FM - Demodulation

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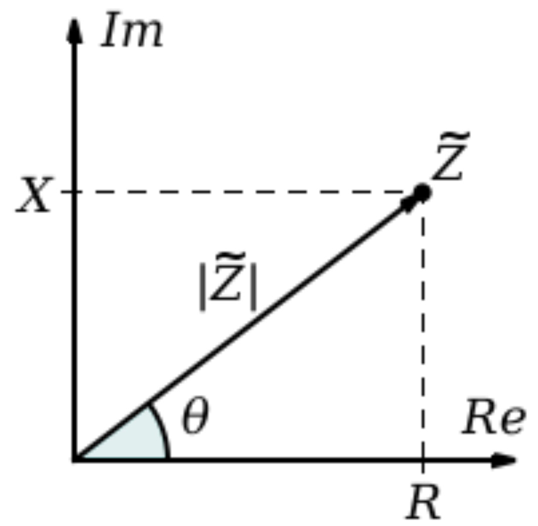
Recall: FM Demodulation

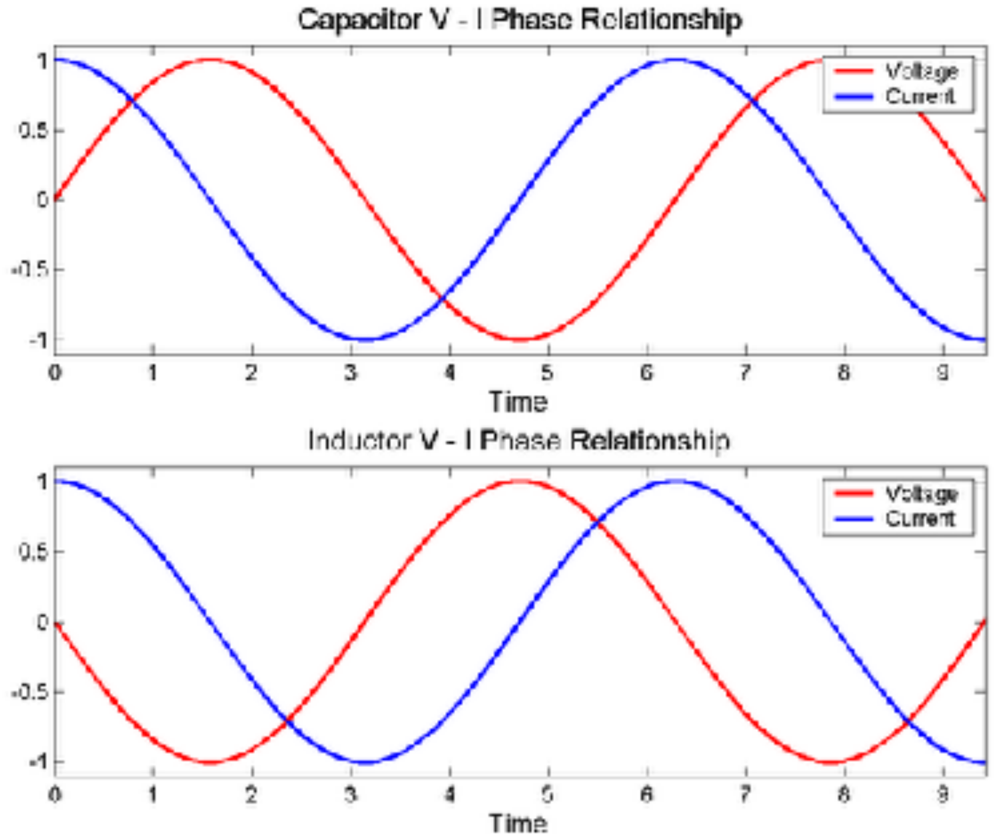
- AM demodulators:
  - cannot be used due to the constant amplitude of FM signals
- FM dedicated demodulator
  - Phase detector
  - The Foster-Seeley discriminator
  - Ratio detector
  - Quadrature detector

Impedance - Capacity and Inductor



- Complex Impedance (do you recall 'phasor'?)

$$Z = R + jX$$



The impedance of an ideal resistor is purely real and is referred to as a *resistive impedance*:

$$Z_R = R$$

In this case, the voltage and current waveforms are proportional and in phase.

