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※ **Your answer must be provided with descriptions how to get the answer.**

1. (7 points) Find the volume of the solid obtained by rotating the region bounded by $y = 9x\ln x$, $x = e$, and $y = 0$ about the $y-$axis.

2. A curve is defined by the parametric equations

$$x = (1-\cos\theta)\cos\theta, \quad y = (1-\cos\theta)\sin\theta.$$

(a) (5 points) Find the total length of the curve.

(b) (6 points) Find the average value of the function $f(t) = \int_0^t \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$ on the interval $[0, \pi]$.

3. (a) (6 points) Find a Cartesian equation for the tangent line to the polar curve $r = \cos 3\theta$ when $\theta = \frac{\pi}{3}$.

(b) (6 points) Find the area of the region that lies inside both curves $r = \sin 3\theta$ and $r = \cos 3\theta$.

4. (a) (5 points) Determine whether the series $\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n+n^7}}$ converges or diverges.

(b) (7 points) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ is convergent. If it is convergent, find the sum of the series.

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5. Consider power series $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2} x^n$.

(a) (7 points) Find the radius of convergence of the power series

(b) (5 points) For what values of x does the series converges absolutely?

6. Let $f(x) = \sin^2 x \cos^2 x$.

(a) (7 points) Find the Maclaurin series for $f(x)$ and also its convergence interval.

(b) (7 points) Find the sum of series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(4\pi)^{2n+1}}{(2n+1)!}$.

7. (7 points) Estimate $\sin(0.5)$ within 0.0001 (do not evaluate).

8. (5 points) Evaluate of determinants by reduction to triangular form.

$$\begin{vmatrix} 2 & 0 & -2 & 3 \\ 4 & 5 & 4 & 0 \\ 0 & 5 & 6 & -1 \\ 0 & 5 & 2 & 1 \end{vmatrix}.$$