COUNTING II

Murat Osmanoglu

Combinations II

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  4A
  4O
  4P

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4A	3A-1P
4O	3A-1O
4P	3O-1P
	3O-1A
	3P-1O
	3P-1A

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4A	3A-1P	2A-2P
4O	3A-1O	2A-2O
4P	3O-1P	2O-2P
	3O-1A	
	3P-1O	
	3P-1A	

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4A	3A-1P	2A-2P	2A-1P-1O
4O	3A-1O	2A-2O	2O-1A-1P
4P	3O-1P	2O-2P	2P-1A-1O
	3O-1A		
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	C		D		F
CCDDF	XX	|	XX	|	X

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CCDDF	XX	$\mid$	XX	$\mid$	X
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FFFFF					
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CCCCD	XXXX	I	X	I	

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$$\frac{7!}{2!.5!}$$

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$$\frac{7!}{2!.5!} = C(7,5)$$

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5 combinations of 6 objects : $\binom{6+5-1}{5} = \binom{10}{5}$

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x_1 $\quad$ x_2 $\quad$ x_3 $\quad$ x_4

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x_1		x_2		x_3		x_4
	|	x	|		|	xxxxxx

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	|	x	|		|	xxxxxx
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x_1		x_2		x_3		x_4
	|	x	|		|	xxxxxx
x	|	xx	|	x	|	xxx
xxxx	|		|	xxx	|	

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$x_1 \qquad x_2 \qquad x_3 \qquad x_4$

```
      |  x  |     | xxxxxx
  x   |  xx |  x  |  xxx
 xxxx |     | xxx |
```

7 combinations of 4 objects

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x_1	x_2	x_3	x_4
	x		xxxxxx
x	xx	x	xxx
xxxx		xxx	

7 combinations of 4 objects

$$\binom{7+4-1}{7} = \binom{10}{7}$$

Pigeonhole Principle

- Assume there are 6 pigeonholes but 7 pigeons, and the pigeons are placed to pigeonholes.

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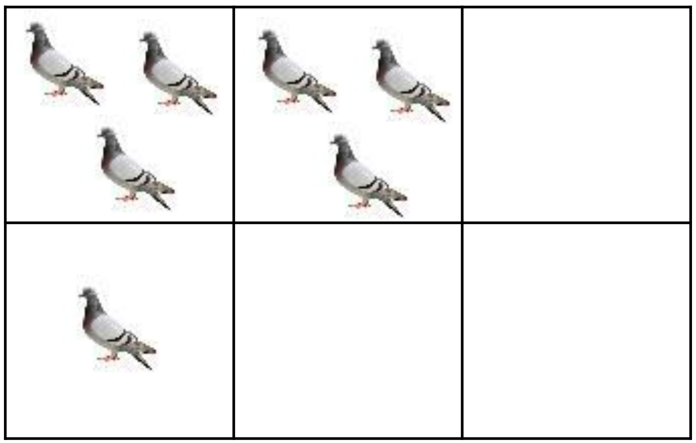

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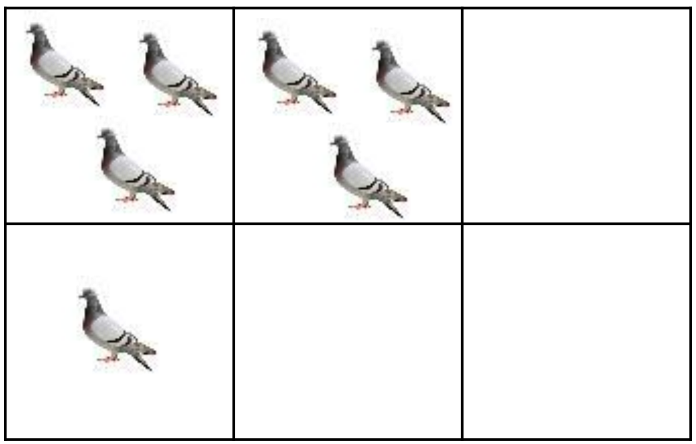

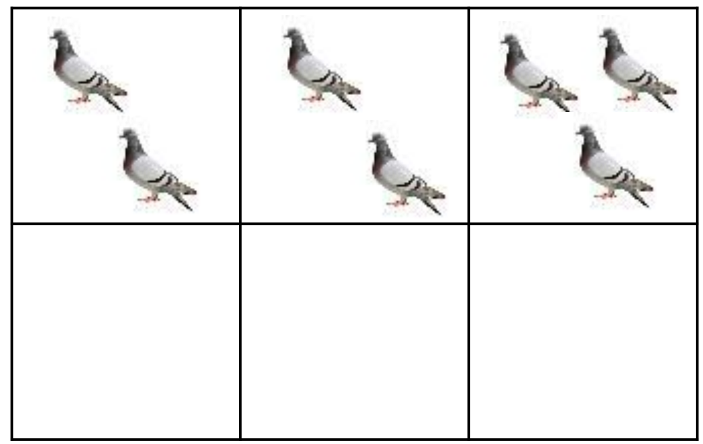

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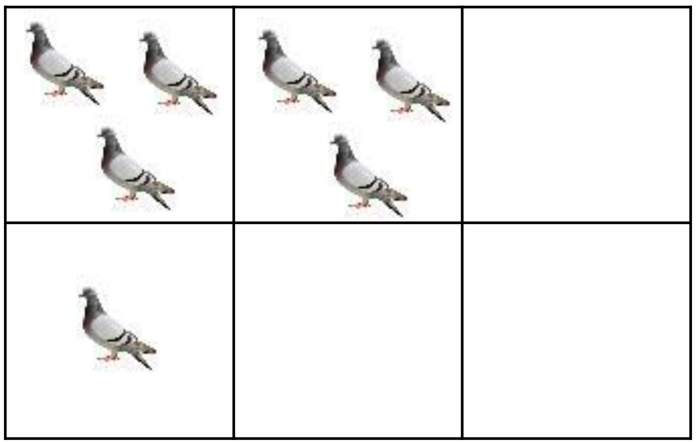

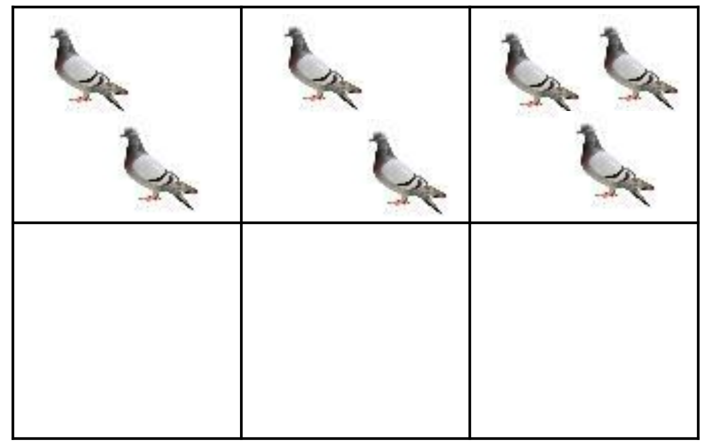

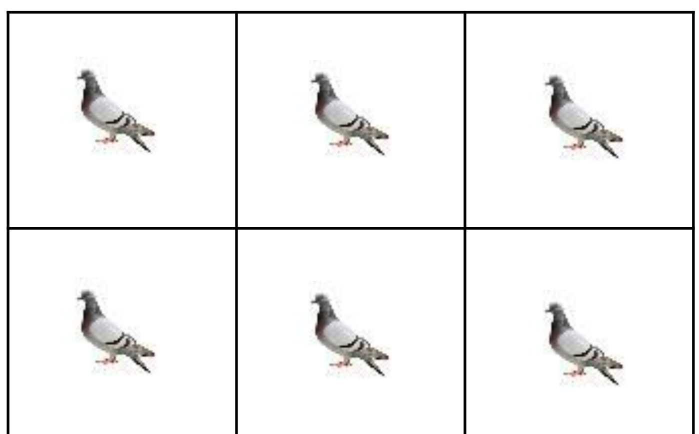

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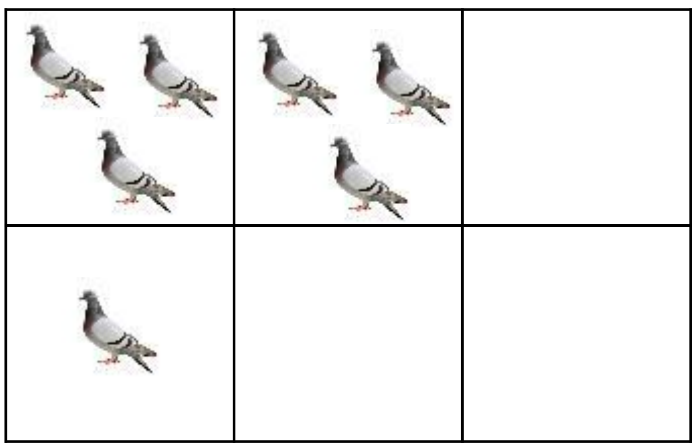

