RANDOM VARIABLES: DISCRETE / CONTINUOUS

A random variable associates a real number with each outcome in the sample space. We can go to Mathematical World from the real World by the random variables. Now; we can study easier with the $\mathbb{R}$ numbers in the mathematical World. Random variables are denoted by the upper case letters X, Y, Z. Random Variables values are denoted by the lower case letters x, y, z . X random variable is shown as;

$$X \ : \ \Omega \ \rightarrow \ \mathbb{R}$$

$$w \ \rightarrow X(w)$$

where:

D_X: The set of X values. There are two types of random variables: Discrete and Continuous Random Variables.

Definitions:

Discrete Random Variable: If D_X is **finite** or **countable** (infinite countable), X is called discrete.

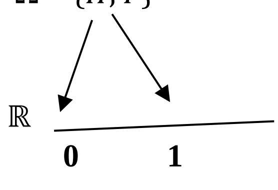
Continuous Random Variable: If D_X consists of an **interval** or **intervals** , X is called continuous.

Examples:

1-The experiment : Flip a coin

X: Number of tail.

$\Omega = \{H, T\}$



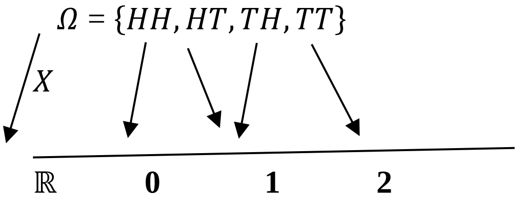
2-The experiment : Flip a coin two times. **GEI**

X: Number of tails.

Sample space :

$\Omega = \{TT, TH, HT, HH\}$

$n(\Omega) = 4$ (Number of elements)



$D_X = \{0,1,2\}$ Örnek olarak olasılıklar.

$P(X > 2) = 0$ $P(X \geq 1) = P(X = 1) + P(X = 2) = P(\{TY, YT\}) + P(\{YY\}) = 3/4$

3- Student:

The experiment : Flip a coin three times. Show the random variable, similar to the examples. Define some probabilities and calculate them.

4 Student: (Homework)

The experiment : Flip two coins at the same time. Show the random variable, similar to the examples. Define some probabilities and calculate them.

Now, we study with the discrete random variable firstly. **GEI

Definition:

Probability function:

When X is a **discrete random variable;** the function

$f_X(x) = P(X = x), x \epsilon D_X$

is called **probability function (pf) of** X random variable.

Example 1.

$\boldsymbol{D_x = \{0, 1\}} \rightarrow \boldsymbol{D_x}$**countable infinite** $\boldsymbol{X}$ **discrete.**

$f_X(x) = P(X = x)$

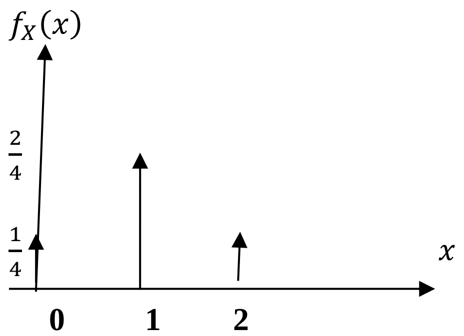
$X = x$	0	1
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{2}$

Example 2.

$\boldsymbol{D_x = \{0, 1, 2\}}$

$f_X(x) = P(X = x)$

$X = x$	0	1	2
$P(X = x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$



Example: **GEI**

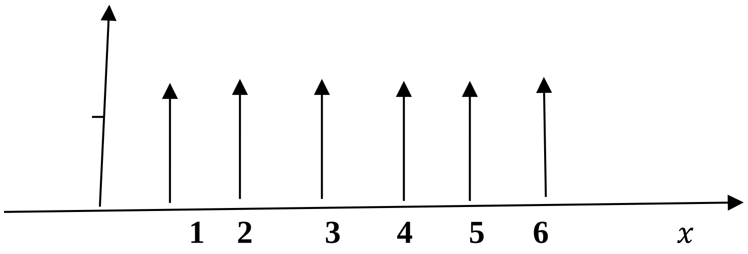
The experiment : A dice is tossed

X : the number of the surface points

$D_x = \{1,2,3,4,5,6\}$

$f(x) = P(X = x) = \frac{1}{6}$ $\quad x \in \boldsymbol{D_x} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}, \mathbf{6}\} \rightarrow \boldsymbol{D_x}$**sayılabilir sonlu,** $\boldsymbol{X}$ **kesikli**

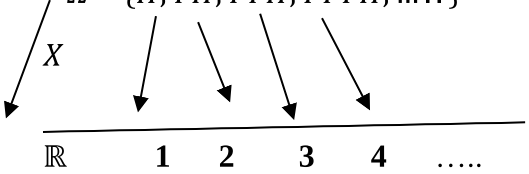
the graphic of function



Example: Flip a coin until the first head.

