CALCULUS I
2013 Spring Final Exam

Dept. or School		Year		proctor	
Student ID		Name			

※ **Your answer must be provided with descriptions how to get the answer.**

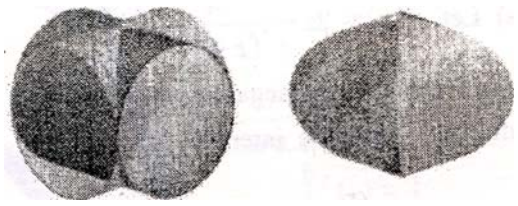
1. (4 points) Evaluate improper integral $\int_{-1}^{\infty}\left(\frac{x^4}{1+x^6}\right)^2 dx$ if it converges.

2. (6 points) Let $f(x)=2x-\frac{4}{(x+1)^2}$

(1)(3 points) Find the nonnegative number(s) b such that the average value of f on the interval $[0,b]$ is 2.

(2)(3 points) Find the average value of the derivative f' on the interval $[0,2]$.

3. (5 points) Find the volume common to two circular cylinders, each with radius a if the axes of the cylinders intersect at right angles.



4. (6 points) Let $L(a)=\int_1^a g(t)dt$ be a form of the length of the curve $y=x+\sqrt{x}$ from the point $(1,2)$ to the point $(a,a+\sqrt{a})$.

(1) (3 points) Find the function $g(t)$ in $L(a)$.

(2)(3 points) By using Simpsons's Rule with $n=2$, estimate the length $L(2)$ of the curve from $(1,2)$ to the point $(2,2+\sqrt{2})$.

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5. (6 points) A curve called the folium of Descartes is defined by the parametric equations $x = \dfrac{3t}{1+t^3}$, $y = \dfrac{3t^2}{1+t^3}$.

(1)(3 points) Show that the polar equation of the curve can be written in the form $r = \dfrac{3\sec\theta\tan\theta}{1+\tan^3\theta}$.

(2)(3 points) Find the area enclosed by the loop of the curve.

6.(5 points) Prove or disprove that if $\sum_{n=1}^{\infty} a_n$ is convergent then $\sum_{n=1}^{\infty} a_n^2$ is convergent.

7.(8 points) Let $f(x)=\sinh^{-1}x$.

(1)(2 points) Find the Maclaurin series for $f(x)$.

(2)(3 points) Determine the radius of convergence of the series in (1).

(3)(3 points) Find the value of

$$\frac{1}{\sqrt{2}}+\sum_{n=1}^{\infty}(-1)^n\frac{f^{(2n+1)}(0)}{(2n+1)!}\left(\frac{1}{\sqrt{2}}\right)^{2n+1}$$