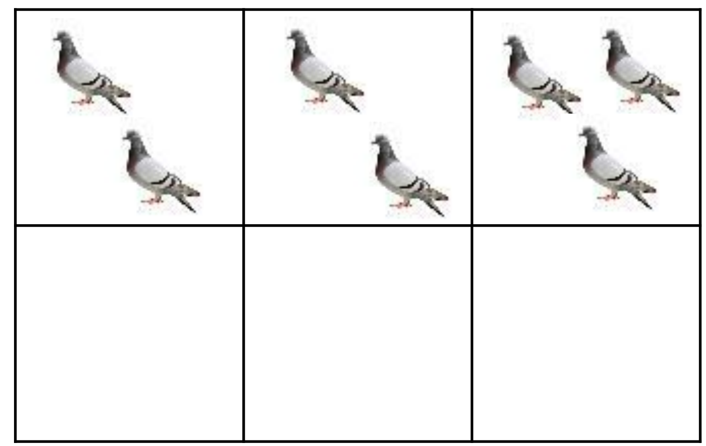

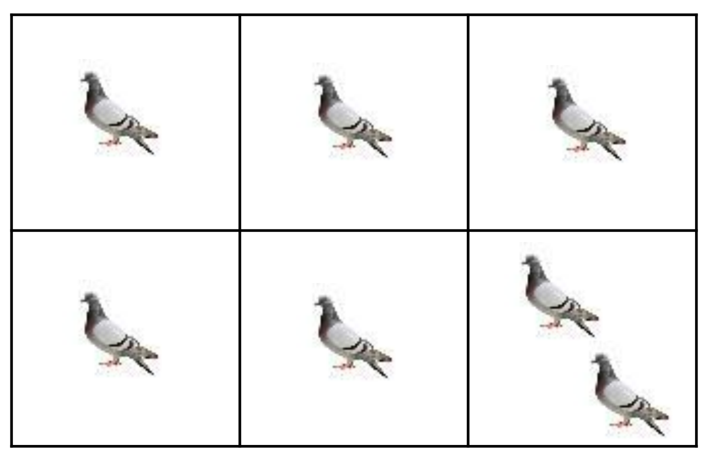

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  - $\lceil N/k \rceil = Q + 1$

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Consider a standard deck of 52 cards:
At least how many cards should be chosen to guarantee that at least 3 cards are chosen from the same suit ?

Pigeonhole Principle

- Among 50 people, there are at least $[50/12] = 5$ people born in the same month

- Assume there are 5 possible grades: A, B, C, D, E. If we want at least 6 students to get same grade on midterm, what should the minimum number of students be ?

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all diamonds + all spades + all hearts

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At least how many cards should be chosen to guarantee that at least 3 clubs are chosen ? (think about the worst case to guarantee that)

all diamonds + all spades + all hearts + 3 clubs = 42

Pigeonhole Principle

- Let (x_i, y_i) , $i = 1, 2, 3, 4, 5$ be a set of five distinct points with integer coordinates in the *xy* plane. Show that the midpoint of the line joining at least one pair of these points has integer coordinates.

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$$(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4), (x_5, y_5)$$

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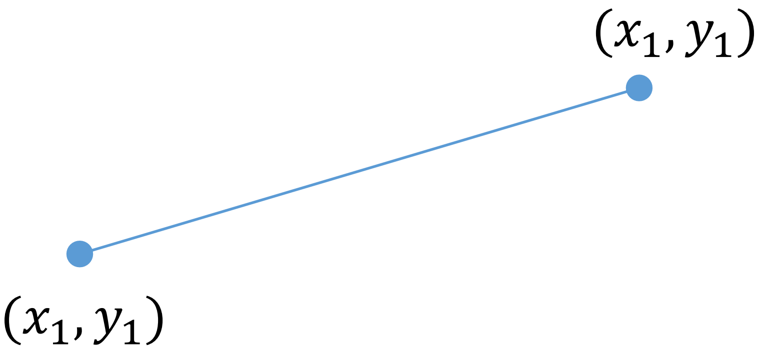

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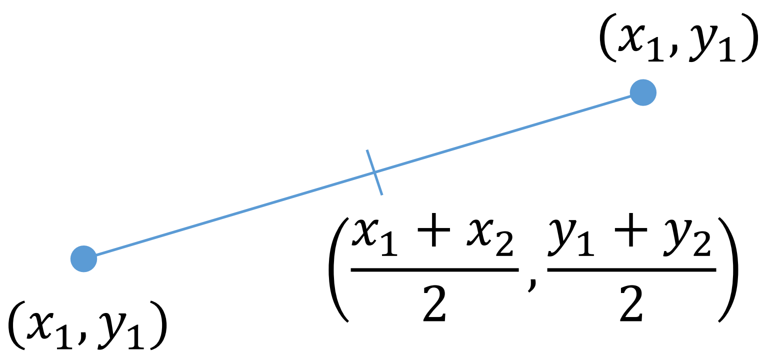

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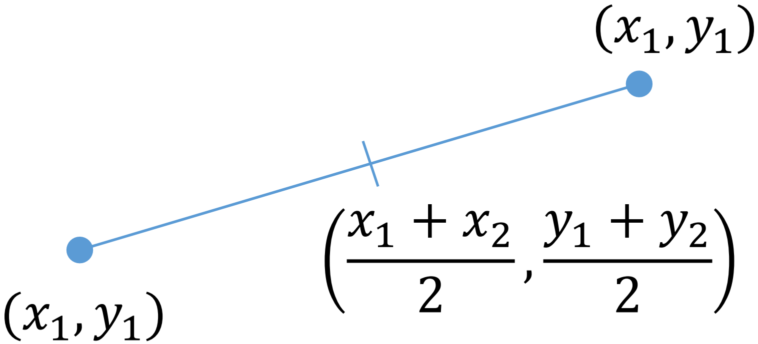

- In which condition, $\frac{x_1+x_2}{2} \epsilon Z$

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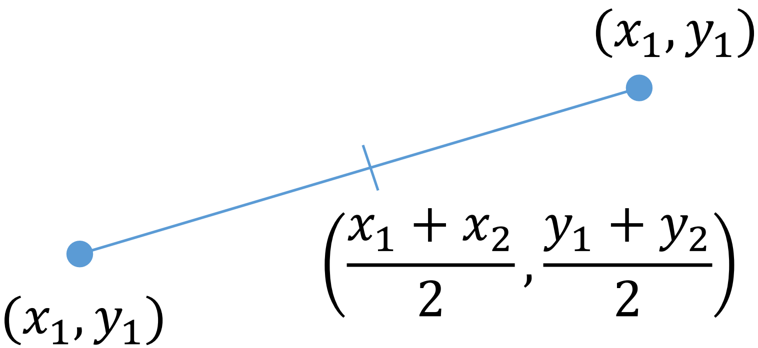

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- $\frac{E+E}{2}$ or $\frac{O+O}{2}$

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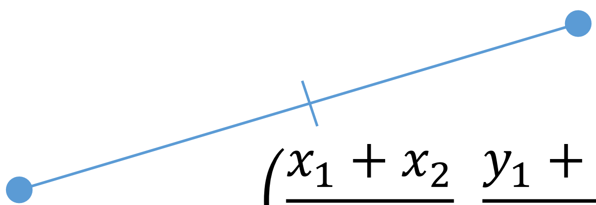

- In which condition, $\frac{x_1+x_2}{2} \in Z$
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- if there are (E, O) and (E, O), then $\left(\frac{\mathrm{E+E}}{2}, \frac{\mathrm{O+O}}{2}\right)$ will be integer

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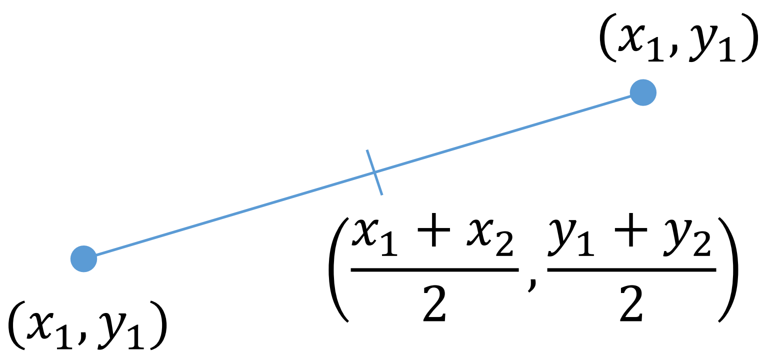

(E, O)	(O, E)	(E, E)	(O, O)

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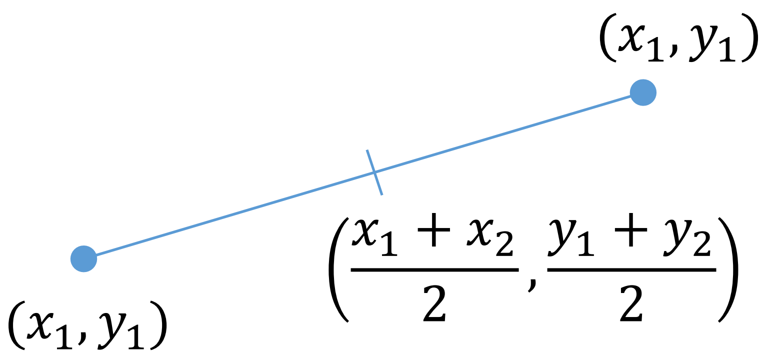

(E, O)	(O, E)	(E, E)	(O, O)

