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Improving Efficiency of Autonomous Vehicles by V2V Communication

Changliu Liu, Chung-Wei Lin, Shinichi Shiraishi, and Masayoshi Tomizuka

***Abstract*— Autonomous vehicles are widely regarded as a promising technology to improve the safety of transportation systems. However, the efficiency of vehicles may be compromised to ensure safety when there are large uncertainties in perception and prediction of the behaviors of other road participants due to limitations in sensors. To remedy this problem, vehicle to vehicle (V2V) communication is applied to improve efficiency of autonomous vehicles during interactions with other vehicles. By requiring the vehicles to communicate their intentions with one another, the efficiency of the vehicles can be improved in terms of smaller variations in their speed profiles and smaller delay as demonstrated in the simulations.**

I. Introduction

Autonomous vehicles are widely regarded as a promising technology to improve the safety of transportation systems as they can avoid accidents caused by human drivers' mistakes. However, the behavior of an autonomous vehicle depends on how it perceives and predicts the surrounding world, which is constrained by its sensing capabilities. When there are large uncertainties in perception and prediction, the vehicle's behavior tends to be conservative, especially during interactions with other road participants.

Although the accuracy of perception and prediction can be improved through extensive training and better driver modeling [1], it may not outperform direct communication. In particular, vehicle to vehicle (V2V) communication can compensate the deficiency in perception as well as reduce the uncertainties in predicting other vehicles' behaviors. For example, in Fig.1, the vehicle on the right lane intends to turn right, but moves left a little in order to increase the turning radius. However, from the view of the vehicle on the left lane, the right vehicle seems likely to change lane to the left as shown by the uncertainty tube. In order to be safe, the left vehicle may slow down to yield the right vehicle in case it changes lane. This unnecessary yield is inefficient, and can be avoided if the vehicles can communicate their intentions to others. With the development of DSRC [2], vehicle to vehicle (V2V) and vehicle to infrastructure (V2I) communication modules will become standard components in new vehicle models. In this paper, we exploit the benefits of communication to improve efficiency of autonomous vehicles, especially during on-road driving.

In literature, the benefits of V2V communication are widely exploited in corporative adaptive cruise control (CACC) [3] to minimize inter-vehicle distance and increase platoon density. However, as the information exchanged in CACC is the vehicles's current states or accelerations, these strategies still may not reduce the uncertainties in predicting others' behaviors in complicated environments, such as intersections. With the introduction of V2I communication, vehicles can exchange future information such as the time to cross the intersection with the infrastructure. Efficient intersection management can be achieved as discussed in [4], [5], [6] and [7]. However, these strategies require investment on the infrastructure.

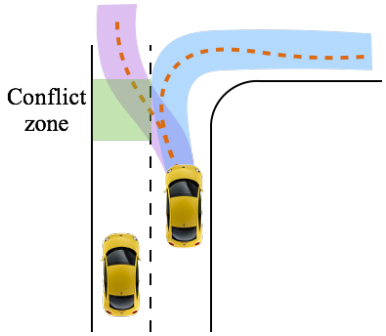

Fig. 1: Uncertainties in other vehicles' behaviors.

The authors proposed a distributed method in [8] for intersection management by requiring vehicles to exchange information of their intentions such as intended maneuvers and time to occupy the intersection via V2V communication. The method works in any traffic condition, not limited to a two-vehicle case as discussed in [9]. It has been demonstrated through macroscopic traffic simulation in [8] that this strategy performs better than conventional traffic management methods such as traffic light or all-way stop control as well as existing distributed conflict resolution mechanisms [10] in terms of smaller traffic delay time and larger traffic throughput. In this paper, this method will be extended to improve the efficiency of autonomous vehicles in diverse on-road driving scenarios. Moreover, the microscopic benefits of communication will be illustrated, e.g., how the efficiency of individual vehicle may be improved by V2V communication.

The remainder of the paper is organized as follows. The mathematical problem for autonomous driving is formulated in Section II, and the inefficiency in a distributed multi-vehicle system will be pointed out. The communication protocol and strategy to improve efficiency are discussed in Section III. Simulation studies are presented in Section IV. Section V discusses potential difficulties in implementing the proposed communication protocol and concludes the paper.