Ideal inductors and capacitors have a purely imaginary *reactive impedance*:

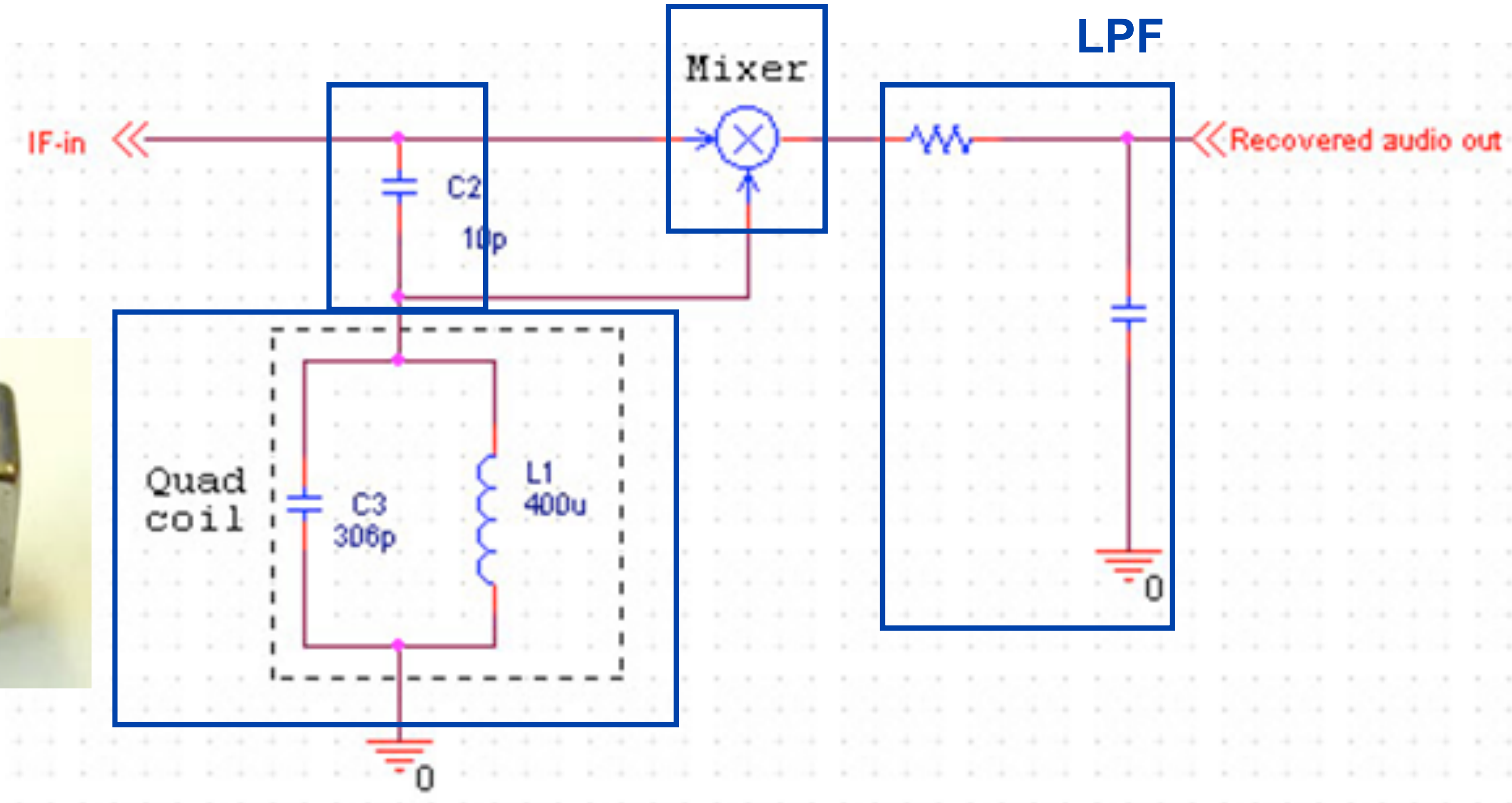
the impedance of inductors increases as frequency increases;

$$Z_L = j\omega L$$

the impedance of capacitors decreases as frequency increases;

