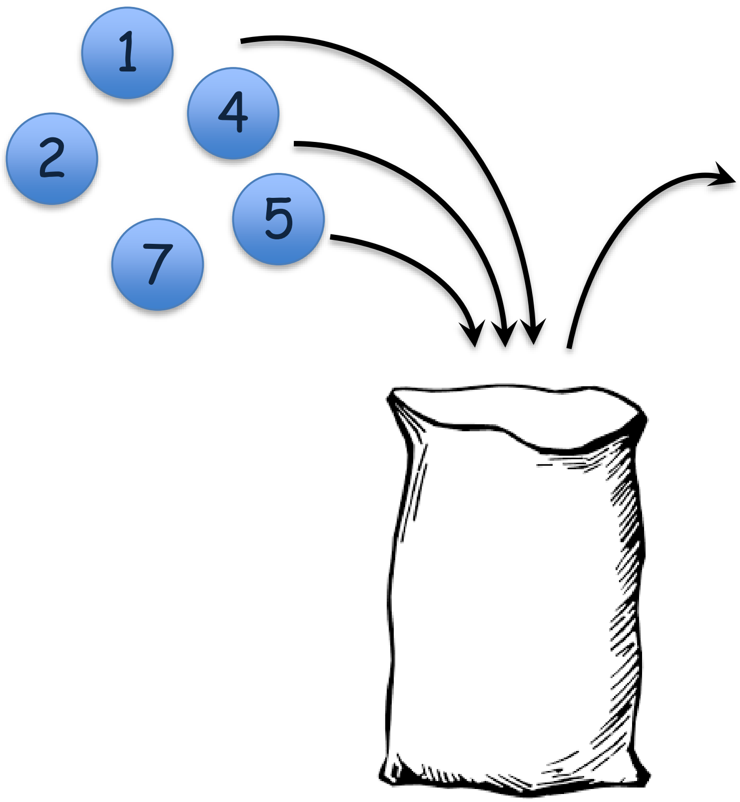
How many 3-digits numbers can you create with the picked numbers ?

- If you leave them to the bag after you pick

$$\underline{5} \times \underline{5} \times \underline{5} = 125$$

- If you keep them after you pick

Counting



you pick one ball at a time

How many 3-digits numbers can you create with the picked numbers ?

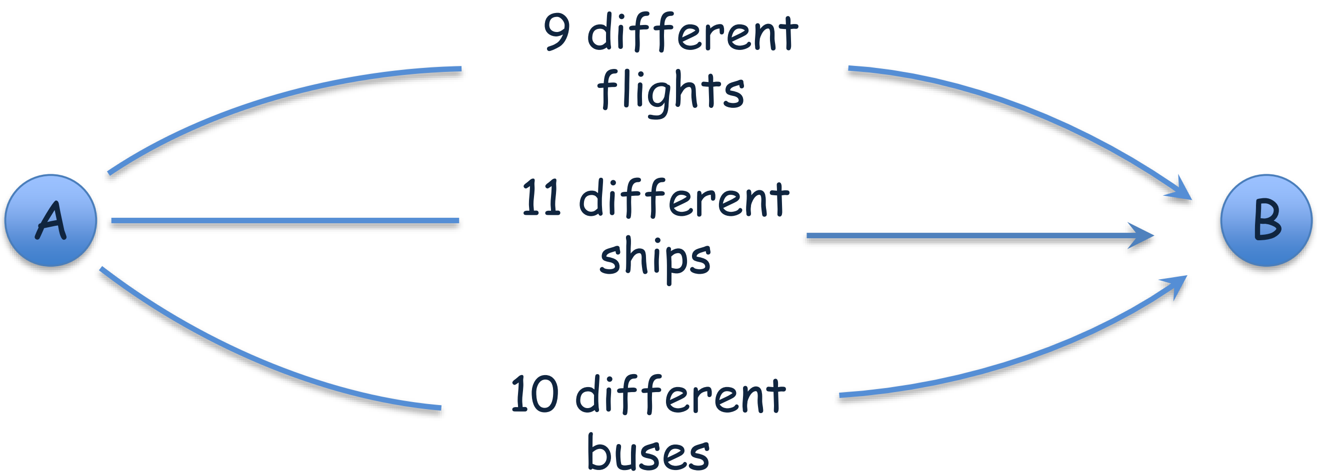
- If you leave them to the bag after you pick

$$\underline{\ 5\ } \times \underline{\ 5\ } \times \underline{\ 5\ } = 125$$

- If you keep them after you pick

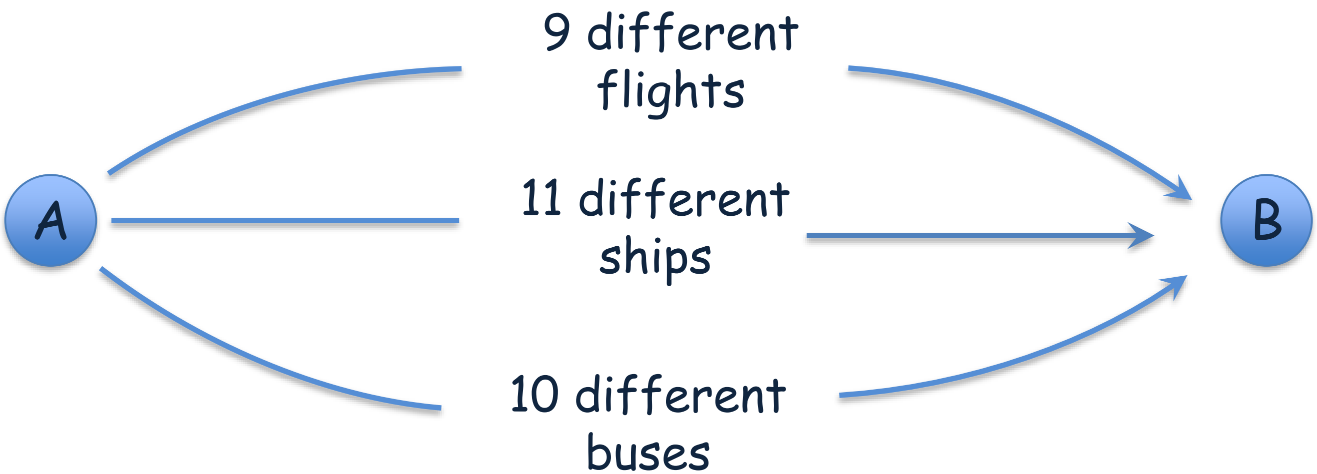
$$\underline{\ 5\ } \times \underline{\ 4\ } \times \underline{\ 3\ } = 60$$

Counting



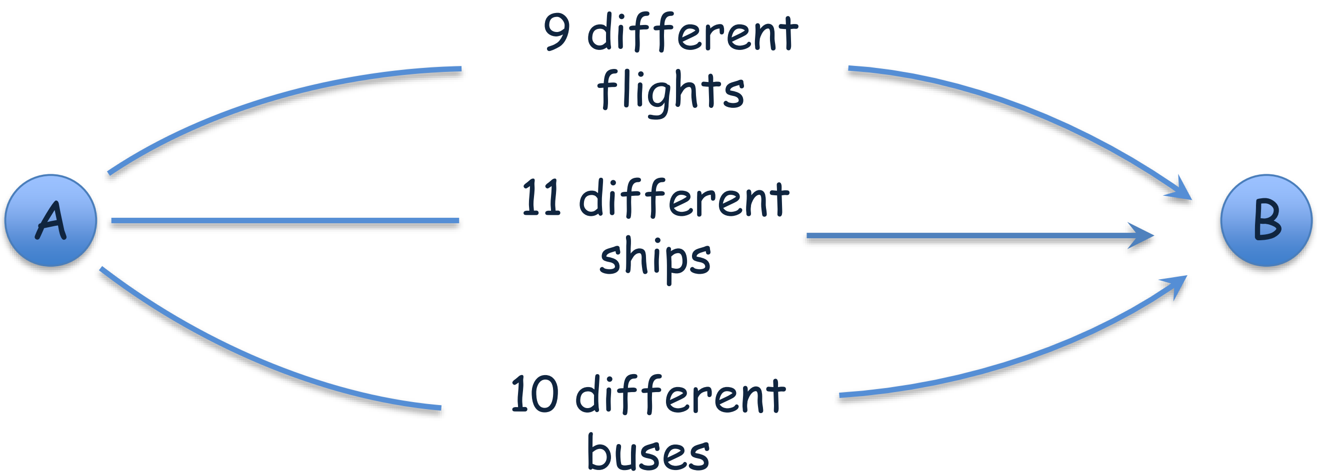
9 different flights
11 different ships
10 different buses
A
B

Counting



In how many different ways can you go from the city A to the city B ?

