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# EE405A

## Vehicle Control

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# Experiment Objectives

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In this week, you will do the following:

- Understand Vehicle Model (Kinematic)
- Learn how to design Vehicle Control
  - ☐ Longitudinal Control (PID Control)
  - ☐ Lateral Control (Pure Pursuit, Stanley Method)
- Programming Assignment :
  - ☐ Design your path following, speed controller.
  - ☐ Reference codes (simulator, controller) will be provided.

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# Vehicle Model

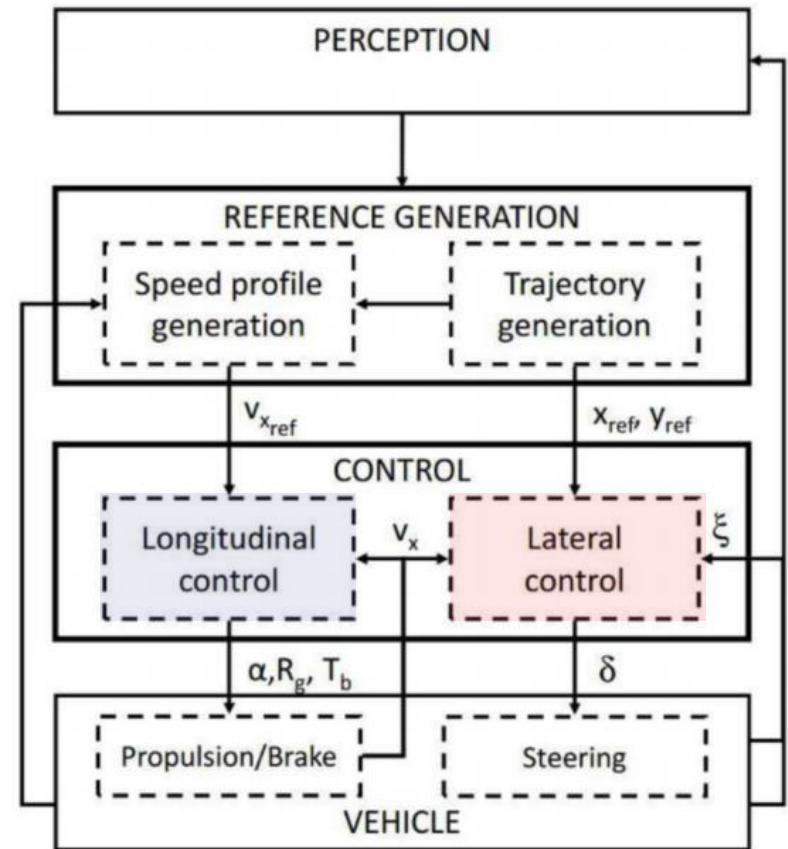
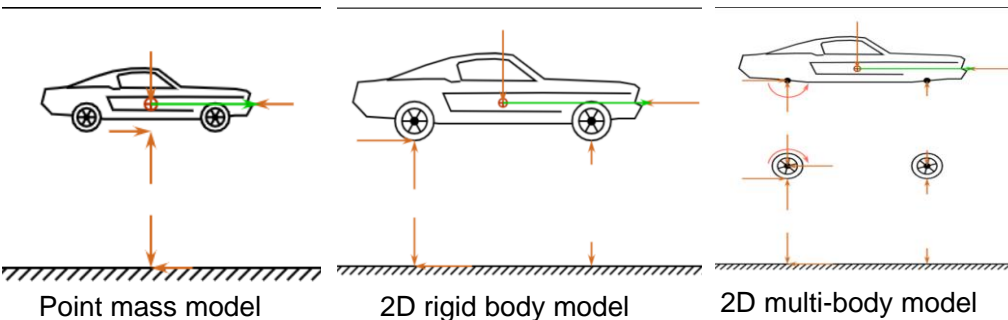
# Vehicle Model

## ➤ Autonomous vehicle controls

- ❑ Longitudinal control
  - : Speed control with acceleration and braking
- ❑ Lateral control
  - : Steering wheel or angle of tires control