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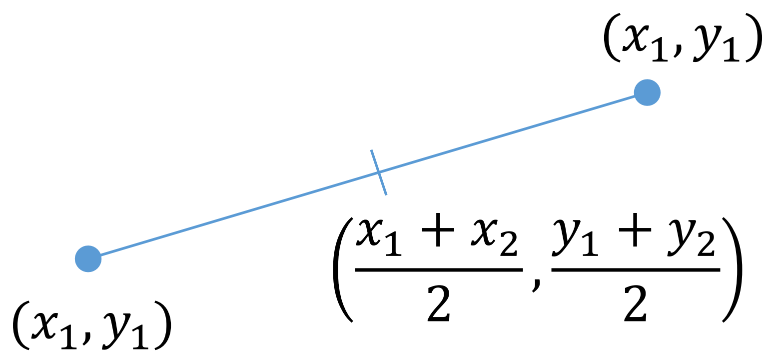

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(E, O)	(O, E)	(E, E)	(O, O)
(x_1, y_1)	(x_2, y_2)	(x_3, y_3)	(x_4, y_4)

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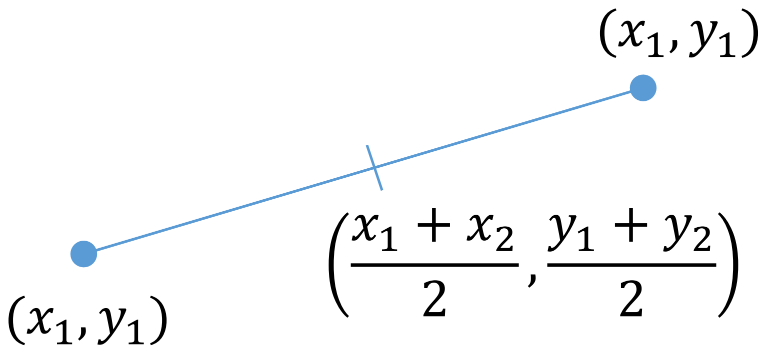

(E, O)	(O, E)	(E, E)	(O, O)
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Pigeonhole Principle

How many ordered pairs of integers (a, b), are needed to guarantee that there are two ordered pairs (a_1, b_1) and (a_2, b_2) such that $a_1 \ mod\ 5 = a_2 \ mod\ 5$ and $b_1 \ mod\ 5 = b_2 \ mod\ 5$?

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- $a \;\; mod \;\; 5 = R$ where R is remainder of the division $(a \,/\, 5)$

- How many remainders are there when an integer is divided by 5 ?

  0, 1, 2, 3, 4

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  5 x 5 = 25

Pigeonhole Principle

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  0, 1, 2, 3, 4

- How many possible pairs of remainders are there ? $(a \; mod \; 5, b \; mod \; 5)$

  5 x 5 = 25

Thus, there should be 26 pairs of remainders so that some two pairs (a_1, b_1) and (a_1, b_1) will have same pair of remainders,

$$a_1 \; mod \; 5 = a_2 \; mod \; 5 \text{ and } b_1 \; mod \; 5 = b_2 \; mod \; 5$$

Principles of Inclusion & Exclusion

Assume there 80 students enrolled to freshman engineering program this year. 30 of them also enrolled in Physics, 25 of them also enrolled in Introdution to Programming, and 10 of them also enrolled in both Physics and Intoduction to Programming courses. How many of them neither taking Physics nor taking Introdution to Programming ?

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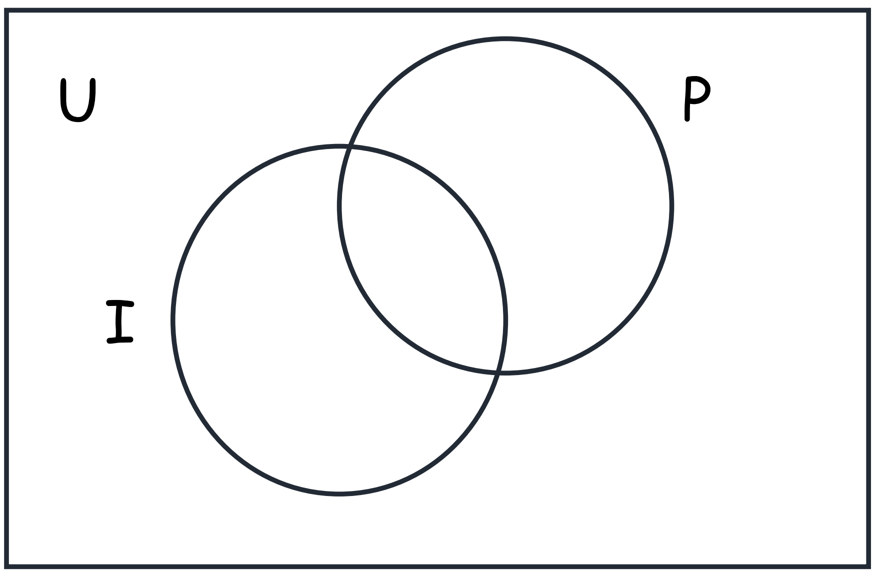

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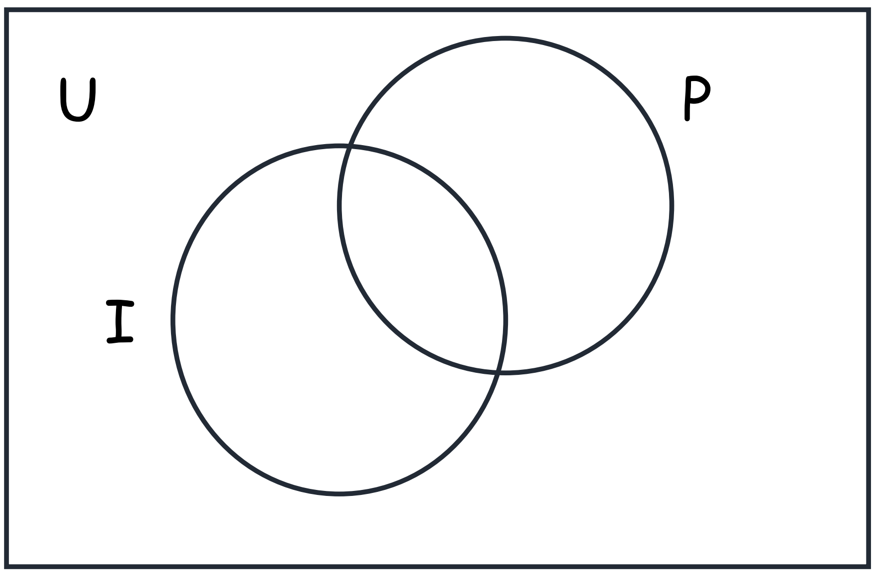

$|U| = 80$

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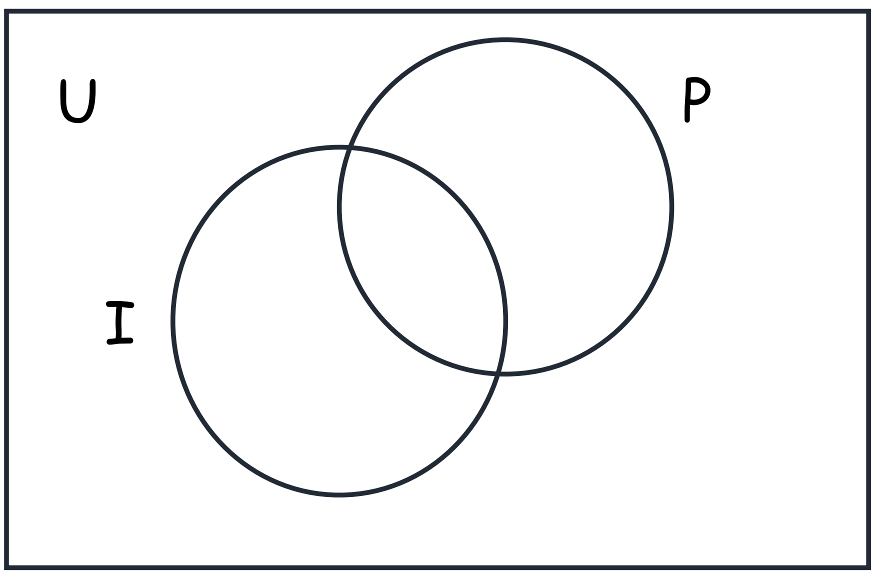

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$$|I \cup P| = |I| + |P| - |I \cap P|$$

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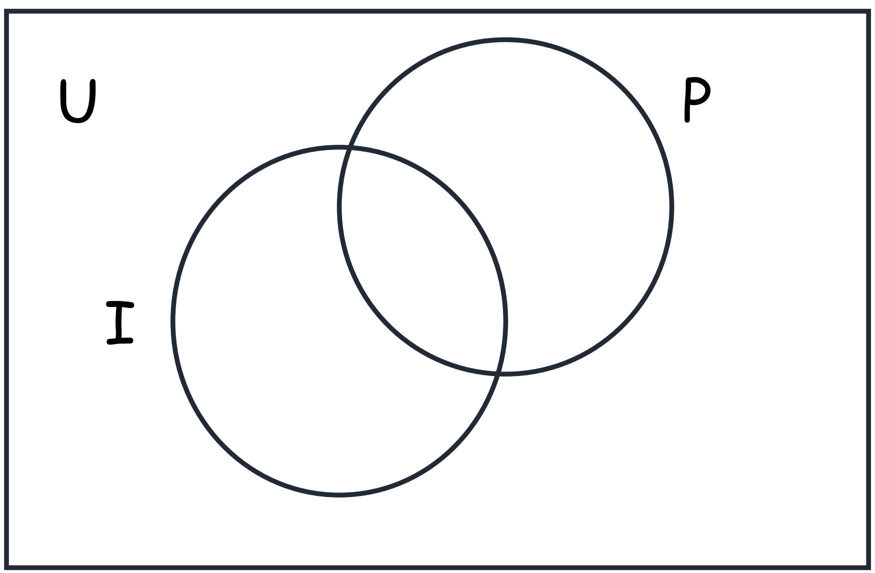