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II. Problem Formulation

In this section, the motion planning problem for individual vehicle will be formulated first, followed by the discussion of inefficiency in the multi-vehicle system. Every vehicle has a unique index i.

A. The Vehicle Perspective

Consider vehicle i. Denote its state at time t as $x_i(t)$ and its intention as G_i. The state x_i includes the position, heading and velocity of the vehicle. The intention G_i refers to its target lane. If the target lane is the same as the current lane, then the vehicle intends to follow its current lane. If the two lanes are not identical, then the vehicle intends to change lane. Before entering an intersection, G_i refers to vehicle i's target lane after passing the intersection.

Since individual vehicle has only local view and local information, vehicle i only considers the road participants in its neighborhood $\mathcal{N}_i$ when making driving decisions, where $\mathcal{N}_i$ is a collection of indices of surrounding vehicles. At time t_0, given the initial state $x_i(t_0)$, the trajectory $x_i(t)$ for $t > t_0$ needs to be computed. Hence the motion planning problem for vehicle i is formulated as,

$$\min_{x_i} \ J(x_i, G_i), \tag{1a}$$

$$\text{s.t. } \dot{x}_i(t) \in \Gamma(x_i(t)), \tag{1b}$$

$$d(x_i(t), \hat{x}_j^i(t)) \geq d_{min}, \forall j \in \mathcal{N}_i, \hat{x}_j^i(t) = h_i(\hat{G}_j^i). \tag{1c}$$

Equation (1a) is the cost function, which evaluates the performance of the trajectory, e.g., $J = \int_{t_0}^{t_0+T} \mathcal{L}(x_i(t), G_i)dt + \mathcal{S}(x_i(t_0+T), G_i)$ where $T > 0$ is the planning horizon; $\mathcal{L}(x_i(t), G_i)$ is the run-time cost; and $\mathcal{S}(x_i(t_0+T), G_i)$ is the terminal cost. Equation (1b) is the feasibility constraint to ensure that the planned trajectory can be tracked by a low level controller, e.g., $\Gamma(x_i) := \{\dot{x}_i | \exists u_i, \text{s.t.} \dot{x}_i = f(x_i, u_i)\}$ where $\dot{x}_i = \partial x_i / \partial t$, u_i is the vehicle control input (wheel angle and throttle torque) and f describes the vehicle dynamics. Equation (1c) is the safety constraint that requires the minimum distance to any surrounding vehicle to be greater than a threshold $d_{min} > 0$, where function d measures the minimum distance between two vehicles. $\hat{x}_j^i(t)$ is the estimate of $x_j(t)$ made by vehicle i, which is a function of the estimated intention $\hat{G}_j^i$ of vehicle j. The function h_i is the prediction model, which can be learned from data. Note that the estimates $\hat{x}_j^i(t)$ and $\hat{G}_j^i$ should be considered as random variables which distribute over a bounded set. And the constraint (1c) should be satisfied for all possible values of the estimates. In Fig.1, from the view of the left vehicle, the estimated intention of the right vehicle $\hat{G}_j^i$ can take two values (left turn and right turn), hence $\hat{x}_j^i(t)$ distributes in the two tubes that corresponds to the two values of $\hat{G}_j^i$. Problem (1) needs to be solved in every time step when new information is obtained. Methods to solve problem (1) is discussed in [11].

B. The System Perspective

Suppose there are N road participants in the system, indexed from 1 to N. The system state at time step t is denoted as $x(t) := [x_1(t); \ldots; x_N(t)]$. All vehicles in the system solve problem (1) for themselves. While all vehicles would like to stay safe as specified in constraint (1c), their intentions may conflict with one another. The system objective is to ensure that the intentions of vehicles are satisfied efficiently and safely. The optimization problem from the system perspective is,

$$\min_{x} \ \sum_{i=1}^{N} w_i J(x_i, G_i), \tag{2a}$$

$$\text{s.t. } \dot{x}_i(t) \in \Gamma(x_i(t)), \forall i, \tag{2b}$$

$$d(x_i(t), x_j(t)) \geq d_{min}, \forall i, j, i \neq j, \tag{2c}$$

where the objective function is a weighted sum of the cost function for every road participant. The weights w_i's represent priorities, e.g., w_i is larger for vehicles on the main street since the needs of these vehicles should be addressed first. The feasibility constraint (2b) is inherited from (1b). Equation (2c) is the safety constraint for the system. For simplicity, define the safe set $\mathcal{X}$ as

