CALCULUS II
2013 Fall Midterm Exam

Dept. or School		Year		proctor	
Student ID		Name			

✱ **Your answer must be provided with descriptions how to get the answer.**

1.(5 point) Let $\left\{\overrightarrow{v_n}\right\}_{n=1}^{\infty}$ be a sequence of vectors generated by the recurrence relation : $\overrightarrow{v_n}$ be the vector projection of $\overrightarrow{v_{n-1}}$ onto $\overrightarrow{v_{n-2}}$ for $n \geq 3$, where two initial $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ are vectors with $\left|\overrightarrow{v_1}\right| = 2$, $\left|\overrightarrow{v_2}\right| = 3$, and $\overrightarrow{v_1} \bullet \overrightarrow{v_2} = 5$.

Compute $\sum_{n=1}^{\infty} \left|\overrightarrow{v_n}\right|$.

2.(5 point) Suppose a surface is generated by rotating the parabola $z = 4y^2, x = 0$ about the z-axis.

Write an equation of the surface in spherical coordinates.

3.(6 point) Let L be the line of intersection of the planes $x+y+z=c$ and $cx+cy+z=c$, where c is a real number, not equal to 1.

(1)(3 point) Find parametric equations for L.

(2)(3 point) Find the point on the line L that is closest to the point $(1,2,3)$.

4.(6 point) Let C be the curve represented by

$$\boldsymbol{r}(t)=\left\langle 2t^3-t^2+2,\ \frac{3}{2}t^2,\ 3t^3-1\right\rangle,\ t<0.$$

Find the curvature at a point P that has normal plane of C parallel to the plane $4x-3y+3z+5=0$.

(Note : The normal plane is parallel to $4x-3y+3z+5=0$.)

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5.(6 point) Let a function f be defined on $\mathbb{R}^2$ by

$$f(x,y)=\begin{cases}\dfrac{xy^2}{\sqrt{x^4+y^2}} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

(1)(3 point) Investigate the continuity of f at (0, 0).

(2)(3 point) Do the partial derivative $D_y f(0,0)=\dfrac{\partial}{\partial y}f(0,0)$ exist? If it exists, find it by **the definition of partial derivative**.

6.(6 point) Let $z=\sin^{-1}(x+y)+\tanh(xy)$

If x changes from 0.6 to 0.61 and y changes from 0 to 0.01, find the value of dz.

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7.(6 point) If $z=f(x+\sqrt{2}t)+g(x-\sqrt{2}t)$, find the value of k satisfying the equation

$$\frac{\partial^2 z}{\partial t^2}=k\frac{\partial^2 z}{\partial x^2}$$