Tree Path Labeling of Path Hypergraphs

A Generalization of Consecutive Ones Property

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Motivation

Combinatorial algorithms

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Matrix reordering problems

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Interval labeling ⇋ **Consecutive Ones Property**

[KKLV10] **[FG65, Tuc72, BL76, NS09]**

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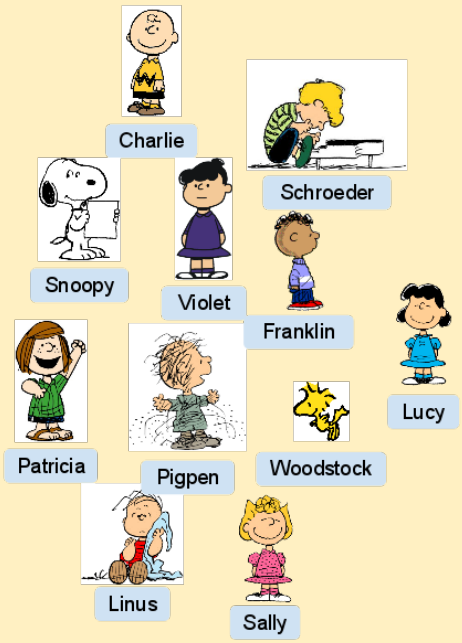
Tree Path Labeling

An Illustration

An Illustration
of Tree Path Labeling problem

Study Group Accommodation problem



Students

$$\mathbb{B} = \{\textbf{Ch}, \textbf{Sa}, \textbf{Fr}, \textbf{Sc}, \textbf{Lu}\}$$
$$\mathbb{T} = \{\textbf{Pa}, \textbf{Pi}, \textbf{Vi}, \textbf{Ch}\}$$
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Study groups

Study Group Accommodation problem

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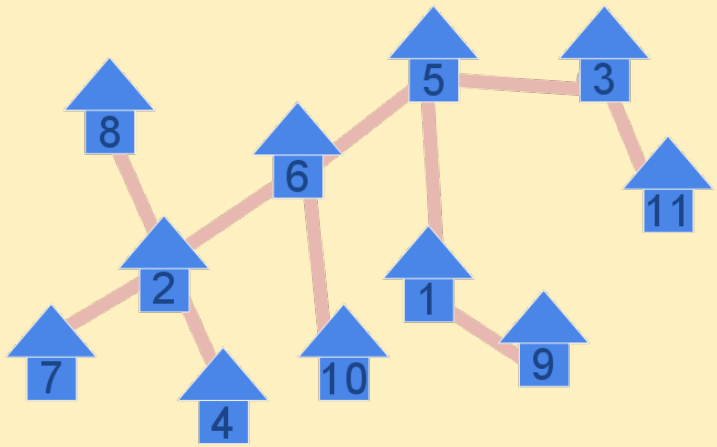
Study groups



Infinite Loop residential block

Study Group Accommodation problem

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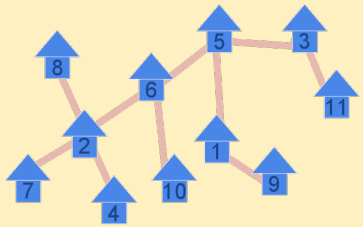
Study groups



Infinite Loop residential block

- A student may be in more than one study group but will be in at least one.
- There are equal number of single occupancy apartments in *Infinite Loop*.
- Streets connecting them do not form loops.

The problem

How should the students be allocated apartments such that students in each group should inhabit a (continuous) path?

Allocate Paths to Study Groups

tree path labeling

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tree path labeling

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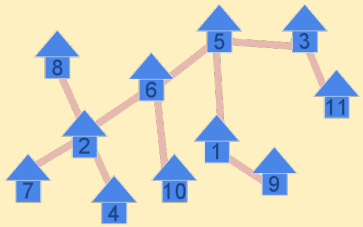
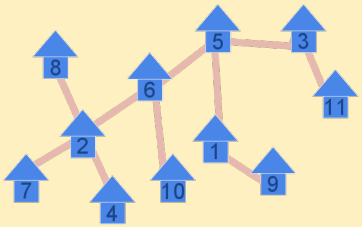