$$Z_C = \frac{1}{j\omega C}$$

Quadrature Detector

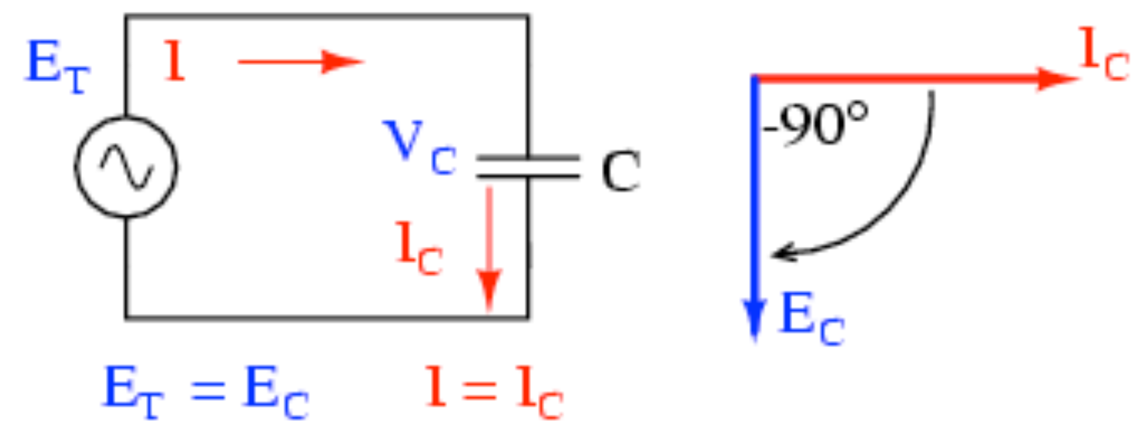


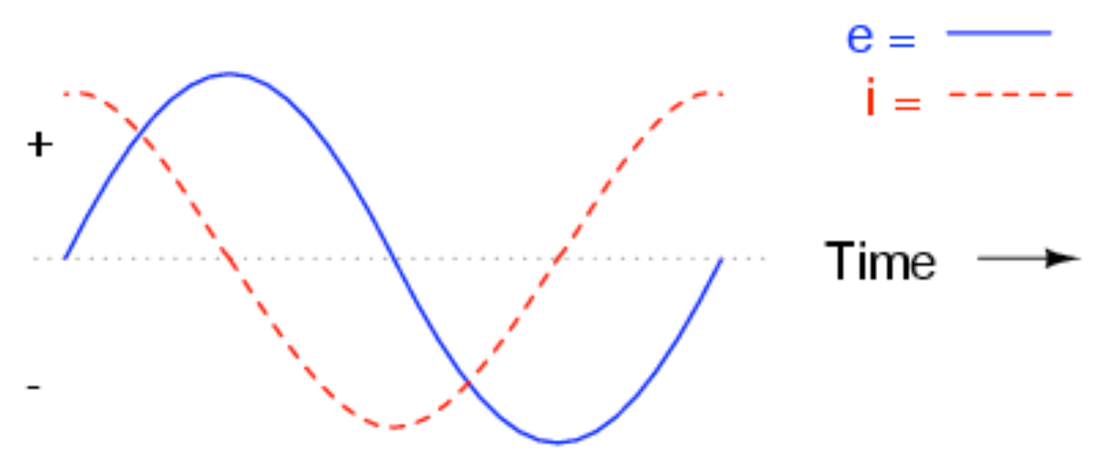
- High-reactance capacitor (C2): produce two signals with 90 degree phase difference (see next slide)
- Phase-shifted signal is then applied to LC-tuned resonant at carrier frequency (L1 and C3)
- Frequency changes will then produce an additional leading or lagging phase shift into the mixer

Recall: Capacitor and Phase

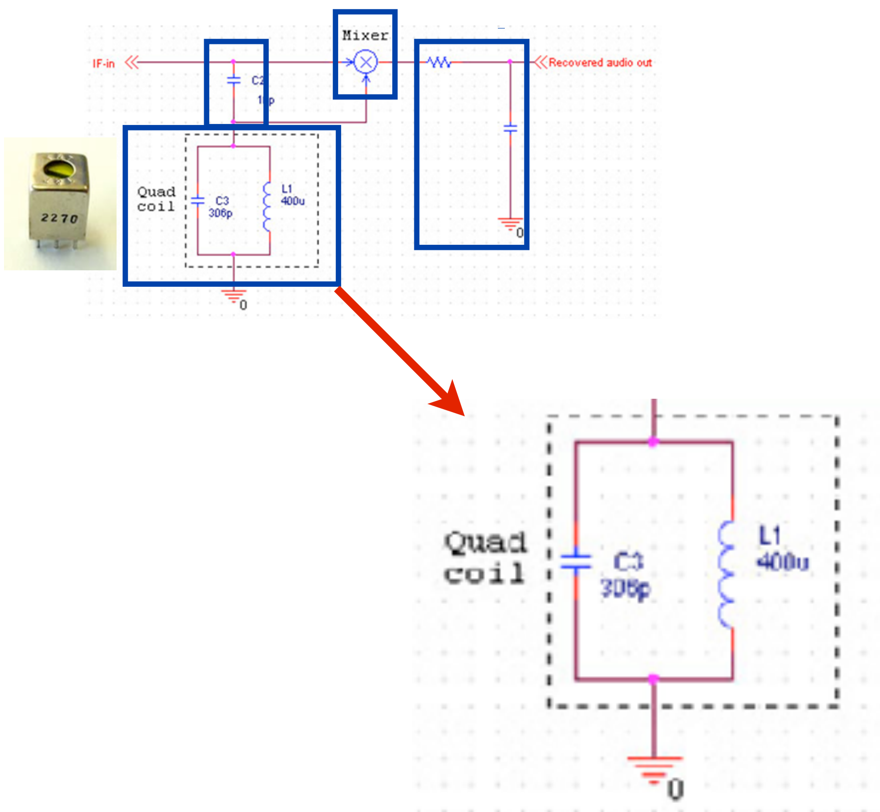
- Flow of electrons “through” capacitor is directly proportional to *rate of change* of voltage across capacitor.
- In Math,

