Leverage Turbulence measuring and probing

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Goal

Assimilating Observed data into dynamical model to reconstruct high precision data

- Review
  - 2019 Stanford University + Sandia Lab using 4D-var to assimilate a jet flow with Re 13500 into LES to eliminate numerical sensitivity of turbulence
  - 2022 Johns Hopkins University and Maryland University uses Ensemble-variational method to assimilate wall-pressure data in LES simulation under Mach 6
  - 2024 Aoyama Gakuin University and Nagoya University using 2D PIV data to tune RANs model parameters under Mach 2

- Our Target:
  - Aiming on Turbulence field reconstruction on Inertia range
  - Proposing new Computational model and data assimilation approaches
  - Demonstrate the potential of precision reconstruction of flow field property in highly non-linear flow situations with coarse measurement.
  - Make engineering compatible measuring tool chains

Innovations in CFD

Using nested multiscale to arrive at DNS accuracy while maintain a near LES cost

- Governing equation

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u = \nu \Delta u + f$$
$$\nabla \cdot u = 0$$

- Periodic boundary with Isotropic turbulence initialization (Rogallo)
- Nested multiscale phase

$$\frac{\partial \boldsymbol{\theta}^{\epsilon}}{\partial t} + (\mathbf{u}^{\epsilon} \cdot \nabla)\, \boldsymbol{\theta}^{\epsilon} = \mathbf{0}$$
$$\boldsymbol{\theta}^{\epsilon}\big|_{t=0} = \mathbf{x}$$

$$\boldsymbol{\theta}^{\epsilon} = \overline{\boldsymbol{\theta}}(t, \mathbf{x}, \tau) + \epsilon \widetilde{\boldsymbol{\theta}}(t, \overline{\boldsymbol{\theta}}, \tau, \mathbf{z}) \quad \mathbf{z} = \frac{\overline{\boldsymbol{\theta}}}{\epsilon}, \quad \tau = \frac{t}{\epsilon}, \quad \tilde{\boldsymbol{\theta}}(\boldsymbol{z}) = \tilde{\boldsymbol{\theta}}(\boldsymbol{z} + \mathbf{1}) \quad \int \tilde{\boldsymbol{\theta}}(\boldsymbol{z}) d\boldsymbol{z} = 0$$

- Soul: The multiscale structure is convected by mean flow and inducing cell (sub-grid) homogenization problem, which homogenizes mean flow.

Algorithm

- Step 1. At t = 0 and $\tau = 0$, we have

$$\boldsymbol{\theta}_{int} = \mathbf{x}, \quad \mathbf{u}_{int} = \mathbf{U}, \quad \mathbf{w}_{int} = \mathbf{W}, \quad \boldsymbol{\Theta}_{\text{int}} = \mathbf{0}, \quad \mathcal{A} = \mathcal{I}.$$

- Step 2. Solve cell problem for (w, q)

$$\partial_\tau \mathbf{w} + D_z \mathbf{w} \mathcal{A} \mathbf{w} + \mathcal{A}^\top \nabla_z q - \frac{\nu}{\epsilon} \nabla \cdot \left(\mathcal{A} \mathcal{A}^\top \nabla_z \mathbf{w} \right) = \mathbf{0}$$
$$\left(\mathcal{A}^\top \nabla_z \right) \cdot \mathbf{w} = 0$$
$$\mathbf{w}\big|_{\tau=\tau_m} = \mathbf{w}_{int}$$

$$\partial_\tau \boldsymbol{\Theta} + \left(\mathcal{I} + D_z \boldsymbol{\Theta} \right) D_x \overline{\boldsymbol{\theta}} \mathbf{w} = \mathbf{0}.$$
$$\boldsymbol{\Theta}\big|_{\tau=t=0} = \boldsymbol{\Theta}_{\text{int}} \ .$$

