SIGNALS and SYSTEM
Lecture Notes and Examples (1)

* Signal is a physical quantity that varies with time, space or any other independent variable(s) and convey information.

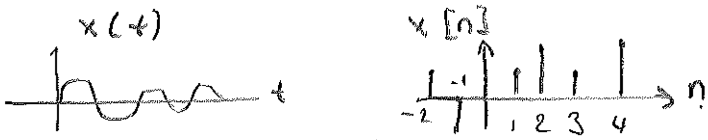
* Noise is a signal which carries unwanted information.

* If a signal varies with one independent variable, it is called one dimensional signal. If signal varies with two or more independent variables, it is called multi-dimensional signal.

* Signals are represented with mathematical functions

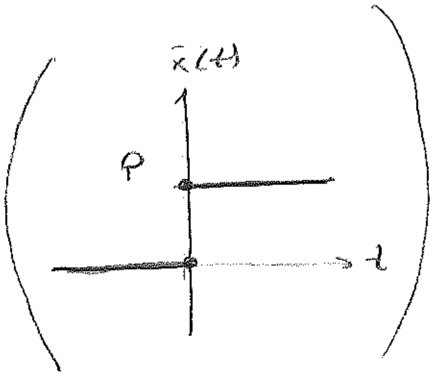
* Continious time signal: Defined for all the values of t and signals represented as $x(t)$, $y(t)$

* Discrete time signal: Defined only at discrete intervals of time. and represented as $x[n]$



⇒ Examples about basic signals and operations

① Plot the graph of step signal and write the function:



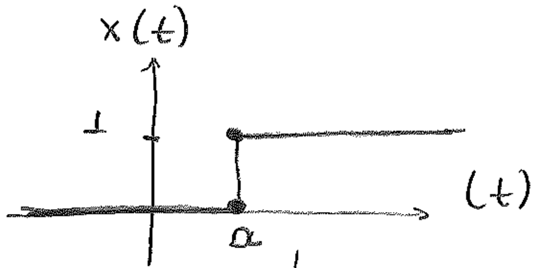
$$Pu(t) = \begin{cases} P & t>0 \\ 0 & t<0 \end{cases}$$

↳ step signal is not defined @ $t=0$

↳ If amplitude of signal $P=1$ then the signal is called as unit step signal

② Plot the graph of $x(t) = u(t-a)$ where $u(t)$ is a unit step signal $(a>0)$

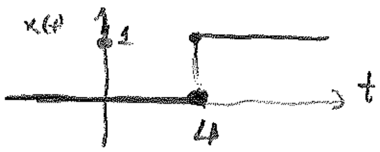
$$u(t-a) = \begin{cases} 1 & t-a>0 \\ 0 & t-a<0 \end{cases} \Rightarrow u(t-a) = \begin{cases} 1 & t>a \\ 0 & t<a \end{cases}$$



(Discontiniuty is shifted here to a)

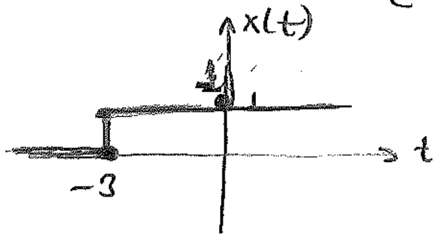
↳ The signal is shifted (right hand side shifting) by a units.

③ Plot the graph of $x(t) = u(t-4)$ where $u(t)$ is a unit step function



3 $x(t) = u(t+3)$ Plot the graph of $x(t)$

$$u(t+3) = \begin{cases} 1 & t+3>0 \\ 0 & t+3<0 \end{cases} \Rightarrow \begin{cases} 1 & t>-3 \\ 0 & t<-3 \end{cases}$$



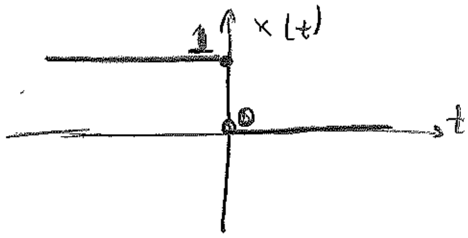
(The signal is shifted to the left hand side)

4 $x(t) = u(-t)$ Plot the graph of $x(t)$

$u(-t)$ means reversal or mirror around vertical axis:

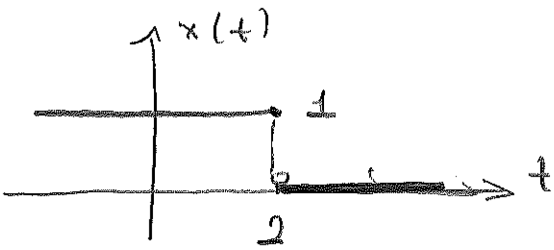
$$u(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases} \Rightarrow u(-t) = \begin{cases} 1 & -t>0 \\ 0 & -t<0 \end{cases}$$

$$u(-t) = \begin{cases} 1 & t<0 \\ 0 & t>0 \end{cases} \Rightarrow$$



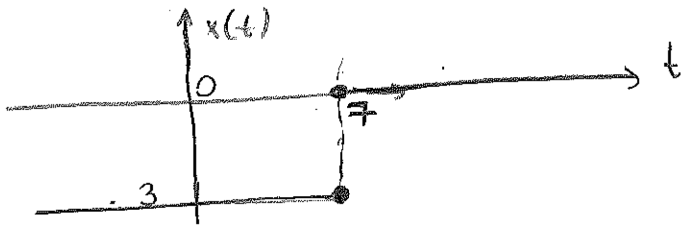
5 $x(t) = u(-t+2)$ Plot the graph

$$x(t) = \begin{cases} 1 & -t+2>0 \longrightarrow -t>-2 \rightarrow t<2 \\ 0 & -t+2<0 \longrightarrow t>2 \end{cases}$$



(6) Plot the graph of signal $x(t) = -3u(-t+7)$

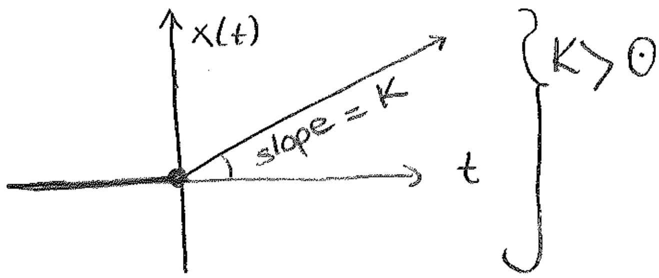
$$-3u(-t+7) = \begin{cases} -3 & -t+7>0 \\ 0 & -t+7<0 \end{cases} \Rightarrow \begin{cases} -3 & t<7 \\ 0 & t>7 \end{cases}$$



(7) Plot the graph of signal $Kr(t)$ where $r(t)$ is a ramp signal.

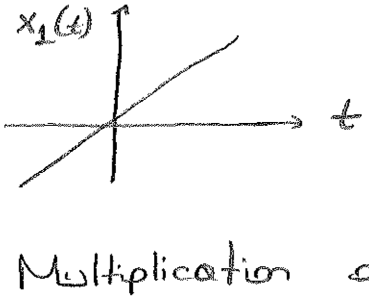
$$Kr(t) = \begin{cases} Kt & t>0 \\ 0 & t<0 \end{cases}$$

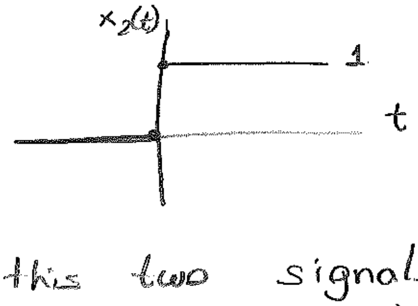
$\hookrightarrow Kr(t)\big|_{t=0} = 0$



(8) Plot the graph of signal $x(t) = t\,u(t)$ where $u(t)$ is unit step signal.

