Signals

Operations on signals
- Amplitude
  - Scaling
  - Addition
  - Multiplication Subtraction
- Time
  - Scaling
  - Reversing
  - Shifting

SIGNALS and SYSTEMS

Discrete Time / Continious Time

Even / Odd Signals

Periodic / Non Periodic

Deterministic / Nondeterministic (Random)

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LECTURE 03

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TOPICS

1. Energy and Power Signals
2. Real and Imaginary Signals
3. Classification of Systems
4. Linear Time Invariant systems → LTI

3.1 Energy signals and power signals

Energy Signals

- A signal is said to be an energy signal if the total energy transmitted is finite.
- The total energy transmitted to load (finite duration):
  - ✓ E=$\int_{-T}^{T} x^2(t)dt$ (finite duration)
  - ✓ E=$\lim_{T\to\infty} \int_{-T}^{T} x^2(t)dt$ (infinite duration)
- The power of an energy signal is 0

If a signal is energy signal
Power of that signals = 0

Power Signal

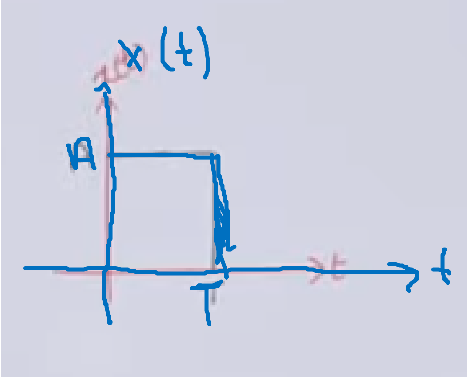
- A signal is said to be a power signal if the total power transmitted is finite.
- The total power transmitted to load (infinite duration):
  - P=(1/T)$\int_{-T/2}^{T/2} x^2(t)dt$ (finite duration)
  - P=$\lim_{T\to\infty} (\frac{1}{T}) \int_{-T/2}^{T/2} x^2(t)dt$ (infinite duration)
- The energy of a power signal is ∞

power → energy
∞

$\boxed{\text{A signal can be either ENERGY or POWER. Not both. But there are some signal}}$

neither energy nor power signal

Eg – Check for energy and power of given signal



$$x(t) = \begin{cases} A & 0<t<T \\ 0 & \text{else} \end{cases} \rightarrow \text{finite duration signal.}$$

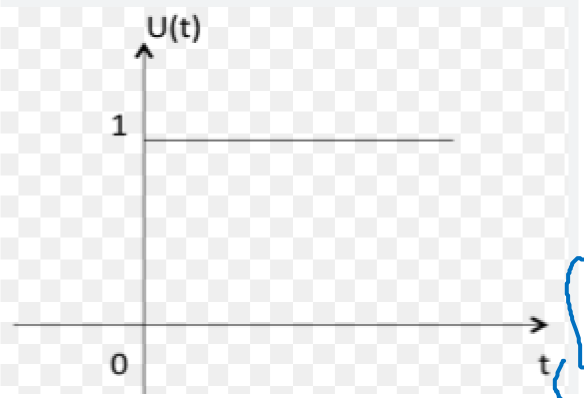
$$\boxed{E = \int_{-T}^{T} x^2(t)\, dt} \Rightarrow \int_0^T x^2(t)\, dt \Rightarrow \int_0^T A^2\, dt$$

$$A^2 t \Big|_0^T = A^2 T - \cancel{A^2 \cdot 0} = \underline{A^2 T} \text{ joules}$$

$\rightarrow$ finite

<u>ENERGY SIGNAL</u> $\longrightarrow$ <u>The power of this signal is $= 0$</u>

Eg – Check energy and power of unit step signal



$0 \longrightarrow \infty$

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

$$E = \lim_{T\to\infty} \int_{-\infty}^{\infty} 1^2 \, dt \Rightarrow \lim_{T\to\infty} \int_{0}^{\infty} 1^2 \, dt$$

$$= \lim_{T\to\infty} t \Big]_0^{\infty} \Longrightarrow \infty$$

This is not an ENERGY signal

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 1^2 \, dt \longrightarrow \lim_{T\to\infty} \frac{1}{2T} t \Big]_0^{T} \to \lim_{T\to\infty} \frac{1}{2T} (T-0)$$

