Tree Path Labeling of Path Hypergraphs

A Generalization of Consecutive Ones Property

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as part of **M. S.** by Research
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Consecutive Ones $\longrightarrow$ Path Labeling

The motivation

[get a better image!]

m1.pdf

[A 30 sec slide on COP? TBD IN THE END]

An Illustration

[say something less apologetic!]

A Study Group Housing problem

[CHUCK THE TEXTUAL SET DEFINITIONS. ONLY DIAGRAMS.]

- A set of n students arrive for a summer course, say $\{$**Pa**, **Pi**, **Sn**, **Wo**, **Vi**, **Li**, **Ch**, **Sa**, **Fr**, **Sc**, **Lu**$\}$, $n = 11$ [a venn diagram of just the universe]

A Study Group Housing problem

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- A set of n students arrive for a summer course, say $\{\textbf{Pa}, \textbf{Pi}, \textbf{Sn}, \textbf{Wo}, \textbf{Vi}, \textbf{Li}, \textbf{Ch}, \textbf{Sa}, \textbf{Fr}, \textbf{Sc}, \textbf{Lu}\}$, $n = 11$ [a venn diagram of just the universe]
- They form m study groups, say $\{\mathbb{B}, \mathbb{T}, \mathbb{W}, \mathbb{F}\}$, $m = 4$ [a venn diagram of the grouping]

A Study Group Housing problem

- A student may be in more than one study group but will be in at least one, say

  [repeat the venn diagram of the grouping (before this bullet)]

$$
\begin{aligned}
\mathbb{B} &= \{\textbf{Ch}, \textbf{Sa}, \textbf{Fr}, \textbf{Sc}, \textbf{Lu}\} \\
\mathbb{T} &= \{\textbf{Pa}, \textbf{Pi}, \textbf{Vi}, \textbf{Ch}\} \\
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\end{aligned}
$$

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\end{aligned}$$

- There are n single occupancy apartments in *Infinite Loop*.

  [image of infinite loop]

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\end{aligned}$$

- There are n single occupancy apartments in *Infinite Loop*.

  [image of infinite loop]

- Streets connecting them do not form loops.

A Study Group Housing problem

The problem

How should the students be allocated apartments such that each study group has the least distance to travel for a discussion?

A Study Group Housing problem

[update to the example in synopsis doc]

a1.pdf

A Study Group Housing problem

[update to the example in synopsis doc]

a2.pdf

A Study Group Housing problem

[update to the example in synopsis doc]

a3.pdf

Tree Path Labeling of Set Systems

The combinatorial problem terminology

[SPLIT THIS INTO FOUR SLIDES WITH CORRESPONDING PREVIOUS IMAGES REPEATED]

Terminology

- The set of study groups $\{\mathbb{B}, \mathbb{T}, \mathbb{W}, \mathbb{F}\} \to$ Hypergraph

Tree Path Labeling of Set Systems

The combinatorial problem terminology

[SPLIT THIS INTO FOUR SLIDES WITH CORRESPONDING PREVIOUS IMAGES REPEATED]

Terminology

- The set of study groups $\{\mathbb{B}, \mathbb{T}, \mathbb{W}, \mathbb{F}\} \rightarrow$ Hypergraph
- *Infinite Loop* block $\rightarrow$ Target tree

Tree Path Labeling of Set Systems

The combinatorial problem terminology

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- The set of study groups $\{\mathbb{B}, \mathbb{T}, \mathbb{W}, \mathbb{F}\} \rightarrow$ HYPERGRAPH
- *Infinite Loop* block $\rightarrow$ TARGET TREE
- Study group path allocation $\rightarrow$ TREE PATH LABELING (TPL)

Tree Path Labeling of Set Systems

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Terminology

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- *Infinite Loop* block $\longrightarrow$ TARGET TREE
- Study group path allocation $\longrightarrow$ TREE PATH LABELING (TPL)
- The apartment allocation $\longrightarrow$ PATH HYPERGRAPH ISOMORPHISM

Tree Path Labeling of Set Systems

The combinatorial problem

[DELETE.]

Terminology [contd.]

There *exists* an apartment allocation that “fits” the route mapping

Tree Path Labeling of Set Systems

The combinatorial problem

[DELETE.]

Terminology [contd.]

There *exists* a hypergraph isomorphism that “fits” the TPL

Tree Path Labeling of Set Systems

The combinatorial problem

[DELETE.]

Terminology [contd.]

There *exists* a hypergraph isomorphism that "fits" the TPL
$\longrightarrow$ the TPL is FEASIBLE

Tree Path Labeling of Set Systems

The combinatorial problem

[DELETE.]

Terminology [contd.]

There *exists* a hypergraph isomorphism that “fits” the TPL
$\longrightarrow$ the TPL is FEASIBLE

There *exists* an apartment allocation that gives the optimal route mapping

Tree Path Labeling of Set Systems

The combinatorial problem

[DELETE.]

Terminology [contd.]

There *exists* a hypergraph isomorphism that “fits” the TPL
$\longrightarrow$ the TPL is FEASIBLE

There *exists* a hypergraph isomorphism that gives paths/adjacent vertices in tree

Tree Path Labeling of Set Systems

The combinatorial problem

[DELETE.]

Terminology [contd.]

There *exists* a hypergraph isomorphism that "fits" the TPL
$\longrightarrow$ the TPL is FEASIBLE

There *exists* a hypergraph isomorphism that gives paths/adjacent vertices in tree
$\longrightarrow$ the hypergraph is a PATH HYPERGRAPH

Tree path labeling of path hypergraphs

The two problems

1

Characterization of a *feasible TPL* and finding the certificate for feasibility i.e. *hypergraph isomorphism*

Tree path labeling of path hypergraphs

The two problems

1

Characterization of a *feasible TPL* and finding the certificate for feasibility i.e. *hypergraph isomorphism*

2

Computation of a *feasible TPL* if any

[DELETE]

1

Characterization of feasible TPL

Given

i. a set system or hypergraph $\mathcal{F}$,

ii. a feasible TPL $\ell : \mathcal{F} \to \mathcal{P}$ where $\mathcal{P}$ is a path system from tree T and $supp(\mathcal{P}) = V(T)$,

what is the hypergraph isomorphism

$$\underline{\phi : supp\,(\mathcal{F}) \to supp\,(\mathcal{P})}$$

such that the induced labeling $\underline{\ell_\phi = \ell}$?

[DELETE]

2

Computing a feasible TPL

Given hypergraph $\mathcal{F}$ with certain properties and a k-subdivided star T, <u>can we find a feasible TPL ℓ to T?</u>

1

Characterization of feasible TPL

The characterization

ICPPL + a filtering algorithm

[a]: [TBD Write the theorem]

The characterization

ICPPL + a filtering algorithm

[a]: [TBD Write the theorem]

[DELETE]

2

Computing a feasible TPL

Given hypergraph $\mathcal{F}$ with certain properties and a k-subdivided star T, <u>can we find a feasible TPL ℓ to T?</u>

Special case

Interval assignment problem / COP

1. T is a path $\Longrightarrow$ paths in T are intervals [a: [quick illustration]]
2. Only pairwise intersection cardinality needs to be preserved $\Longrightarrow$ ICPIA [NS09]
3. Higher level intersection cardinalities preserved by **Helly Property** – [Gol04]
4. *filter_1*,*filter_2* do not need the the **exit** conditions. [a: [is this cryptic?]]

This problem is equivalent to Consecutive Ones Property of binary matrices [NS09]

Path Labeling $\longrightarrow$ Graph Isomorphism

Application

[get a better image!]

m1.pdf

Path Labeling → Graph Isomorphism

Application

[get a better image!]

m2.pdf

Path Labeling $\longrightarrow$ Graph Isomorphism

Application

[get a better image!]

m3.pdf

Thank You

Q & A

References

[improve - add some jazz. this is a notional slide only for offline reference.]

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