- Step 3. Update large scale solution

$$\partial_t \mathbf{u} + \left(\mathbf{u} \cdot \nabla_x \right) \mathbf{u} + \nabla_x p + \nabla_x \cdot \langle [\mathbf{w} \otimes \mathbf{w}]_\Delta^* \rangle - \nu \Delta \mathbf{u} = \mathbf{f}$$
$$\nabla_x \cdot \mathbf{u} = 0$$
$$\mathbf{u}\big|_{t=t_m} = \mathbf{u}_{int}$$

$$\partial_t \boldsymbol{\theta} + \left(\mathbf{u} \cdot \nabla_x \right) \boldsymbol{\theta} + \epsilon \nabla_x \cdot \langle [\boldsymbol{\Theta} \otimes \mathbf{w}]_\Delta^* \rangle = \mathbf{0}$$
$$\boldsymbol{\theta}\big|_{t=t_m} = \boldsymbol{\theta}_{\text{int}}$$

- Step 4. Go back to step 2 and start over

$$\boldsymbol{\theta}_{\text{int}} = \boldsymbol{\theta}\big|_{t=t_{m+1}}, \quad \mathbf{u}_{int} = \mathbf{u}\big|_{t=t_{m+1}}, \quad \mathbf{w}_{int} = \mathbf{w}\big|_{\tau=\tau_{m+1}},$$
$$\boldsymbol{\Theta}_{\text{int}} = \boldsymbol{\Theta}\big|_{\tau=\tau_{m+1}}, \quad \mathcal{A} = D_x\boldsymbol{\theta}\big|_{t=t_{m+1}}.$$

- Adaptive Technique: Solve the cell problem when needed
  - The Jacobian of inverse flow map $D_x\boldsymbol{\theta}$ determines the significance of cell problem

$$\partial_\tau \mathbf{w} + (D_x\boldsymbol{\theta}\mathbf{w}\cdot\nabla_z)\,\mathbf{w} + D_x\boldsymbol{\theta}^\top\nabla_z q - \frac{\nu}{\epsilon}\nabla_z\cdot\left(D_x\boldsymbol{\theta}D_x\boldsymbol{\theta}^\top\nabla_z\mathbf{w}\right) = \mathbf{0},$$
$$\left(D_x\boldsymbol{\theta}^\top\nabla_z\right)\cdot\mathbf{w} = 0$$
$$\mathbf{w}\big|_{\tau=t=0} = \mathbf{W}(\mathbf{x},\mathbf{z}).$$

  - Evaluate $G = \left\|(D_x\theta)_n^T(D_x\theta)_n - I\right\|$ at time t = n as the standard
- Strength: No parameter tuning, DNS comparable accuracy. ϵ determines the background mesh density.

Result

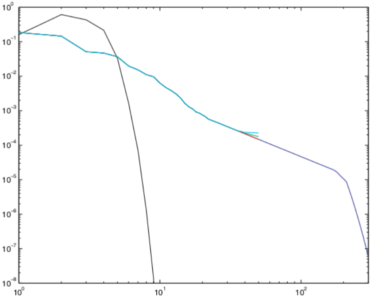

FIG. 5.9. *Comparison of energy spectra. The black curve corresponds to the energy spectrum at $t = 0$. The other curves correspond to the energy spectra at $t = 30$, which are DNS (blue); $P = 128$ (red); $P = 64$ (green); $P = 32$ (light blue).*

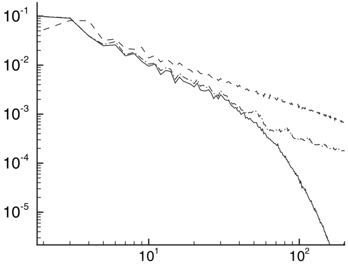

FIG. 5.6. *Spectrum of velocity component u_1. (Dashed line), $t = 0.0$; (solid line), DNS, $t = 20.0$; (dashed-dotted line), multiscale model, $t = 20.0$.*

Caltech

Innovations in Data assimilation

Algorithm

Result

Questions?

Thanks!!!!!!!!!!!!