CALCULUS II
2014 Fall Midterm Exam

Dept. or School		Year		proctor	
Student ID		Name			

※ Your answer must be provided with descriptions how to get the answer.

1. Let the three points be $A(1,1,1)$, $B(1,1,2)$, $C(3,-1,2)$.

(1) (3 points) Find the value of t so that the four points $A, B, C,$ and $D(t,t^2,t^3)$ lie in the same plane.

Sol.) 스칼라 삼중적을 이용

$\overrightarrow{AB}\cdot(\overrightarrow{AC}\times\overrightarrow{AD})=0$ 를 만족하는 t를 구하면 된다.

$\overrightarrow{AB}=\langle 0,0,1\rangle$

$\overrightarrow{AC}=\langle 2,-2,1\rangle$

$\overrightarrow{AD}=\langle t-1,t^2-1,t^3-1\rangle$

$$\overrightarrow{AB}\cdot(\overrightarrow{AC}\times\overrightarrow{AD})=\begin{vmatrix}0&0&1\\2&-2&1\\t-1&t^2-1&t^3-1\end{vmatrix}=2(t^2-1)+2(t-1)=0$$

$(t-1)(t+2)=0$

$\therefore t=1,-2.$

$t=1$: $D(1,1,1)=A(1,1,1)$ ×

$t=-2$: $D(-2,4,-8)$ O.K.

(2) (3 points) Find the volume of the tetrahedron whose points are $A, B, C,$ and $D(1,2,3)$ (Hint. A tetrahedron is a solid with four points $A, B, C,$ and D, and four triangular faces).

$$\overrightarrow{AB}\cdot(\overrightarrow{AC}\times\overrightarrow{AD})=\begin{vmatrix}0&0&1\\2&-2&1\\0&1&2\end{vmatrix}=2$$

사면체의 부피 $V=\frac{1}{6}$ 평행육면체의 부피

$=\frac{1}{6}\left|\overrightarrow{AB}\cdot(\overrightarrow{AC}\times\overrightarrow{AD})\right|$

$=\frac{1}{3}$

2.(1)(3 points) Find parametric equations for the line l in which the two planes $3x-6y-2z-3=0$ and $2x+y-2z-2=0$ intersect.

Sol.) 교선 위의 한점 $P(1,0,0)$

교선의 방향벡터를 $\vec{v}$ 라 하면

$\vec{v}\perp\langle 3,-6,-2\rangle$

& $\vec{v}\perp\langle 2,1,-2\rangle$

$\Rightarrow \vec{v}\parallel\langle 3,-6,-2\rangle\times\langle 2,1,-2\rangle$

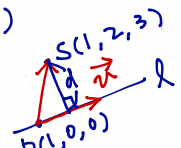
$\therefore \vec{v}=\langle 14,+2,15\rangle$

$\therefore$ 교선 l의 매개방정식

$$\begin{cases}x=1+14t\\y=2t\\z=15t\end{cases},\ t\in\mathbb{R}$$

(2)(3 points) Find the distance from the point S(1,2,3) to the line l.

Sol.)



평행사변형 면적

$*\ |\overrightarrow{PS}\times\vec{v}|=|\vec{v}|\cdot d$

$\overrightarrow{PS}=\langle 0,2,3\rangle$

$\vec{v}=\langle 14,2,15\rangle$

$$d=\frac{|\overrightarrow{PS}\times\vec{v}|}{|\vec{v}|}=\frac{|\langle 24,+42,-28\rangle|}{\sqrt{425}}$$

$$=\frac{2\sqrt{781}}{5\sqrt{17}}$$

3.(7 points) The point $(4, \frac{\pi}{2}, \frac{2}{3}\pi)$ is given in spherical coordinates. Find an equation in spherical coordinates for the largest sphere that passes through the point $(4, \frac{\pi}{2}, \frac{2}{3}\pi)$ and is such that each of the points (ρ, θ, ϕ) inside the sphere satisfies the condition

$$\rho^2 \leq 40 + 2\rho[\sin\phi(\cos\theta + \sqrt{3}\sin\theta) - \cos\phi]$$

Sol.) 서정방의아님!

