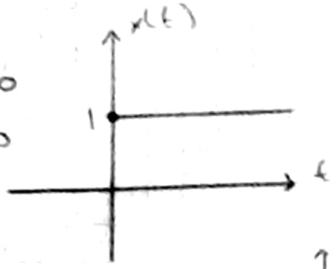
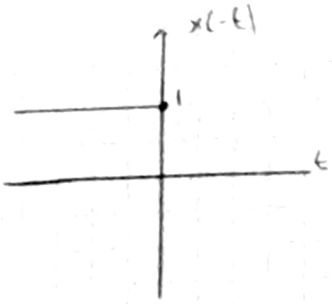
Homework 1

1) The function $x(t)$ is shown in the figure. Even and odd parts of this unit step function?

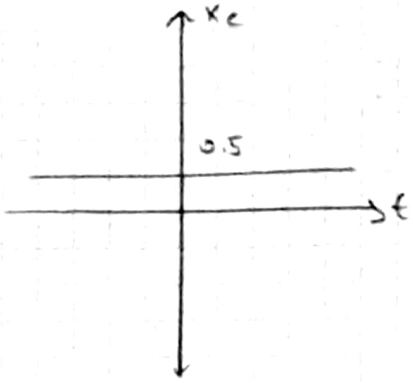


$$x(t) = \begin{cases} 0, & t<0 \\ 1, & t\geq 0 \end{cases}$$

$$\text{even part} = \frac{x(t)+x(-t)}{2}$$

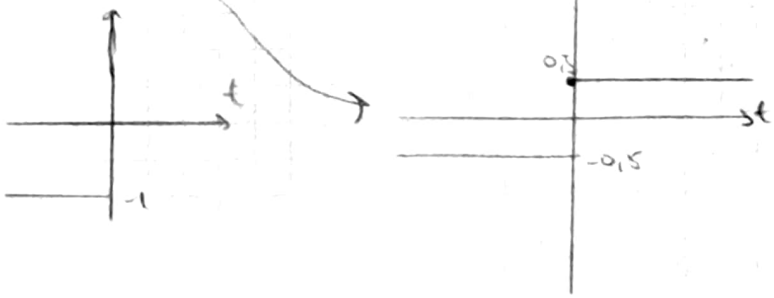


$$x(-t) \begin{cases} 0, & t>0 \\ 1, & t<0 \end{cases} \Rightarrow$$



$$\text{odd part} = x_o = \frac{x(t)-x(-t)}{2}$$

$$-x(-t) \begin{cases} 0, & t>0 \\ -1, & t<0 \end{cases} \Rightarrow$$



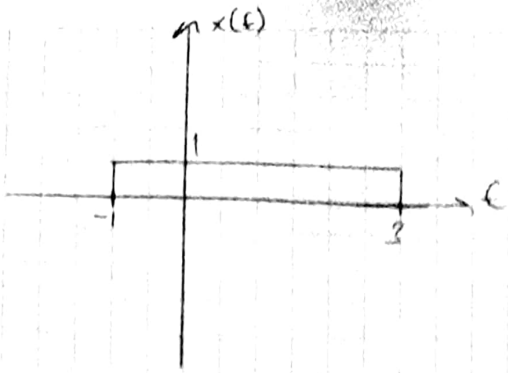
2) The fundamental period of $x(t) = 2\sin \underbrace{2\pi}_{w_1} t + 3\sin \underbrace{3\pi}_{w_2} t$

$$w_1 = 2\pi = \frac{2\pi}{T_1} \Rightarrow T_1 = \frac{2\pi}{2\pi} = 1$$

$$w_2 = 3\pi = \frac{2\pi}{T_2} \Rightarrow T_2 = \frac{2\pi}{3\pi} = \frac{2}{3}$$

$$LCM\left(1, \frac{2}{3}\right) = 2 \text{ sec}$$

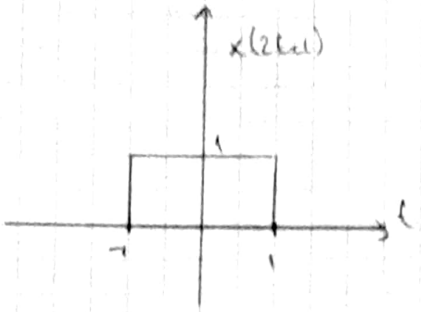
3)



$x(2t+1)$?

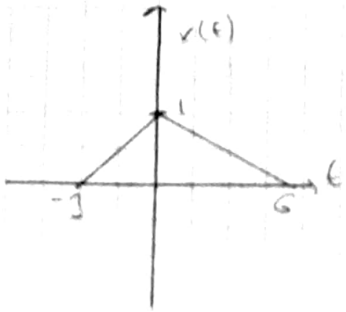
$$x(t) = \begin{cases} 0, & t<-1 \\ 1, & -1 \le t \le 3 \\ 0, & t>3 \end{cases}$$

$$x(2t+1) = \begin{cases} 0, & 2t+1<-1 = 2t<-2 \\ 1, & -1 \le 2t+1 \le 3 = -2 \le 2t \le 2 \\ 0, & 2t+1>3 = 2t>2 \end{cases}$$



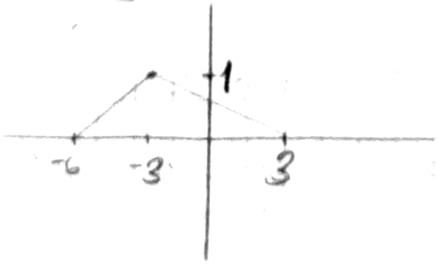
$$x(2t+1) = \begin{cases} 0, & t<-1 \\ 1, & -1 \le t \le 1 \\ 0, & t>1 \end{cases}$$

4)

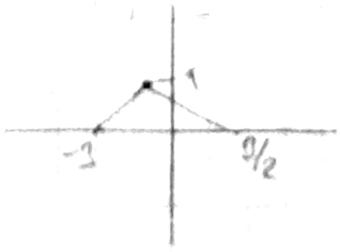


$3x(-2t+3)$

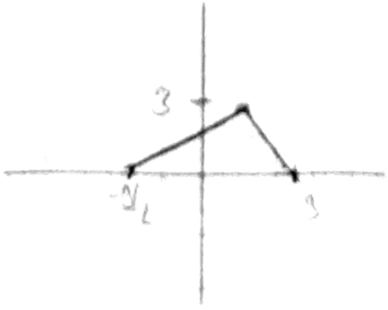
- Time shifting 3 left



- Time compression



- Mirror about y and 3x scaling



5) $x(t) = 3\,\mathrm{rect}\left(\frac{2t}{T}\right)$

$$A\,\mathrm{rect}\left(\frac{t}{\tau}\right) = \begin{cases} A, & |t| < \tau/2 \\ 0, & |t| > \tau/2 \end{cases}$$

$$3\,\mathrm{rect}\left(\frac{2t}{T}\right) = \begin{cases} 3, & |2t| < \tau/2 = |t| < \tau/4 \\ 0, & |2t| > \tau/2 = |t| > \tau/4 \end{cases}$$