Mathematical Proof Techniques

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Proofs

- Valid arguments that establish the truth of mathematical statements

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argument : sequence of sentences (propositions); premises at the beginning and conclusion at the end

Proofs

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- If you have a password, then you can log onto the network.

- You have a password

- Therefore,
  you can log onto the network

Proofs

- An argument is called valid if the truthness of all its premises implies that the conlusion is true

- If you have a password, then you can log onto the network. $p \rightarrow q$
- You have a password p
- Therefore, you can log onto the network q

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline q \end{array}$$

Proofs

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline q \end{array}$$

Proofs

Modus Ponens

$p \rightarrow q$

p

---

q

Proofs

Modus Ponens

$p \rightarrow q$

p

---

q

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$
1	1	0	0	1
1	0	1	1	1
0	1	1	0	1
0	0	1	0	1

Proofs

Modus Ponens

- If $\sqrt{5} > \sqrt{3}$, then $(\sqrt{5})^2 > (\sqrt{3})^2$.

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- We know that $\sqrt{5} > \sqrt{3}$ p

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- We know that $\sqrt{5} > \sqrt{3}$ p
- So, $(\sqrt{5})^2 > (\sqrt{3})^2$

Proofs

Modus Ponens

- If $\sqrt{5} > \sqrt{3}$, then $(\sqrt{5})^2 > (\sqrt{3})^2$. $\quad$ $p \to q$
- We know that $\sqrt{5} > \sqrt{3}$ $\quad$ p
- So, $(\sqrt{5})^2 > (\sqrt{3})^2 \;\to\; 5 > 3$ $\quad$ q

Proofs

- To prove $\forall x\ (P(x) \rightarrow Q(x))$, show that $P(c) \rightarrow Q(c)$ is true for an arbitrary element c of the domain.

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   Socrates is a man

   Socrates is mortal

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$P(x)$: x is a man

$Q(x)$: x is mortal

Proofs

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1) All men are mortal $\qquad \forall x\,(P(x) \rightarrow Q(x))$

   Socrates is a man $\qquad$ P(Socrates)

   Socrates is mortal $\qquad$ Q(Socrates)

P(x) : x is a man

Q(x) : x is mortal

Proofs

- To prove $\forall x\ (P(x) \to Q(x))$, show that $P(c) \to Q(c)$ is true for an arbitrary element c of the domain.

- To prove $P(c) \to Q(c)$, show that $Q(c)$ is true if $P(c)$ is true

Proofs

- To prove $\forall x\,(P(x) \rightarrow Q(x))$, show that $P(c) \rightarrow Q(c)$ is true for an arbitrary element c of the domain.
- To prove $P(c) \rightarrow Q(c)$, show that $Q(c)$ is true if $P(c)$ is true (p → q is true unless p is true but q is false)

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Direct Proof

Proofs

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Direct Proof

- To prove p → q is true, first assume p is true, then show that q must also be true.

Proofs

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- Thus, if p is true, then q must also be true, so that

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- To prove $P(c) \to Q(c)$, show that $Q(c)$ is true if $P(c)$ is true (p → q is true unless p is true but q is false)

Direct Proof

- To prove p → q is true, first assume p is true, then show that q must also be true.
- Thus, if p is true, then q must also be true, so that the combination of p true and q false never occurs

Proofs

Direct Proof

If n is odd integer, then n^2 is odd integer.

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If $\underbrace{n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

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$p \rightarrow q$

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Direct Proof

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$p \rightarrow q$ assume p is true

Proofs

Direct Proof

If $\underbrace{n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

p → q

assume p is true

$$n = 2k + 1, \exists k \in Z$$

Proofs

Direct Proof

If $\underbrace{n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

$p \rightarrow q$ assume p is true

$n = 2k + 1, \exists k \in Z$

$n^2 = (2k + 1)^2$

Proofs

Direct Proof

If $\underbrace{n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

$p \rightarrow q$ assume p is true

$$n = 2k + 1, \exists k \in Z$$
$$n^2 = (2k+1)^2$$
$$n^2 = 4k^2 + 4k + 1$$

Proofs

Direct Proof

If $\underbrace{n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

p → q

assume p is true

$$n = 2k + 1, \exists k \in Z$$
$$n^2 = (2k + 1)^2$$
$$n^2 = 4k^2 + 4k + 1$$
$$n^2 = 2(2k^2 + 2k) + 1$$

Proofs

Direct Proof

If n is odd integer, then n^2 is odd integer.