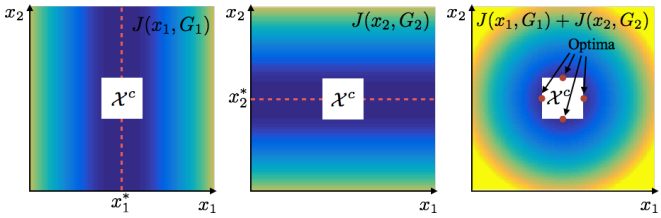
$$\mathcal{X} := \{x | d(x_i, x_j) \geq d_{min}, \forall i, j, i \neq j\}. \tag{3}$$

The system objective provides a measure of efficiency of the vehicles from the system level. When all vehicles solve the optimization (1) at the same time, it is a simultaneous game. It is ideal that the distributed solutions in (1) match the system optima in (2). Since no vehicle has the incentive to change its trajectory in the system optima, the system optima is indeed a Nash Equilibrium in the distributed system. However, due to uncertainties in the estimates $\hat{x}_j^i$ and $\hat{G}_j^i$ in (1c), it is hard to attain the Nash Equilibrium for the distributed system, which may cause inefficiency as will be discussed below.

C. Inefficiency in the Multi-Vehicle System

Suppose the vehicles are equipped with cooperative adaptive cruise control (CACC) modules for safe and efficient car following. Let x_i^* be the optimal trajectory that minimizes $J(x_i, G_i)$ only considering the dynamic feasibility (1b) and car following constraint $d(x_i(t), x_{\mathcal{F}_i}(t)) \geq d_{min}$ where $\mathcal{F}_i$ denotes the front vehicle of vehicle i, but not the collision avoidance constraint with other vehicles in (1c).

If the optimal trajectories do not intersect or overlap with one another, we say that the vehicles do not have spacial conflicts. Otherwise, there are spacial conflicts. In Fig.1, there is a spacial conflict if the right vehicle changes lane to the left, and there is no spacial conflict if the right vehicle turns right. Among the scenarios with spacial conflicts, we say that the vehicles do not have temporal conflicts if $[x_1^*(t); \ldots; x_N^*(t)] \in \mathcal{X}$ for all t. Otherwise, there are temporal conflicts. We call the locations that spacial conflicts take place as conflict zones, which can either be fixed or flexible in the 2D space. For example, in an intersection, the



(a) V1's perspective (b) V2's perspective (c) System perspective

Fig. 2: The generalized Chicken game.

location of a conflict zone is determined by the intersection of two incoming lanes, hence fixed in the 2D space. However, the conflict zone between a lane-change vehicle and the vehicle in the target lane is not fixed in the 2D space, which indeed depends on where the lane-change vehicle enters the target lane. Since the vehicles are assumed to be equipped with CACC, there is no temporal conflict among vehicles that follow the same lane. Hence we only define the conflict zones for vehicles from different lanes. A more detailed classification of the conflict zones is discussed in [12].

In the ideal case, vehicles should execute the optimal trajectory x_i^*'s if there is no spacial or temporal conflicts, since individual optima align with system optima in those cases. However, due to uncertainties in the predictions of others as shown in Fig.1, vehicle i may not dare to execute its optimal trajectory x_i^*.

