CALCULUS II 2014 Fall Final Exam	Dept. or School		Year		proctor	
	Student ID		Name			

※ **Your answer must be provided with descriptions how to get the answer.**

1. Let $f(x,y) = e^{2x-x^2-2y^2}$ and $g(x,y) = |\nabla f(x,y)|$.

(1)(4 points) Find the maximum value of $g(x,y)$.

(2)(2 points) At which point(s) (x,y) and in the direction of which unit vector(s) $\hat{u}$ does the maximum value for the directional derivative $D_{\hat{u}} f(x,y)$ occur?

2.(4 points) Let $f(x,y) = e^{2x-x^2-2y^2}$ and

$$R = \{(x,y) \mid -2 \leq x \leq 0,\ -2 \leq y \leq 0\}.$$

By dividing R into four equal squares, calculate the smallest and largest Riemann sums for approximating $\iint_R f(x,y)\,dA$.

3. Evaluate the following iterated integrals.

(1)(3 points) $\int_{-1}^{1}\int_{-1}^{x}(1+x\sin^2 y+y^3\cos^4 x)\,dydx$

$+\int_{-1}^{1}\int_{-1}^{y}(2+x\sin^2 y+y^3\cos^4 x)\,dxdy$

(2)(3 points) $\int_{0}^{\infty}\int_{0}^{\infty}(x^2+y^2)\,e^{-(x^2+y^2)^2}\,dy\,dx$

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4.(6 points) Let $F(x,y,z)=2zi+yj-xk$ and a curve C be obtained by the part of the intersection of the surfaces $z=x^2$ and $x^2+y^2=4$ with non-negative y-coordinate and non-positive x-coordinate and oriented so that z-coordinate increases as one travel along C.

Evaluate $\int_C F \cdot dr$.

5. Let a vector field

$$F(x, y) = < \cos x + \frac{y^2}{1+x^2} \,,\, \frac{2y}{1+y^2} + 2y\tan^{-1}x >$$

and a curve $C: r(t) = < t^2, 2t >, 0 \leqq t \leqq 1$ be given.

(1)(3 points) Find a potential function f such that $\nabla f = F$.

(2)(3 points) Evaluate a line integral $\int_C F \cdot dr$.

6.(6 points) Find the area of the part of the surface $f(x,y) = 2y + x^2$ that lies above the region $R = \{(x,y) \mid y \leq x \leq 1,\ 0 \leq y \leq 1\}$.

7.(6 points) Verify that Green's Theorem is true for the line integral $\int_C xy^2dx - x^2y\,dy$, where C consists of the parabola $y = x^2$ from $(-1,1)$ to $(1,1)$ and the line segment from $(1,1)$ to $(-1,1)$.