# Sets

Murat Osmanoglu

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- $x \in A$, $x$ is an element of the set $A$
- $x \notin A$, $x$ is not an element of the set $A$

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- $Z = \{2, -22, 12, 0, 43, -1287, . . .\}$
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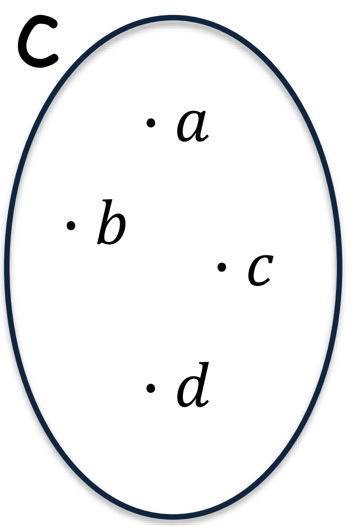
- Venn Diagram



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- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

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- of A is

$A = \{x | x = 4k + 1 \ for \ some \ k \in Z \}$ ,
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Show that whether the sets A and B are equal or not.

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Thus, A=B.

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- If |S|=n, then |P(S)|=$2^n$

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- The complement of A, denoted by $\overline{\mathrm{A}}$, contains elements that are in U but not in A.

$$\overline{\mathrm{A}} = \{x \in U | x \notin A\}$$

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- $\overline{A \cup B} = \bar{A} \cap \bar{B}$ (De Morgan)
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  $p \vee 0 \equiv p$
  $p \wedge 1 \equiv p$

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  $\sim(\sim p) \equiv p$

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  $\sim(p \vee q) \equiv \sim p \wedge \sim q$
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$$\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$$

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$$(x \notin \mathrm{A}\cup\mathrm{B}) \leftrightarrow \sim((x \in A) \vee (x \in B))$$

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Show that $\overline{\mathrm{A} \cup \mathrm{B}} = \overline{\mathrm{A}} \cap \overline{\mathrm{B}}$.

($\overline{\mathrm{A} \cup \mathrm{B}} \subseteq \overline{\mathrm{A}} \cap \overline{\mathrm{B}}$) assume $x \in \overline{\mathrm{A} \cup \mathrm{B}}$, then $(x \notin \mathrm{A} \cup \mathrm{B})$

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