Tree Path Labeling of Set Systems
A Generalization of Consecutive Ones Property

Anju Srinivasan

as part of **M. S.** by Research
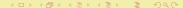advised by **Dr. N. S. Narayanaswamy**
CSE, IITM, Chennai - 36

18 Oct 2011
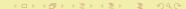
An Illustration

Caveat

- A very simplistic example.

- Aims only to introduce the combinatorial problem of TPL.

Introduction
○●○○
An Illustration

Characterization
○○

Computation of TPL
○

Conclusion
○○

A Study Group Housing problem

- A set of *n* students arrive for a summer course, say
  $\{a, b, c, d, e, f, g, h, i, j, k\}$, $n = 11$.

Introduction
○●○○
An Illustration

Characterization
○○

Computation of TPL
○

Conclusion
○○

A Study Group Housing problem

- A set of n students arrive for a summer course, say $\{a, b, c, d, e, f, g, h, i, j, k\}$, $n = 11$.
- They form m study groups, say $\{R, B, O, G\}$, $m = 4$

Introduction
○●○○
An Illustration

Characterization
○○

Computation of TPL
○

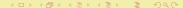Conclusion
○○

A Study Group Housing problem

- A set of n students arrive for a summer course, say $\{a, b, c, d, e, f, g, h, i, j, k\}$, $n = 11$.
- They form m study groups, say $\{R, B, O, G\}$, $m = 4$
- A student may be in more than one study group but **will** be in at least one, say

Introduction
○○●○

Characterization
○○

Computation of TPL
○

Conclusion
○○

An Illustration

A Study Group Housing problem

- A set of n students arrive for a summer course, say $\{a, b, c, d, e, f, g, h, i, j, k\}$, $n = 11$.
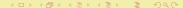- They form m study groups, say $\{R, B, O, G\}$, $m = 4$
- A student may be in more than one study group but **will** be in at least one, say
  - $R = \{g, h, i, j, k\}$

Introduction
○●○○
An Illustration

Characterization
○○

Computation of TPL
○

Conclusion
○○

A Study Group Housing problem

- A set of n students arrive for a summer course, say $\{a, b, c, d, e, f, g, h, i, j, k\}$, $n = 11$.
- They form m study groups, say $\{R, B, O, G\}$, $m = 4$
- A student may be in more than one study group but **will** be in at least one, say
  - $R = \{g, h, i, j, k\}$
  - $B = \{a, b, e, g\}$
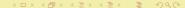
A Study Group Housing problem

- A set of n students arrive for a summer course, say $\{a, b, c, d, e, f, g, h, i, j, k\}$, $n = 11$.
- They form m study groups, say $\{R, B, O, G\}$, $m = 4$
- A student may be in more than one study group but **will** be in at least one, say
  - $R = \{g, h, i, j, k\}$
  - $B = \{a, b, e, g\}$
  - $O = \{c, b, d\}$

A Study Group Housing problem

- A set of n students arrive for a summer course, say $\{a, b, c, d, e, f, g, h, i, j, k\}$, $n = 11$.
- They form m study groups, say $\{R, B, O, G\}$, $m = 4$
- A student may be in more than one study group but **will** be in at least one, say
  - $R = \{g, h, i, j, k\}$
  - $B = \{a, b, e, g\}$
  - $O = \{c, b, d\}$
  - $G = \{e, f, g, i\}$

Introduction
○●○○
An Illustration

Characterization
○○

Computation of TPL
○

Conclusion
○○

A Study Group Housing problem

- A set of n students arrive for a summer course, say $\{a, b, c, d, e, f, g, h, i, j, k\}$, $n = 11$.
- They form m study groups, say $\{R, B, O, G\}$, $m = 4$
- A student may be in more than one study group but **will** be in at least one, say
  - $R = \{g, h, i, j, k\}$
  - $B = \{a, b, e, g\}$
  - $O = \{c, b, d\}$
  - $G = \{e, f, g, i\}$
- There are n single occupancy apartments in the university campus for their accommodation.
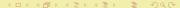
Introduction
○●○○
An Illustration