## ➤ Autonomous vehicle controls

- ❑ Point mass model
- ❑ 2D rigid body model
- ❑ 2D multi-body model

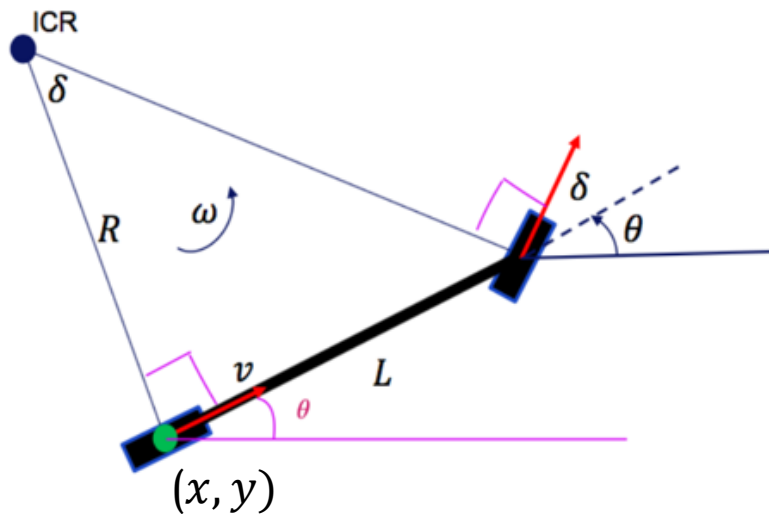


Block diagram for a vehicle control system

# Vehicle Model (Kinematic)

## ➤ Bicycle model (Rear axle centered)

- ❑ A car is assumed to drive in a circle with a fixed steering angle. No slip to the sides.
- ❑ From observation, we get the equation describing the relationship between steering angle  $\delta$  and the corresponding turning radius  $R$ , given its wheelbase length  $L$ .



✓ Velocity

$$\begin{aligned}\dot{x} &= v \cos \psi \\ \dot{y} &= v \sin \psi\end{aligned}$$

✓ Instantaneous Center of Rotation (ICR)

$$\tan \delta = \frac{L}{R} \quad v = R\omega = R\dot{\psi}$$

✓ Acceleration

$$\dot{v} = a$$

$$\Rightarrow \dot{\psi} = \frac{v}{L} \tan \delta$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \cos \psi \\ v \sin \psi \\ \frac{v}{L} \tan \delta \\ a \end{bmatrix}$$

State :  $\{x, y, \psi, v\}$   
Control input :  $\{\delta, a\}$

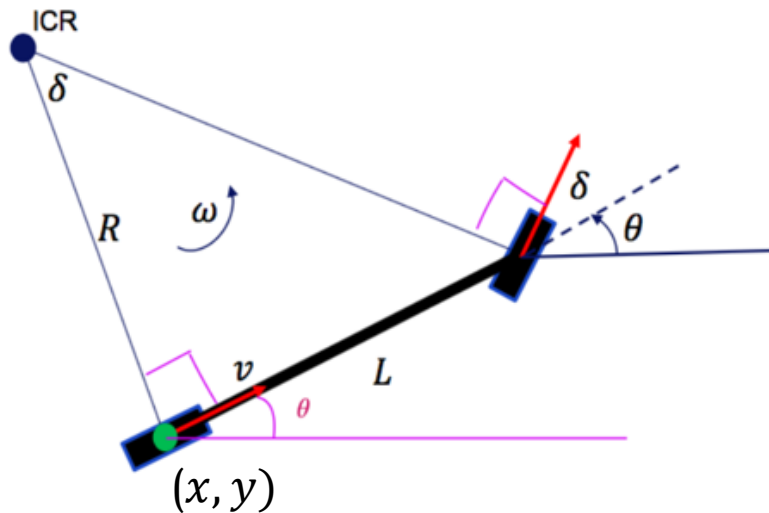
$x$  : position x  
 $y$  : position y  
 $\psi$  : yaw angle  
 $v$  : velocity

$\delta$  : steering angle  
 $a$  : acceleration

# Vehicle Model (Kinematic)

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$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{v} \end{bmatrix}$$

Discrete model in code

$$\begin{aligned}x_{t+1} &= x_t + \dot{x}_t \cdot dt \\ y_{t+1} &= y_t + \dot{y}_t \cdot dt \\ \psi_{t+1} &= \psi_t + \dot{\psi}_t \cdot dt \\ v_{t+1} &= v_t + \dot{v}_t \cdot dt\end{aligned}$$

