New Vector Spaces

- All 3x3 matrices : M
  - Does it form a vector space?

    add
    MULTIPLY by SCALAR
    ZERO MATRIX : additive identity

  - Dimension of M

    $\Downarrow$

    R^9

    vectors in 9 dimension.

* B/W image of 1M Pixel Camera

  $R^{1024 \times 1024}$: 1024×1024 dimension

  How about for color image?

  $3 \times 1024 \times 1024$ dimension

* Basis of M

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \cdots \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

9 basis vectors

* Subspaces of M

U : all upper triangular 3x3

S : all symmetric 3x3

- Are they subspaces ?
- What ~~are~~ dimensions ?
- Basis ?

* $S \cap U = D$: diagonal 3×3

Subspace ?
dimension ?
BASIS ?

* $S \cup U$: UNION OF symmetric upper triangular 3×3

Subspace ?
dimension ?
basis ?

* $S + U$ = anything in S + anything in U $\quad 3\times3$

$\overset{?}{=} M \quad 3\times3.$

$\text{DIM}(S+U) = 9$

$= \text{Dim}(S) + \text{Dim}(U) - \text{Dim}(S\cap U)$

$= 6+6-3 = 9.$

Differential Equations

$$\frac{d^2y}{dx^2} + y = 0$$

$y = \cos x$, $y = \sin x$, $y = e^{ix}$

$$y_{complete} = C_1 \cos x + C_2 \sin x$$

$C_1, C_2 \in R^1$

✓ Linear combination of 2 basis functions $\cos x$, ~~cos~~ $\sin x$

✓ Dim (solution space) = 2.

NULL SPACE ?

- $$\frac{d^2y}{dx^2} + y = f(x)$$

$$y_{complete} = f_p(x) + C_1 \cos x + C_2 \sin x$$

- when $f(x) = b$,

$$y_{complete} = b + C_1 \cos x + C_2 \sin x$$

- when $f(x) = ax+b$,

$$y_{complete} = ax+b + C_1 \cos x + C_2 \sin x$$

- when $f(x) = ax^2+bx+c$,

$$y_{complete} = ax^2+bx+c-2a + C_1 \cos x + C_2 \sin x.$$

Example

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \text{ in } R^4$$

S = all v in R^4 with

$$v_1 + v_2 + v_3 + v_4 = 0$$

- Is this subspace?

$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$A = [1, 1, 1, 1]$

$S = N(A)$ — Yes subspace.

- $C(A) = 1$ Dimension $\Rightarrow R^1$

  basis $[1]$

  $\text{Rank}(A) = r = 1.$

- $N(A)$ in R^4

  dimension $= n - r = 4 - 1$
  $= 3$

  <u>BASIS</u>

  $$\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

- Row Space of A $= C(A^T)$ in R^4

  Dimension $= 1$

  BASIS $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

- $N(A^T)$

  $A^T x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

  Dimension $= 0$ (?)

  BASIS $= [0]$