

CALCULUS I 2013 Spring Midterm Exam	Dept. or School		Year		proctor	
	Student ID		Name			
※ Your answer must be provided with descriptions how to get the answer.						
1. (4 points) For a positive number a , find $\lim_{x \rightarrow \infty} \left(\frac{a^{1/x} + a^{-1/x}}{2} \right)^x$.						
2.(4 points) Evaluate $\int_1^3 \sqrt{x} \int_0^{\sqrt{x}} \frac{1}{1+t^2} dt \, dx$.						

3.(5 points) Find an equation of the tangent line to the curve $y \sin 2x = x \cos 2y$ at the point $(\pi/2, \pi/4)$.

4.(5 points) Evaluate $\int_{\ln(\sqrt{6}+1)}^{\ln(2\sqrt{2}+1)} \frac{e^{4t} - 3e^{3t} + 3e^{2t} - e^t}{\sqrt{e^{2t} - 2e^t - 3}} dt$

<div> <div>CALCULUS I</div> <div>2013 Spring Midterm Exam</div> </div>	Dept. or School		Year		proctor	
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<div> <div>5 (1)(3 points) Express the following limit as a form of definite integral.</div> <div> $\lim_{n \rightarrow \infty} \left[\frac{1}{n} \cos \left(\ln \left(1 + \frac{1}{n} \right) \right) + \frac{1}{n} \cos \left(\ln \left(1 + \frac{2}{n} \right) \right) + \cdots + \frac{1}{n} \cos \left(\ln \left(1 + \frac{n}{n} \right) \right) \right]$ </div> <div> <div>(2)(3 points) Use (1) to find the limit</div> <div> $\lim_{n \rightarrow \infty} \left[\frac{1}{n} \cos \left(\ln \left(1 + \frac{1}{n} \right) \right) + \frac{1}{n} \cos \left(\ln \left(1 + \frac{2}{n} \right) \right) + \cdots + \frac{1}{n} \cos \left(\ln \left(1 + \frac{n}{n} \right) \right) \right]$ </div> </div> </div>	<div> <div>6. Let $f(x) = x \sqrt{4 - x^2} + 4 \sin^{-1} \left(\frac{x}{2} \right)$ and $g(x) = \cos^{-1} \left(\frac{x}{2} \right)$.</div> <div>(1)(3 points) Differentiate $f(x)$.</div> <div>(2)(3 points) Solve the equation $\frac{d^2}{dx^2} f(x) + \frac{d}{dx} g(x) = 0$.</div> </div>					

7. If $n \geq 2$ is a positive integer and $x > 0$,

(1) (3 points) Find the maximum value of x^{1/x^n} over the interval $1 \leq x \leq e$.

(2) (3 points) Find $\lim_{x \rightarrow \infty} x^{1/x^n}$ for every integer n .

(3)(4 points) For what values of a is the following equation true?

$$\lim_{x \rightarrow \infty} \left[x^{1/x^n} \left(\frac{x+a}{x-a} \right)^x \right] = e.$$