$|\mathrm{U}| = 80$

$|\mathrm{I} \cup \mathrm{P}| = |\mathrm{I}| + |\mathrm{P}| - |\mathrm{I} \cap \mathrm{P}|$

$|\mathrm{I}| = 25, |\mathrm{P}| = 30,$

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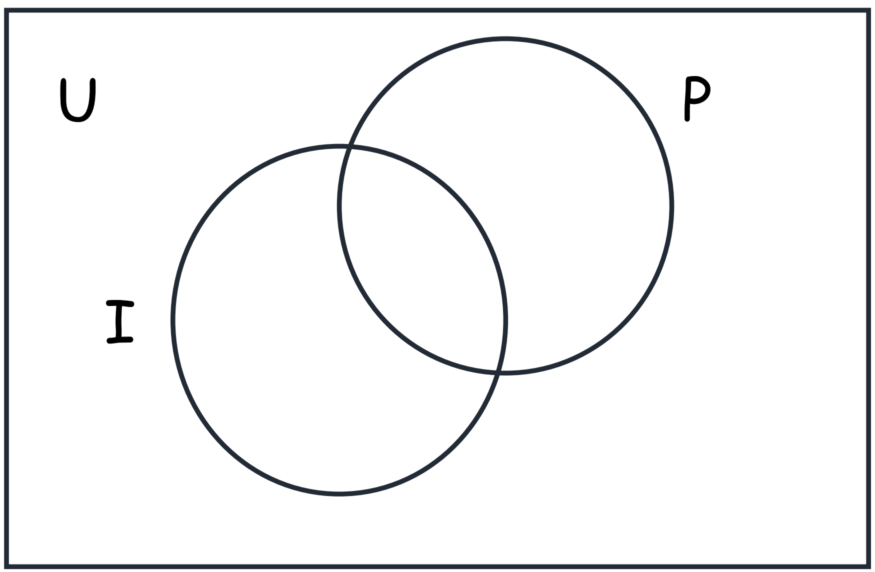

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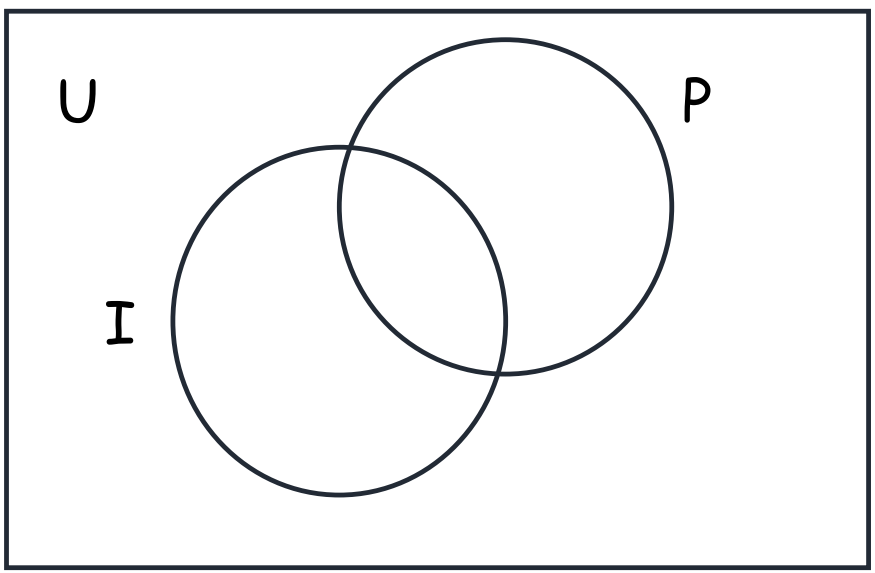

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$|\mathrm{I}| = 25,\ |\mathrm{P}| = 30,\ |\mathrm{I} \cap \mathrm{P}| = 10$

$|\overline{\mathrm{I}} \cap \overline{\mathrm{P}}| = ?$

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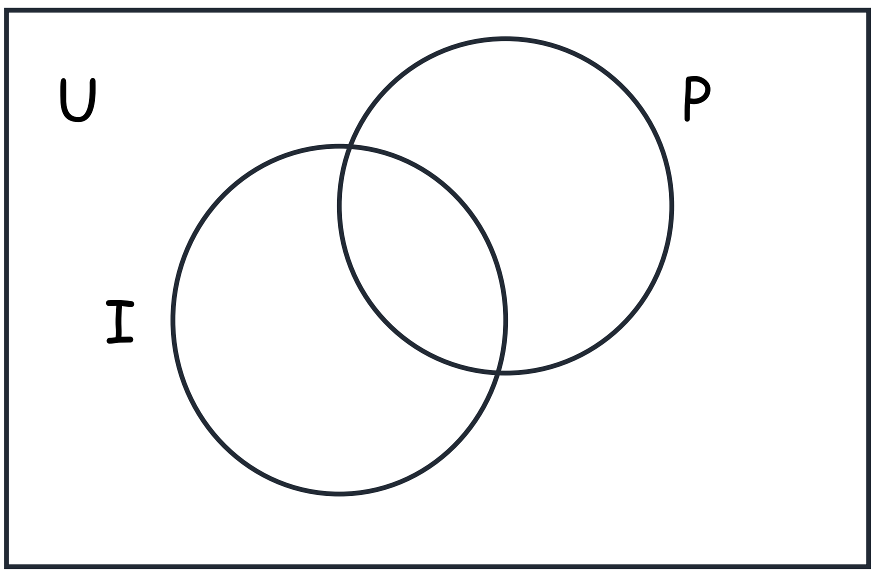

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$$|I| = 25,\ |P| = 30,\ |I \cap P| = 10$$

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$$|\bar{I} \cap \bar{P}| = |\overline{I \cup P}|$$

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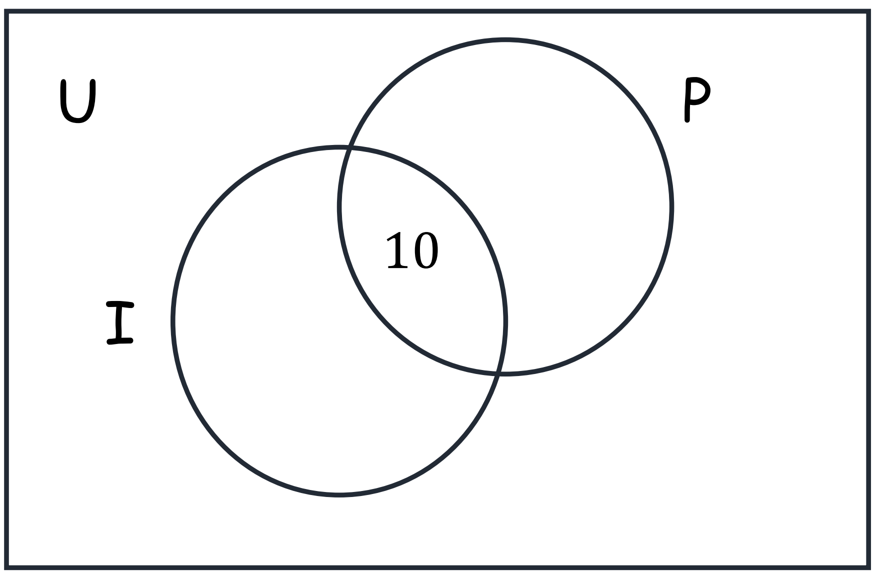

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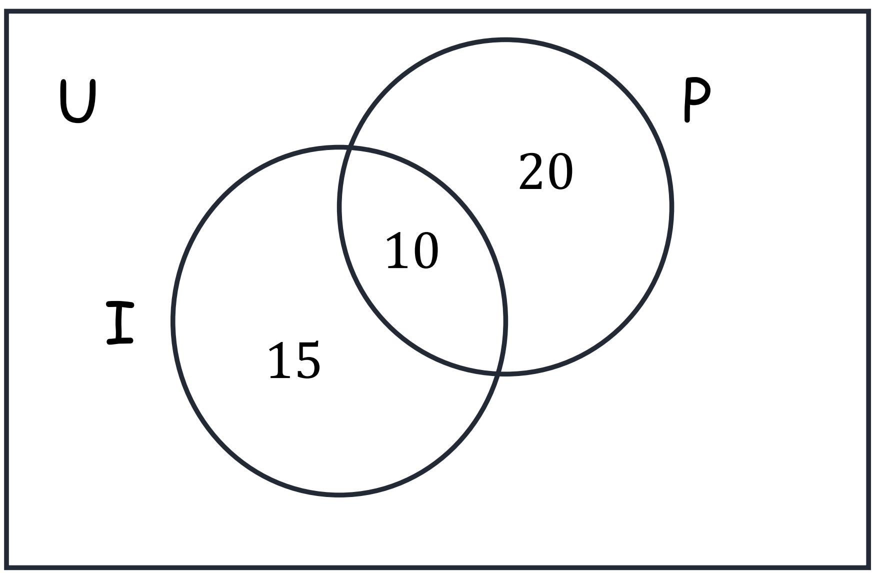