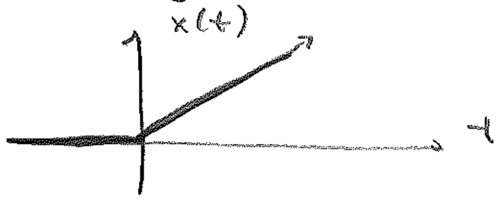
Solution: We have two signals: $x_1 = t$ $x_2 = u(t)$





Multiplication of this two signals can be represented as

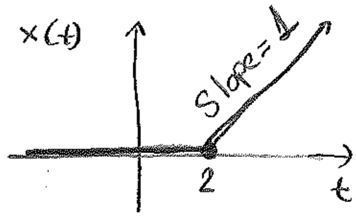
$$t\,u(t) = \begin{cases} t.1 & t>0 \\ t.0 & t<0 \end{cases}$$



(9) Plot the graph of $x(t) = r(t-2)$ where r is the ramp signal.

$$r(t-2) = \begin{cases} t-2 & t-2>0 \\ 0 & t-2<0 \end{cases} \longrightarrow r(t-2) = \begin{cases} (t-2) & t>2 \\ 0 & t<2 \end{cases}$$

↳ Becouse the amplitude here directly depend on time we use $(t-2)$ in amplitude part.

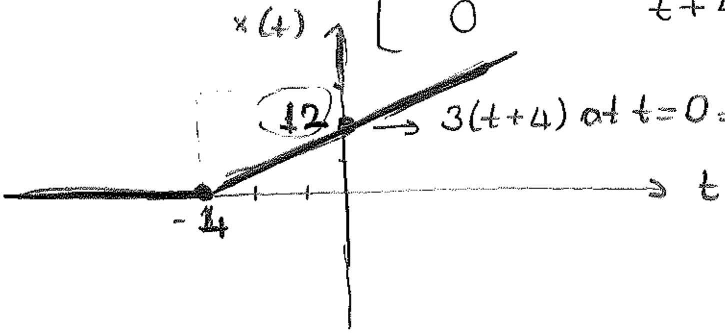


* If we want to write $x(t) = r(t-2)$ in terms of unit step signal.

$$r(t-2) = (t-2)\, u(t-2)$$

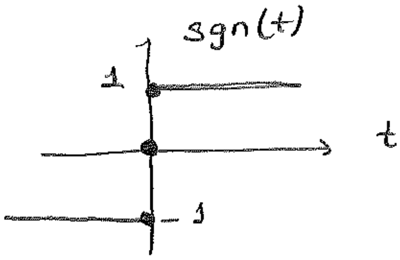
(10) Plot the graph of $x(t) = 3r(t+4)$ (r = ramp signal)

$$3r(t+4) = \begin{cases} 3(t+4) & t+4>0 \\ 0 & t+4<0 \end{cases} = \begin{cases} 3(t+4) & t>-4 \\ 0 & t<-4 \end{cases}$$



→ $3(t+4)$ at $t=0 \Rightarrow 12$.

(11) Plot graph of signum function $sgn(t)$



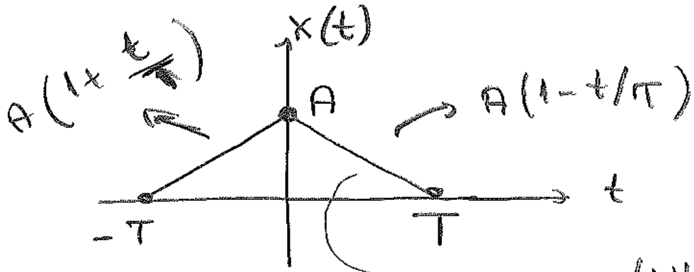
$$sgn(t) = \begin{cases} 1 & t>0 \\ 0 & t=0 \\ -1 & t<0 \end{cases}$$

** $sgn(t) = 2u(t) - 1$

$sgn(t) = u(t) - u(-t)$

(12) Plot the graph of triangular function and the function definition

$$x(t) = A\,\text{tri}\,(t/T) = \begin{cases} A\left(1-\frac{|t|}{T}\right) & |t| \geq T \\ 0 & |t| < T \end{cases} \quad \text{or} \quad \begin{cases} A\left(1+\frac{t}{T}\right) & -T<t<0 \\ A\left(1-\frac{t}{T}\right) & 0<t<T \\ 0 & |t|>T \end{cases}$$

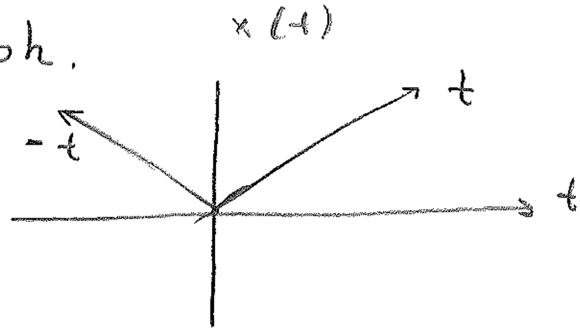


What is the area ; $\frac{(2T)\,A}{2} = AT$

$AT = 1$ → unit triangular

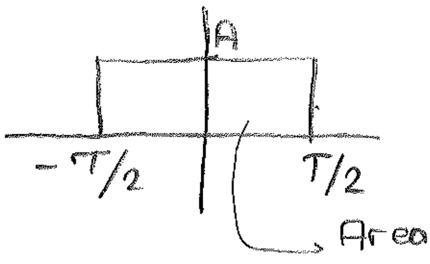
(13) $x(t) = |t|$ Plot the graph.

$$x(t) = |t| = \begin{cases} t & t>0 \\ -t & t<0 \end{cases}$$



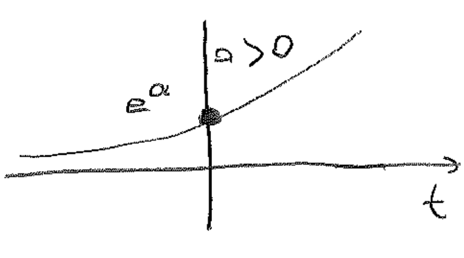
(14) Plot the rectangular function rect(t)

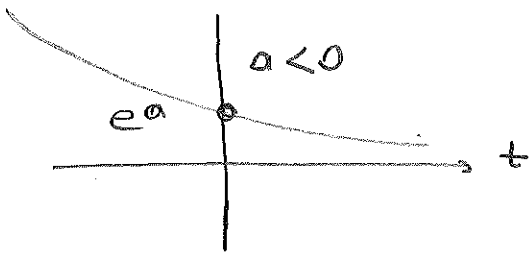
$$A\,\text{rec}\left(\frac{t}{T}\right) = \begin{cases} A & |t| < T/2 \\ 0 & |t| > T/2 \end{cases}$$



→ Note that; if area $AT = 1$ will become unit rectangular function

(15) $x(t) = e^{at}$, Plot $x(t)$ if $a>0$ and $a<0$





Note: Exponential signals can be real exponential signals or complex exponential signals.

e^{at} is real exponential

$e^{\pm j\omega t} \longrightarrow$ periodic signal. (complex exponential signal)

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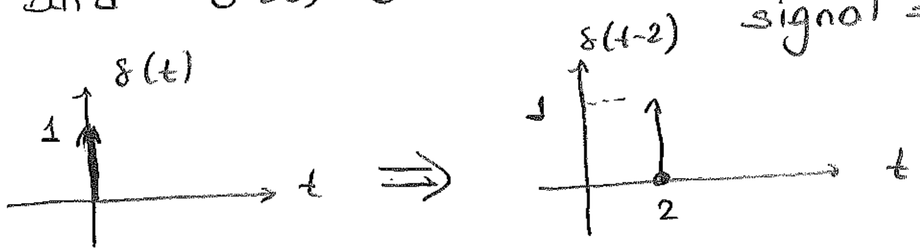
16) The value of $\int_{-\infty}^{\infty} e^{-t}\, \delta(2t-2)\, dt$ where