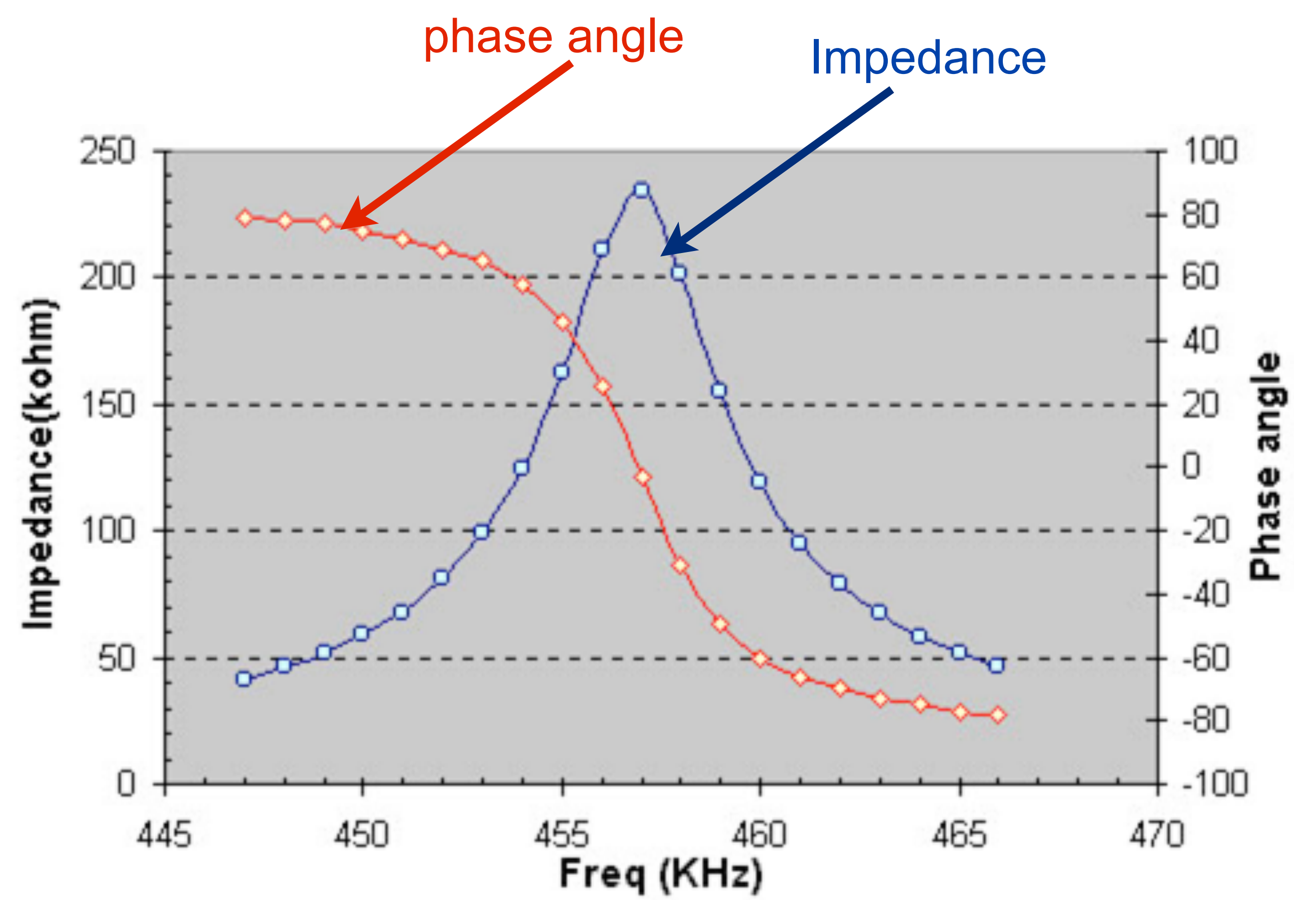
$$i = C\frac{dV}{dt}$$





Impedance and Phase of LC tuned Circuit



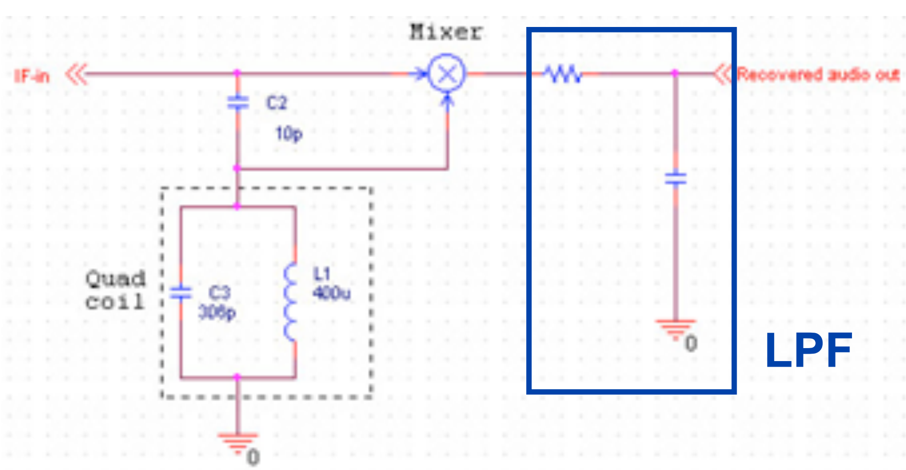


- Facts:
  - Phase (red curve): 0 at resonance
  - Frequency lower than 457KHz: phase positive
  - Frequency higher than 457KHz: phase negative
- In conclusion: If frequency changes, phase will also vary and output voltage (audio signal) too

Property

- **RULE:** multiplication of two periodic signals with same frequency produces DC voltage that is directly proportional to signal phase difference
- Example:

$$\begin{aligned}\sin(x)\sin(90^\circ + x + \theta) &= \sin(x)\cos(x+\theta)\\ &= \frac{1}{2}(\sin(2x+\theta) - \sin(\theta)) \rightarrow^{LPF} -\frac{1}{2}\sin\theta \overset{\text{Taylor Expansion}}{\approx} -\frac{1}{2}\theta\end{aligned}$$



- Finally, we can recover signal that is proportion to frequency changes!

Note - Taylor Expansion

- Taylor series of a function *f(x)* in *a* is given by

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f^{(3)}(a)}{3!}(x-a)^3+\cdots$$

or

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

- Examples:

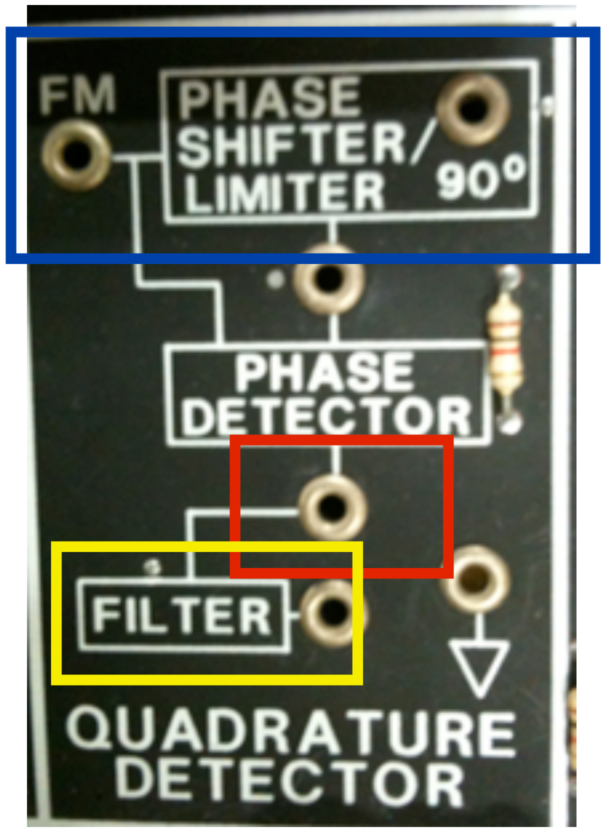
$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