$$\underbrace{n \text{ is odd integer}}_{p}, \quad \text{then} \quad \underbrace{n^2 \text{ is odd integer}}_{q}$$

p → q

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$$n = 2k + 1, \exists k \in Z$$
$$n^2 = (2k + 1)^2$$
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$$n^2 = 2m + 1, \exists m \in Z$$

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$$n = 2k + 1, \exists k \in Z$$
$$n^2 = (2k + 1)^2$$
$$n^2 = 4k^2 + 4k + 1$$
$$n^2 = 2(2k^2 + 2k) + 1$$
$$n^2 = 2m + 1, \exists m \in Z$$

q is also true

Proofs

Direct Proof

If a and b are odd integers, then $a + b$ is even integer.

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers}}_{p}$, then $\underbrace{a + b \text{ is even integer}}_{q}$.

Proofs

Direct Proof

$$\text{If } \underbrace{a \text{ and } b \text{ are odd integers,}}_{p} \text{ then } \underbrace{a+b \text{ is even integer.}}_{q}$$

$p \rightarrow q$ assume p is true

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers,}}_{p}$ then $\underbrace{a+b \text{ is even integer.}}_{q}$

$p \rightarrow q$ assume p is true

$a = 2x + 1$ and $b = 2y + 1 \ \exists x, y \in Z$

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers}}_{p}$, then $\underbrace{a + b \text{ is even integer}}_{q}$.

p → q

assume p is true

$a = 2x + 1$ and $b = 2y + 1$ $\exists x, y \in Z$

$a + b = 2x + 1 + 2y + 1$

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers,}}_{p}$ then $\underbrace{a + b \text{ is even integer.}}_{q}$

$p \rightarrow q$ assume p is true

$a = 2x + 1$ and $b = 2y + 1 \; \exists x, y \in Z$

$a + b = 2x + 1 + 2y + 1$

$a + b = 2x + 2y + 2$

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers}}_{p}$, then $\underbrace{a + b \text{ is even integer}}_{q}$.

$p \rightarrow q$ assume p is true

$a = 2x + 1$ and $b = 2y + 1\ \exists x, y \in Z$

$a + b = 2x + 1 + 2y + 1$

$a + b = 2x + 2y + 2$

$a + b = 2(x + y + 1)$

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers,}}_{p}$ then $\underbrace{a + b \text{ is even integer.}}_{q}$

$p \rightarrow q$ assume p is true

$a = 2x + 1$ and $b = 2y + 1\ \exists x, y \in Z$

$a + b = 2x + 1 + 2y + 1$

$a + b = 2x + 2y + 2$

$a + b = 2(x + y + 1)$

$a + b = 2m, \exists m \in Z$

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers,}}_{p}$ then $\underbrace{a+b \text{ is even integer}}_{q}$.

$p \rightarrow q$ assume p is true

$a = 2x + 1$ and $b = 2y + 1 \; \exists x, y \in Z$

$a + b = 2x + 1 + 2y + 1$

$a + b = 2x + 2y + 2$

$a + b = 2(x + y + 1)$

$a + b = 2m, \exists m \in Z$

q is also true

Proofs

Direct Proof

If m and n are perfect squares, then $m.n$ is also a perfect square.

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If $\underbrace{m \text{ and } n \text{ are perfect squares}}_{p}$, then $\underbrace{m.n \text{ is also a perfect square.}}_{q}$

$p \rightarrow q$

Proofs

Direct Proof

If $\underbrace{m \text{ and } n \text{ are perfect squares}}_{p}$, then $\underbrace{m.n \text{ is also a perfect square.}}_{q}$

$p \to q$ assume p is true

Proofs

Direct Proof

$$\text{If } \underbrace{m \text{ and } n \text{ are perfect squares}}_{p}\text{, then } \underbrace{m.n \text{ is also a perfect square.}}_{q}$$

p → q assume p is true

$m = x^2$ and $n = y^2$, $\exists x, y \in Z$

Proofs

Direct Proof

If $\underbrace{m \text{ and } n \text{ are perfect squares}}_{p}$, then $\underbrace{m.n \text{ is also a perfect square.}}_{q}$

$p \rightarrow q$ assume p is true

$m = x^2$ and $n = y^2$, $\exists x, y \in Z$

$m.n = x^2y^2$

Proofs

Direct Proof

If $\underbrace{m \text{ and } n \text{ are perfect squares}}_{p}$, then $\underbrace{m.n \text{ is also a perfect square.}}_{q}$

p → q   assume p is true

$m = x^2$ and $n = y^2$, $\exists x, y \in Z$

$m.n = x^2 y^2$

$m.n = (x.y)^2$

Proofs

Direct Proof

If $\underbrace{m \text{ and } n \text{ are perfect squares}}_{p}$, then $\underbrace{m.n \text{ is also a perfect square}}_{q}$.

p → q   assume p is true

$m = x^2$ and $n = y^2$, $\exists x, y \in Z$

$m.n = x^2 y^2$

$m.n = (x.y)^2$

$m.n = k^2$, $\exists k \in Z$

Proofs

Direct Proof

If m and n are perfect squares, then $m.n$ is also a perfect square.

