Communications Lab. Fall 2017

Phase Shift Keying (PSK)

Prof. Hyunggon Park Lecture Note

1 Key Terms

- phase, BPSK, QPSK
- correlator, matched filter, phase locked loop (PLL)

2 Phase Shift Keying (PSK) - Modulation

- Phase:
  - Offset in the displacement from a specified reference point at time $t = 0$
  - For a sinusoidal signal,

$$x(t) = A \cdot \sin(2\pi f t + \theta)$$

  - Example: phase shift between cosine and sine functions is $\pi/2$ because

$$\cos(2\pi f t) = \sin(2\pi f t + \pi/2)$$

- Terminology: in-phase and quadrature-phase
  - Consider two sinusoidal signals:

$$A(t)\sin(2\pi ft + \phi(t)) = I(t)\sin(2\pi ft) + Q(t)\sin\left(2\pi ft + \frac{\pi}{2}\right)$$
$$A(t)\cos(2\pi ft + \phi(t)) = I(t)\cos(2\pi ft) + Q(t)\cos\left(2\pi ft + \frac{\pi}{2}\right)$$

    where

$$I(t) = A(t)\cos(\phi(t)), \quad Q(t) = A(t)\sin(\phi(t))$$

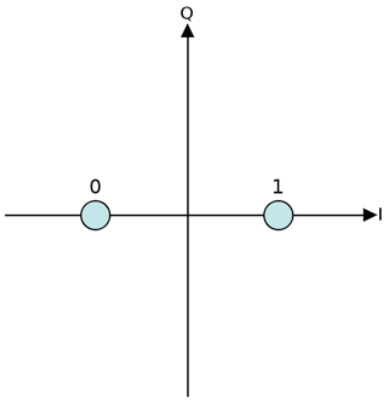
  - Same phase with carrier: in-phase component, $I(t)$
  - $2\pi/4$ out of phase from carrier: quadrature-phase component: $Q(t)$

- PSK - Phase Shift Keying sends digital data
  - Changing phase of a carrier frequency
  - Selecting a finite set of phases

$$s_b(t) = A\cos(2\pi ft + \phi(t))$$

3 BPSK - Binary PSK

- Concept:
  - Use two phases
  - Q: What would be the best phase selection?
  - A:
  - Q: Why?
  - A:
  - Constellation:



- Expression:

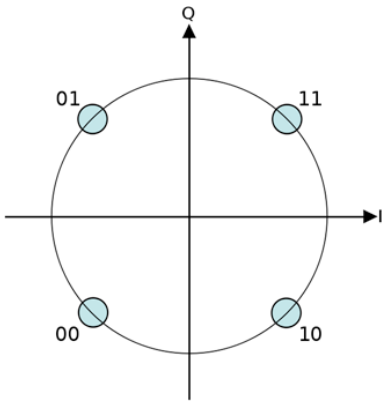
$$s_i(t) = A\cos(2\pi ft + \pi(1-i)),\ i = 0, 1$$
$$s_0(t) = A\cos(2\pi ft + \pi) = -A\cos(2\pi ft)$$
$$s_1(t) = A\cos(2\pi ft)$$

- Basis function:

$$\phi(t) = A\cos(2\pi ft)$$

4 QPSK - Quadrature PSK

- Concept:
  - Use four phases
  - Q: What would be the best phase selection?
  - A:
  - Q: Why?
  - A:
  - Constellation:



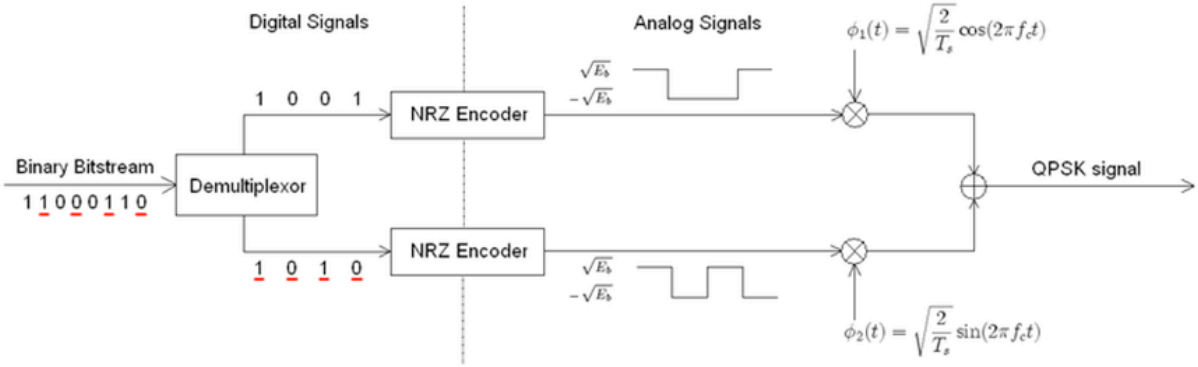
- Expression:

$$s_i(t) = A\cos\left(2\pi ft + \frac{\pi}{4}(2i-1)\right),\ i = 1, 2, 3, 4.$$

- Basis functions:

$$\phi_1(t) = A\cos(2\pi ft),\ \phi_2(t) = A\sin(2\pi ft)$$

- QPSK considered as two independent BPSKs



5 Phase Shift Keying (PSK) - Demodulation

- Orthogonality (Review):
  - Two functions f and g are orthogonal if their inner product is zero for $f \neq g$.

$$\langle f, g \rangle = \int_a^b f(x)g(x)dx = 0$$

  - Example: $\cos\theta$ and $\sin\theta$ are orthogonal

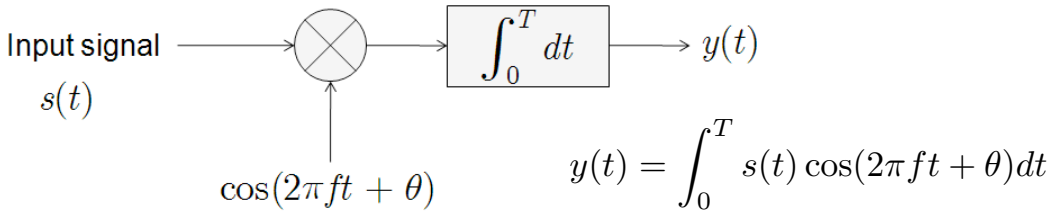
$$\begin{aligned}\langle \cos\theta, \sin\theta \rangle &= \int_0^{2\pi} \cos\theta \sin\theta d\theta \\ &= \int_0^{2\pi} \frac{1}{2}\sin 2\theta d\theta = 0\end{aligned}$$

  - Example:

$$\begin{aligned}\langle \sin\theta, \sin\theta \rangle &= \int_0^{2\pi} \sin\theta \sin\theta d\theta \\ &= -\frac{1}{2}\int_0^{2\pi} \{\cos 2\theta - \cos 0\} = \pi \neq 0\end{aligned}$$

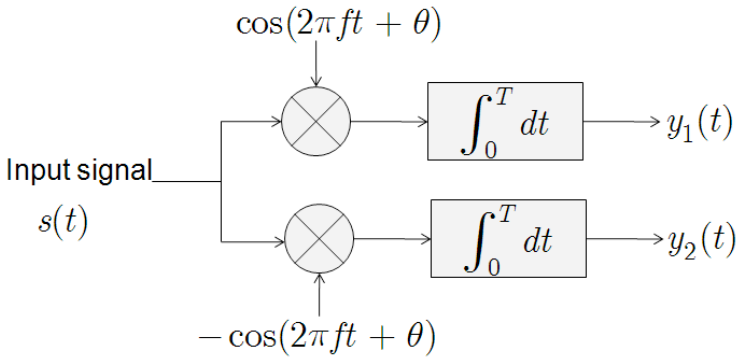
$$\begin{aligned}\langle \cos\theta, \cos\theta \rangle &= \int_0^{2\pi} \cos\theta \cos\theta d\theta \\ &= \frac{1}{2}\int_0^{2\pi} \{\cos 2\theta + \cos 0\} = \pi \neq 0\end{aligned}$$