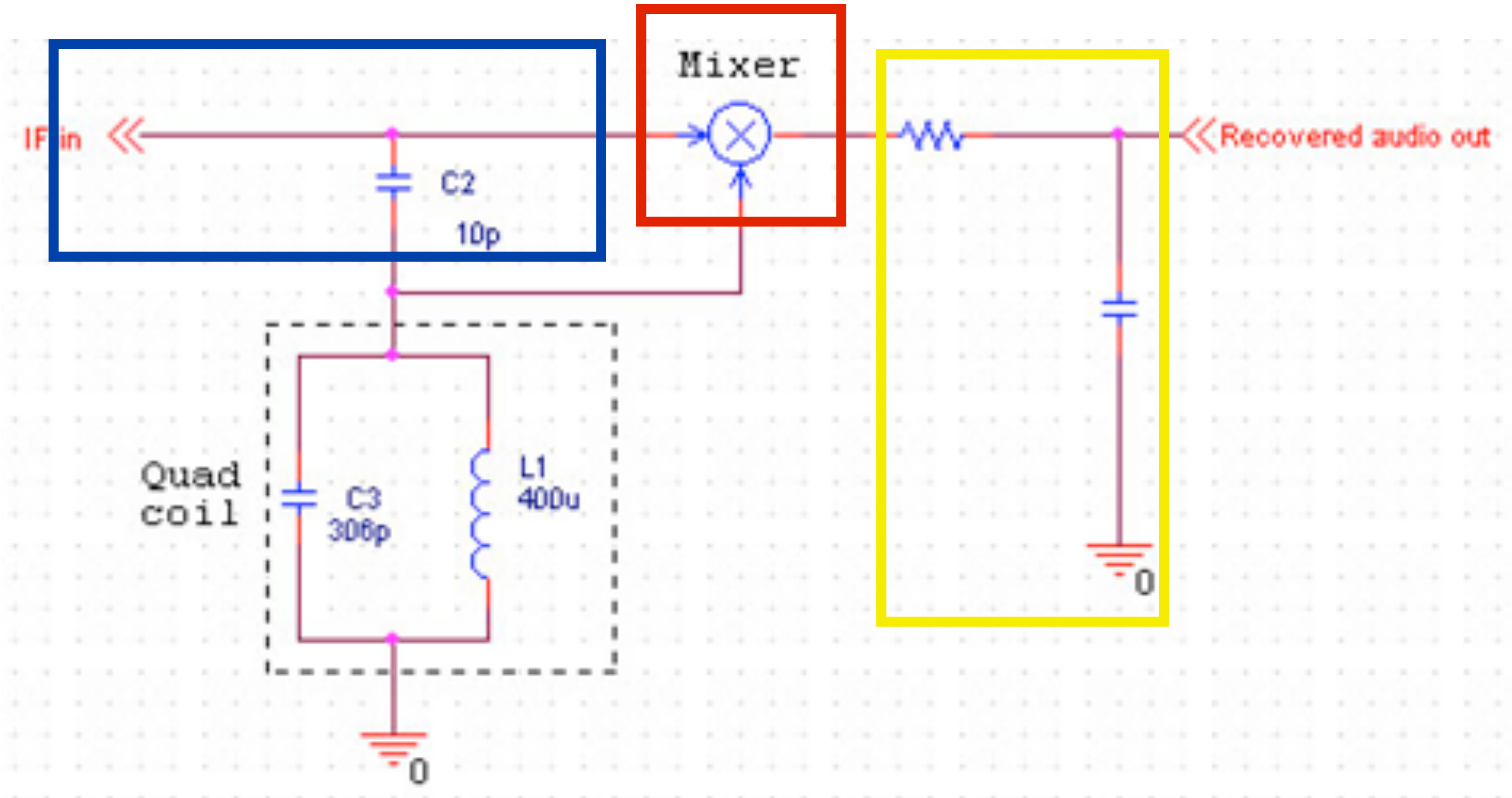
$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots$$