$x$  : position x

$y$  : position y

$\psi$  : yaw angle

$v$  : velocity

$\delta$  : steering angle

$a$  : acceleration

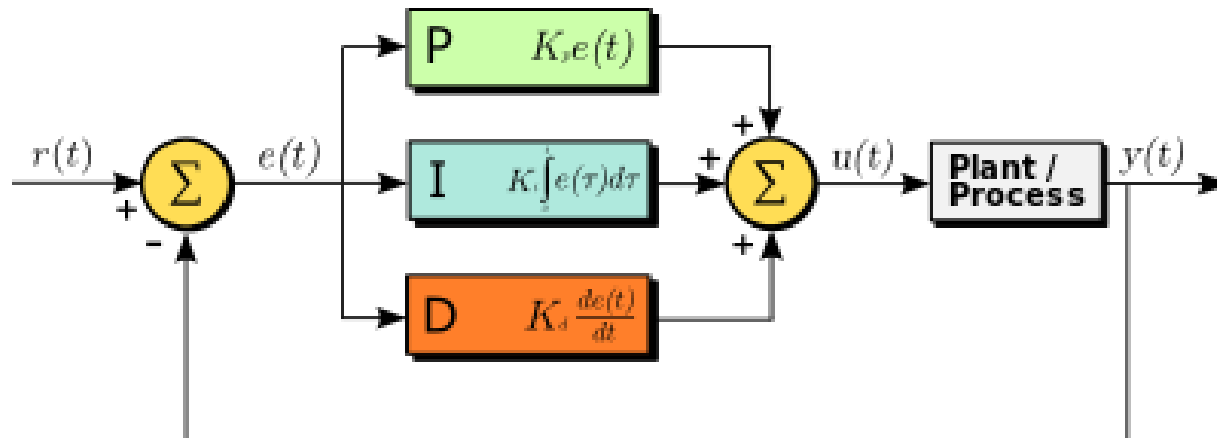
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# Vehicle Control

# Vehicle Control (PID Controller)

## ➤ Proportional-Integral-Derivative Controller (PID Controller)

- ❑ PID Controller consists of **three terms**: proportional(P), integral(I) and derivative(D) term.
- ❑ Each term has a control gain:  $K_P$  gain,  $K_I$  gain,  $K_D$  gain.
- ❑ **P-term** is proportional to the error,  $r(t) - y(t)$ .
- ❑ **I-term** accounts for past error values and integrates them over time.
- ❑ **D-term** estimates the future trend of the error, based on its current rate of change.



A block diagram of a PID controller in a feedback loop.