When there are temporal conflicts among vehicles, the scenario runs into a generalized Chicken game[1] [4]. Executing the optimal trajectory x_i^* can be considered as the choice of "go". Vehicles will collide with each other if none of them "yields", i.e., executing a trajectory different than x_i^*. The situation for $N = 2$ is illustrated in Fig.2. The cost functions and the optimal trajectories x_1^* and x_2^* for vehicle 1 (V1) and vehicle 2 (V2) are shown in Fig.2a and Fig.2b respectively. The darker the color, the smaller the cost. The white square is the infeasible set $\mathcal{X}^c$ (the complement of $\mathcal{X}$), which can be regarded as infinite cost. The pair (x_1^*, x_2^*) does not belong to $\mathcal{X}$. Fig.2c illustrates the system objective in (2a) with $w_1 = w_2$. The system optima are marked as red dots. From the system perspective, either of the vehicles needs to yield. However, the system optima is hard to achieve. In current design of autonomous vehicles, $\hat{x}_j^i$ is estimated based on data from local sensors. In order to account for uncertainties in the estimation, the behavior of an autonomous vehicle tends to be conservative. The system may be trapped in a situation that all vehicles decide to slow down to yield, which is very inefficient. A mechanism is needed to resolve the conflict and break the symmetry.

[1]The game of chicken is a model of conflict for two vehicles in game theory. Each vehicle has two choices: yield or go. If the other vehicle yields, the ego vehicle receives higher payoff to go. If the other vehicle does not yield, the ego vehicle receives higher payoff to yield. The desired case is that one vehicle yields and the other goes. However, as this is a simultaneous game, the vehicles do not know the other vehicle's plan in advance. Hence it is hard for the vehicles to decide whether to yield or to go.

III. Improving Efficiency via Communication

In order to overcome the inefficiency discussed above, vehicles need to be certain about whether there are spacial or temporal conflicts with others. Moreover, a mechanism is needed to resolve the conflicts.

A. Communication Protocol and Strategy

In order for vehicles to decide whether there are spacial conflicts, the intentions G_i's should be broadcasted. For example, if two vehicles in different lanes intend to follow their current lanes, then there is no spacial conflict. With the broadcasted G_i, the scenario in Fig.1 can be avoided. Among vehicles that have spacial conflicts, the conflict zone C_l should be identified first. Then the time to occupy the conflict zone should be broadcasted in order to determine whether there are temporal conflicts.

If vehicle i does not have any temporal conflict with others, it just executes the optimal trajectory x_i^*. Otherwise, its trajectory needs to be modified considering the collision avoidance constraint (1c) according to the broadcasted information.

A common strategy to resolve temporal conflicts is "first enter first go", i.e., whoever arrives first on a conflict zone should go first. Consider the case in Fig.1. Suppose the right vehicle intends to change lane to the left and will arrive at the conflict zone (the intersection of the trajectories of the two vehicles) first. Hence the right vehicle can execute its optimal trajectory x^*, while the left vehicle will slow down to yield the right vehicle. However, this strategy may create deadlocks, i.e., all vehicles decide to yield the others, since it is possible that vehicle A arrives earlier in some conflict zone, while vehicle B arrives earlier in another conflict zone. In [8], a tie breaking mechanism is introduced by assigning a priority score to all vehicles. The priority score provides a total ordering of the vehicles such that once a deadlock is detected, the vehicle with the highest priority score will not yield others even if it will arrive later than some vehicle in some conflict zone. In this way, conflicts can be resolved safely and efficiently.

However, the limitation of the method discussed in [8] is that all vehicles are required to be connected and use the same strategy, which is hard to achieve in real world scenarios. In the following discussion, the method will be generalized to account for diverse on-road driving scenarios.

B. Motion Planning Problem Considering Communication

For vehicle i, divide its surrounding vehicles intro three groups:

- $\mathcal{S}_i$: vehicles that communicate with vehicle i and use the same communication protocol and strategy;
- $\mathcal{D}_i$: vehicles that communicate with vehicle i but use different communication protocols and strategies;
- $\mathcal{N}_i \setminus (\mathcal{D}_i \cup \mathcal{S}_i)$: vehicles in the neighborhood of vehicle i but do not communicate with vehicle i.

