CALCULUS I
2014 Spring Final Exam

Dept. or School		Year		proctor	
Student ID		Name			

※ **Your answer must be provided with descriptions how to get the answer.**

1.(5 points) Find the volume of the solid obtained by rotating the region bounded by the curve $y=\dfrac{\sin(\ln x)}{x}$ where $e^{\frac{\pi}{4}} \leqq x \leqq e^{\frac{\pi}{2}}$ and the line $y=0$ about the y-axis .

2. Let C be a curve in the plane given by $x=\cos\theta-\sin\theta\cos\theta$, $y=\sin\theta-\sin^2\theta$, $0 \leqq \theta \leqq 2\pi$.

(a)(3 points) Find the length of C.

(b)(3 points) Find the area of the region enclosed by C.

3.(4 points) Find the number b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is 3.

4. Find all values of p for which the following series convergent. (※You must give a reason.)

(a)(3 points) $\sum_{n=1}^{\infty} \frac{1}{n^p}$

(b)(3 points) $\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n^p}\right)$

4

CALCULUS I 2014 Spring Final Exam	Dept. or School		Year		proctor	
	Student ID		Name			

5. Let $f(x) = \ln\left(\frac{1+x}{1-x}\right)$. Answer the questions (a) and (b).

(a)(4 points) Find a power series representation for $f(x)$.

(b)(2 points) By using your answer in (a), find the sum of the series $\sum_{n=0}^{\infty} \frac{1}{(2n+1)617^{2n+1}}$.

6. Let $f(x) = \ln(1+3x)$. Answer the questions (a) and (b).

(a)(4 points) Find a Taylor polynomial $T_3(x)$ representing up to degree 3 for $f(x)$ centered at $a = 1$.

(b)(3 points) From your answer $T_3(x)$ in (a), estimate maximum error of $|f(x) - T_3(x)|$ where $\frac{1}{3} \leqq x \leqq \frac{5}{3}$.

7.(6 points) Find x to solve the equation

$$\begin{vmatrix} 1 & -1 & 2 & -1 \\ -3 & 4 & 1 & -1 \\ 2 & -5 & -3 & 8 \\ -2 & x & -4 & 1 \end{vmatrix} = 154$$