# Vehicle Control (PID Controller)

## ➤ Proportional-Integral-Derivative Controller (PID Controller)

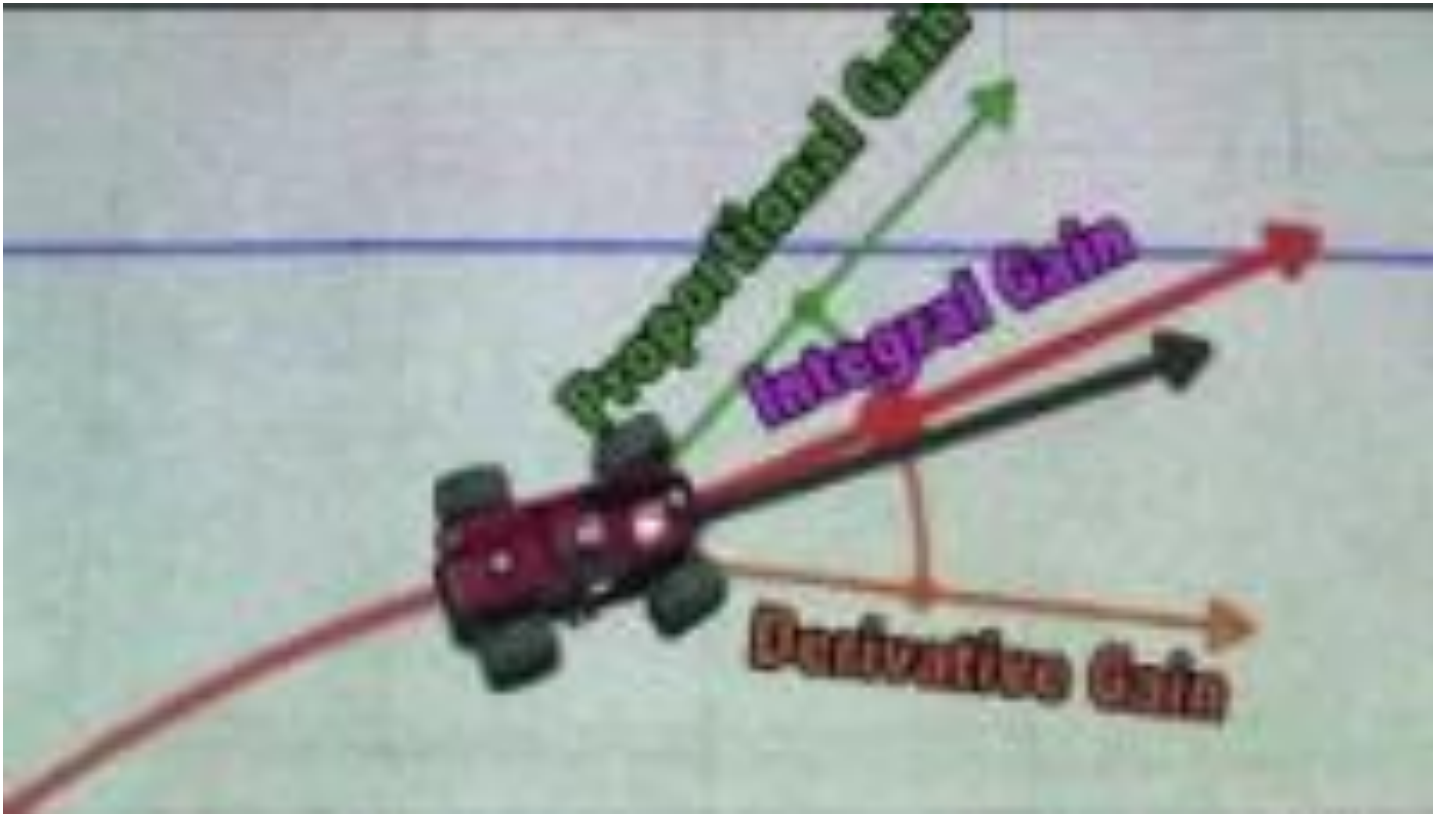
- ❑ A brief introduction of PID Control



Reference : <https://www.youtube.com/watch?v=UR0hOmjaHp0>

# Vehicle Control (PID Controller)

- **Proportional-Integral-Derivative Controller (PID Controller)**
  - ❑ How P, I, D gains affect the performance of the vehicle

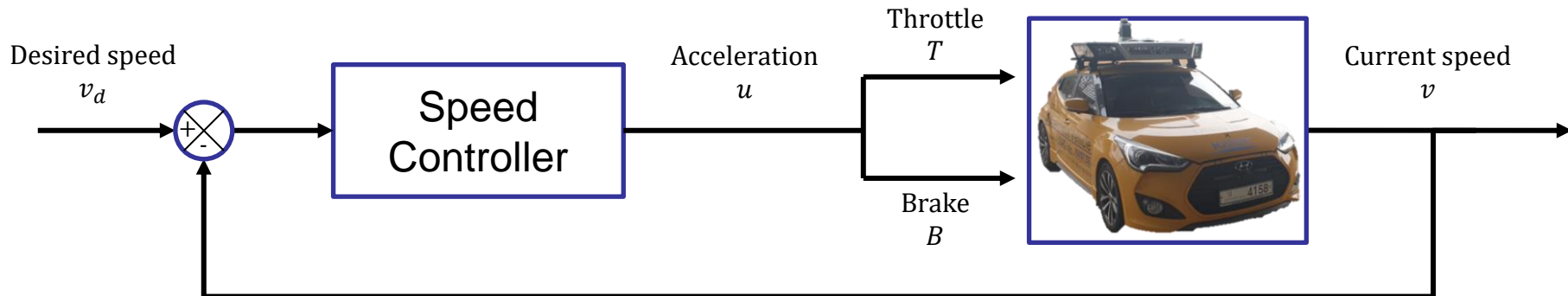


Reference : <https://www.youtube.com/watch?v=4Y7zG48uHRo>

# Vehicle Control (Longitudinal)

## ➤ Longitudinal Control

- ❑ For maintaining the desired speed of a vehicle, a **longitudinal(speed) controller** should be designed.
- ❑ A feedback control system is used to minimize an error between **current** and **desired** speed.
- ❑ The control value  $u$  is mapped to throttle  $T$  or brake  $B$  pedal position.



- ❑ **PID controller** can be used for the speed control.

$$u = \underbrace{K_P(v_d - v)}_{\text{Proportional Term}} + \underbrace{K_I \int_0^t (v_d - v) dt}_{\text{Integral Term}} + \underbrace{K_D \frac{d(v_d - v)}{dt}}_{\text{Derivative Term}}$$

# Vehicle Control (Lateral)

## ➤ Geometry for Lateral Control

Vehicle states

$(x_r, y_r, \psi_r)$  : x, y, yaw of the ego vehicle's reference point

$\delta$  : Steering angle

The reference point can be whether:

- Rear/Front axle
- Center point (Center of Gravity, CoG)