Solution: $\delta(t)$ = Dirac Delta function = Continous Time Impulse Signal

$$\Rightarrow \int_{-\infty}^{\infty} \delta(t)\, dt = 1 \quad \text{(Area of this impulse signal is 1)}$$

and $\delta(t) = 0$ if $t \neq 0$ (width of the signal = 0)



Note: $\int_{-\infty}^{\infty} f(t)\, \delta(t-t_0)\, dt = f(t)\big|_{t=t_0} = f(t_0)$

$$\delta(at) = \frac{1}{|a|}\, \delta(t)$$

$$= \int_{-\infty}^{\infty} e^{-t}\, \delta(2t-2)\, dt$$

$$\delta(2(t-1)) = \frac{1}{2}\, \delta(t-1)$$

$$= \int_{-\infty}^{\infty} e^{-t} \left(\frac{1}{2}\, \delta(t-1)\right) dt = \frac{1}{2}\, e^{-t}\Big]_{t=1} = \frac{e^{-1}}{2}$$

$1/2e$

17) Calculate the results?

a) $\displaystyle\int_{-5}^{+5} e^{-t}\, \underbrace{\delta(t-1)}_{\delta}\, dt$

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$$

$$= \int_{-5}^{5} e^{-t}\,\delta(t-1) = e^{-1} = \underline{\underline{1/e}}$$

b) $\displaystyle\int_{-5}^{-2} e^{-t}\, \underbrace{\delta(t-1)}_{\delta}\, dt$

↳ $\delta(t-1)=1$ only $t=1$
But interval for the integral is from -5 to -2 and it does not include 1. So the result is $\underline{\underline{0}}$ (zero)

18) The value of the integral:

$\displaystyle\int_{-2}^{6} e^{-2t}\, \underbrace{\delta(t-1)}_{\delta}\, dt$

integration interval includes $t=1$

↳ $\delta(t-1)=1$ in only $t=1$.. Integration interval can be thougt as $\displaystyle\int_{-\infty}^{\infty}$ Then

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0) \Longrightarrow \int_{-2}^{6} e^{-2t}\,\delta(t-1) = e^{-2t}\Big|_{t=1} = e^{-2}$$

19) Let $\delta(t)$ denote the Delta function (impulse function) the value of integral is?

$$\int_{-\infty}^{\infty} \cos\left(\frac{3t}{2}\right) \quad \delta(t)$$

Solution: $\cos\left(\frac{3t}{2}\right)\Big|_{t=0} = \cos\left(\frac{3.0}{2}\right) = \underline{\underline{1}}$

NOTE: The Dirac Delta function $\delta(t)$ is defined as:

$$\delta(t) = \begin{cases} \infty & t=0 \\ 0 & \text{otherwise} \end{cases} \Bigg\} \text{ for continious time signal.}$$

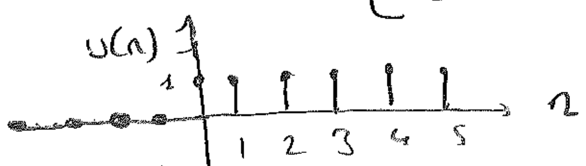
And the area $\int_{-\infty}^{\infty} \delta(t)\, dt$ always equals to $\underline{\underline{1}}$

20) Plot unit step signal graph for continious time (CT) and discrete time (DT)

CT $\Rightarrow$ $u(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$

$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n<0 \end{cases}$ (DT)

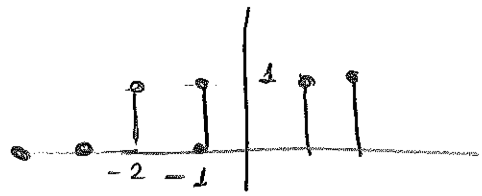
not defined for t = 0



defined for n = 0

21) Plot the graph for $u[n+2]$ (unit step for discrete time)

$u[n+2] = \begin{cases} 1 & n+2 \geq 0 \\ 0 & n+2 < 0 \end{cases}$

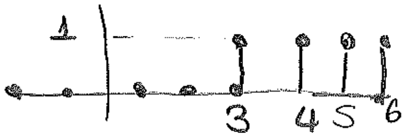


(Left hand shifted (LHS)

Called as delayed signal

22) Plot the graph for $u[n-3]$ (DT Unit Step)

$$u[n-3] = \begin{cases} 1 & n-3 \geq 0 \\ 0 & n-3 < 0 \end{cases}$$



(Right hand shifted (RHS) Called as Advanced signal)

NOTE : $x(t) \rightarrow$ LHS by two units means $\longrightarrow x(t+2)$
(LHS = Left hand shift)

$x[n] \rightarrow$ LHS by two units $\rightarrow x(n+2)$ } Advanced signal

$x(t) \rightarrow$ RHS by three units $\longrightarrow x(t-3)$
(RHS = Right hand shift)

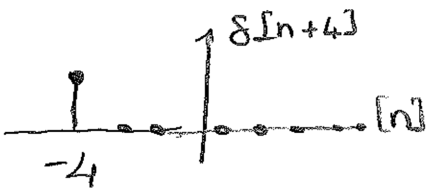
$x[n] \rightarrow$ RHS by 3 units $\longrightarrow x(n-3)$ } Delayed signal

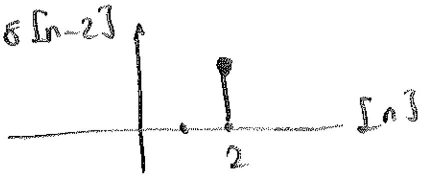
23) Unit impulse sequence OR unit sample sequence?

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

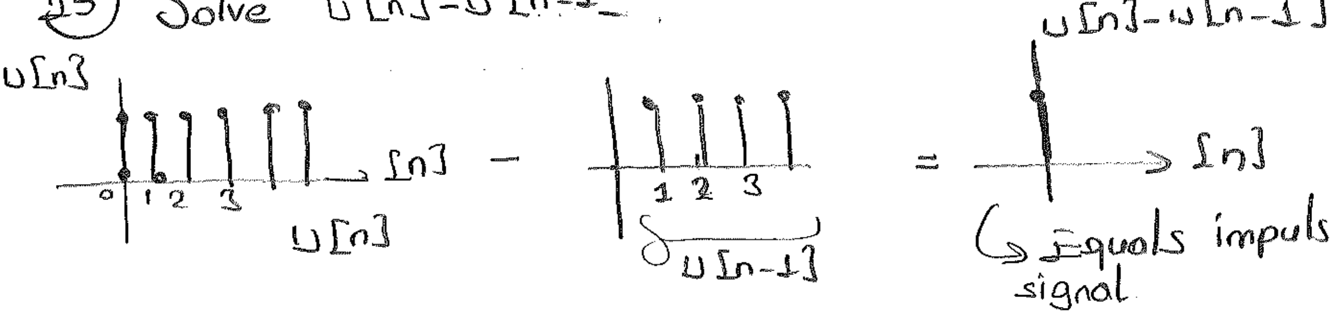
($\delta(t)$ magnitude is $= \infty$
by $\delta[n]$ magnitude is 1)

24) Plot $\delta[n+4]$ and $\delta[n-2]$



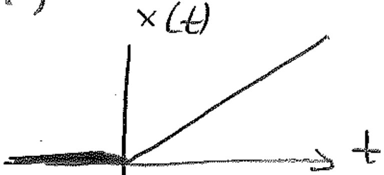


25) Solve $u[n] - u[n-1]$



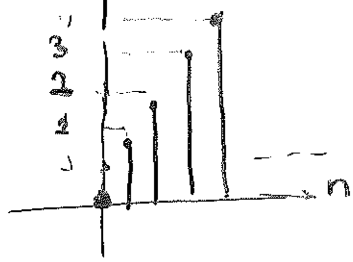
(26) Plot ramp function for both Cont. Time (CT) and Discrete Time (DT)

$$r(t) = \begin{cases} t & t > 0 \\ 0 & t < 0 \end{cases}$$



There is discontinuity in $t = 0$

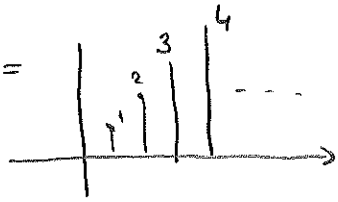
$$r[n] = \begin{cases} n & n \geq 0 \\ 0 & t < 0 \end{cases}$$