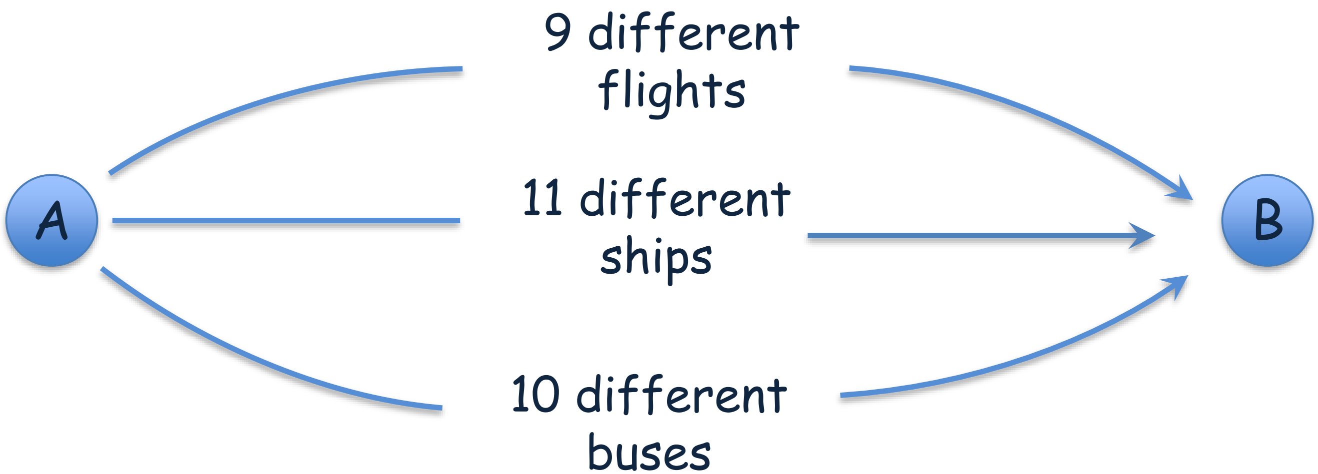
Counting



In how many different ways can you go from the city A to the city B ?

$$9 + 11 + 10 = 30$$

Counting

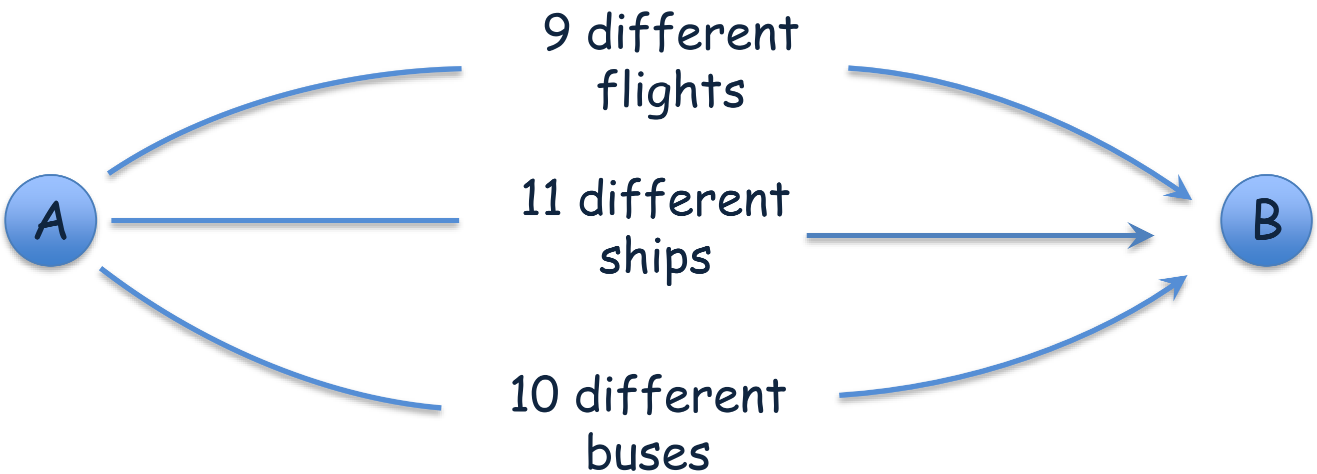


In how many different ways can you go from the city A to the city B ?

$$9 + 11 + 10 = 30$$

In how many different ways can you go to B and come back to A ?

Counting



In how many different ways can you go from the city A to the city B ?

$$9 + 11 + 10 = 30$$

In how many different ways can you go to B and come back to A ?

$$30 \times 30 = 900$$

Counting

- You prepare a meal for your friends

Counting

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- There are 5 kinds of bagels, 7 kinds of sandwiches, 6 drinks (hot coffee, hot tea, iced tea, cola, orange juice, apple juice)

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- There are 5 kinds of bagels, 7 kinds of sandwiches, 6 drinks (hot coffee, hot tea, iced tea, cola, orange juice, apple juice)
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How many different meals can you prepare?

(bagel, hot) or (sand, cold)

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(bagel, hot) or (sand, cold)

$$5 \times 2 \quad + \quad 7 \times 4 \;=\; 38$$

Counting

- How many different licence plates can be created if a plate contains of 2 digits indicating the city the car registered followed by 3 uppercase Turkish letters followed by 3 digits ?

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$$\underbrace{\_\_\_\_}_{\text{indicating the city}} \quad \underbrace{\_\_\_\_ \;\; \_\_\_\_ \;\; \_\_\_\_}_{\text{Turkish letters}} \quad \underbrace{\_\_\_\_ \;\; \_\_\_\_ \;\; \_\_\_\_}_{\text{numbers}}$$

Counting

- How many different licence plates can be created if a plate contains of 2 digits indicating the city the car registered followed by 3 uppercase Turkish letters followed by 3 digits ?

$$\underbrace{\underline{\quad 81 \quad}}_{\text{indicating the city}} \times \underbrace{\underline{\qquad}\ \ \underline{\qquad}\ \ \underline{\qquad}}_{\text{Turkish letters}} \quad \underbrace{\underline{\qquad}\ \ \underline{\qquad}\ \ \underline{\qquad}}_{\text{numbers}}$$

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$$\underbrace{\underline{81}}_{\text{indicating the city}} \times \underbrace{\underline{23} \times \underline{23} \times \underline{23}}_{\text{Turkish letters}} \times \underbrace{\underline{\quad} \;\; \underline{\quad} \;\; \underline{\quad}}_{\text{numbers}}$$

