Tree Path Labeling of Path Hypergraphs

Generalization of the Consecutive-ones Property

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as part of **M. S.** by Research
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CSE, IITM, Chennai - 36

25 March 2013

An Illustration

- To introduce the combinatorial problem of TPL.

Study Group Accommodation problem

- A set of n **students** arrive for a summer course, say $\{a, b, c, d, e, f, g, h, i, j, k\}$, $n = 11$.

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- There are n single occupancy **apartments** in the university campus for their accommodation.
- All these apartments are placed such that streets connecting them do not form loops - **streets form a tree**

Study Group Accommodation problem

The problem

How should the students be allocated apartments such that:

- students of each study group are neighbours?

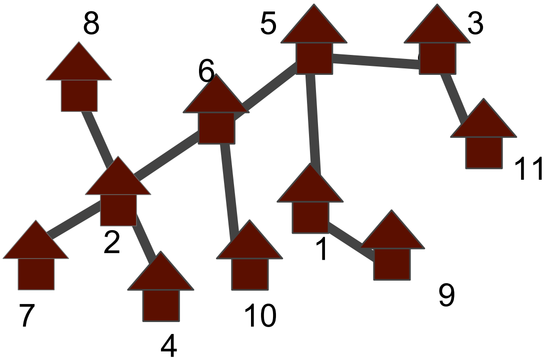
Study Group Accommodation problem

The problem

How should the students be allocated apartments such that:

- students of each study group are neighbours?
- i.e. a study group forms **a path in the tree**.

Study Group Accommodation problem



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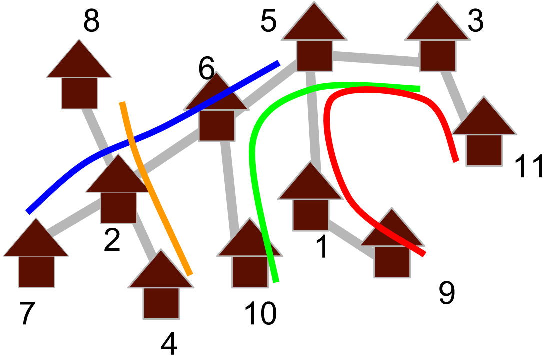
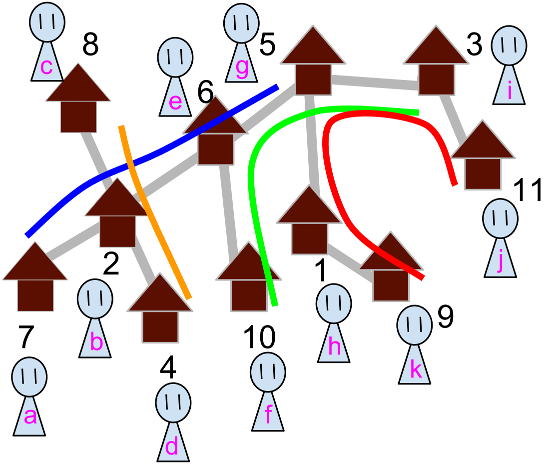


$R = \{g, h, i, j, k\}$
$\rightarrow \{9, 1, 5, 3, 11\}$
$B = \{a, b, e, g\}$
$\rightarrow \{7, 2, 6, 5\}$
$O = \{c, b, d\}$
$\rightarrow \{4, 2, 8\}$
$G = \{e, f, g, i\}$
$\rightarrow \{10, 6, 5, 3\}$

Study Group Accommodation problem



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Tree Path Labeling of Path Hypergraphs

- The set of study groups → **Set system / Hypergraph**

Tree Path Labeling of Path Hypergraphs

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- The streets with apartments $\rightarrow$ **Target tree**

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- The streets with apartments → **Target tree**
- The path mapping to study groups → **Tree Path Labeling (TPL)**
- The apartment allocation → **Path Hypergraph Isomorphism**

