
Linear Algebra

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Summary So Far

- Eigenvalue and eigenvectors

- Null space, column space, and orthogonal complement in \mathbb{R}^n

- Diagonalization and eigendecomposition

- Linear transformation via eigendecomposition

$$Ax = \lambda x \rightarrow \det(A - \lambda I)x = 0$$

차원 합

$$\begin{cases} \text{Nul } A \subset \mathbb{R}^2 \\ \text{Row } A \subset \mathbb{R}^2 \end{cases}$$

②
 \hookrightarrow Nul A 가 차원 1개면
 Row A 도 차원 1개,
 Row A 가 차원 2개면
 Nul A 는 0개

$$A \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A^T \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Col A 가 차원 2개면
 Nul A 는 차원 1개

$$\begin{cases} \text{Nul } A^T \subset \mathbb{R}^3 \\ \text{Col } A^T \subset \mathbb{R}^3 \end{cases}$$

$$D = V^{-1}AV \Leftrightarrow A = VDV^{-1}$$

동치관계

$$A = VDV^{-1}x$$



Linear Transformation via A^k

- Now, consider recursive transformation $A \times A \times \cdots \times A \mathbf{x} = A^k \mathbf{x}$.
- If A is diagonalizable, A has eigendecomposition

$$A = VDV^{-1}$$

- $A^k = (VDV^{-1})(VDV^{-1}) \cdots (VDV^{-1}) = VD^kV^{-1}$
- D^k is simply computed as

$$D^k = \begin{bmatrix} \lambda_1^k & 0 & \cdots & 0 \\ 0 & \lambda_2^k & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n^k \end{bmatrix}$$