X : the number of flips

$\Omega = \{H, TH, TTH, TTTH, \ldots\}$



$\boldsymbol{D_x} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots\} \rightarrow \boldsymbol{D_x}$ **countable infinite,** $\boldsymbol{X}$ **discrete**

Properties of the probability function (pf): X discrete random variable and $f(x)$ is its probability function.

1. $f(x) > 0, \; x \epsilon D_x$

2. $\sum_{x \epsilon D_x} f(x) = 1$

Examples: **GEI**

1. The probability function of X random variable is given as;

$$f(x) = cx^2 \quad D_X = \{-2, -1, 1, 2\}$$

a. c =?
b. Obtain the probability function, table, graphic.
c. Calculate the probabilities.

$P(X > 2)$ =? $P(X \geq 1)$ =? $P(0 < X \leq 2)$ =?

Solution:

a $.\sum_{-2}^{2} cx^2 = 1 \rightarrow c = \frac{1}{10}$

b. The Probability function.

$$f(x) = \begin{cases} \dfrac{4}{10}, & x = -2 \\ \dfrac{1}{10}, & x = -1 \\ \dfrac{1}{10}, & x = 1 \\ \dfrac{4}{10}, & x = 2 \end{cases}$$

$X = x$	-2	-1	1	2
$P(X = x)$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{4}{10}$

c. $P(X > 2) = 0$, $P(X \geq 1) = P(X = 1) + P(X = 2) = f(1) + f(2) = \frac{5}{10}$

Student: **GEI**

2. The probability function of X random variable is given as;

$$f(x) = c \qquad D_X = \{-1,0,1,2\}$$

d. $c =?$
e. Obtain the probability function, table, graphic.
f. Calculate the probabilities.

$$P(X > 2) =? \quad P(X \geq 1) =? \quad P(0 < X \leq 2) =?$$

*When X is **continuous** random variable (D_X is uncountable) **GEI**

$$P(a < X < b) = P(a \leq X \leq b) = \int_a^b f(x)dx$$

where;

$f(x)$ is called **probability density function (pdf)** of X random variable.

Properties of the probability density function (pdf): X continuous random variable and $f(x)$ is its probability density function.

1. $f(x) \geq 0$

2. $\int_{-\infty}^{\infty} f(x)dx = 1$

Examples:

X is continuous random variable, its pdf is given as;

1. $f(x) = \begin{cases} cx^2, \dots 0 < \mathrm{x} < 1 \\ 0, \dots other\ wises \end{cases}$

a. $c = ?$
b. $P(0 < X \leq 0.5) = ?$
c. $P(X \geq 1) = ?$
d. $P(X \leq 1) = ?$

Solution:

a. $\int_{-\infty}^{\infty} f(x)dx = 1 \rightarrow \int_{-\infty}^{0} f(x)dx + \int_0^1 f(x)dx + \int_1^{\infty} f(x)dx = 1$

$\int_{-\infty}^{0} 0dx + \int_0^1 cx^2dx + \int_1^{\infty} 0dx = 1 \rightarrow 0 + \int_0^1 cx^2dx + 0 = 1 \rightarrow c = 3$

b. $P(0 < X \leq 0.5) = \int_0^{0.5} 3x^2dx = 1/8$

c. $P(X \geq 1) = \int_1^{\infty} f(x)dx = \int_1^{\infty} 0\, dx = 0$ **GEI**

d. $P(X \leq 1) = \int_0^1 f(x)dx = 1$

2.Student:

X is continuous random variable, its pdf is given as;

$$f(x) = \begin{cases} cx^2, \ldots -1 \leq \mathrm{x} \leq 1 \\ 0, \ldots other\ wises \end{cases}$$

e. $c =?$

b. $P(X \leq 0) =?$

$P(X \leq 2) =?$

$P(X \geq 2) =?$

$P(0 < X \leq 1) = ?$

$P(X \leq 1.5) =?$

$P(0 < X \leq 2) = ?$