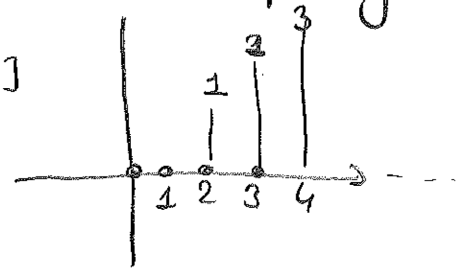
There is no discontinuity

(27) $r[n] - r[n-1]$ ($r[n]$ = Discrete ramp signal)

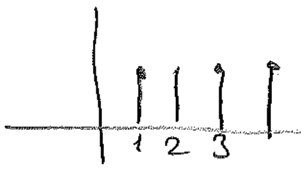
$r[n] =$



$r[n-1]$



$r[n] - r[n-1] =$

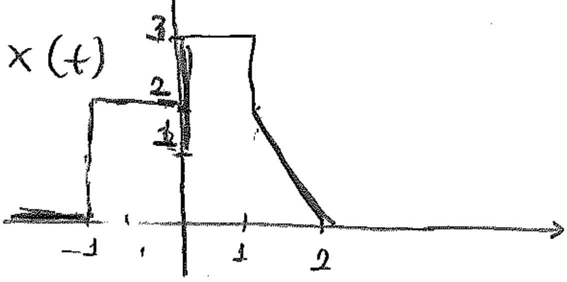


$\rightarrow u[n-1]$

NOTE : $r[n] - r[n-1] = u[n-1]$

$r[n+1] - r[n] = u[n]$

(28) $x(t)$



Plot signals

(a) $x(t-2)$

(b) $x(t+3)$

(c) $-x(t)$

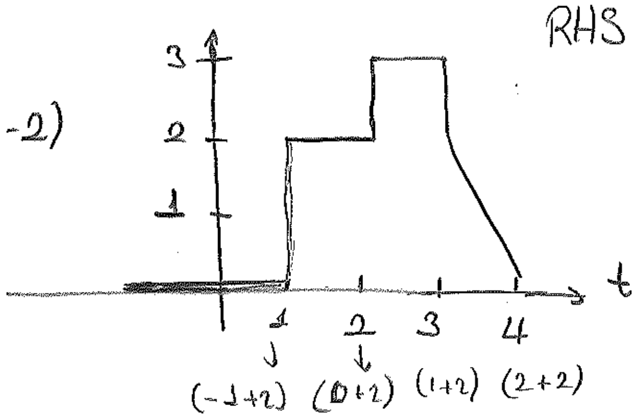
(d) $x(t/3)$

(e) $x(2t)$

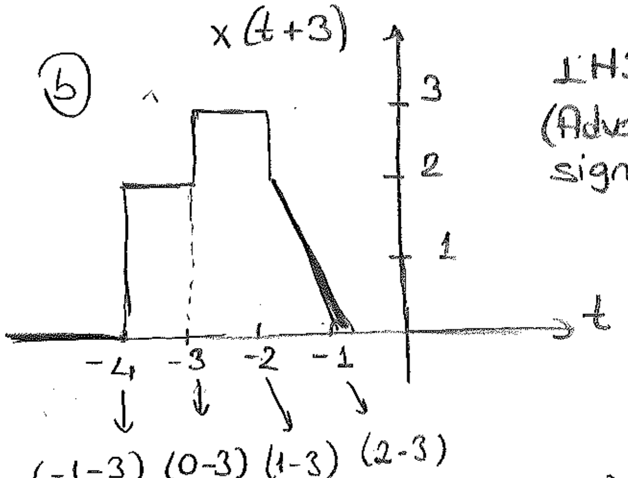
(f) $x(-t)$

RHS (Delayed signal)

(a) $x(t-2)$

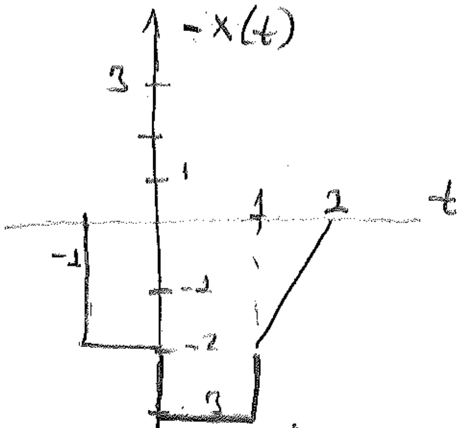


(b) $x(t+3)$



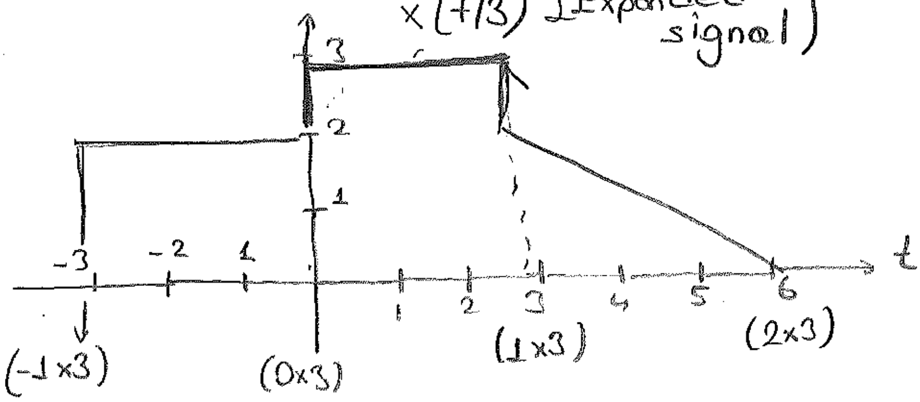
LHS (Advanced signal)

(c) $-x(t)$

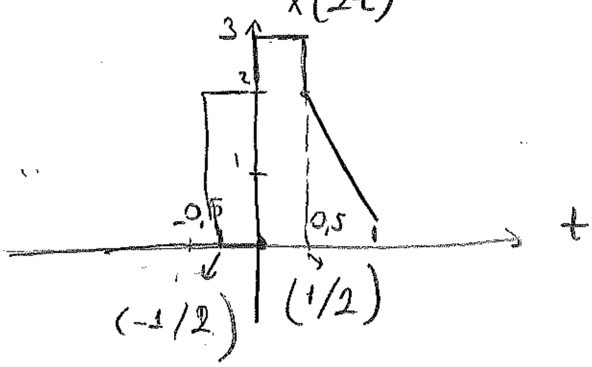


Amplitude Reversal

(d) $x(t/3)$ [Expanded signal)

