DSP HW2

1. Determine the impulse response of the system described by

$$y[n] - \frac{5}{2}y[n-1] + y[n-2] = x[n-1]$$

for all possible regions of convergence.

2. Given an causal system described by $y[n] = \frac{1}{2}y[n-1] + x[n]$, compute its response to the following inputs:

(a) $x[n] = e^{j(\pi/4)n}$, $-\infty < n < \infty$

(b) $x[n] = e^{j(\pi/4)n}u[n]$,

(c) $x[n] = (-1)^n$, $-\infty < n < \infty$

(d) $x[n] = (-1)^n u[n]$.

3. Consider a causal system with input $x[n]$ and output $y[n]$. If the input is given by

$$x[n] = -(1/3)(1/2)^n u[n] - (4/3)2^n u[-n-1],$$

the output has a z-transform given by

$$Y(z) = \frac{1 - z^{-2}}{\left(1 - \frac{1}{2}z^{-1}\right)(1 - 2z^{-1})}.$$

(a) Determine the z-transform of the input $x[n]$.

(b) Find all possible choices for the impulse response of the system.

4. Determine zero-input, zero-state, transient, and steady-state responses of the system

$$y[n] = \frac{1}{4}y[n-1] + x[n] + 3x[n-1], \ n \geq 0$$

to the input $x[n] = e^{j\pi n/4}u[n]$ with $y[-1] = 2$.

5. Let $x_1[n]$ and $x_2[n]$ be periodic sequences which fundamental periods N_1 and N_2, respectively. Under what conditions is the sum $x[n] = x_1[n] + x_2[n]$ periodic, and what is its fundamental period N if it is periodic?

6. Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period:
(a) $x_1(t) = 2\cos(3\pi t) + 3\sin(4t)$
(b) $x_2[n] = 4\cos(0.1\pi n)$
(c) $x_3(t) = 3\sin(3000\pi t) + 5\sin(2000\pi t)$
(d) $x_4[n] = 2\cos(n/11) + 5\sin(n/31)$
(e) $x_5[n] = [\cos(\pi n/5) + 2\sin(\pi n/6)]\sin(\pi n/2)$

7. Determine and plot the magnitude and phase spectra of the following periodic sequences:
(a) $x_1[n] = 4\cos(1.2\pi n + 60°) + 6\sin(0.4\pi n - 30°)$
(b) $x_3[n] = \{1,1,0,1,1,1,0,1\}$, (one period)
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8. Given that $x[n]$ is a periodic sequence with fundamental period N and Fourier coefficients a_k, determine the Fourier coefficients of the following sequences:
(a) $x[n-n_0]$
(b) $x[n] - x[n-1]$
(c) $(-1)^n x[n]$ (N even)

(d) $(-1)^n x[n]$ (N odd)

9. Let $h[n]$ and $x[n]$ be periodic sequences with fundamental period N and Fourier coefficients a_k and b_k , respectively.

(a) Show that the Fourier coefficients c_k of $y[n]=h[n]x[n]$ are given by

$$c_k = \sum_{l=0}^{N-1} a_l b_{k-l} = \sum_{l=0}^{N-1} b_l a_{k-l}$$

(b) Verify the result in (a) using the periodic sequences ($N=8$)
$$h[n] = \sin(3\pi n/4) \quad \text{and} \quad x[n] = \{1,1,1,1,0,0,0,0\}$$