From vehicle i's perspective, if vehicle j in $\mathcal{S}_i$ has spacial conflicts with vehicle i in a conflict zone C_l, then the order of passing should be determined by the "first enter first go"

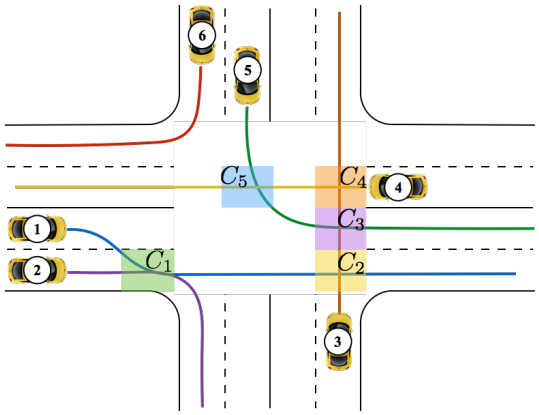
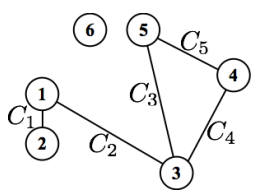
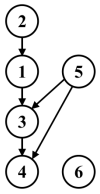


Fig. 3: A four-way intersection.



(a) Graph of spacial conflict.



(b) Graph of passing order.

Fig. 4: The topological graphs.

strategy with tie breaking. If vehicle i needs to yield vehicle j, then it should enter C_l at a time $\mathbb{T}_{i,l,j}$ later than the broadcasted exit time of vehicle j. There is no constraint for vehicle i if it does not need to yield vehicle j. Then the desired time for vehicle i to enter C_l is

$$\mathbb{T}_{i,l} = \max_{j \in \mathcal{S}_i} \mathbb{T}_{i,l,j}. \tag{4}$$

For vehicle j in C_i, vehicle i can directly obtain its intention G_j through communication, and predict its trajectory $\hat{x}_j^i$ under G_j using the prediction model h_i. Regarding the three groups of vehicles, the constraint (1c) in the motion planning problem (1) can be modified as

$$x_i(t) \notin C_l, \forall t \leq \mathbb{T}_{i,l}, \forall l, \tag{5a}$$

$$d(x_i(t), \hat{x}_j^i(t)) \geq d_{min}, \forall j \in \mathcal{D}_i, \hat{x}_j^i(t) = h_i(G_j), \tag{5b}$$

$$d(x_i(t), \hat{x}_j^i(t)) \geq d_{min}, \forall j \in \mathcal{N}_i \setminus (\mathcal{D}_i \cup \mathcal{S}_i), \hat{x}_j^i(t) = h_i(\hat{G}_j^i), \tag{5c}$$

where (5a) is for vehicles in $\mathcal{S}_i$, (5b) for $\mathcal{D}_i$, and (5c) for $\mathcal{N}_i \setminus (\mathcal{D}_i \cup \mathcal{S}_i)$. Applying the new constraints, vehicle i can avoid the inefficiency induced by misinterpretation with respect to vehicles in $\mathcal{D}_i$. Moreover, since vehicle i can reach a consensus with vehicles in $\mathcal{S}_i$, the inefficiency implied by the Chicken game can also be avoided with respect to vehicles in $\mathcal{S}_i$.

C. Example

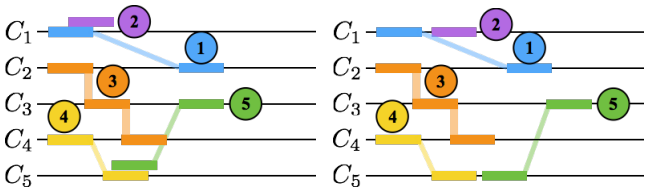
In this part, the proposed method will be illustrated through an example in a four-way intersection as shown in Fig.3. There are two incoming lanes and two outgoing lanes in every leg of the intersection. There are six vehicles in the environment indexed 1 to 6, whose intentions and trajectories are shown in the figure.