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$|I| = 25,\ |P| = 30,\ |I \cap P| = 10$

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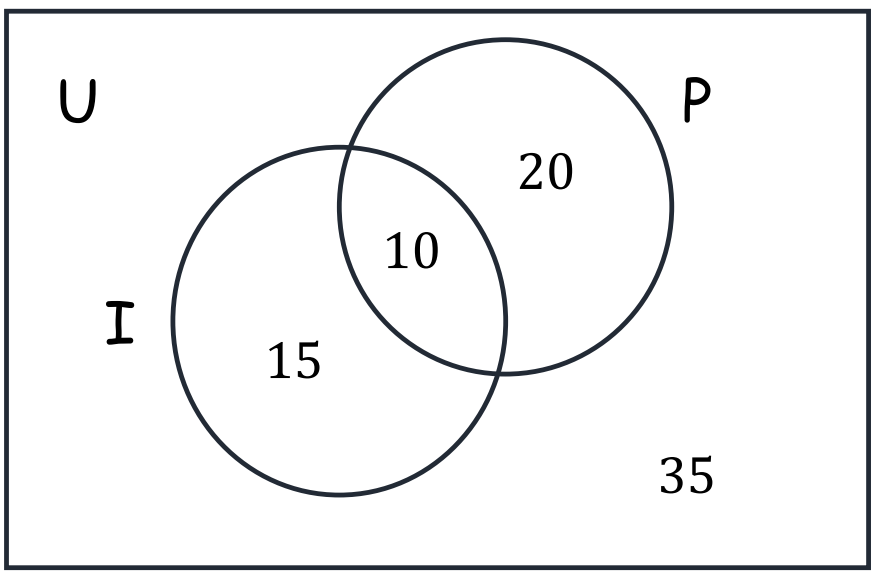

$|\mathrm{U}| = 80$

$|\mathrm{I} \cup \mathrm{P}| = |\mathrm{I}| + |\mathrm{P}| - |\mathrm{I} \cap \mathrm{P}|$

$|\mathrm{I}| = 25,\ |\mathrm{P}| = 30,\ |\mathrm{I} \cap \mathrm{P}| = 10$

$|\overline{\mathrm{I}} \cap \overline{\mathrm{P}}| = ?$

$|\overline{\mathrm{I}} \cap \overline{\mathrm{P}}| = |\overline{\mathrm{I} \cup \mathrm{P}}| = |\mathrm{U}| - |\mathrm{I} \cup \mathrm{P}|$

Principles of Inclusion & Exclusion

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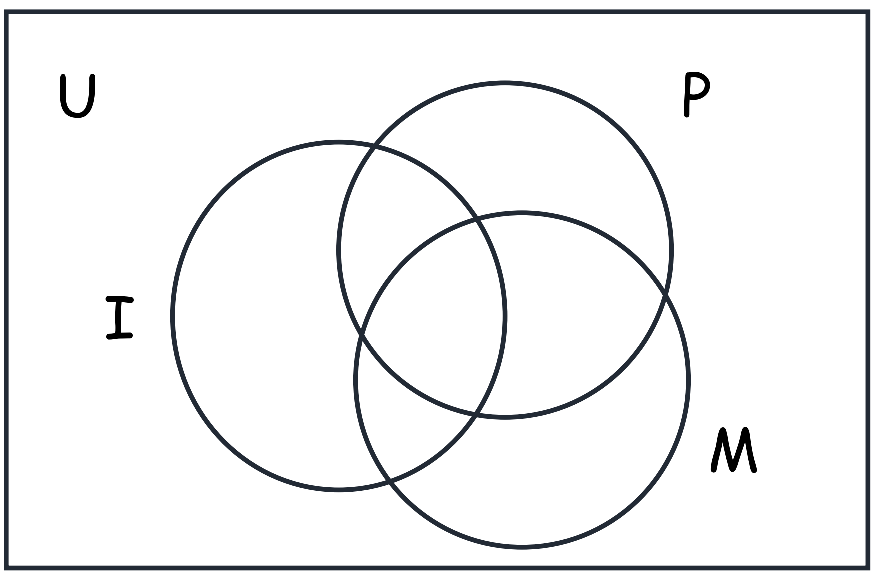
20 of them taking Math, 5 of them taking both Math and Intro, 15 of them taking Math and Physics, 3 of them taking all. What is the number of students not taking any of them ?



$$|\mathrm{U}| = 80$$

$$|\mathrm{I} \cup \mathrm{P} \cup M| = |\mathrm{I}| + |\mathrm{P}| + |M| - |\mathrm{I} \cap \mathrm{P}|$$
$$- |\mathrm{I} \cap M| - |\mathrm{M} \cap \mathrm{P}|$$
$$+ |\mathrm{I} \cap \mathrm{P} \cap M|$$

$$|\mathrm{I}| = 25,\ |\mathrm{P}| = 30,\ |\mathrm{I} \cap \mathrm{P}| = 10$$

Principles of Inclusion & Exclusion

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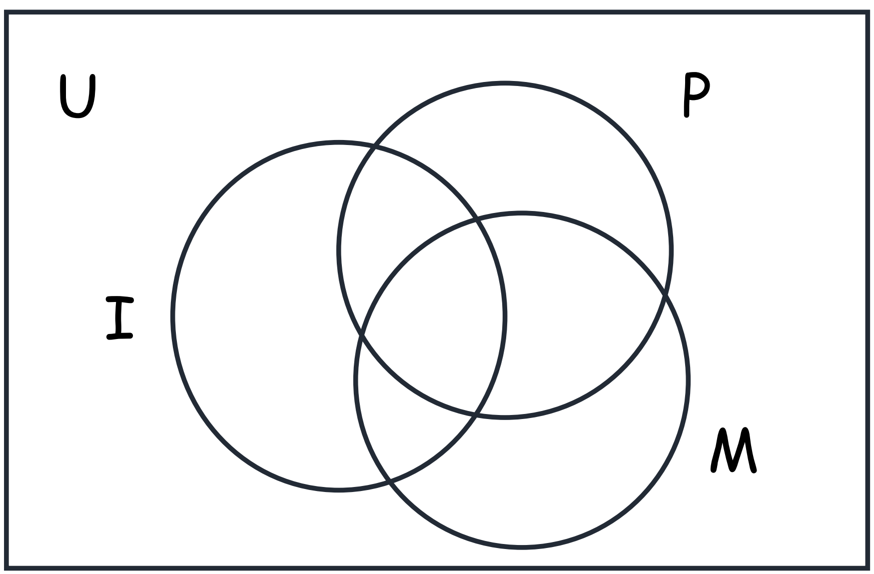

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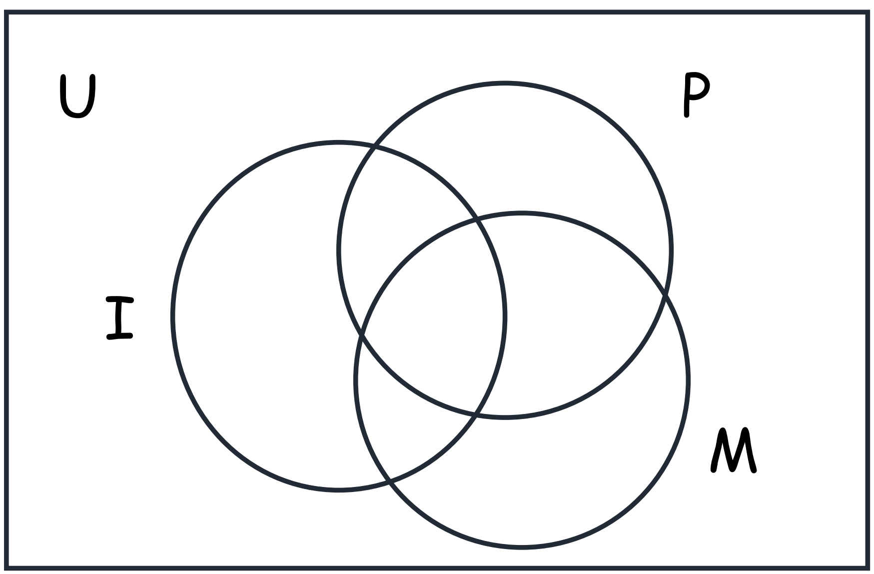

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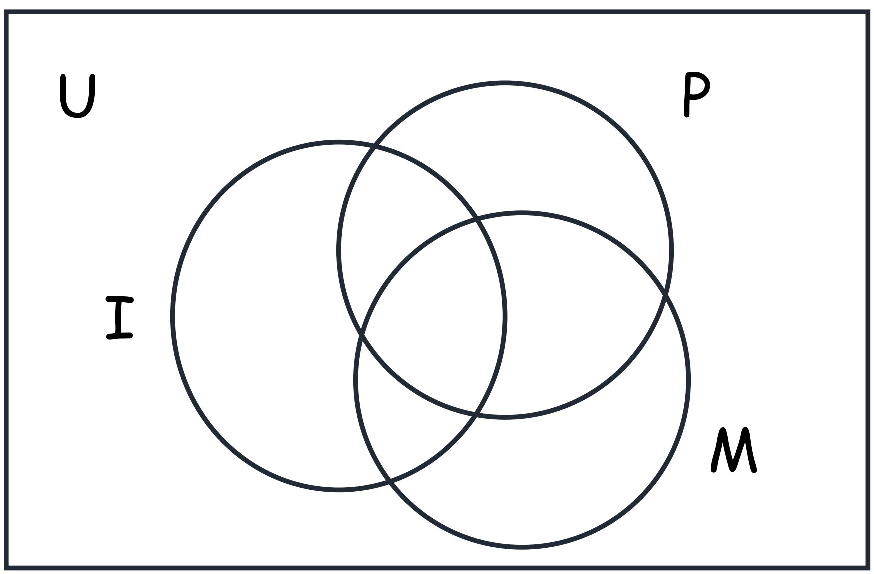

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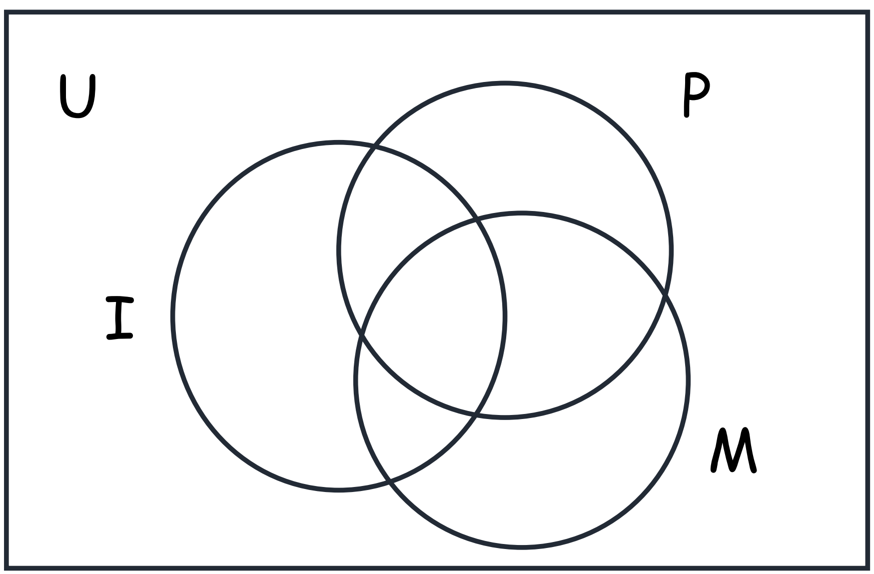

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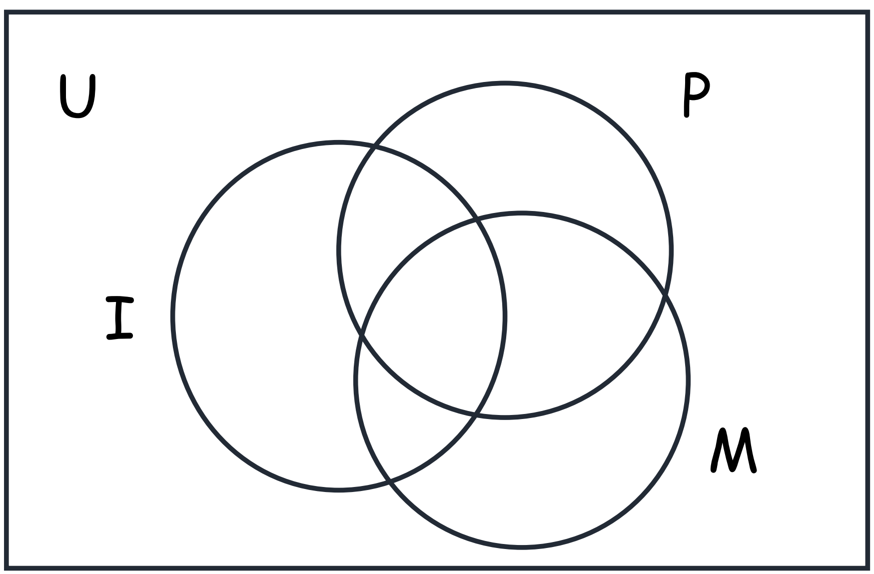

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$$|\mathrm{I} \cup \mathrm{P} \cup M| = 30 + 25 + 20 - 10$$
$$-5 - 15 + 3$$

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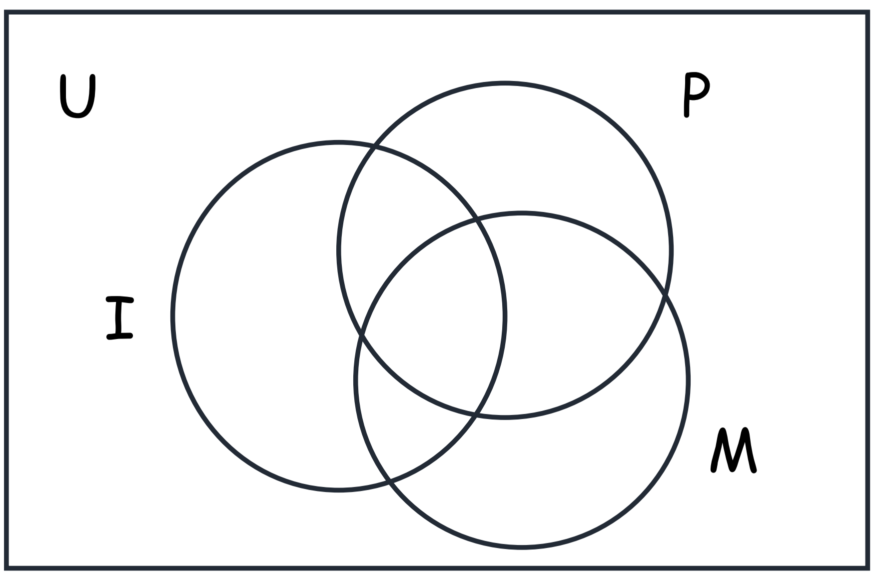

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$$|\mathrm{I} \cup \mathrm{P} \cup M| = 30 + 25 + 20 - 10 \\ -5 - 15 + 3 = 48$$

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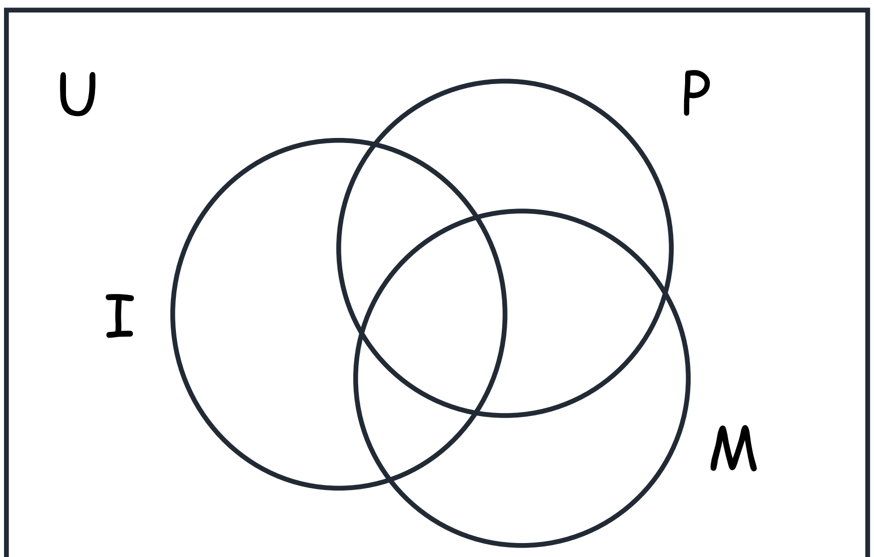

$$|\mathrm{U}| = 80$$

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$$|\mathrm{I} \cap \mathrm{P} \cap M| = 3$$

$$|\mathrm{I} \cup \mathrm{P} \cup M| = 30 + 25 + 20 - 10 - 5 - 15 + 3 = 48$$

$$|\overline{\mathrm{I} \cup \mathrm{P} \cup M}| = 80 - 48 = 32$$

Principles of Inclusion & Exclusion

- Find the number of positive integers sitrictly less than 101 that is not divisible by 2, 3, and 5 ?

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$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

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$$A \cap B = \{x \in Z |\ 1 \leq x \leq 100 \text{ and x is divisble by } 6\}$$

Principles of Inclusion & Exclusion

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$$|\bar{A} \cap \bar{B} \cap \bar{C}| = ?$$

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$$A \cap B = \{x \in Z |\ 1 \leq x \leq 100 \text{ and x is divisble by } 6\}$$
$$A \cap C = \{x \in Z |\ 1 \leq x \leq 100 \text{ and x is divisble by } 10\}$$

Principles of Inclusion & Exclusion

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$$C = \{x \in Z \mid 1 \leq x \leq 100 \text{ and x is divisble by } 5\}$$

$$|\bar{A} \cap \bar{B} \cap \bar{C}| = ?$$

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$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$A \cap B = \{x \in Z \mid 1 \leq x \leq 100 \text{ and x is divisble by } 6\}$$
$$A \cap C = \{x \in Z \mid 1 \leq x \leq 100 \text{ and x is divisble by } 10\}$$
$$B \cap C = \{x \in Z \mid 1 \leq x \leq 100 \text{ and x is divisble by } 15\}$$

Principles of Inclusion & Exclusion

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$$C = \{x \in Z |\ 1 \le x \le 100 \text{ and x is divisble by } 5\}$$

$$|\bar{A} \cap \bar{B} \cap \bar{C}| = ?$$

$$|\bar{A} \cap \bar{B} \cap \bar{C}| = |\overline{A \cup B \cup C}| = |U| - |A \cup B \cup C| = ?$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$A \cap B = \{x \in Z |\ 1 \le x \le 100 \text{ and x is divisble by } 6\}$$
$$A \cap C = \{x \in Z |\ 1 \le x \le 100 \text{ and x is divisble by } 10\}$$
$$B \cap C = \{x \in Z |\ 1 \le x \le 100 \text{ and x is divisble by } 15\}$$
$$A \cap B \cap C = \{x \in Z |\ 1 \le x \le 100 \text{ and x is divisble by } 30\}$$

Principles of Inclusion & Exclusion

- Find the number of positive integers sitrictly less than 101 that is not divisible by 2, 3, and 5 ?