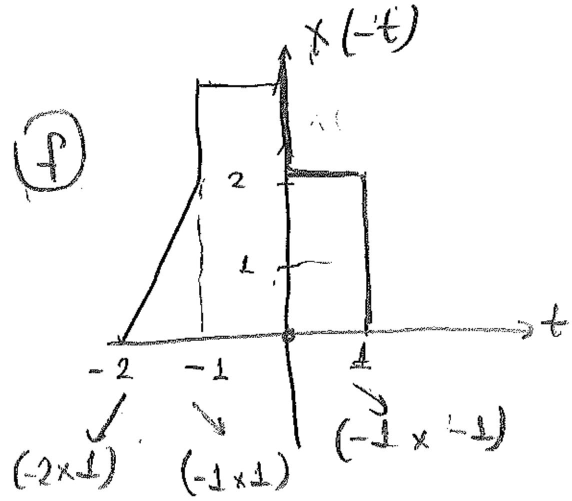


(e) $x(2t)$



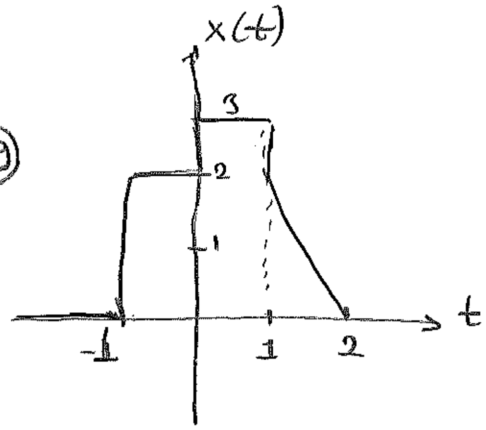
Compressed signal

(f) $x(-t)$



Time Reversal

29

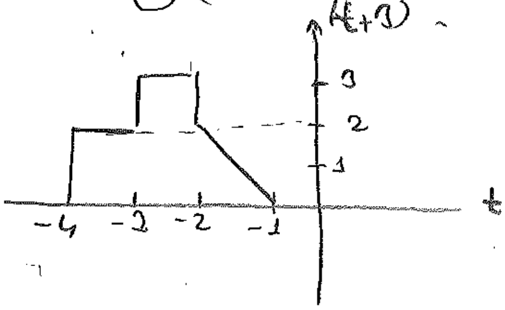


Plot $-3x(-2t+3)$

① $t+3$
② $2t+3$
③ $(-2t+3)$
④ $3x(-2t+3)$
⑤ $-3x(-2t+3)$

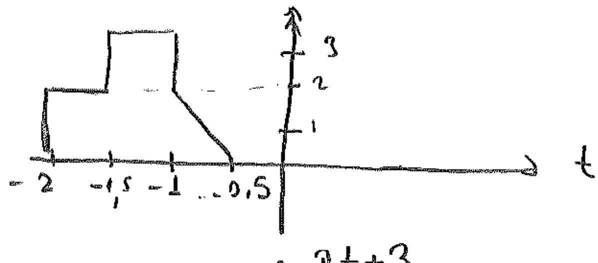
① $t+3$

$-1-3=-4 \rightarrow 2$
$0-3=-3 \rightarrow 3$
$1-3=-2 \rightarrow \cancel{2}$
$2-3=-1 \rightarrow 0$



② $2t+3$

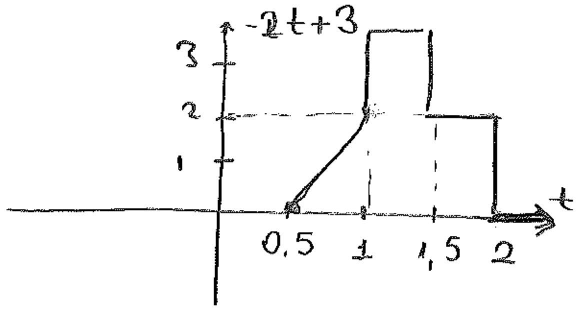
$-4/2=-2 \; (2)$
$-3/2=-1.5 \; (3)$
$-2/2=-1 \; (2)$
$-1/2=-0.5 \; (0)$



③ $-2t+3$

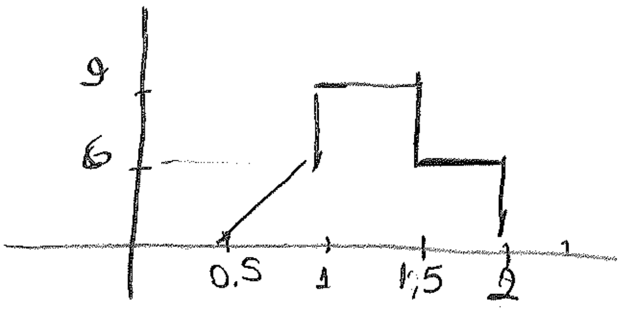
$t=-t$

$-2 \rightarrow 2 \; (2)$
$-1{,}5 \rightarrow 1{,}5 \; (3)$
$-1 \rightarrow 1 \; (2)$
$-0.5 \rightarrow 0.5 \; (0)$



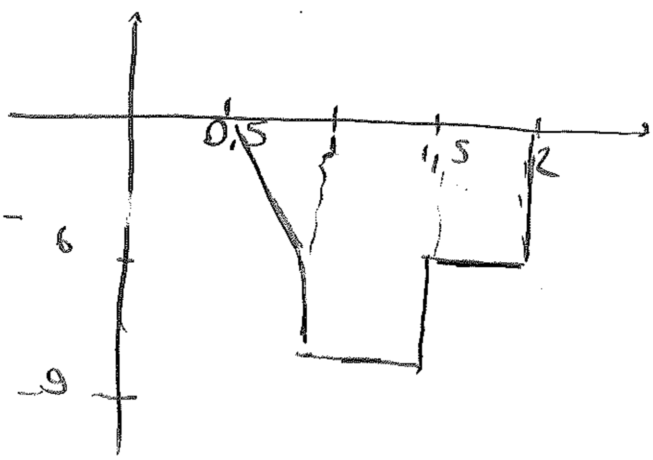
④ $3x(-2t+3)$

$2 \rightarrow (2\times 3=6)$
$1{,}5 \rightarrow (3\times 3=9)$
$1 \rightarrow (2\times 3=6)$
$0.5 \rightarrow 0$

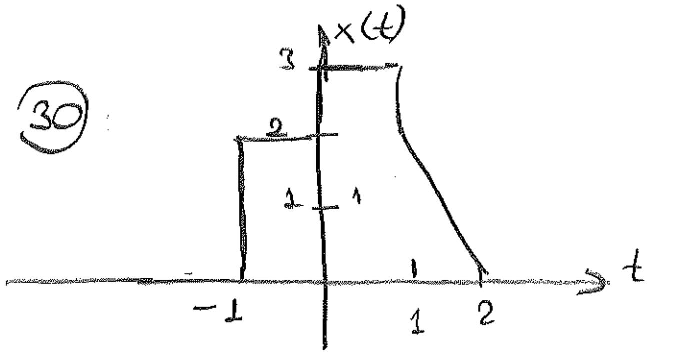


⑤ Amplitudes

$+2 \rightarrow -6$
$1{,}5 \rightarrow -9$
$1 \rightarrow 6$
$0{,}5 \rightarrow 0$



30



Plot

$-\frac{2}{5}x(-3t-4)$

In example 4, all step are plotted. In this example we will plot at the end

① $t-4$.
Right Shift

$-1+4 \to 3$ (2)
$0+4 = 4$ (3)
$1+4 = 5$ (2)
$2+4 = 6$ (0)

② $+3t-4$
Time Compression

$3/3 \to (2)$
$4/3 \to (3)$
$5/3 \to (2)$
$6/3 \to (0)$

③ $-3t-4$ (Reversal)

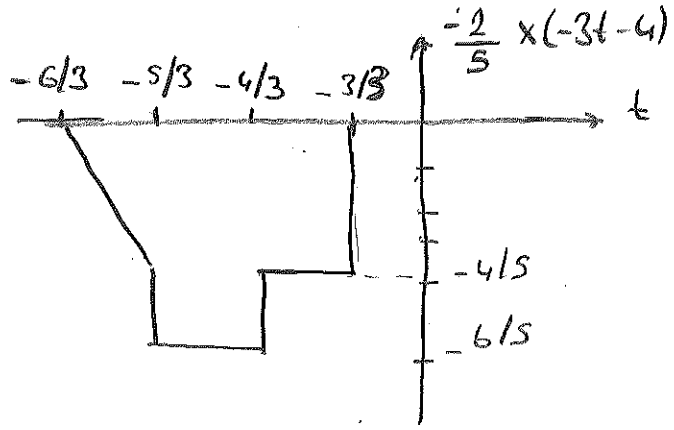
$-3/3 \to 2$
$-4/3 \to 3$
$-5/3 \to 2$
$-6/3 \to 0$

④ $2/5\ x(-3t-4)$
Amplitude scaling

$-3/3 \to 2 \cdot 2/5 = 4/5$
$-4/3 \to 3 \cdot 2/5 = 6/5$
$-5/3 \to 2 \cdot 2/5 = 4/5$
$-6/3 \to 0 \cdot 2/5 = 0$