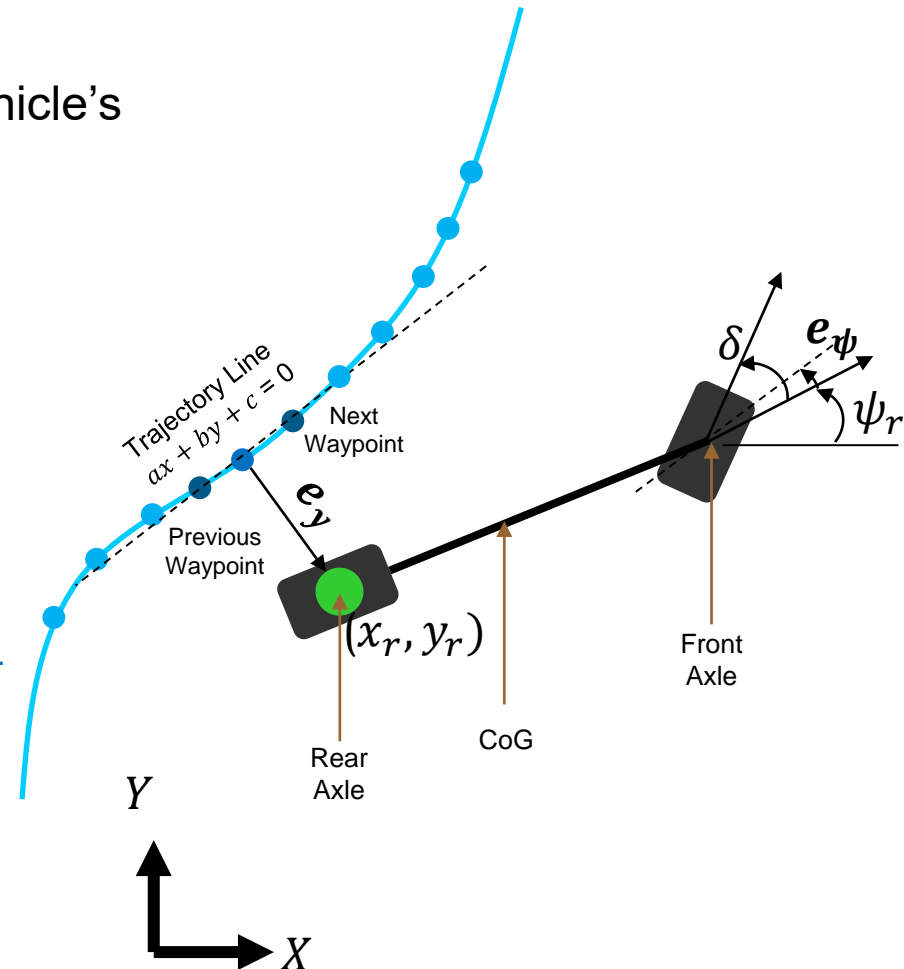
Cross track error

$$e_y = \frac{ax_c + by_c + c}{\sqrt{a^2 + b^2}}$$

Or the distance between the ego and **closest waypoint**.

Heading error

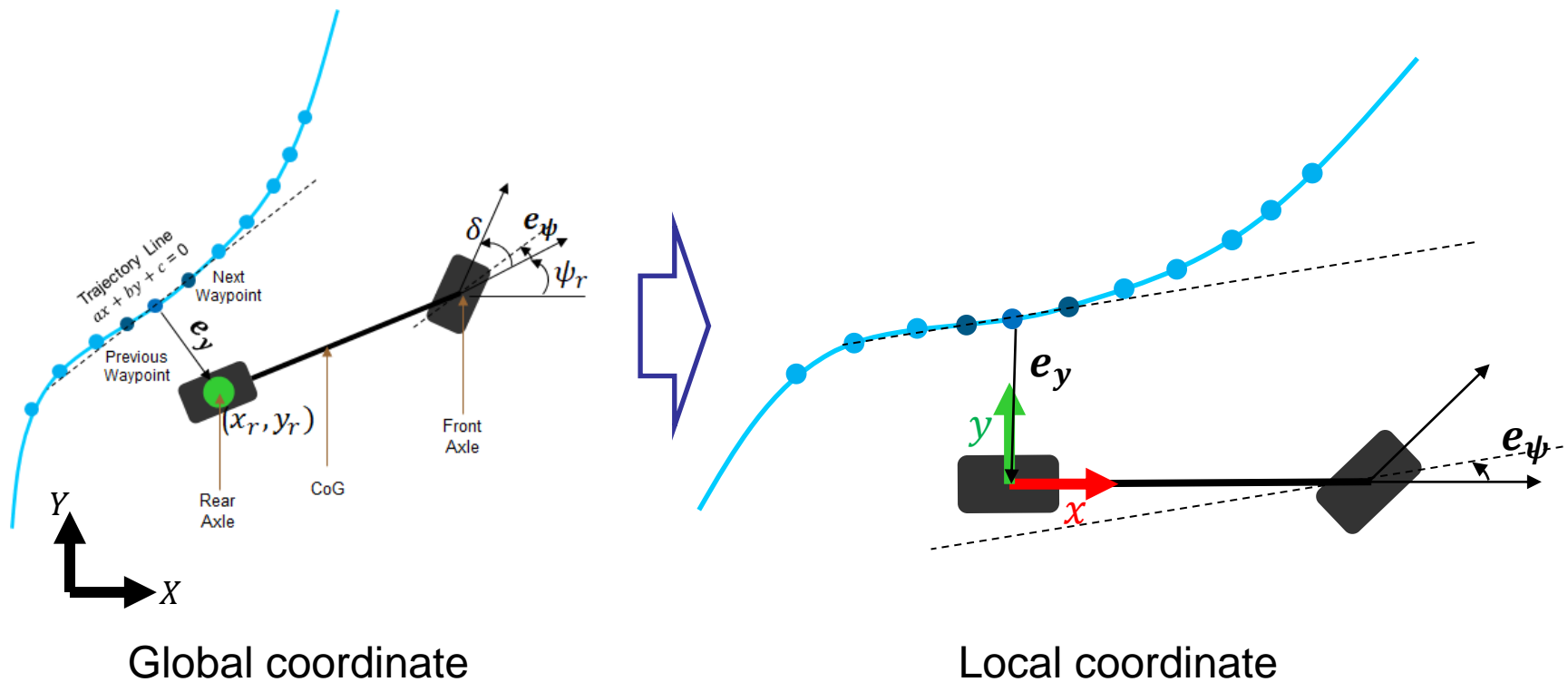
$$e_\psi = \tan^{-1}\left(\frac{-a}{b}\right) - \psi_r$$