Study groups - $\mathbb{B}, \mathbb{T}, \mathbb{W}, \mathbb{F}$

Allocate Paths to Study Groups

tree path labeling



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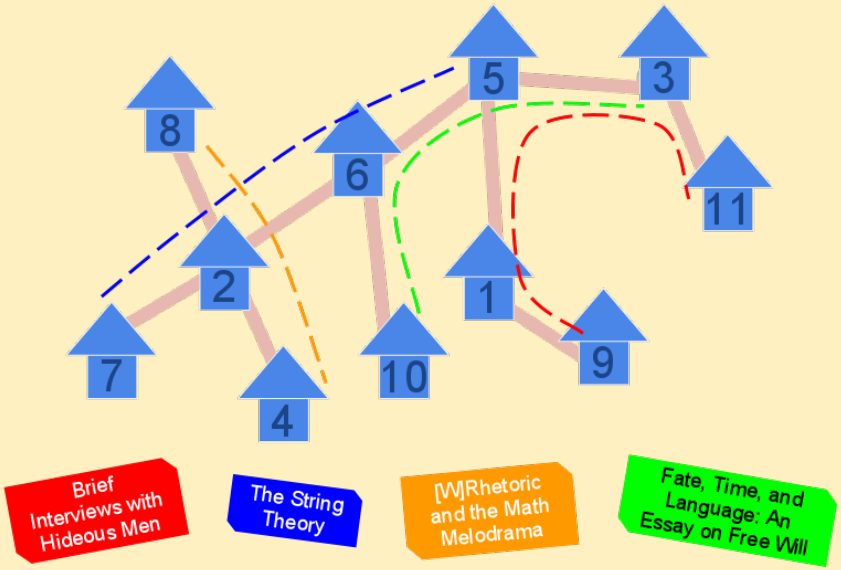
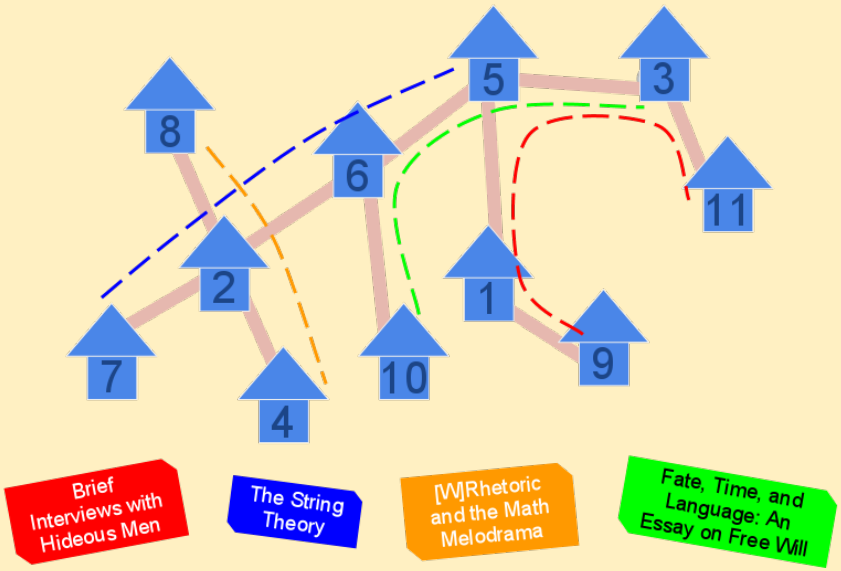


Study groups - $\mathbb{B}$, $\mathbb{T}$, $\mathbb{W}$, $\mathbb{F}$

Allocate Paths to Study Groups

tree path labeling - feasible?



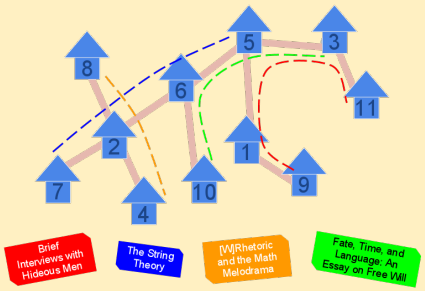
Is this feasible?