$\underbrace{\text{$m$ and n are perfect squares}}_{p}$, then $\underbrace{\text{$m.n$ is also a perfect square.}}_{q}$

$p \rightarrow q$

assume p is true

$m = x^2$ and $n = y^2$, $\exists x, y \in Z$

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$m.n = k^2, \exists k \in Z$

q is also true

Proofs

Proof by Contraposition

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- Instead of proving $p \rightarrow q$, prove logically equivalent proposition $\sim q \rightarrow \sim p$

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Proof by Contraposition

- Instead of proving $p \rightarrow q$, prove logically equivalent proposition $\sim q \rightarrow \sim p$ --WHY?

Proofs

Proof by Contraposition

If $3n + 2$ is an odd integer, then n is odd integer

Proofs

Proof by Contraposition

If $\underbrace{3n + 2 \text{ is an odd integer}}_{p}$, then $\underbrace{n \text{ is odd integer}}_{q}$

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If $\underbrace{3n + 2 \text{ is an odd integer}}_{p}$, then $\underbrace{n \text{ is odd integer}}_{q}$

$p \rightarrow q$ assume p is true

Proofs

Proof by Contraposition

If $\underbrace{3n + 2 \text{ is an odd integer}}_{p}$, then $\underbrace{n \text{ is odd integer}}_{q}$

p → q   assume p is true

$$3n + 2 = 2k + 1, \exists k \in Z$$

Proofs

Proof by Contraposition

If $\underbrace{3n + 2 \text{ is an odd integer}}_{p}$, then $\underbrace{n \text{ is odd integer}}_{q}$

$p \rightarrow q$ assume p is true

$$3n + 2 = 2k + 1, \exists k \in Z$$
$$3n = 2k - 1$$

Proofs

Proof by Contraposition

$$\text{If } \underbrace{3n+2 \text{ is an odd integer}}_{p}\text{, then } \underbrace{n \text{ is odd integer}}_{q}$$

$p \rightarrow q$ assume p is true

$$3n + 2 = 2k + 1, \exists k \in Z$$
$$3n = 2k - 1$$
$$n = \frac{2k-1}{3}$$

Proofs

Proof by Contraposition $p \rightarrow q \equiv \sim q \rightarrow \sim p$

If $3n + 2$ is an odd integer, then n is odd integer

Proofs

Proof by Contraposition

$p \rightarrow q \equiv \sim q \rightarrow \sim p$

If $3n + 2$ is an odd integer, then n is odd integer

If n is not odd integer, then $3n + 2$ is not odd integer

Proofs

Proof by Contraposition $p \to q \equiv \sim q \to \sim p$

If $3n + 2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n + 2 \text{ is not odd integer}}_{\sim p}$

Proofs

Proof by Contraposition $\quad p \rightarrow q \equiv {\sim}q \rightarrow {\sim}p$

If $3n + 2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n + 2 \text{ is not odd integer}}_{\sim p}$

assume ~q is true

Proofs

Proof by Contraposition $\quad p \to q \equiv {\sim}q \to {\sim}p$

If $3n + 2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n + 2 \text{ is not odd integer}}_{\sim p}$

assume ~q is true $\qquad n = 2k, \exists k \in Z$

Proofs

Proof by Contraposition

$p \rightarrow q \equiv {\sim}q \rightarrow {\sim}p$

If $3n+2$ is an odd integer, then n is odd integer

$$\text{If } \underbrace{n \text{ is not odd integer}}_{\sim q}\text{, then } \underbrace{3n+2 \text{ is not odd integer}}_{\sim p}$$

assume ~q is true

$$n = 2k, \exists k \in Z$$
$$3n+2 = 6k+2$$

Proofs

Proof by Contraposition $p \rightarrow q \equiv \sim q \rightarrow \sim p$

If $3n+2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n+2 \text{ is not odd integer}}_{\sim p}$

assume ~q is true

$$n = 2k, \exists k \in Z$$
$$3n+2 = 6k+2$$
$$3n+2 = 2(3k+1)$$

Proofs

Proof by Contraposition $\quad p \rightarrow q \equiv \sim q \rightarrow \sim p$

If $3n + 2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n + 2 \text{ is not odd integer}}_{\sim p}$

assume ~q is true

$$n = 2k, \exists k \in Z$$
$$3n + 2 = 6k + 2$$
$$3n + 2 = 2(3k + 1)$$
$$3n + 2 = 2m, \exists m \in Z$$