# Vehicle Control (Lateral)

## ➤ Geometry for Lateral Control

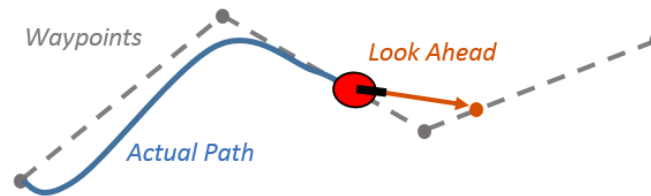
Coordinate transformation from global to local frame



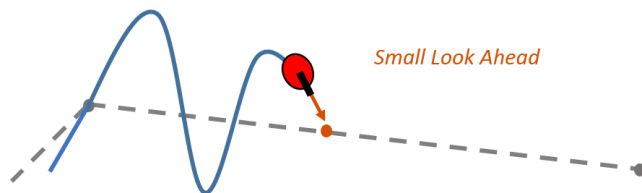
# Vehicle Control (Lateral)

## ➤ Look Ahead Distance

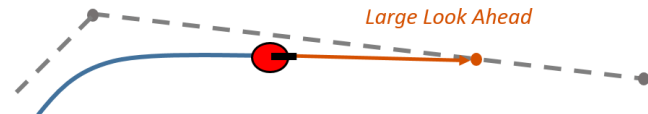
- ❑ Look ahead distance is one of the main tuning parameters for the controller.
- ❑ The look ahead distance is how far along the path the robot should look to compute control commands.



- ❑ The effect of changing the distance can change how the robot tracks the path.
- ❑ Usually, closer distance during slow speed; farther distance during fast speed, for stability.



- Fast recovery
- Large oscillation



- Slow recovery
- Small oscillation

Reference : <https://kr.mathworks.com/help/nav/ug/pure-pursuit-controller.html>

# Vehicle Control (Lateral)

## ➤ Pure pursuit method

: The pure pursuit method consists of geometrically calculating the curvature of a circular arc that connects the rear axle location to a goal point on the path ahead of the vehicle. The goal point is determined from a **look-ahead distance** from the current rear axle position of the vehicle to the desired path.

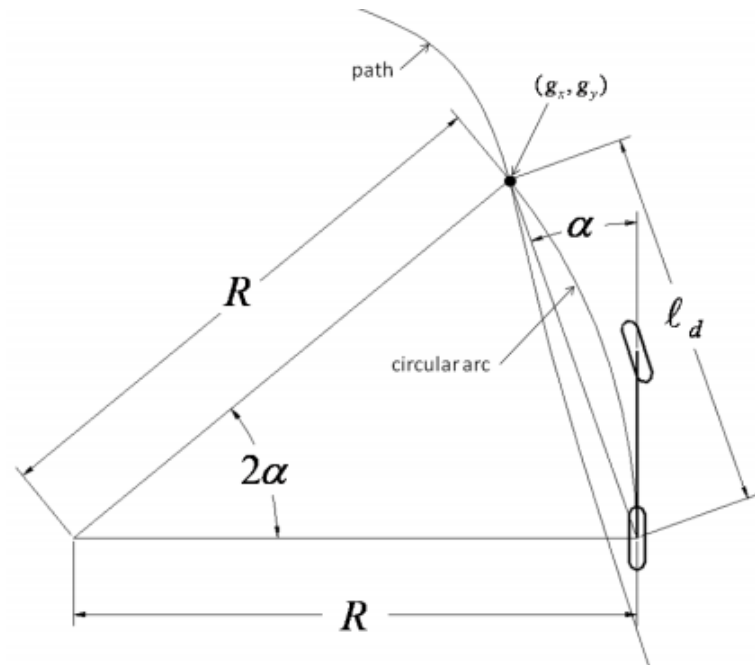
Using the equation of bicycle model,

$$\frac{\ell_d}{\sin(2\alpha)} = \frac{R}{\sin\left(\frac{\pi}{2} - \alpha\right)}$$

