Problem A
A Chair Problem

Time limit: 1 second
Memory limit: 256 megabytes

Problem Description

It's September and your little brother starts elementary school. For better ergonomics, the teacher has to know who are taller or shorter to select their tables and chairs. One noticeable thing is some pupils have difficulty telling who's taller if they're of the same height standing on different chairs. To help the pupils understand, the teacher makes them compare their heights several times and notes the results on a sheet. There are n pupils, whose heights are H_1, H_2, $\ldots$, and H_n, conducting m comparisons in the class. There are many chairs, each of which has height either C_1, C_2, $\ldots$, or C_k, while $C_0 = 0$ denotes the ground. Since some pupils may report the results incorrectly, the teacher needs you to check if there is any error on the sheet.

Input Format

The first line contains two integers n and m. Each of the next m lines contains four integers i, u, j, v indicating that $H_i + C_u \leq H_j + C_v$.

Output Format

Please output `YES` if there exists a set of positive real numbers H_1, H_2, $\ldots$, H_n and C_1, C_2, $\ldots$, C_k such that all inequations are satisfied. Otherwise, output `NO`.

Technical Specification

- $2 \leq n \leq 10^3$
- $1 \leq m \leq 10^5$
- $k = 1$
- $1 \leq i \leq n$, $1 \leq j \leq n$, $i \neq j$
- $0 \leq u \leq k$, $0 \leq v \leq k$

Sample Input 1

```
2 1
1 0 2 1
```

Sample Output 1

```
YES
```

Sample Input 2

```
3 2
1 1 2 0
2 1 1 0
```

Sample Output 2

```
NO
```

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Problem B
Both Chairs

Time limit: 1 second

Memory limit: 256 megabytes

Problem Description

It's September and your little brother starts elementary school. For better ergonomics, the teacher has to know who are taller or shorter to select their tables and chairs. One noticeable thing is some pupils have difficulty telling who's taller if they're of the same height standing on different chairs. To help the pupils understand, the teacher makes them compare their heights several times and notes the results on a sheet. There are n pupils, whose heights are H_1, H_2, $\ldots$, and H_n, conducting m comparisons in the class. There are many chairs, each of which has height either C_1, C_2, $\ldots$, or C_k, while $C_0 = 0$ denotes the ground. Since some pupils may report the results incorrectly, the teacher needs you to check if there is any error on the sheet.

Input Format

The first line contains two integers n and m. Each of the next m lines contains four integers i, u, j, v indicating that $H_i + C_u \leq H_j + C_v$.

Output Format

Please output `YES` if there exists a set of positive real numbers H_1, H_2, $\ldots$, H_n and C_1, C_2, $\ldots$, C_k such that all inequations are satisfied. Otherwise, output `NO`.

Technical Specification

- $2 \leq n \leq 500$
- $1 \leq m \leq 10^4$
- $k = 2$
- $1 \leq i \leq n$, $1 \leq j \leq n$, $i \neq j$
- $0 \leq u \leq k$, $0 \leq v \leq k$

Sample Input 1

```
2 1
1 0 2 1
```

Sample Output 1

```
YES
```

Sample Input 2

```
3 2
1 1 2 0
2 1 1 0
```

Sample Output 2

```
NO
```

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Problem C
Chairs

Time limit: 1 second
Memory limit: 256 megabytes

Problem Description

It's September and your little brother starts elementary school. For better ergonomics, the teacher has to know who are taller or shorter to select their tables and chairs. One noticeable thing is some pupils have difficulty telling who's taller if they're of the same height standing on different chairs. To help the pupils understand, the teacher makes them compare their heights several times and notes the results on a sheet. There are n pupils, whose heights are H_1, H_2, $\ldots$, and H_n, conducting m comparisons in the class. There are many chairs, each of which has height either C_1, C_2, $\ldots$, or C_k, while $C_0 = 0$ denotes the ground. Since some pupils may report the results incorrectly, the teacher needs you to check if there is any error on the sheet.

Input Format

The first line contains two integers n and m. Each of the next m lines contains four integers i, u, j, v indicating that $H_i + C_u \leq H_j + C_v$.

Output Format

Please output `YES` if there exists a set of positive real numbers H_1, H_2, $\ldots$, H_n and C_1, C_2, $\ldots$, C_k such that all inequations are satisfied. Otherwise, output `NO`.

