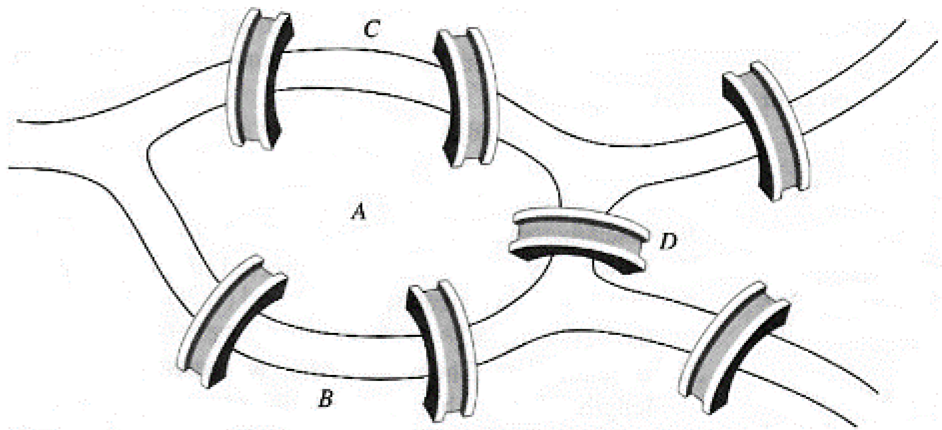
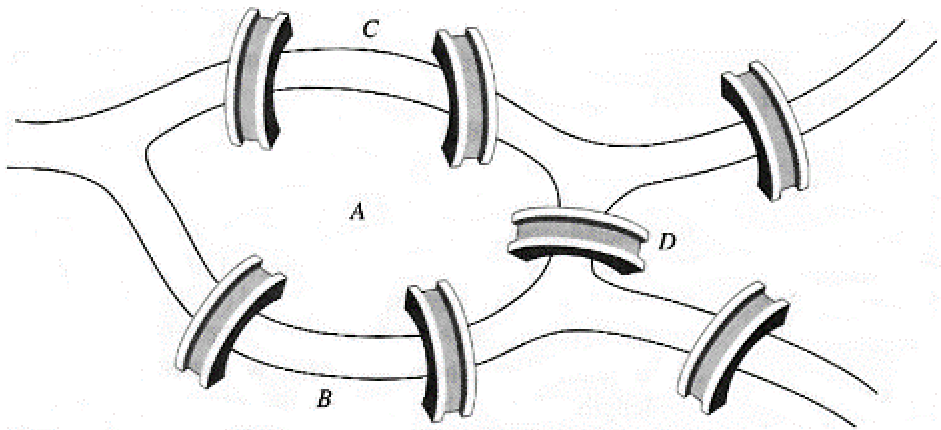
Graphs

Graph Theory

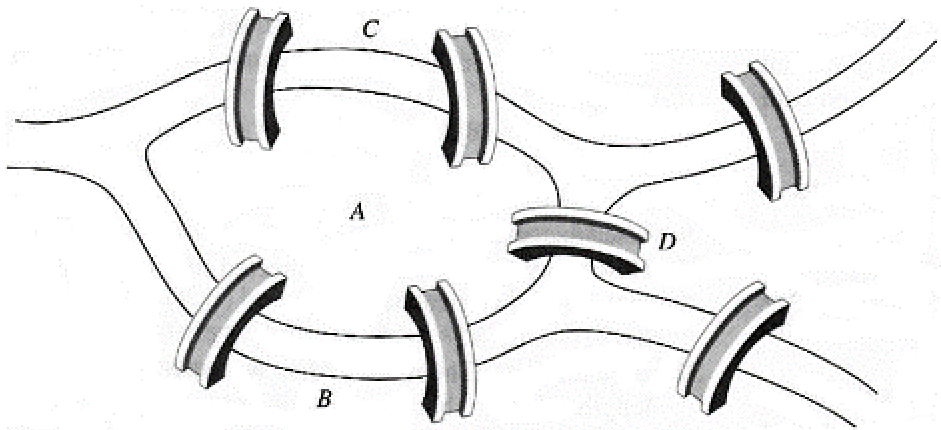


Graph Theory



- Königsberg was a city in Germany in 18th century. There was a river named Pregel that divided the city into four distinct regions.

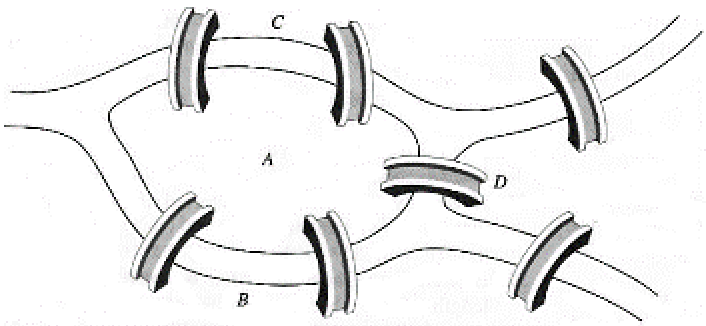
Graph Theory



- Königsberg was a city in Germany in 18th century. There was a river named Pregel that divided the city into four distinct regions.
- There was a natural question for the people of Königberg :

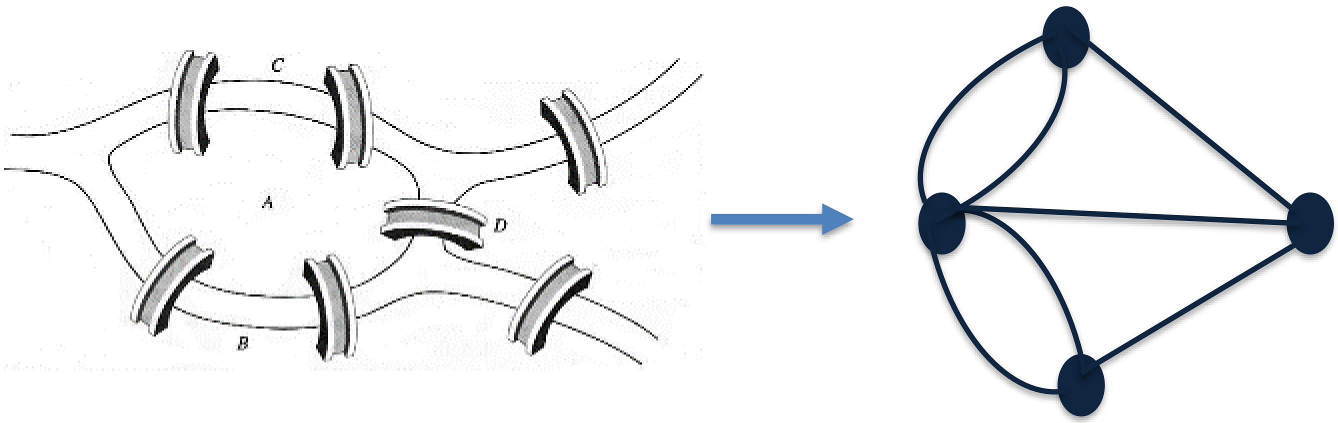
'Is it possible to take a walk around the city that crosses each bridge exaactly once?'

Graph Theory



- The problem was solved by Swiss mathematician Leonard Euler. His works are considered as the beginning of Graph Theory.

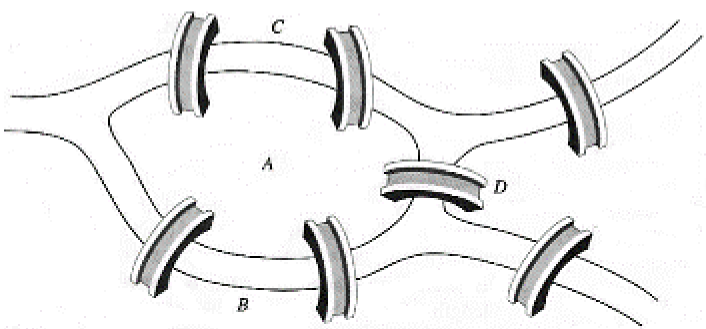
Graph Theory

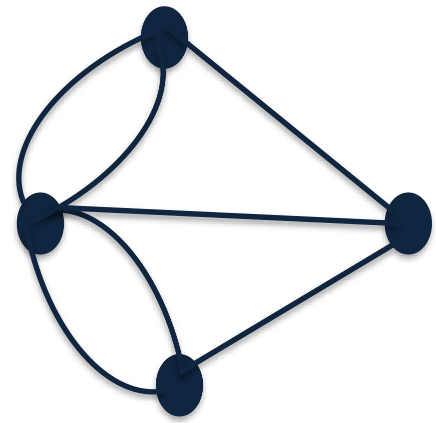


- The problem was solved by Swiss mathematician Leonard Euler. His works are considered as the beginning of Graph Theory.

- Euler represented four distinct lands with four points (or nodes), and seven bridges with seven lines connecting those points.

Graph Theory





- The problem was solved by Swiss mathematician Leonard Euler. His works are considered as the beginning of Graph Theory.

- Euler represented four distinct lands with four points (or nodes), and seven bridges with seven lines connecting those points.

'Can you find a path that includes every edge exactly once?'
'Is the given graph traversable?'

Graph Theory

$$G = (V, E)$$

Graph Theory

$$G = (V, E)$$

set of nodes (or vertices)

set of edges (or arc)

Graph Theory

$G = (V, E)$

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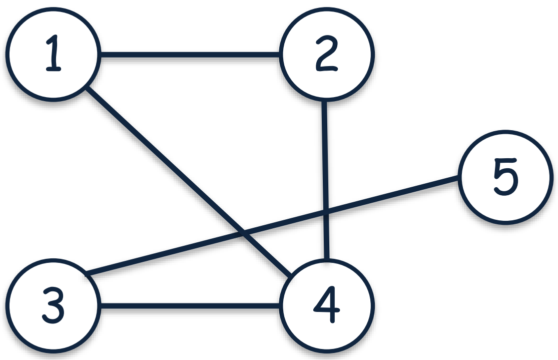
set of edges (or arc)



Graph Theory

$$G = (V, E)$$

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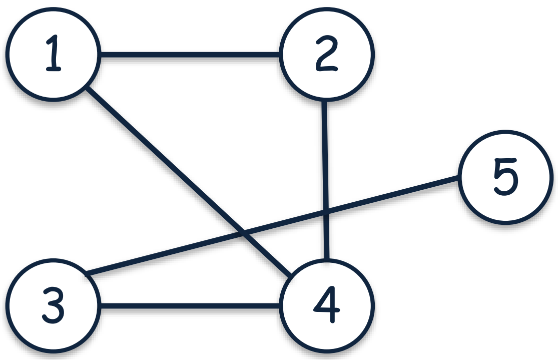

- $V = \{1, 2, 3, 4, 5\}$

Graph Theory

$$G = (V, E)$$

set of nodes (or vertices)

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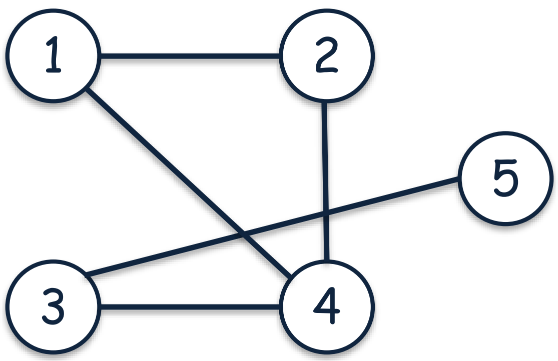
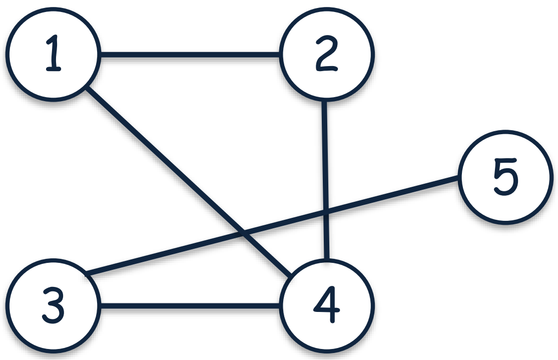


- $V = \{1, 2, 3, 4, 5\}$
- $E \subseteq V \times V$

Graph Theory

$$G = (V, E)$$

set of nodes (or vertices) set of edges (or arc)



- $V = \{1, 2, 3, 4, 5\}$
- $E \subseteq V \times V$

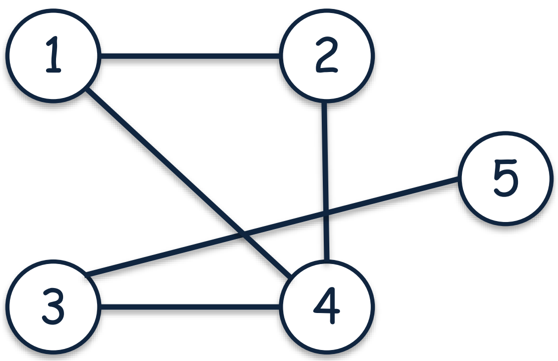
$$(1, 2) \in E$$

Graph Theory

$$G = (V, E)$$

set of nodes (or vertices)

set of edges (or arc)



- $V = \{1, 2, 3, 4, 5\}$
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starting node

ending node

Graph Theory

$G = (V, E)$

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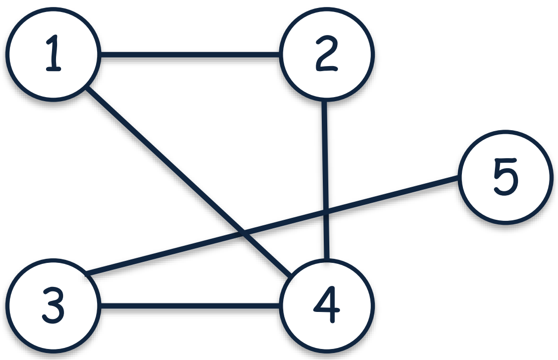


- $V = \{1, 2, 3, 4, 5\}$
- $E \subseteq V \times V$

$(1, 2) \in E$

starting node

ending node

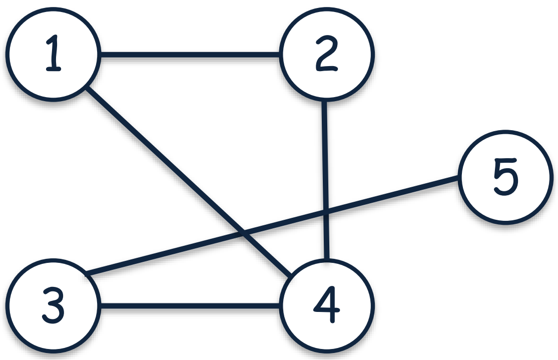
- $E = \{(1, 2), (2, 4), (4, 3), (1, 4), (3, 5)$
  $(2, 1), (4, 2), (3, 4), (4, 1), (5, 3)\}$

Graph Theory

$$G = (V, E)$$

set of nodes (or vertices)

set of edges (or arc)



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ending node

- $E = \{(1,2), (2,4), (4,3), (1,4), (3,5)$
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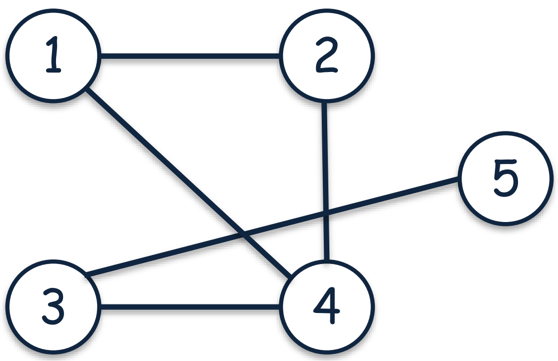
- If $(1,2) \in E$, 1 and 2 are adjacent vertices.

Graph Theory

G = (V, E)

set of nodes (or vertices)

set of edges (or arc)



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$(1, 2) \in E$

starting node

ending node

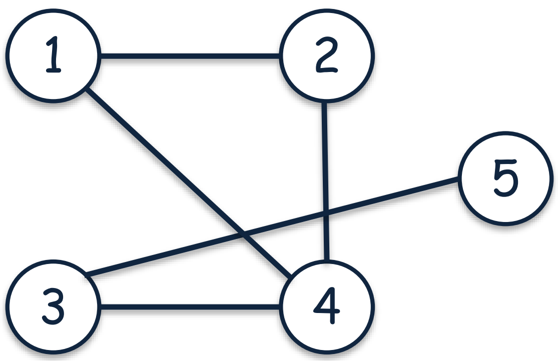
- $E = \{(1, 2), (2, 4), (4, 3), (1, 4), (3, 5)$
  $(2, 1), (4, 2), (3, 4), (4, 1), (5, 3)\}$
- If $(1, 2) \in E$, 1 and 2 are adjacent vertices.
- adj(4) = {1, 2, 3}

Graph Theory

$$G = (V, E)$$

set of nodes (or vertices)

set of edges (or arc)



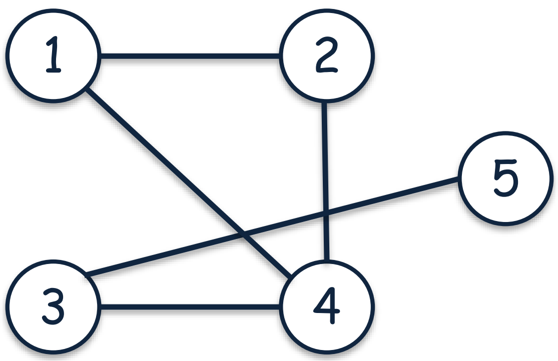
undirected graph

Graph Theory

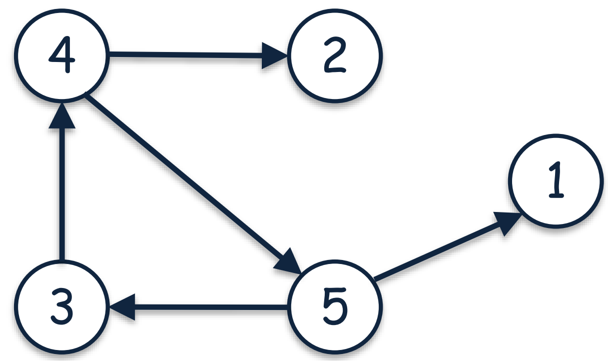
$$G = (V, E)$$

set of nodes (or vertices)

set of edges (or arc)



undirected graph



directed graph

Graph Theory

$$G = (V, E)$$

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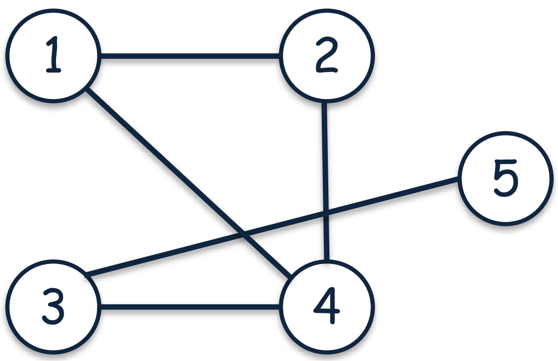
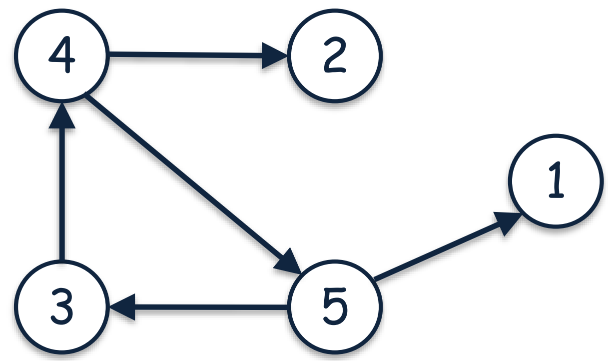


undirected graph



directed graph

deg(v)= # of edges at that vertex

Graph Theory

$$G = (V, E)$$

set of nodes (or vertices)

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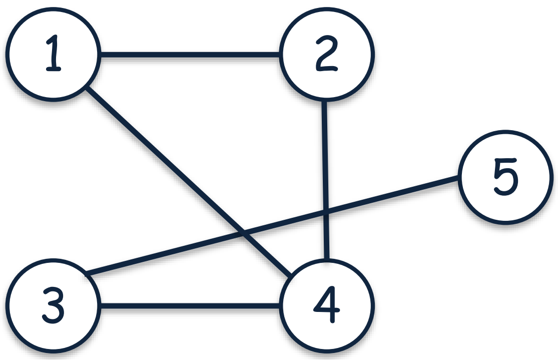
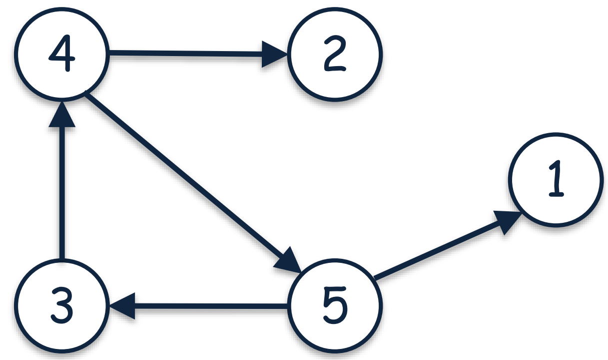


undirected graph



directed graph

deg(v)= # of edges at that vertex

deg(1)=2

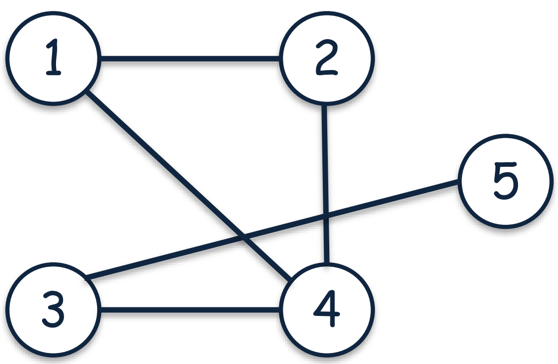
deg(4)=3

Graph Theory

$$G = (V, E)$$

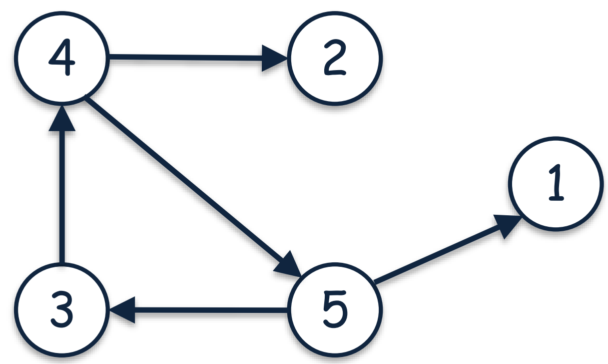
set of nodes (or vertices)

set of edges (or arc)



undirected graph

$deg(v)$= # of edges at that vertex



directed graph

$deg^{in}(v)$ = # of incoming edges

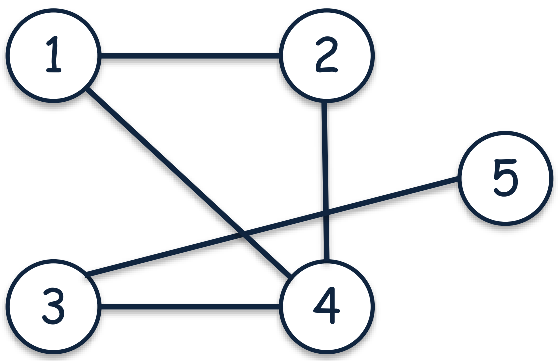
$deg^{out}(v)$ = # of outgoing edges

Graph Theory

$$G = (V, E)$$

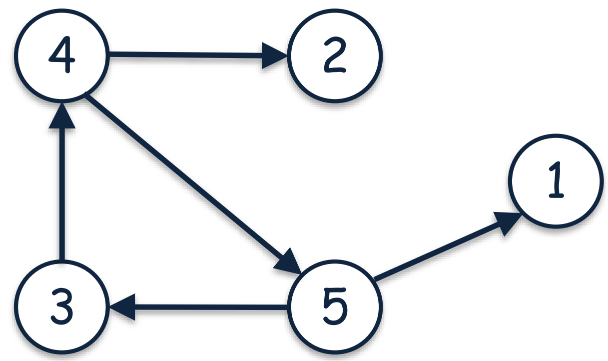
set of nodes (or vertices)

set of edges (or arc)



undirected graph

$deg(v)$= # of edges at that vertex



directed graph

$deg^{in}(v)$ = # of incoming edges

$deg^{out}(v)$ = # of outgoing edges

$deg^{in}(5) = 1$

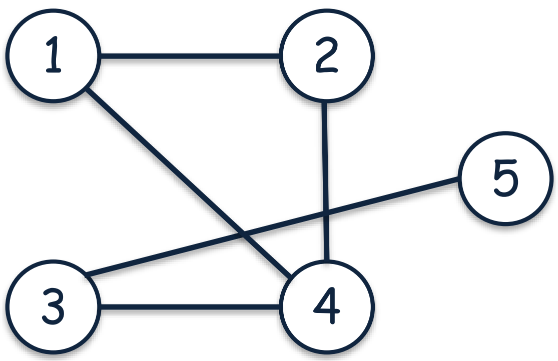
$deg^{out}(4) = 2$

Graph Theory

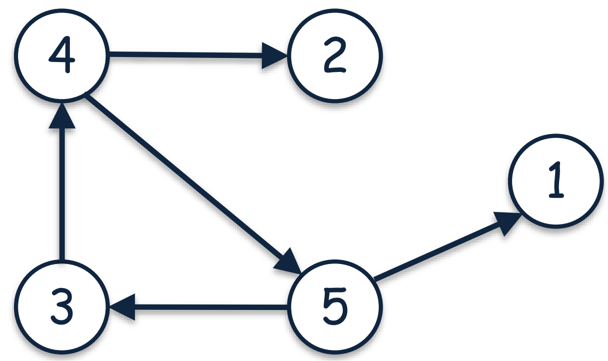
$G = (V, E)$

set of nodes (or vertices)

set of edges (or arc)



undirected graph



directed graph

deg(v)= # of edges at that vertex

Σ deg(v) = 2 |E|

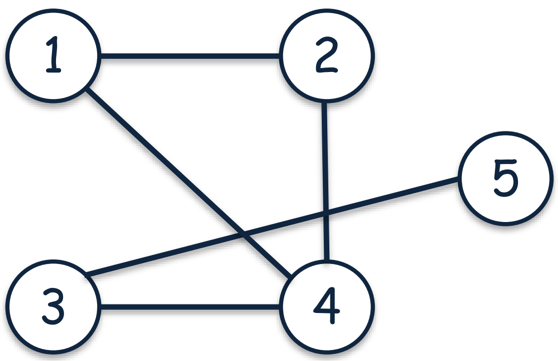
deg^{in} (v) = # of incoming edges

deg^{out} (v) = # of outgoing edges

Graph Theory

$$G = (V, E)$$

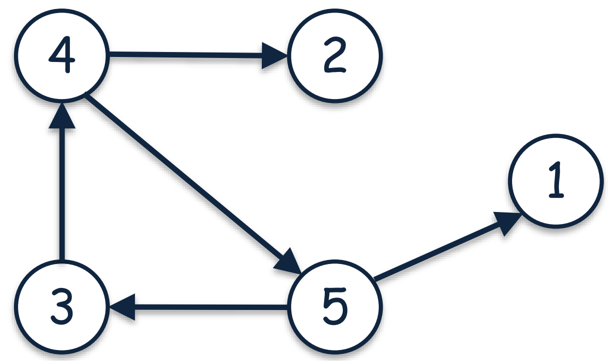
set of nodes (or vertices) set of edges (or arc)



undirected graph

deg(v)= # of edges at that vertex

$$\Sigma \deg(v) = 2\,|E|$$



directed graph

$\deg^{in}(v)$ = # of incoming edges

$\deg^{out}(v)$ = # of outgoing edges

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Graph Theory

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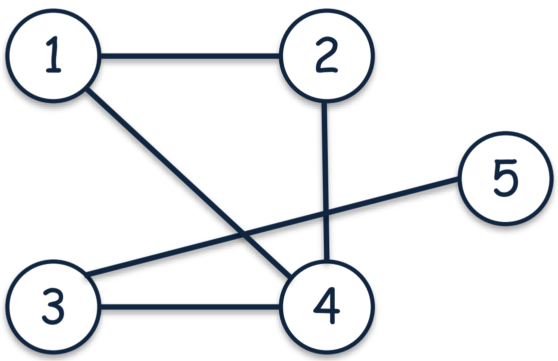
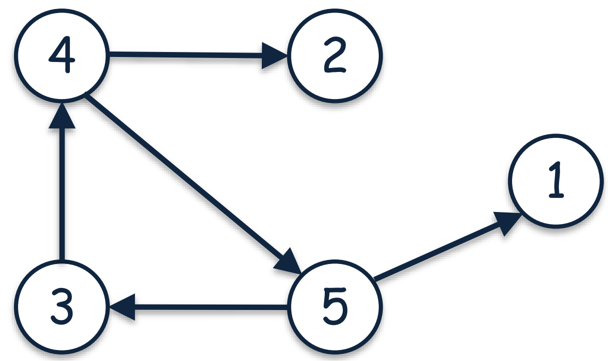


undirected graph



directed graph

deg(v)= # of edges at that vertex

$$\Sigma \, deg(v) = 2\,|E|$$

- a vertex v is called odd vertex if deg(v) is odd
- a vertex v is called even vertex if deg(v) is even

$deg^{in}(v)$ = # of incoming edges

$deg^{out}(v)$ = # of outgoing edges

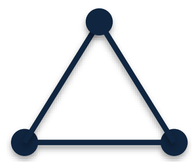
$$\Sigma \, deg^{in}(v) = \Sigma \, deg^{out}(v) = |E|$$

Graph Theory

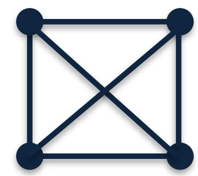
Complete Graphs

K_1

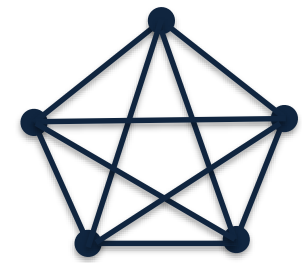
K_2



K_3



K_4



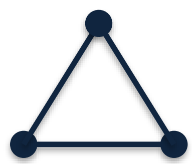
K_5

Graph Theory

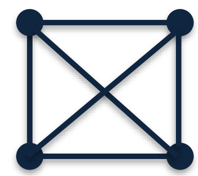
Complete Graphs

K_1

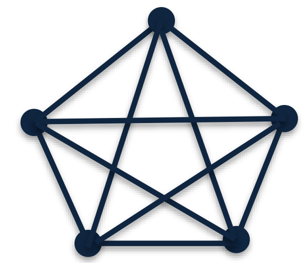
K_2



K_3

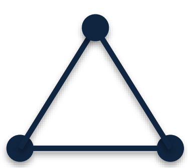


K_4

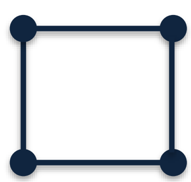


K_5

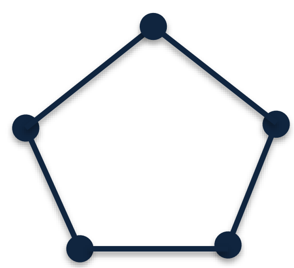
Cycle Graphs



C_3



C_4



C_5

Graph Theory

- a subgraph of a graph $G = (V, E)$ is a graph $H = (W, F)$ such that $W \subseteq V$ and $F \subseteq E$.

Graph Theory

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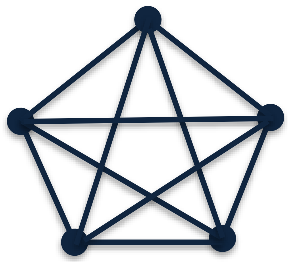

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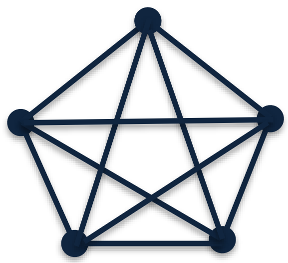
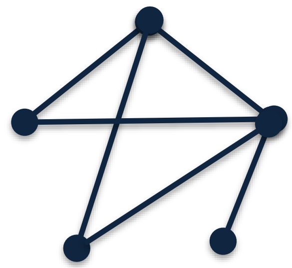


K_5



$G_1 \subseteq K_5$

Graph Theory

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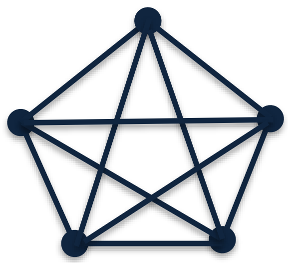
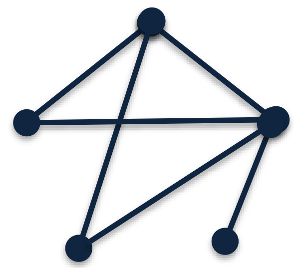


K_5



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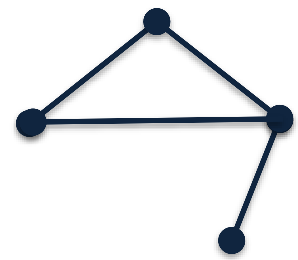

$G_2 \subseteq K_5$

Graph Theory

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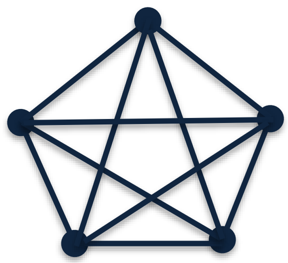
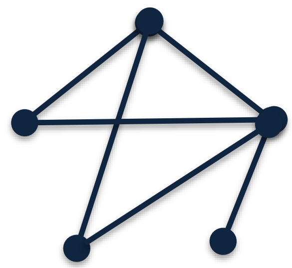


K_5



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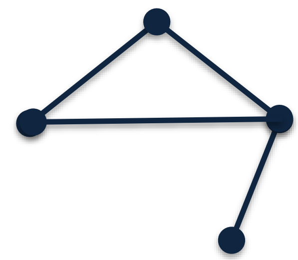
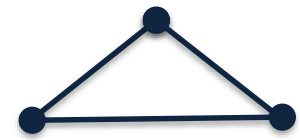


$G_2 \subseteq K_5$



$G_3 \subseteq K_5$

Graph Theory

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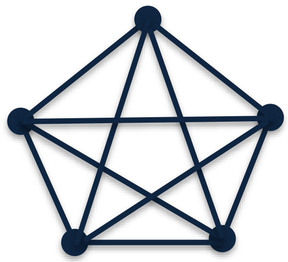
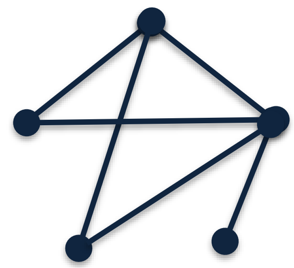
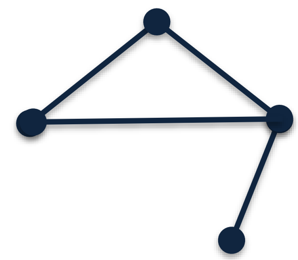


K_5

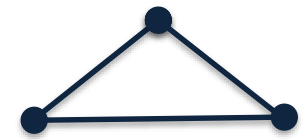


$G_1 \subseteq K_5$



$G_2 \subseteq K_5$

$G_2 \subseteq G_1$



$G_3 \subseteq K_5$

$G_3 \subseteq G_2$

Graph Theory

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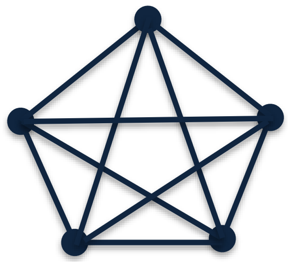
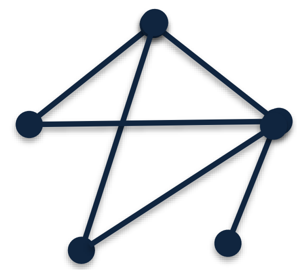
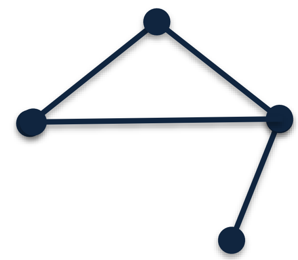


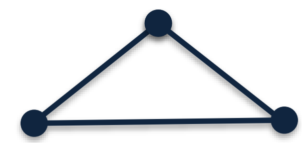
K_5



$G_1 \subseteq K_5$



$G_2 \subseteq K_5$
$G_2 \subseteq G_1$



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- $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, then $G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$

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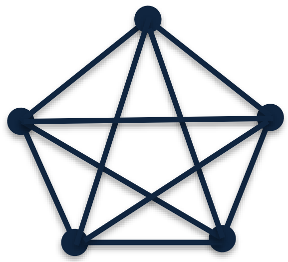
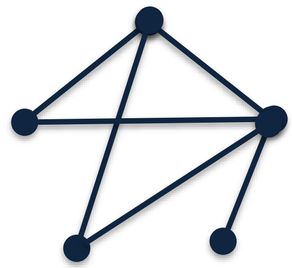