$$\frac{\ell_d}{2\sin(\alpha)\cos(\alpha)} = \frac{R}{\cos(\alpha)}$$

$$\frac{\ell_d}{\sin(\alpha)} = 2R$$

$$\delta(t) = \tan^{-1}\left(\frac{2L\sin(\alpha(t))}{\ell_d}\right)$$



Pure pursuit geometry

Reference : [https://www.ri.cmu.edu/pub\\_files/2009/2/Automatic\\_Steering\\_Methods\\_for\\_Autonomous\\_Automobile\\_Path\\_Tracking.pdf](https://www.ri.cmu.edu/pub_files/2009/2/Automatic_Steering_Methods_for_Autonomous_Automobile_Path_Tracking.pdf)

# Vehicle Control (Lateral)

## ➤ Stanley method

: The Stanley method is the path tracking approach used by Stanford University's autonomous vehicle entry in the DARPA Grand Challenge, Stanley. The Stanley method is a nonlinear feedback function of the cross track error and heading error.

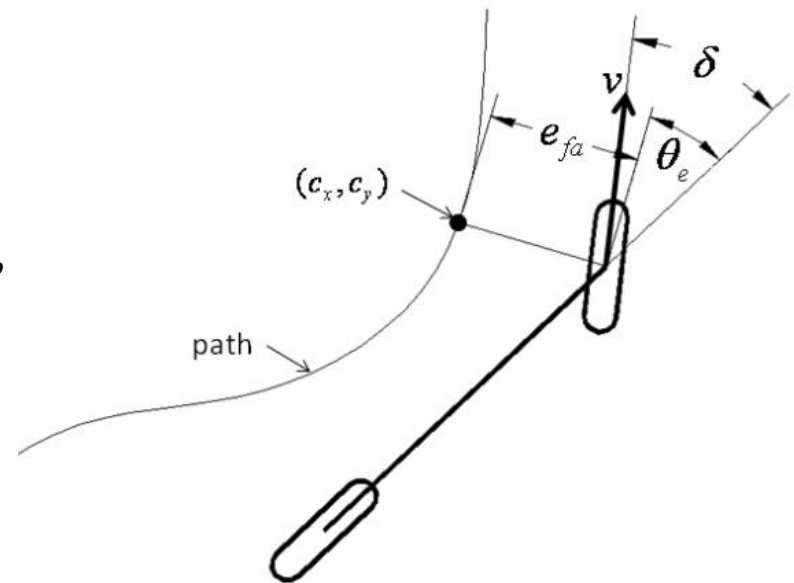
$$\theta_e = \theta - \theta_p,$$

where  $\theta$  is the heading of the vehicle and  $\theta_p$  is the heading of the path at  $(c_x, c_y)$

$$\delta(t) = \theta_e(t) + \tan^{-1} \left( \frac{ke_{fa}(t)}{v_x(t)} \right)$$

Heading error term

Position error term



Stanley method geometry

Reference : [https://www.ri.cmu.edu/pub\\_files/2009/2/Automatic\\_Steering\\_Methods\\_for\\_Autonomous\\_Automobile\\_Path\\_Tracking.pdf](https://www.ri.cmu.edu/pub_files/2009/2/Automatic_Steering_Methods_for_Autonomous_Automobile_Path_Tracking.pdf)



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# Programming Assignment

# Programming Assignment

## ➤ Control your own vehicle in a simple vehicle simulator

❑ Feel free to use and check the following ROS packages.

- ✓ Link : <https://github.com/hynkis/EE405A/tree/main/Week5/Materials>
- ✓ ROS-based simple vehicle simulator (**eurecarr\_vehicle\_sim/simulate\_dynamics.py**)
  - Kinematic bicycle model-based simulator.
- ✓ Reference code for vehicle control (**waypoint\_follower/controller.py**)
  - The control interface has been already implemented.  
(subscribe to vehicle states and publish control commands)
  - See how to implement
    - Subscribing to an Odometry message.
    - Coordinate transform (from global to local)
    - Calculating cross track error ('error\_y'), heading error ('error\_yaw').
- ✓ Waypoint visualizer (**waypoint\_follower/ wpt\_loader.py**)
  - Visualize a pre-built waypoint trajectory in Rviz.