Allocate Apartments to Students

path graph isomorphism/feasibility bijection

Allocate Apartments to Students

path graph isomorphism/feasibility bijection



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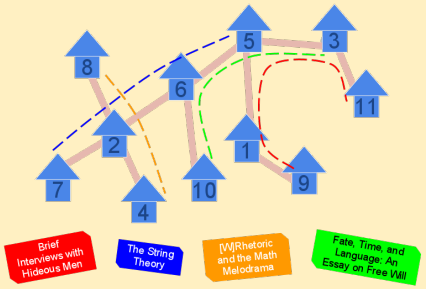
Allocate Apartments to Students

path graph isomorphism/feasibility bijection



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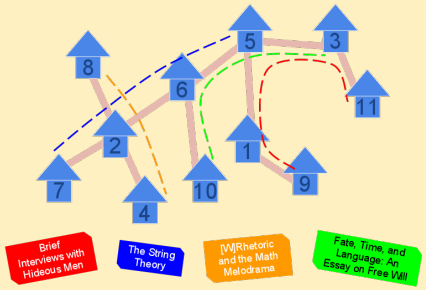
\end{aligned}$$

In this case, **is feasible**.

Allocate Apartments to Students

path graph isomorphism/feasibility bijection



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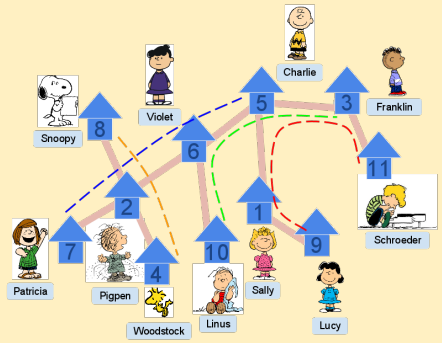

Basic terminology

a crash course on the TPL machinery

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The set of study groups $\{\mathbb{B}, \mathbb{T}, \mathbb{W}, \mathbb{F}\} \rightarrow$ Hypergraph

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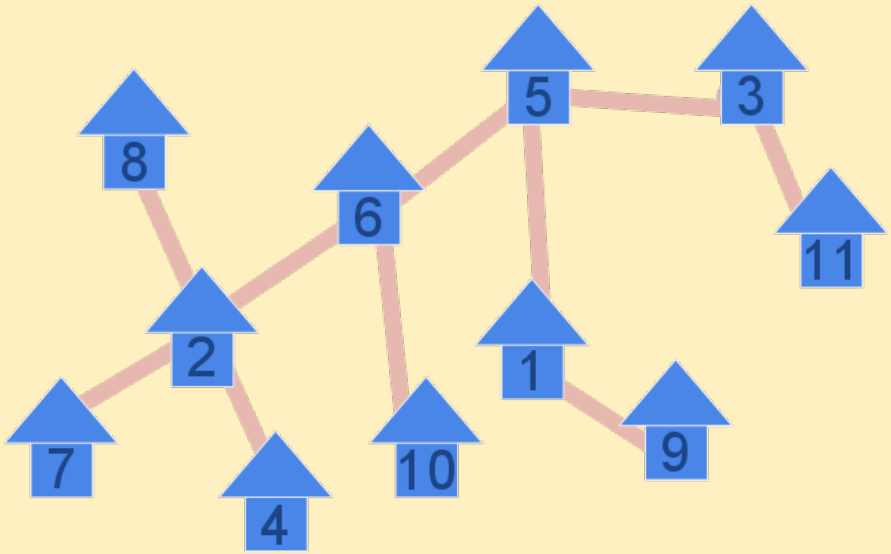