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$$C = \{x \in Z|\ 1 \le x \le 100 \text{ and x is divisble by } 5\}$$

$$|\bar{A} \cap \bar{B} \cap \bar{C}| = ?$$

$$|\bar{A} \cap \bar{B} \cap \bar{C}| = |\overline{A \cup B \cup C}| = |U| - |A \cup B \cup C| = ?$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

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$$B \cap C = \{x \in Z|\ 1 \le x \le 100 \text{ and x is divisble by } 15\}$$
$$A \cap B \cap C = \{x \in Z|\ 1 \le x \le 100 \text{ and x is divisble by } 30\}$$

$$|A \cup B \cup C| = 50 + 33 + 20 - 16 - 10 - 6 + 3 = 74$$

Principles of Inclusion & Exclusion

- Find the number of positive integers sitrictly less than 101 that is not divisible by 2, 3, and 5 ?

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$$|\bar{A} \cap \bar{B} \cap \bar{C}| = ?$$

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$$B \cap C = \{x \in Z|\ 1 \le x \le 100 \text{ and x is divisble by } 15\}$$
$$A \cap B \cap C = \{x \in Z|\ 1 \le x \le 100 \text{ and x is divisble by } 30\}$$

$$|A \cup B \cup C| = 50 + 33 + 20 - 16 - 10 - 6 + 3 = 74$$
$$|\bar{A} \cap \bar{B} \cap \bar{C}| = 100 - 74 = 26$$

Principles of Inclusion & Exclusion

- In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns 'car', 'dog', 'pun', and 'byte' occurs ?

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Let's define a set A, that contains all permutations of 26 letters in which the pattern car occurs.

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In similar way, B for dog, C for pun, and D for byte

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$|A| = 24!,$

Principles of Inclusion & Exclusion

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$|\mathrm{A}| = 24!, \ |\mathrm{B}| = 24!, \ |\mathrm{C}| = 24!, \ |\mathrm{D}| = 23!$

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In similar way, B for dog, C for pun, and D for byte

$|\mathrm{A}| = 24!,\ |\mathrm{B}| = 24!,\ |\mathrm{C}| = 24!,\ |\mathrm{D}| = 23!$
$|\mathrm{A} \cap \mathrm{B}| = 22!, |\mathrm{A} \cap C| = 22!, |\mathrm{A} \cap D| = 21!,$
$|\mathrm{B} \cap C| = 22!, |\mathrm{B} \cap D| = 21!, |\mathrm{C} \cap D| = 21!$

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Principles of Inclusion & Exclusion

- In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns 'car', 'dog', 'pun', and 'byte' occurs ?

. . . efcardxyz . . . . . . efcarxdogbus . . .

Let's define a set A, that contains all permutations of 26 letters in which the pattern car occurs.
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$$\begin{aligned}|A \cup B \cup C \cup D| = {} & |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| \\ & -|B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| \\ & +|A \cap C \cap D| + |B \cap C \cap D| - |A \cap B \cap C \cap D|\end{aligned}$$

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$$|U| - |\mathrm{A} \cup \mathrm{B} \cup C \cup D| = 26! - K$$

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- Six married couples are seating around a round table. In how many arrangements does no wife sit next to her husband ?

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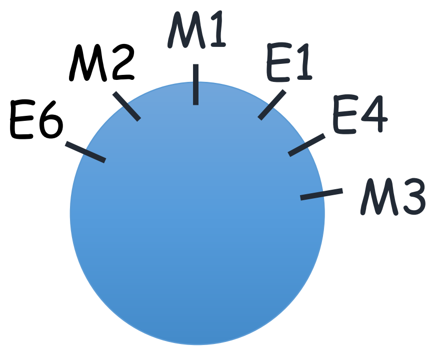

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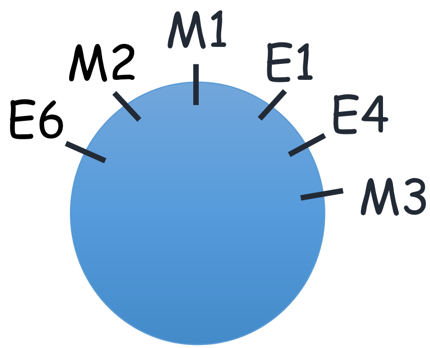

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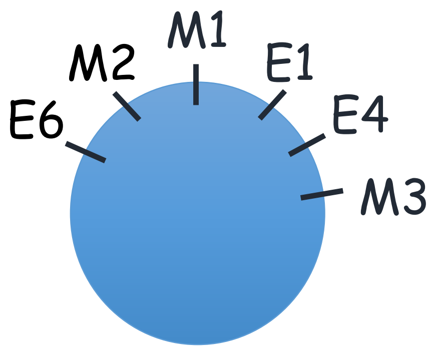

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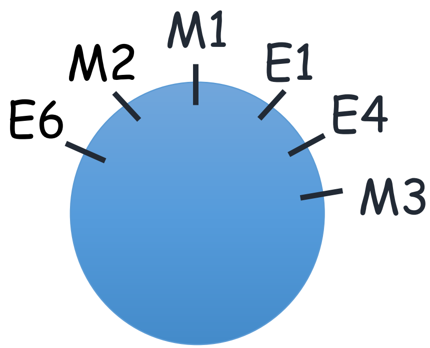

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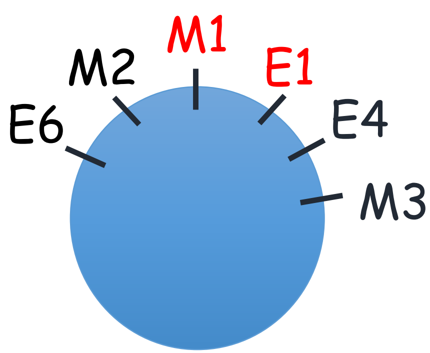

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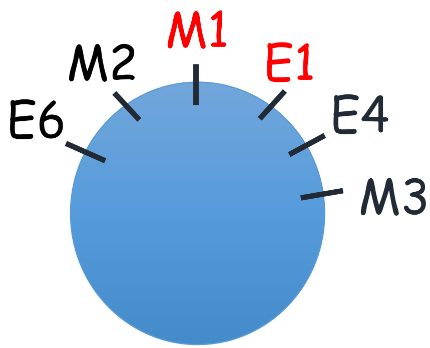

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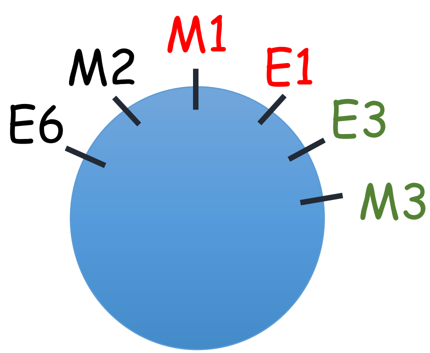

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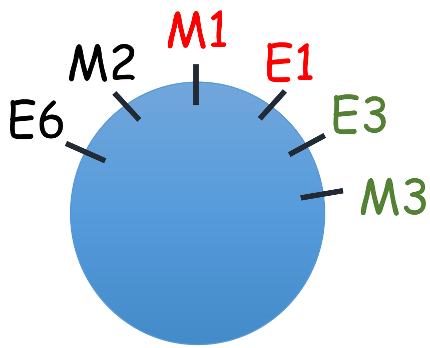

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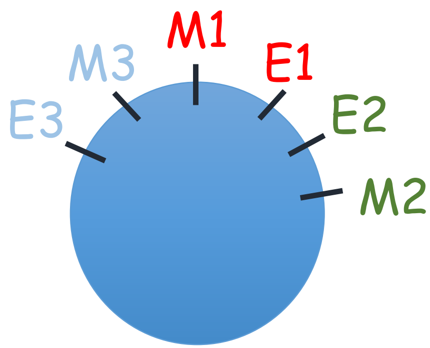

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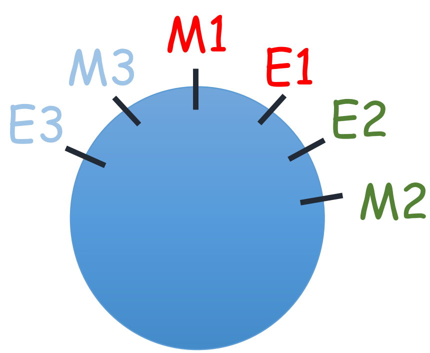

