SVD: Singular Value Decomposition

* Eigen Value Decomposition $A_{n \times n}$

$$AV = \lambda V = V\lambda$$

$\downarrow$ independent eigenvectors

$$A = V \lambda V^{-1}$$

$\downarrow$ orthogonal eigenvectors

$$A = Q \lambda Q^T$$

* Singular Value Decomposition : $A_{m \times n}$

$$A = UDV^T$$

U: left singular vectors (AA^T)

V: right singular vectors (A^TA)

Moore-Penrose Pseudo inverse

$$Ax = y \quad \text{for } A_{m \times n}$$

$$\downarrow$$

$$x = By \quad B_{n \times m}$$

① $A = [\quad]$ ⇒ may have No solutions

② $A = [\quad]$ ⇒ may have many solutions

③ $A = [\quad]$ ⇒ may have One solution

* SVD

$$Ax = y \qquad\qquad A = UDV^T$$

$$\downarrow \qquad\qquad \downarrow$$

$$x = By \qquad\qquad A^+ = VD^+U^T$$

$$D^+ = \begin{bmatrix} 1/\sigma_1 & & & \\ & 1/\sigma_2 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}$$

$$\downarrow$$

$$x = By = A^+y$$

* if $A = \begin{bmatrix} & & & \end{bmatrix}$: more columns than rows

$$Ax = y \longrightarrow x = A^{+}y$$

Minimum Euclidean Norm $\|x\|_2$ among all possible solutions.

* if $A = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$: more rows than columns

$$Ax = y \longrightarrow x = A^{+}y$$

As close as possible to y in terms of Euclidean norm $\|Ax - y\|_2$