In the case when all vehicles are connected and using the same strategy, a vehicle can determine a set of vehicles that it has spacial conflicts with based on the broadcasted information. The relationship of spacial conflicts forms an indirected graph as shown in Fig.4a. The nodes in the graph represent the vehicles. There is a link between two nodes if there is a spacial conflict between the two vehicles. Node 6 is isolated in the graph since vehicle 6 does not have any spacial conflict with others. Hence vehicle 6 can execute the optimal trajectory x_6^*. For other vehicles, we still need to check whether there are temporal conflicts. The broadcasted time slots are shown as the thick bars in Fig.5a, where the horizontal axis represents time. There is a temporal conflict between vehicle 1 and vehicle 2 at C_1, as well as between vehicle 4 and vehicle 5 at C_5. There is no time conflict for vehicle 3, hence the optimal trajectory x_3^* can be executed. The trajectories of vehicles that involve in temporal conflicts need to be modified according to (5a). Since vehicle 1 arrives at C_1 earlier than vehicle 2, vehicle 1 does not need to change its trajectory. But vehicle 2 needs to yield vehicle 1 by passing C_1 after vehicle 1 as shown in Fig.5b. Similarly, vehicle 4 can execute its optimal trajectory x_4^*, while vehicle 5 needs to yield vehicle 4 in C_5 as shown in Fig.5b. Note that the vehicles are not required to construct the whole graph in Fig.4a and Fig.5. They just need local information to make decisions. Once the local decisions are made, the indirected conflict graph in Fig.4a can be transformed into a directed graph in Fig.4b, which shows the passing order in the corresponding conflict zones.

Consider the case that some vehicles are not connected. For example, vehicle 1 is communicating with vehicle 2 and vehicle 3, but not with vehicles 4, 5 and 6. In vehicle 1's motion planning problem, constraint (5c) applies for vehicles 4 to 6 where their intentions and trajectories need to be estimated. Some of the possible future trajectories of vehicles 4 to 6 may intersect with vehicle 1's trajectory, which introduces more "conflict zones" as shown in Fig.6. The uncertainties of the trajectories in the time domain are shown in Fig.7a. Under those uncertainties, there are temporal conflicts between vehicle 1 and vehicle 4, 5 or 6. If vehicle 1 does not change its path, in order to satisfy constraint (5c), it needs to slow down as shown in Fig.7b. However, when there is communication, vehicle 1 does not need to slow down as shown in Fig.5.

IV. Simulation Results

The method is simulated in a four-way intersection in a $50m \times 50m$ square shown in Fig.8. There are six vehicles in the environment, all of which intend to go straight. At time $0s$, they are on the boarder of the environment as shown in Fig.8. The desired longitudinal speed for each vehicle is shown in Table I. There are eight conflict zones. The sampling time is $0.1s$ in the system and all vehicles are synchronized.



(a) With temporal conflicts. (b) Temporal conflicts resolved.

Fig. 5: The time to occupy conflict zones.

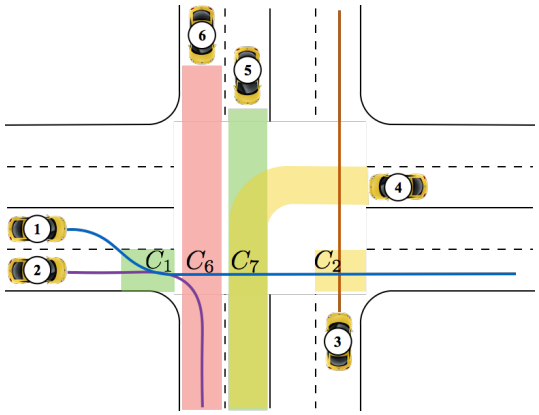
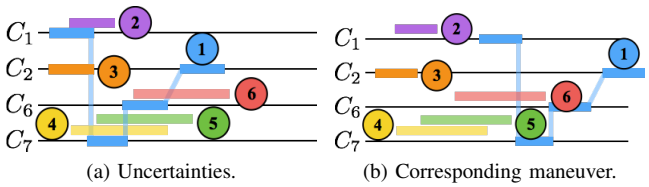


Fig. 6: Illustration of the uncertainty perceived by vehicle 1.



(a) Uncertainties. (b) Corresponding maneuver.

Fig. 7: The uncertainty perceived by vehicle 1 in the time domain.

