Image Compression Based on Singualr Value Decomposition

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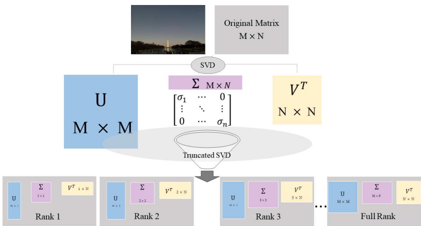
Introduction

Nowadays, images are stored and transmitted in and through digital devices such as computers, laptops, or smartphones. As digital imaging technology is growing continuously, high-resolution images with high visual quality are more commonly produced and utilized in the corresponding fields. [1] Since these high-resolution images require a large amount of storage space due to their big data size, it has become necessary to resolve the problems in data storage and transmission caused by the big data size of the images.[2] The image compression schemes are normally divided into two big category: the lossless compression and the lossy compression.The lossless compression doesn't change the data of the original image which means there is no degradation in image quality, but has less compression ratio than the lossy compression. As a result, there is a trade-off between the level of compression and the quality of the image after reconstruction. [3] Since it is essential to maintain rigid similarity of the compressed image with the original image, to reduce the storage that the image takes, and to lesson the time it takes to transmit the image data in the medical field, the development in specialized-medical lossy compression technique is in need. This paper focuses on the lossy compression using SVD. The objective of this paper is to propose an optimal rank for each medical images.

Methods

1. Singular Value Decomposition



2. Image Compression

Since every medical image is stored and transmitted through digital devices, the image matrix represents the image that's displayed on the device. Every digital image has its own pixel matrix that has a value assigned from 0 to 255 for each pixel according to the characteristic of the image. Thus, with the fact that images are stored in the form of matrices in the digital devices, we can apply Singular Value Decomposition to compress the medical image. Once we extract the target image in matrix form, the image pixel matrix A can be factorized into three matrices U, Σ, V so that the number of the singular values, or the rank, could be controlled by the experimenter. The truncated SVD works as follows:

Let k be the rank selected to be tested. Then, Σ becomes $k \times k$ matrix, U becomes $m \times k$ matrix, and V becomes $n \times k$ matrix. Then, the size of the truncated image becomes the sum of these three matrices.

$$\text{size of the rank k truncation image} = k(n+1+m) \qquad (1)$$

3. Applications : Medical Images

In this paper, 4 MRI images and 4 X-ray images have been used for the image compression. The images that were not originally square-sized are zero-padded for equivalent condition. We use these zero-padded medical images to apply the Singular Value Decomposition as described in the diagram above. Afterwards, we execute the low rank approximation and reconstruct the truncated image.

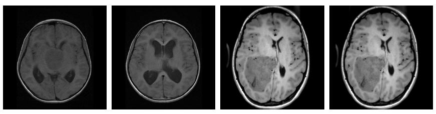


Fig. 1: These are 4 MRI images used in this paper

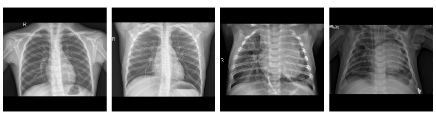


Fig. 2: These are 4 X-ray images used in this paper

Results

The optimal rank for medical image compression based on SVD has been chosen based on SSIM in this paper. Two thresholds are set: SSIM > 0.95 and SSIM > 0.99.

1 MRI Images

For the optimization based on SSIM > 0.95, corresponding PSNR values ranged from 37 dB to 40 dB and it has tendency to increase as the original rank is higher. The ratio $\frac{r_{opt}}{r_{ori}}$ show that about 11% of the singular values are used in optimization for 4 MRI images.

	r_{ori}	r_{opt}	PSNR	$\frac{r_{opt}}{r_{ori}}$
1-(a)	473	55	37.9593872818052	0.1162790698
1-(b)	473	59	38.561411231756	0.1247357294
1-(c)	511	60	39.7522287499315	0.1174168297
1-(d)	512	60	39.6538543950581	0.1171875

