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※ **Your answer must be provided with descriptions how to get the answer.**

1. Let f be a function of 2 variables that has continuous partial derivatives and let $f(5,3)=4$. The rate of change of f at $A(5,3)$ in the direction $i+j$ is $4\sqrt{2}$ and the rate of change at A in the direction $-3i+4j$ is -2.

(a) (4 points) In what direction does f have the maximum rate of change at A? What is this maximum rate of change?

(b) (6 points) If $g(s,t,u)=sf(t+2u, tu)$, find the maximum rate of change at $(2,3,1)$ and the direction in which it occurs.

2. (6 points) Find the angle of intersection between the sphere $x^2+y^2+z^2=6$ and the paraboloid $z=x^2+y^2$ at the point $(1,1,2)$.

3. (6 points) Find and classify the critical points of the function
$$f(x,y)=xy(x+y-3).$$

4. (a) (6 points) Evaluate $\int_{-1}^{0}\int_{\cos^{-1}y}^{\pi} e^{\sin x}\,dx\,dy$.

(b) (6 points) The cardioid $r=1+2\cos\theta$ has one inner loop. Find the area of the inner loop using a double integral.

5. (6 points) Find the volume of the solid enclosed by the paraboloids $z=2x^2+y^2$ and $z=8-x^2-2y^2$, inside the cylinder $x^2+y^2=1$.

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6. (6 points) Find the area of the surface with parametric equations

$$x = 3\cos\theta + \cos\phi\cos\theta,\ y = 3\sin\theta + \cos\phi\sin\theta,\ z = \sin\phi$$

$$(0 \le \phi \le 2\pi,\ 0 \le \theta \le 2\pi).$$

7. (6 points) Evaluate $\int_C [3x^2y^2 + 2\cos(2x+y)]\,dx + [2x^3y + \cos(2x+y)]\,dy$ where C is the upper half of the circle $x^2+y^2=1$ from $(1, 0)$ to $(-1, 0)$.

8. Let C consist of the line segment C_1 from $(0,0)$ to $(\sqrt{6}, \sqrt{3}-1)$, the arc C_2 of the curve $x^2y-y^3=4$ from $(\sqrt{6}, \sqrt{3}-1)$ to $(\sqrt{6},2)$.

(a) (4 points) Find a gradient vector field ∇f where $f(x,y)=x^2y-y^3-4$.

(b) (6 points) Evaluate $\int_{C=C_1 \cup C_2} \nabla f \cdot dr$.

(c) (6 points) Evaluate

$$\int_{C_2} \frac{2xy}{\sqrt{(2xy)^2+(x^2-3y^2)^2}}\,dx + \frac{x^2-3y^2}{\sqrt{(2xy)^2+(x^2-3y^2)^2}}\,dy.$$

9. Let C be a plane curve consisting the upper half of circle $x^2+y^2=8$ from $(2\sqrt{2},0)$ to $(-2\sqrt{2},0)$ and the upper half of ellipse $\frac{x^2}{8}+\frac{y^2}{2}=1$ from $(-2\sqrt{2},0)$ to $(2\sqrt{2},0)$.

(a) (6 points) Use a line integral to find the area enclosed by the given curve C.

(b) (6 points) Let a vector field F be given by $F(x,y)=<e^{-y}, 5x-xe^{-y}>$ and a plane curve C be given in (a).
Then use Green's theorem to calculate a line integral $\int_C F \cdot dr$.