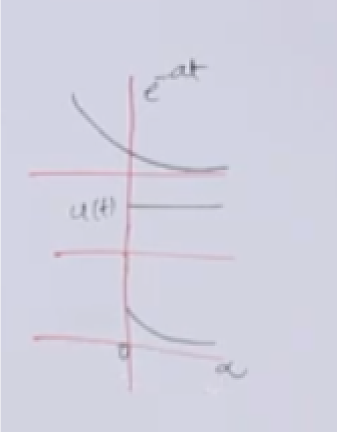
$$P = \lim_{T\to\infty} \frac{1}{2} = \frac{1}{2} \text{ watt}$$

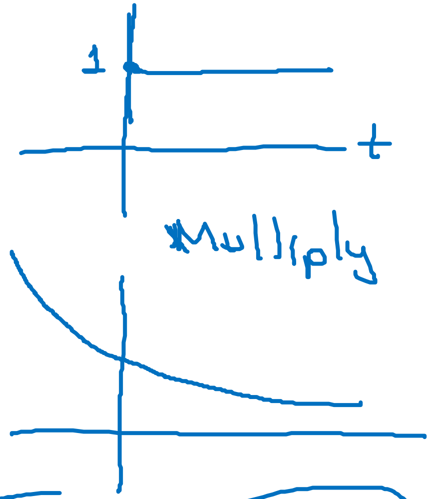
finite

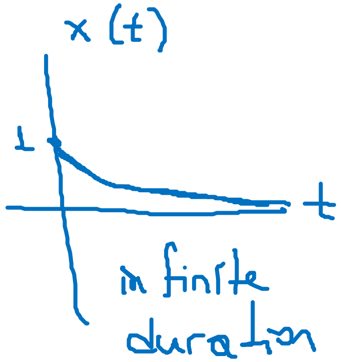
UNIT STEP SIGNAL IS A POWER SIGNAL

THE ENERGY of A power signal is ∞

Eg – Check energy and power of signal x(t)= $u(t)e^{-at}$







$$E = \lim_{T\to\infty} \int_0^T \left[e^{-at} \cdot u(t)\right]^2 dt$$

$$E = \lim_{T\to\infty} \int_0^T e^{-2at} \, dt$$

$$E = \lim_{T\to\infty} \left. \frac{e^{-2at}}{2a} \right]_0^T$$

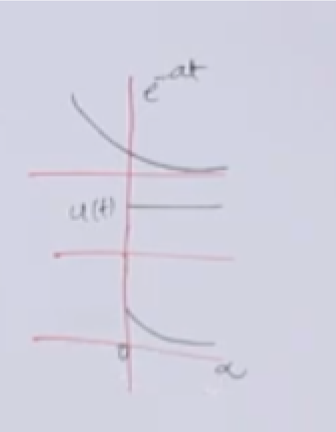
$$E = \lim_{T\to\infty} \frac{e^{-2a(T)}}{-2a} - \frac{e^{-2a \cdot 0}}{-2a}$$

$$0 \quad \frac{e^{-2a\infty}}{-2a} - \frac{1}{-2a} = \frac{1}{2a} \text{ Joules}$$

(finite amount of energy)

ENERGY SIGNAL → POWER of x(t) = 0

Eg : Calculate the energy of the sgibal x(2t) where $x(t) = u(t)e^{-5t}$



$x(t) = u(t)e^{-5t} \rightarrow$ Energy of $x(2t)$

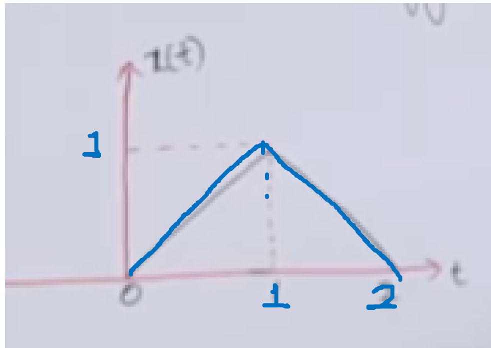
If energy of $x(t) = E$ then
energy of $x(bt) = E/b$