K_5



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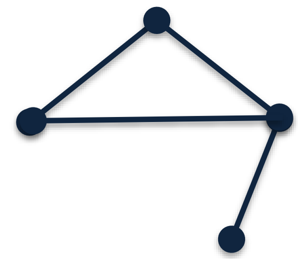

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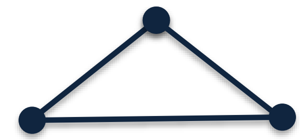

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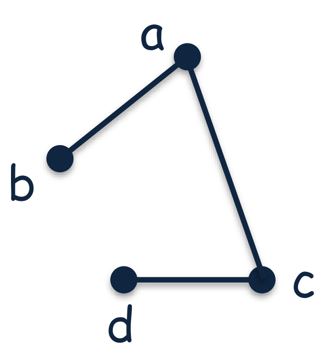

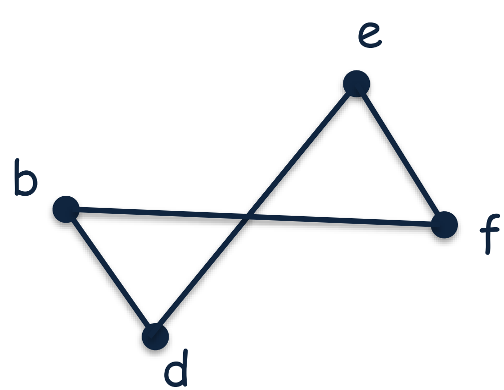

Graph Theory

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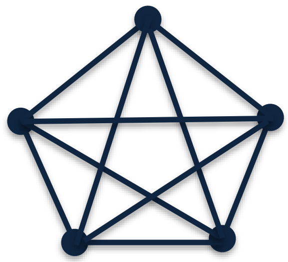
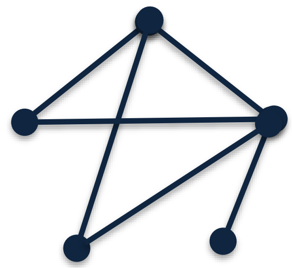


K_5



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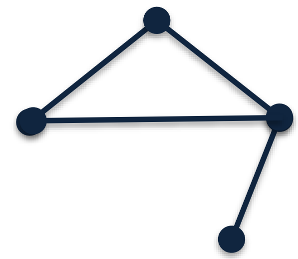

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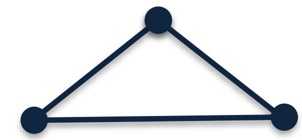

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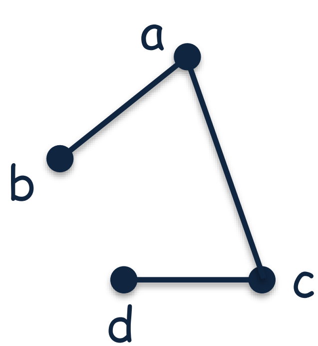
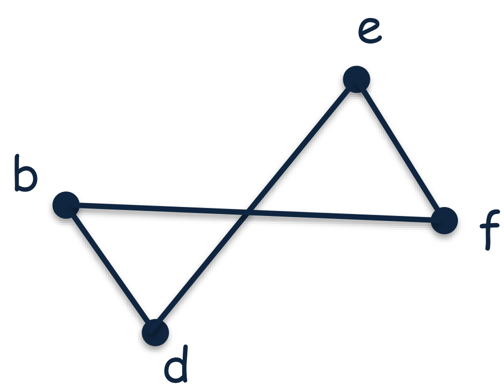
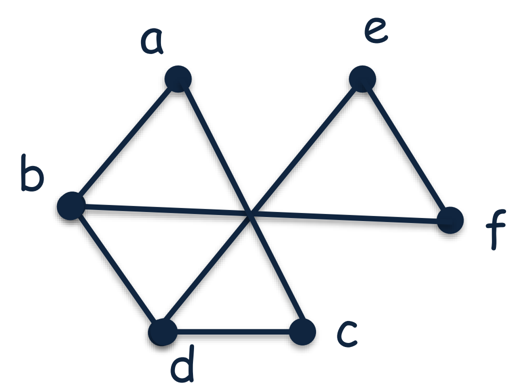
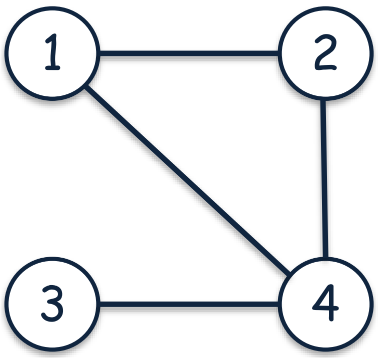


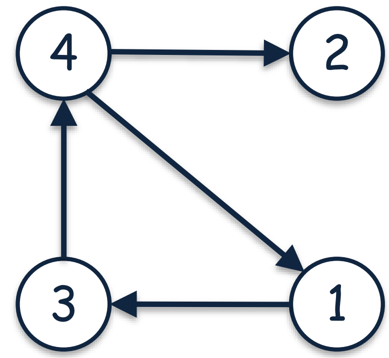




Representation



Adjacency List



Adjacency List

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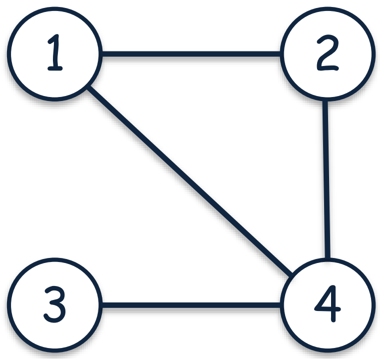
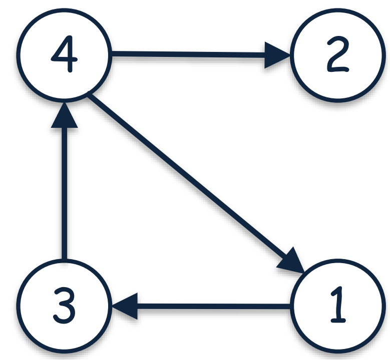




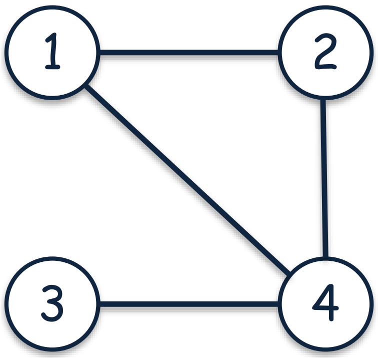
Adjacency List

1 - 2,4
2 - 1,4
3 - 4
4 - 1,2,3

Adjacency List

1 - 3
2 -
3 - 4
4 - 1,2

Representation

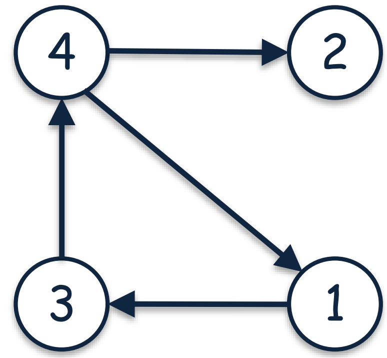


Adjacency List

1 - 2,4
2 - 1,4
3 - 4
4 - 1,2,3

Adjacency Matrix

	1	2	3	4
1	0	1	0	1
2	1	0	0	1
3	0	0	0	1
4	1	1	1	0



Adjacency List

1 - 3
2 -
3 - 4
4 - 1,2

Adjacency Matrix

	1	2	3	4
1	0	0	1	0
2	0	0	0	0
3	0	0	0	1
4	1	1	0	0

Representation

Adjacency List **Adjacency Matrix**

Representation

	Adjacency List	Adjacency Matrix
• retrieving all neighbors of a given node u	O(deg(u))	O(|V|)

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Representation

	Adjacency List	Adjacency Matrix
• retrieving all neighbors of a given node u	O(deg(u))	O(|V|)
• given nodes u and v, checking if u and v are adjacent	O(deg(u))	O(1)
• space	O(|E|+|V|)	O(|V|^2)

Representation

	Adjacency List	Adjacency Matrix
• retrieving all neighbors of a given node u	$O(deg(u))$	$O(\|V\|)$
• given nodes u and v, checking if u and v are adjacent	$O(deg(u))$	$O(1)$
• space	$O(\|E\|+\|V\|)$	$O(\|V\|^2)$

If graph is sparse, use adjacency list;
if graph is dense, use adjacency matrix

Isomorphism

- Two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if there exists a bijection f from V_1 to V_2 such that a and b are adjacent in G_1 if and only if f(a) and f(b) are adjacent in G_2 for all $a, b \in V_1$

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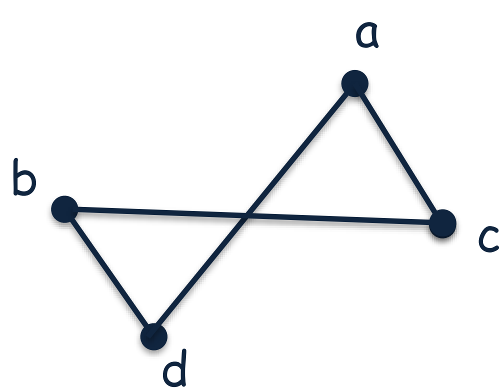

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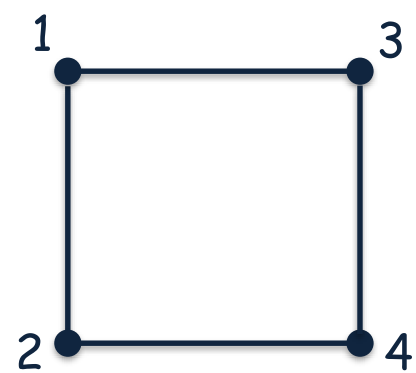

$G_2 = (V_2, E_2)$

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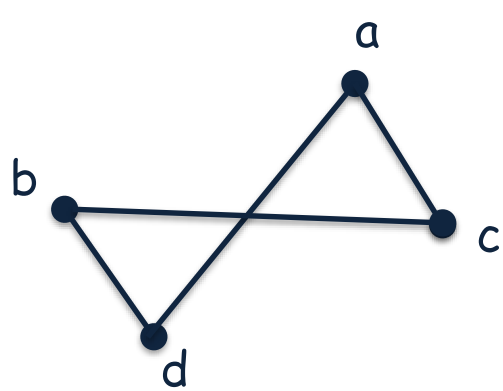

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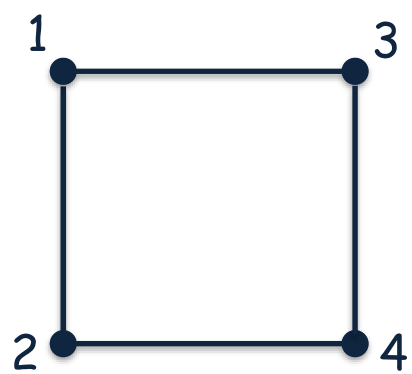

$G_2 = (V_2, E_2)$

- f : $V_1 \to V_2$, f(a) = 1, f(b) = 4, f(c) = 3, f(d) = 2

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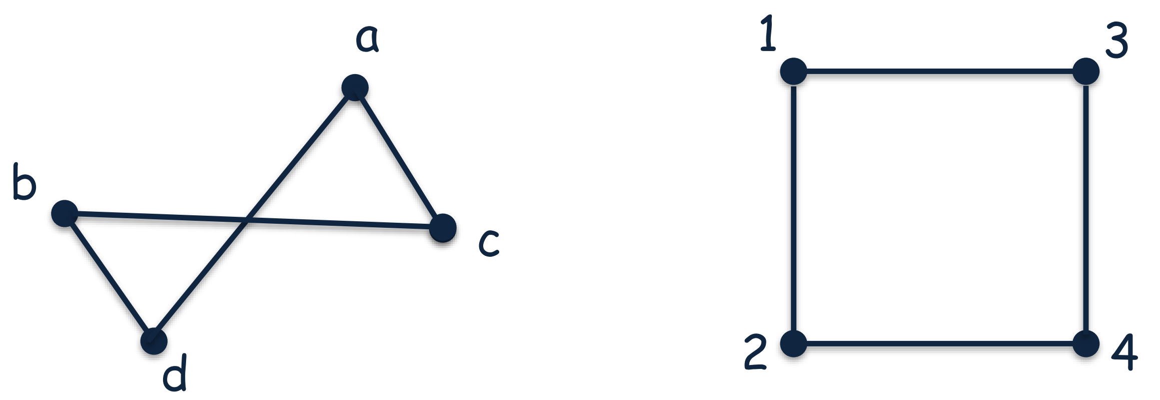

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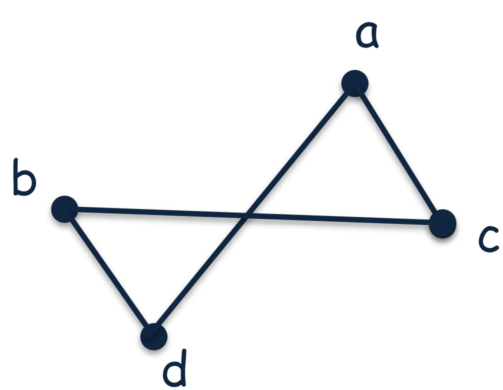

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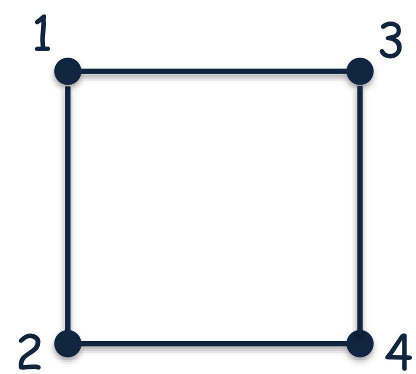

$G_2 = (V_2, E_2)$

- f : $V_1 \rightarrow V_2$, f(a) = 1, f(b) = 4, f(c) = 3, f(d) = 2

a and c are adjacent in G_1, f(a) = 1 and f(c) = 3 are adjacent in G_2
a and d are adjacent in G_1, f(a) = 1 and f(d) = 2 are adjacent in G_2

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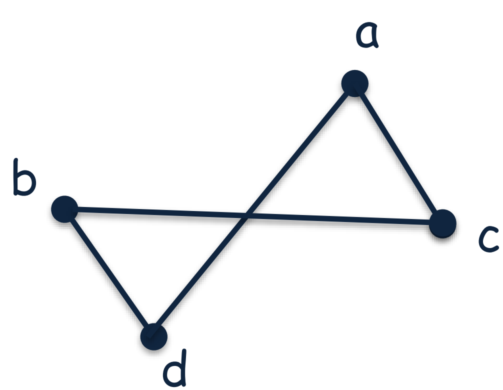

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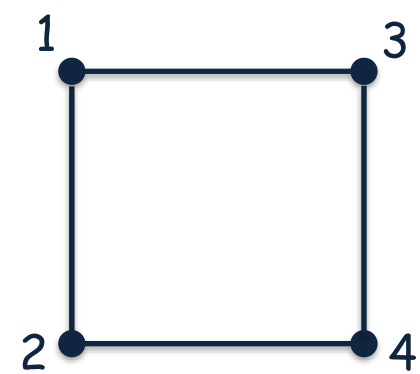

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$$A_{G_1} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \qquad A_{G_2} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

$G_1 = (V_1, E_1)$ $\qquad$ $G_2 = (V_2, E_2)$

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Isomorphism

- Isomorphic graphs must have same number of edges

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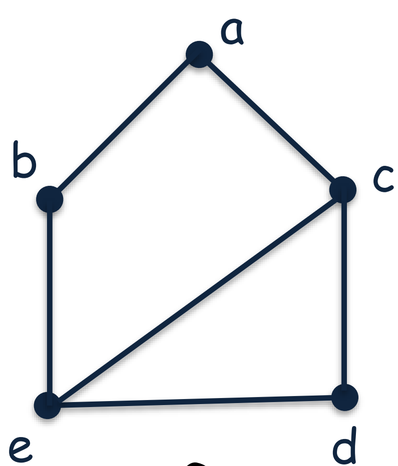
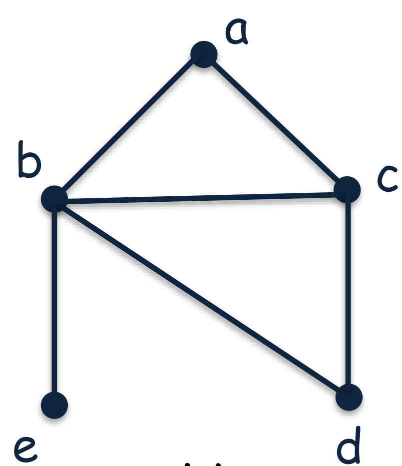


G



H

Isomorphism

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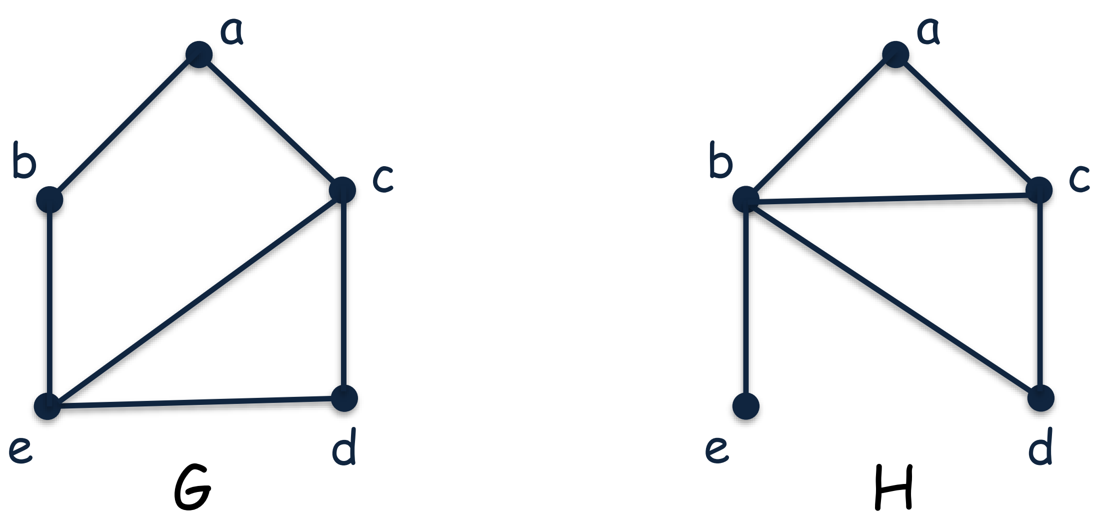

- G and H both have 5 vertices and 6 edges

Isomorphism

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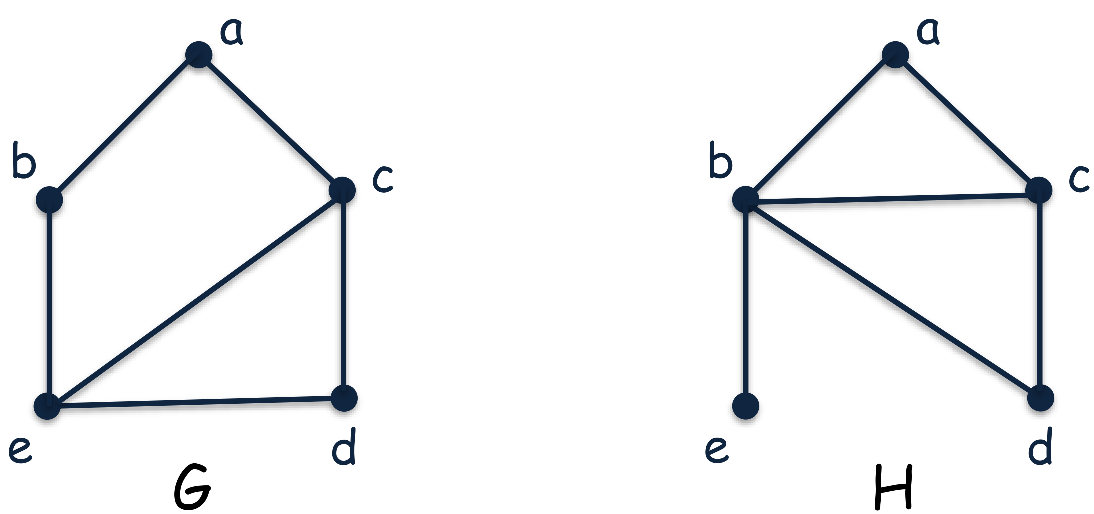

- G and H both have 5 vertices and 6 edges
- G has 3 vertices of degree two and 2 vertices of degree three

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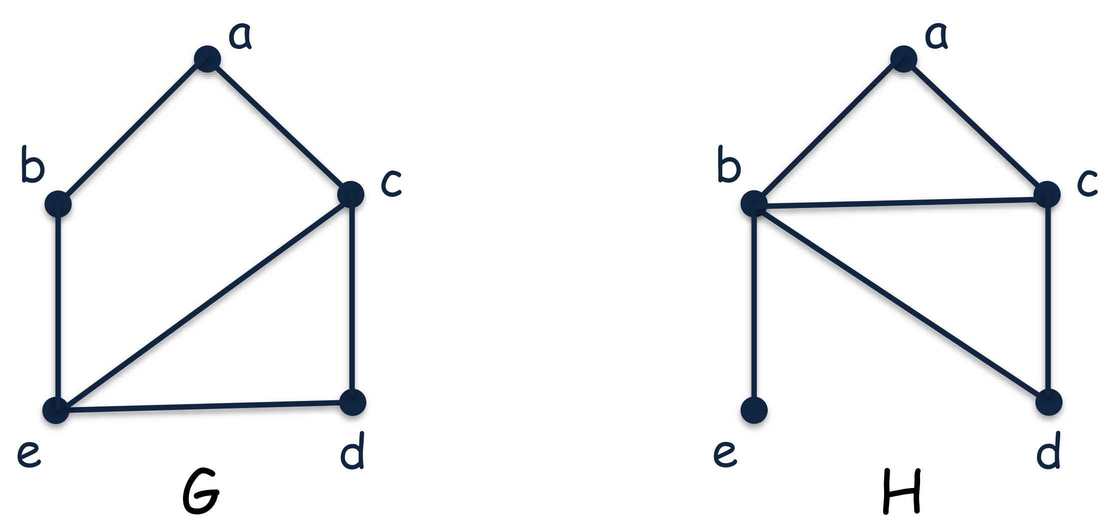

- G and H both have 5 vertices and 6 edges
- G has 3 vertices of degree two and 2 vertices of degree three
  H has 1 vertex of degree one, 2 vertices of degree two, 1 vertex of degree three, and 1 vertex of degree 4

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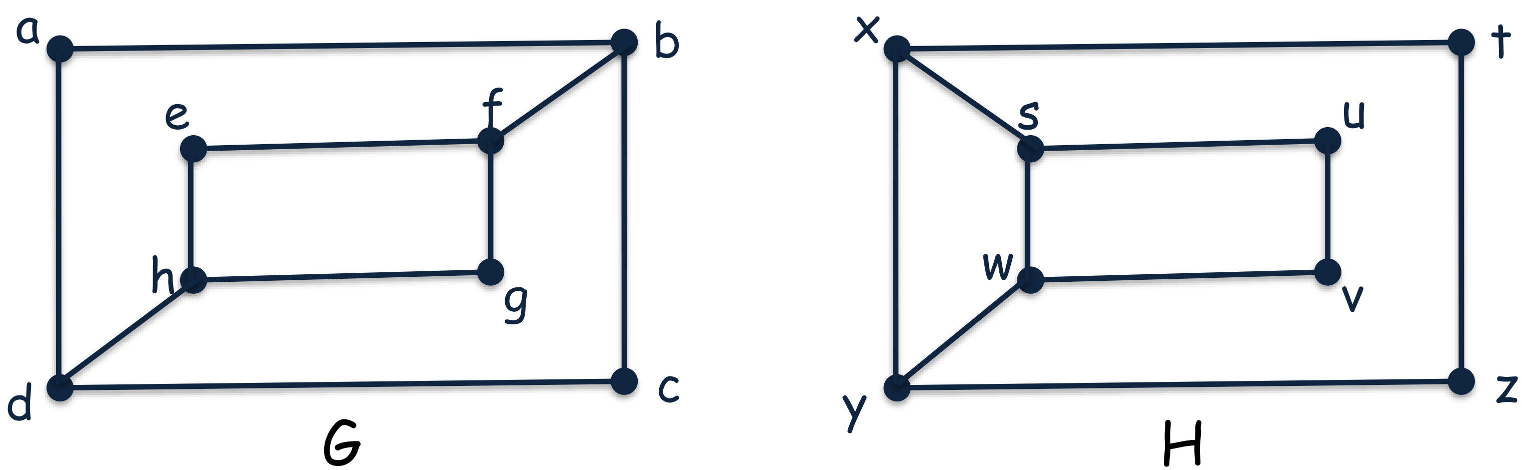

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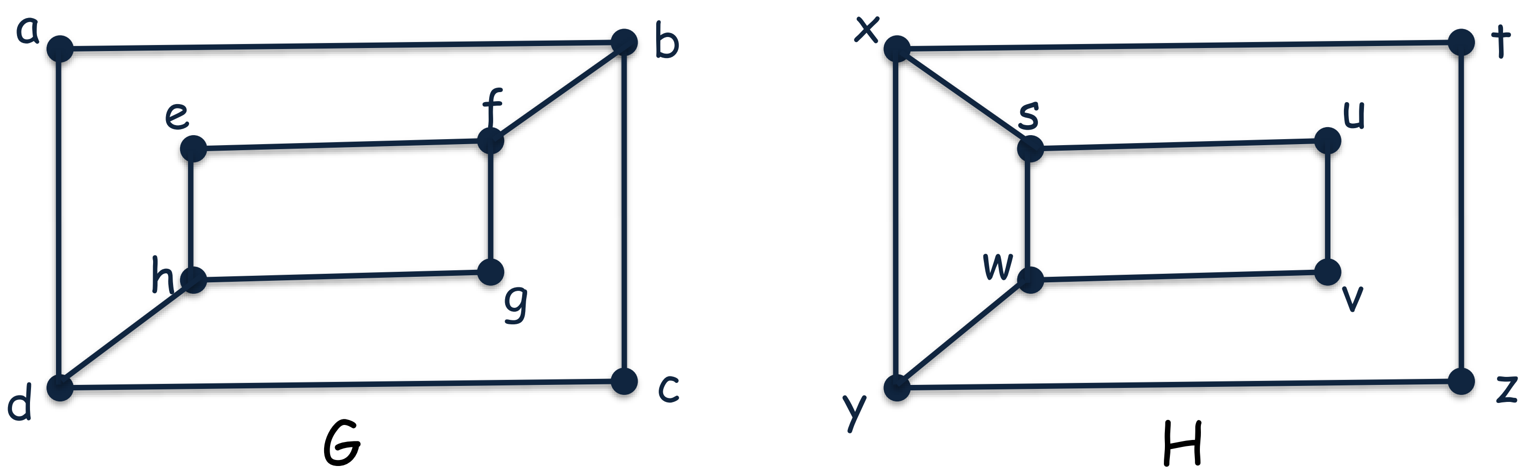

- G and H both have 8 vertices and 10 edges

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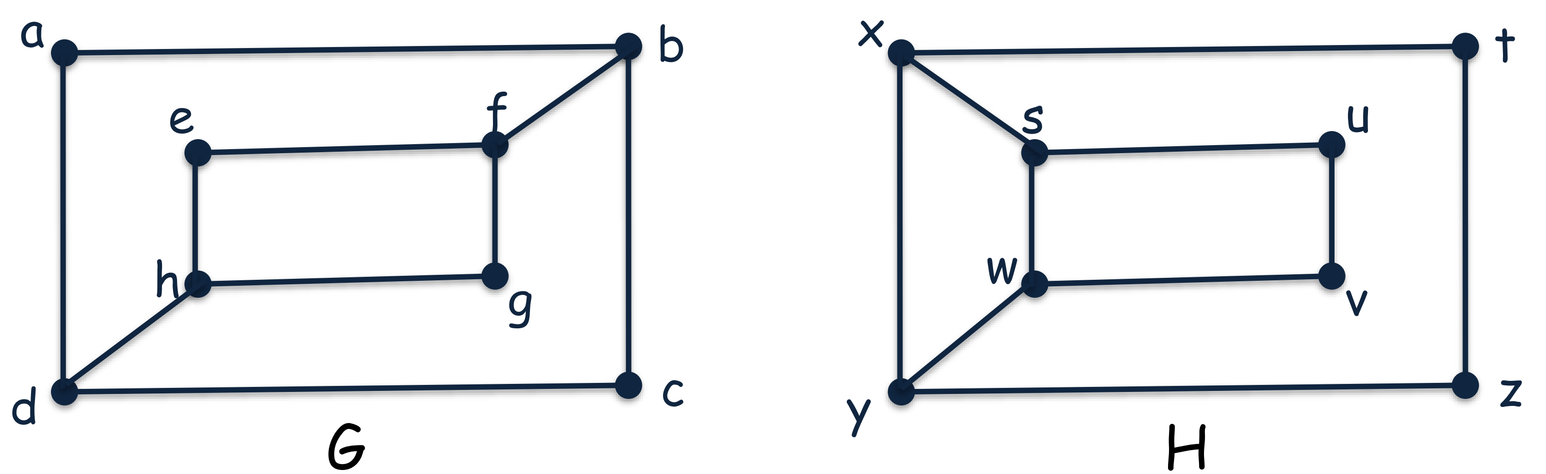

- G and H both have 8 vertices and 10 edges
- G has 4 vertices of degree two and 4 vertices of degree three

Isomorphism

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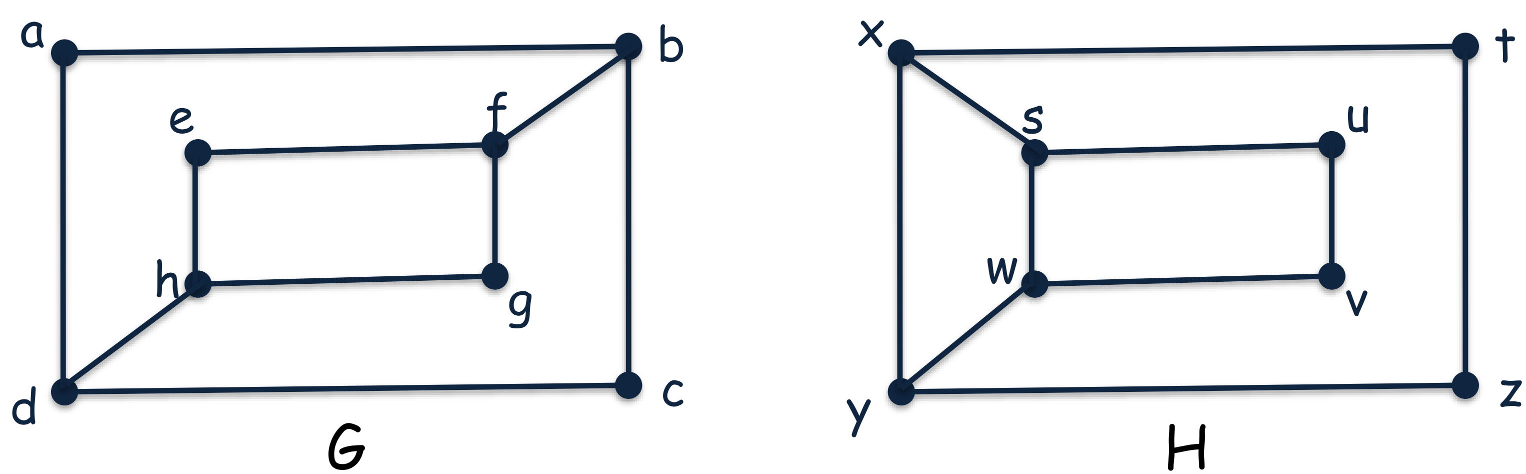

- G and H both have 8 vertices and 10 edges
- G has 4 vertices of degree two and 4 vertices of degree three
  H has 4 vertices of degree two and 4 vertices of degree three

Isomorphism

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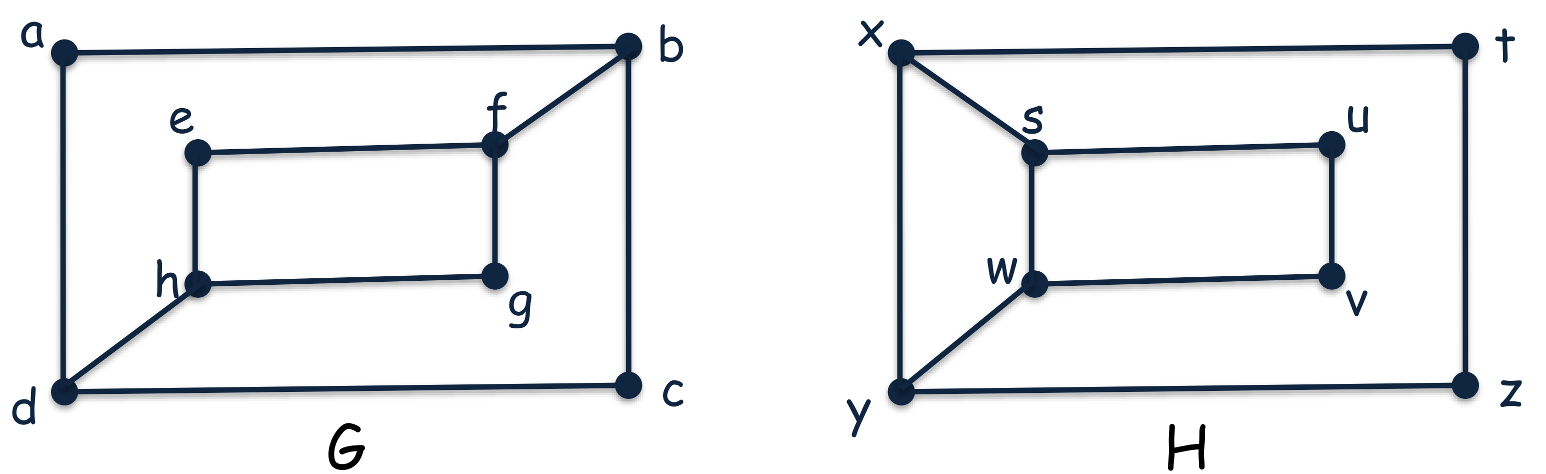
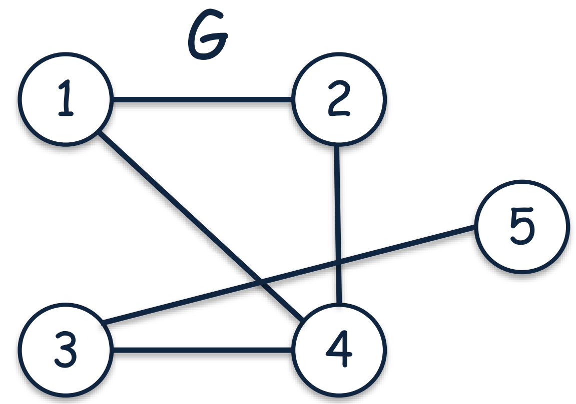


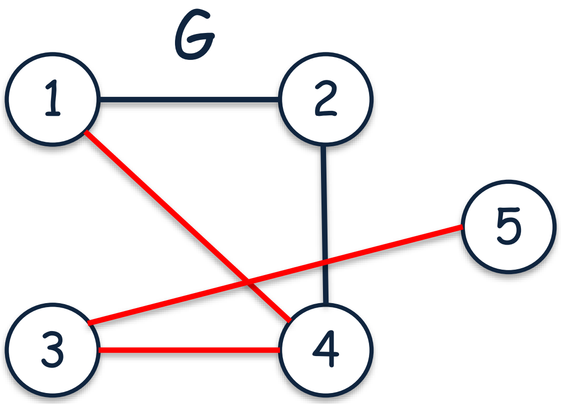
- G and H both have 8 vertices and 10 edges
- G has 4 vertices of degree two and 4 vertices of degree three
  H has 4 vertices of degree two and 4 vertices of degree three
- One of the odd vertices (s) in H has 2 adjacent odd vertices (w and x)
  We don't have such case in G

Connectivity



- a path in a graph is a sequence of nodes v_1, v_2, …, v_k such that (v_i, v_j) is an edge in the graph.
  a path is simple if all nodes are distinct