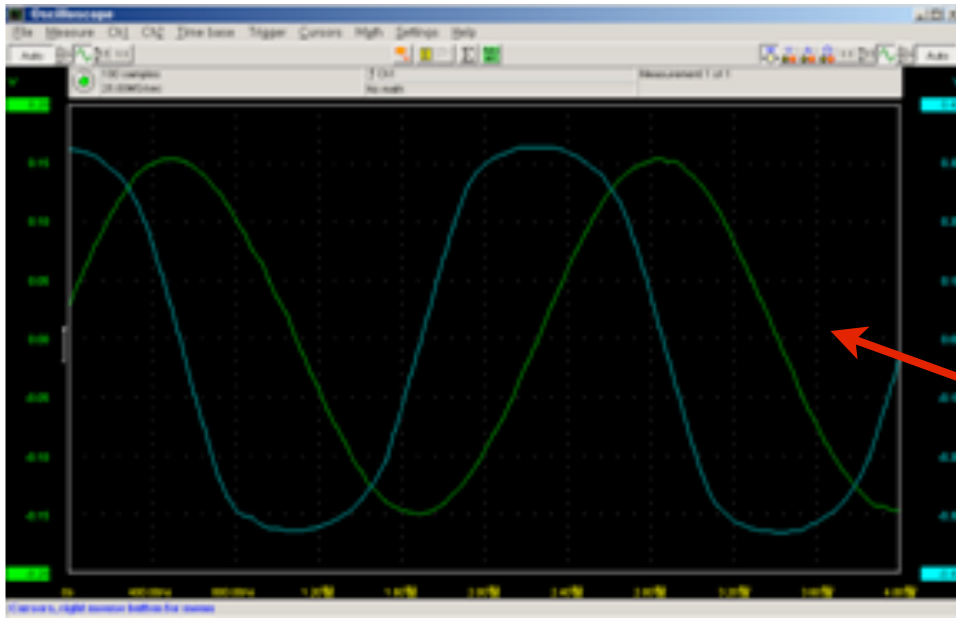
In Experiments,

- In circuit board:

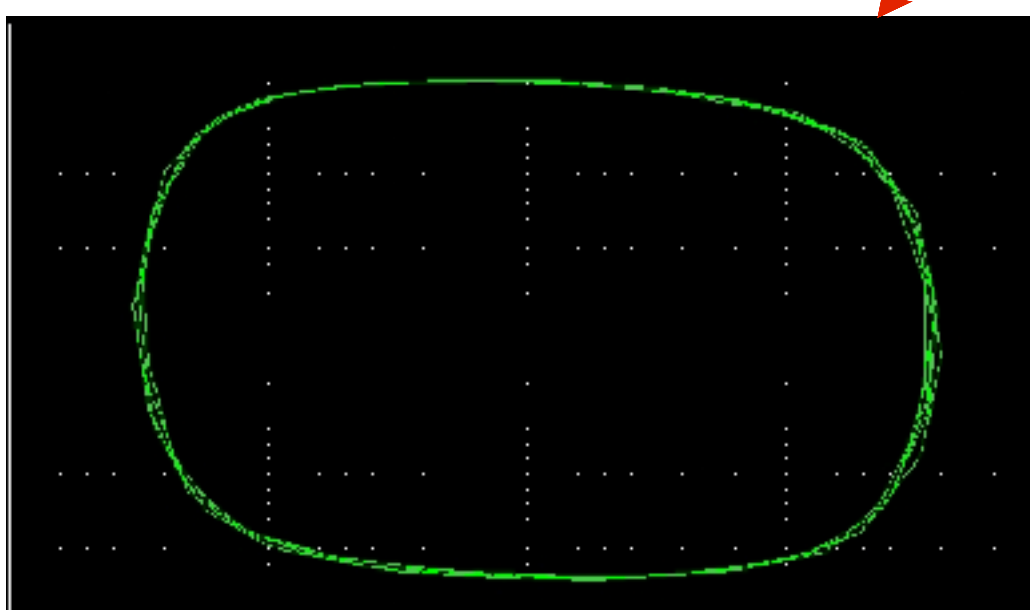


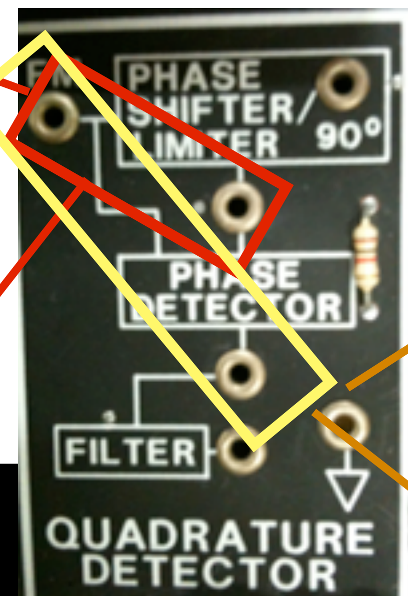


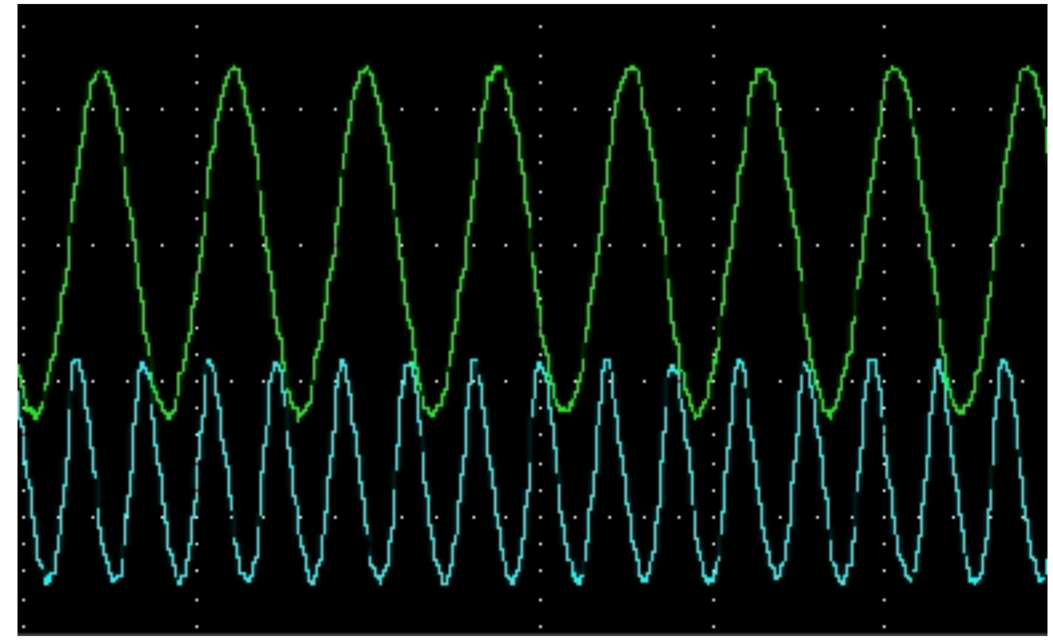
Illustrative Results:



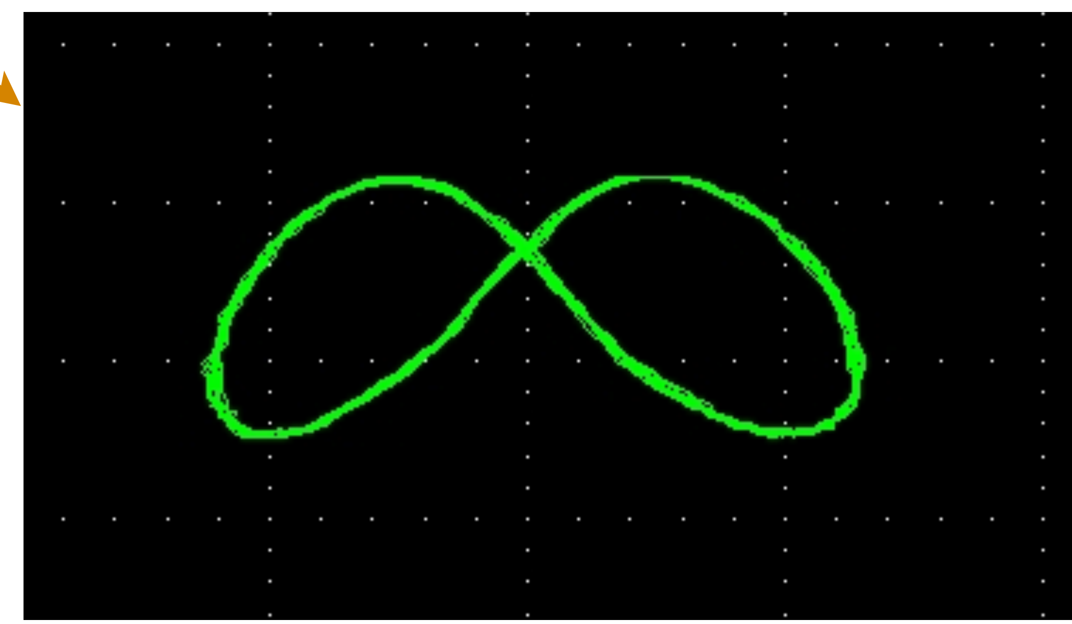
$$\sin(x)\sin(90^\circ + x + \theta)$$







$$\frac{1}{2}(\sin(2x+\theta) - \sin(\theta))$$



Illustrative Results: