1.Definitions :

1.Statistical Experiment: is a planned activity whose set of the results (outcomes) is certain**;** but it can't be said before that; which outcome occurs.

1.2.Sample Space: is the set of all possible outcomes of the experiment**.** It is denoted by S or Ω .

1.3.Event: is a subset of the sample space. For an n element set;

The number of the events: 2^n

EXAMPLES:

1- The experiment : Flip a coin (tossing of a coin)

Sample Space :

$\Omega = \{H, T\}$

Head Tail

Events :

$A_1 = \{H\}$, The filp is H.

$A_2 = \{T\}$, The filp is T.

$A_3 = \{\emptyset\}$

$A_4 = \{H, T\}$, The filp is H or T

2- The experiment : Flip a coin two times.

Sample space :

$\Omega = \{TT, TH, HT, HH\}$

$n(\Omega) = 4$ (Number of elements)

Events:

$A = \{TT\}$

$A = \{TH\}$

$A = \{HT\}$

$A = \{HH\}$

$A = \{TT, TH\}$

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$A = \Omega$

A: The first flip is head.

$A = \{HT, HH\}$

Number of Events: $2^4 = 16$

B: The second flip is tail.

$B = \{TT, HT\}$

C: The first flip is tail, the second is head.

$C = \{TH\}$

D: In two times, at least one time tail.

$D = \{TT, TH, HT\}$

3-Flip a coin three times.

Sample space :

$\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(\Omega) = 8$ (Number of elements)

Number of Events: $2^8 = 64$

Event A: 1st flip is heads.

$$A = \{HHH, HHT, HTH, HTT\}$$

4-Flip the two coins at the same time.

Sample space :

$\Omega = \{TT, TH, HH\}$

$n(\Omega) = 3$ (Number of elements)

TT means: Two flips are observed as tails.

TH means: One flip is head one is tail (HT is the same as the TH, either of them can be used)

HH means: Two flips are observed as heads.

Events:

$A = \{TT\}$

$A = \{TH\}$

$A = \{HT\}$

$A = \{HH\}$

$A = \{TT, TH\}$

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$A = \Omega$

Number of Events: $2^3 = 8$

Student:

5. Flip the three coins at the same time. Write the sample space, define the events.

$\Omega =$……………………

$n(\Omega) =$ ……………………

Events:

The number of events:

6. Flip a dice:

<u>Sample space :</u>

$\Omega = \{1,2,3,4,5,6\}$

<u>Events:</u>

$A = \{1\}$

$A = \{2\}$

$A = \{3\}$

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$E = \{1,2,3,4,5,6\}$

Number of Events: $2^6 = 64$

7. Flip a dice two times.

Sample space :

$\Omega = \{(1,1),(1,2),\ldots,(6,6)\}$

$n(\Omega) = 36$

Number of Events: 2^{36}

Student:

8.Flip the two dices at the same time.

$\Omega =$……………………

$n(\Omega) =$ ……………………

Events:

The number of events: ……………..

Event/Set Operations:

1.The complement of an event A: The set of all elements of S not in A. It is denoted as A^C

2.The intersection of two events A and B: The set of all elements in both A and B. It is denoted as $A \cap B$.

3.The union of two events A and B: The set of all elements in either A and B. It is denoted as $A \cup B$.

4. If $A \cap B = \emptyset$; two events are **mutually exclusive.**

Example:

Experiment: Flip a coin three times.

$\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

A: The first flip is head.

$A = \{HHH, HHT, HTH, HTT\}$

$A^c = \{THH, THT, TTH, TTT\}$

B: The 3. flip is tail.

$B = \{HHT, HTT, THT, TTT\}$

$A \cap B$: The first flip is head, third is tail.

$A \cap B = \{HHT, HTT\} \rightarrow \boldsymbol{A}, \boldsymbol{B}$ **are not mutually exclusive.**

$A \cup B$: The first flip is head or third is tail.

$A \cup B = \{HHH, HHT, HTH, HTT, THT, TTT\}$

Probablity of an Event :

The probability of an event A is given as;

$P(A)=\frac{n}{N}$

where

n : the number of points in cluster A.

N : the number of points in Ω sample space.

The Probability Axioms:

For any event A;

1. $0 \le P(A) \le 1$.
2. $P(\Omega) = 1$
3. The probability of the complement of event A (A^c) ;

$P(A^c) = 1 - P(A)$

4. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
5. If A , B are **mutually exclusive;**

$P(A \cup B) = P(A) + P(B)$

(Because; $P(A \cap B) = P(\emptyset) = 0$)

Independent Events: For any event A, B $(A, B \neq \emptyset)$ If the probability $P(A \cap B)$ can be written as ;

$P(A \cap B) = P(A).P(B)$

The two events are **independent** from each other.

Note: If A, B are mutually exclusive, they can't be independent.

Examples:

1.The experiment : Flip a coin two times.

Sample space :

$\Omega = \{TT, TH, HT, HH\}$; $N = 4$

A: The 1.flip is tail

$A = \{TT, TH\},\ n = 2$

$$P(A) = \frac{n}{N} = \frac{2}{4}$$

B: The 2.flip is head

$B = \{TH, HH\}$

$$P(A) = \frac{n}{N} = \frac{2}{4}$$

$A \cap B = \{TH\}$

$$P(A \cap B) = \frac{n}{N} = \frac{1}{4}$$

$\boldsymbol{A, B}$ **are not mutually exclusive,**

$A \cap B \neq \emptyset$;

$P(A \cap B) = P(A).P(B)$

$$\frac{1}{4} = \frac{1}{2}.\frac{1}{2}$$

$\boldsymbol{A, B}$ **are independent from each other.**

2.The experiment : Flip a coin three times.

$\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Student:

A: At most two flips are heads

$A =$

$P(A)$=

B**:** At At most two flips are tails.

$B =$

$P(B)$=

Are they independent events ?

Student:

3.The experiment : Flip the three coins at the same time.

Define some events and calculate the probabilities.

4.The experiment : Flip a dice two times.

$\Omega = \{(1,1), (1,2), \dots, (6,6)\}$

A: The sum of the surface points is seven

$A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}\ n = 6$

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

B: The sum of the surface points is odd number

$B = \{(1,2), (1,4), (1,6), (2,1), (2,3), (2,5), \dots (6,5)\}$

$$P(A) = \frac{18}{36} = \frac{1}{2}$$