⑤ $-2/5\ x(-3t-4)$

$-3/3 \to -4/5$
$-4/3 \to -6/5$
$-5/3 \to -4/5$
$-6/3 \to 0$



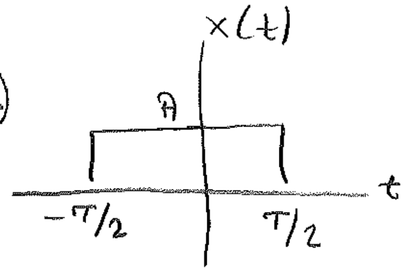
NOTE : A signal which satisfies the equation $x(t) = x(-t)$ for all time t is said to be EVEN signal

Eg : $x(t) = \cos(t)$

$$x(-t) = \cos(-t) = \cos t$$

$$x(-t) = x(t)$$ (even signal)

31



$$x(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)$$

$$x(t) = x(-t)$$

(mirror image around y-axis)

NOTE : Even signals are ALWAYS symmetrical about y-axis

NOTE : A signal which satisfies the equation $x(t) = -x(-t)$ is ODD signal

Eg : $x(t) = \sin t$

$$x(-t) = \sin(-t)$$

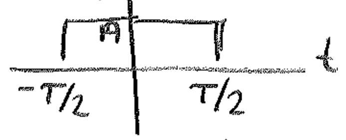
$$x(-t) = \underbrace{-\sin(t)}_{-x(t)}$$

$$\underline{x(-t) = -x(t)} \quad \text{or} \quad \underline{x(t) = -x(-t)}$$

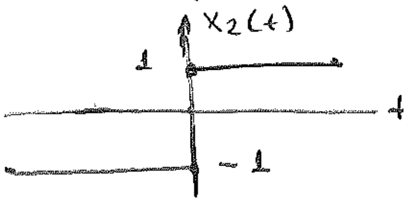
32 $x(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)\mathrm{sgn}(t)$ (sgn(t) is signum f, rect is rectangular f)

We have two signals multiplied:

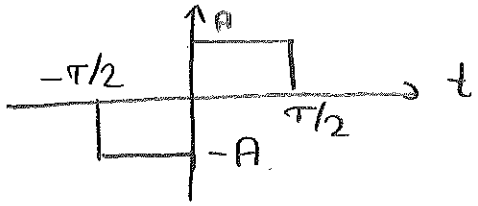
$x_1(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)$



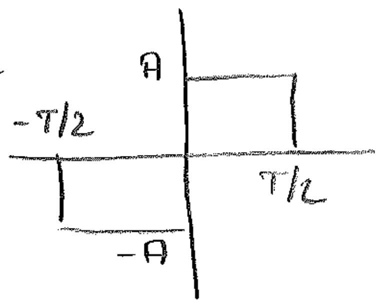
$x_2(t) = \mathrm{sgn}(t)$



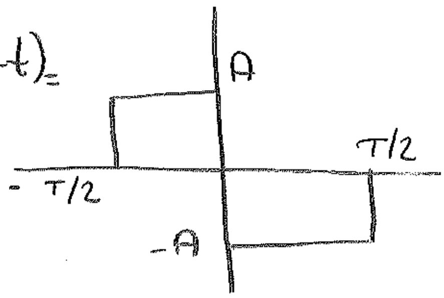
$x_1(t) \cdot x_2(t)$



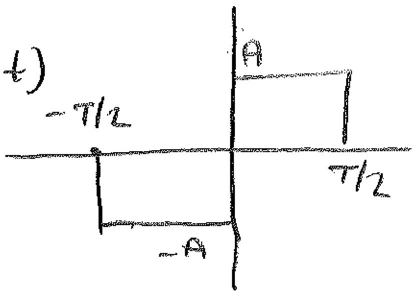
$x(t) =$



$x(-t) =$



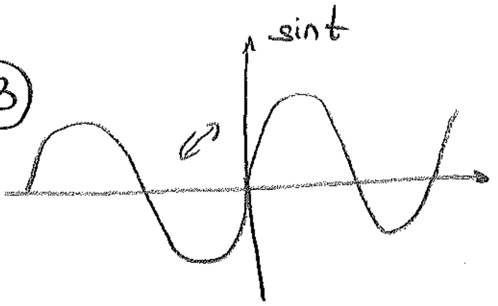
$-x(-t)$



$x(t) = -x(-t)$ ODD signal

NOTE: ODD signal is symmetric around origin

33



sin t is also symmetric about the origin
$(x(t) = -x(-t))$

NOTE: For discrete time; definitions are same.

$x(t) = x(-t)$ C.Time, $x[n] = x[-n]$ D.Time } EVEN

$x(t) = -x(-t)$ C.T, $x[n] = -x[-n]$ D.T } ODD

NOTE: Any signal $x(t)$ can be expressed as sum of two signals out of which one is completely <u>even</u> and other is completely <u>odd</u>

$$x(t) = x_e(t) + x_o(t)$$

$$x(t) = x_e(t) + x_o(t) \quad \text{(Eqn 1)}$$
$$x(-t) = x_e(-t) + x_o(-t) \quad \text{(Eqn 2)}$$

$x_o(-t) = -x_o(t)$ (for ODD signals)

$x_e(-t) = x_e(t)$ (for EVEN signals)

By using Eqn1 and Eqn2 we can define ODD and even parts of signals

Eqn1 + Eqn2 $\Rightarrow$

$$x(t) = x_e(t) + x_o(t) \quad [\text{Eqn1}]$$
$$x(-t) = x_e(t) - x_o(t) \quad [\text{Eqn2}]$$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

Same equations are valid for discrete time

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

(34) Find even and odd components of

$$x(t) = 1 - 3t - 5t^2 + 4t^3 - 6t^4$$

Solution: Let's use formulas

$$x(t) = 1 - 3t - 5t^2 + 4t^3 - 6t^4$$

$$x(-t) = 1 - 3(-t) - 5(-t)^2 + 4(-t)^3 - 6(-t)^4$$

$$= 1 + 3t - 5t^2 - 4t^3 + 6t^4$$

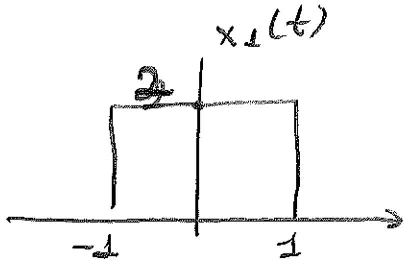
$$x_e = \frac{x(t) + x(-t)}{2} = \left.\begin{matrix} 1 - \cancel{3t} - 5t^2 + \cancel{4t^3} - 6t^4 \\ + \; 1 + \cancel{3t} - 5t^2 - \cancel{4t^3} - 6t^4 \end{matrix}\right\} \frac{2 - 10t^2 - 12t^4}{2}$$

$$x_e(t) = 1 - 5t^2 - 6t^4$$

$$x_o = \frac{x(t) - x(-t)}{2} = \left.\begin{matrix} 1 - 3t - 5t^2 + 4t^3 - 6t^4 \\ - 1 - 3t + 5t^2 + 4t^3 + 6t^4 \end{matrix}\right\} \frac{-6t + 8t^3}{2}$$

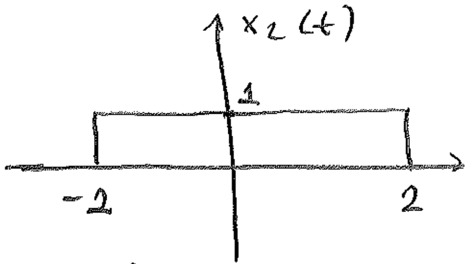
$$x_o = -3t + 4t^3$$

(35) $x_1(t) =$



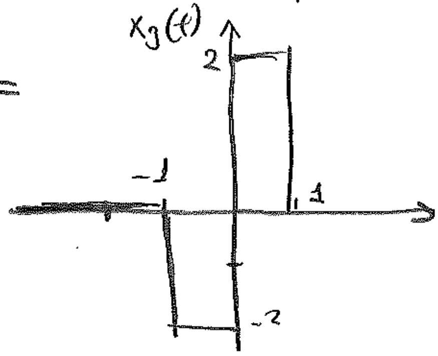
$x_1(t) = 2\,\mathrm{rect}\left(\frac{t}{2}\right)$