Technical Specification

- $2 \leq n \leq 44$
- $1 \leq m \leq 4444$
- $k = 4$
- $1 \leq i \leq n$, $1 \leq j \leq n$, $i \neq j$
- $0 \leq u \leq k$, $0 \leq v \leq k$

Sample Input 1

```
2 1
1 0 2 1
```

Sample Output 1

```
YES
```

Sample Input 2

```
3 2
1 1 2 0
2 1 1 0
```

Sample Output 2

```
NO
```

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Problem D
Temperature Difference

Time limit: 10 seconds
Memory limit: 256 megabytes

Problem Description

Gaia travels among planets via (bidirectional) wormholes. She starts from planet X and plans to reach planet Y. There may or may not exist a wormhole that connects planets X and Y directly. Therefore, on some occasions, it is necessary to traverse along a sequence of wormholes rather than just a single one. Some intermediate planets other than X and Y are visited while transferring from one wormhole to another. Given the temperature of all planets, Gaia would like to search for a sequence of wormholes from planet X to Y that minimizes the temperature difference between the hottest visited planets and the coldest visited one. For the sake of simplicity, we assume that planet X has temperature X (K) for each X.

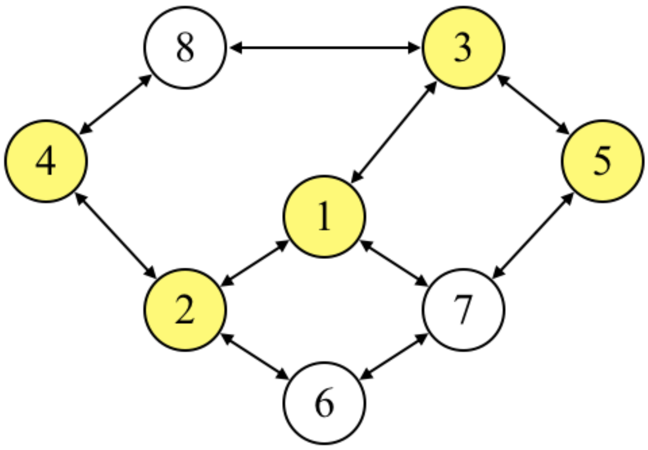

Figure 1: If $X = 4$ and $Y = 5$, then the best route for Gaia to reach Y from X is $4 \to 2 \to 1 \to 3 \to 5$ because it has the smallest temperature difference 4.

Input Format

In the first line, three integers n, m, k are given. n is an integer in $[1, 3000]$ that denotes the number of planets, and m is an integer in $[1, n(n-1)/2]$ that denotes the number of wormholes. k is an integer in $[1, 20]$ that specifies the number of queries. The n planets are numbered from 1 to n. Then the description of the m wormholes follows. Each wormhole is specified by the identifier of the end-planets, u and v for some $u \neq v \in \{1, 2, \ldots, n\}$. Then the description of the k queries follows. Each query gives an (X, Y) pair. You may assume that the planets are connected.

Output Format

For each query, output the smallest temperature difference on a unique line.

Sample Input 1

```
8 10 3
4 8 8 3 3 5
4 2 2 1 1 7 7 5
2 6 6 7
1 3
4 5
1 6
8 8
```

Sample Output 1

```
4
5
0
```

Problem E
P-adic Equation

Time limit: 1 second
Memory limit: 256 megabytes

Problem Description

Kurt's got some equations to solve, help him out! You will be given the following items:

- A prime p, integer $2 \leq n \leq 200$, integer $1 \leq k$
- A square matrix $A \mod p$ of size $n \times n$
- An integer vector $0 \leq b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$ of size $n \times 1$
- An integer k

Matrix A is guaranteed to be invertible mod p. Try to solve for $x \mod p^k$ where

$$Ax \equiv b \mod p^k$$

It is guaranteed that $p^k < 2^{31}$ and $\forall b_i : 0 \leq b_i < 2^{31}$

Input Format

The first line contains p, n, k seperated by single blanks. The second line contains $b_1, \ldots, b_n$ the elements of b. Then for the next n lines, the i-th line contains $a_{i,1}, a_{i,2}, \ldots, a_{i,n}$, where $a_{i,j}$ is the i-th row j-th column element of A.

Output Format

Output a line containing $x_1, \ldots x_n$ seperated by blanks.

