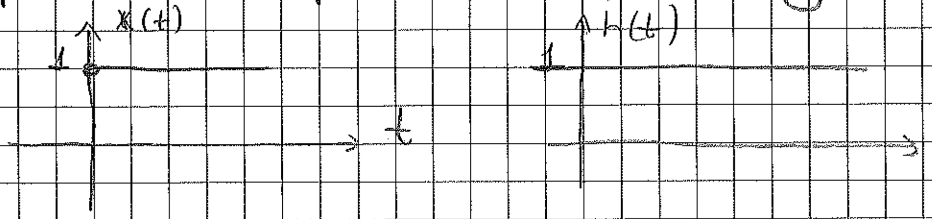
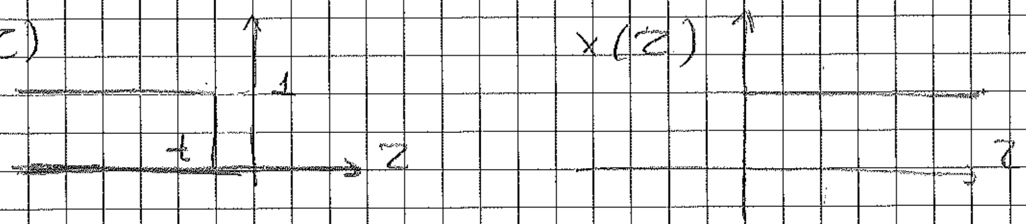
# SOME EXAMPLE PROBLEMS ABOUT CONVOLUTION

**Q.1** Find the output $y(t)$ of an LTI system for the input and impulse response given




Solution:




$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$t \leq 0 \qquad y(t) = 0$ (No overlap)

$t > 0 \qquad y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau = \int_{-\infty}^{t} 1\, d\tau = t$

$$y(t) = \begin{cases} 0 & t < 0 \\ t & t > 0 \end{cases} = \text{Ramp signal.}$$

HINT: You can see the short cut property of step function $x(t) * u(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

**Q.2** Convolution of $x(t+5)$ with impulse function $\delta(t-7)$ is equal to ?

$x(t+5) * \delta(t-7)$

$x(t) * \delta(t) \Rightarrow x(t)$ (When you convolute $x(t)$ with $\delta(t)$ then you get $x(t)$)

$\downarrow$

$x(t+5) * \delta(t-7) \Rightarrow x(t - [-5+7])$

$= x(t-2)$

Q.3 $x(-t) * \delta(-t-t_0) = ?$

$$x(t) * \delta(t) = x(t)$$

$$x(t-0) * \delta(t+t_0) = x(t-(0-t_0))$$
$$= x(t+t_0)$$

$$x(at) * h(at) = \frac{1}{|a|} y(at)$$

$$x((-1)(t)) * \delta((-1)(t+t_0)) = \frac{1}{|-1|} x((-1)(t+t_0))$$
$$= x(-t-t_0) \quad (a=-1)$$

Q.4 The impulse response of a system is $h(t) = \delta(t-0.5)$. If two such systems are cascaded, the output (response) of the overall system will be ? (Ref. IES-2001)

Solution: In a cascaded system we convolute transfer functions.

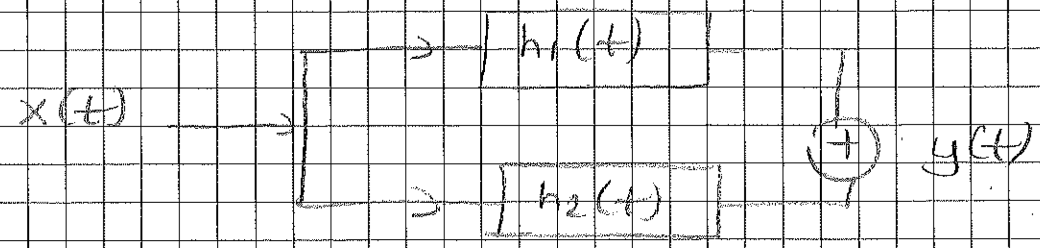
$$x(t) * \delta(t-t_1) = x(t-(0+t_1)) = x(t-t_1)$$

$$x(t-t_1) * \delta(t-t_1) = x(t-(t_1+t_1)) = x(t-2t_1)$$

$t_1 = 0.5$ in our problem. So result is

$$y(t) = x(t-2\underbrace{t_1}_{0.5}) = x(t-1)$$

Q.5 Consider the parallel combination of two LTI systems shown in the figure




The impulse responses of the systems are

$h_1(t) = 2\delta(t+2) - 3\delta(t+1)$

$h_2(t) = \delta(t-2)$

If the input $x(t)$ is a unit step signal, then the energy of $y(t)$ is ??

