Midterm

Midterm should be done individually (You should write the solution by yourself). Solutions must be submitted electronically as .pdf file before **3.30 pm on Wednesday August 18**. Collaboration is allowed/encouraged on problems, however each student must independently complete their own write-up. **No credit will be given to solutions obtained verbatim from the Internet or other sources.**

1. (15p) What value is returned by the following algorithm? What is its basic operation? How many times is the basic operation executed? Give the worst-case running time of the algorithm using Big Oh notation.

```
ACONCAGUA(n)
input : an integer n
r ← 0
for i = 1 to n
    for j = i to n
        for k = 1 to j
            r ← r + 2
return r
```

2. (15p) A celebrity among a group of n people is a person who knows nobody but is known by everybody else. Design a decrease-and-conquer algorithm that identifies a celebrity among n people, or outputs "no celebrity identified" if there is no such person, by only asking the question "Do you know him/her?" to people. Analyze the efficiency of the algorithm.

Employ your id to calculate a specific number that will be used in the further questions as follows *('14290519' will be used here as an example to show you how the number is calculated):*

- multiply your id with 987654321

$$14290519 * 987654321 = 14114092839682599$$

- remove all the zeros from the resulting number

$14114092839682599 \longrightarrow 1411492839682599$

Note that you can use the website https://www.calculator.net/big-number-calculator.html to perform such calculations.

3. (40p) For the questions, first construct a directed graph using the resulting number as follows:

- consider each consecutive two numbers as an edge in the graph

$1 \longrightarrow 4 \longrightarrow 1 \longrightarrow 1 \longrightarrow 4 \longrightarrow 9 \longrightarrow 2 \longrightarrow 8 \longrightarrow 3 \longrightarrow 9 \longrightarrow 6 \longrightarrow 8 \longrightarrow 2 \longrightarrow 5 \longrightarrow 9 \longrightarrow 9$

- remove the reflexive edges (having same starting and ending nodes) from the graph

$1 \longrightarrow 4 \longrightarrow 1 \longrightarrow 4 \longrightarrow 9 \longrightarrow 2 \longrightarrow 8 \longrightarrow 3 \longrightarrow 9 \longrightarrow 6 \longrightarrow 8 \longrightarrow 2 \longrightarrow 5 \longrightarrow 9$

Note that for the duplicate edges, your consider only one of them for the graph. For the duplicate $1 \longrightarrow 4 \longrightarrow 1 \longrightarrow 4 \longrightarrow \cdots$, *we used only one of them in the graph.*

a) Show the **adjacency list** of your graph.

b) Apply breadth-first search (BFS) algorithm to your graph using any vertex as the source vertex. **For the answer, just draw the BFS tree** that you obtain after the algorithm terminates (to get full credit, you should write the final keys near to the corresponding vertices in the graph as shown in the slides).

c) Apply depth-first search (DFS) algorithm to your graph using any vertex as the source vertex. **For the answer, just draw the DFS tree** that you obtain after the algorithm terminates (to get full credit, you should write the discovery-finish times of the vertices near to the corresponding vertices in the graph as shown in the slides).

4. (15p) Given a sequence of n integers $x_1, x_2, \dots, x_n$, the product-sum is defined as the largest sum of multiplications of adjacent elements in the sequence. Each element can be matched (multiplied) **at most** one of its neighbors.

For example:

for the sequence 1, 2, 3, 1; the product-sum will be $1 + 2\times3 + 1 = 8$
for the sequence 1,4,3,2,3; the product-sum will be $1 + 4\times3 + 2\times3 = 19$

Assume you have a brute-force algorithm, Product-SUM, that **outputs all the possible sums of multiplications of adjacent elements** for a given sequence together with the corresponding result, and **marks one of them as the product-sum**.

Determine the output generated by the algorithm Product-SUM given the last 5 digits of the resulting number as input *(for the student id '14290519', the last 5 digits of the resulting number will be* 1411492839682599 ⟶ 82599 ⟶ 8,2,5,9,9*)*

5. (15p) For the question,

- first remove all the 1s from the resulting number

  1411492839682599 ⟶ 4492839682599

- cut out the last 3 digits and assign them to the letters A, B, C, respectively.

  $5 \longrightarrow A, \quad 9 \longrightarrow B, \quad 9 \longrightarrow C$

- put the numbers in place of the corresponding letters in the following recurrence relation and solve it using Master Theorem

$$T(n) = \begin{cases} 1 & , \quad if \ n = 1 \\ A.T\left(\frac{n}{B}\right) + n^C & , \quad if \ n > 1 \end{cases}$$

Note that first employ your student id to calculate your specific number, then create your own question set. Don't use the numbers provided here.