① $\rho^2 \leq 40 + 2\rho\sin\phi\cos\theta + 2\sqrt{3}\rho\sin\phi\sin\theta - 2\rho\cos\phi$

$\Leftrightarrow x^2+y^2+z^2 \leq 40 + 2x + 2\sqrt{3}y - 2z$

$\Leftrightarrow (x-1)^2+(y-\sqrt{3})^2+(z+1)^2 \leq 45$

② $(4, \frac{\pi}{2}, \frac{2\pi}{3}) \Leftrightarrow (x,y,z) = (0, 2\sqrt{3}, -2)$

③ 두점 $(0, 2\sqrt{3}, -2)$, $(1, \sqrt{3}, -1)$ 을 지나는 직선 $\begin{cases} x = t \\ y = 2\sqrt{3} - \sqrt{3}t \\ z = -2 + t \end{cases}$

$\Rightarrow$ ①의 구와 이 있는 점으로 $(0, 2\sqrt{3}, -2)$ 와 가장 먼 거리에 있는 점 구하기

$(t-1)^2 + (\sqrt{3} - \sqrt{3}t)^2 + (t-1)^2 = 45$

$\Rightarrow t = 4, -2$

$\therefore (4, -2\sqrt{3}, 2)$

④ 구하고자 하는 구의 중심 $(2, 0, 0)$

반지름 $\sqrt{4+12+4} = \sqrt{20}$

⑤ 구의 방정식

$(x-2)^2 + y^2 + z^2 = 20$

$\Leftrightarrow x^2+y^2+z^2-4x = 16$

$\Leftrightarrow \rho^2 - 4\rho\sin\phi\cos\theta = 16$

정답

4.(7 points) Let C be the curve of intersection of the plane $y = \sqrt{3}z$ and the cylinder $x^2 + 4z^2 = 4$.
Find parametric equations for the tangent line to the curve C at the point $(-2, 0, 0)$.

Sol.) $C \Leftrightarrow \vec{r}(t) = \langle 2\cos t, \sqrt{3}\sin t, \sin t \rangle$

$P(-2,0,0) \Leftrightarrow t = \pi$

접선 l의 방향벡터 $\vec{v}$

$\vec{v} // \vec{r}'(\pi) = \langle 0, -\sqrt{3}, -1 \rangle$

$\therefore$ 접선 l의 매개방정식

$\begin{cases} x = -2 \\ y = \sqrt{3}s \\ z = s \end{cases}$, $s \in \mathbb{R}$

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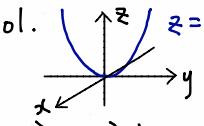
5.(7 points) The parabola $z=4y^2, x=0$ is rotated about the z-axis. Find an equation of the resulting surface in spherical coordinates.

시험범위아님!

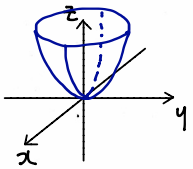
Sol. 

$z=4y^2$: 포물선 (in plane $x=0$)

$\Rightarrow$ z축 회전

원포물곡면 (paraboloid)

$z=4(x^2+y^2)$



$\Rightarrow$ 구면방정식으로 변환하면

$\rho\cos\phi = 4\rho^2\sin^2\phi$

6. Let a surface S be given by $z=f(x,y)$ where $f(x,y)=x(y^2+x^2)^{-\frac{3}{2}}e^{\sin((x-1)^2y)}$.

(1)(3 points) Find a tangent plane of S at (1,0,1).

① $f(x,0)=x^{-2}$, $x>0$

$\Rightarrow f_x(1,0)=-2x^{-1}\Big|_{x=1}=-2$

② $f(1,y)=(y^2+1)^{-\frac{3}{2}}$

$\Rightarrow f_y(1,0)=-\frac{3}{2}(y^2+1)^{-\frac{5}{2}}\cdot 2y\Big|_{y=0}$

$=0$

∴ 접평면의 방정식

$z=-2(x-1)+1$

(2)(4 points) Using the linearization $L(x,y)$ at (1,0), approximate the value of $f(1.01,-0.03)$.

$L(x,y)=-2(x-1)+1$

$f(1.01,-0.03)$

$\approx L(1.01,-0.03)=-0.02+1$

$=0.98$