<u>Solution</u>:

$$y(t) = x(t) * h_1(t) + x(t) * h_2(t)$$

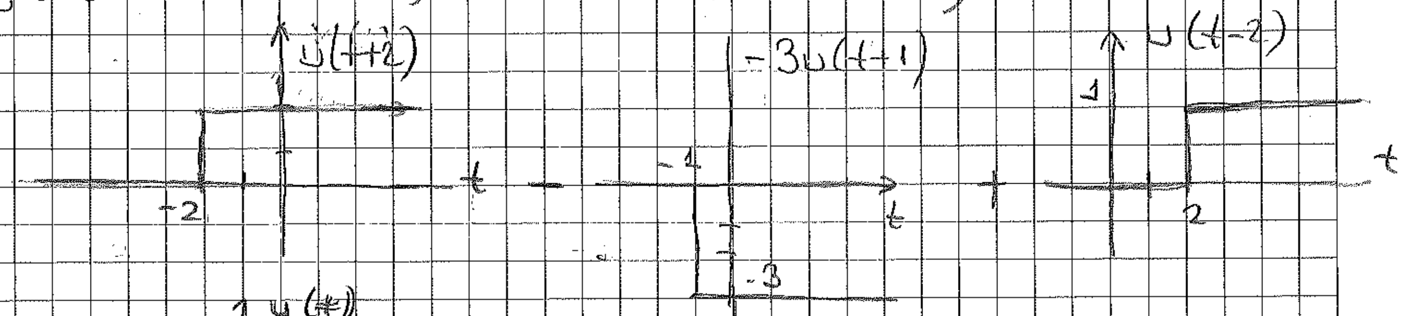
$$= \underbrace{x(t) * [2\delta(t+2) - 3\delta(t+1)]} + x(t) * [\delta(t-2)]$$

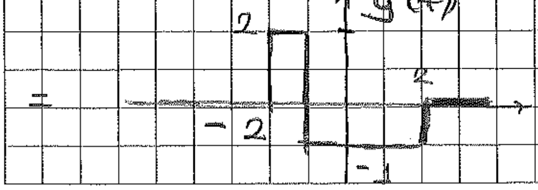
$$y(t) = \big(x(t) * 2\delta(t+2)\big) - \big(x(t) * 3\delta(t+1)\big) + \big(x(t) * \delta(t-2)\big)$$

$$y(t) = 2x(t+2) - 3x(t+1) + x(t-2)$$

If $x(t)$ is unit step signal, then

$$y(t) = 2u(t+2) - 3u(t+1) + u(t-2)$$



$$E = \int_{-\infty}^{\infty} |y(t)|^2 \, dt$$

$$E(y(t)) = 0 + \int_{-2}^{-1} (2)^2 dt + \int_{-1}^{2} (-1)^2 dt + 0$$

$$= 0 + 4t \Big]_{-2}^{-1} + t \Big]_{-1}^{2} + 0$$

$$= 4(-1-(-2)) + (2-(-1))$$

$$= 4.1 + 3 = 7 \text{ joules}$$

Q.6 Let $u(t)$ be the step function. Calculate waveform that corresponds the convolution of $\{u(t) - u(t-1)\}$ with $u(t) - u(t-2)$

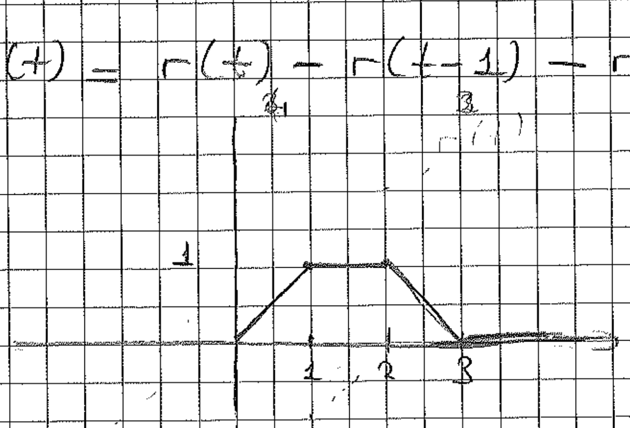
$$[u(t) - u(t-1)] * u(t) - [u(t) - u(t-1)] * u(t-2)$$

$$x(t) * u(t) = \int_{-\infty}^{t} \underbrace{x(t)}_{u(t)} \, dt = t = r(t)$$

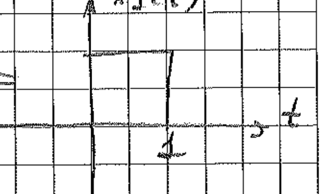
$$\underbrace{u(t) * u(t)}_{r(t)} - \underbrace{u(t) * u(t-1)}_{r(t-1)} - \underbrace{u(t) * u(t-2)}_{r(t-2)} - \underbrace{u(t-1) * u(t-2)}_{r(t-(1+2))}$$

$$r(t-2) - r(t-3)$$

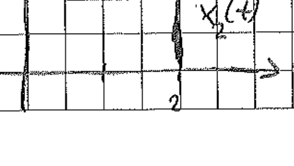
$$y(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$$