$x_2(t) =$

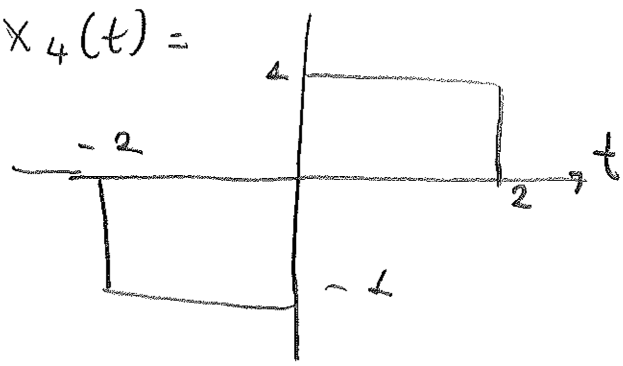


$x_2(t) = \mathrm{rect}\,(t/4)$

$x_3(t) =$



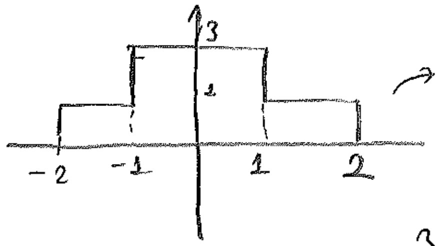
$x_4(t) =$



Given the signals in Eq 35 Check the signals below. Are they even or odd

1) $x_{R_1}(t) = x_1(t) + x_2(t)$

$\quad (E) + (E)$



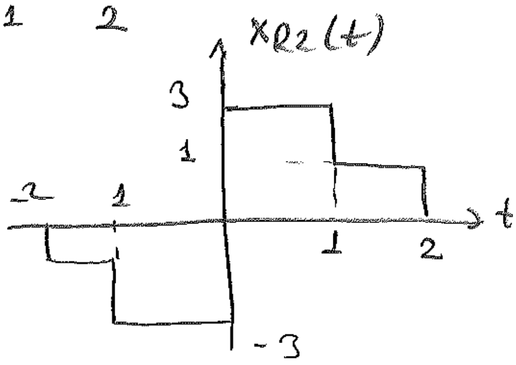
Even

$x_{R_1}(t) \Rightarrow$ Even signal.

2) $x_{R_2}(t) = x_3(t) + x_4(t)$

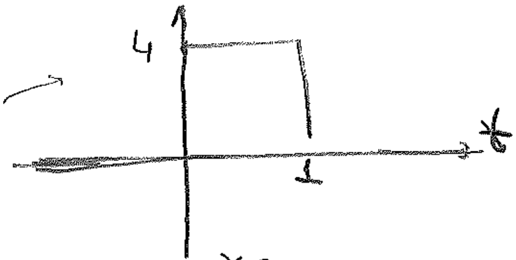
$x_3(t) \Rightarrow$ ODD $\quad x_4(t) \Rightarrow$ ODD

$x_{R_2}(t) \Rightarrow$ ODD signal



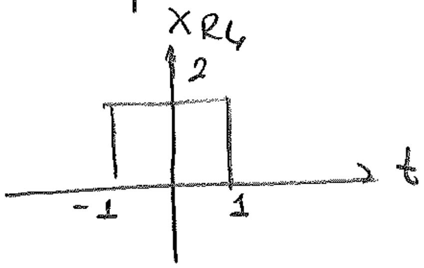
3) $x_{R3} = x_1(t) + x_3(t)$ (Even + Odd)

∴ The resultant signal x_{R3} is Neither ODD nor EVEN



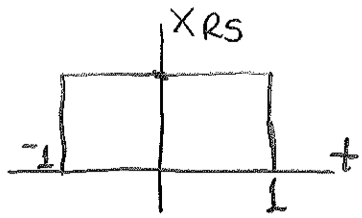
4) $x_{R4} = x_1(t) . x_2(t)$ (Even * Even)

The resultant signal is EVEN



5) $x_{R5} = x_3(t)\, x_4(t)$ (ODD. ODD)

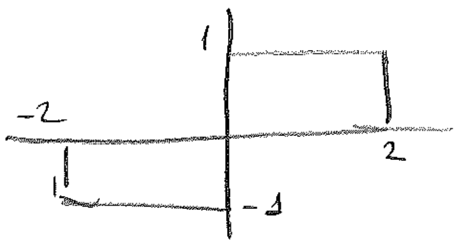
x_{R5} is EVEN



6) $x_{R6} = x_2(t)\, x_4(t)$

EVEN * ODD

x_{R6} = ODD



NOTE : For every signal

1. Even + Even = Even
2. Odd + Odd = Odd
3. Even + Odd = Neither Even Nor ODD
4. Even * Even = Even
5. Odd * Odd = Even
6. Even × Odd = Odd.

NOTE : Area of odd signal

$$\int_{-\infty}^{\infty} odd = 0$$

Area off even signal

$$\int_{-\infty}^{\infty} even = 2\int_{0}^{\infty} even$$

NOTE : For odd signals

$$x(-t) = -x(t)$$
$$x(0) = -x(0)$$
$$2x(0) = 0$$
$$x_0 = 0$$

$x(0)$ (x zero) is always equals to zero in odd signals.

NOTE : All these conclusions and expressions given above are also valid for discrete time signals.

36 The period of the signal

$x(t) = 8\sin(0.8\pi t + \pi/4)$

$\omega_0 = \frac{2\cancel{\pi}}{T} = 0.8\cancel{\pi} \quad T = 2/0.8 = 2.5 \text{ sec.}$

NOTE : $A\sin(\omega t + \phi)$, $A\cos(\omega t + \phi)$, $Ae^{\pm j(\omega t + \phi)}$ } These are all periodic signals, angular frequency ω (rad/s) $\omega = \frac{2\pi}{T} \quad T = \frac{2\pi}{\omega}$

37 $x(t) = 8\sin(0.8\pi t + \frac{\pi}{4}) + 5\cos(0.6\pi t + \frac{\pi}{6})$

Check the periodicity of $x(t)$

$\omega_1 = \frac{2\pi}{T_1} = 0.8\pi \quad T_1 = \frac{2}{0.8} = 2.5 \text{sec}$

$\omega_2 = \frac{2\pi}{T_2} = 0.6\pi \quad T_2 = \frac{2}{0.6} = \frac{10}{3} \text{ sec}$

$T_1/T_2 = \frac{25/10}{10/3} = \frac{3.25}{10.10} = \frac{75}{100} \in \mathbb{R}$ $x(t)$ is a periyodic signal

NOTE ; If T_1/T_2 is a rotional number then resultant signal is periodic.

37 Check the periodicity $x(t) = 8\sin(0.8t + \frac{\pi}{4}) + 5\cos(0.6\pi t + \frac{\pi}{5})$

$\omega_1 = \frac{2\pi}{T_1} = 0.8 \quad T_1 = \frac{2\pi}{0.8} = \frac{20\pi}{8} \text{ sec}$ $\quad \frac{10\pi}{8} \cdot \frac{6}{10}$

$\omega_2 = \frac{2\pi}{T_2} = 0.6\pi \quad T_2 = \frac{2\pi}{0.6\pi} = \frac{20}{6} \text{ sec}$

$\frac{T_1}{T_2} = \frac{20\pi/8}{20/6} = \frac{6\pi}{8} \notin \mathbb{R} \rightarrow$ $x(t)$ is <u>not</u> periodic

38) which of the following signals are periodic?

a) $x(t) = e^{8\pi j t}$

Periodic complex exponential signal

$$\omega = \frac{2\pi}{T} = 8\pi \qquad T = \frac{2\pi}{8\pi} = \frac{1}{4} \text{ sec}$$

b) $y(t) = \underbrace{\cos 2t \cdot \cos 4t}$

we have to convert this multiplication to addition. There is a simple formula.

$$(\cos a)(\cos b) = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)$$

$$= \frac{1}{2}\left(\cos(6t) + \cos(2t)\right)$$

$$\omega_1 = \frac{2\pi}{T_1} = 6 \qquad T_1 = \frac{2\pi}{6}$$

$$\omega_2 = \frac{2\pi}{T_2} = 2 \qquad T_2 = \frac{2\pi}{2}$$

$$\frac{T_1}{T_2} = \frac{2\pi/6}{2\pi/2} = 3 \in R$$

periodic.