Connectivity



5, 3, 4, 1 is a simple path in *G*

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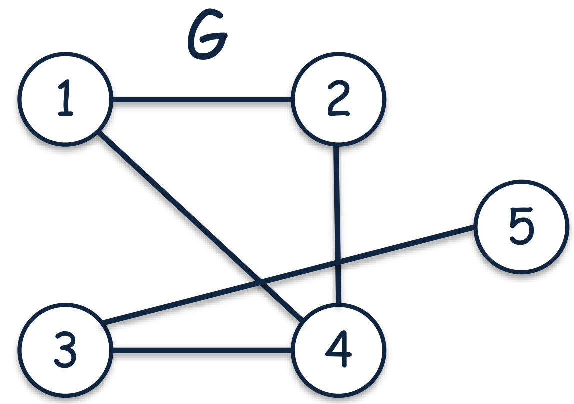
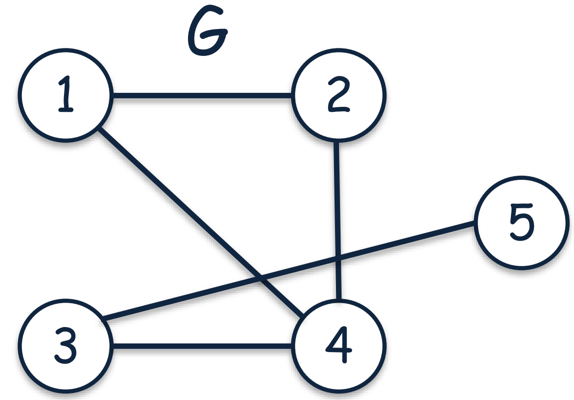
Connectivity



- a path in a graph is a sequence of nodes $v_1, v_2, \ldots, v_k$ such that (v_i, v_j) is an edge in the graph.
  a path is simple if all nodes are distinct

- nodes u and v are called connected if there is a path between them. A graph is connected if there is a path between every pair of nodes

Connectivity



- a path in a graph is a sequence of nodes $v_1, v_2, \ldots, v_k$ such that (v_i, v_j) is an edge in the graph.
  a path is simple if all nodes are distinct
- nodes u and v are called connected if there is a path between them. A graph is connected if there is a path between every pair of nodes
- a cycle is a path $v_1, v_2, \ldots, v_k$ such that $v_1 = v_k$. A cycle is simple if first k-1 nodes are distinct

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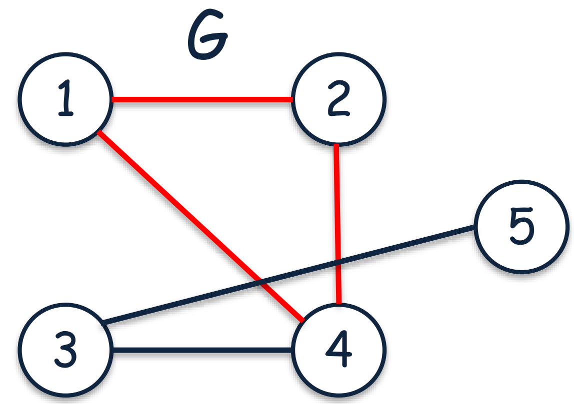

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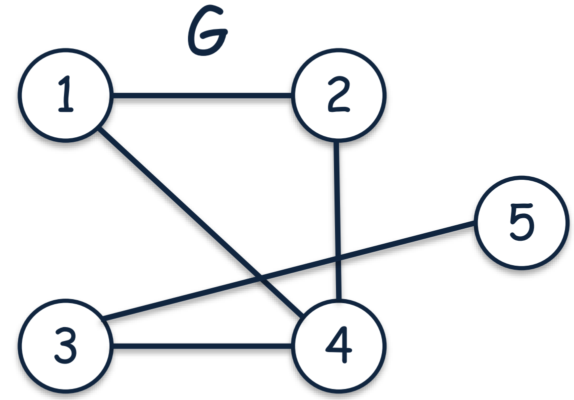

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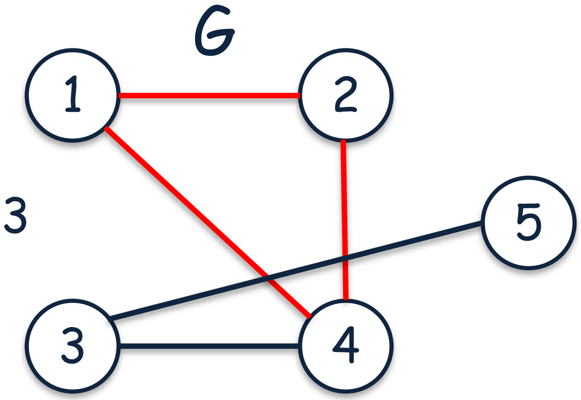
Connectivity

4, 1, 2, 4 is a simple cycle with length 3



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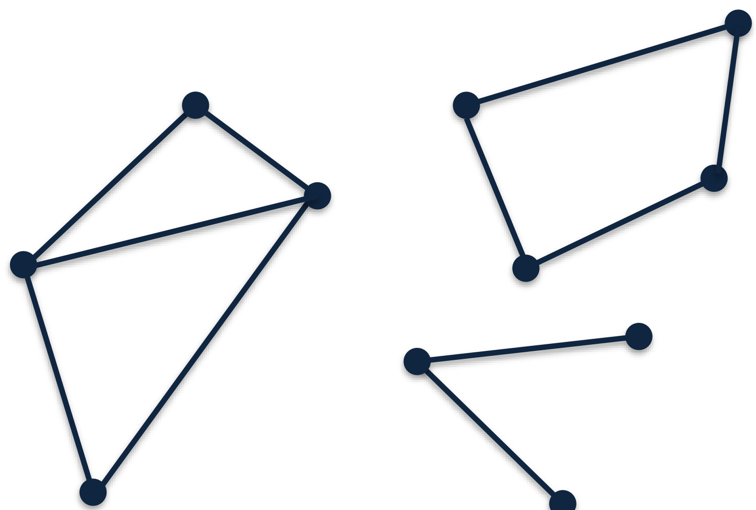

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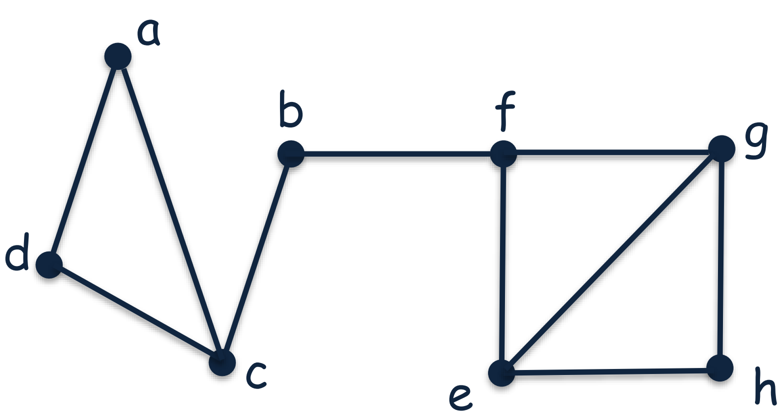

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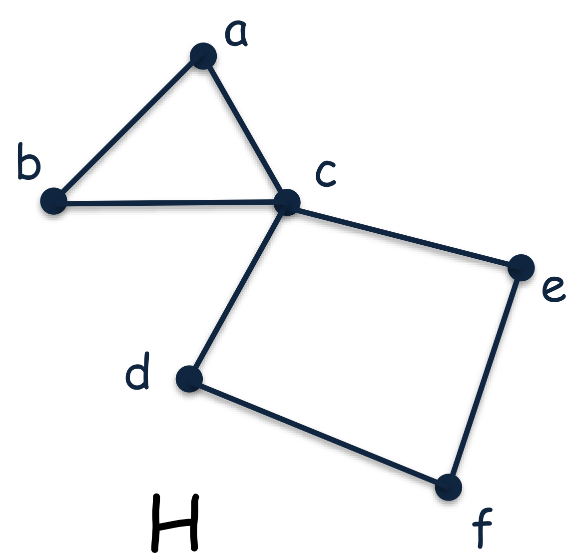

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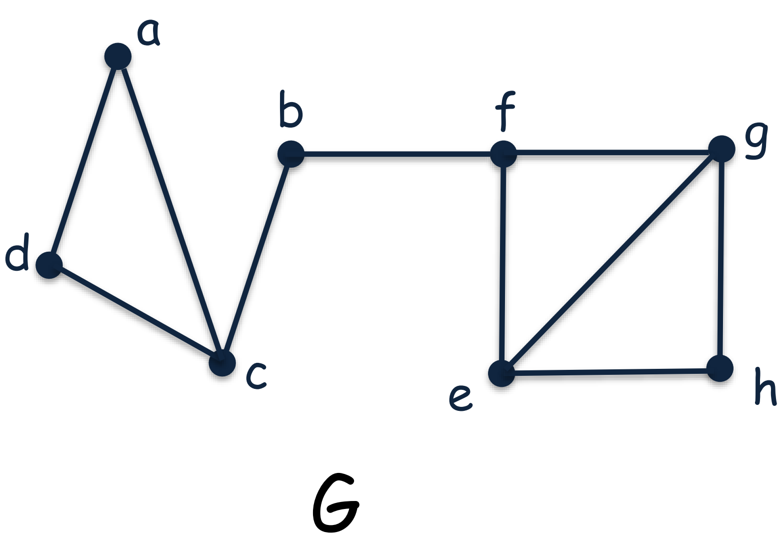

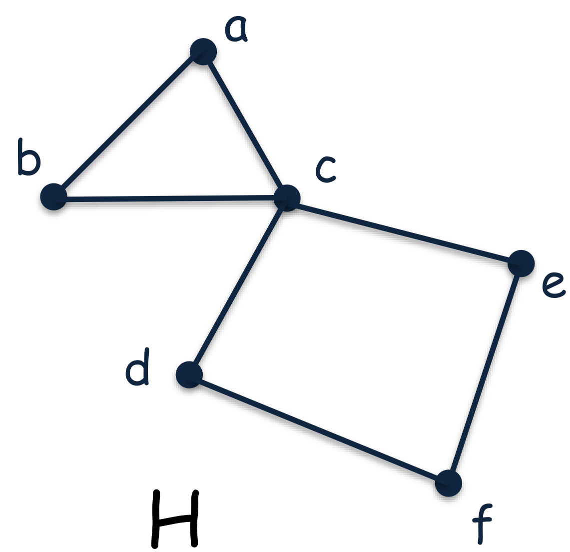

cut vertices : {b, c, f}
cut edges : {(b, f), (c, b)}

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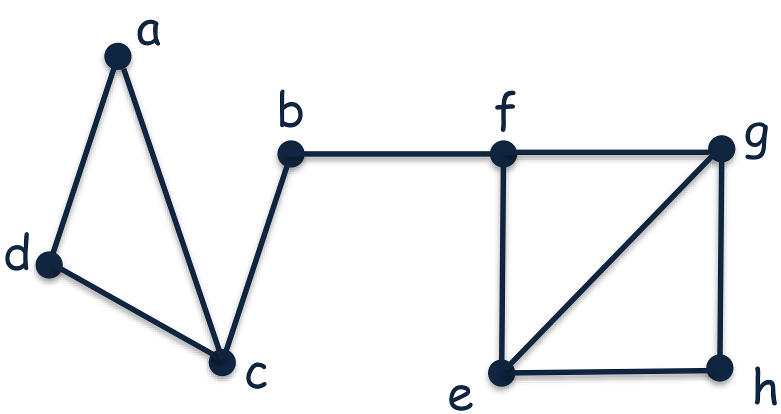

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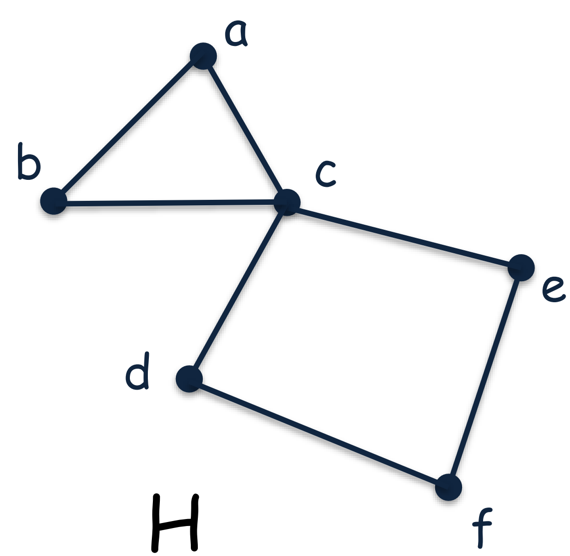

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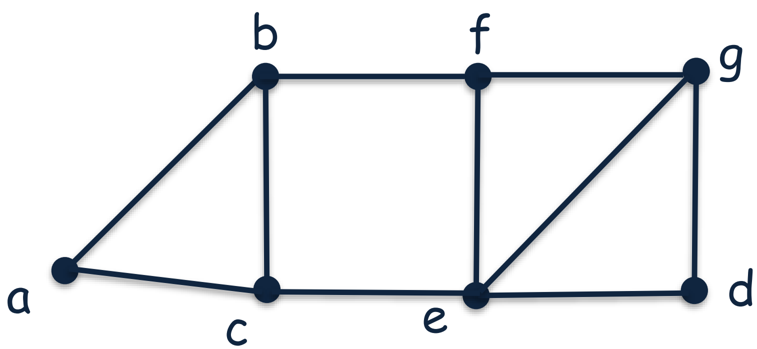

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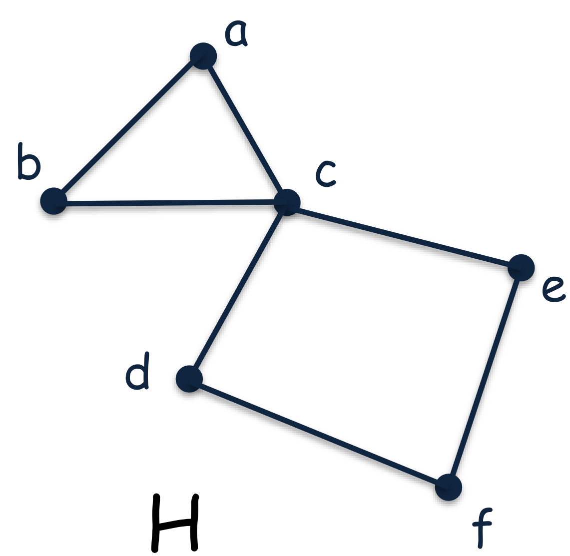

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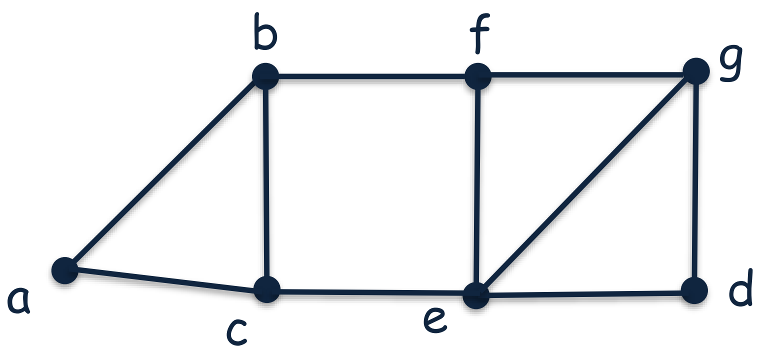

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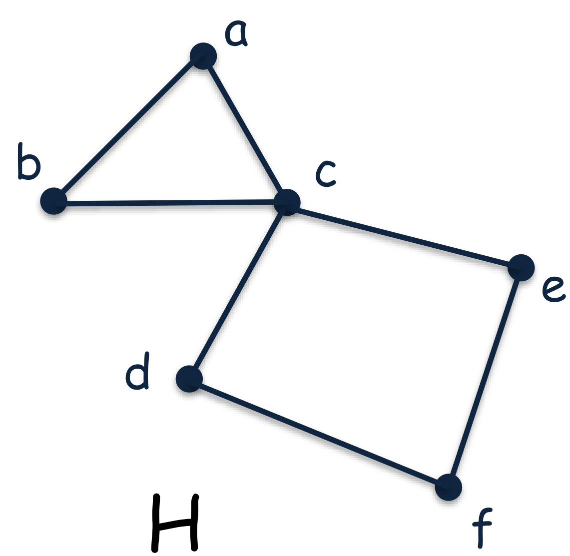

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vertex cut: {b, c} or {f, e}
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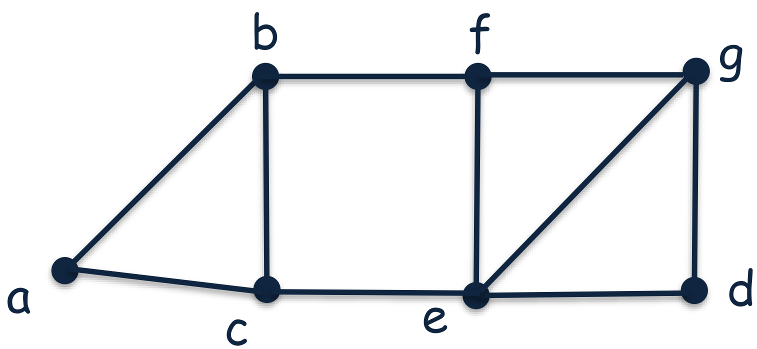

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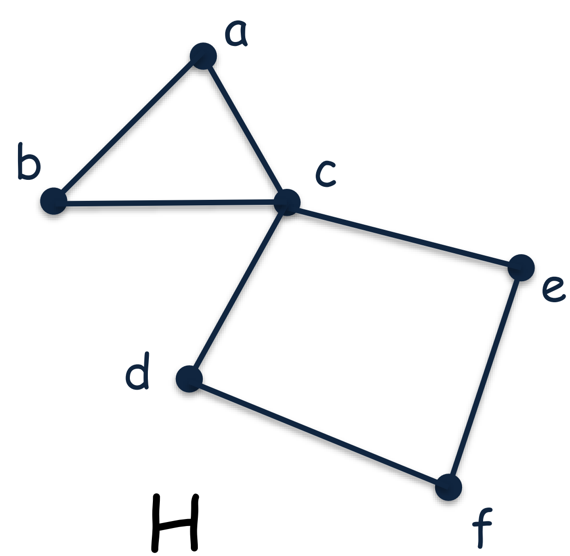

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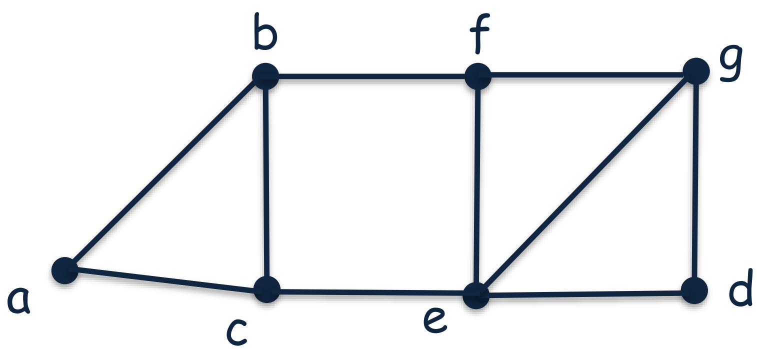

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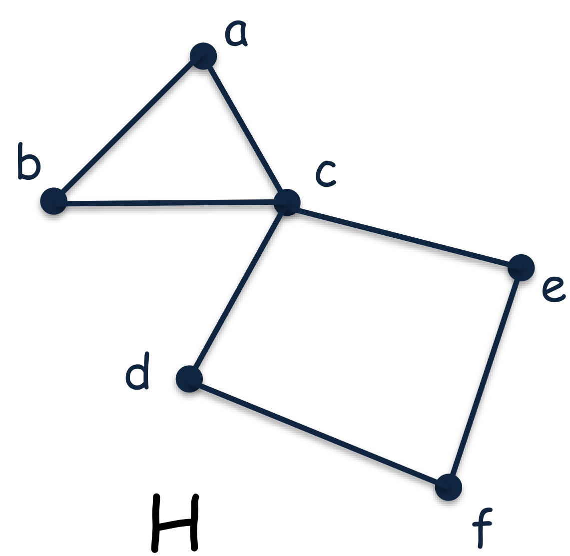

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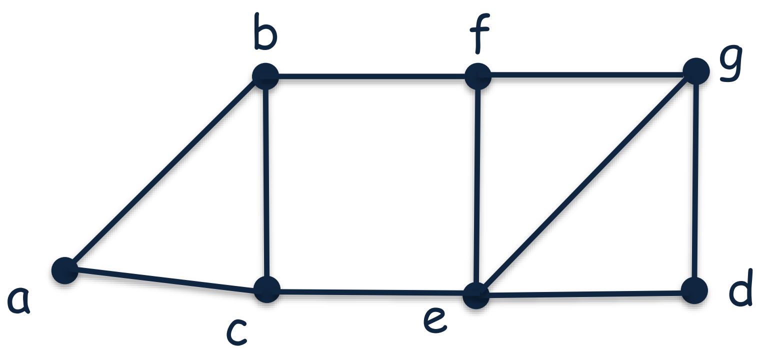

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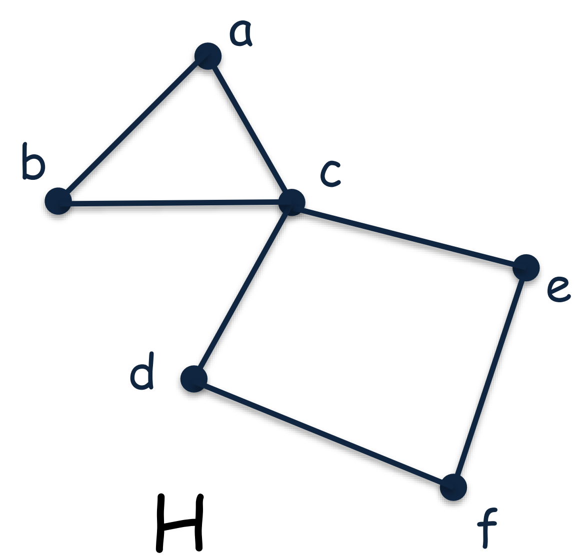

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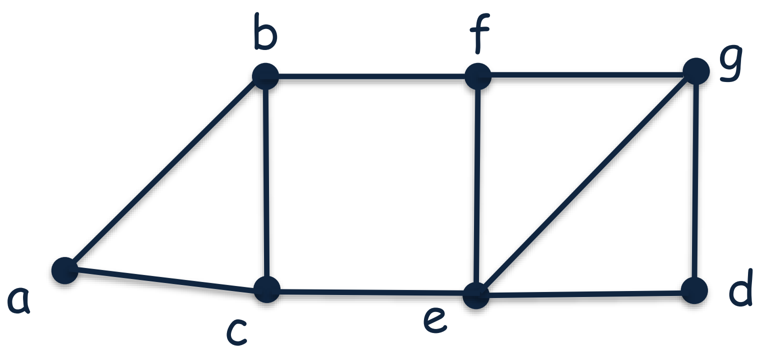

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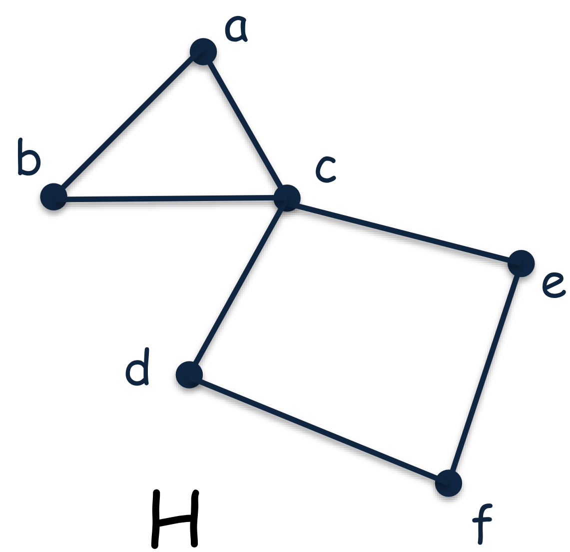

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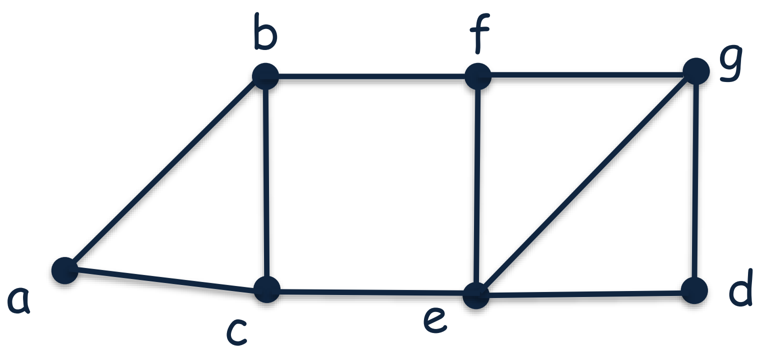

G

$\kappa(G) = 2$
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vertex cut: {b, c} or {f, e}
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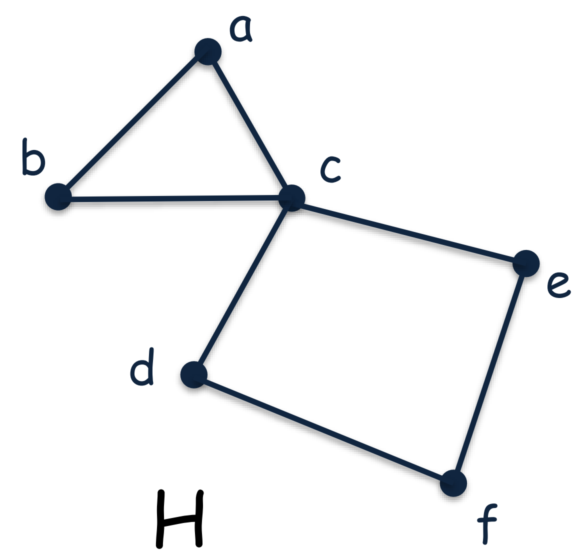
no cut vertex and no cut edge



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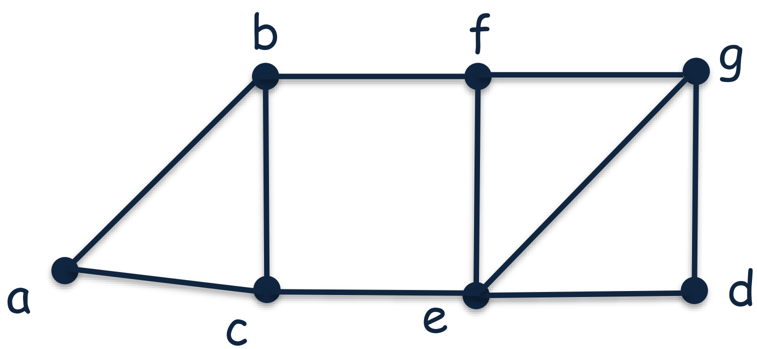

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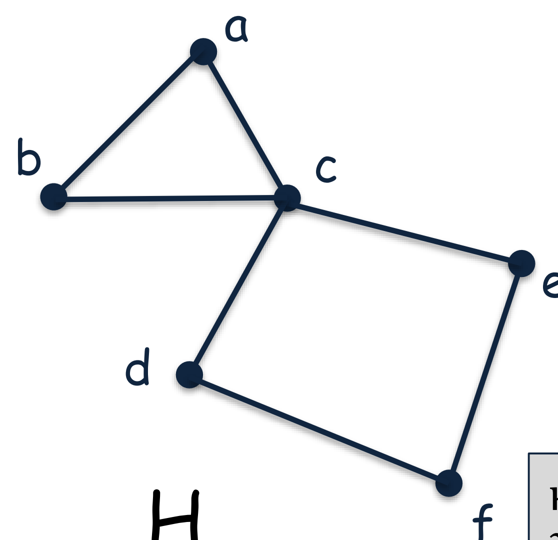

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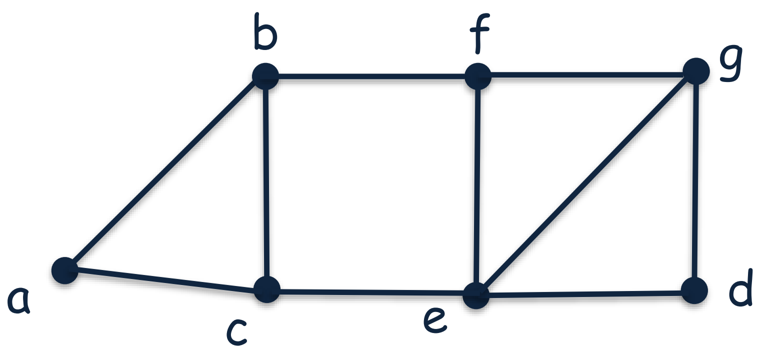

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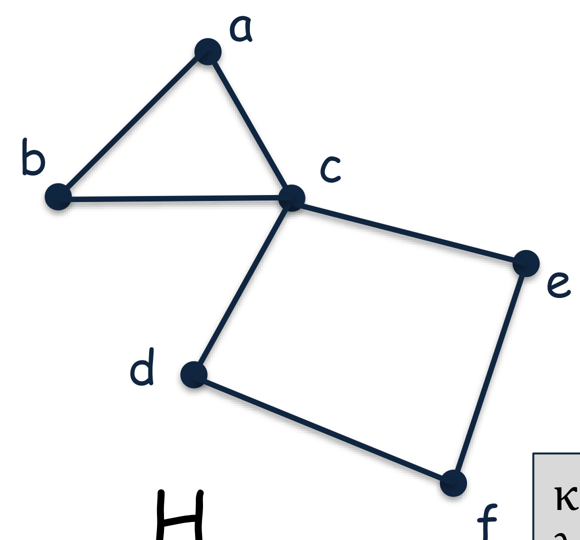

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$$\kappa(G) \leq \lambda(G) \leq min_{v\in V} \deg(v)$$

Isomorphism

- Isomorphic graphs must have same number of edges
- The degrees of the vertices in isomorphic graphs must be same
- They must have same amount of simple circuits of length k

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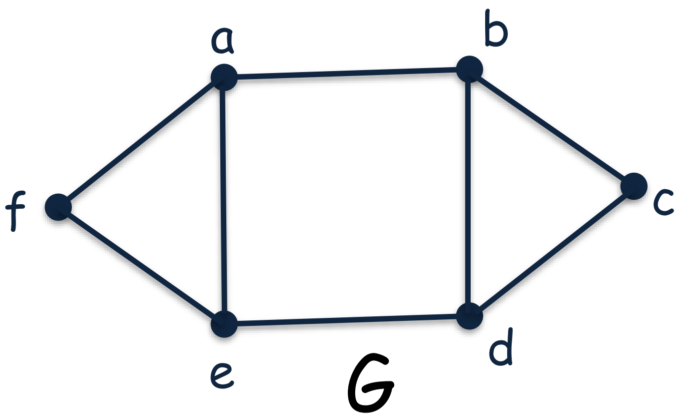

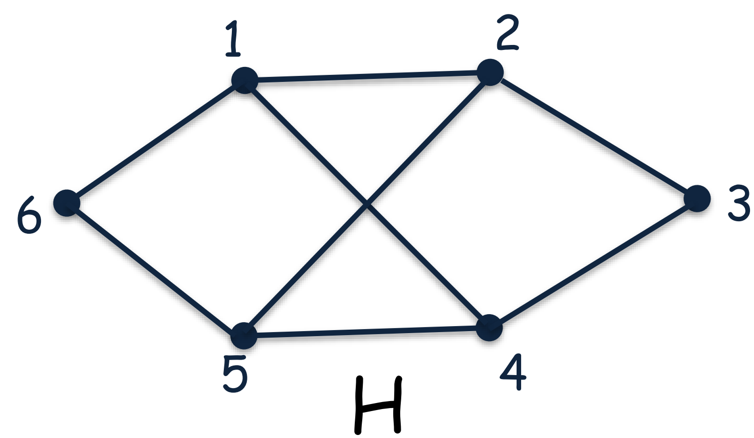

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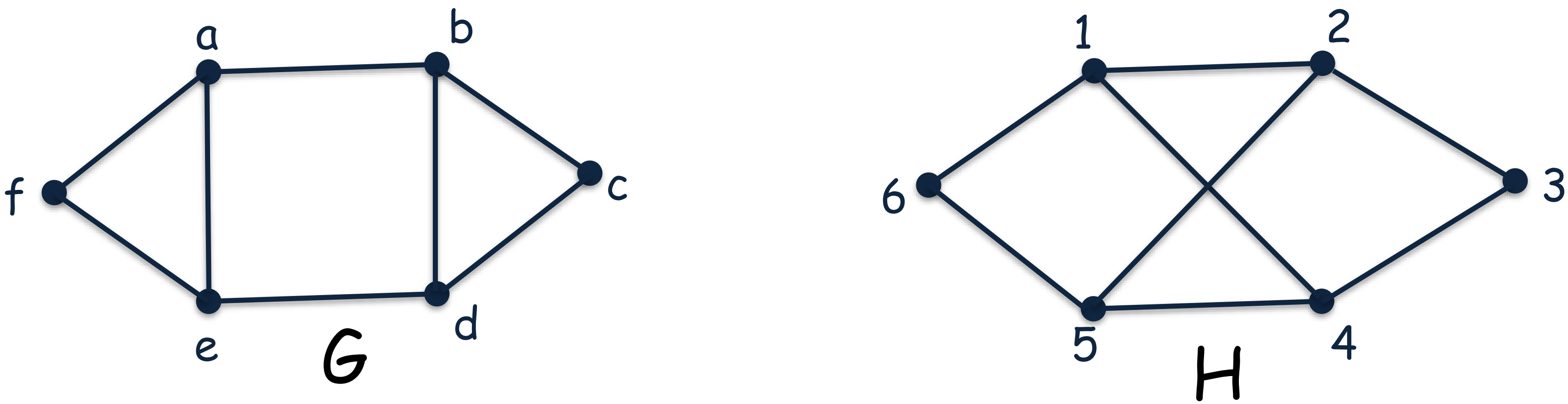

- G and H both have 6 vertices and 8 edges

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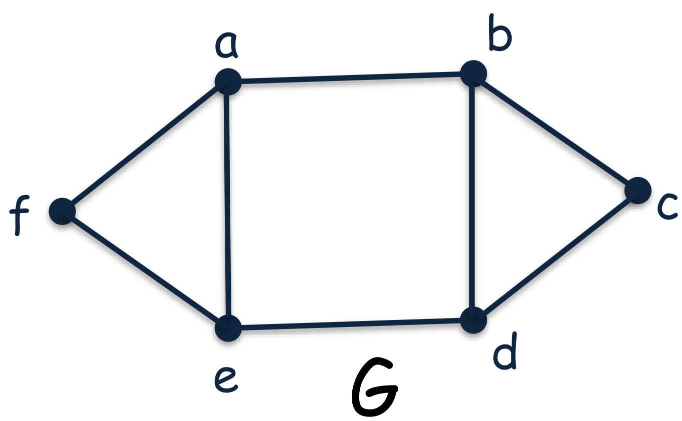

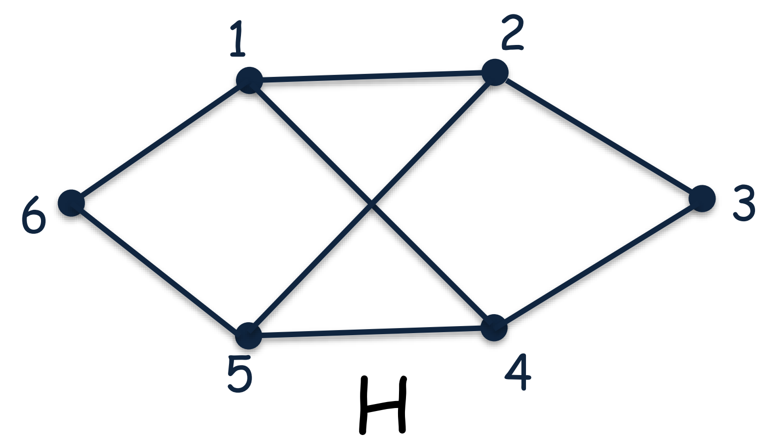

- G and H both have 6 vertices and 8 edges
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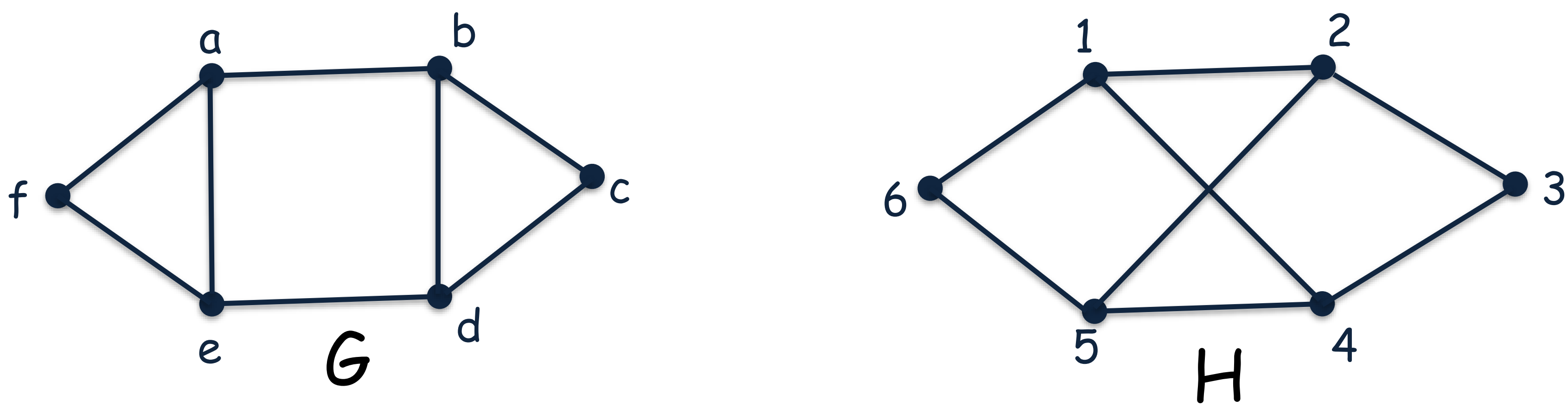

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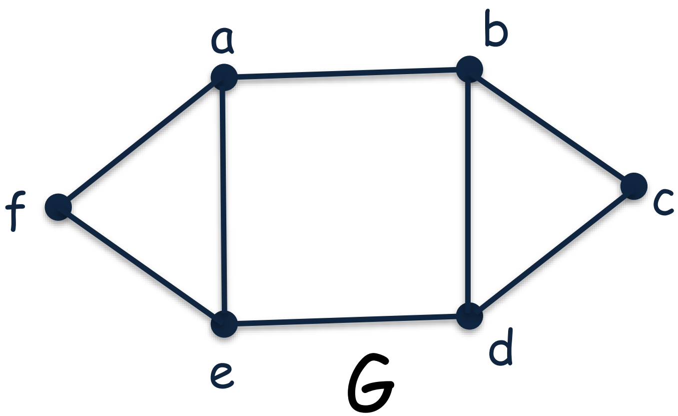
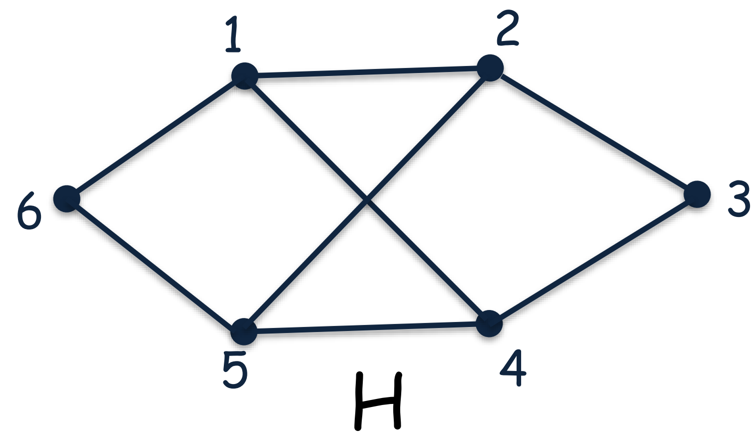
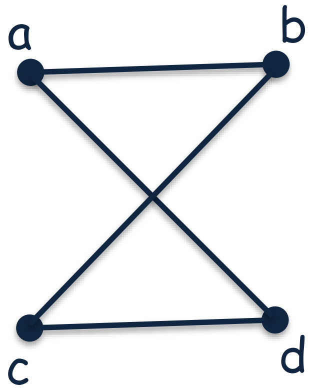




- G and H both have 6 vertices and 8 edges
- G has 2 vertices of degree two and 4 vertices of degree three
  H has 2 vertices of degree two and 4 vertices of degree three
- G has two simple circuits of length three; however, H has no simple circuit of length three

Connectivity



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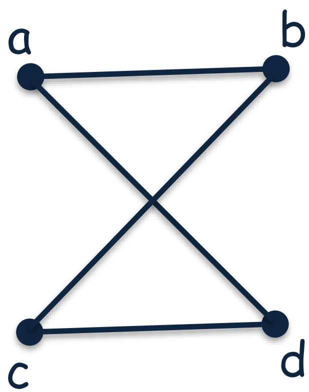

- How many paths of length two from a to c ?