- Correlator



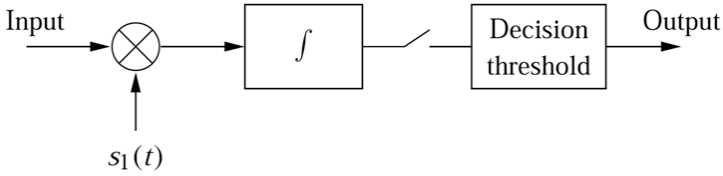
  - If $s(t) = \cos(2\pi ft), \theta = 0 \rightarrow y(t) > 0$
  - If $s(t) = \sin(2\pi ft), \theta = 0 \rightarrow y(t) = 0$

- BPSK Decoding

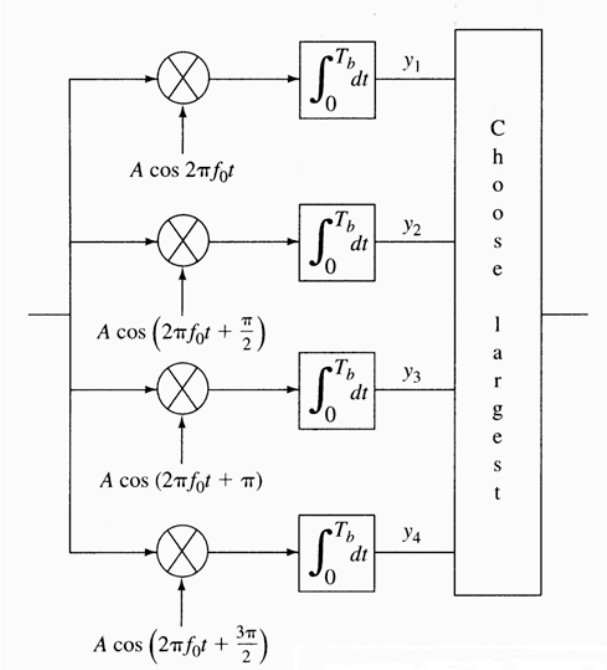


  - Input signals:
$$s_0(t) = A\cos(2\pi ft + \pi),\ s_1(t) = A\cos(2\pi ft),$$
  - Decision rule:
$$\hat{y}(t) = \max(y_1(t), y_2(t))$$

  - Simple BPSK Decoding Block Diagram

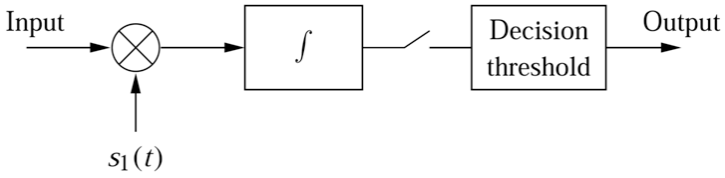


- QPSK Decoding: Simple extension of BPSK decoding

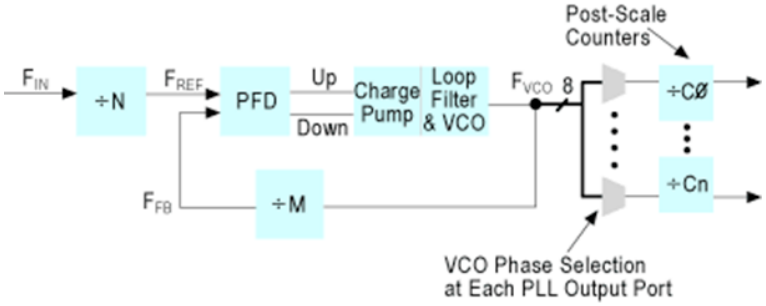


  - Input signals:
$$s_0(t) = A\cos(2\pi f t + \pi),\ \ s_1(t) = A\cos(2\pi f t),$$
  - Decision rule:
$$\hat{y}(t) = \max(y_1(t), y_2(t))$$

  - Simple BPSK Decoding Block Diagram



- PLL - Phase Locked Loop
  - Generate output signal whose phase is related to phase of the input "reference" signal.



  - PFD (phase frequency detector): detects difference in phase and frequency of two input signals