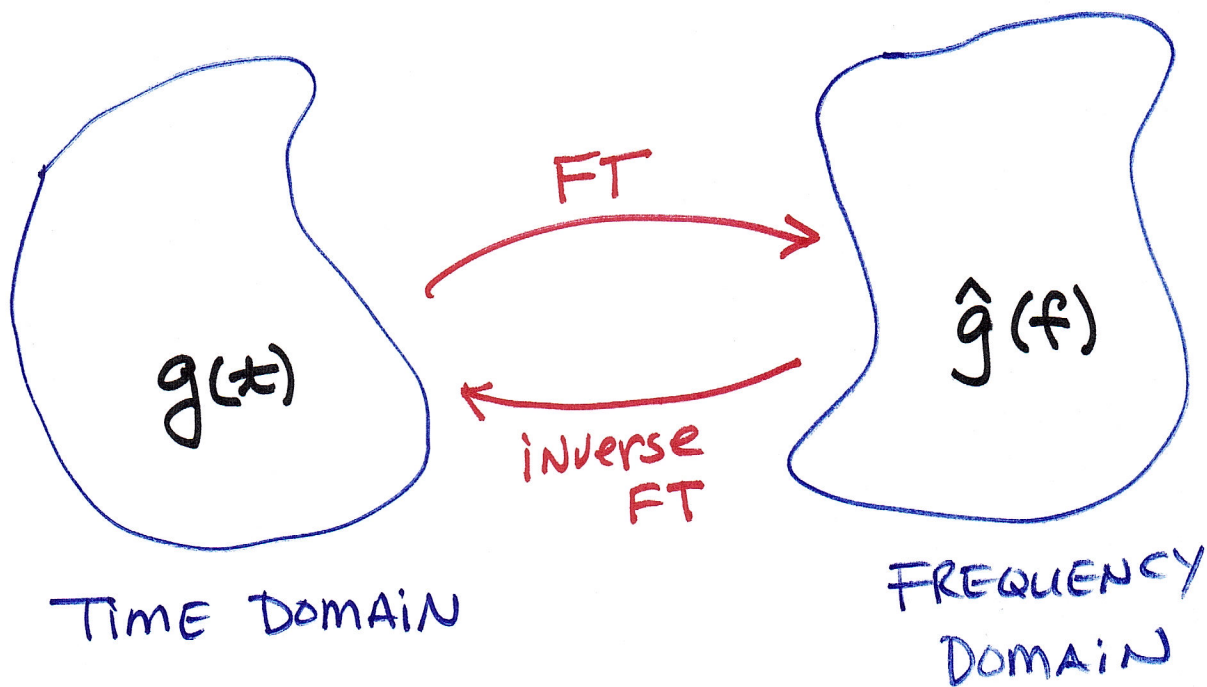


<Continuous>

Fourier Transform



$$\hat{g}(f) = \int_{-\infty}^{\infty} g(t) e^{-2\pi i t f} dt \quad \text{for ANY real } f.$$

t : time in seconds

f : frequency in Hz

$$g(t) = \int_{-\infty}^{\infty} \hat{g}(f) e^{2\pi i f t} df \quad \text{FOR ANY Real } t.$$

①

• Complex vectors

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} \in \mathbb{C}^n$$

* Length

$$\mathbf{z}^T \mathbf{z} = [z_1, \dots, z_n] \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix}$$

NO Good.

X

$$[1 \ i] \begin{bmatrix} 1 \\ i \end{bmatrix} = 1 + i^2 = 0$$

$$|\mathbf{z}|^2 = \overline{\mathbf{z}}^T \mathbf{z} = [\bar{z}_1, \dots, \bar{z}_n] \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix}$$

○

$$\Downarrow$$

$$\mathbf{z}^H \mathbf{z}$$

$$= |z_1|^2 + |z_2|^2 + \dots + |z_n|^2$$

$$\left| \begin{bmatrix} 1 \\ i \end{bmatrix} \right|^2 = [1 \ -i] \begin{bmatrix} 1 \\ i \end{bmatrix} = 2$$

(2)

* Inner Product

$$y^H x = \bar{y}^T x = \bar{y}_1 x_1 + \bar{y}_2 x_2 + \dots + \bar{y}_n x_n$$

Complex MATRICES

Symmetric $A^T \neq A$ No good

$$A^H = \bar{A}^T = \begin{bmatrix} 2 & 3+i \\ 3-i & 5 \end{bmatrix}$$

* Real Eigenvalues

* Perpendicular q_1, q_2, \dots, q_n

$$\bar{q}_i^T q_j = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

$$\parallel$$

$$q_i^H q_j$$

$$Q^H Q = I$$

orthonormal \equiv \uparrow unitary.

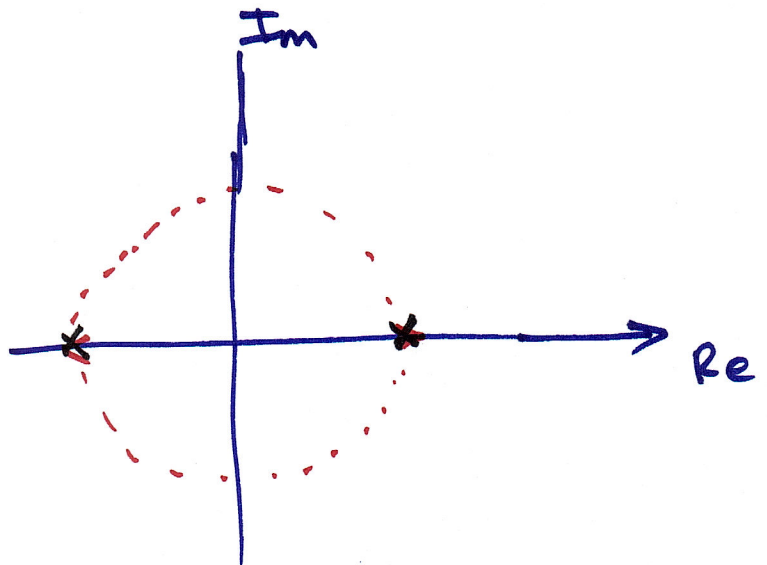
< COMPLEX NUMBER >

③

$$\bullet e^{i \cdot 2\pi \cdot k/N} = \cos(2\pi \cdot \frac{k}{N}) + i \sin(2\pi \frac{k}{N})$$
$$= W_N^k$$

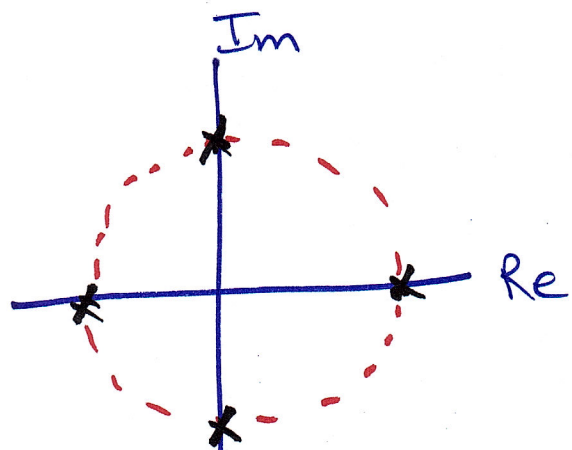
• For $N=2$

$$W_2^0, W_2^1, W_2^2, \dots$$



• For $N=4$

$$W_4^0, W_4^1, W_4^2, \dots$$



• For $N=8$

$$W_8^0, W_8^1, \dots$$