Infinite Loop residential block $\longrightarrow$ Target tree

Basic terminology

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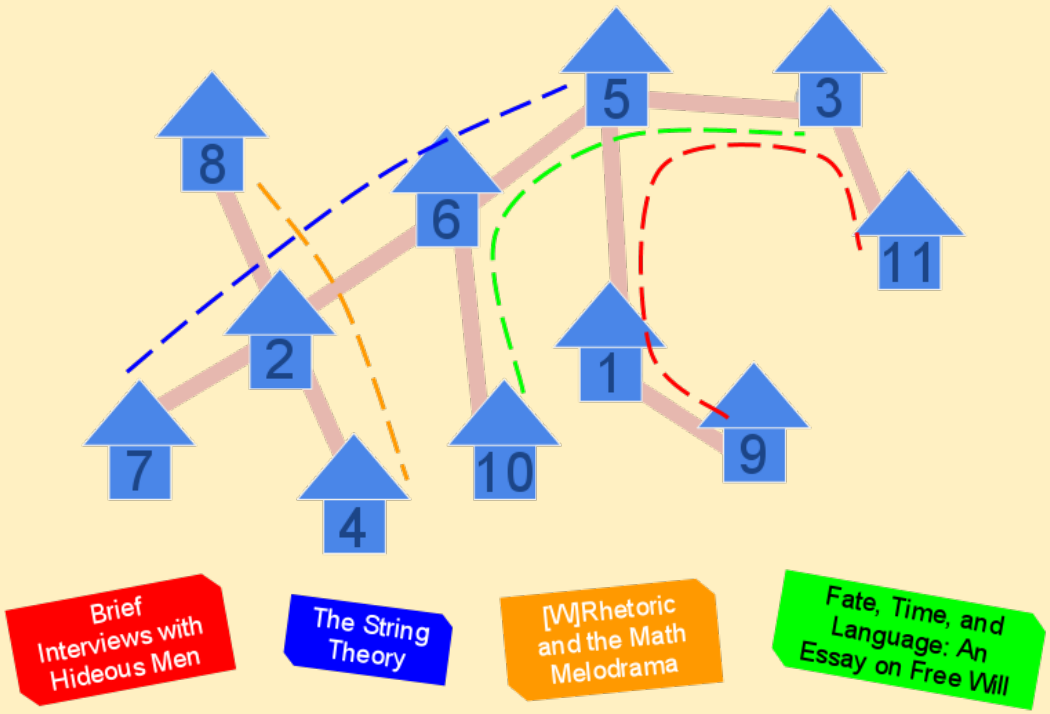

Study group path allocation → Tree Path Labeling

Basic terminology

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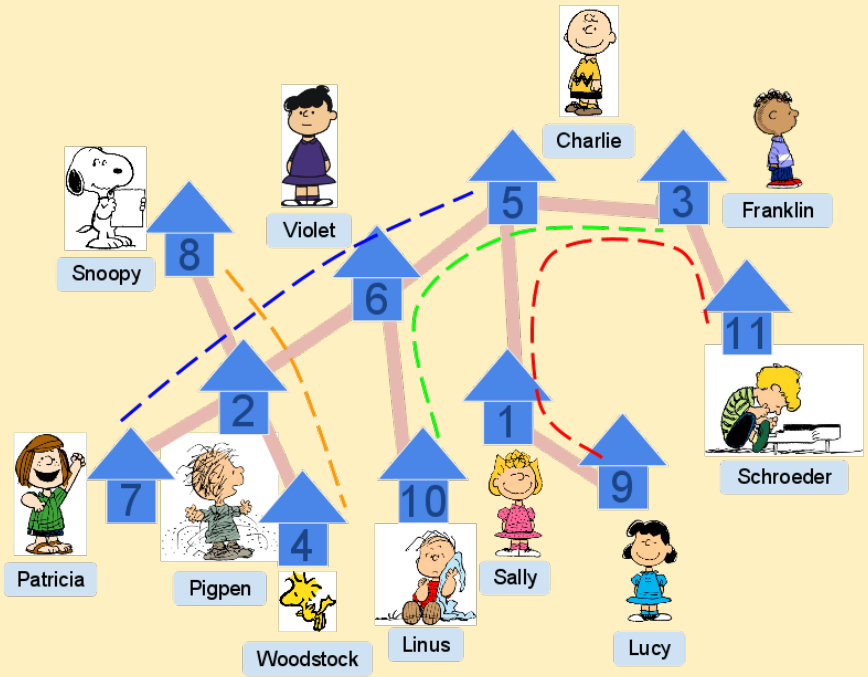