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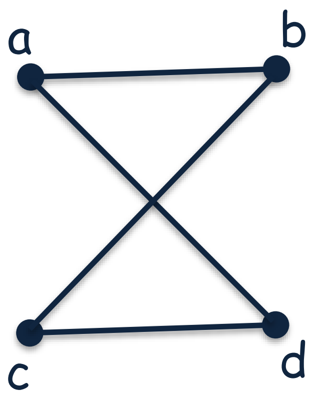

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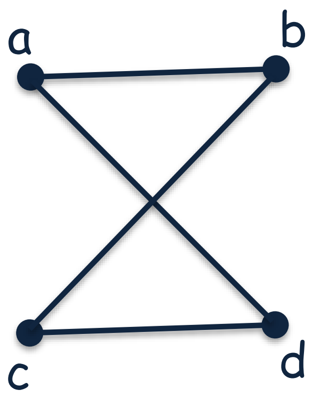

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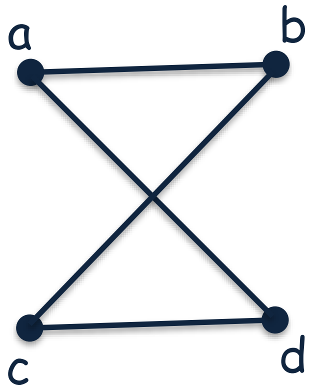

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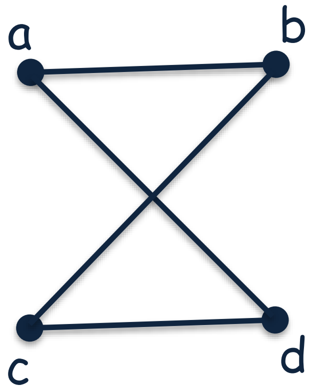
- Given a graph $G = (V, E)$ together with the adjacency matrix A, the number of different paths of length m from v_i to v_j will be the (i, j)-th entry of A^m

Connectivity



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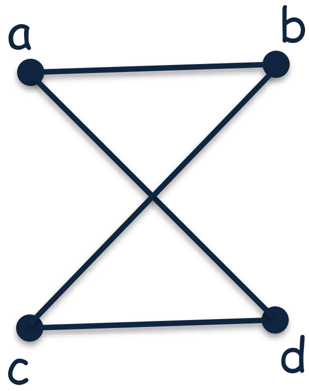
Connectivity



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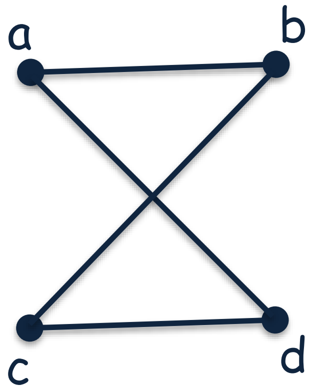

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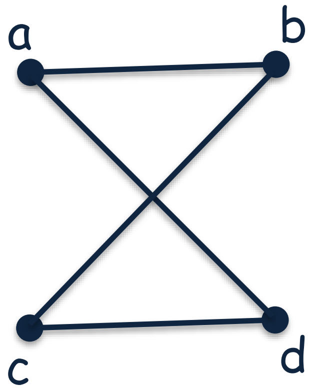

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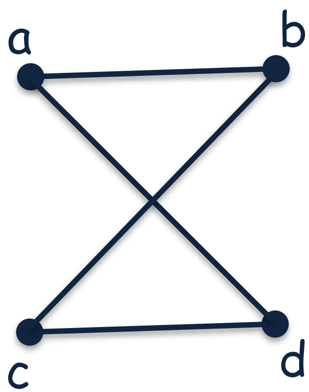

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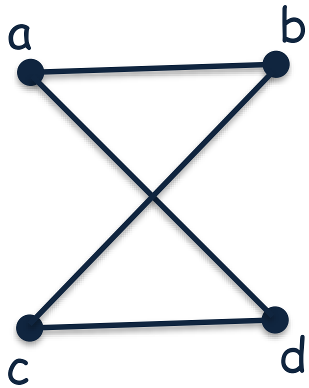

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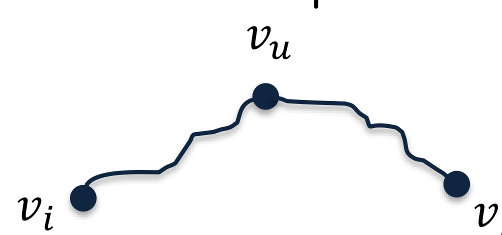

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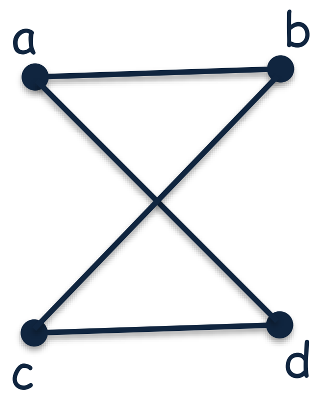

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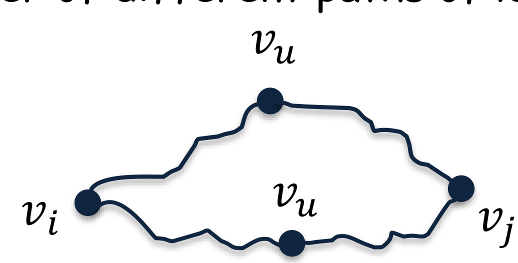

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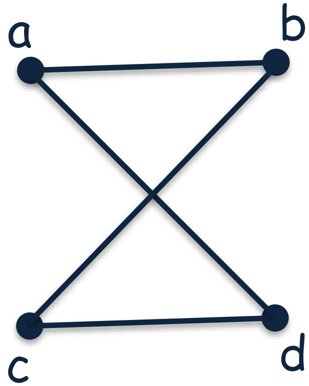

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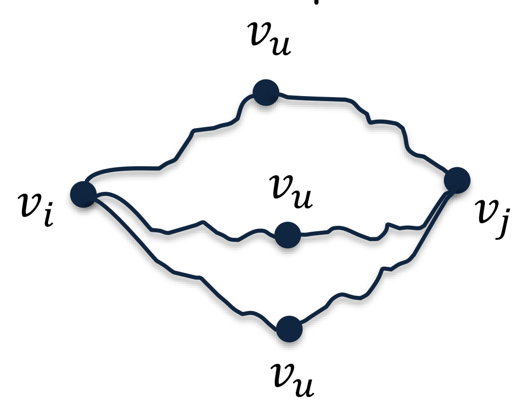

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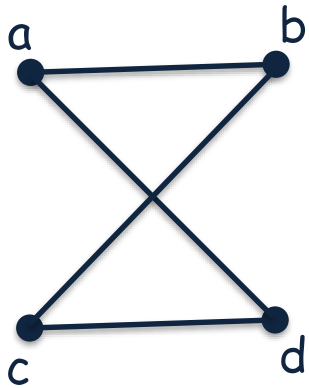

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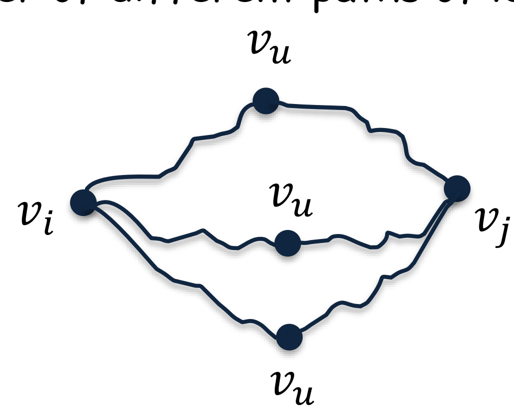
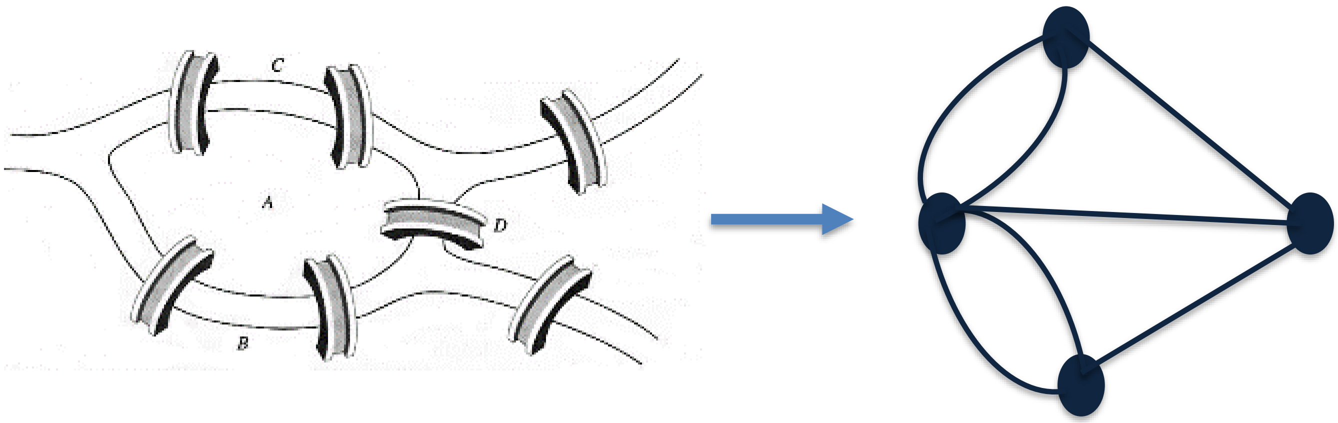


c_{ij} : the number of different paths of length (k+1) from v_i to v_j

Euler Paths and Circuits



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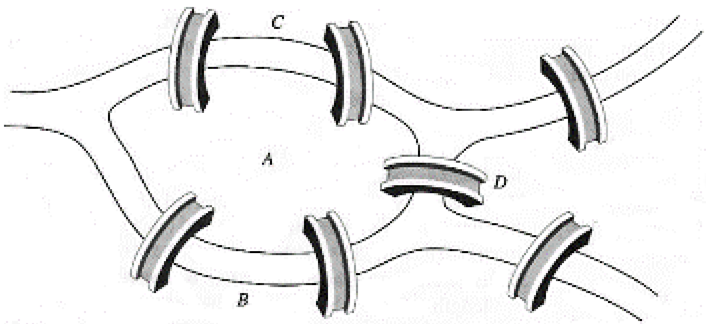

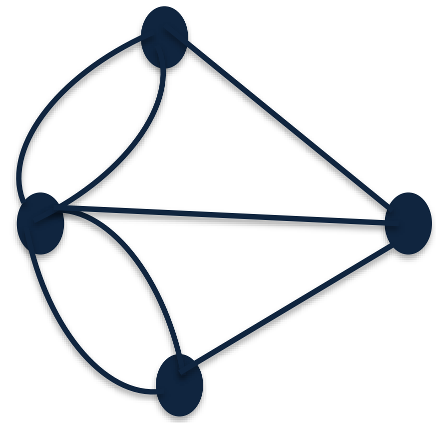

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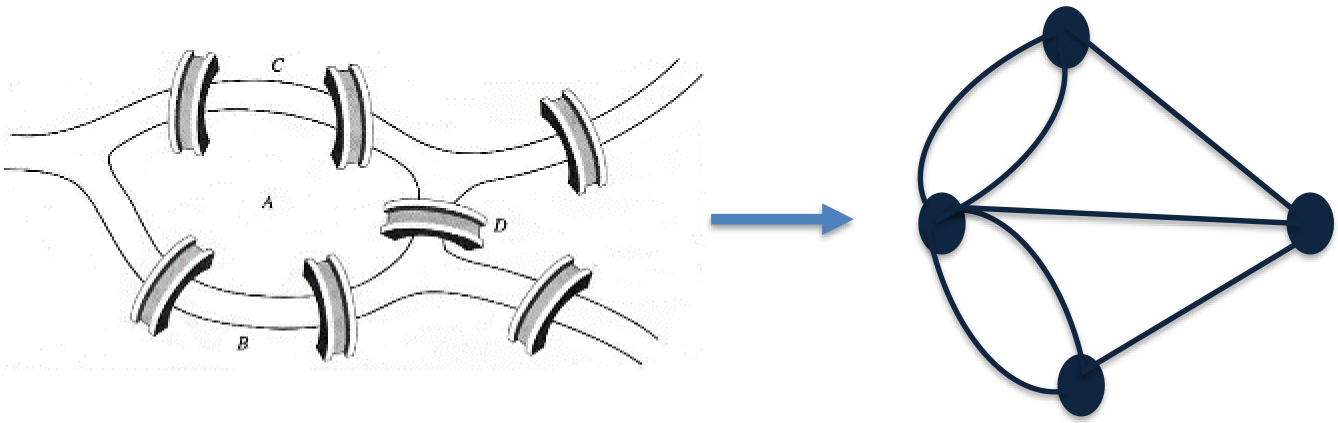

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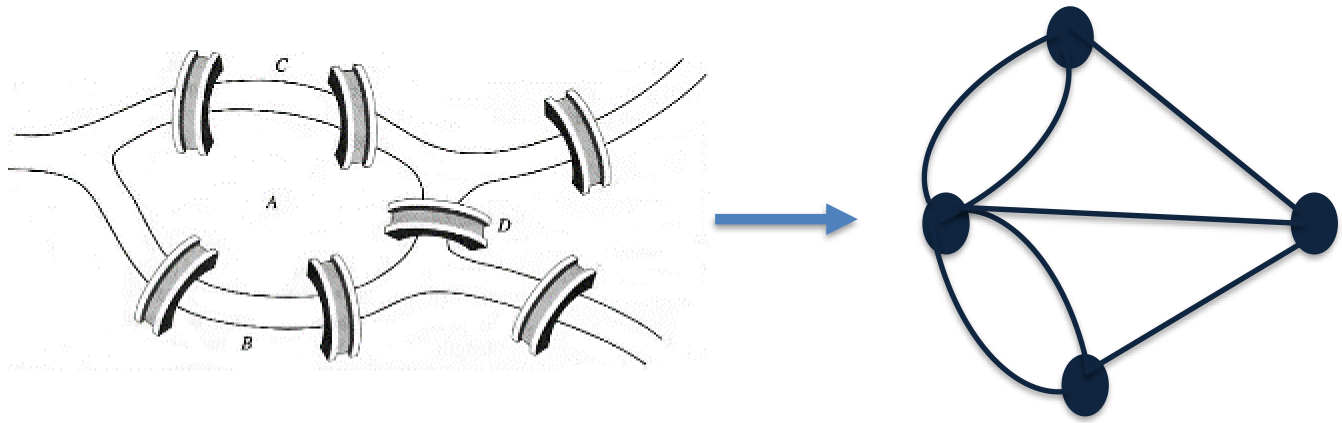
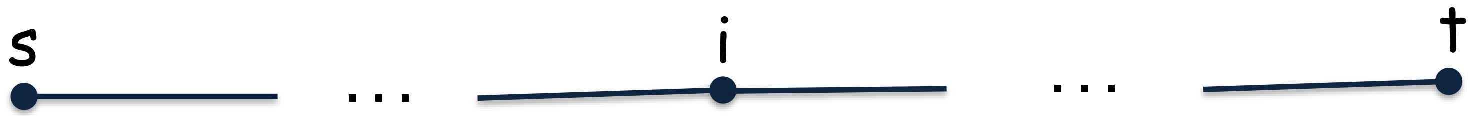


- Euler circuit is a simple circuit that contains every edge of G.
- Euler path is a simple path that contains every edge of G
- Does this graph have an Euler path or Euler circuit?

Euler Paths and Circuits



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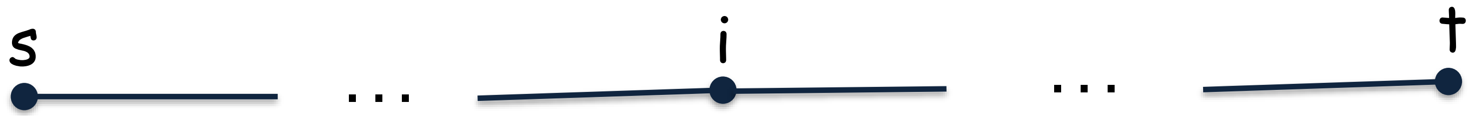
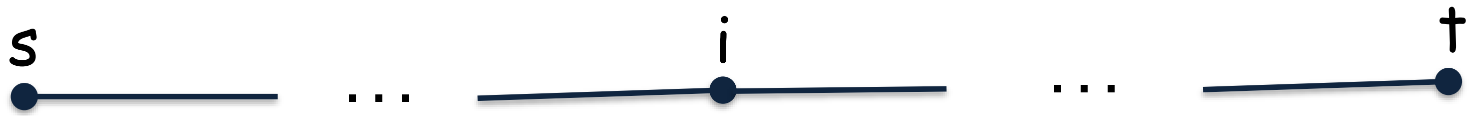


- when you pass a vertex, you add two to the degree of it.

Euler Paths and Circuits



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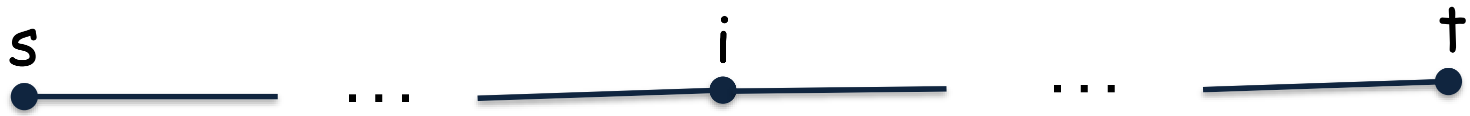
- the degree of starting node and ending node just one or odd number

Euler Paths and Circuits



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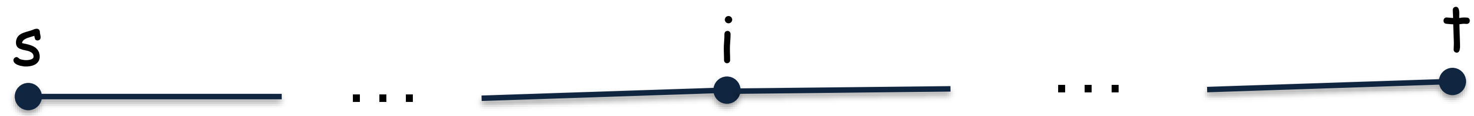
Euler Paths and Circuits



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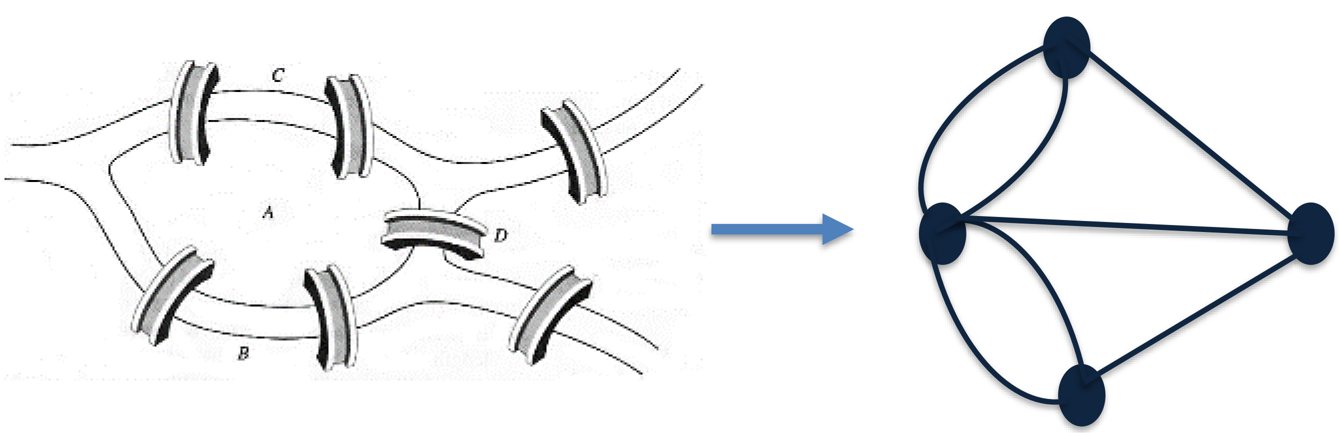

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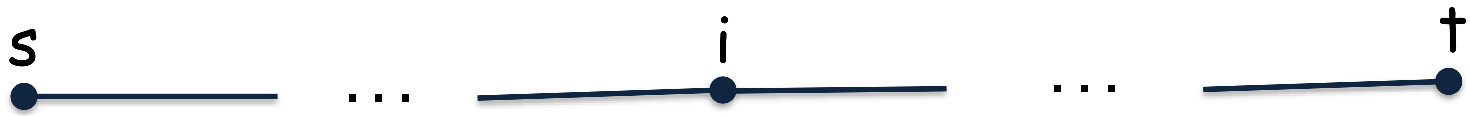

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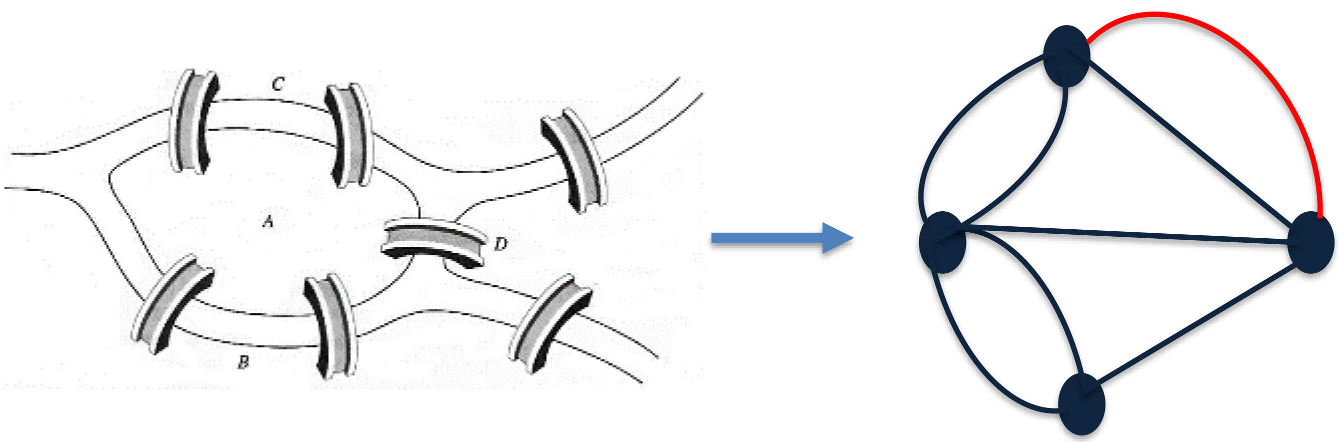

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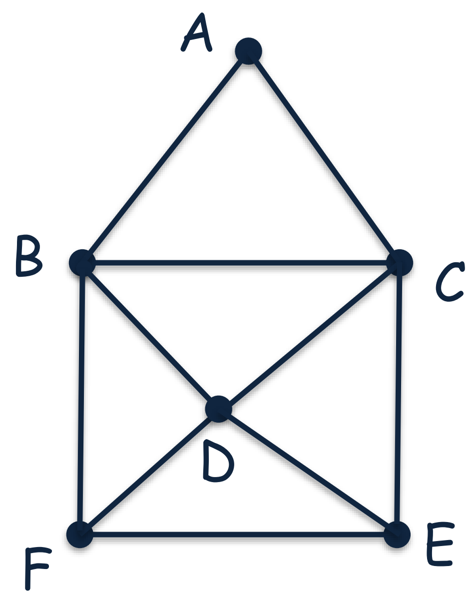

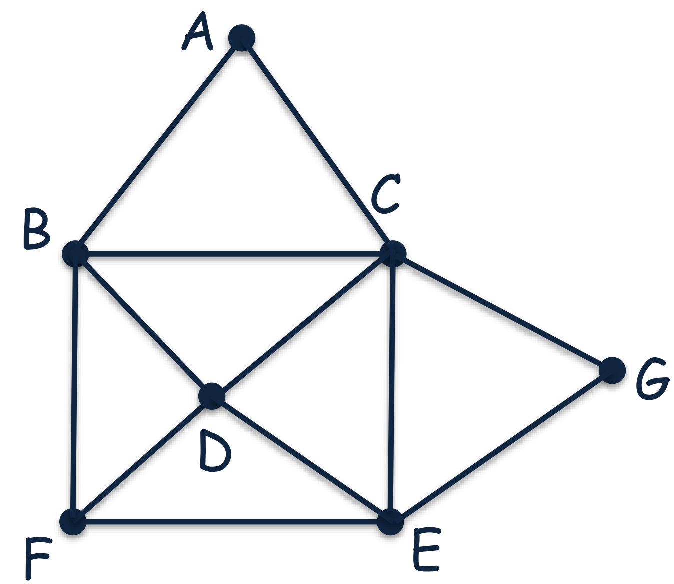

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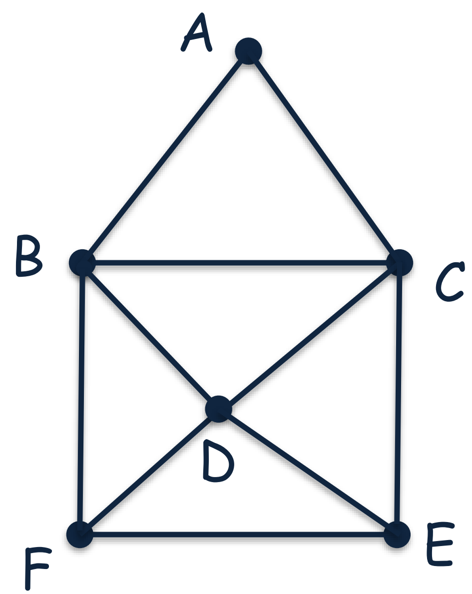
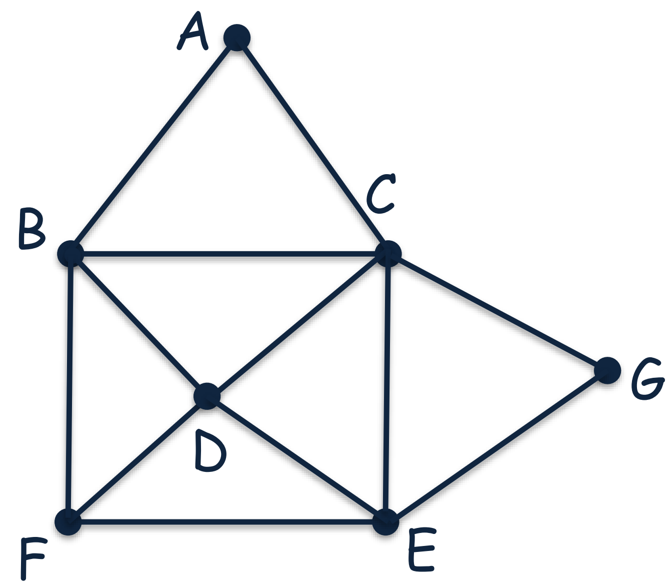


F-B-A-C-B-D-F-E-D-C-E



F-B-D-E-G-C-E-F-D-C-A-B-C

Hamilton Paths and Circuits

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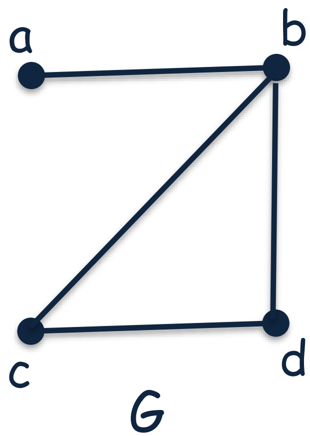

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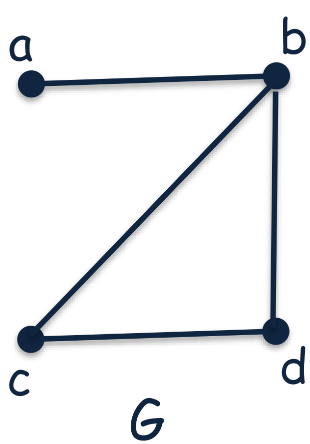

- Does G contain a Hamilton path or circuit ?