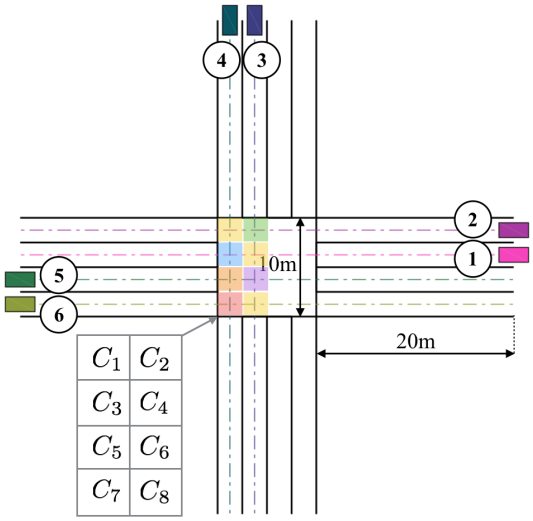


Fig. 8: The simulation environment at time $0s$.

The trajectories of the vehicles when all of them use the proposed strategy are shown in Fig.9. At time $0s$, the time slots to occupy the conflict zones are broadcasted as shown in Fig.11a. There are temporal conflicts between vehicle 2 and vehicle 3 in C_2, between vehicle 1 and vehicle 3 in C_4 and between vehicle 4 and vehicle 5 in C_5. Hence vehicle 3 and vehicle 4 slow down to yield the other vehicles. In the next time step, the expected time slots are broadcasted again as shown in Fig.11b. The temporal conflicts in C_2, C_4 and C_5 are resolved. However, new temporal conflicts are created in C_1 and C_3 as a consequence. Then vehicle 1 and vehicle 2 slow down to yield vehicle 4 as shown in the broadcasted time slots in Fig.11c. At time $0.3s$ in Fig.11d, vehicle 3 further slows down to yield vehicle 1 and vehicle 2 again. Then all temporal conflicts are resolved. The executed speed profiles of the vehicles are shown in Fig.10. The delay for every vehicle is computed as the difference between the actual travel time and the traffic-free travel time ($50m/v^r$), which is shown in the row "Delay-C" in Table I. Due to numerical truncations in simulation, the delay can be negative as shown for vehicle 6.

When the vehicles are not communicating with each other, the unmanaged intersection functions as a four-way-stop intersection, i.e., all vehicles stop before entering the intersection. The speed profiles of the vehicles in this case are shown in Fig.12 which have larger variations than the profiles in Fig.10. The delays also increase as shown in the row "Delay-NC" in Table I. The smallest delay is 0.92s, which is greater than the largest delay 0.79s when there is communication. This is due to the fact that communication enables vehicles to adjust their speed profiles in advance, hence avoid unnecessary stops.

V. Discussion and Conclusion

In this paper, we discussed methods to improve efficiency of autonomous vehicles through V2V communication. In the proposed communication protocol, the future information of a vehicle needed to be broadcasted in addition to current information such as current state. If rough future information (e.g., intention G_i) was communicated, it helped to determine spacial conflicts and avoid unnecessary reactions to vehicles that the ego vehicle did not have spacial conflict with. If detailed future information (e.g., time to occupy conflict zones) was communicated, it helped to form consensus among vehicles regarding the passing order in the conflict zones. The motion planning problem that evaluated information from communications was discussed. In the simulation result, it was verified that the efficiency of the autonomous vehicles was improved in the sense of smaller variation in the speed profile and smaller delay time.

Nonetheless, the proposed method is prune to network delay and attacks. For example, a vehicle may lie about its

TABLE I: The statistics in the simulation.