Proofs

Proof by Contraposition

$p \to q \equiv \sim q \to \sim p$

If $3n + 2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n + 2 \text{ is not odd integer}}_{\sim p}$

assume ~q is true

$n = 2k, \exists k \in Z$

$3n + 2 = 6k + 2$

$3n + 2 = 2(3k + 1)$

$3n + 2 = 2m, \exists m \in Z$

~p is also true

Proofs

Proof by Contraposition

$p \to q \equiv \sim q \to \sim p$

Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.

Proofs

Proof by Contraposition

$p \rightarrow q \equiv {\sim}q \rightarrow {\sim}p$

Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.

If $\underbrace{x < 50 \text{ and } y < 50}_{\sim q}$, then $\underbrace{x + y < 100}_{\sim p}$

Proofs

Proof by Contraposition

$p \to q \equiv \sim q \to \sim p$

Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.

If $\underbrace{x < 50 \text{ and } y < 50}_{\sim q}$, then $\underbrace{x + y < 100}_{\sim p}$

assume ~q is true

Proofs

Proof by Contraposition $p \to q \equiv \sim q \to \sim p$

Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.

If $\underbrace{x < 50 \text{ and } y < 50}_{\sim q}$, then $\underbrace{x + y < 100}_{\sim p}$

assume ~q is true $x < 50$ and $y < 50$

Proofs

Proof by Contraposition $\quad p \rightarrow q \equiv \sim q \rightarrow \sim p$

Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.

If $\underbrace{x < 50 \text{ and } y < 50}_{\sim q}$, then $\underbrace{x + y < 100}_{\sim p}$

assume ~q is true $\quad x < 50$ and $y < 50$

$$x + y < 100$$

Proofs

Proof by Contraposition $\quad p \rightarrow q \equiv \sim q \rightarrow \sim p$

Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.

If $\underbrace{x < 50 \text{ and } y < 50}_{\sim q}$, then $\underbrace{x + y < 100}_{\sim p}$

assume ~q is true $\quad x < 50$ and $y < 50$

$x + y < 100$

~p is also true

Proofs

Proof by Contradiction

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$$\sim p \rightarrow q$$

$$\rightarrow F \equiv T$$

Proofs

Proof by Contradiction

- To prove that 'p is true', find a contradiction q such that ~p → q is true.

$$\sim p \rightarrow q$$

$$F \rightarrow F \equiv T$$

Proofs

Proof by Contradiction

- To prove that 'p is true', find a contradiction q such that $\sim p \rightarrow q$ is true.

$$\sim p \rightarrow q$$

$$F \rightarrow F \equiv T$$

$$q \equiv r \wedge \sim r$$

Proofs

Proof by Contradiction

- To prove that 'p is true', find a contradiction q such that $\sim p \rightarrow q$ is true.

$$\sim p \rightarrow q$$

$$F \rightarrow F \equiv T$$

$$q \equiv r \wedge \sim r \equiv 0$$

Proofs

Proof by Contradiction

- To prove that 'p is true', find a contradiction q such that $\sim p \rightarrow q$ is true.

$$\sim p \rightarrow q$$

$$F \rightarrow F \equiv T$$

- assuming '~p is true' leads us a contradiction

$$q \equiv r \wedge \sim r \equiv 0$$

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

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- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$y = \frac{a}{b}$ and $x + y = \frac{c}{d}$, $\exists\ a, b, c, d \in Z$

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$y = \frac{a}{b}$ and $x + y = \frac{c}{d}$, $\exists\ a, b, c, d \in Z$

There is no integers e,f such that $x = \frac{e}{f}$

Proofs

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There is no integers e,f such that $x = \frac{e}{f}$ (the proposition r)

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Proof by Contradiction

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$y = \frac{a}{b}$ and $x + y = \frac{c}{d}$, $\exists\ a, b, c, d \in Z$
There is no integers e,f such that $x = \frac{e}{f}$ (the proposition r)

$$x + y = \frac{c}{d}$$

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$y = \frac{a}{b}$ and $x + y = \frac{c}{d}$, $\exists\, a, b, c, d \in Z$
There is no integers e,f such that $x = \frac{e}{f}$ (the proposition r)