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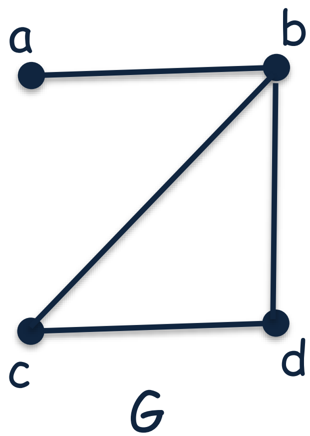

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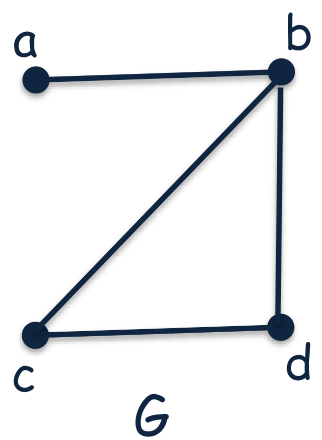

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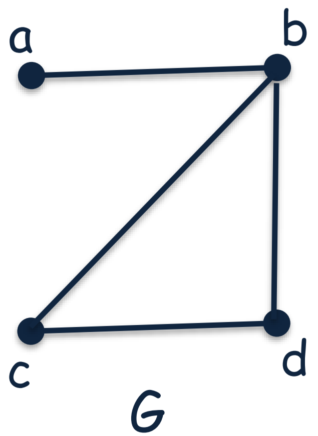

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a graph with a vertex of degree one cannot have a Hamilton circuit

SSSP

- given a weighted graph $G=(V,E)$ and a source vertex s in V, find the shortest path from s to every other vertex in V

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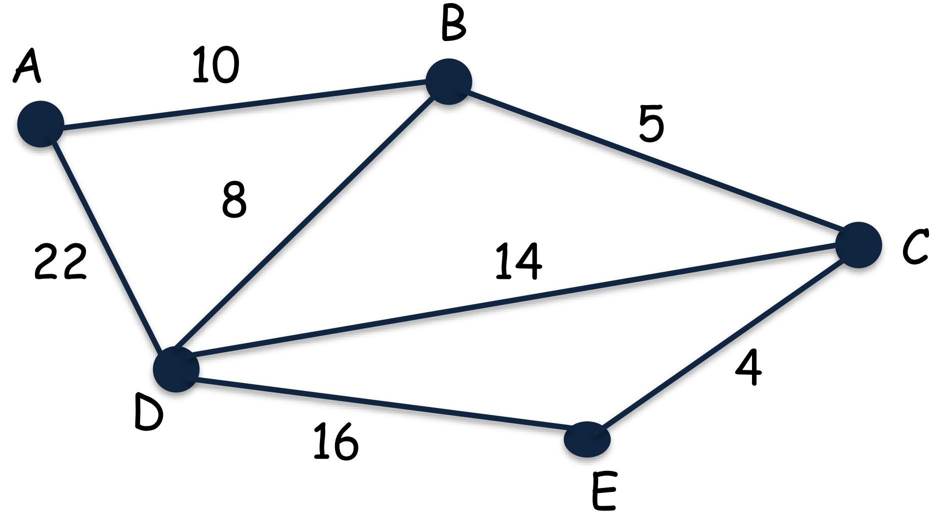

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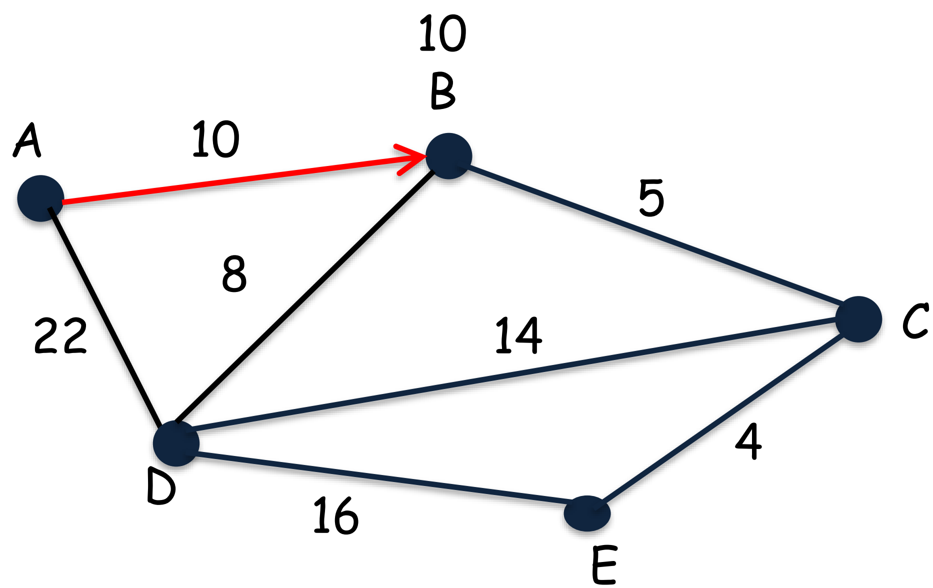

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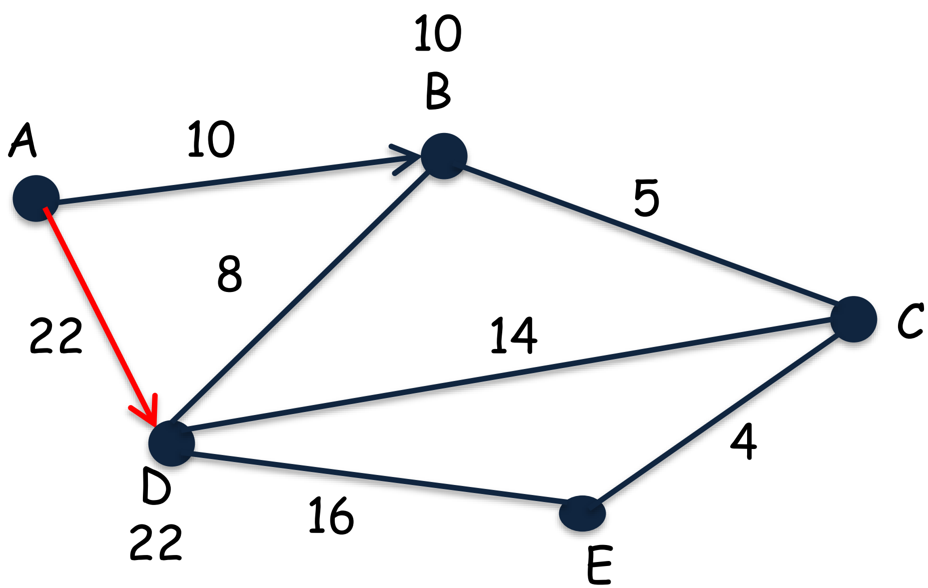

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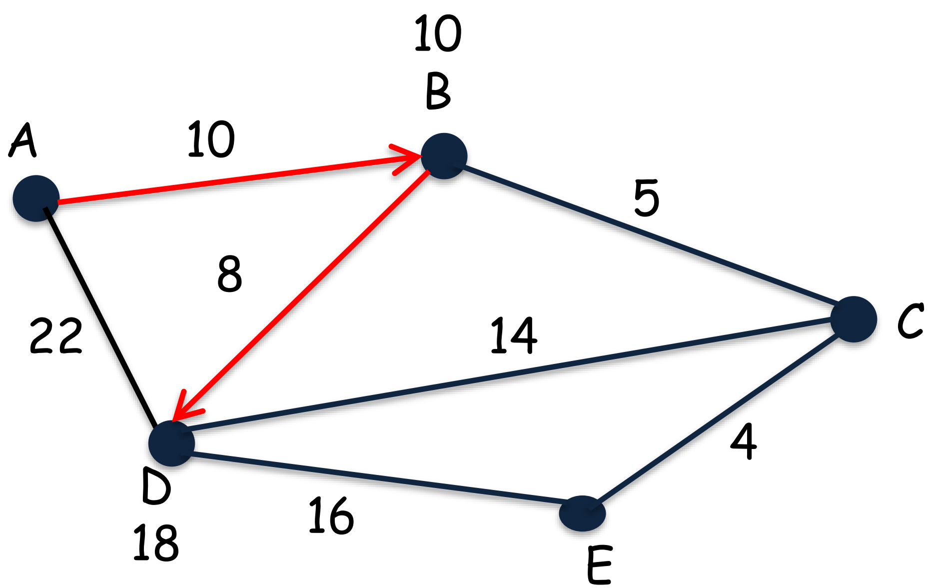

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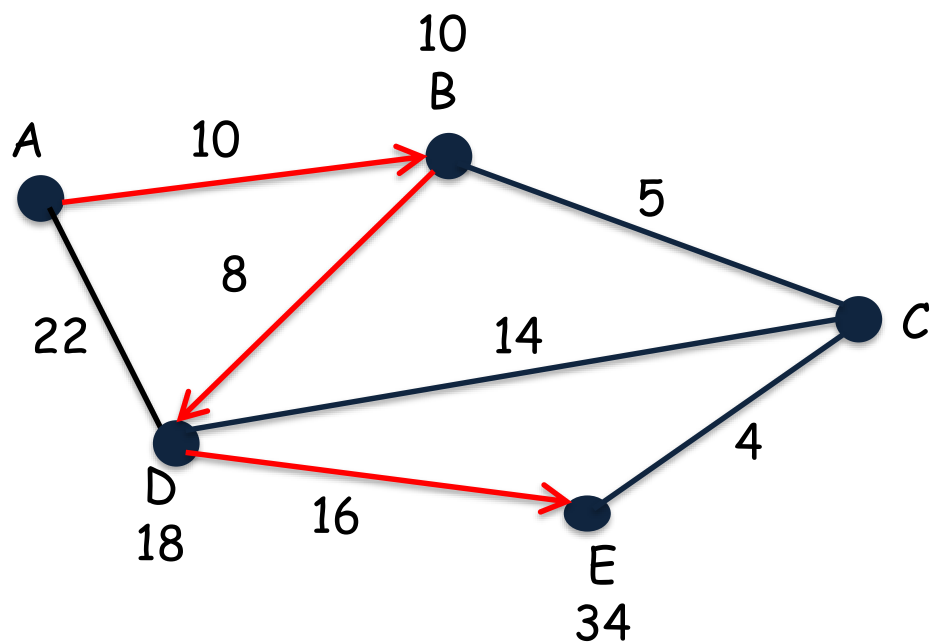

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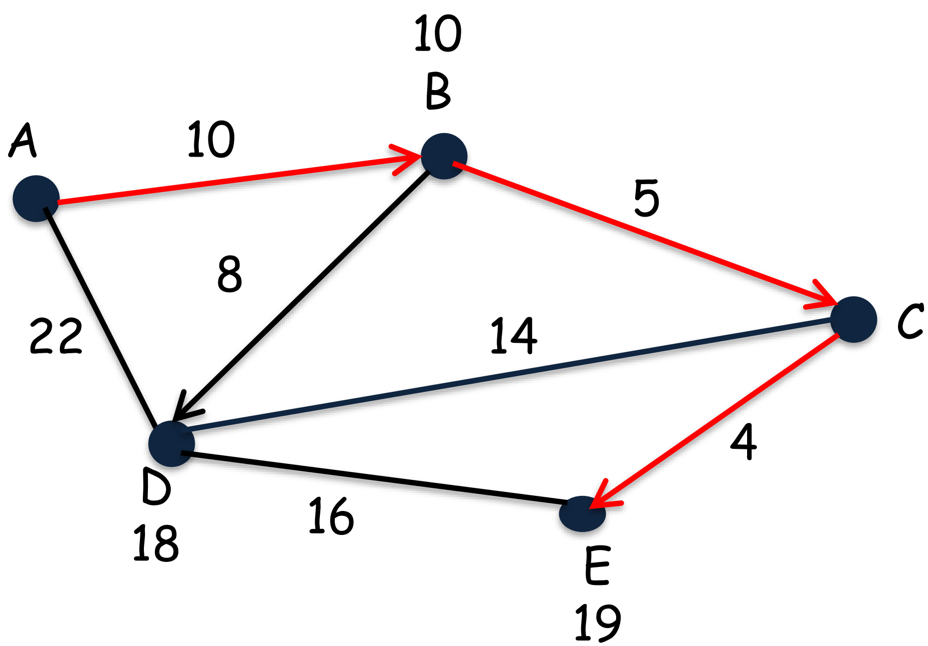

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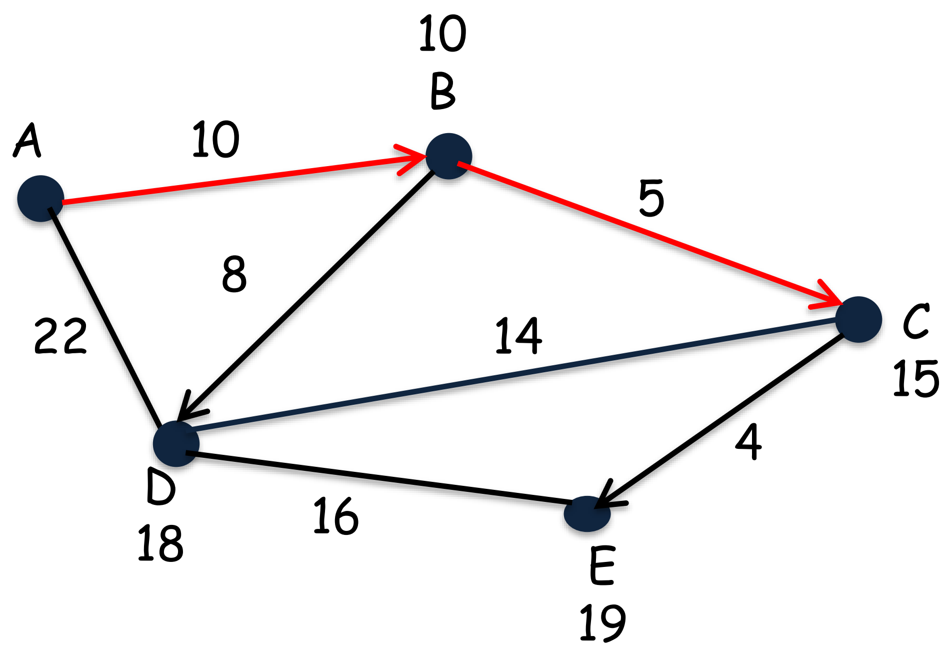

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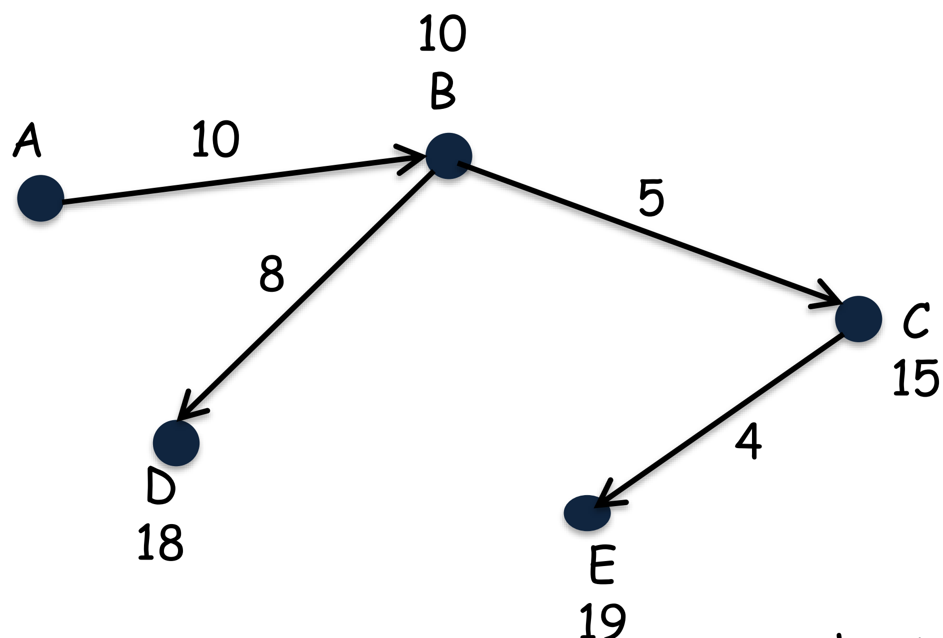

shortest-paths tree

SSSP

- given a weighted graph $G=(V,E)$ and a source vertex s in V, find the shortest path from s to every other vertex in V

- Three cases :

SSSP

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  - the weight of each edge fixed as 1

    --BFS--

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  - the weight of each edge non-negative
    --Dijkstra—

SSSP

- given a weighted graph G=(V,E) and a source vertex s in V, find the shortest path from s to every other vertex in V

- Three cases :

  - the weight of each edge fixed as 1

    --BFS--

  - the weight of each edge non-negative

    --Dijkstra—

  - the weight of each can be negative

    --Belmann/Ford--

Relaxation

- For each vertex v in V, initialize two parameters :

Relaxation

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Initialize (G, s)

```
for each vertex v i V
    v.dis = ∞
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s.dis = 0
```

Relaxation

- relaxing an edge (u,v) : testing whether the shortest path to the vertex v can be improved by going through the vertex u

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Relax(u, v)

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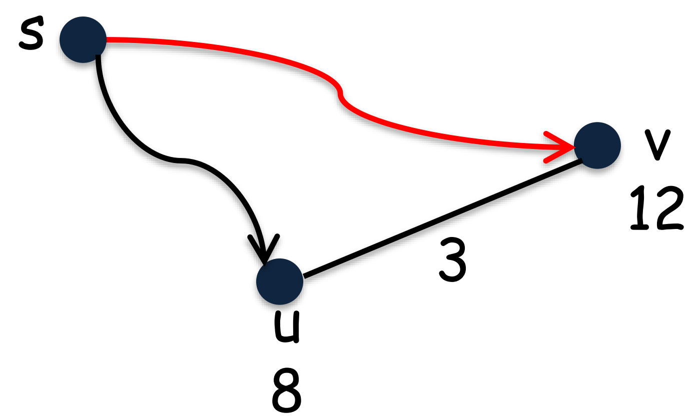

Relaxation

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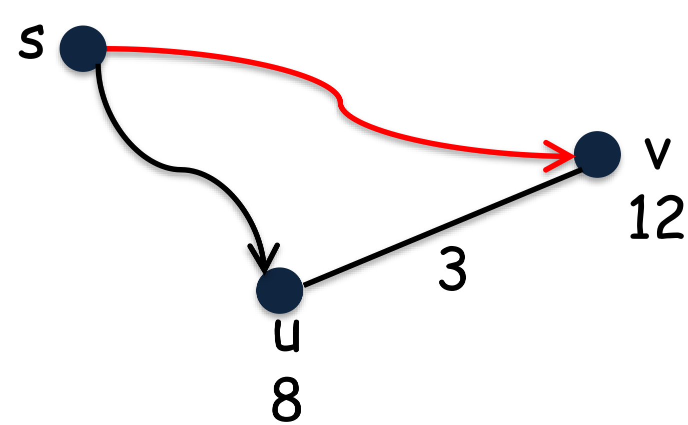

v.dis > u.dis + w(u,v)
12 > 8 + 3

Relaxation

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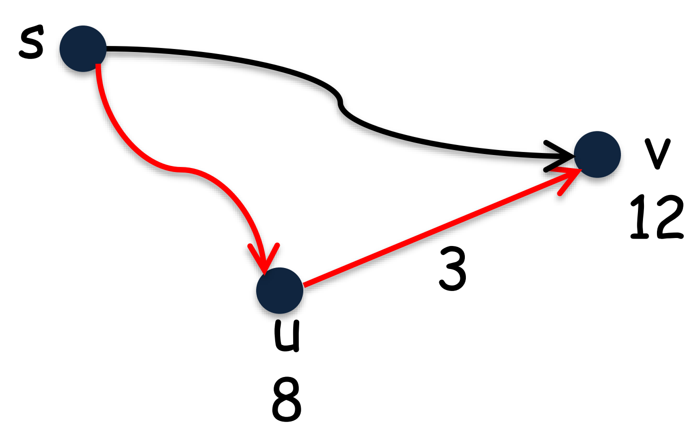


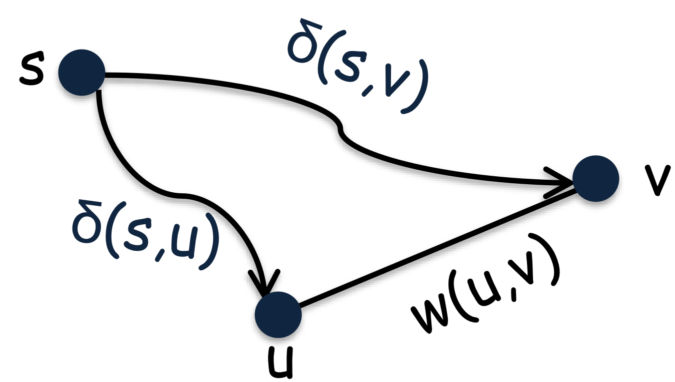
v.dis > u.dis + w(u,v)
12 > 8 + 3

→

v.dis = u.dis + w(u,v)
v.dis = 11
v.par = u

Relaxation

- Let δ(s,v) be the weight of the shortest path from source to the vertex v (after the termination of the program)

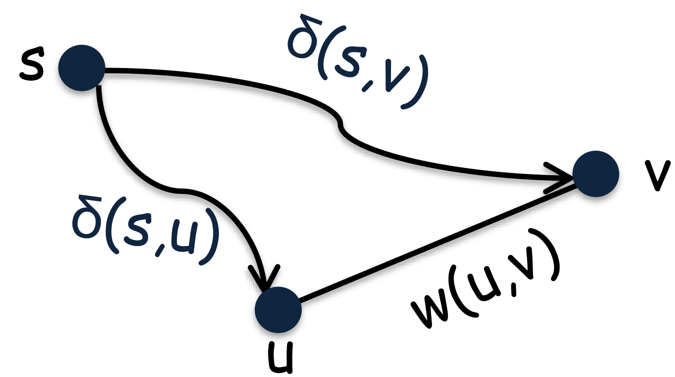


Relaxation

- Let $\delta(s,v)$ be the weight of the shortest path from source to the vertex v (after the termination of the program)

- For any edge (u,v) in E,

$$\delta(s,v) \leq \delta(s,u) + w(u,v)$$



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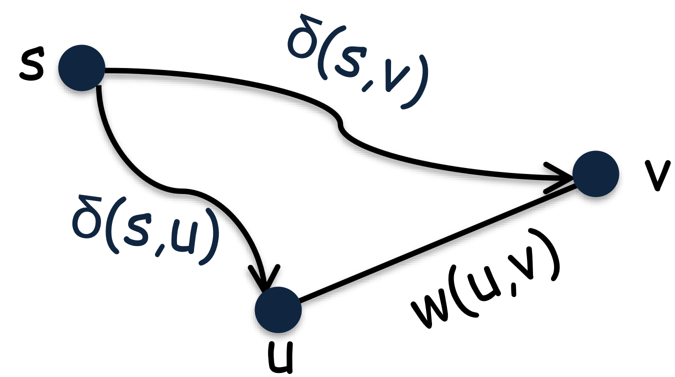
- For all vertices v in V,

$$v.dis \ge \delta(s,v)$$



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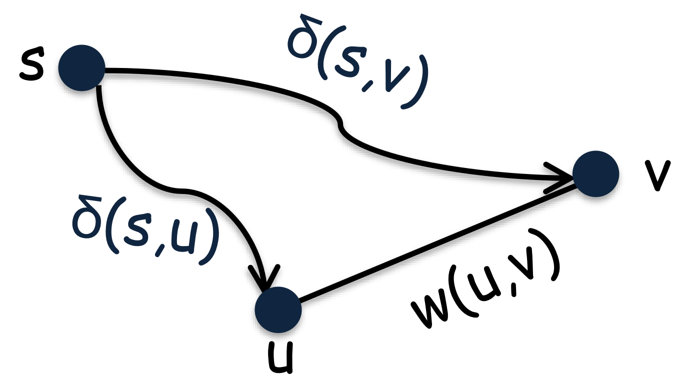