$x(t) = \frac{1}{2a} \rightarrow \frac{1}{2.5} = \frac{1}{10}$ $\quad b = 2$

$a \rightarrow 5$

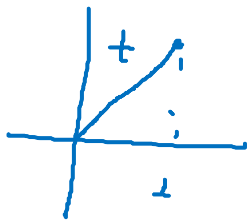
$x(bt) = x(2t) = \frac{E}{b} = \frac{1/10}{2} = \frac{1}{20}$ Joules

Eg : Determine the total energy of the signal shown in the figure. (triangular signal)

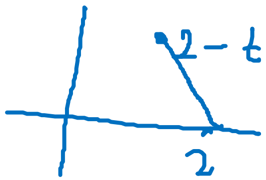


$$x(t) = A\left(1 - \frac{|t|}{T}\right)$$

$x_1(t)$



$x_2(t)$



$$E = \int_{-T}^{T} x^2\,dt = \int_0^1 t^2\,dt + \int_1^2 \underbrace{(2-t)^2}_{4-4t+t^2}\,dt$$

$$E = \left.\frac{t^3}{3}\right|_0^1 + \left.4t - \frac{4t^2}{2} + \frac{t^3}{3}\right|_1^2$$

$$\frac{1}{3} + \left(4\cdot 2 - \frac{4}{2}\cdot 4 + \frac{2^3}{3}\right) - \left(4 - \frac{4}{2} + \frac{1}{3}\right) = \frac{2}{3} \text{ Joules}$$

finite

ENERGY SIGNAL. POWER of $x(t) = 0$

3.2 Real And Imaginary Signals

Real Signals

- A signal is said to real when it satisfies the condition
- $X(t) = X^*(t)$
- Example: x(t)= cost, sint, At (signals that have no imaginary component)
- For a real signal imaginary part will be 0 (zero)

Complex conjugate: $4-7j \longrightarrow 4+7j$

$8+2j \longrightarrow 8-2i$

Real:

$x(t) = x(t)$

$x(t) = \cos t + 0j$

Imaginary Signal

- A signal is said to be imaginary when it satisfies the condition:
  - $X(t) = -X^*(t)$
- Example: x(t)= jbt / For this signal (jbt)* = -jbt = -x(t)
- For an imaginary signal, real part must be zero.

$x(t) = jbt \Rightarrow 0+jbt \Rightarrow -jbt$

$\sin t + j0$ Real

$0 + j\sin t$ Imag

3.2 Real And Imaginary Signals

Two important signals (Complex signals)

- $X(t) = e^{jw0t} = \cos w_0 t + j \sin w_0 t$
- $X(t) = e^{-jw0t} = \cos w_0 t - j \sin w_0 t$
- These two have both real and imaginary parts.
- These are complex signals.

$e^{jw_0t} = \cos w_0 t + j \sin w_0 t$

$e^{-jw_0t} \Rightarrow \cos w_0 t - j \sin w_0 t$

To make the signal imaginary / real

- If you want to make the complex signal above imaginary; you have to make the real part 0
- If you want to make the complex signal above real; you have to make the imaginary part 0

3.3. Classification of Systems

.Basically, systems are classified into 7 different types

- Classification is based on the input and output characteristics of the systems

1- Lineer and non-linear systems	If a system satisfies superposition theorem.
2- Time variant – time invariant systems	input/output characteristic
3-Linear time-Variant (LTV) – Linear time invariant (LTI) systems	
4-Static and dynamic system	Dynamic system has memory
5-Causal and non-causal system	→ output depend only on
6-Invertible and non-invertible system	→ Response of the Output
7-Stable and unstable system	stable boundry in output

- Next week, we will discuss
  - Classification of systems one by one
  - Example file for energy signals will be uploaded.

- Please check our lessons from ekampus.ankara.edu.tr

Thank You