The apartment allocation → Path Hypergraph Isomorphism

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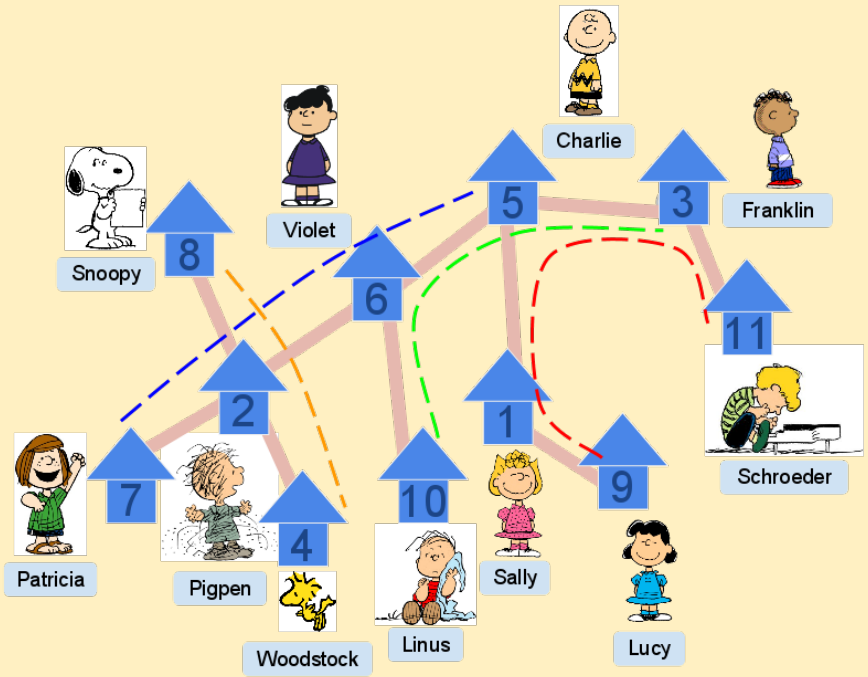

The apartment allocation → Path Hypergraph Isomorphism

The problems studied

1. Compute Feasible Path Labeling

Computation of a feasible tree path labeling (FTPL) if any.

2. Compute k-subdivided Star Path Labeling

Computation of an FTPL if any, if target tree is a k-subdivided star.

3. Feasible Tree Path Labeling

Characterization of an FTPL and finding the feasibility bijection/hypergraph isomorphism

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3.

Characterization

- Three way intersection cardinality preservation
- Filtering and pruning algorithm

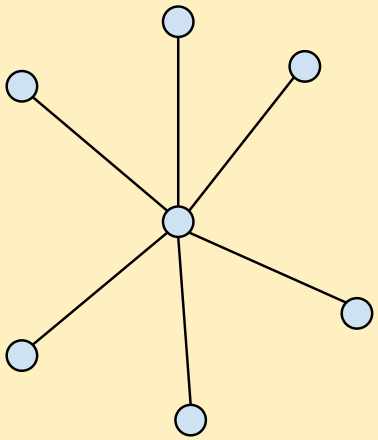
Special case

Interval assignment problem / COP

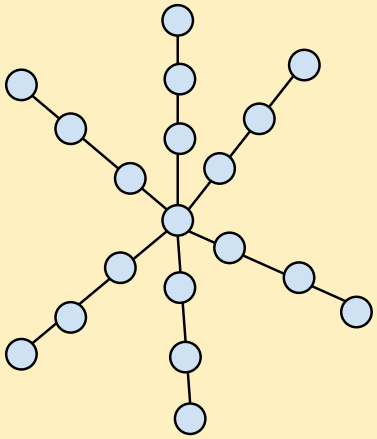
1. T is a path $\Longrightarrow$ paths in T are intervals
2. Only pairwise intersection cardinality needs to be preserved $\Longrightarrow$ ICPIA [NS09]
3. Higher level intersection cardinalities preserved by **Helly Property** – [Gol04]
4. *filter_1*,*filter_2* do not need the the **exit** conditions.

This problem is equivalent to Consecutive Ones Property of binary matrices [NS09]

2.

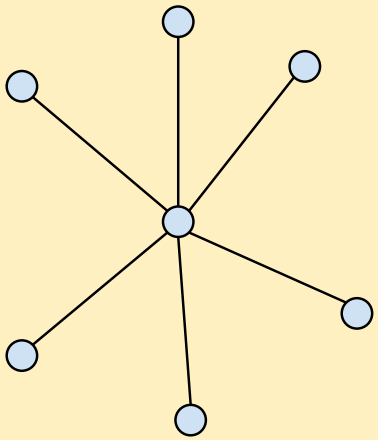


(a)



(b)

2.