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$$\underbrace{\underline{81}}_{\text{indicating the city}} \times \underbrace{\underline{23} \times \underline{23} \times \underline{23}}_{\text{Turkish letters}} \times \underbrace{\underline{10} \times \underline{10} \times \underline{10}}_{\text{numbers}}$$

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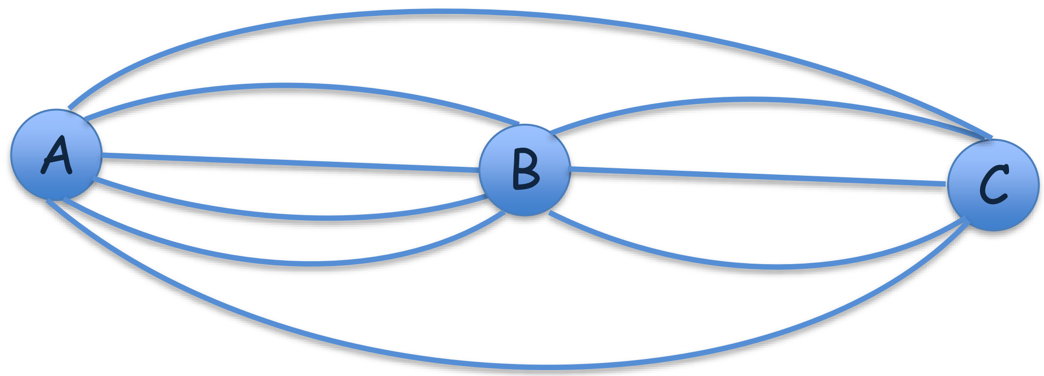
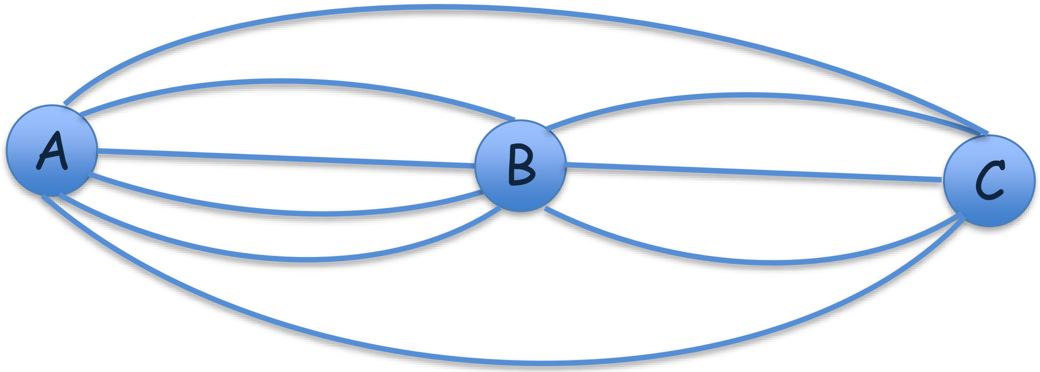
Counting

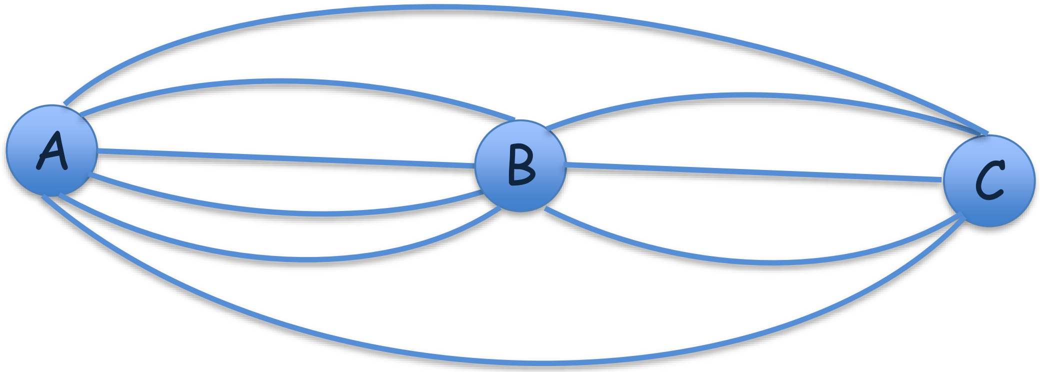


Counting



In how many different ways can you go from the city A to the city C ?

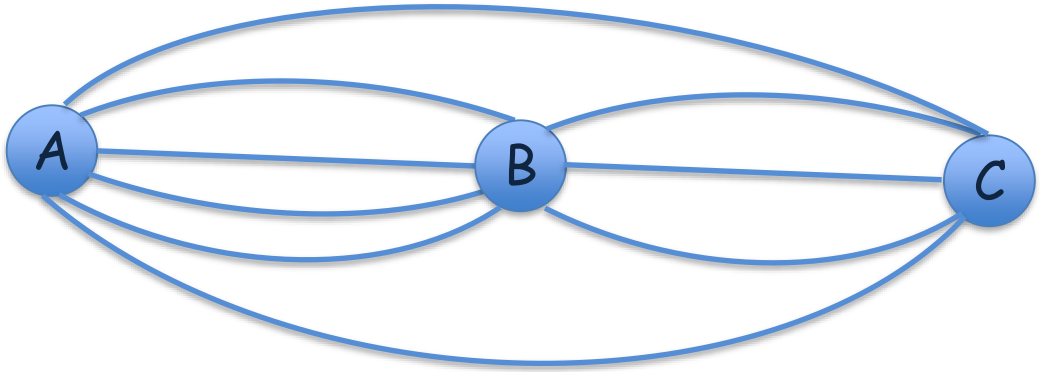
Counting



In how many different ways can you go from the city *A* to the city *C* ?

$$4 \times 3 + 2 = 14$$

Counting

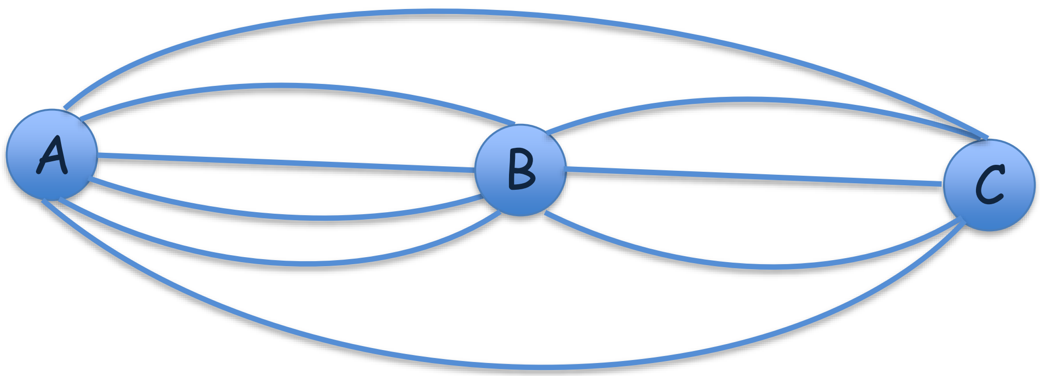


In how many different ways can you go from the city A to the city C ?

$$4 \times 3 + 2 = 14$$

In how many different ways can you go to C and come back to A ?