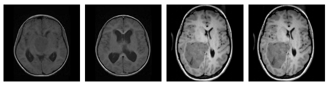


Fig. 4: These are 4 MRI images optimized with SSIM > 0.95

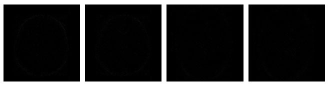


Fig. 5: These are MRI images' difference images

For the optimization based on SSIM > 0.99, corresponding PSNR values ranged from 46 dB to 48 dB and it has tendency to increase as the original rank is higher. The ratio $\frac{r_{opt}}{r_{ori}}$ show that about 20% of the singular values are used in optimization for 4 MRI images.

	r_{ori}	r_{opt}	PSNR	$\frac{r_{opt}}{r_{ori}}$
1-(a)	473	100	47.7405542704281	0.2114164905
1-(b)	473	103	47.5215627756651	0.2177589852
1-(c)	511	91	47.0891781543897	0.1780821918
1-(d)	512	90	46.9322872134835	0.17578125

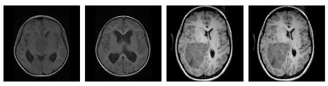


Fig. 7: These are 4 MRI images optimized with SSIM > 0.99

Fig. 8: These are MRI images' difference images

2 X-ray Images

For the optimization based on SSIM > 0.95, corresponding PSNR values ranged from 40 dB to 42 dB and it has tendency to increase as the original rank is higher. The ratio $\frac{r_{opt}}{r_{ori}}$ show that about 10% to 15% of the singular values are used in optimization for 4 X-ray images.

	r_{ori}	r_{opt}	PSNR	$\frac{r_{opt}}{r_{ori}}$
2-(a)	1317	208	41.1560756946181	0.1579347
2-(b)	1509	242	41.332631388226	0.160371107
2-(c)	762	78	40.641793985265	0.102362205
2-(d)	680	67	40.1265089685621	0.098529412

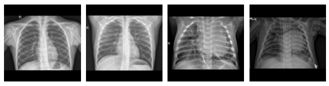


Fig. 10: These are 4 X-ray images optimized with SSIM > 0.95

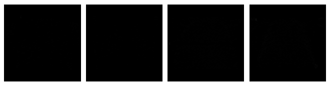


Fig. 11: These are X-ray images' difference images

For the optimization based on SSIM > 0.99, corresponding PSNR values are around 48 dB and it has tendency to increase as the original rank is higher. The ratio $\frac{r_{opt}}{r_{ori}}$ show that about 30% to 37% of the singular values are used in optimization for 4 X-ray images.

	r_{ori}	r_{opt}	PSNR	$\frac{r_{opt}}{r_{ori}}$
2-(a)	1317	208	48.2505290660966	0.374335611
2-(b)	1509	242	48.383441712362	0.378396289
2-(c)	762	78	48.1003301244956	0.284776903
2-(d)	680	67	48.4582524720168	0.292647059

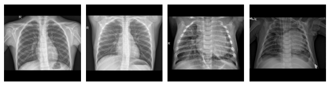


Fig. 13: These are 4 X-ray images optimized with SSIM > 0.99

Fig. 14: These are X-ray images' difference images

3 Comparison: MRI vs X-ray

From the results above, it is shown that the overall ratio $\frac{r_{opt}}{r_{ori}}$ is higher in X-ray. It is interpreted that MRI stores more significant information on the first few singular values σ than X-ray. The PSNR values ranged higher about 5 dB than MRI in SSIM > 0.95 compression and about 1 dB than MRI in SSIM > 0.99 compression. As a result, there are more perceptible differences in MRI images than X-ray images.

Conclusions

For MRI images, the optimal rank of truncation in SVD image compression would be 10% to 20% of the original rank. And for X-ray images, the optimal rank of truncation in SVD image compression would be 10% to 40% of the original rank. From the results above, we could observe that image compression using Singular value Decomposition worked more effectively in X-ray images for SSIM > 0.95 truncation. And it is analyzed that the image compression based on SVD worked more effectively in MRI images for SSIM > 0.99 truncation. This paper also proposes the significance of the number of the singular values in SVD providing valid metrics.

References

References

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