$x + y = \frac{c}{d} \quad \rightarrow \quad x + \frac{a}{b} = \frac{c}{d}$

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$y = \frac{a}{b}$ and $x + y = \frac{c}{d}$, $\exists\, a, b, c, d \in Z$
There is no integers e,f such that $x = \frac{e}{f}$ (the proposition r)

$x + y = \frac{c}{d} \;\rightarrow\; x + \frac{a}{b} = \frac{c}{d} \;\rightarrow\; x = \frac{c}{d} - \frac{a}{b}$

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$y = \frac{a}{b}$ and $x + y = \frac{c}{d}$, $\exists\ a, b, c, d \in Z$
There is no integers e,f such that $x = \frac{e}{f}$ (the proposition r)

$$x + y = \frac{c}{d} \;\rightarrow\; x + \frac{a}{b} = \frac{c}{d} \;\rightarrow\; x = \frac{c}{d} - \frac{a}{b} \;\rightarrow\; x = \frac{e}{f}\ ,\ \exists\ e, f \in Z$$

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$y = \frac{a}{b}$ and $x + y = \frac{c}{d}$, $\exists\ a, b, c, d \in Z$

There is no integers e,f such that $x = \frac{e}{f}$ (the proposition r)

$x + y = \frac{c}{d} \;\rightarrow\; x + \frac{a}{b} = \frac{c}{d} \;\rightarrow\; x = \frac{c}{d} - \frac{a}{b} \;\rightarrow\; x = \frac{e}{f}$, $\exists\ e, f \in Z$ (~r)

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

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There is no integers e,f such that $x = \frac{e}{f}$ (the proposition r)

$x + y = \frac{c}{d}$ $\rightarrow$ $x + \frac{a}{b} = \frac{c}{d}$ $\rightarrow$ $x = \frac{c}{d}$ - $\frac{a}{b}$ $\rightarrow$ $x = \frac{e}{f}$, $\exists\ e, f \in Z$ (~r)

~p → (r ∧ ~r) : assuming '~p is true' leads us a contradiction.

Proofs

Proof by Contradiction

- Prove that if $3n + 2$ is an odd integer, then n is odd integer

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- Prove that if $\underbrace{3n + 2 \text{ is an odd integer}}_{p}$, then $\underbrace{n \text{ is odd integer}}_{q}$

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Assuming 'p → q is not true' leads us a contradiction.

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- Prove that if $\underbrace{3n + 2 \text{ is an odd integer}}_{p}$, then $\underbrace{n \text{ is odd integer}}_{q}$

Assuming 'p → q is not true' leads us a contradiction.

$\sim(p \rightarrow q)$

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$3n + 2$ is an odd integer and n is even integer. (p ∧ ~q)

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$n = 2k, \exists\, k \in Z.$

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$n = 2k, \exists\, k \in Z$. So $3n+2 = 6k+2$

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$n = 2k, \exists\, k \in Z$. So $3n + 2 = 6k + 2 = 2(3k + 1)$

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$n = 2k, \exists\, k \in Z$. So $3n+2 = 6k+2 = 2(3k+1) = 2m, \exists\, m \in Z$

$3n+2$ is an even integer. (Contradiction!)

Proofs

Proof of Equivalence (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

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n is odd integer if and only if $5n + 4$ is odd integer

Proofs

Proof of Equivalence (to prove two statements p and q are equal, the statement of the form p↔q should be proved)

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$p \rightarrow q$ (direct proof)

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assume p is true

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$n = 2k + 1, \exists k \in Z$

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$5n + 4 = 10k + 9$

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p → q (direct proof)

assume p is true

$n = 2k + 1, \exists k \in Z$

$5n + 4 = 10k + 9$

$5n + 4 = 2(5k + 4) + 1$

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Proof of Equivalence (to prove two statements p and q are equal, the statement of the form $p \leftrightarrow q$ should be proved)

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$q \rightarrow p$ (proof by contraposition)

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$$\underbrace{\text{n is odd integer}}_{p} \text{ if and only if } \underbrace{5n+4 \text{ is odd integer}}_{q}$$

$\boxed{\sim p \rightarrow \sim q}$

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$n = 2k, \exists k \in Z$

$5n + 4 = 10k + 4$

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$q \rightarrow p$ (proof by contraposition)

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$\sim q$ is true