Tree Path Labeling of Path Hypergraphs

There *exists* an apartment allocation that “fits” the path mapping

Tree Path Labeling of Path Hypergraphs

There *exists* a hypergraph isomorphism that “fits” the TPL

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$\Rightarrow$ the TPL is FEASIBLE
There *exists* an apartment allocation that gives some study group path mapping

Tree Path Labeling of Path Hypergraphs

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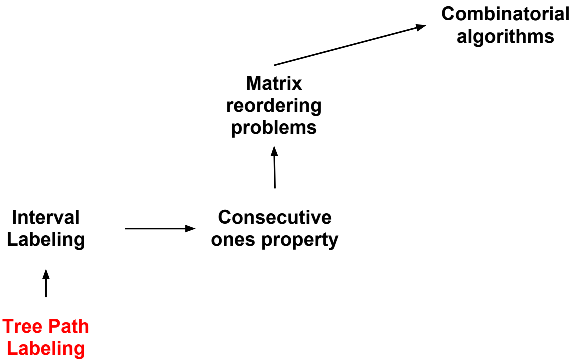
$\Rightarrow$ the TPL is FEASIBLE
There *exists* a hypergraph isomorphism that gives at least one feasible TPL

Tree Path Labeling of Path Hypergraphs

There *exists* a hypergraph isomorphism that "fits" the TPL
$\Rightarrow$ the TPL is FEASIBLE
There *exists* a hypergraph isomorphism that gives at least one feasible TPL
$\Rightarrow$ the hypergraph is a PATH HYPERGRAPH

Consecutive Ones → Path Labeling

The motivation



2

Computing a feasible TPL

Given hypergraph $\mathcal{F}$ with certain properties and a k-subdivided star T, <u>can we find a feasible TPL ℓ to T?</u>

1

Characterization of feasible TPL

The characterization

ICPPL + a filtering algorithm

The characterization

ICPPL + a filtering algorithm

2

Computing a feasible TPL

Given hypergraph $\mathcal{F}$ with certain properties and a k-subdivided star T, can we find a feasible TPL ℓ to T?

Special case

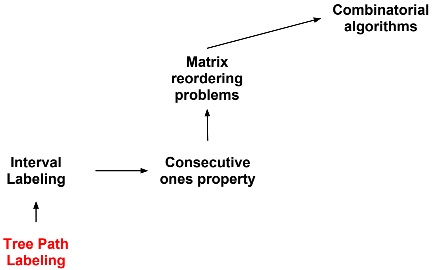
Interval assignment problem / COP

1. T is a path $\Longrightarrow$ paths in T are intervals
2. Only pairwise intersection cardinality needs to be preserved $\Longrightarrow$ ICPIA [NS09]
3. Higher level intersection cardinalities preserved by **Helly Property** – [Gol04]
4. *filter_1*,*filter_2* do not need the the **exit** conditions.

This problem is equivalent to Consecutive Ones Property of binary matrices [NS09]

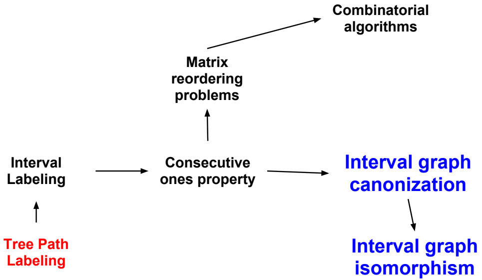
Path Labeling → Graph Isomorphism

Application



Path Labeling $\rightarrow$ Graph Isomorphism

Application



Path Labeling → Graph Isomorphism

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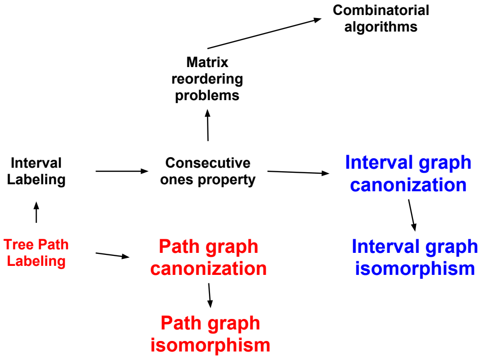

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