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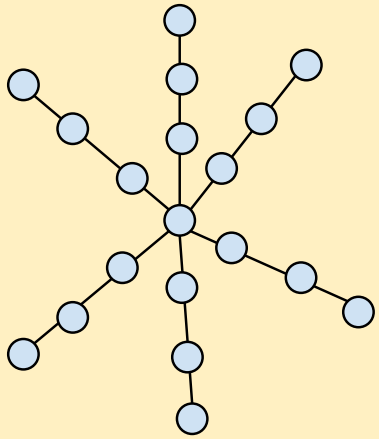

(b)

Compute TPL on k subdivided stars

- each rays of the k sub star are independent intervals when root is excluded.
- each ray is considered independently as interval assignment problem

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Combinatorial algorithms

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Matrix reordering problems

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Interval labeling ⇋ **Consecutive Ones Property**

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Tree Path Labeling

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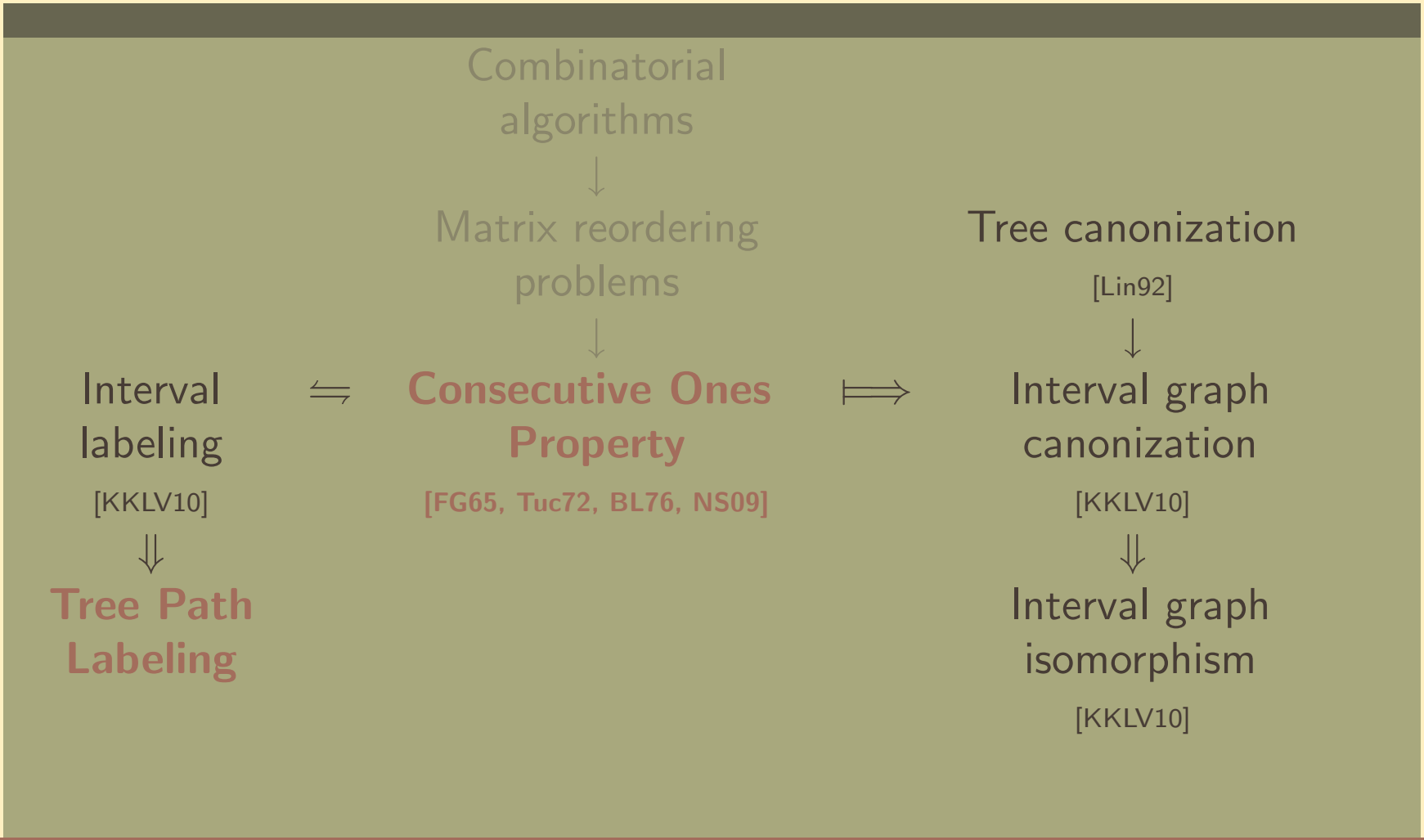
Combinatorial algorithms
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[KKLV10]
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Tree canonization
[Lin92]
Interval graph canonization
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Interval graph isomorphism
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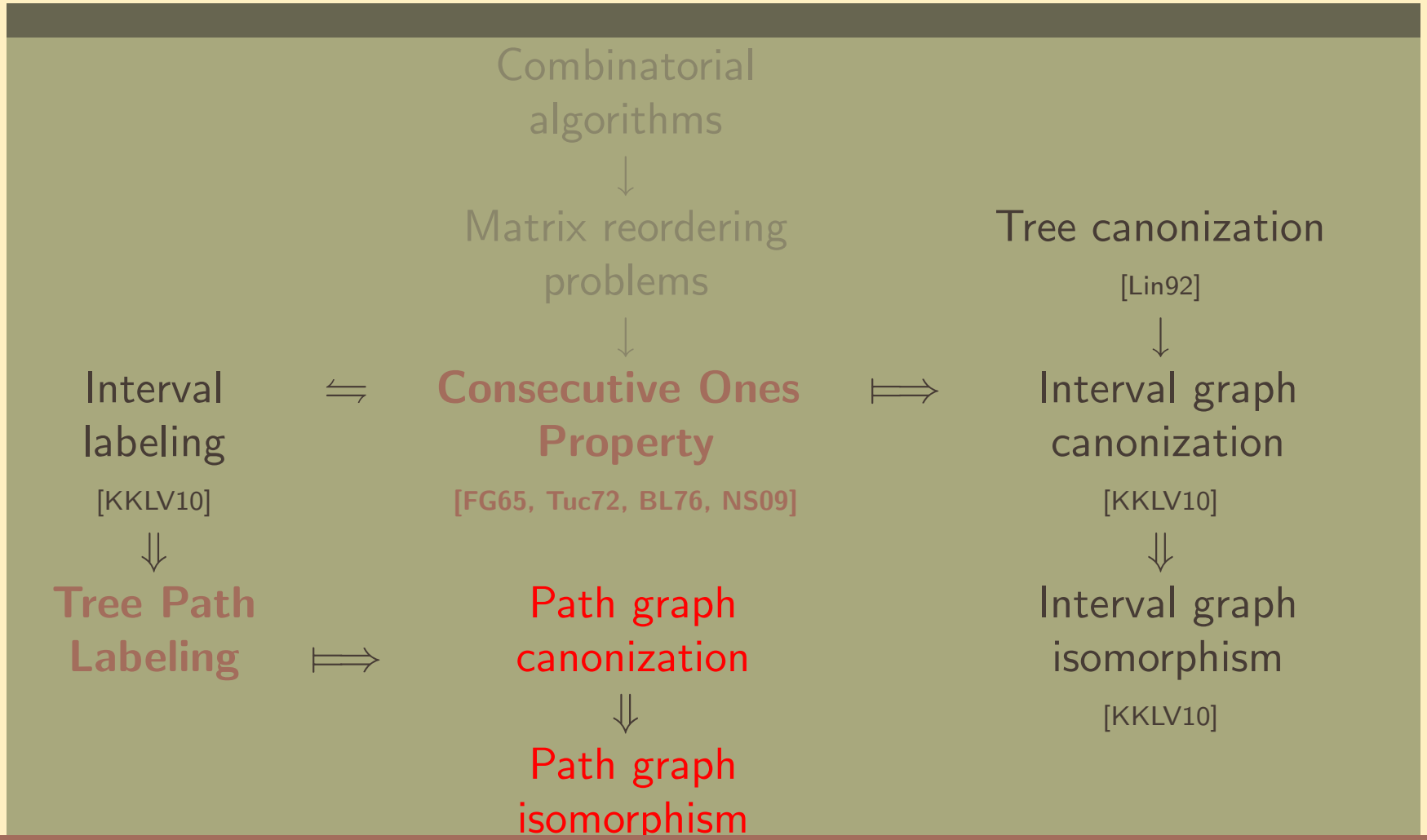
Motivation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Thank You

Q & A

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