Vehicle	1	2	3	4	5	6
v^r (m/s)	12.54	13.83	10.40	13.28	12.91	13.04
Delay-C (s)	0.31	0.39	0.79	0.23	0.03	-0.03
Delay-NC (s)	3.85	0.92	4.94	2.27	9.97	8.20

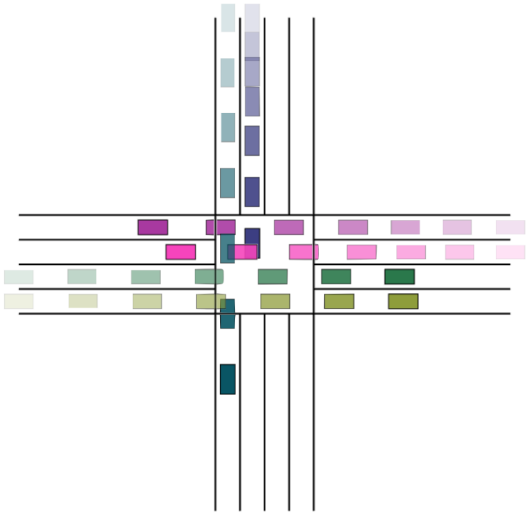


Fig. 9: The configurations of the environment under communication at time $0s, 0.5s, \ldots, 3s$.

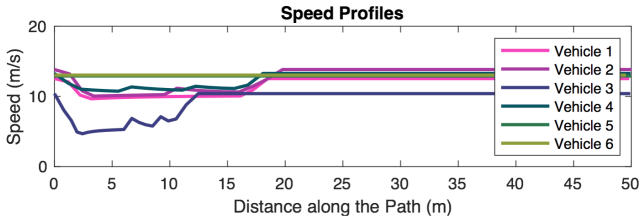


Fig. 10: The speed profiles for the vehicles under communication.

intention or estimated time slot to occupy the conflict zone in order to make other vehicles yield. Another problem is that if a vehicle does not receive updated information of another vehicle, the two vehicles may operate on different information set, hence hard to reach consensus.

In the future, more realistic scenarios which consider network deficiencies will be considered. Moreover, the properties of the multi-vehicle systems with only partial connectivity will be studied theoretically.

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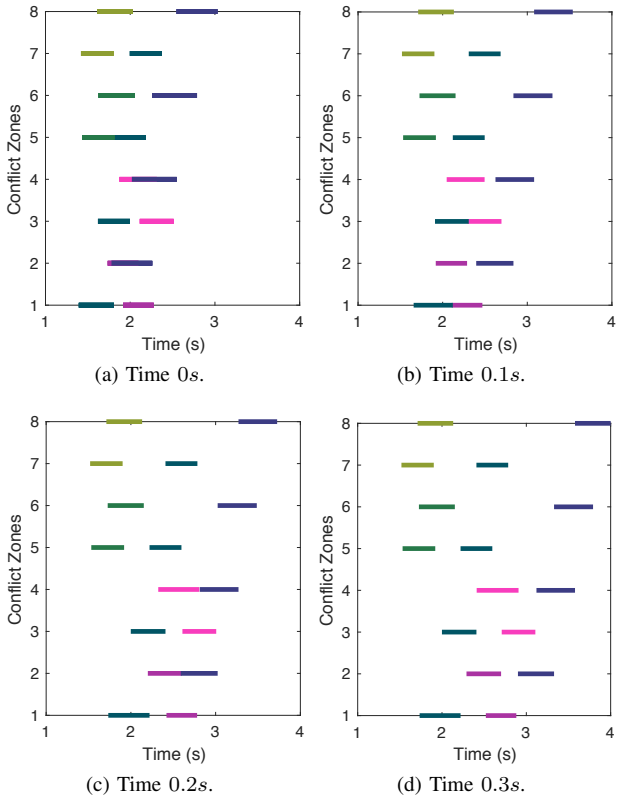

(a) Time $0s$. (b) Time $0.1s$.

(c) Time $0.2s$. (d) Time $0.3s$.

Fig. 11: The broadcasted time slots to occupy the conflict zones of all vehicles.

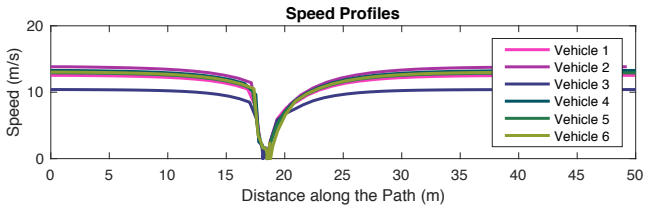

Fig. 12: The speed profiles for the vehicles without communication.

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