Digital SIgnal Processing Homework 1.
Due date: 10th Oct. before class

1. Compute and plot the response of the following systems :

$$y[n] = \frac{n}{n+1} y[n-1] + x[n], \qquad y[-1] = 0$$

$$y[n] = 0.9\, y[n-1] + x[n], \qquad y[-1] = 0$$

to the inputs $x[n] = \delta[n]$ and $x[n] = \delta[n-5]$, for $0 \le n \le 20$, and comment upon the obtained results.

2. A 5-point moving average filter computes a simple average over five input samples at each n.
(a) Determine the difference equation for this filter.
(b) Determine and plot the impulse response $h[n]$.
(c) Draw the system block diagram.

3. The input $x[n] = \{1, \underset{\uparrow}{3}, 2, -1\}$ is applied to the LTI system described by the impulse response $h[n] = 2(0.8)^n,\ 0 \le n \le 6$.
(a) Using the convolution as a superposition of scaled ands hifted replicas, determine $y[3]$.
(b) Illustrate the above calculation graphically.

4. Test which of the following systems are linear, time-invariant, causal, and stable.
(a) $y[n] = x[-n]$ (Time-flip)
(b) $y[n] = \log(|x[n]|)$ (Log-magnitude)
(c) $y[n] = x[n] - x[n-1]$ (First-difference)
(d) $y[n] = \text{round}\ \{x[n]\}$ (Quantizer)

5. Consider the following discrete-time system

$$y[n] = 10x[n]\cos(0.25\pi n + \theta)$$

where θ is a constant. Check if the system is

(a) Linear.

(b) Time invariant.

(c) Causal.

(d) Stable.

6. Compute the step response of a system with impulse response $h[n] = b a^n u[n]$. Choose b so that $s[n]$ approaches the level of $u[n]$ for large values of n.

7. Determine, with justification, whether each of the following statements is true of false regarding discrete-time LTI systems.

(a) A system is causal if the step response $s[n]$ is zero for $n < 0$.

(b) If the impulse response $h[n] \neq 0$ is periodic, then the output is always periodic.

(c) A cascade connection of a stable and an unstable system is always unstable.

(d) The inverse of a causal system is a noncausal system.

(e) A system with infinite-duration impulse response is unstable.

(f) If $|h[n]|$ is finite at each n, then the system is stable.

8. Determine the z-transform and sketch the pole-zero plot with the ROC for each of the following sequences:

(a) $x[n] = \left(\frac{1}{2}\right)^n (u[n] - u[n-10])$,

(b) $x[n] = \left(\frac{1}{2}\right)^{|n|}$,

(c) $x[n] = 5^{|n|}$,

(d) $x[n] = \left(\frac{1}{2}\right)^n \cos(\pi n/3) u[n]$.

9. Use the method of partial fraction expansion to determine the sequences corresponding to the following z-transforms :

(a) $X(z) = \dfrac{1 - \frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})}$, all possible ROCs.

(b) $X(z) = \dfrac{1-z^{-1}}{1-\frac{1}{4}z^{-1}}$, $x[n]$ is causal.

(c) $X(z) = \dfrac{1}{(1-0.5z^{-1})(1-0.25z^{-1})}$, $x[n]$ is absolutely summable.

10. Compute $y[n] = h[n] * x[n]$ for $h[n] = a^n u[n]$ and $x[n] = u[-n-1]$ using z-transform.