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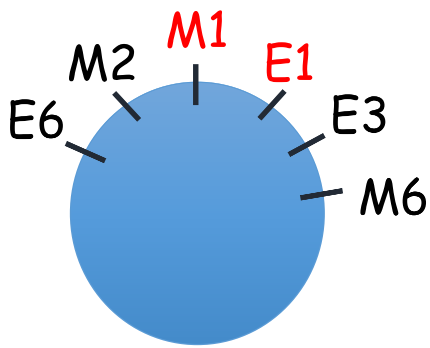

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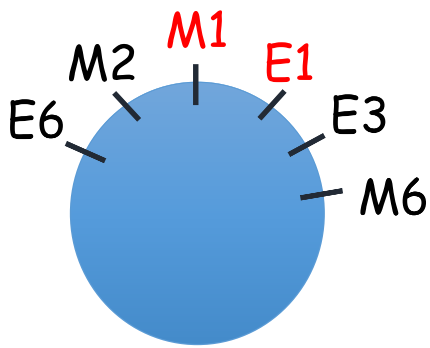

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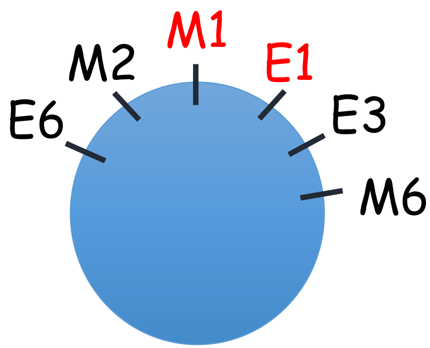

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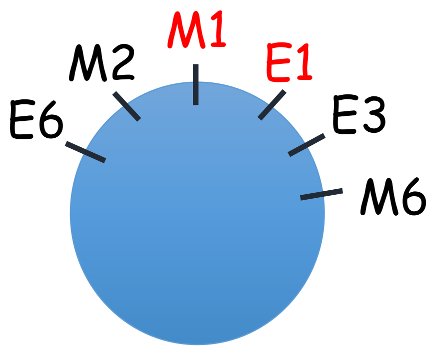

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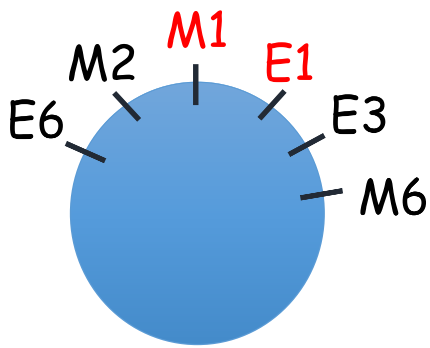

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Principles of Inclusion & Exclusion

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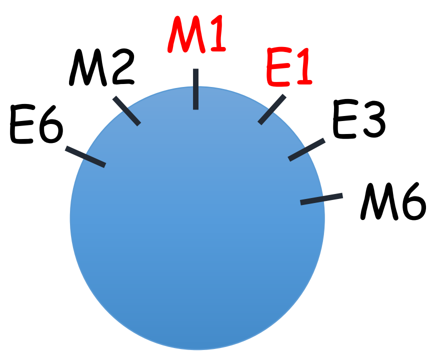

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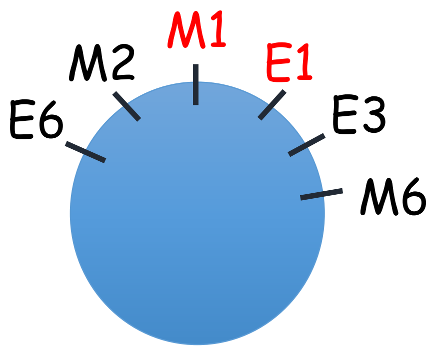

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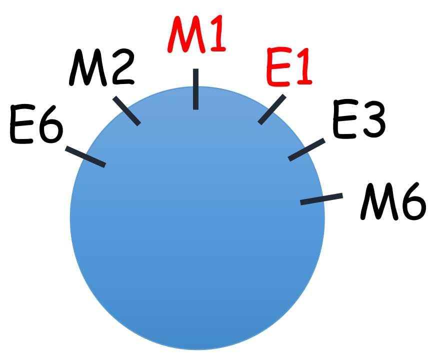

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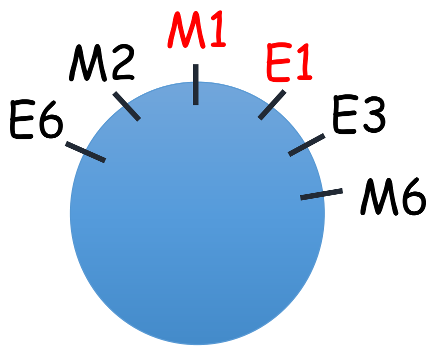

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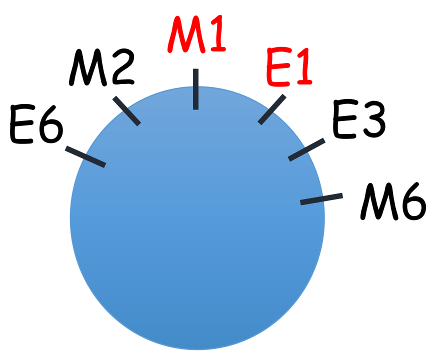

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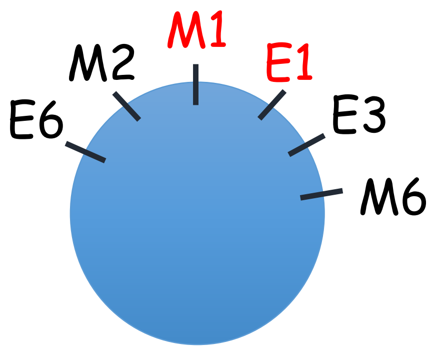

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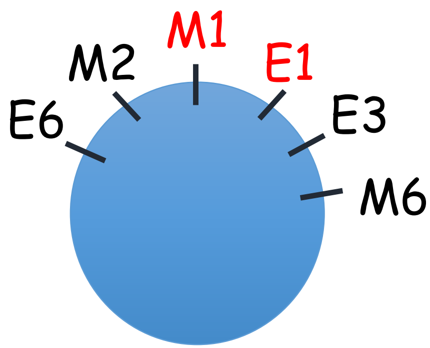

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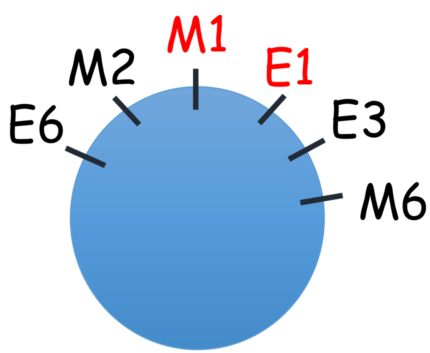

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$\mathrm{S_i} \cap \mathrm{S_j} = \left\{(x_1, x_2, x_3, x_4) \middle| x_1 + x_2 + x_3 + x_4 = 18,\ x_i, x_j > 7\right\}$

Principles of Inclusion & Exclusion

- $x_1 + x_2 + x_3 + x_4 = 18$ where $x_i \leq 7$ for $1 \leq i \leq 4$. How many different non-negative integer solution sets are there ?

  (5, 3, 4, 6), (7, 6, 5, 0), (7, 7, 3, 1), . . .

$\mathrm{S_i} = \{(x_1, x_2, x_3, x_4) | x_1 + x_2 + x_3 + x_4 = 18,\ x_i > 7\}$ $\binom{13}{10}$

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$|\mathrm{S_i} \cap \mathrm{S_j} \cap \mathrm{S_k}| = 0$

Principles of Inclusion & Exclusion

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$|\mathrm{S_i} \cap \mathrm{S_j} \cap \mathrm{S_k}| = 0$ and $|\mathrm{S_1} \cap \mathrm{S_2} \cap \mathrm{S_3} \cap \mathrm{S_4}| = 0$

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$\left|\mathrm{S_i} \cap \mathrm{S_j} \cap \mathrm{S_k}\right| = 0$ and $|\mathrm{S_1} \cap \mathrm{S_2} \cap \mathrm{S_3} \cap \mathrm{S_4}| = 0$

$|\mathrm{S_1} \cup \mathrm{S_2} \cup \mathrm{S_3} \cup \mathrm{S_4}| = 4\binom{13}{10} - \binom{4}{2}\binom{5}{2}$ and $|U| = \binom{21}{18}$

Principles of Inclusion & Exclusion

- $x_1 + x_2 + x_3 + x_4 = 18$ where $x_i \leq 7$ for $1 \leq i \leq 4$. How many different non-negative integer solution sets are there ?

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- solve the equation $x_1 + x_2 + x_3 + x_4 = 2$
- then add 8 to x_i and x_j in the solution to find the elements of the set $S_i \cap S_j$

$S_i \cap S_j \cap S_k = \{(x_1, x_2, x_3, x_4) | x_1 + x_2 + x_3 + x_4 = 18,\ x_i, x_j, x_k > 7\}$
$|S_i \cap S_j \cap S_k| = 0$ and $|S_1 \cap S_2 \cap S_3 \cap S_4| = 0$

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$$|U| - |S_1 \cup S_2 \cup S_3 \cup S_4| = \binom{21}{18} - 44$$