# Programming Assignment

## ➤ Control your own vehicle in a simple vehicle simulator

### ❑ Design your waypoint following controller (waypoint\_follower/controller.py)

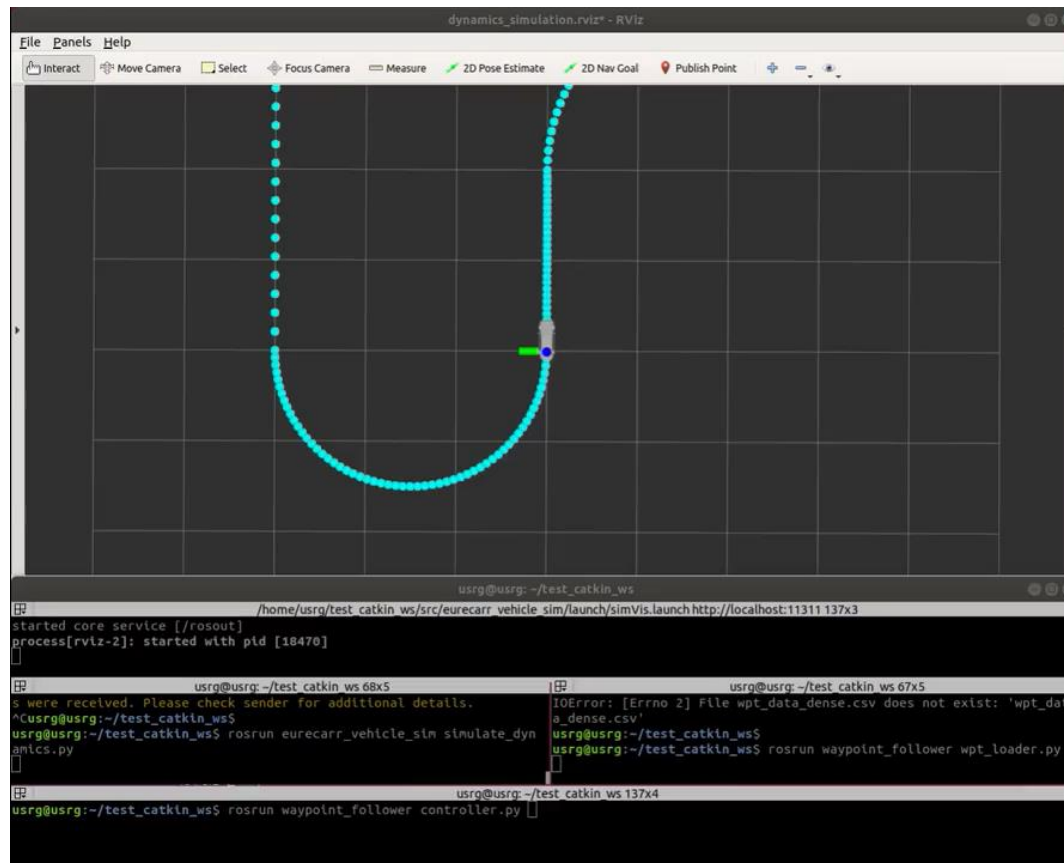
- ✓ Tune the P-controller in the reference code.
  - Currently, all the gains ('kp\_y', kp\_yaw') are 0.
  - You need to find proper controller gains.
  - See 'TODO' in the reference code (controller.py).
- ✓ Implement a PI-controller
  - Try to implement PI controller.
  - You need to compute additional integral of the errors.
- ✓ (Optional) Implement a Pure Pursuit or Stanley Method.

### ❑ Design your speed controller (waypoint\_follower/controller.py)

- ✓ Tune the P-controller in the reference code.
  - Currently, the gain ('kp\_v') is 0.
  - You need to find proper controller gains.

# Programming Assignment

- **Control your own vehicle in a simple vehicle simulator**
  - ❑ Design your waypoint following controller (waypoint\_follower/controller.py)
    - ✓ Demonstration



# Programming Assignment

➤ Send followings to [hynkis@kaist.ac.kr](mailto:hynkis@kaist.ac.kr) until 21.04.14

☐ Your ROS package (waypoint\_follower)

☐ Your Report

✓ Write **what you have learned** this week.

✓ You can use both **KOR/ENG** in your report.

✓ **Discuss** the following topics:

- Difference between P and PI control you implemented.
- The reason for the large cross track error when driving corners.
- Techniques to minimize the cross track error.  
(hint: look ahead distance, velocity).

☐ Please **zip your ROS package and Report** with the following filename.

EE405A\_[lecture\_date(YYMMDD)]\_[Student ID]\_[Full name]

(e.g., EE405A\_210402\_20215169\_Hyunki\_Seong.zip)

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# Q & A

Email : [hynkis@kaist.ac.kr](mailto:hynkis@kaist.ac.kr)