Response to M.S. Thesis Review Comments Thesis Title: *Generalization of the Consecutive-ones Property*

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1 Reviewers

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The reviewers careful comments are acknowledged and appreciated. Responses to the same are given in following sections.

2 Response to Reviewer 1

2.1 Comments on corrections

Responses to the comments in review document Page 2 items 0 to 7.

0. Comment: **In thesis Page 7, Definition 1.3.7: After $x_m \leq a$, you need to add $x_m \neq a$.**
   Response: True, earlier definition did not accomodate the fact $x_m \leq x_m$. Suggested change made in Page 7 to the definiton of Poset.
1. Comment: **In thesis Page 7, Definition 1.3.8: E is a set of "unordered" pairs. You need to add the word "unordered"**
   Response: Suggested change made in to definition of Undirected Graph Page 7 Definition 1.3.8 (1).

2. Comment: **In thesis Page 8, A tree is a *connected* and acyclic graph**
   Response: The word "connected" has been added to the definition of Tree in Page 7 Definition 1.3.8 (3).

3. Comment: **In thesis Page 8, Definition 1.3.10 you need to add that one is looking at intervals of integers. Also define isomorphism between graphs for the sake of completeness**.
   Response: Definition of Interval graph has been changed to say "a set of intervals (of integers)" in Page 8, Definition 1.3.10. Graph isomorphism definition has been added in Page 7, Definition 1.3.8(4) as follows:

   A ***graph isomorphism*** *between two graphs* G_1 *and* G_2 *is a bijection* $\phi : V(G_1) \to V(G_2)$ *such that* $u, v \in V(G_1)$ *are adjacent in* G_1 *if and only if* $\phi(u), \phi(v)$ *are adjacent in* G_2*. If such a bijection exists, it is said that* G_1 *is* ***isomorphic*** *to* G_2*, denoted by* $G_1 \cong G_2$*.*

4. Comment: **In thesis Page 8, Definition 1.3.10 definition of path graph: You need to emphasise that $\mathcal{P}$ forms a set system**
   Response: This change has been made - "a set system of paths $\mathcal{P}$" in Page 8 Definition 1.3.10(2).

5. Comment: **In thesis Page 12, third para of Section 1.5: you need to write it more precisely and carefully conveying the idea. As it is it is not clear what you are saying.**
   Response: The problem Compute feasible tree path labeling and its result have been described formally and precisely in Chapter 3 (New results). The aforementioned paragraph describes the same informally. These informal sentences have been footnoted with the corresponding definitions and theorems from Chapter 3. This change is in Page 12.

6. Comment: **In thesis Page 28, Definition 7 (of binary operator /): it is not defined precisely. What do you mean by "b is a new element added.."?**.
   Response: This was in thesis Page 26. A/B is created by removing the elements of second operand set B from the first operand set A and introducing a new element *not* in the universe to A to represent set B. This is a set operation mentioned in [MM96] (they use "B" to denote the new element to represent set B which is also confusing) which is eventually used to decompose permutation of elements in A to permutation of elements in A/B and permutation of elements in B. Perhaps it would be clearer if b is replaced by x_B. This change has been made in Page 26 Definition 2.2.4 (7) and other mentions of this operation "/" Page 27 item (iii) and

h has been replaced with x_H in Corollary 2.2.8. There was also a typo in Page 27 paragraph under Corollary 2.2.8 item (ii) where "$U \setminus H$" was written instead of U/H. This has also been corrected.

7. Comment: **In thesis Page 39 Definition 2.3.1: the definition of $A_\mathcal{F}$ is not clear. Where is the dependence on $\mathcal{F}$?**
   Response: This was in thesis Page 36. It is true, there is no dependence in $\mathcal{F}$. It is only dependent on the universe U of $\mathcal{F}$. All occurences of $A_\mathcal{F}$ has been changed to A_U in Page 36.

2.2 Other comments

The following are a few observations mentioned in the review document with my responses/changes.

1. Observation: (in Page 1, second last paragraph) **The result on characterization of TPL in the thesis is a variant of the 1978 JCT(B) result due to Fournier who showed that two hypergraphs $H_1 = (V_1, X)$ and $H_2 = (V_2, Y)$ such that $|X| = |Y| = m$ are isomorphic iff there is a bijection ϕ on $I = \{1, ..., m\}$ s.t. for every $L \subset I, |\bigcap_{i \in L} X_i| = |\bigcap_{j=\phi(i), i \in L} Y_j|$**
   Response: This is an important observation and was not cited in the thesis. Also [Fou80] generalizes [BR72] and [FG65] by characterizing the isomorphism of two hypergraphs by means of equicardinality of certain edge intersections and the exclusion of certain pairs of subhypergraphs. TPL characterization is a special case of this with one of the hypergraphs having hyperedges that are paths from a tree and the characterization only uses edge intersections of at most 3 hyperedges. This has been added in the Conclusions chapter Page 72 under *Graph isomorphism and logspace canonization.*

2. Observation: (in Page 2, first paragraph) **A k-subdivided ~~tree~~ [star] is a tree with a single central vertex with a number of paths each of k edges emanating from it.**
   Response: k-subdivided star as defined in this thesis (Section 3.2.4, Page 43) is such that each of the aforementioned paths are of length "$k + 1$" edges since each such path has k nodes apart from the central node and the leaf node, i.e. $k + 2$ nodes. No change in the thesis.

3 Response to Reviewer 2

1. Observation: **The definition of k-subdivided star is not given in the thesis. What I understood is that it has a root node v of degree k, the degree of all other nodes is at most 2, and each path from root to leaf is $k+2$.**
   Response: The definition of k-subdevided star was given in Section 3.2.4, Definition 3.2.6. Perhaps this has not been sufficiently referred to, hence references to this definition has been made in Chapter 1 Page 13 second last paragraph where it is first referred and in Chapter 3 Page 39 second paragraph where COMPUTE k-SUBDIVIDED STAR PATH LABELING problem is defined.

   The reviewer's understanding is correct except that the root node can have any degree, thus a k-subdivided star is a collection of graphs with degree of all non-root nodes as at most 2, each path from root to leaf is $k+2$ and the root having any degree. See thesis Figure 1.5 in page 13 for examples.

2. Comment: **The proof of Lemma 3.3.4 is unclear.**
   Response: The proof was correct but terse. It has been elaborated to as follows:

   Lemma: Let $(\mathcal{F}, \ell)$ be an ICPPL and $\ell(S_i) = P_i$, $S_i \in \mathcal{F}$, $1 \le i \le 4$. Then, $|\cap_{i=1}^4 S_i| = |\cap_{i=1}^4 P_i|$.

   Proof: Consider the set of set intersections with S_1 $\mathcal{F}' = \{S_2 \cap S_1, S_3 \cap S_1, S_4 \cap S_1\}$ and let ℓ' be a tree path labeling of $\mathcal{F}'$ such that $\ell'(S_i \cap S_1) = P_i \cap P_1, 2 \le i \le 4$. Clearly, $P_2 \cap P_1$, $P_3 \cap P_1$, and $P_4 \cap P_1$ are subpaths of path P_1, thus equivalent to intervals. Due to the three way intersection cardinality preservation property of the ICPPL $(\mathcal{F}, \ell)$, this new tree path labeling $(\mathcal{F}', \ell')$ preserves pairwise intersection cardinalities. Now by applying Lemma 3.3.1 to the sets in $\mathcal{F}'$ and their corresponding path images due to ℓ', it follows that $|\cap_{i=1}^4 S_i| = |\cap_{i=1}^4 P_i|$.

3. Comment: **While studying Feasible Tree Path Labeling Problem in Section 3.3, two filters are mentioned and their pseudocodes given. If possible, state the problem in words. ... Explicitly state the roles of these two filters in the algorithm with proper figures.**
   Response: The problems are described in words but they were part of a long narrative. It has been summarised now in Page 44 before going into details of the algorithms as follows:

Filter 1. `filter common leaf` *Refine $\mathcal{F}$ such that the resulting labeling will not have paths that share a leaf thus each leaf being unique to a path. This is done* *by breaking the paths into subpaths and their corresponding preimage sets as described in Algorithm 1.*

Filter 2. `filter fix leaf` *Find the element in universe U that maps to each leaf in T as described in Algorithm 2.*

Remove the leaves from T and their corresponding preimages from U and call the filters again. This is repeated until the resulting truncated tree is a path. The remaining mapping can be found using ICPIA.

Hopefully this makes the role of the filter algorithms clear. I felt that describing details of the pseudocode in words in the narrative would be tedious and redundant and the pseudocode has comments that describe what is being done whenever it is not obvious. Regarding figures, Section 3.4 in page 52 has an example worked out in detail with figures. This was added with the intention of illustrating the filter algorithms.

4. Comment: **It is shown that TREE PATH LABELING PROBLEM on arbitrary trees can be solved polynomial time if the two subproblems, identified in section 3.6 in the thesis are polynomial time solvable. The formal definition of these two subproblems are not given. However, in page 71 item numbers 1 and 2 the subproblems are described with several notations, where the motivation is unclear.**
Response: The prime submatrices partial order theory and notations are required for a formal definition. The two subproblems are defined now as Find Overlap Component Partition Subtrees and Find MUB Feasible TPL and motivation is given in Page 61 before setting up the theory and formal defintion is given in page 66 after setting up the theory required to define it.

Find Overlap Component Partition Subtrees

`Input`	*A hypergraph $\mathcal{F}$ with its in-tree partitions $\{\mathbb{P}_1, \ldots, \mathbb{P}_r\}$ and a tree T.*
`Question`	*Compute a partition of T into subtrees $\{T_1, \ldots, T_r\}$ such that there exists an ICPPL for $mub(\mathbb{P}_i)$ from subtree T_i for all $i \in [r]$ and there exists an ICPIA for all other prime submatrices in $\mathbb{P}_i$ as claimed in Lemma 3.6.8.*

Find MUB Feasible TPL

`Input`	*An in-tree partition $\mathbb{P}_i$ of a hypergraph $\mathcal{F}$ and a subtree T_i of a tree T.*
`Question`	*Compute a feasible TPL for $mub(\mathbb{P}_i)$ from subtree T_i.*

5. Comment: **... it is shown that consecutive 1s property testing for a ~~graph~~ [hypergraph] is solvable in LOGSPACE. *This is a very important contribution.***

   Response: Since this is a conclusion from earlier results, it has not been mentioned as a completely “new” result. It is mentioned in the abstract as a derived result.

References

[BR72] C Berge and R Rado. Note on isomorphic hypergraphs and some extensions of whitney’s theorem to families of sets. *Journal of Combinatorial Theory, Series B*, 13(3):226 – 241, 1972.

[FG65] D. R. Fulkerson and O. A. Gross. Incidence matrices and interval graphs. *Pac. J. Math.*, 15:835–855, 1965.

[Fou80] J.-C. Fournier. Isomorphismes d’hypergraphes par intersections équicardinales d’arêtes et configurations exclues. *J. Comb. Theory, Ser. B*, 29(3):321–327, 1980.

[MM96] J. Meidanis and E. G. Munuera. A theory for the consecutive ones property. In *Proc. of the III South American Workshop on String Processing*, volume 88, pages 194–202, Recife, Brazil, 1996.