$\underline{T_1} = 3T_2$

c) $z(t) = \cos 2t + \cos 3t + \cos 5t$

$$\omega_1 = \frac{2\pi}{T_1} = 2 \qquad T_1 = \frac{2\pi}{2} = \pi$$

$$\omega_2 = \frac{2\pi}{T_2} = 3 \qquad T_2 = \frac{2\pi}{3} = \frac{2\pi}{3}$$

$$\omega_3 = \frac{2\pi}{T_3} = 5 \qquad T_3 = \frac{2\pi}{5} = \frac{2\pi}{5}$$

$$\frac{T_1}{T_2} = \frac{\pi}{2\pi/3} = \frac{3}{2}$$

$$2T_1 = 3T_2$$

$$2(\pi) = 3\left(\frac{2\pi}{3}\right)$$

$$2\pi = 2\pi$$

So period of summation is $T_{12} = 2\pi$

$T_{12} = 2\pi$ and $T_3 = \frac{2\pi}{5}$

$$T_{12}/T_3 = \frac{2\pi}{2\pi/5} = 5$$

$$\boxed{T_{12} = 5T_3 = 2\pi}$$

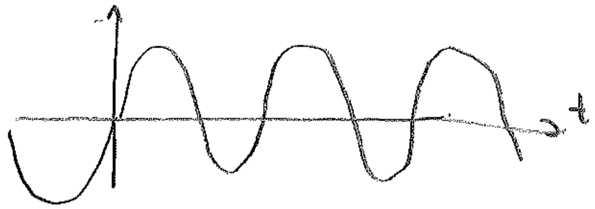
Periodic.

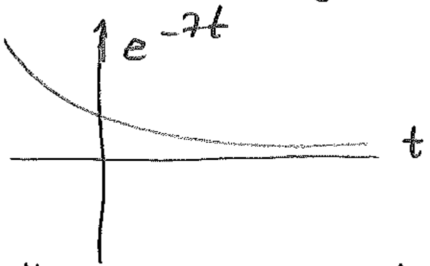
(d) $s(t) = e^{-7t} \sin 10\pi t$

$\sin 10\pi t$ ↳ periodic signal

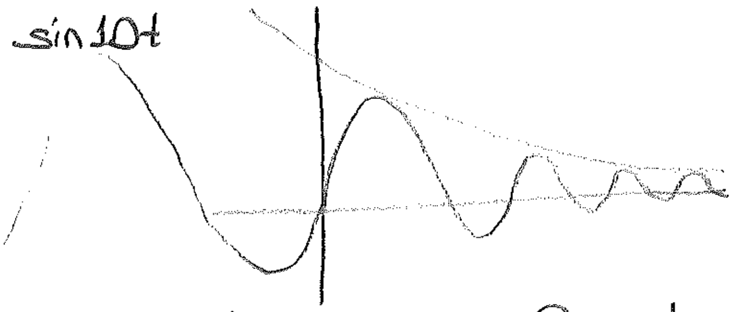
e^{-7t} ↳ Decaying exponential signal.

$\sin 10t$





The exponential signal will be an envelope for the $\sin 10t$



This signal is not repeating itself. Amplitudes are changing

The signal is NOT Periodic

(39) We added three periodic signals. The angular frequencies are

(a) $\omega_1 = 2, \ \omega_2 = 3, \ \omega_3 = 5$

(b) $\omega_1 = 24 \quad \omega_2 = 48, \ \omega_3 = 36$

Check the periodicity and find the period of resultant signal.

SOLUTION: We can use Greatest Common Divisor Method here;

(a)
$$\left.\begin{aligned} \omega_1 &= 2 \ (1) \\ \omega_2 &= 3 \ (1) \\ \omega_3 &= 5 \ (1) \end{aligned}\right\} \quad \begin{aligned} \omega_{123} &= 1 \\ T_{123} &= \frac{2\pi}{1} \\ T_{123} &= \pi \end{aligned}$$

GCD = 1

(b)
$$\left.\begin{aligned} \omega_1 &= 24 \\ \omega_2 &= 48 \\ \omega_3 &= 36 \end{aligned}\right\} \quad \begin{aligned} \omega_{123} &= 12 \\ T_{123} &= \frac{2\pi}{12} \\ &= \underline{\pi/6} \end{aligned}$$

40) For a periodic signal

$$v(t) = 30\sin 100t + 10\cos 300t + 6\sin\left(500t + \frac{\pi}{4}\right)$$

The fundamental frequency in rad/s is :

$$\left.\begin{array}{l} \omega_1 = 100 \\ \omega_2 = 300 \\ \omega_3 = 500 \end{array}\right\} \quad \begin{array}{l} GCD = 100 \\ \omega_{123} = 100 \text{ rad/s.} \\ T_{123} = \dfrac{2\pi}{100} = \dfrac{\pi}{50} \text{ sec} \end{array}$$

41) $x[n] = \sin\left(\frac{2\pi}{3} n\right)$ $\quad T = ?$

NOTE: All rules are valid also for Discrete Time

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi/3} = 3 \text{ sec. } (T \text{ is integer})$$

42) $x[n] = \sin\left(\frac{4}{3} n\right)$ $\quad$ check periodicity?

$$\omega = \frac{2\pi}{T} = \frac{4}{3} \Rightarrow T = \frac{6\pi}{4}(m) \quad T \neq \text{integer for any value of } m$$

Non_periodic signal

43) A discrete time signal (n is integer)

$x[n] = \sin(\pi^2 n)$ $\quad$ check periodicity.
If periodic; find T.

$$\omega = \frac{2\pi}{T} = \pi^2 \quad T = \frac{2\pi}{\pi^2} = \frac{2}{\pi}$$

NOTE : If the signal was a continious time signal we can say that the signal is periodic but in discrete time we will add a coefficient m.

$T = \frac{2}{\pi}m \longrightarrow$ The won't be integer for any integer value of m. So the signal is aperiodic.

24) A continious-time function $x(t)$ is periodic with period T. The function is sampled uniformly with a sampling period T_s. In which of the following cases, sampled signal periodic ?

a) $T = \sqrt{2}\, T_s$ b) $T = 1.2 T_s$

c) Always d) Never

$\frac{T}{T_s} \cdot m$ → There must be an integer m which will make the result

In b) $\frac{T}{T_s} = \underline{1{,}2m}$

Another solution. Let's our first signal is $\sin \frac{2\pi}{T_1} t$ when we sample this signal

$$x[n] = \sin\left(\frac{2\pi}{T_1} \underline{nT_s}\right) \qquad \omega = \frac{2\pi T_s}{T_1} = \frac{2\pi}{T}$$

$$T = \frac{T_1}{T_s} \rightarrow \text{Time period of sampled signal.}$$

$T = \frac{T_1}{T_s}(m)$ ⇒ The only value for coefficient m in the options is given in B. For 1,2 we have 10 which will make the result integer.

25) The fundamental period of

$x(t) = 2\cos 6\pi t + 4\cos 5\pi t$ with t expressed in seconds;

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{6\pi} = \frac{1}{3}$$

$$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{5\pi} = \frac{2}{5}$$

$$T_1/T_2 = \frac{1/3}{2/5} = \frac{1}{3} \cdot \frac{5}{2} = \frac{5}{6}$$

$$6T_1 = 5T_2$$

$$6 \cdot T_1 = 6 \cdot \frac{1}{3} = 2 \text{ seconds.}$$