Sample Input 1

```
3 2 1
2 1
1 1
1 2
```

Sample Output 1

```
0 2
```

Sample Input 2

```
3 2 1
0 2
1 1
1 2
```

Sample Output 2

```
1 2
```

Sample Input 3

Sample Output 3

```
3 2 2
0 2
1 1
1 2
```

```
7 2
```

Sample Input 4

```
3 2 3
0 2
1 1
1 2
```

Sample Output 4

```
25 2
```

Problem F
Forming Number

Time limit: 1 second
Memory limit: 256 megabytes

Problem Description

You are given a positive integer n. You have infinite supplies of 1's, additions, multiplications and brackets for composing an expression. What is the least total number of additions and multiplications required to compose an expression evaluated into n?

For example, $n = 9 = (1+1+1) \times (1+1+1)$ can be evaluated with 5 operations and it is impossible to use 4 operations to compose an expression evaluated into 9. Therefore, you should output 5. Note that concatenating 1's is not allowed.

Input Format

First line contains the number of testcases T. Each testcase is a line containing a positive integer n.

Output Format

For each testcase, output a number indicating the minimum total number of additions and multiplications required.

Technical Specification

- $1 \leq T \leq 10^4$
- $1 \leq n \leq 10^5$

Sample Input 1

```
10
1
2
5
11
22
55
100
222
555
1000
```

Sample Output 1

```
0
1
4
7
9
11
13
15
18
20
```

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Problem G
Geometric Center

Time limit: 1 second

Memory limit: 256 megabytes

Problem Description

We say a point (x^*, y^*) is a geometric median of n points $(x_1, y_1), \ldots, (x_n, y_n)$ on the 2D-plane if $\sum_{i=1}^{n} \sqrt{(x^* - x_i)^2 + (y^* - y_i)^2} = \min_{x,y} \sum_{i=1}^{n} \sqrt{(x - x_i)^2 + (y - y_i)^2}$.

It is probably too hard for you to find a geometric median in general. So please try to solve the cases for $n = 4$.

Input Format

The input consists of four lines. The i-th line contains two positive integer x_i and y_i indicating the i-th point is (x_i, y_i).

Output Format

Output x' and y' separated by a blank where (x', y') should be a geometric median.

Technical Specification

- $n = 4$
- $0 \leq x_i \leq 1000$ for $1 \leq i \leq n$
- $0 \leq y_i \leq 1000$ for $1 \leq i \leq n$
- The judge script will consider it is correct if your output x' and y' satisfies:

$$\sum_{i=1}^{n} \sqrt{(x' - x_i)^2 + (y' - y_i)^2} \leq 1.000001 \cdot \min_{x,y} \sum_{i=1}^{n} \sqrt{(x - x_i)^2 + (y - y_i)^2}$$

Sample Input 1

```
0 0
1 0
0 1
1 1
```

Sample Output 1

```
0.5 0.5
```

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Problem H
Construct DAG

Time limit: 1 second
Memory limit: 256 megabytes

Problem Description

This problem is about to construct a directed acyclic graph (DAG) of n vertices and m edges. The graph must have the vertex set be $\{1, \ldots, n\}$. Any edge (u, v) in this graph must satisfy $u < v$. Any vertex u may have at most k out-going edges.

You are given three numbers n, k, a where n is the number of vertices and k is the limit of out-going edges. Please determine whether there is a DAG such that there are at least a^n paths from 1 to n.

Input Format

The input is one line containing three numbers n, k, a.

Output Format

If there is no such DAG, output `-1`. Otherwise, output $n - 1$ lines. The i-th line is a list of vertices that is adjacent to i. Separate the vertices with a blank.

Technical Specification

n and k are no more than 1000, and a is a positive number greater than 1.

Sample Input 1

```
4 3 2
```

Sample Output 1

```
-1
```

Sample Input 2

```
4 3 1
```

Sample Output 2

```
2
3
4
```

Problem I
Ideal Triangle

Time limit: 1 second
Memory limit: 256 megabytes

Problem Description

We say a point (x, y) on the 2D-plane is a lattice point if x and y are integers. We say a triangle is an ideal triangle if all of its vertices are lattice points. You are given a positive n and asked to generate an ideal triangle such that there are exactly n lattice points on its boundary.

Input Format

The input is a line containing a positive integer n.

Output Format

Output three lines which are distinct. Each of them contains two integers x and y indicating (x, y) is the coordinate of an ideal triangle. This ideal triangle must have n lattice points on its boundary.