Characterization
○○

Computation of TPL
○

Conclusion
○○

A Study Group Housing problem

- A set of n students arrive for a summer course, say $\{a, b, c, d, e, f, g, h, i, j, k\}$, $n = 11$.
- They form m study groups, say $\{R, B, O, G\}$, $m = 4$
- A student may be in more than one study group but **will** be in at least one, say
  - $R = \{g, h, i, j, k\}$
  - $B = \{a, b, e, g\}$
  - $O = \{c, b, d\}$
  - $G = \{e, f, g, i\}$
- There are n single occupancy apartments in the university campus for their accommodation.
- All these apartments are placed such that streets connecting them do not form loops

A Study Group Housing problem

The problem

How should the students be allocated apartments such that each study group has the least distance to travel for a discussion?

Introduction
OOOO
An Illustration

Characterization
OO

Computation of TPL
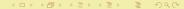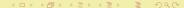O

Conclusion
OO

1

Characterization of feasible TPL

Given

i. a set system or hypergraph $\mathcal{F}$,

ii. a feasible TPL $l : \mathcal{F} \to \mathcal{P}$ where $\mathcal{P}$ is a path system from tree T and $supp(\mathcal{P}) = V(T)$,
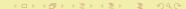
what is the hypergraph isomorphism

$$\phi : supp\,(\mathcal{F}) \to supp\,(\mathcal{P})$$

such that the induced labeling $l_\phi = l$?

Introduction
○○○●
An Illustration

Characterization
○○

Computation of TPL
○

Conclusion
○○

1

Characterization of feasible TPL

Given

  i. a set system or hypergraph $\mathcal{F}$,

  ii. a feasible TPL $l : \mathcal{F} \rightarrow \mathcal{P}$ where $\mathcal{P}$ is a path system from tree T and $supp(\mathcal{P}) = V(T)$,

what is the hypergraph isomorphism

$$\phi : supp\,(\mathcal{F}) \rightarrow supp\,(\mathcal{P})$$

such that the induced labeling $l_\phi = l$?

1

Characterization of feasible TPL

Given

 i.   a set system or hypergraph $\mathcal{F}$,

 ii.   a feasible TPL $l : \mathcal{F} \rightarrow \mathcal{P}$ where $\mathcal{P}$ is a path system from tree T and $supp(\mathcal{P}) = V(T)$,

what is the hypergraph isomorphism

$$\phi : supp\,(\mathcal{F}) \rightarrow supp\,(\mathcal{P})$$

such that the induced labeling $l_\phi = l$?

The characterization
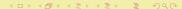
ICPPL + a filtering algorithm

anju s | anjuzabil@gmail.com
CS09S012
tree path labeling – a generalization of consecutive ones property of binary matrices
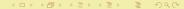
The characterization

ICPPL + a filtering algorithm

Special case

Interval assignment problem / COP

1. T is a path $\implies$ paths in T are intervals
2. Only pairwise intersection cardinality needs to be preserved $\implies$ ICPIA [NS09]
3. Higher level intersection cardinalities preserved by **Helly Property** – [Gol04]
4. *filter_1*, *filter_2* do not need the the **exit** conditions.
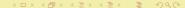
This problem is equivalent to Consecutive Ones Property of binary matrices [NS09]

Path Labeling → Graph Isomorphism

Application

References I

Martin Charles Golumbic.
Algorithmic graph theory and perfect graphs, volume 57 of *Annals of Discrete Mathematics*.
Elsevier Science B.V., 2004.
Second Edition.

Johannes Köbler, Sebastian Kuhnert, Bastian Laubner, and Oleg Verbitsky.
Interval graphs: Canonical representation in logspace.
Electronic Colloquium on Computational Complexity (ECCC), 17:43, 2010.

N.S. Narayanaswamy and R. Subashini.
A new characterization of matrices with the consecutive ones property.
Discrete Applied Mathematics, 157(18):3721–3727, 2009.