Counting



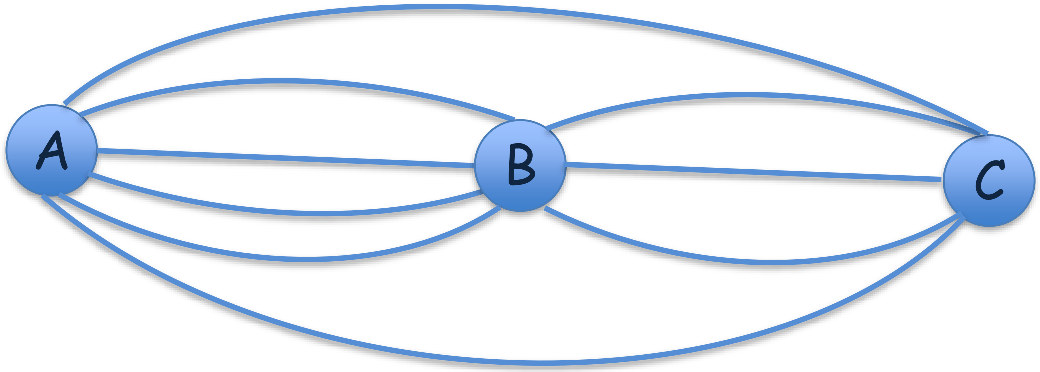
In how many different ways can you go from the city *A* to the city *C* ?

$$4 \times 3 + 2 = 14$$

In how many different ways can you go to *C* and come back to *A* ?

$$14 \times 14 = 196$$

Counting



In how many different ways can you go from the city A to the city C ?

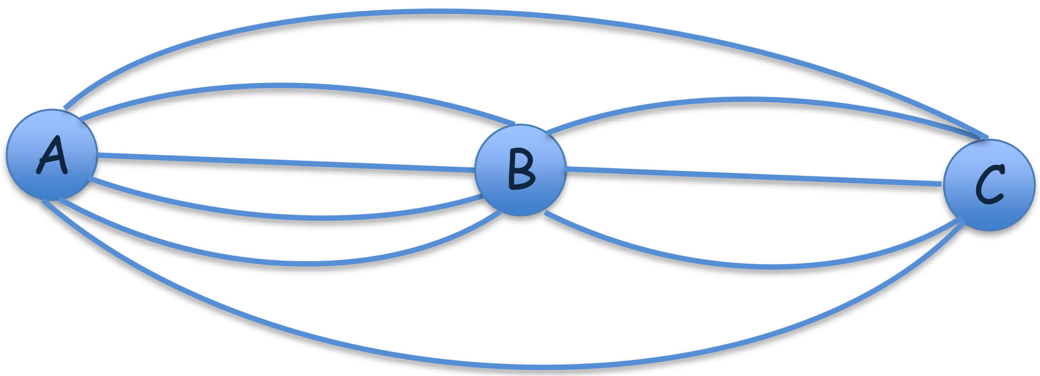
$$4 \times 3 + 2 = 14$$

In how many different ways can you go to C and come back to A ?

$$14 \times 14 = 196$$

In how many different ways can you go to C and come back to A so that you can use same route to come back?

Counting



In how many different ways can you go from the city A to the city C ?

$$4 \times 3 + 2 = 14$$

In how many different ways can you go to C and come back to A ?

$$14 \times 14 = 196$$

In how many different ways can you go to C and come back to A so that you can use same route to come back?

$$14 \times 13 = 182$$

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

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How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

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$$\underline{10}\times \underline{9}\times \underline{8}\times \underline{7}\times \underline{6} = 30240$$

Permutation

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- possible arrangements 13429, 60938, 19082, . . .

$$\underline{10}\times \underline{9}\times \underline{8}\times \underline{7}\times \underline{6} = 30240$$

How many different arrangements can we make for all students ?

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

$$\underline{10}\times \underline{9}\times \underline{8}\times \underline{7}\times \underline{6} = 30240$$

How many different arrangements can we make for all students ?

$$\underline{10}\times \underline{9}\times \underline{8}\times \underline{7}\times \ldots \times \underline{1} = 3628800$$

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

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- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
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$$10 \times 9 \times 8 \times 7 \times 6$$

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

$$10 \times 9 \times 8 \times 7 \times 6 \; \frac{\times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$$

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- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

$$10 \times 9 \times 8 \times 7 \times 6 \, \frac{\times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = \frac{10!}{5!}$$

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

$$10 \times 9 \times 8 \times 7 \times 6 \ \frac{\times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = \frac{10!}{5!}$$

- the number of different permutation of size 5 for 10 objects

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

$$10 \times 9 \times 8 \times 7 \times 6 \; \frac{\times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = \frac{10!}{5!}$$

- the number of different permutation of size 5 for 10 objects
- the number of different permutation of size r for n objects

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

$$10 \times 9 \times 8 \times 7 \times 6 \; \frac{\times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = \frac{10!}{5!}$$

- the number of different permutation of size 5 for 10 objects
- the number of different permutation of size r for n objects

$$P(n,r) = \frac{n!}{(n-r)!}$$

Permutation

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$$8 \times 8 \times 8 \times 8 \times 8 = 32768$$

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$5!\ /\ 3! = 20$

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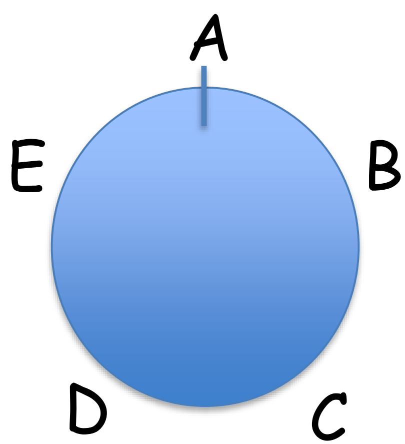

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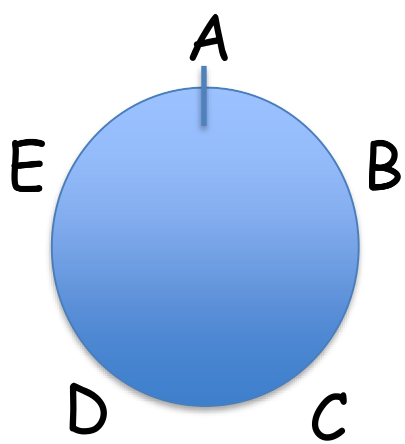

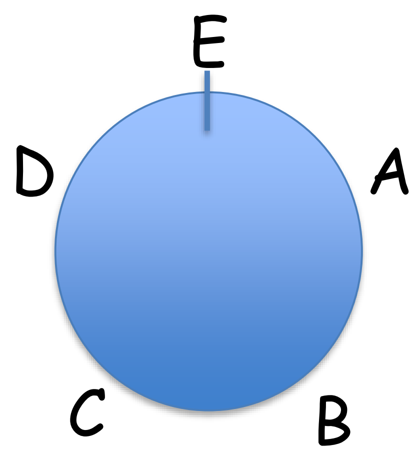

A B C D E
E A B C D

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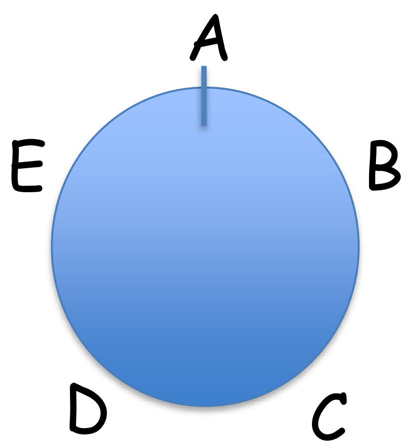

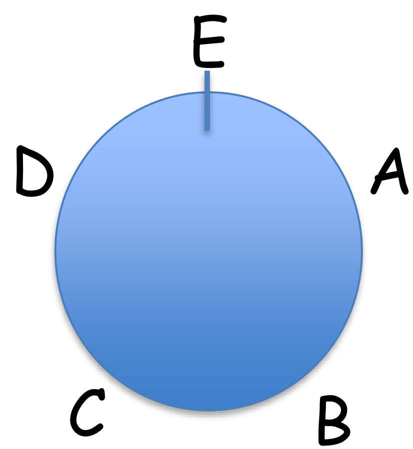

A	B	C	D	E
E	A	B	C	D
D	E	A	B	C
C	D	E	A	B
B	C	D	E	A

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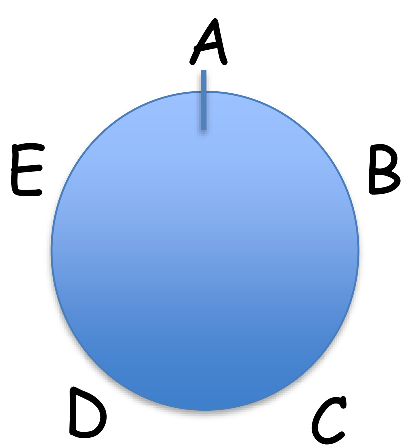

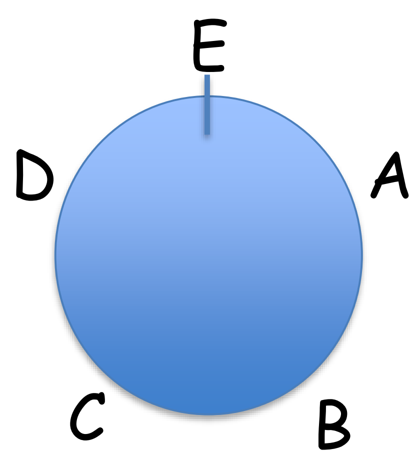

A	B	C	D	E
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D	E	A	B	C
C	D	E	A	B
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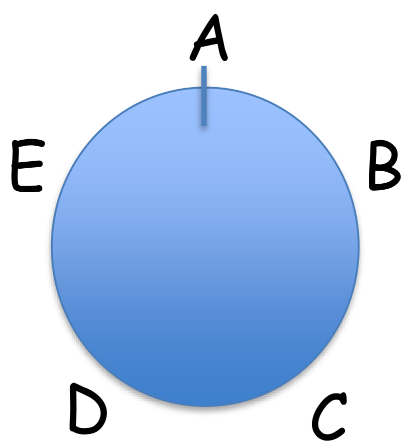

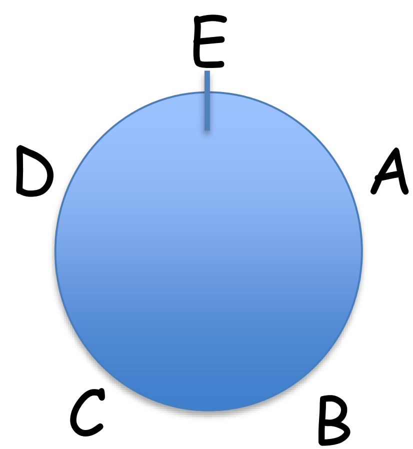

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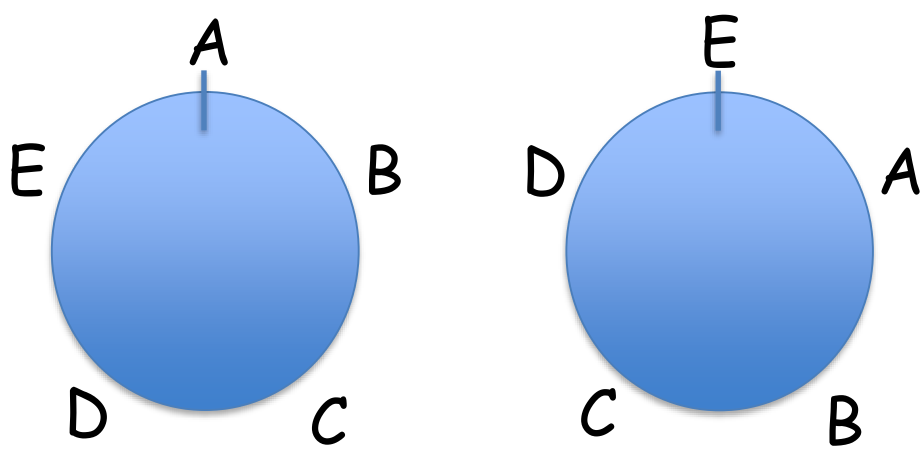

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5 x (# of circular) = (# of linear)

(# of circular) = 5! / 5 = 24

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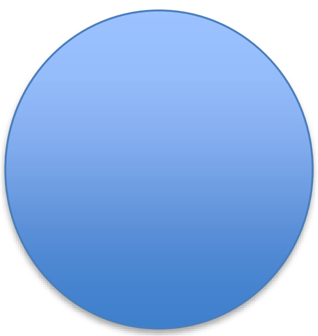

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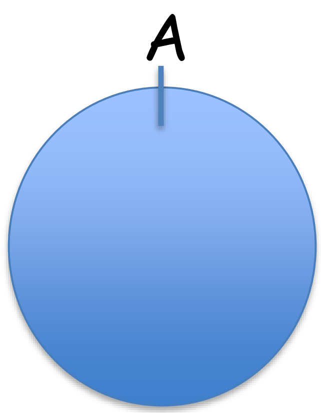

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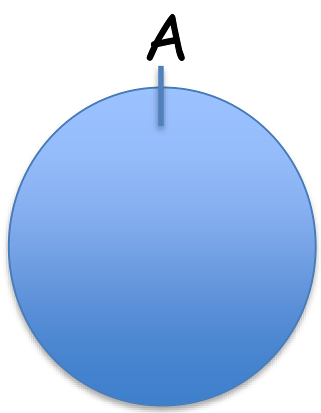

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- 4!

Permutation

- You invite 2 couples for the dinner (3 couples at total)

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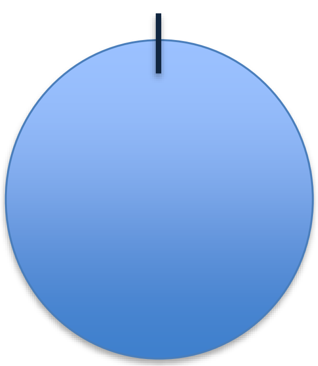

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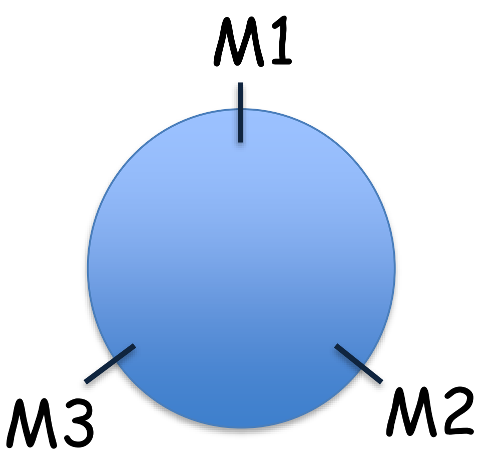

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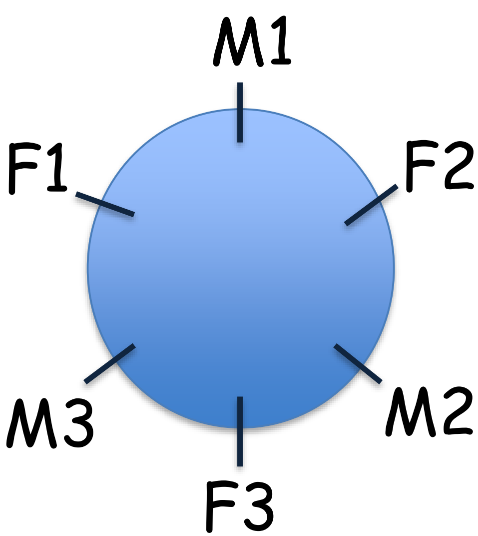

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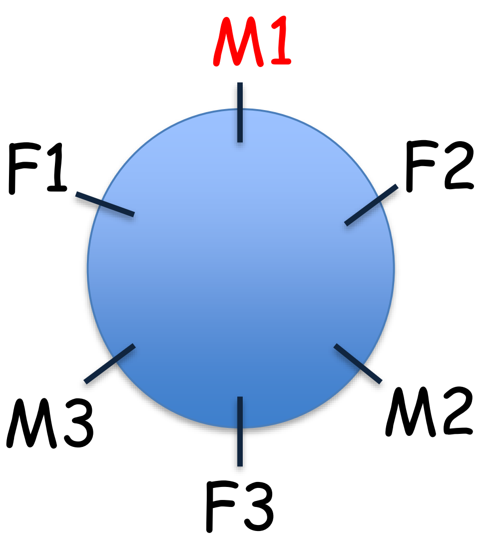

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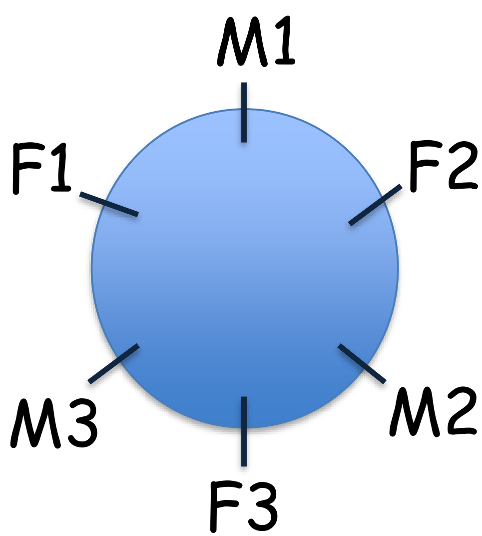

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- $3! \times 2! = 12$

Combinations

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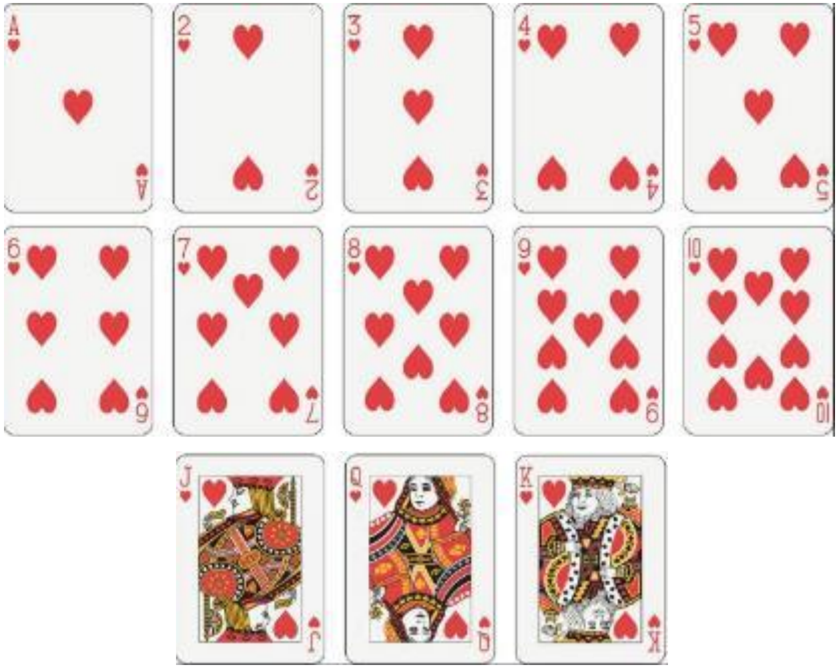

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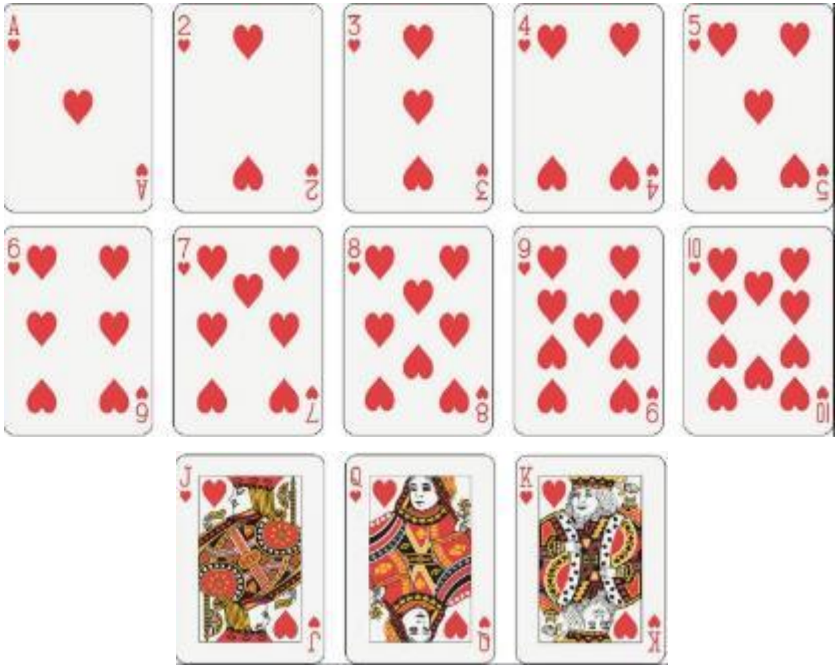

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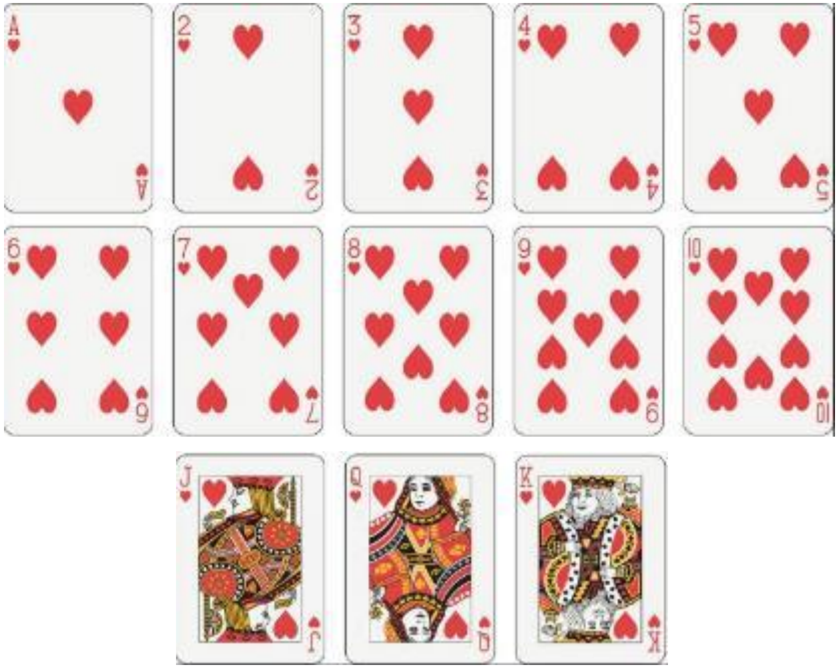

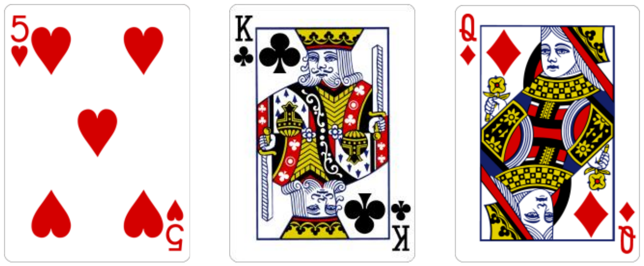

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$$P(52,3) = 52! / (52-3)!$$

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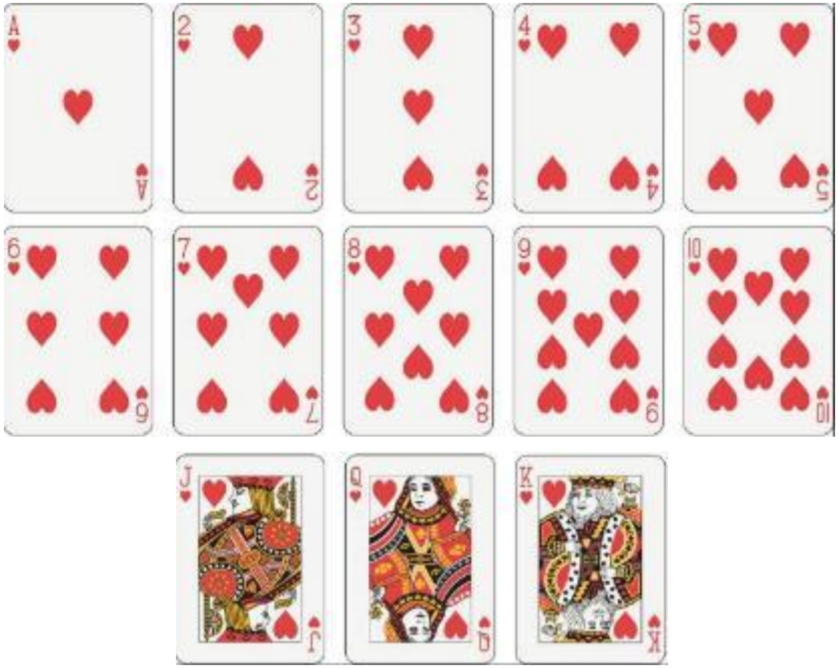

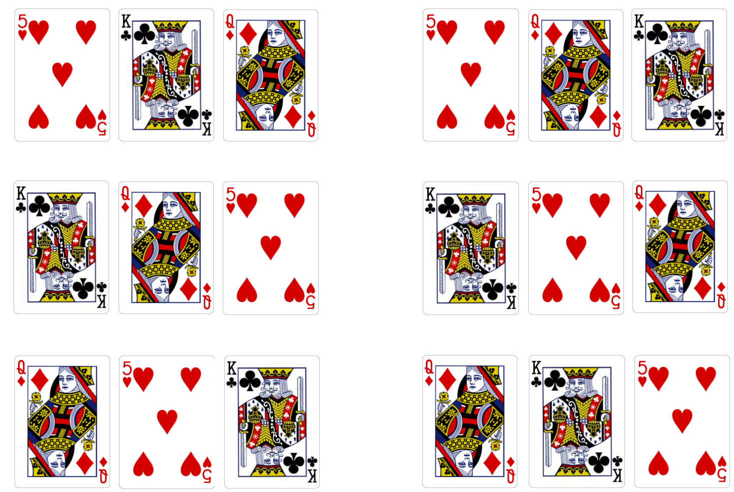

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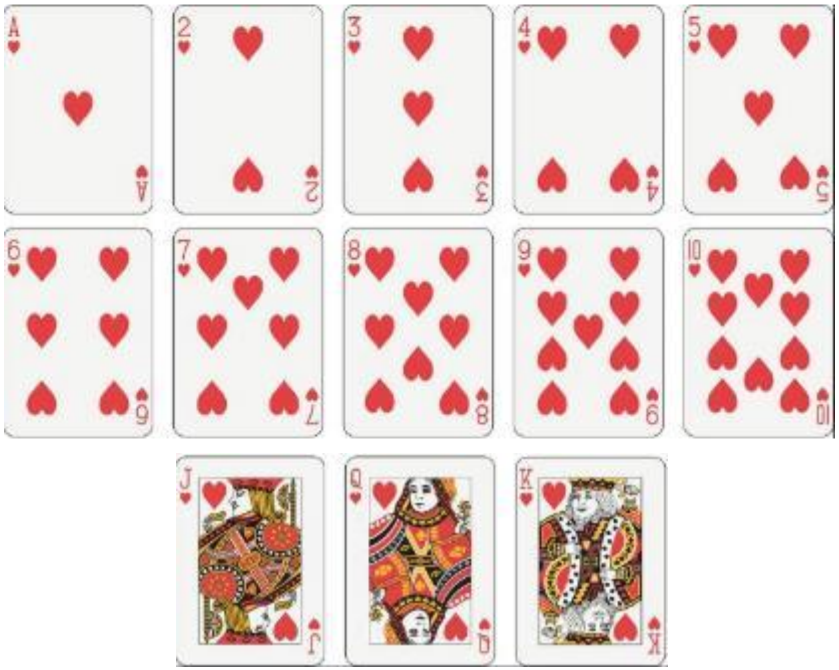

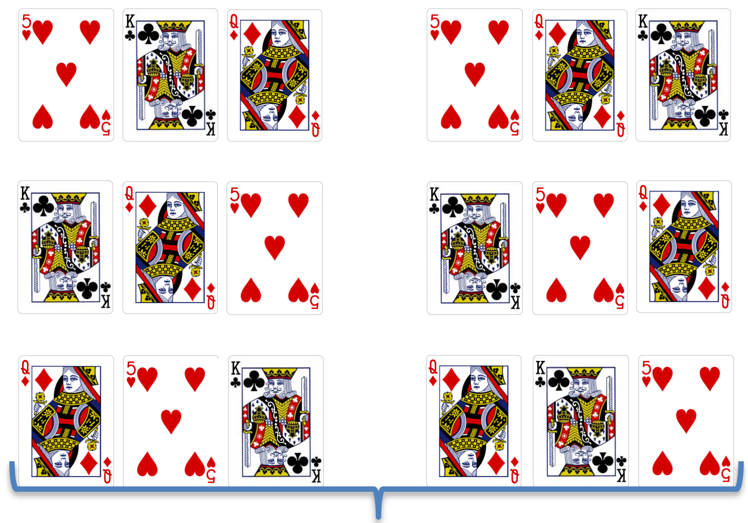

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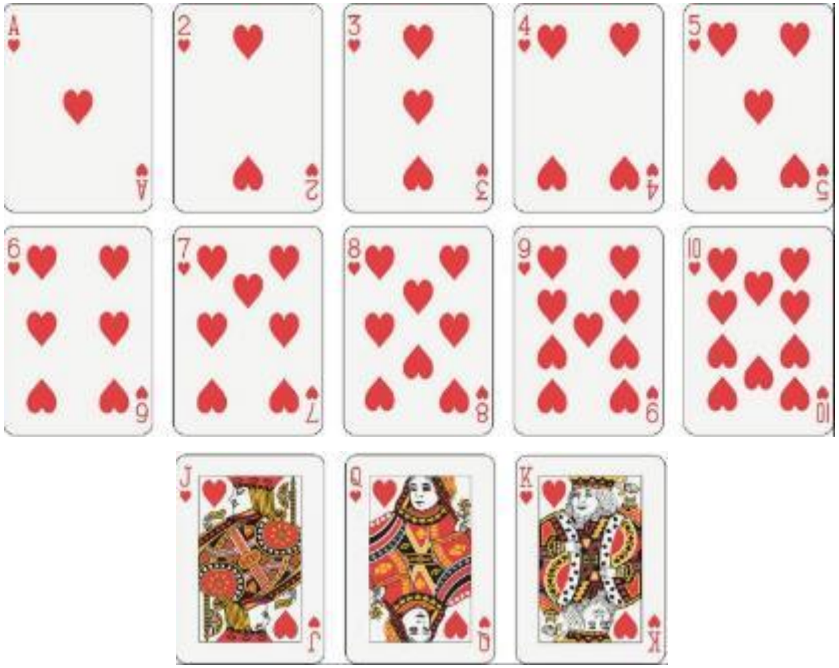

