Decoding and performance of LBC

- **Soft-Decision Decoding.** In Chapters 8 and 9, we have seen that the optimum signal-detection scheme on an additive white Gaussian noise channel is detection based on minimizing the Euclidean distance between the received signal and the transmitted signal. This means that after receiving the output of the channel and passing it through the matched filters, we choose one of the message signals that is closest to the received signal in the Euclidean distance sense.

From Equations (13.2.22) and (13.2.23), we conclude that

(Same as arguments in section 13.1)

$$P_M \leq (M-1)Q\left(\sqrt{\frac{2d_{\min}R_c\mathscr{E}_b}{N_0}}\right). \tag{13.2.24}$$

Equations (13.2.22) and (13.2.24) are bounds on the code word error probability of a coded communication system when optimal demodulation is employed. By optimal demodulation, we mean passing the received signal $r(t)$ through a bank of matched filters to obtain the received vector $\boldsymbol{y}$, and then finding the closest point in the constellation to $\boldsymbol{y}$ in the Euclidean distance sense. This type of decoding that involves finding the minimum Euclidean distance is called *soft-decision decoding*, and requires real number computation.

Example 13.2.5, and 13.2.4

Example 13.2.5

Compare the performance of an uncoded data transmission system with the performance of a coded system using the (7, 4) Hamming code given in Example 13.2.4 when applied to the transmission of a binary source with the rate $R = 10^4$ bits/sec. The channel is assumed to be an additive white Gaussian noise channel, the received power is 1 microwatt and the noise power spectral density is $\frac{N_0}{2} = 10^{-11}$. The modulation scheme for the elements of any code word is binary PSK.

Example 13.2.4

Find the parity-check matrix and the generator matrix of a (7, 4) Hamming code in the systematic form.

$$\boldsymbol{H} = \left[\begin{array}{cccc|ccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]; \qquad \boldsymbol{G} = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right];$$

Solution to Example 13.2.5

- **Solution**

1. If no coding is employed, we have

$$P_2 = Q\left(\sqrt{\frac{2\mathscr{E}_b}{N_0}}\right) = Q\left(\sqrt{\frac{2P}{RN_0}}\right). \qquad (13.2.25)$$

But $\frac{2P}{RN_0} = \frac{10^{-6}}{10^4 \times 10^{-11}} = 10$; therefore,

$$P_2 = Q(\sqrt{10}) = Q(3.16) \approx 7.86 \times 10^{-4}. \qquad (13.2.26)$$

The error probability for four bits will be

$$P_{\text{Error in 4 bits}} = 1 - (1 - p_b)^4 \approx 3.1 \times 10^{-3}. \qquad (13.2.27)$$

Solution to Example 13.2.5

$$P_{M_i} \le M\,Q\left(\sqrt{\frac{4d^{\mathrm{H}}_{\min}\mathscr{E}}{2N_0}}\right)$$

2. If coding is employed, we have $d_{\min} = 3$ and

$$\frac{\mathscr{E}}{N_0} = R_c\frac{\mathscr{E}_b}{N_0} = R_c\frac{P}{RN_0} = \frac{4}{7} \times 5 = \frac{20}{7}.$$

Therefore, the *message* error probability is given by

$$P_M \le (M-1)Q\left(\sqrt{\frac{2d_{\min}\mathscr{E}}{N_0}}\right)$$

$$= 15Q\left(\sqrt{3 \times \frac{40}{7}}\right)$$

$$= 15Q(4.14) \approx 2.6 \times 10^{-4}.$$

We see that using this simple code decreases the error probability by a factor of 12. Of course, the price that has been paid is an increase in the bandwidth required for the transmission of the messages. This bandwidth expansion ratio is given by

$$\frac{W_{\text{coded}}}{W_{\text{uncoded}}} = \frac{1}{R_c} = \frac{7}{4} = 1.75. \qquad \blacksquare$$