CALCULUS II 2015 Fall Midterm Exam	Dept. or School		Year		proctor	
	Student ID		Name			

※ Your answer must be provided with descriptions how to get the answer.

1. For three points $P(2,-3,4)$, $Q(0,1,2)$, and $R(-1,2,0)$ in $\mathbb{R}^3$.

(a) (4 points) Find the area of the triangle whose vertices are P, Q, R.

(b) (4 points) Find the equation of the plane that passes through the points P, Q, R.

(c) (4 points) Find the variable t which P, Q, R, and $S(1,t,2)$ are coplanar.

2. Let α, β, and γ be positive real numbers and $O(0,0,0)$ the origin in $\mathbb{R}^3$. For each $n=1,2,\cdots$, associated are points $A_n(n^\alpha, 2n^\alpha, 0)$, $B_n(-2n^\beta, 0, 3n^\beta)$ and $C_n(0, 2n^\gamma, -3n^\gamma)$.

(a) (4 points) Find $\overrightarrow{OA_1} \times \overrightarrow{OB_1}$.

(b) (6 points) Let V_n be the volume of the tetrahedron whose vertices are O, A_n, B_n, and C_n. Find the condition for the series to $\sum_{n=1}^{\infty}(V_n)^{-1}$ converge.

3. (6 points) Find the symmetric equation of the line which meets perpendicularly with both of the lines:

L_1 : $x=1+t,\ y=1+2t,\ z=2+t$,

L_2 : $x=1+s,\ y=2+s,\ z=1+s \quad (s,t\in\mathbb{R})$.

4. (6 points) Find an equation of the osculating plane of the space curve $\{(t^3, t^2, t) \mid t\in R\}$ at the point $(1,1,1)$.

5. (a) (4 points) Find an equation in a rectangular equation for the surface whose spherical equation is $6\cos\theta=\rho\sin\phi$.

(b) (6 points) Find symmetric equations for the tangent line to the curve of intersection of the equation (a) and the plane $x+y+z=1$ at the point $P(0,0,1)$.

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6. Let C be the curve represented by the vector function $r(t)=\langle \cos t, \sin t, t\rangle$.

(a) (4 points) Find the curvature $\kappa(t)$ of C.

(b) (6 points) Let D be the osculating circle of C at the point $P(0,1,\pi/2)$. And suppose that every point (x,y,z) on D satisfies the equation

$$\frac{(x+a)^2}{b}+\frac{(y+c)^2}{d}=1.$$

Then find the value $a+b+c+d$.

7. Let a function f defined on $\mathbb{R}^2$ by

$$f(x,y)=\begin{cases}\dfrac{x^2-y^2}{\sqrt{x^2+y^2}} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0).\end{cases}$$

(a) (4 points) Compute $f_x(1,0)$ and $f_y(0,0)$.

(b) (6 points) Show that $f(x,y)$ is continuous at $(0,0)$ but $f_x(x,y)$ is not continuous at $(0,0)$.

8. (6 points) Find the linear approximation of the function $f(x,y)=\tan^{-1}(xy)$ at $(1,1)$ and use it to approximate $f(1.01, 0.99)$.

9. (4 points) Find the tangent plane to the surface with parametric equations $r(u,v)=<u^2, v^2, uv>$ at the point $(1,1,1)$.

10. (6 points) Let $f(x,y,z)=xe^{y-z^2}$, $x=uv$, $y=u-v$, and $z=u+v$.

Use the chain rule to find $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$ when $u=3$, $v=-1$.