Technical Specification

- $1 \le n \le 10^{32}$
- $0 \le x \le 10^{64}$
- $0 \le y \le 10^{64}$

Sample Input 1

```
6
```

Sample Output 1

```
0 0
2 2
2 0
```

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Problem J
Joyful Path

Time limit: 1 second
Memory limit: 256 megabytes

Problem Description

You are planing your next family trip to a foreign country. The number of days of this trip will be between d days to D days. This foreign country has n cities (numbered from 0 to $n-1$) such that there are direct flights between your home town and them. The trip can start and end in any of these n cities.

Your family loves taking trains. During the trip, you may stay in a city without any joy or move from one city to another one via train. You notice that there are m train routes via which your family will have some joy.

You numbered these m route from 1 to m. Route i starts from city u_i to city v_i. It gives your family j_i joyness and takes d_i days to travel. Please find the maximum average joyness (the total joyness divided by the number of days of your trip) travel plan.

Input Format

The first line of the input contains four positive integers n, m, d, D. n is the number of cities. m is the number of joyful routes. The length of your family trip can only be d to D days. Then m lines follows. The i-th of them contains four integers u_i, v_i, j_i and d_i described in the problem statement.

Output Format

Output the total joyness J and the length of your family trip L separated by a blank. This must maximize the average joyness. If there are multiple solution, output the one with L minimized.

Technical Specification

- $1 \leq n \leq 100$
- $1 \leq m \leq 1000$
- $1 \leq d \leq D \leq 100$
- $0 \leq u_i < n$ for $1 \leq i \leq m$
- $0 \leq v_i < n$ for $1 \leq i \leq m$
- $1 \leq j_i \leq 1000$ for $1 \leq i \leq m$
- $1 \leq d_i \leq 1000$ for $1 \leq i \leq m$

Sample Input 1

```
4 5 10 10
1 2 3 4
2 3 1 2
3 0 1 3
0 1 4 2
2 0 1 9
```

Sample Output 1

```
8 10
```

Problem K
Ideal Triangle Checker

Time limit: 1 second
Memory limit: 256 megabytes

Problem Description

We say a point (x, y) on the 2D-plane is a lattice point if x and y are integers. We say a triangle is a ideal triangle if all of its vertices are lattice points. You are given an ideal triangle n. Please compute the number of lattice points on its boundary.

Input Format

The input consists of three lines which are distinct. Each of them contains two integers x and y indicating (x, y) is the coordinate of some vertex of the ideal triangle.

Output Format

Output the number of lattice points lying on the boundary of the given ideal triangle.

Technical Specification

- $0 \leq x \leq 10^9$
- $0 \leq y \leq 10^9$

Sample Input 1

```
0 0
2 2
2 0
```

Sample Output 1

```
6
```

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Problem L
Run Length

Time limit: 1 second
Memory limit: 256 megabytes

Problem Description

We say a point (x, y) on the 2D-plane is a lattice point if x and y are integers. We say a triangle is a ideal triangle if all of its vertices are lattice points. You are given an ideal triangle n. Please compute the number of lattice points on its boundary.

Input Format

The input consists of three lines which are distinct. Each of them contains two integers x and y indicating (x, y) is the coordinate of some vertex of the ideal triangle.

Output Format

Output the number of lattice points lying on the boundary of the given ideal triangle.

Technical Specification

- $0 \le x \le 10^9$
- $0 \le y \le 10^9$

Sample Input 1

```
1 1000000000
```

Sample Output 1

```
14570502158
```

Sample Input 2

```
1 5
```

Sample Output 2

```
7
```

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Problem M
Monotone Chain

Time limit: 1 second
Memory limit: 256 megabytes

Problem Description

We say a point (x, y) on the 2D-plane is a lattice point if x and y are integers. We say a triangle is a ideal triangle if all of its vertices are lattice points. You are given an ideal triangle n. Please compute the number of lattice points on its boundary.

Input Format

The input consists of three lines which are distinct. Each of them contains two integers x and y indicating (x, y) is the coordinate of some vertex of the ideal triangle.

Output Format

Output the number of lattice points lying on the boundary of the given ideal triangle.

Technical Specification

- $0 \le x \le 10^9$
- $0 \le y \le 10^9$

Sample Input 1

```
3
```

Sample Output 1

```
0 0
1 1
2 0
```