- If there is no path from s to v, then

$$v.dis = \delta(s,v) = \infty$$

Dijkstra's Algorithm

Dijkstra(G,s)

```
for each u of V
     u.key = ∞
     u.par = nil
s.key = 0
initialize an empty set S
create a minimum priority Q on V
while Q ≠ { }
     u = ExtractMin(Q)
     S = S ∪ {u}
     for each v of Adj(u)
          if  v.dis > u.dis + w(u,v)
               v.dis = u.dis + w(u,v)
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```

Dijkstra's Algorithm

Dijkstra(G,s)

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for each u of V                 Initialize(G,s)
    u.key = ∞                   O(|V|)
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Initialize(G,s)
O(|V|)

O(|V|)

O(|V|.log|V|)

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Initialize(G,s)
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O(|V|.log|V|)

Relax(u,v)
O(1)

Dijkstra's Algorithm

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```

Initialize(G,s)
O(|V|)

O(|V|)

O(|V|.log|V|)

O(|E|.log|V|)

Relax(u,v)
O(1)

Dijkstra's Algorithm

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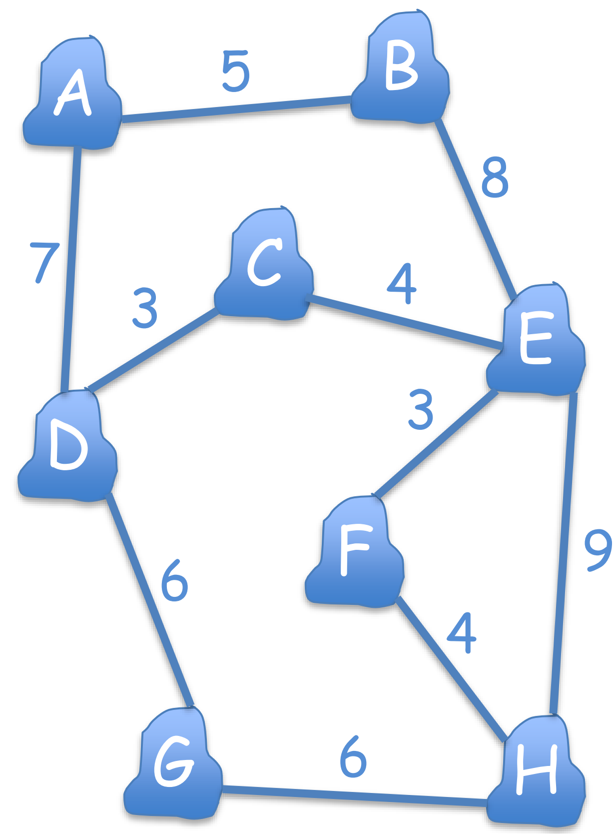

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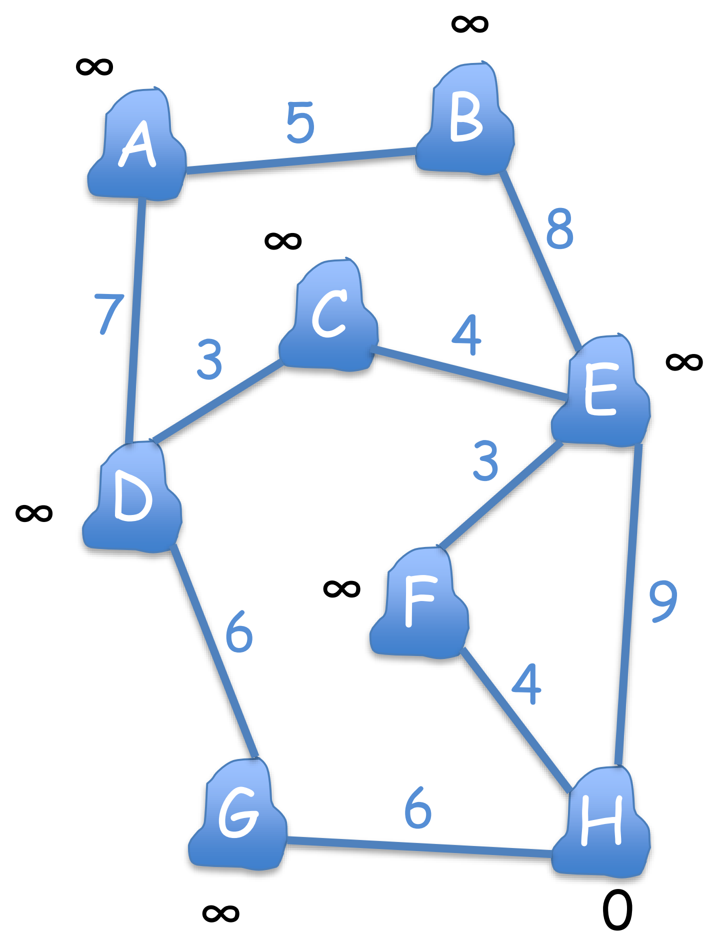

H F G E D C B A
S = { }

Dijkstra's Algorithm

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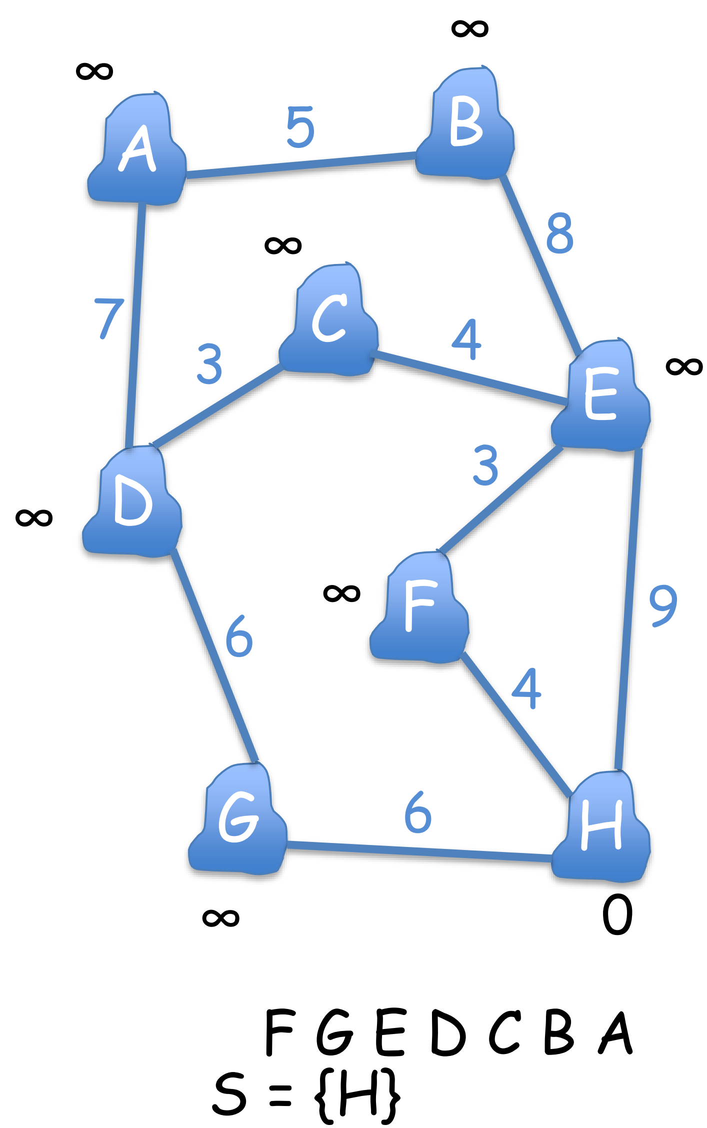

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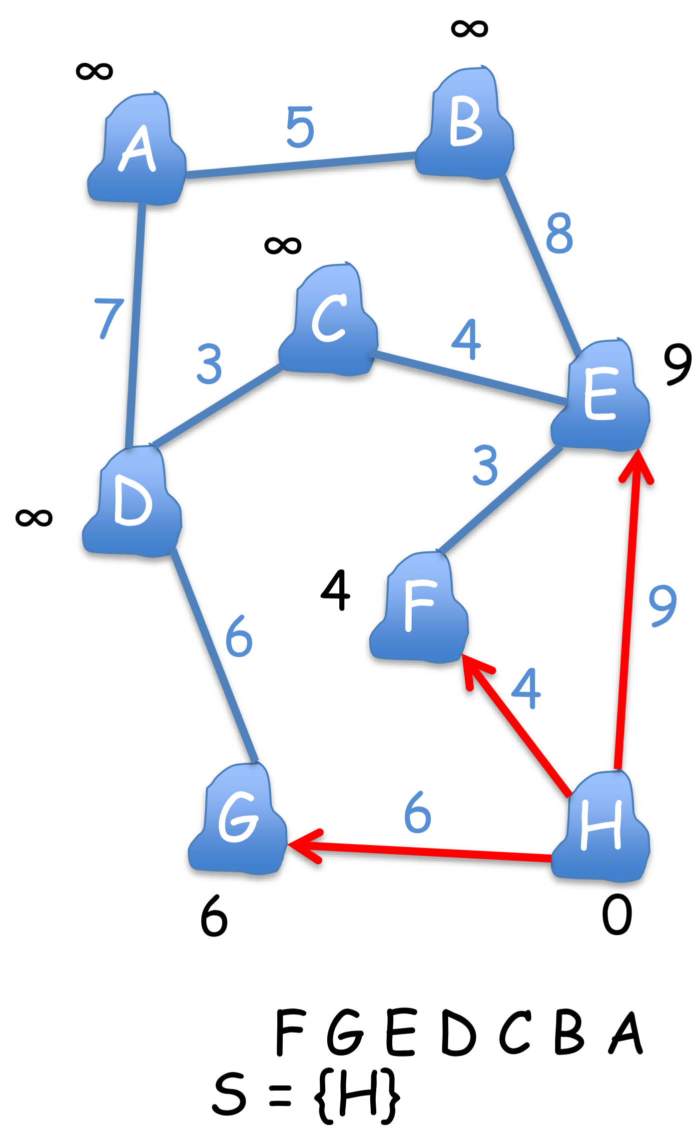

Dijkstra's Algorithm

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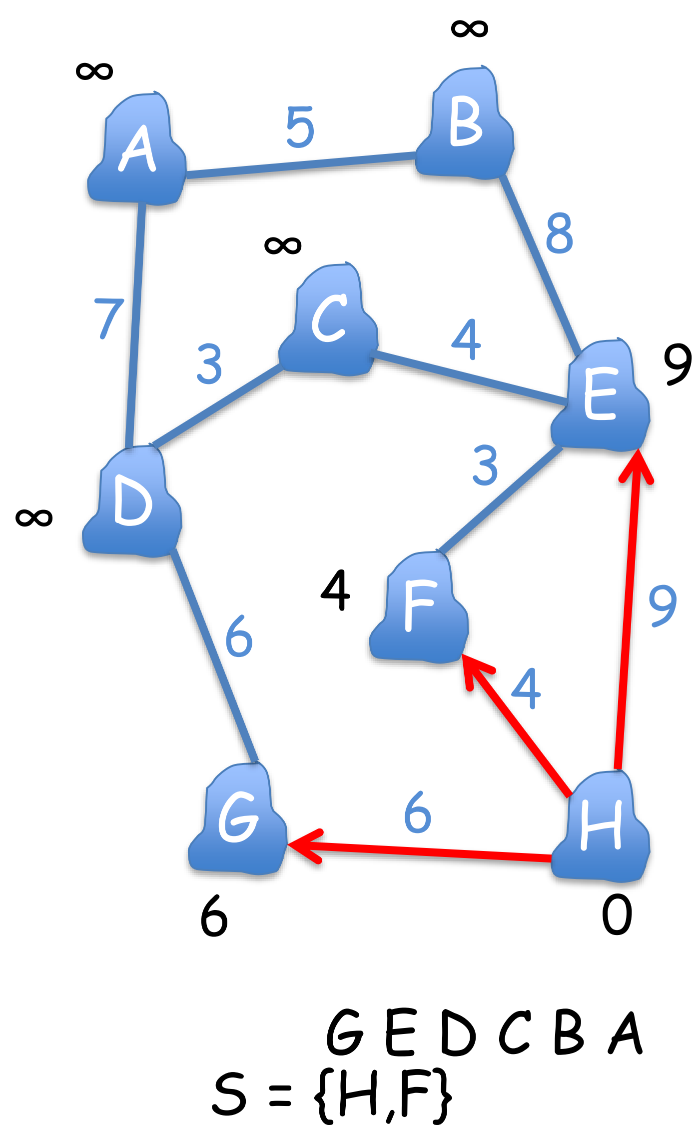

Dijkstra's Algorithm

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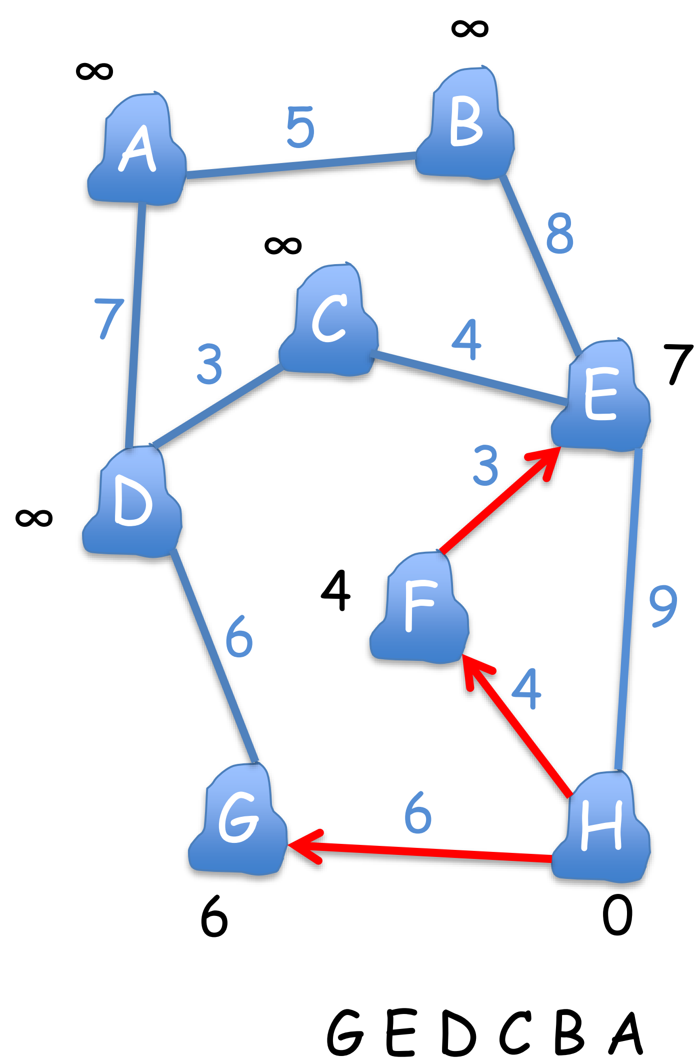

S = {H,F}

Dijkstra's Algorithm

Dijkstra(G,s)

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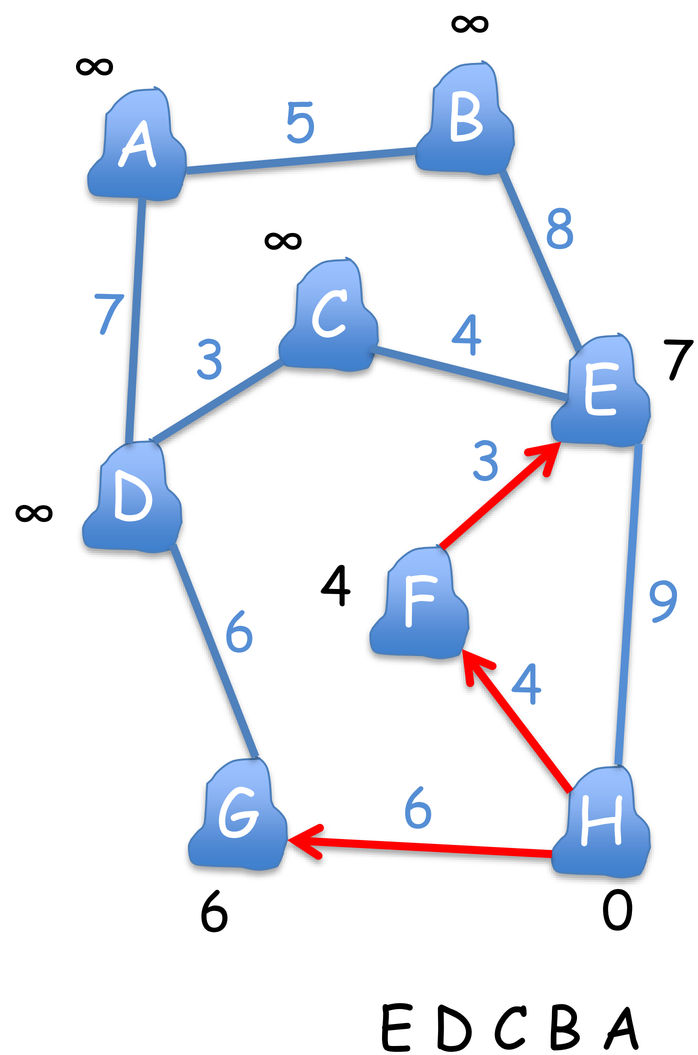

S = {H,F,G}

Dijkstra's Algorithm

Dijkstra(G,s)

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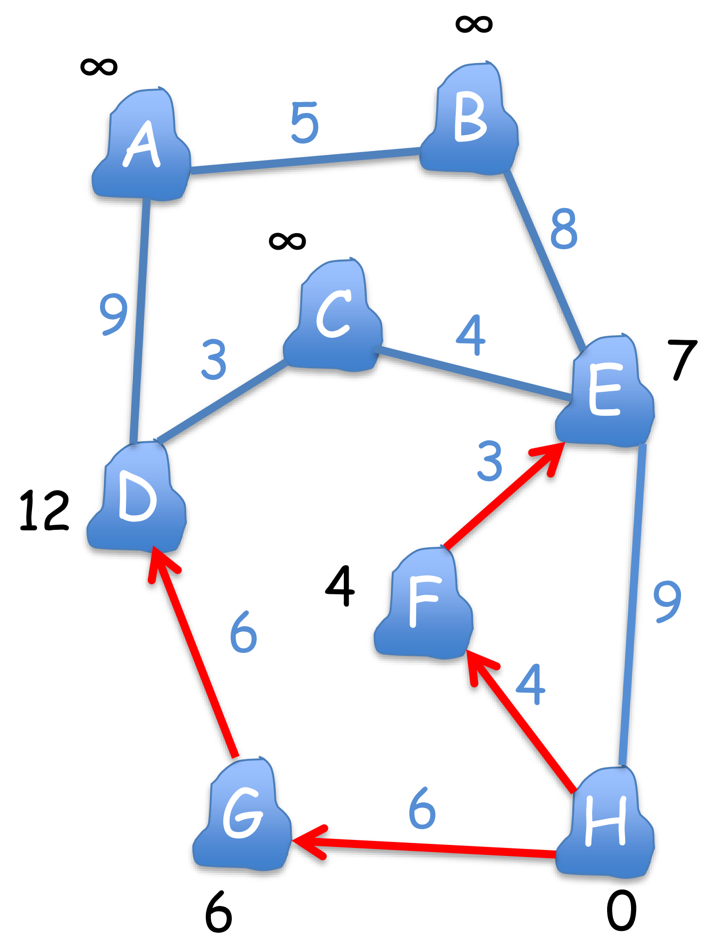

E D C B A
S = {H,F,G}

Dijkstra's Algorithm

Dijkstra(G,s)

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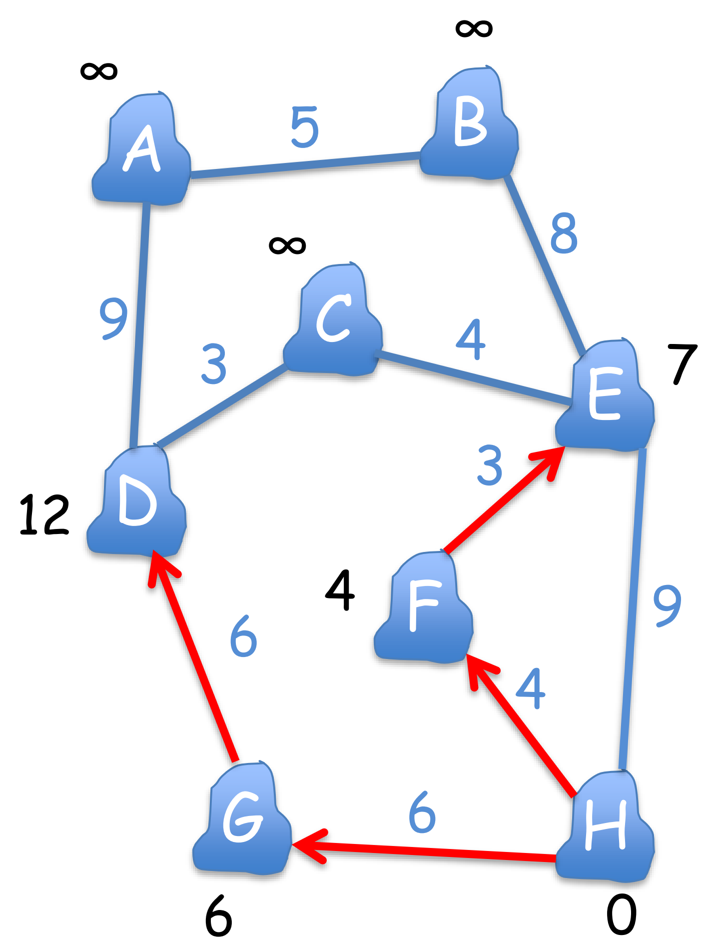

D C B A

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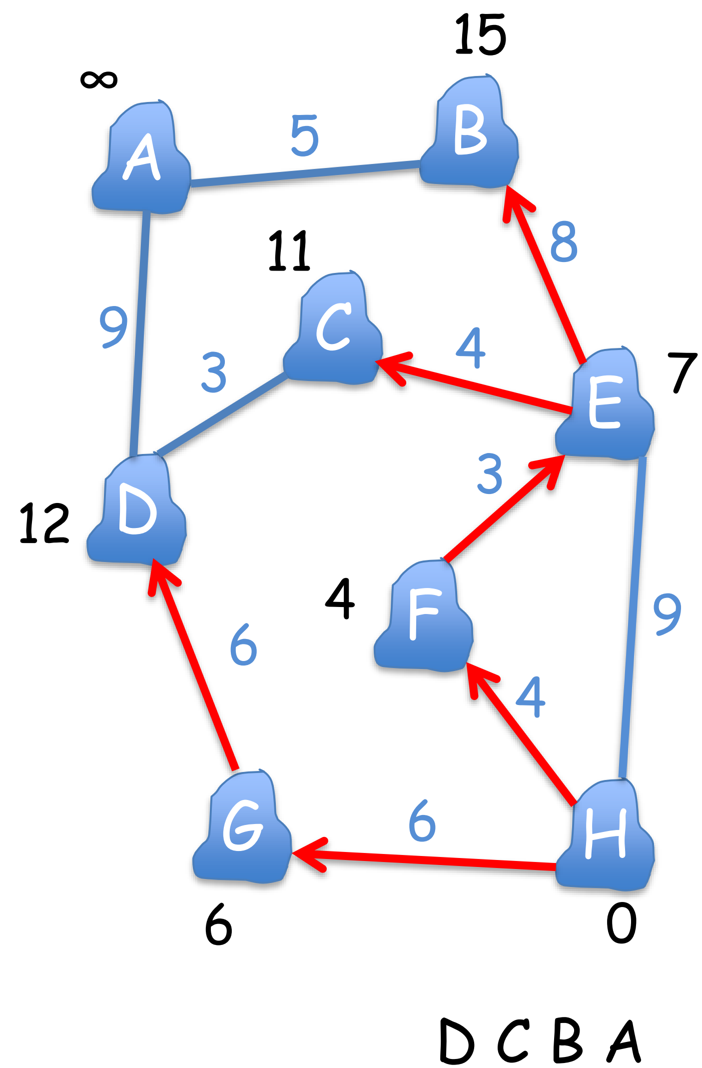

S = {H,F,G,E}

Dijkstra's Algorithm

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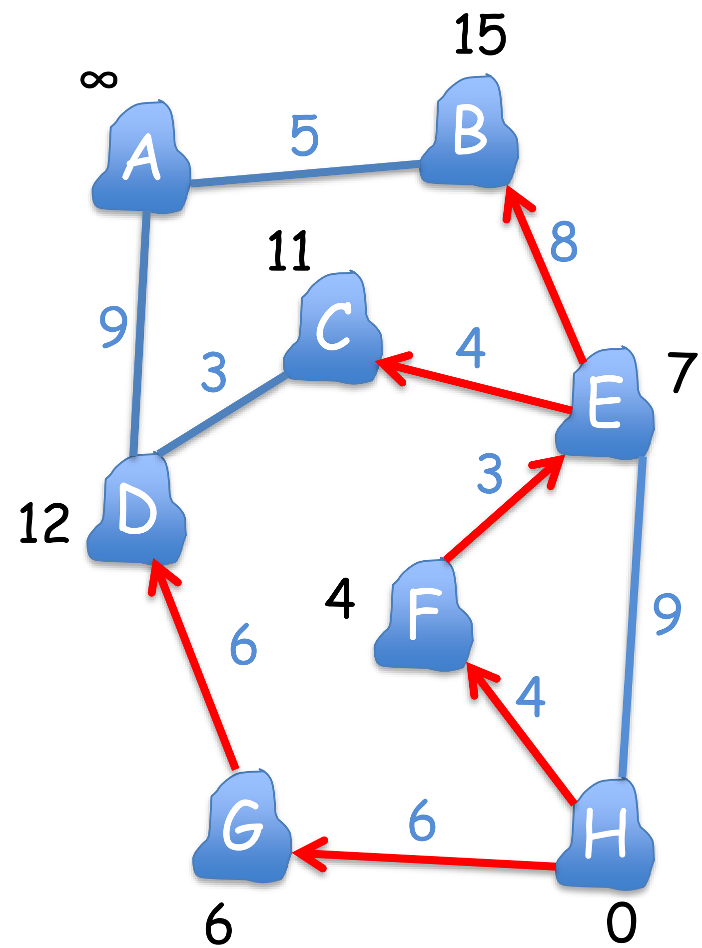

D B A
S = {H,F,G,E,C}

Dijkstra's Algorithm

Dijkstra(G,s)

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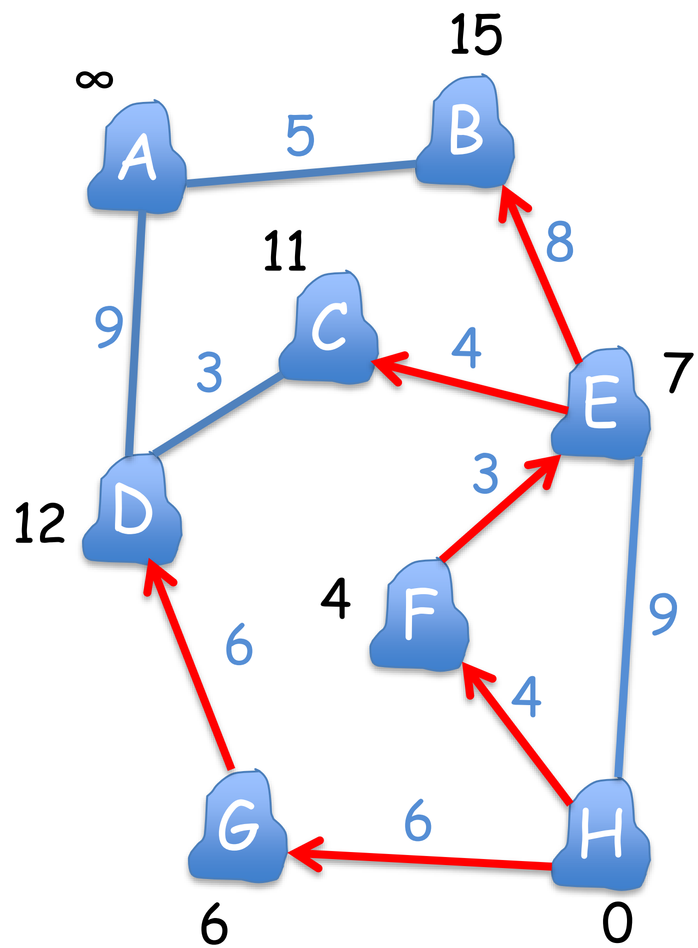

B A

S = {H,F,G,E,C}

Dijkstra's Algorithm

Dijkstra(G,s)

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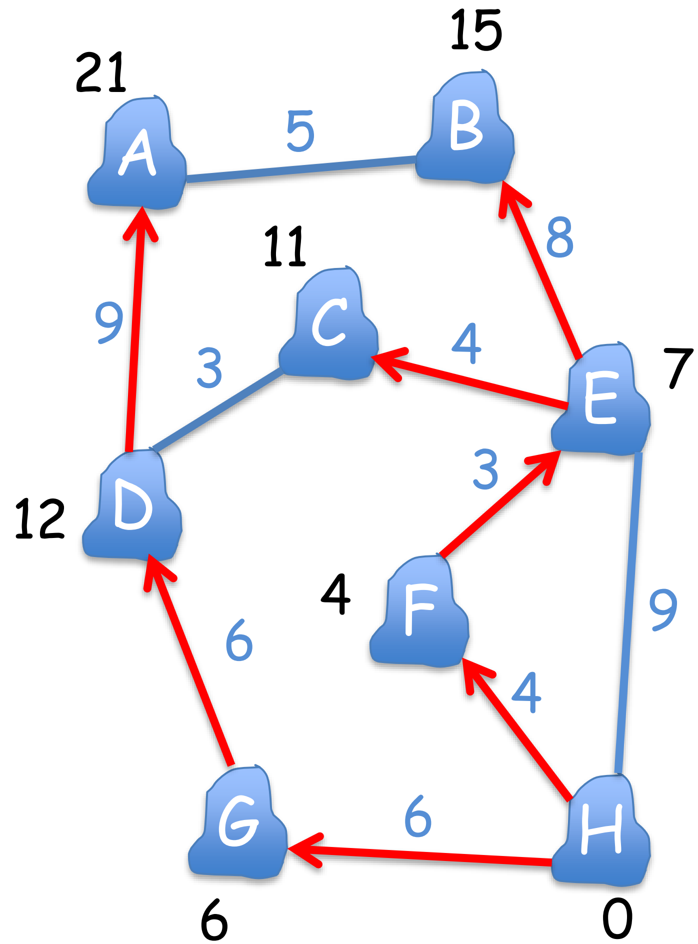

B A

S = {H,F,G,E,C,D}

Dijkstra's Algorithm

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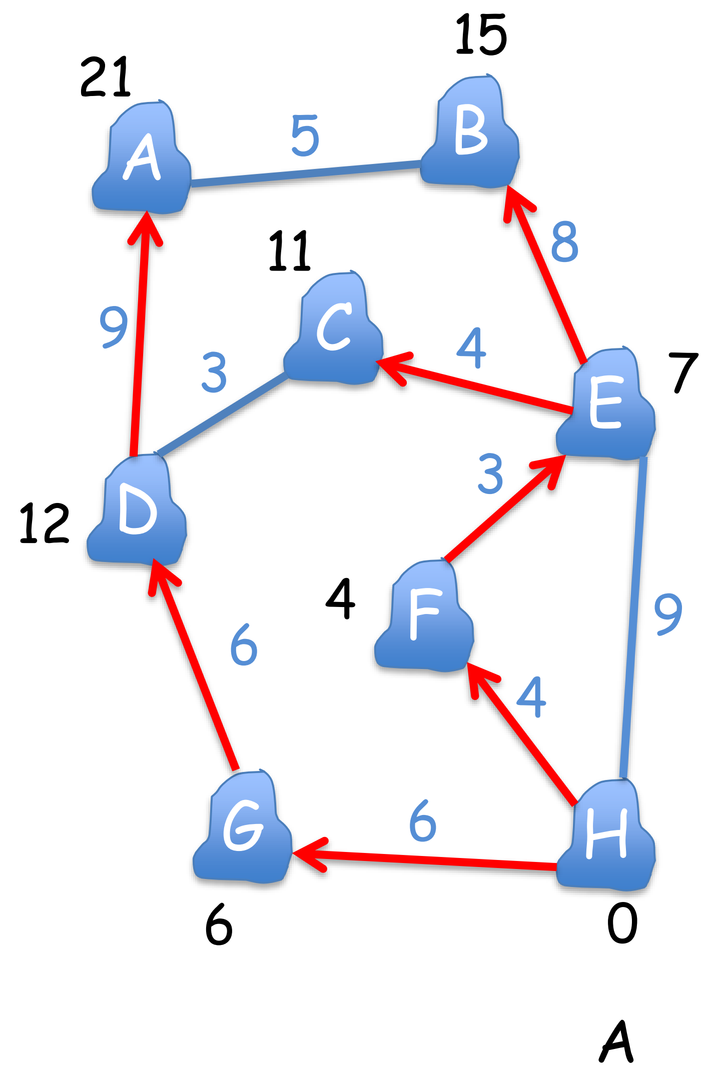

S = {H,F,G,E,C,D,B}

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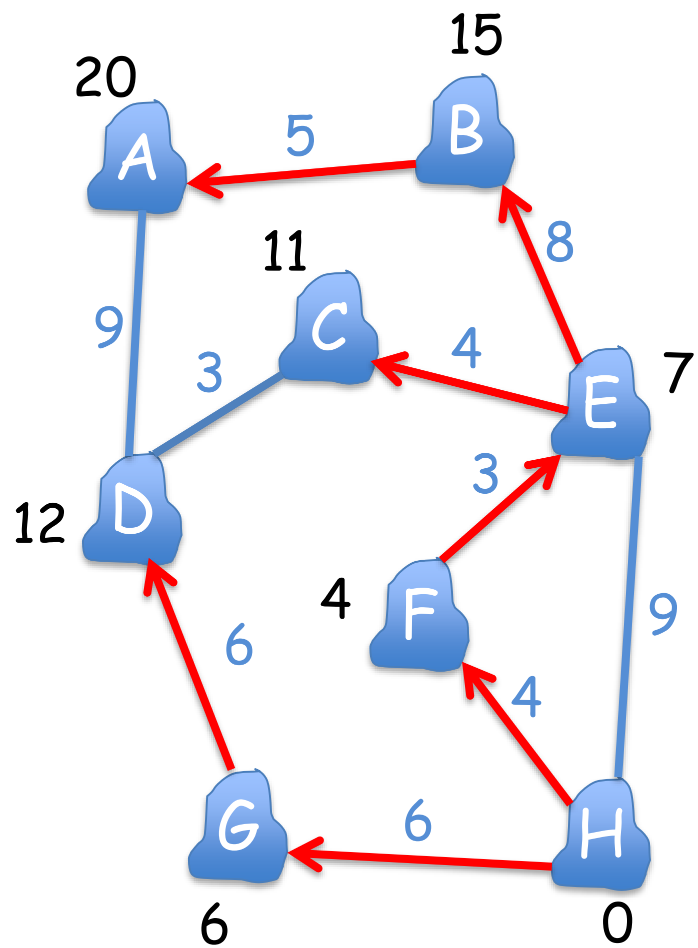

A

S = {H,F,G,E,C,D,B}

Dijkstra's Algorithm

Dijkstra(G,s)

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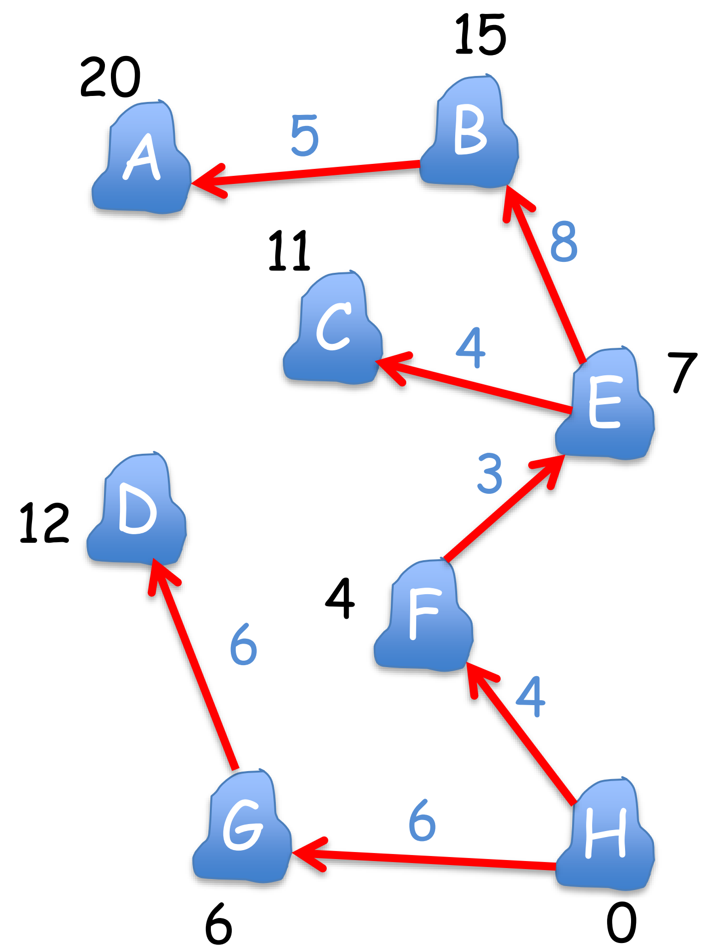

S = {H,F,G,E,C,D,B,A}

Bipartite Graphs

- a simple graph G is called bipartite if its vertex set V can be partitioned into two disjoint subsets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2

Bipartite Graphs

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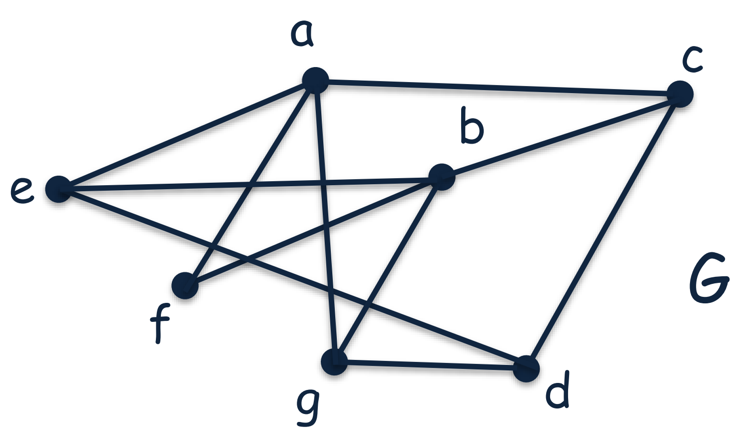

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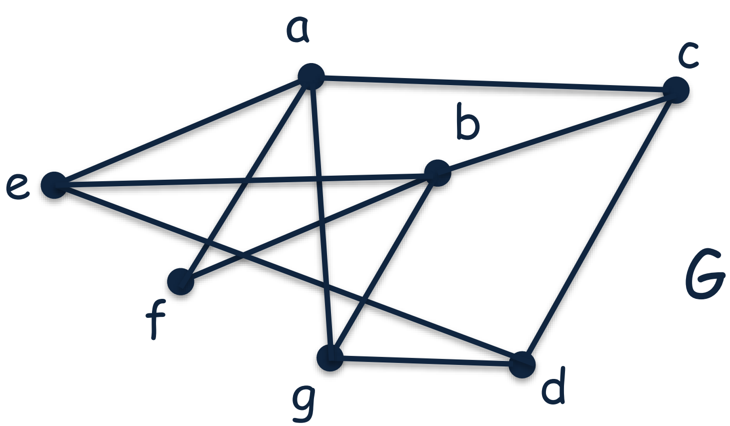
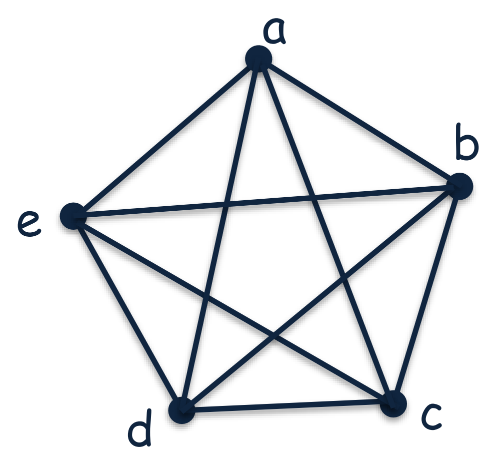


G



K_5

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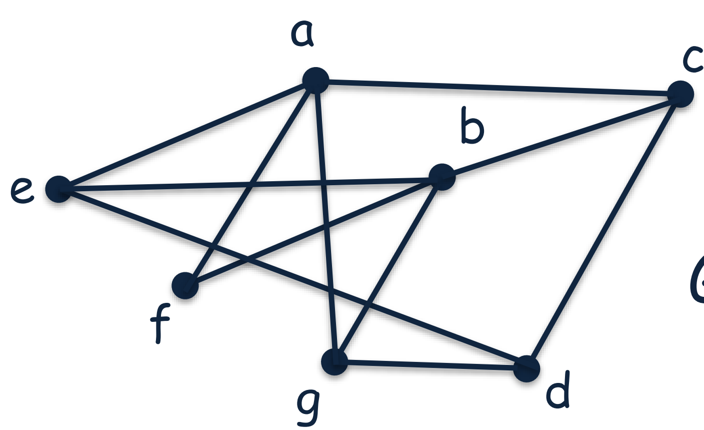

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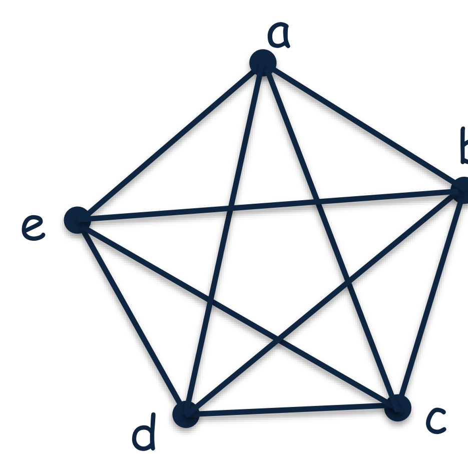
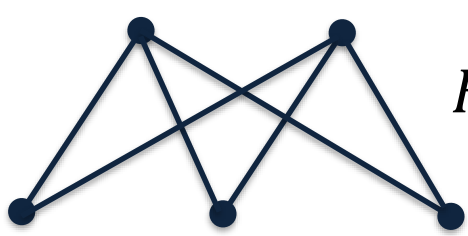


K_5



$K_{2,3}$

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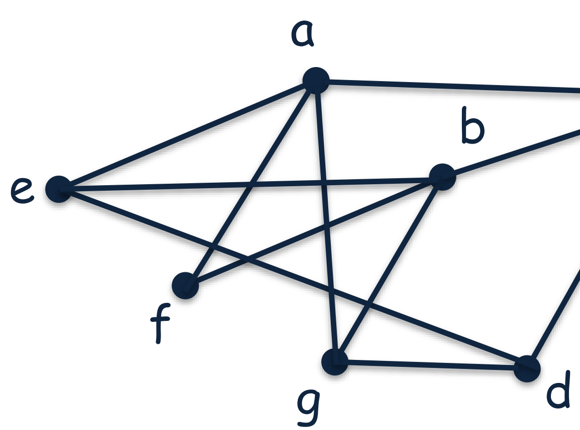
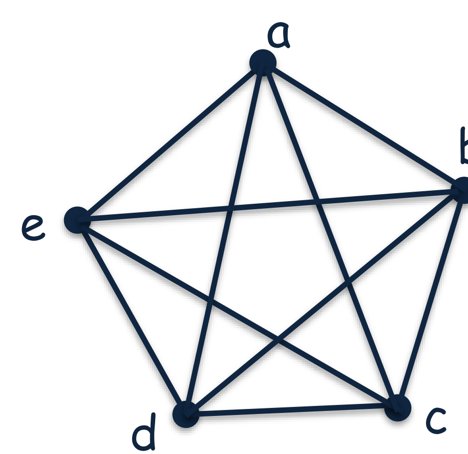
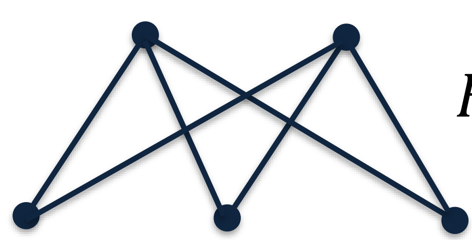


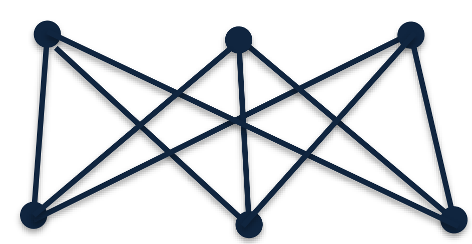
G



K_5



$K_{2,3}$



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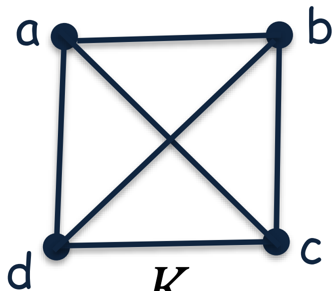
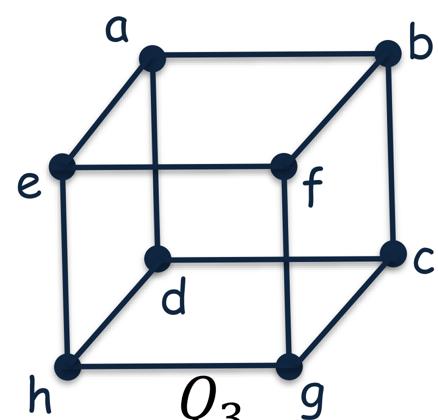
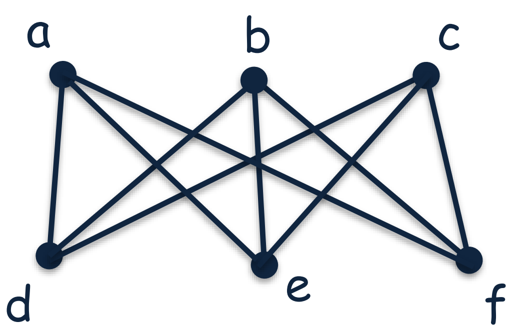


K_4



Q_3



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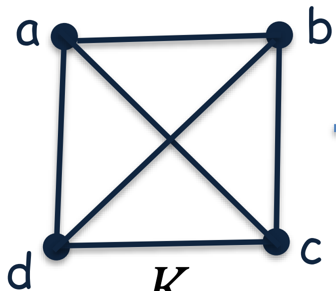

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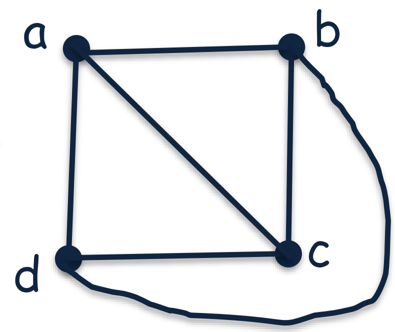

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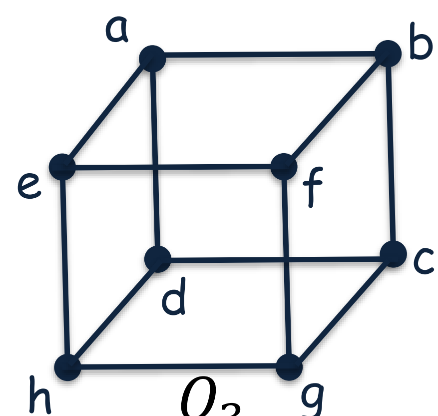

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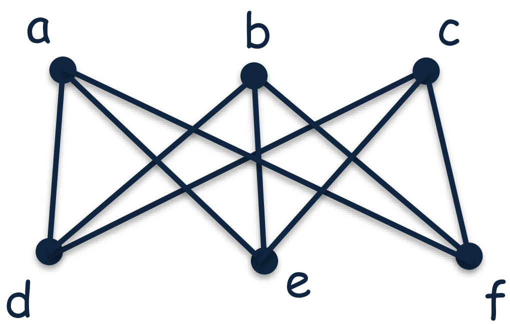

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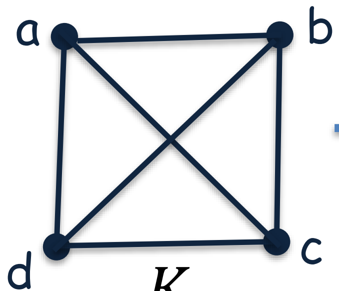

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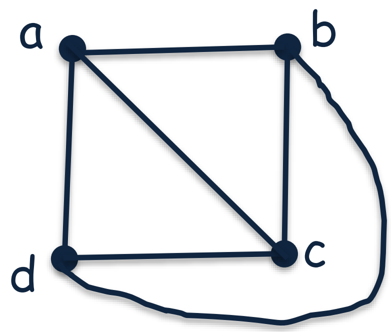

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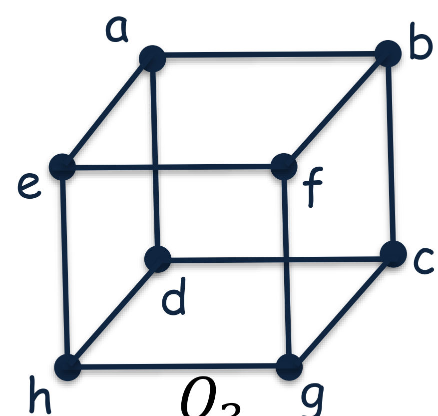

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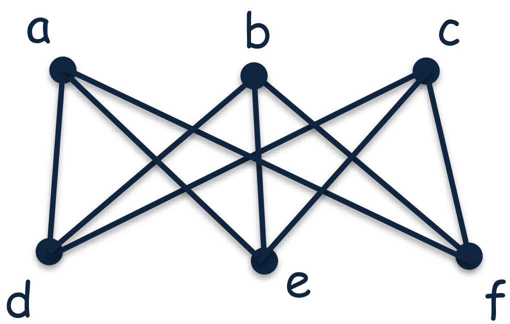
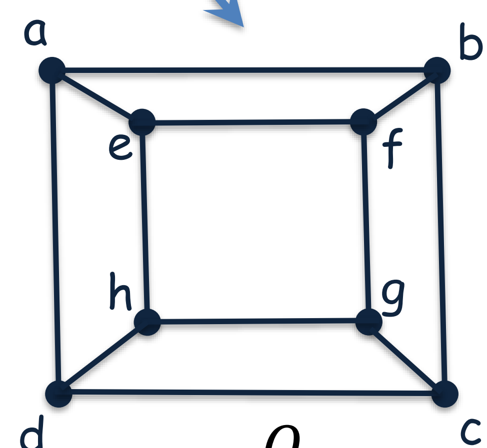


$K_{3,3}$



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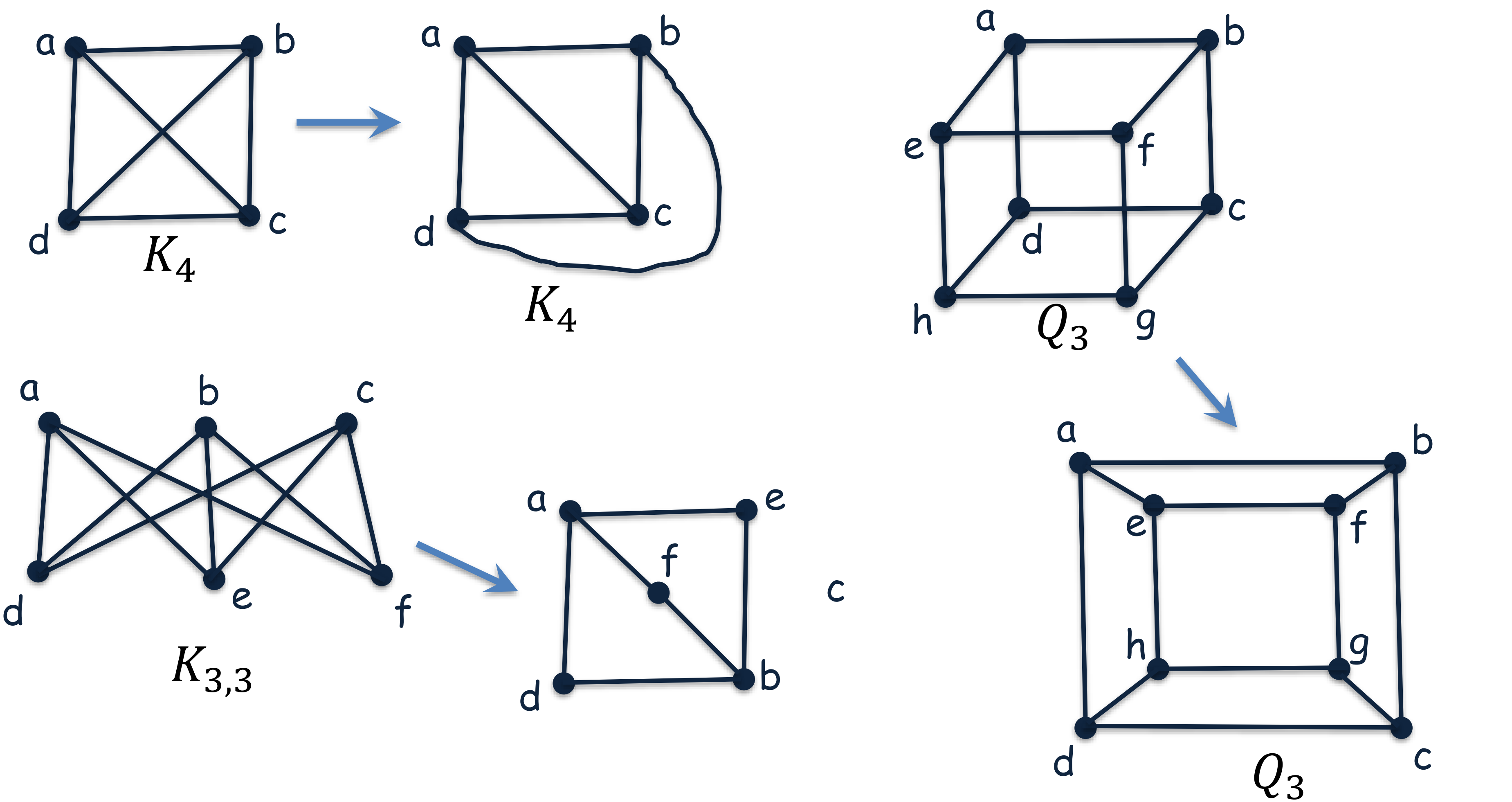

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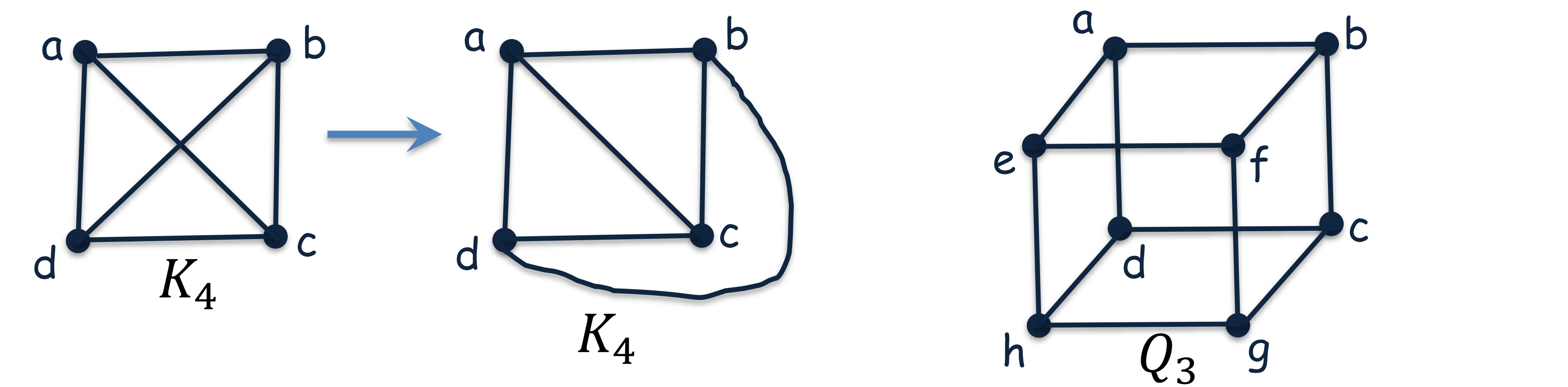

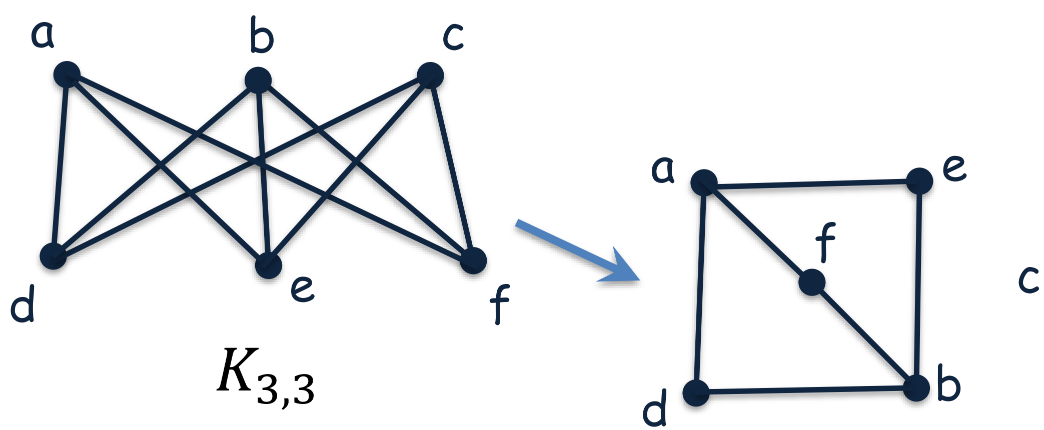

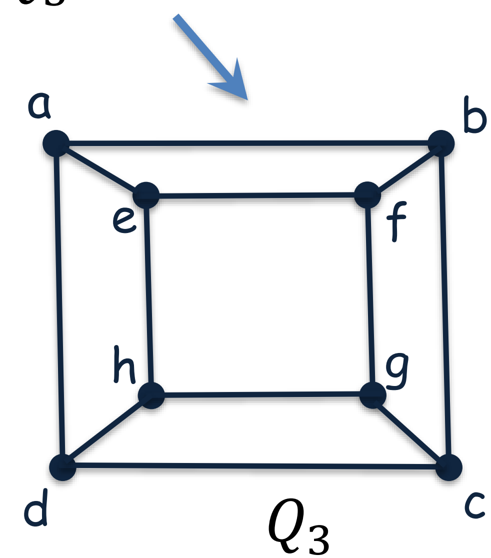

- $K_{3,3}$ cannot be drawn as planar graph

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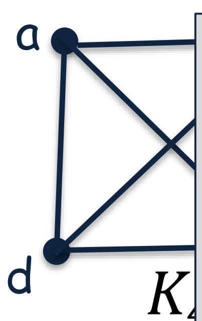
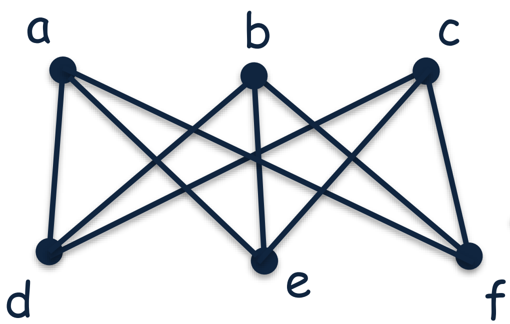


Euler Formula : Let G be connected simple graph with e edges and v vertices. Let r be the number of region in a planar representation of G. Then,

$$r = e - v + 2$$



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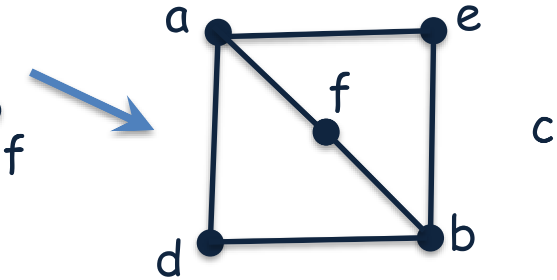

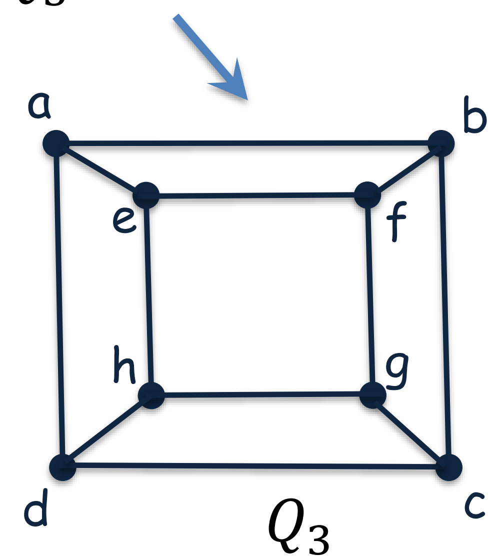

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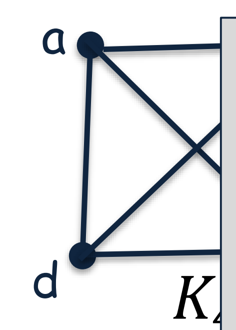

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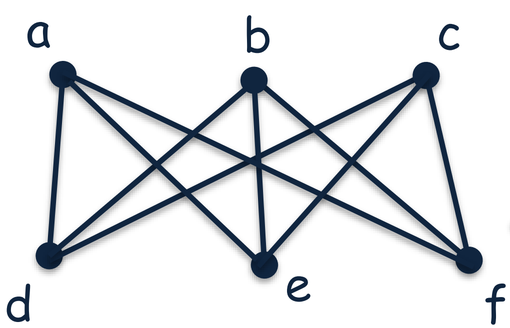

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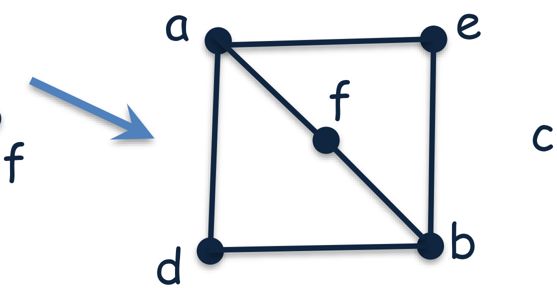

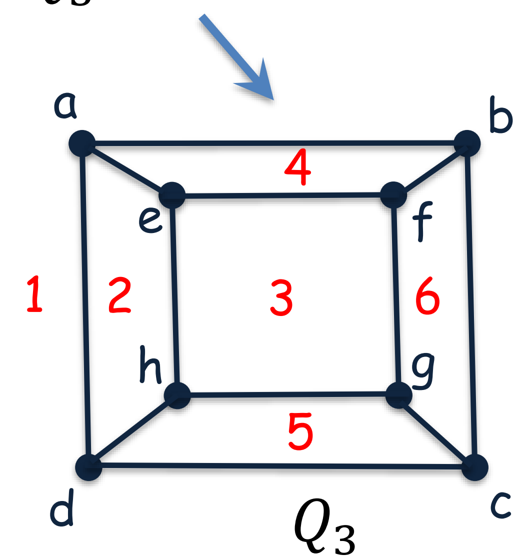

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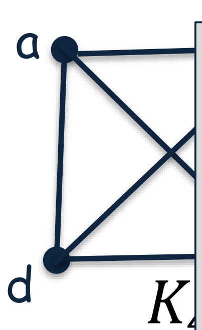

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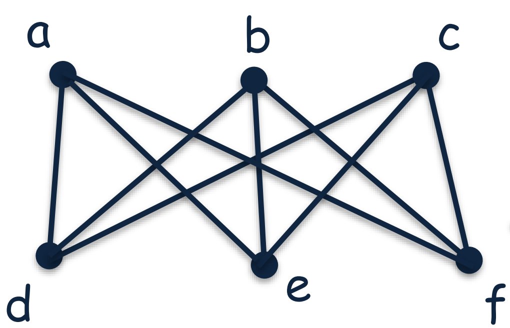
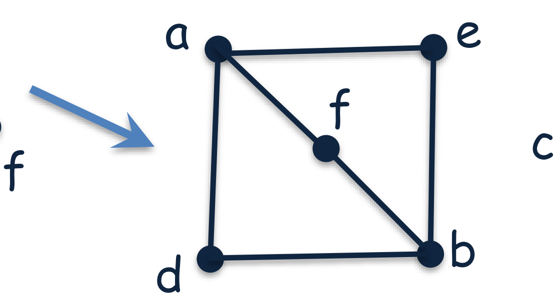
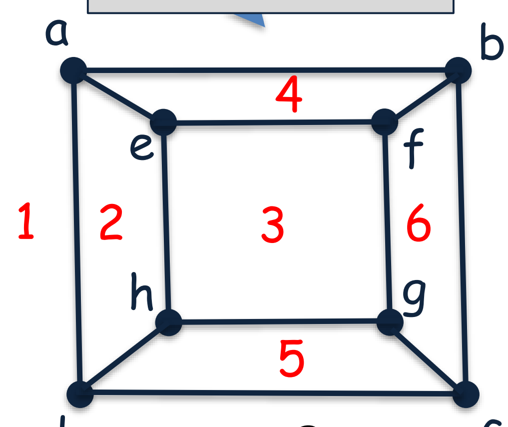


$K_{3,3}$



$6 = 12 - 8 + 2$



Q_3

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Graph Coloring

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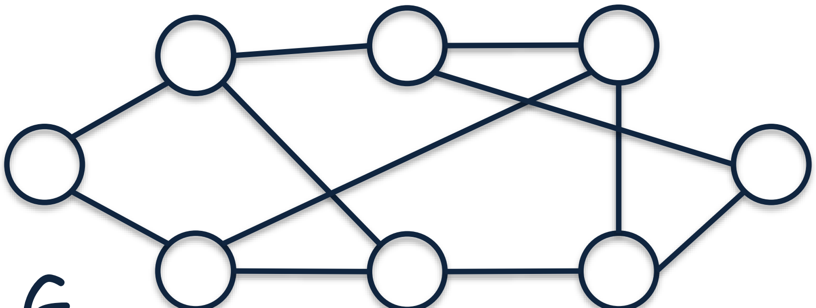
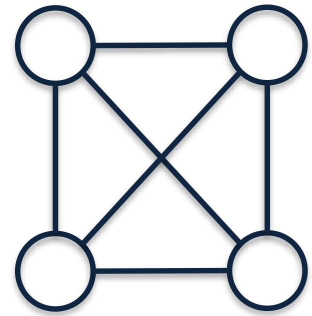
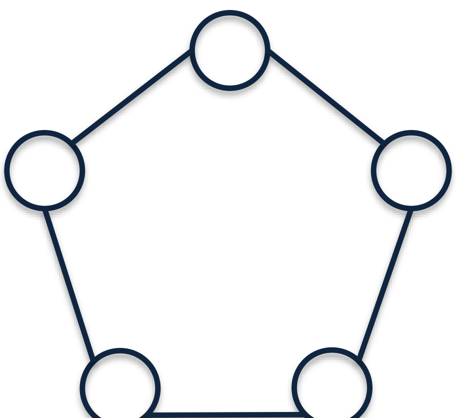


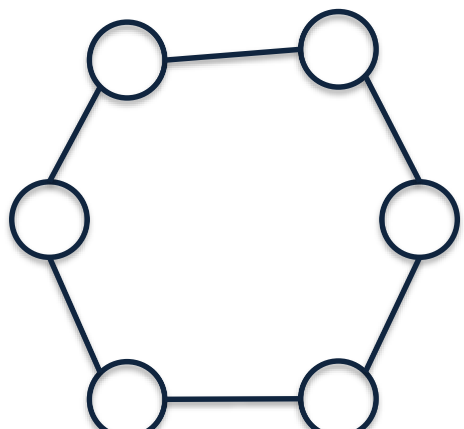
G



K_5



C_5



C_6

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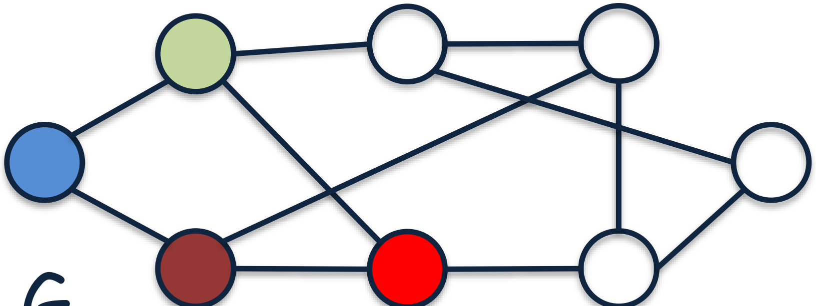
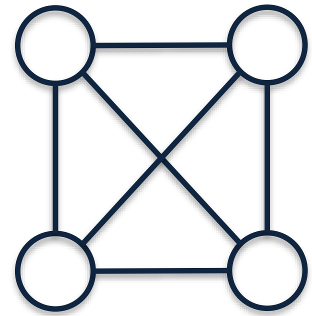


G



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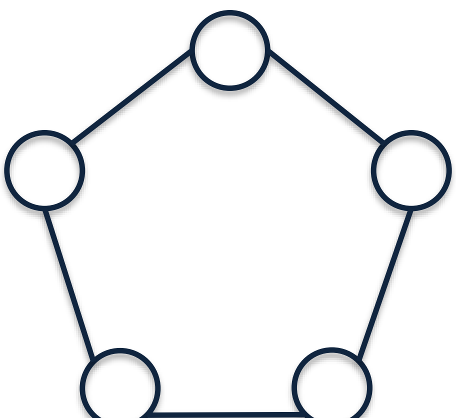
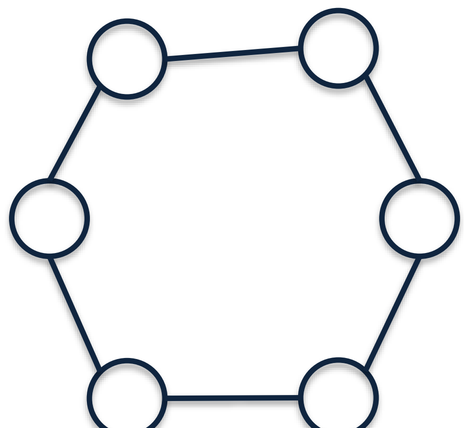


C_5



C_6

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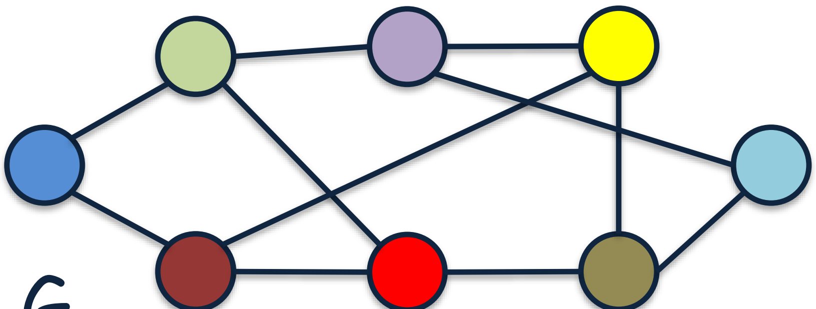
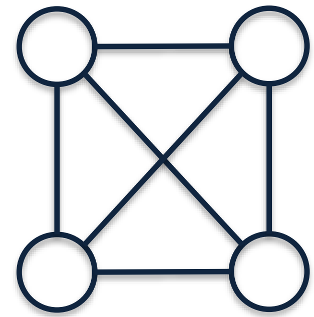
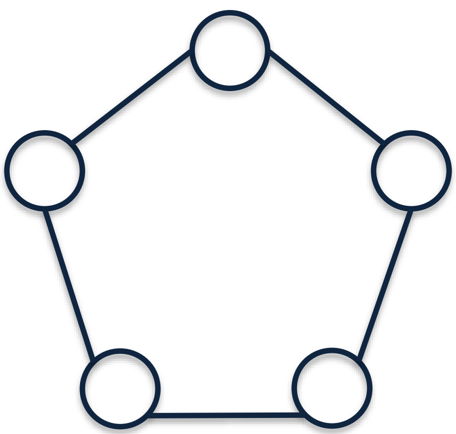


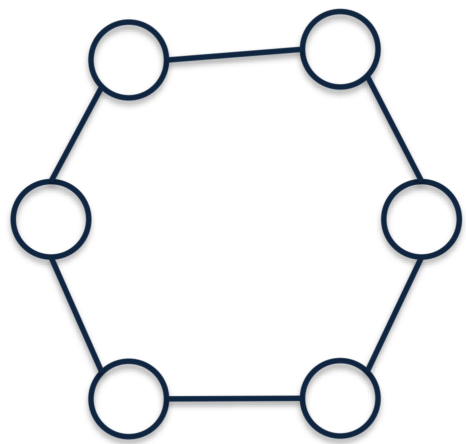
G



K_5



C_5



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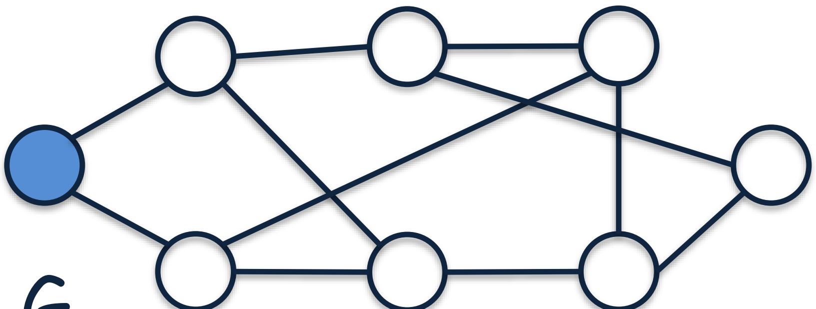
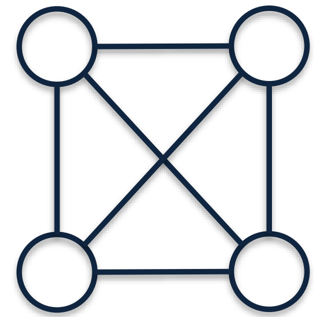
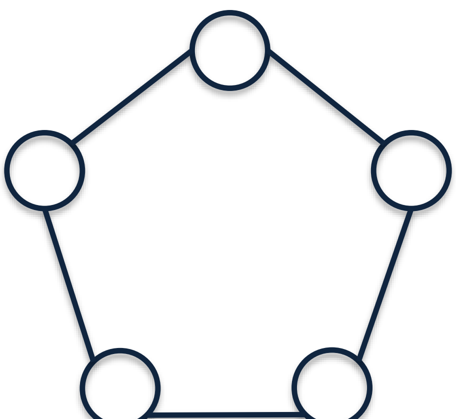


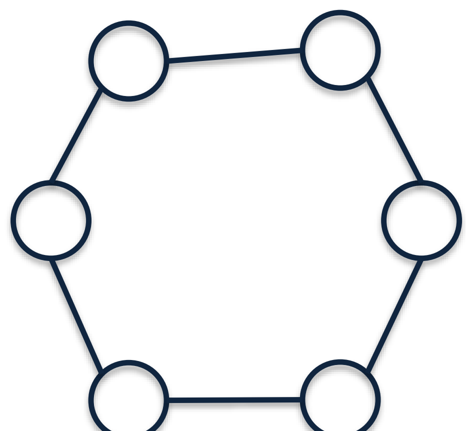
G



K_4



C_5



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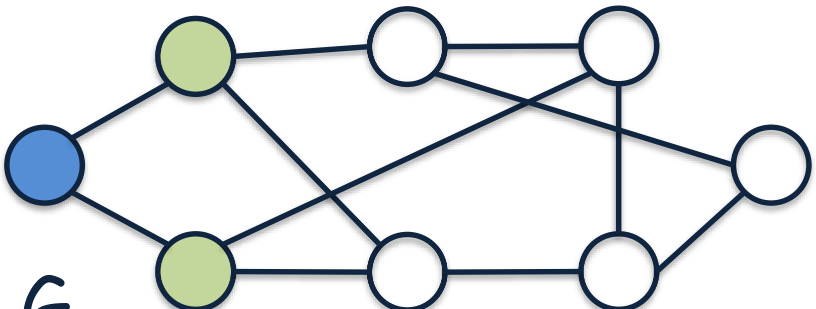
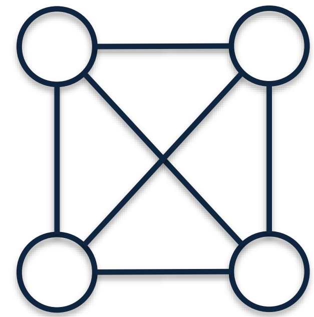


G



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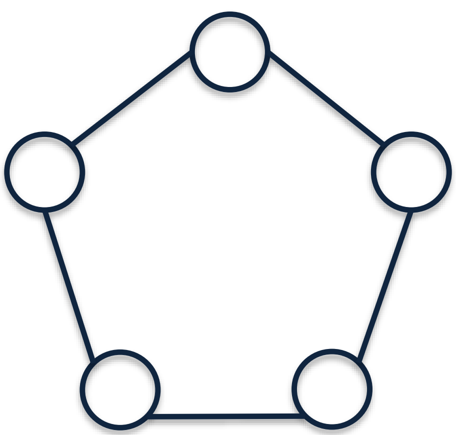

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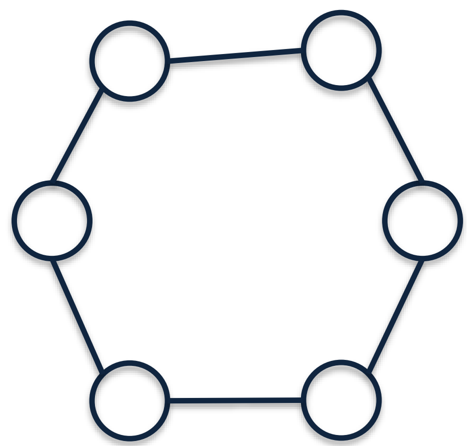

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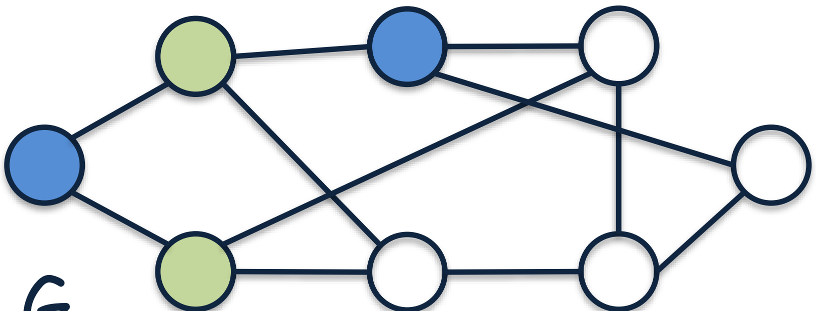

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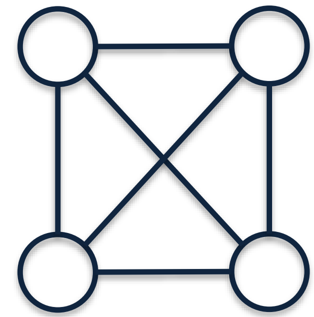

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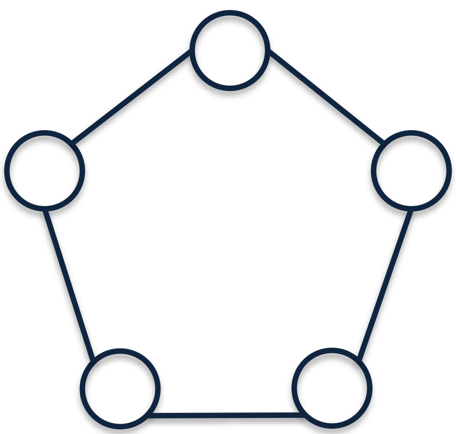

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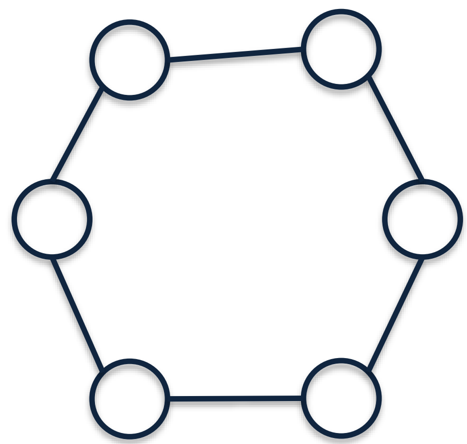

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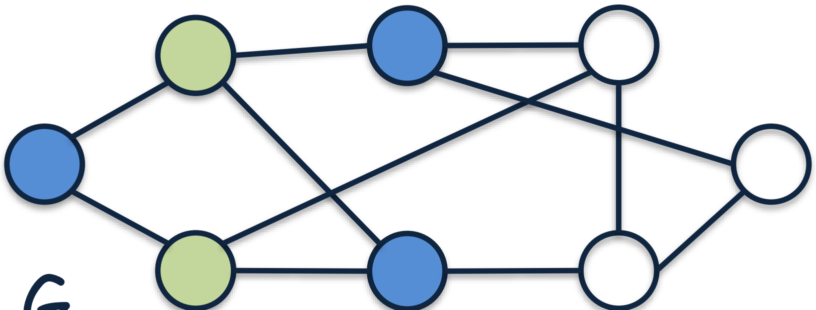

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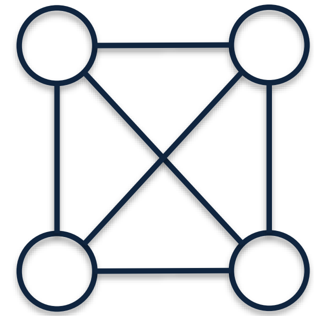
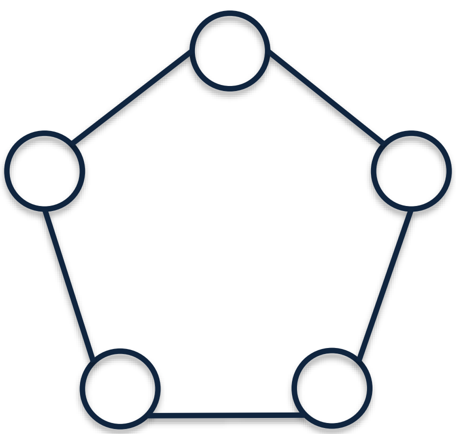
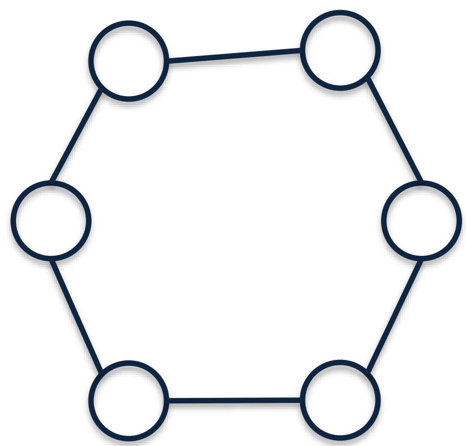


K_4



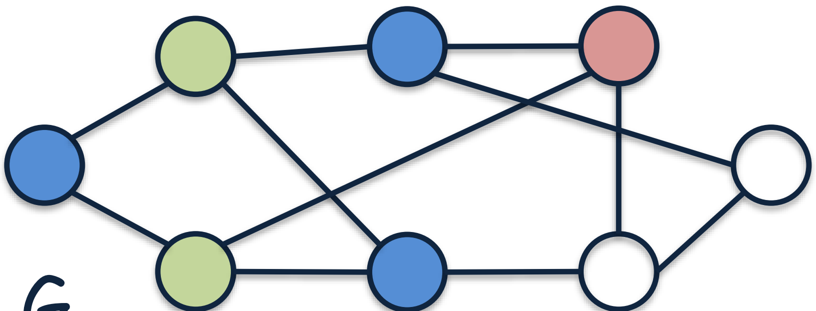
C_5



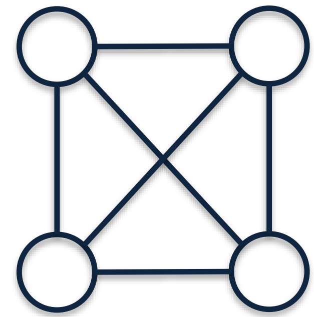
C_6

Graph Coloring

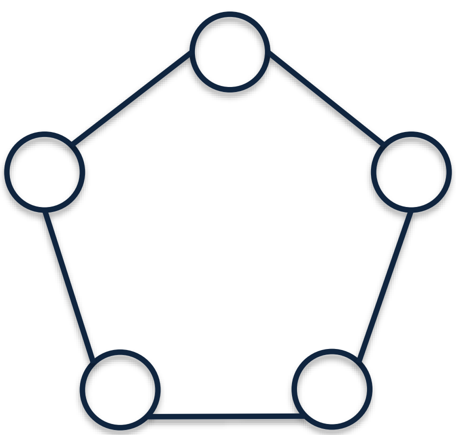
- a coloring of a simple graph is the assignment of a color to each vertex so that no two adjacent vertices are assigned the same color



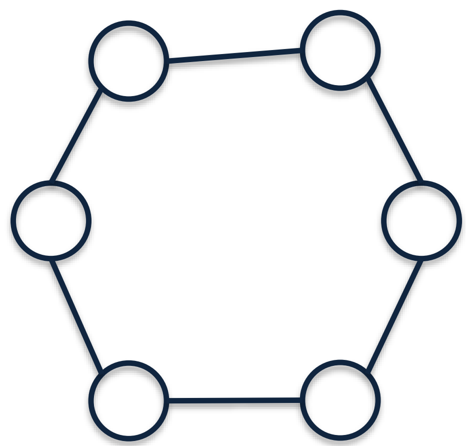
G



K_4



C_5



C_6

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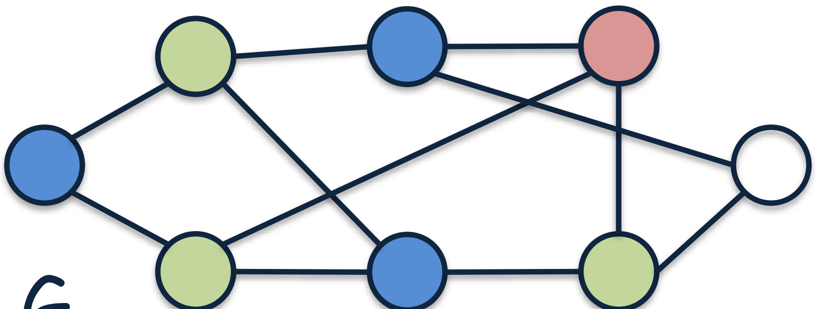
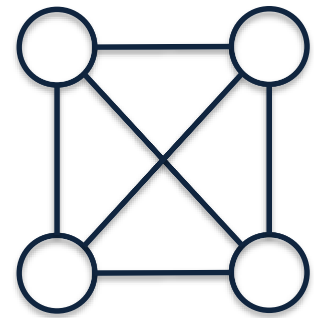
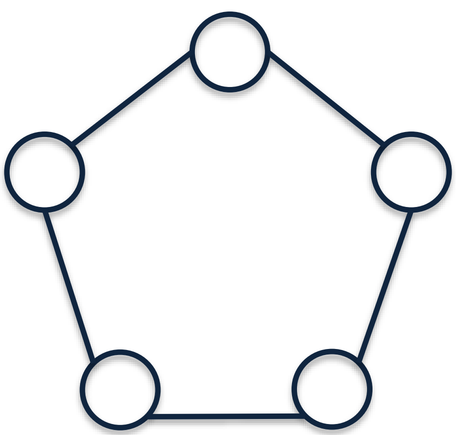


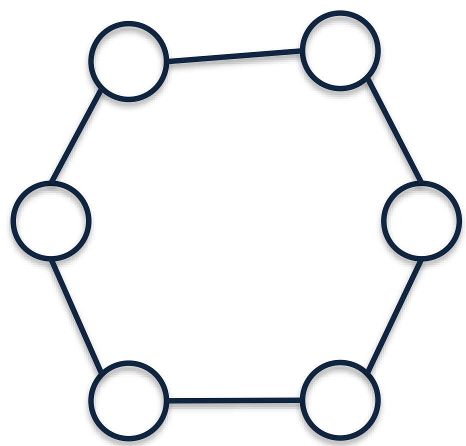
G



K_4



C_5



C_6

Graph Coloring

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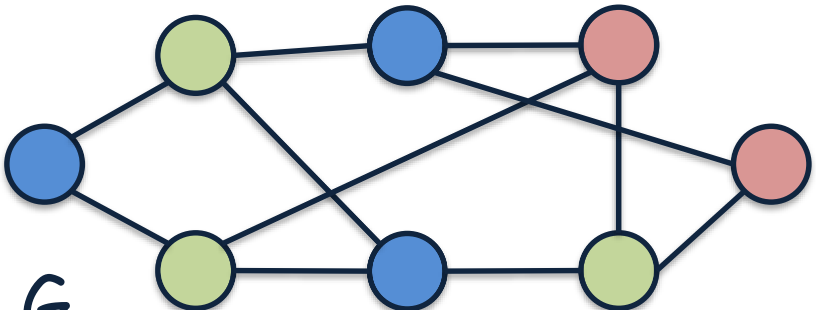
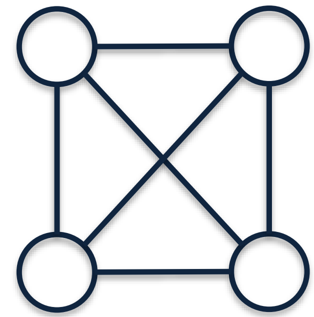
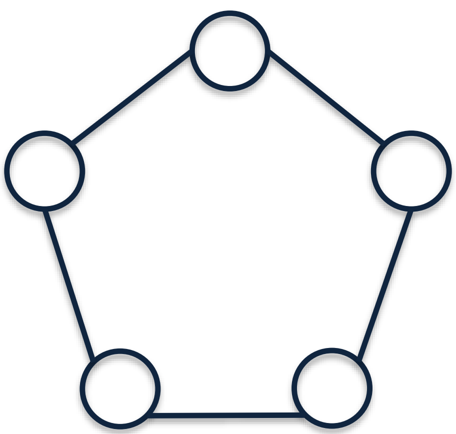


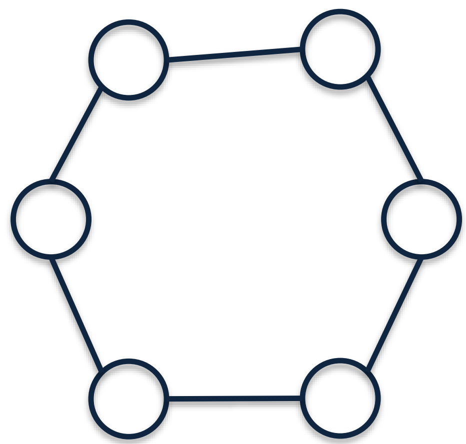
G



K_4



C_5



C_6

Graph Coloring

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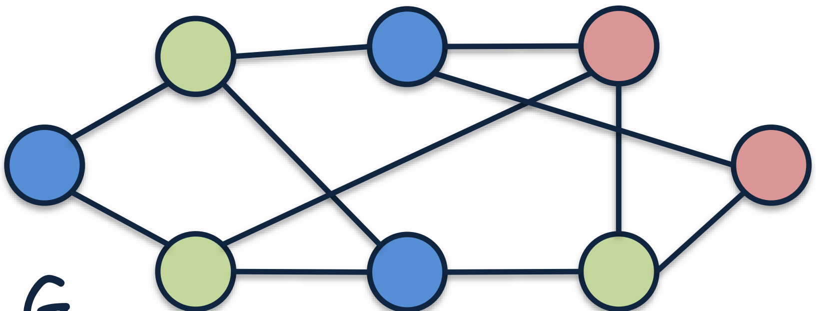
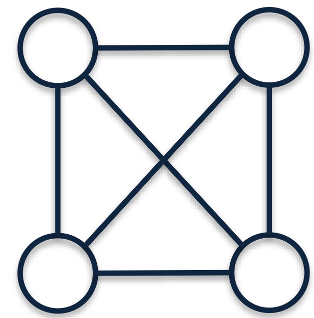
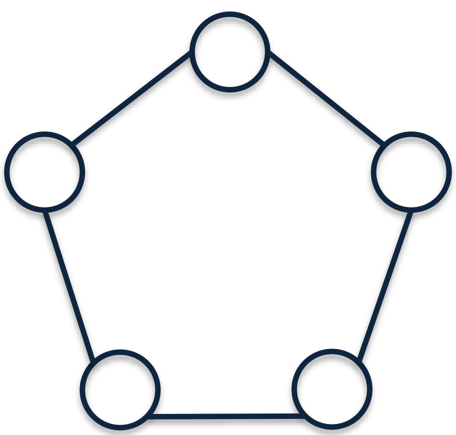


G

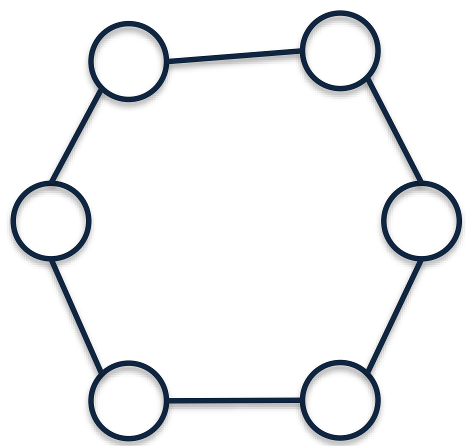


K_4

$\chi(G) = 3$ (chromatic number)



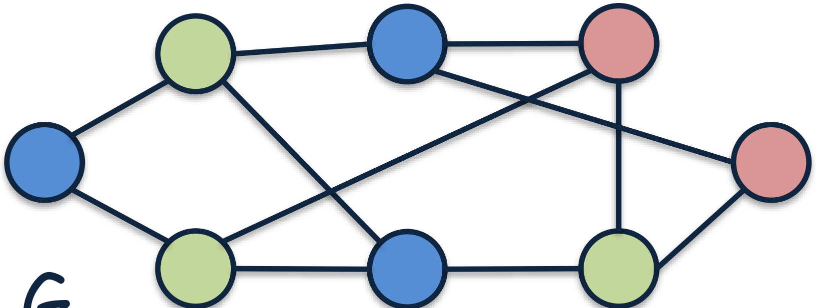
C_5



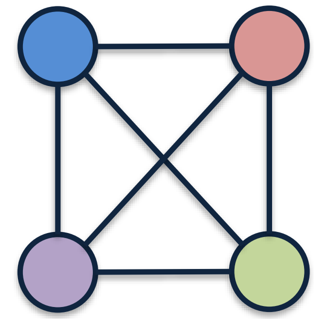
C_6

Graph Coloring

- a coloring of a simple graph is the assignment of a color to each vertex so that no two adjacent vertices are assigned the same color



G



K_4

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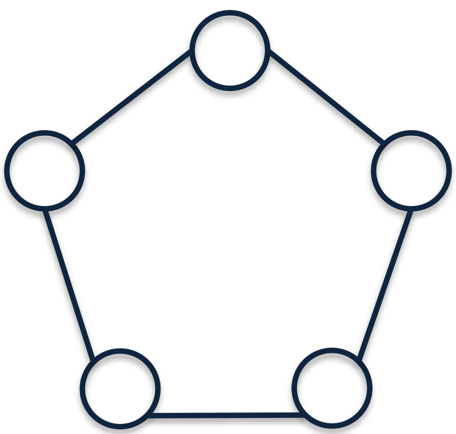
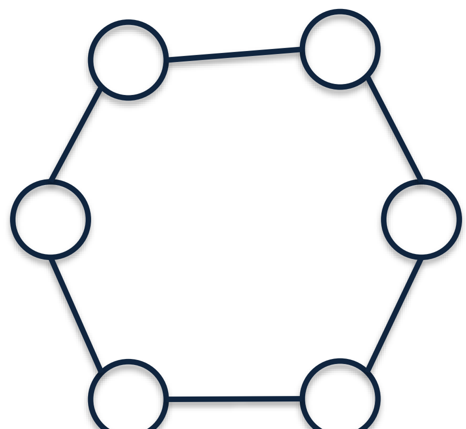


C_5



C_6

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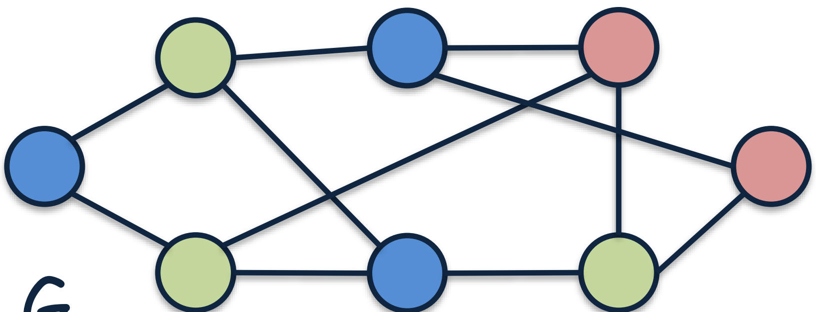
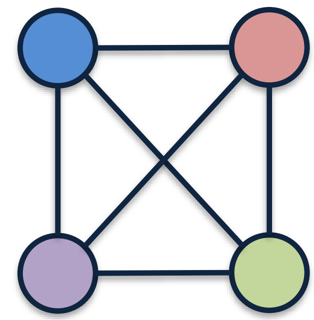


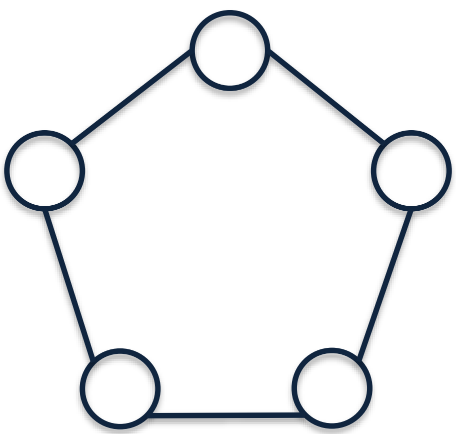
G

$\chi(G) = 3$ (chromatic number)

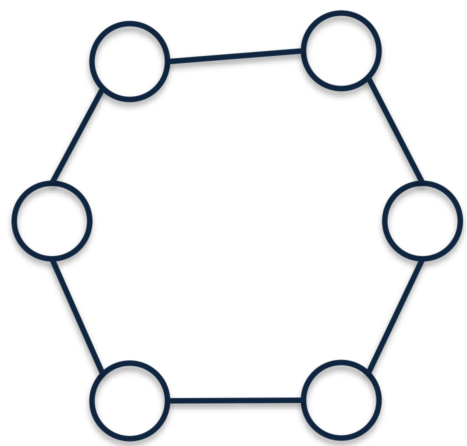


K_4

$\chi(K_4) = 4$



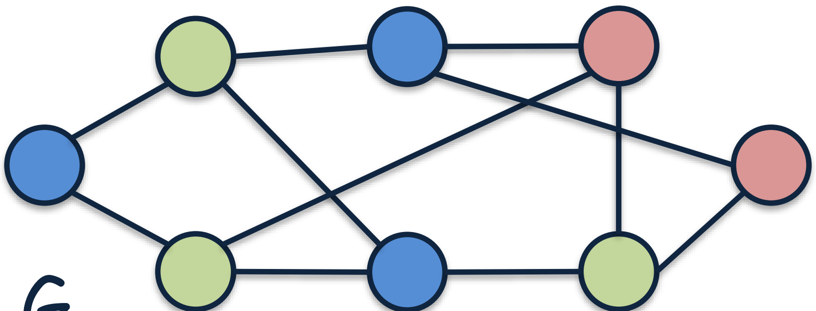
C_5



C_6

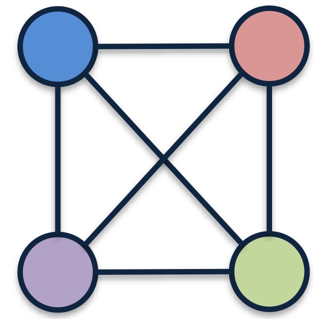
Graph Coloring

- a coloring of a simple graph is the assignment of a color to each vertex so that no two adjacent vertices are assigned the same color



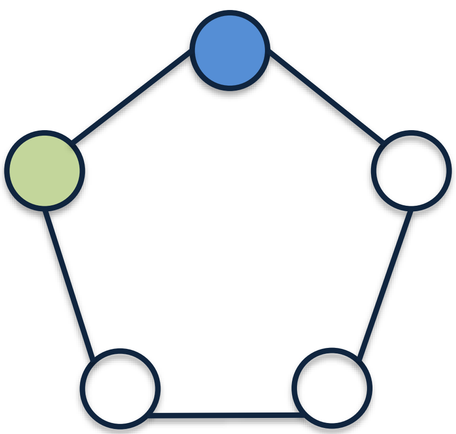
G

$\chi(G) = 3$ (chromatic number)

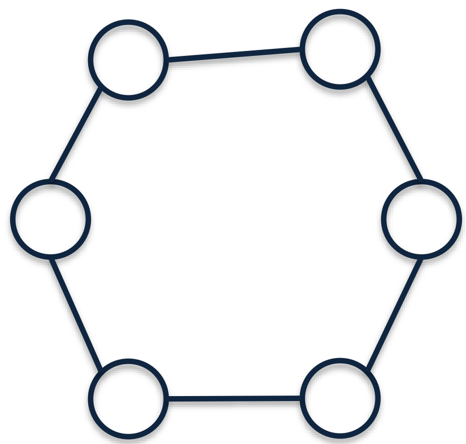


K_4

$\chi(K_4) = 4$



C_5



C_6

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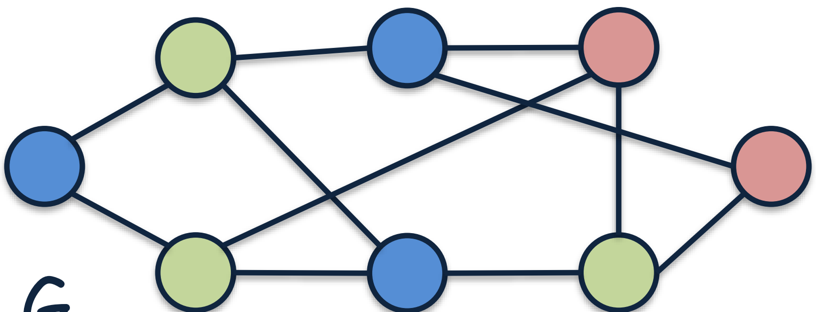
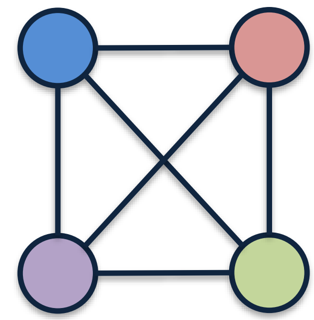


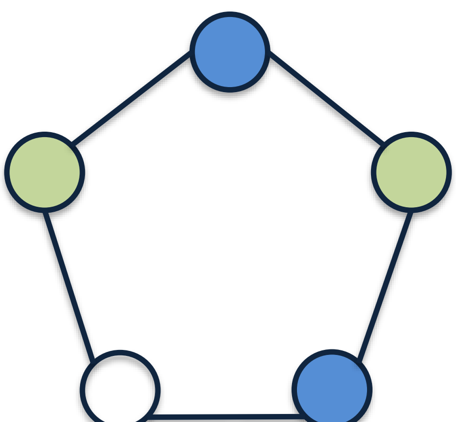
G

$\chi(G) = 3$ (chromatic number)

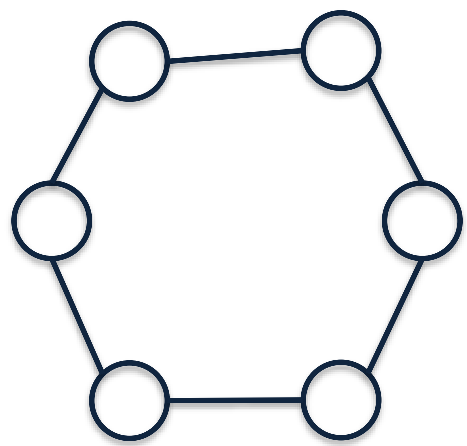


K_4

$\chi(K_4) = 4$



C_5



C_6

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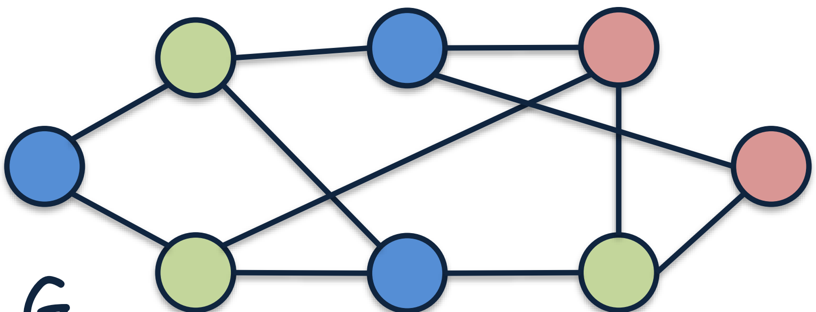

G

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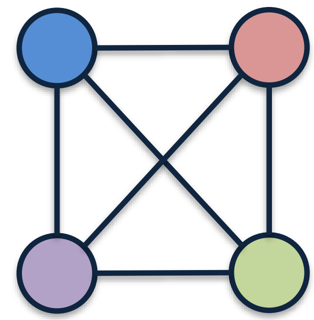

K_4

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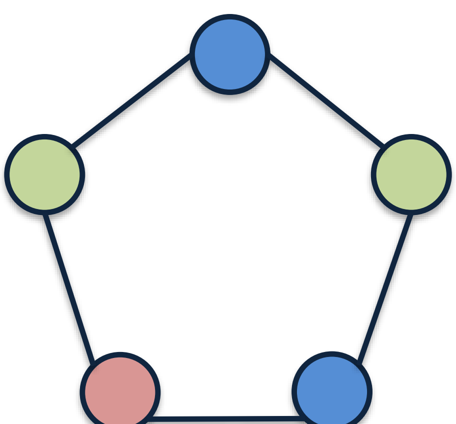
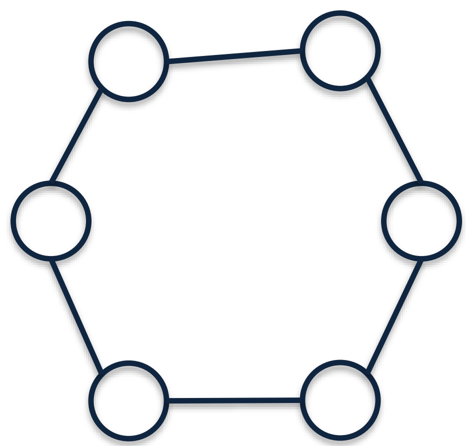


C_5



C_6

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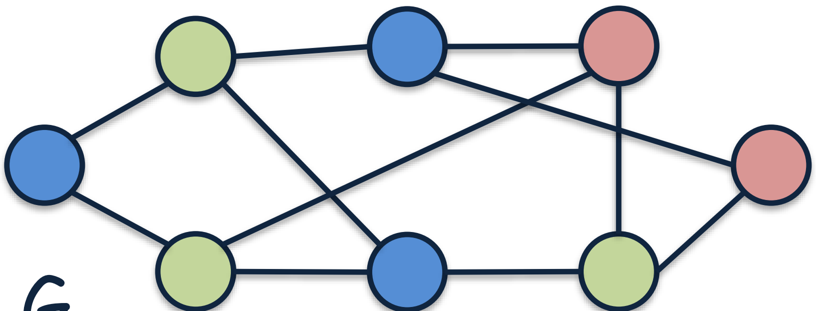

G

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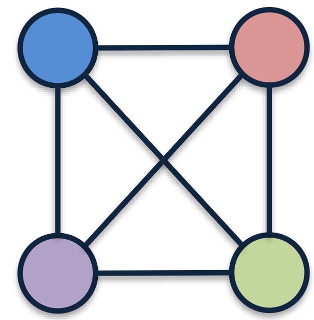
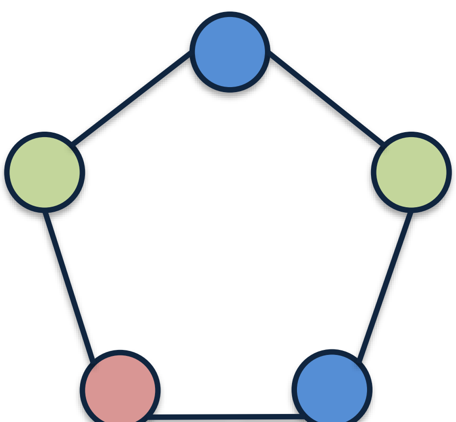
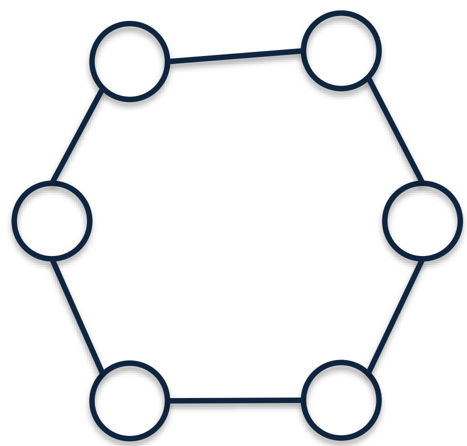


K_4

$\chi(K_4) = 4$



C_5 $\chi(C_5) = 3$



C_6

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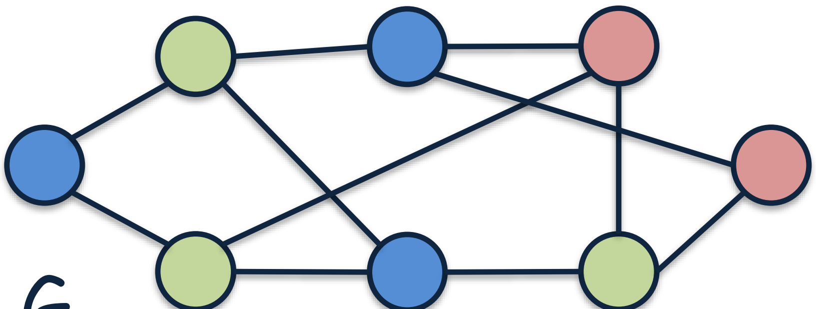

G

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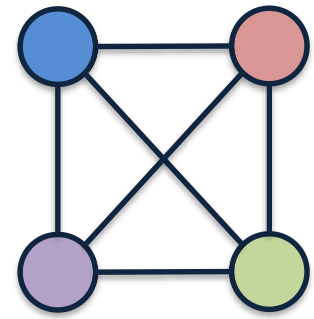

K_4

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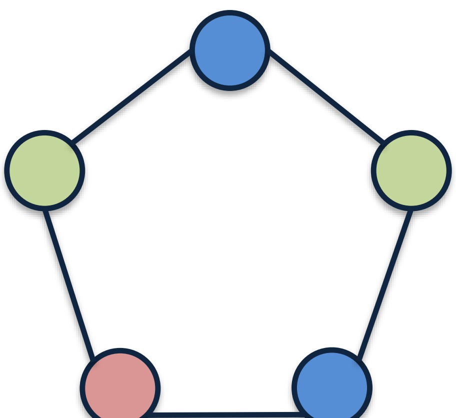
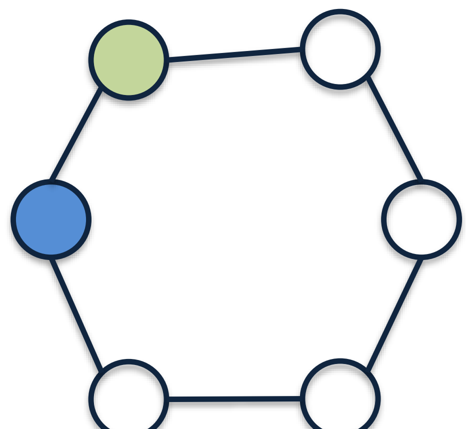


C_5 $\chi(C_5) = 3$



C_6

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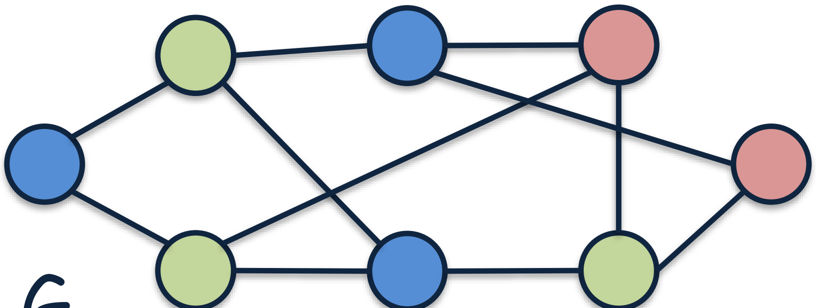
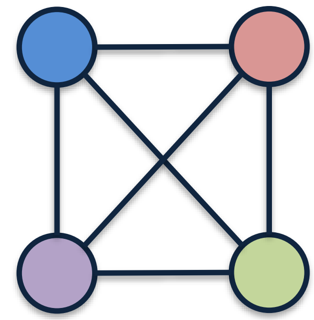


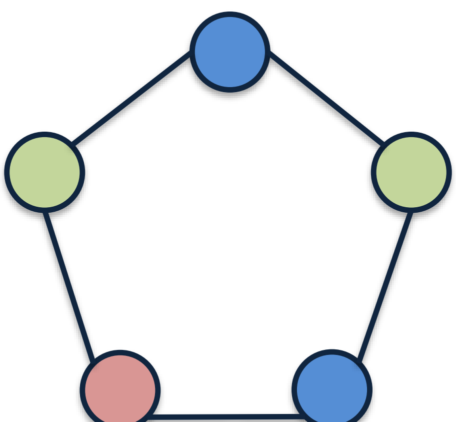
G

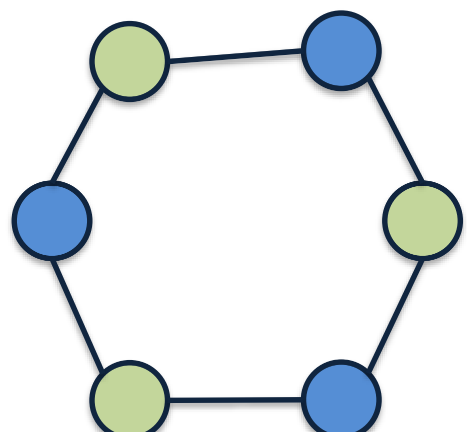
$\chi(G) = 3$ (chromatic number)



K_4

$\chi(K_4) = 4$





C_5 $\quad \chi(C_5) = 3$

C_6

Graph Coloring

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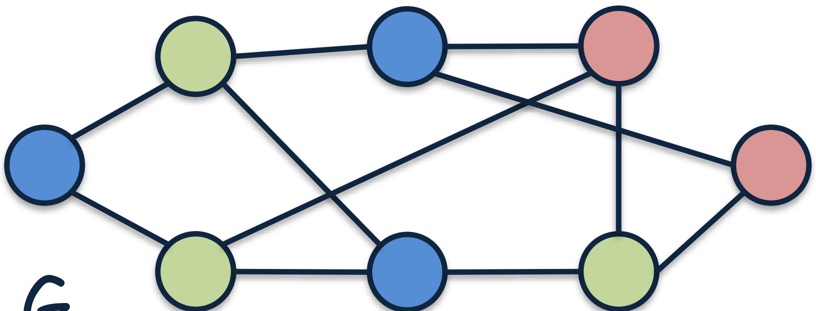
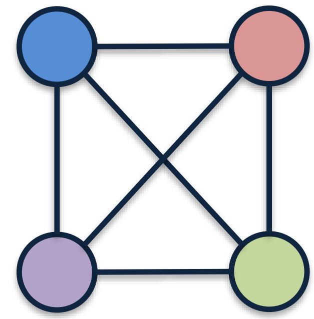


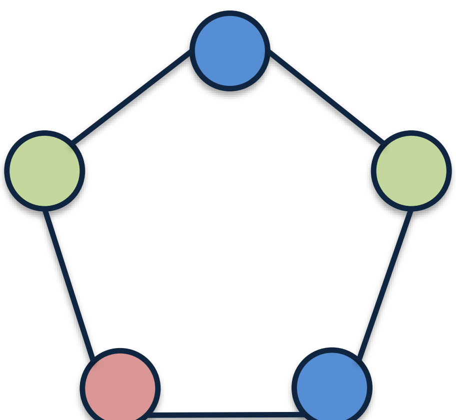
G

$\chi(G) = 3$ (chromatic number)

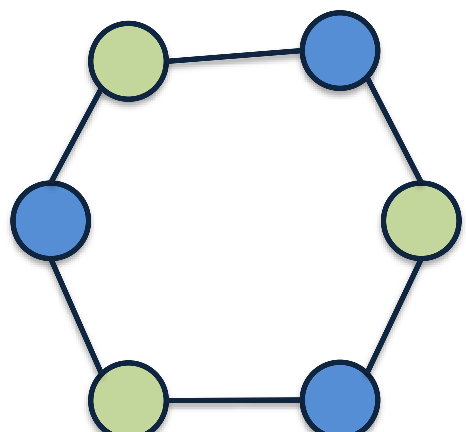


K_4

$\chi(K_4) = 4$



C_5 $\chi(C_5) = 3$



C_6 $\chi(C_6) = 2$

Graph Coloring

1	Mathematics
2	Chemistry
3	English
4	Intro. to Prog.
5	Algorithms
6	Data Structures

Orhan	1, 4	Esengul	2, 5
Bergen	2, 3, 4	Ferdi	6, 3
Muslum	1, 2, 4	Nese	1, 6, 5

Graph Coloring

1	Mathematics
2	Chemistry
3	English
4	Intro. to Prog.
5	Algorithms
6	Data Structures

Orhan	1, 4	Esengul	2, 5
Bergen	2, 3, 4	Ferdi	6, 3
Muslum	1, 2, 4	Nese	1, 6, 5

- determine at least how many time slots required to schedule the final exams in a way that no student has two exams at the same time

Graph Coloring

1	Mathematics
2	Chemistry
3	English
4	Intro. to Prog.
5	Algorithms
6	Data Structures

Orhan	1, 4	Esengul	2, 5
Bergen	2, 3, 4	Ferdi	6, 3
Muslum	1, 2, 4	Nese	1, 6, 5

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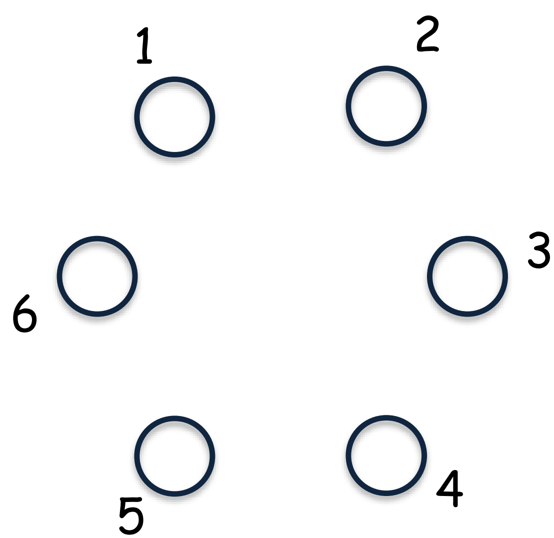

Graph Coloring

1	Mathematics
2	Chemistry
3	English
4	Intro. to Prog.
5	Algorithms
6	Data Structures

Orhan	1, 4	Esengul	2, 5
Bergen	2, 3, 4	Ferdi	6, 3
Muslum	1, 2, 4	Nese	1, 6, 5

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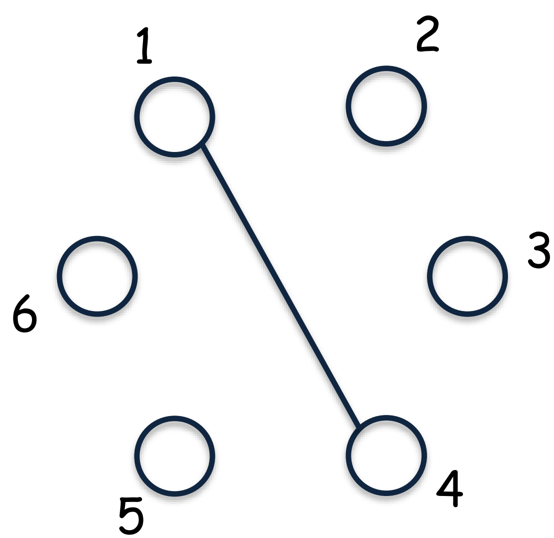

Graph Coloring

1	Mathematics
2	Chemistry
3	English
4	Intro. to Prog.
5	Algorithms
6	Data Structures

Orhan	1, 4	Esengul	2, 5
Bergen	2, 3, 4	Ferdi	6, 3
Muslum	1, 2, 4	Nese	1, 6, 5

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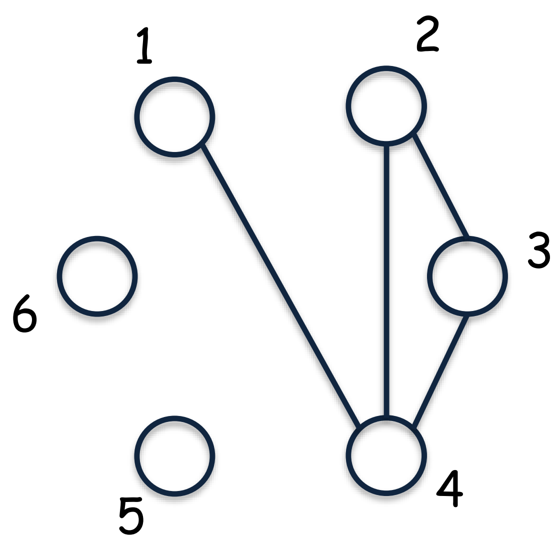

Graph Coloring

1	Mathematics
2	Chemistry
3	English
4	Intro. to Prog.
5	Algorithms
6	Data Structures

Orhan	1, 4	Esengul	2, 5
Bergen	2, 3, 4	Ferdi	6, 3
Muslum	1, 2, 4	Nese	1, 6, 5

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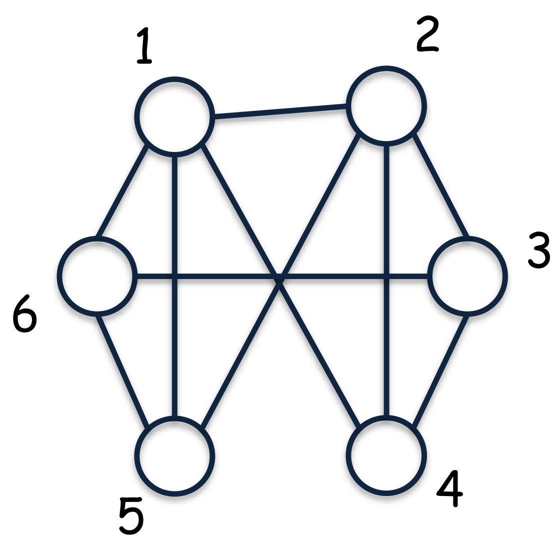

Graph Coloring

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4	Intro. to Prog.
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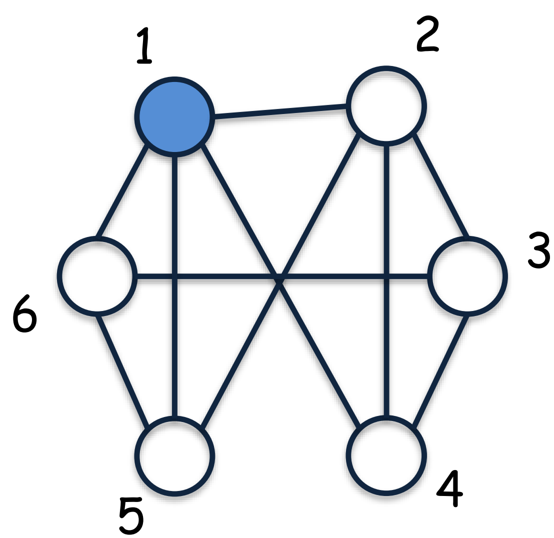

Graph Coloring

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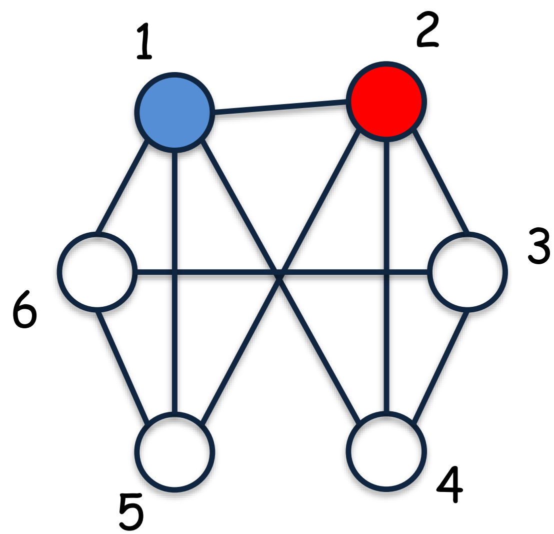
Orhan	1, 4	Esengul	2, 5
Bergen	2, 3, 4	Ferdi	6, 3
Muslum	1, 2, 4	Nese	1, 6, 5

- determine at least how many time slots required to schedule the final exams in a way that no student has two exams at the same time



Graph Coloring

1	Mathematics
2	Chemistry
3	English
4	Intro. to Prog.
5	Algorithms
6	Data Structures

Orhan	1, 4	Esengul	2, 5
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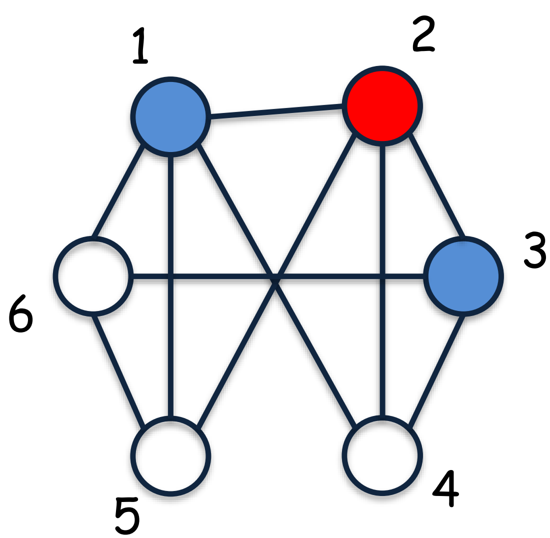

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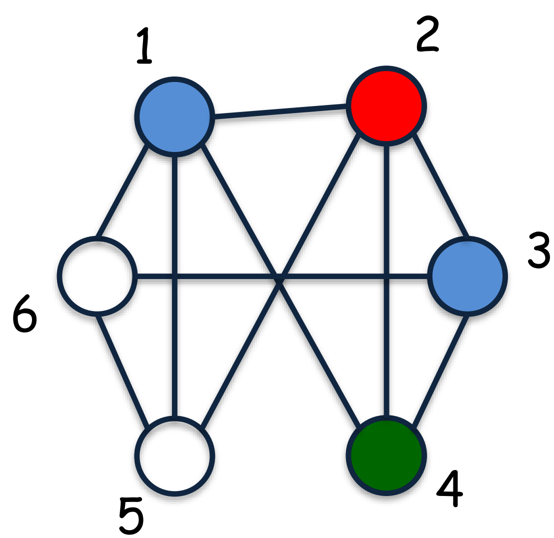

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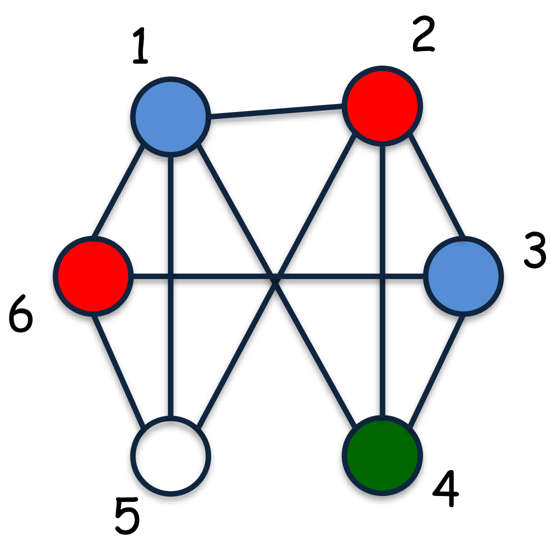

Graph Coloring

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