HINT: If you see the trick: $u(t) - u(t-1) =$

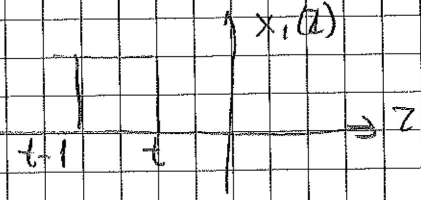



$u(t) - u(t-2) =$



We can also convolve this two signals

$x_1(t) * x_2(t)$




$t<0 \rightarrow y(t) = \underline{\underline{0}}$

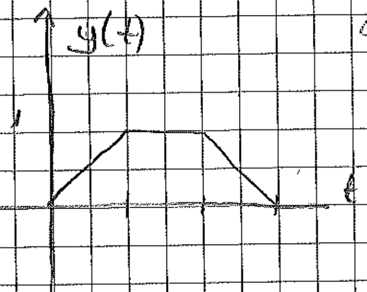
$0<t<1 \qquad \int_0^t 1\,dt = \underline{\underline{t}}$

$1<t<2 \qquad \int_{t-1}^{t} 1\,dt = t\Big]_{t-1}^{t} = \underline{\underline{1}}$

$2<t<3 \qquad \int_{t-1}^{2} 1\,dt = t\Big]_{t-1}^{2} = 3-t$

$t>3 \rightarrow 0$

$$y(t) = \begin{cases} 0 & t<0 \\ t & 0<t\leq 1 \\ 1 & 1<t\leq 2 \\ 3-t & 2<t\leq 3 \\ 0 & t>3 \end{cases}$$




**Q7** $x(t) * \delta(3t-4)$ ?

Solution: We have two properties with delta (impulse) function:

$$x(t) * \delta(t-t_0) = x(t-t_0)$$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$x(t) * \delta\left(3\left(t-4/3\right)\right) \longrightarrow \frac{1}{3}\delta(t-4/3)$$

$$x(t) * \frac{1}{3}\delta(t-4/3) = \frac{1}{3}x(t-4/3)$$

Q.8 The impulse response of continious system is given by $h(t) = \delta(t-1) + \delta(t-3)$

The value of step response at $t = 2$ is?

$y(t) = x(t) * h(t)$

$x(t) * [\delta(t-1) + \delta(t-3)]$

$x(t) * \delta(t-1) + x(t)\,\delta(t-3)$

$y(t) = x(t-1) + x(t-3)$

If $x(t) = u(t)$ then $y(t) = u(t-1) + u(t-3)$

$y(2) = u(2-1) + u(2-3)$

$= u(1) + u(-1)$

$y(2) = 1 + 0 = 1$

Q.9 $x(t) = u(t-3) - u(t-5)$

$h(t) = e^{-3t}u(t)$ Find $y(t) = \frac{d}{dt}x(t) * h(t)$

Solution $\frac{dx(t)}{dt} = \frac{d}{dt}u(t-3) - \frac{d}{dt}u(t-5)$

$= \delta(t-3) + \delta(t-5)$

$y(t) = (\delta(t-3) + \delta(t-5)) * e^{-3t}u(t)$

$= e^{-3t}u(t) * \delta(t-3) + e^{-3t}u(t) * \delta(t-5)$

$y(t) = e^{-3(t-3)}u(t-3) + e^{3(t-5)}u(t-5)$

Q.10 $y(t) = u(t+1) * r(t-2)$

Solution, According to unit step property

$u(t) * x(t) = \int_{-\infty}^{t} x(\tau)\,d\tau \Rightarrow \int_{-\infty}^{t} r(t)\,dt = (t^2/2)u(t)$

Then apply shifting properly $y(t-(-1+2)) = ((t-1)^2/2)u(t-1)$

Q.11 Find the response of the system if

$h(t) = t u(t)$ if the input is $x(t) = u(t-1)$

$y(t) = x(t) * h(t) = (u(t-1)) * t u(t)$

$= \int_{-\infty}^{t} \tau u(\tau)\, d\tau = \frac{t^2}{2} u(\tau)$

For shifting $u(t-1) \rightarrow y(t) = \frac{(t-1)^2}{2} u(t-1)$