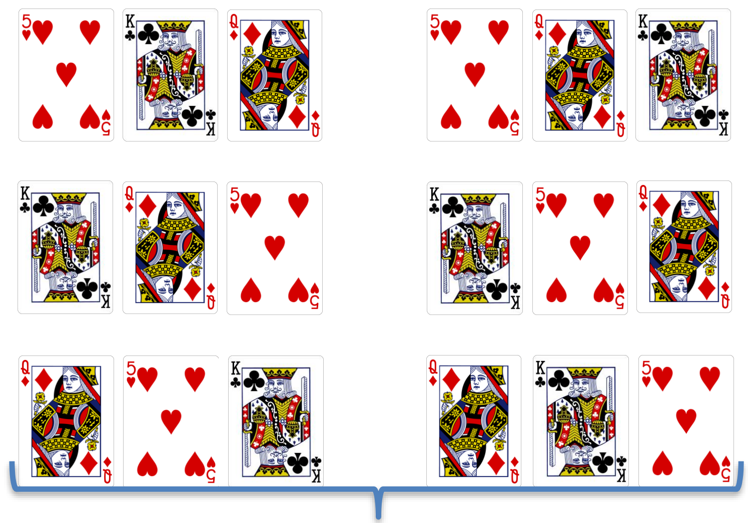

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$52! / [(52-3)! . 3!]$

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$$\binom{5}{4} \text{ x } \binom{5}{3} = \frac{5!}{1!.4!} \text{ x } \frac{5!}{2!.3!} = 50$$

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Combinations

- We are forming soccer teams with 6 players for a tournament from 30 boys and 25 girls. How many different 4 teams (2 for girls and 2 for boys) can you create ?

$$\binom{30}{6}\binom{24}{6}\binom{25}{6}\binom{19}{6}$$

- Suppose you are playing a game with five cards.

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Combinations

$\Sigma = \{0, 1\}$.
Let's use this alphabet to create three digits encoding:

000, 010, 111, 011, . . .

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Assume $x = x_1 x_2 \ldots x_n$, the weight of a given encoding is

$$w(x) = x_1 + x_2 + \ldots + x_n$$

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$$\binom{8}{0} + \binom{8}{2} + \ldots + \binom{8}{8}$$

Combinations

$\Sigma = \{0, 1, 2\}$.
Let's use this alphabet to create three digits encoding:

020, 010, 112, 211, . . .

- How many encodings of length 8 can you create using the alphabet Σ ?

- How many of them contains exactly three 1's ?

Assume $x = x_1x_2...x_n$, the weight of a given encoding is

$$w(x) = x_1 + x_2 + ... + x_n$$

- How many encodings of length 8 have even weight ?

Combinations

$\Sigma = \{0, 1, 2\}$.
Let's use this alphabet to create three digits encoding:

020, 010, 112, 211, . . .

- How many encodings of length 8 can you create using the alphabet Σ ?

$$3 \times 3 \times \ldots \times 3 = 3^8$$

- How many of them contains exactly three 1's ?

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$$w(x) = x_1 + x_2 + \ldots + x_n$$

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$$\binom{8}{0}2^8 + \ldots + \binom{8}{8}2^0$$

Binomial Theorem

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Let x and y be variables and n be non-negative integer, then

$$(x+y)^n$$

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Let x and y be variables and n be non-negative integer, then

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- What is the coefficient of x^5y^2 in the expansion of $(x+y)^7$?

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$$\binom{n}{i} x^{n-i}y^i \Rightarrow \mathrm{n} = 7 \text{ and } i = 2$$

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- What is the coefficient of $x^{10}y^{15}$ in the expansion of $(2x-3y)^{25}$?

Binomial Theorem

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  $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Let x and y be variables and n be non-negative integer, then

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \ldots + \binom{n}{n} y^n$$

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- What is the coefficient of $x^{10}y^{15}$ in the expansion of $(2x-3y)^{25}$?

$$\binom{n}{i} x^{n-i} y^i \ \Rightarrow \mathrm{n} = 25 \text{ and } i = 15 \Rightarrow \binom{25}{15} (2x)^{10} (-3y)^{15}$$

Binomial Theorem

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$$\Rightarrow \binom{25}{15} 2^{10} (-3)^{15} x^{10} y^{15}$$

Binomial Theorem

Let x and y be variables and n be non-negative integer, then

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \ldots + \binom{n}{n} y^n$$

- What is the coefficient of ${x_1}^{n_1} {x_2}^{n_2} \ldots {x_k}^{n_k}$ in the expansion of $(x_1 + x_2 + \cdots + x_k)^n$?

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$$\binom{n}{n_1}$$

Binomial Theorem

Let x and y be variables and n be non-negative integer, then

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$$\binom{n}{n_1} \cdot \binom{n-n_1}{n_2}$$

Binomial Theorem

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Binomial Theorem

Let x and y be variables and n be non-negative integer, then

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Binomial Theorem

Let x and y be variables and n be non-negative integer, then

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \ldots + \binom{n}{n} y^n$$

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- What is the coefficient of $x^3 y^2 z^2$ in the expansion of $(x+y+z)^7$?

Binomial Theorem

Let x and y be variables and n be non-negative integer, then

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- What is the coefficient of ${x_1}^{n_1} {x_2}^{n_2} \ldots {x_k}^{n_k}$ in the expansion of $(x_1 + x_2 + \cdots + x_k)^n$?

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- What is the coefficient of $x^3 y^2 z^2$ in the expansion of $(x+y+z)^7$?

$$\frac{7!}{3!\,2!\,2!} = 210$$

Binomial Theorem

- Prove that $\sum_{i=0}^{n} \binom{n}{i} = 2^n$

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Suppose there is a set A={1, 2, . . ., n}.

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Suppose there is a set A={1, 2, . . ., n}. Let's write the elements of the power set P(A):

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∅

{1}, {2}, {3}, . . ., {n}

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Suppose there is a set A={1, 2, . . ., n}. Let's write the elements of the power set P(A):

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{1}, {2}, {3}, . . ., {n}

{1, 2}, {1, 3}, {1, 4}, . . ., {n-1, n}

Binomial Theorem

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{1}, {2}, {3}, . . ., {n}

{1, 2}, {1, 3}, {1, 4}, . . ., {n-1, n}

{1, 2, 3}, {1, 2, 4}, . . ., {n-2, n-1, n}

. . .

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Binomial Theorem

- Prove that $\sum_{i=0}^{n} \binom{n}{i} = 2^n$

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There is a set A such that |A|=n+1
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A={ . . ., a, . . . }

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create subsets of
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$$
\begin{array}{ccccccc}
 & & & \binom{0}{0} & & & \\
 & & \binom{1}{0} & & \binom{1}{1} & & \\
 & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\
\binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3}
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1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

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Prove that $\sum_{k=1}^{n} \binom{k}{1} = \binom{n+1}{2}$

$$= \binom{2}{2} + \binom{2}{1} + \binom{3}{1} + \ldots + \binom{n-1}{1} + \binom{n}{1}$$

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