

Response to M.S. Thesis Reviewers
*Thesis Title “Generalization of the
Consecutive-ones Property”*

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1 Reviewers

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2 Response to Reviewer 1

2.1 Observations

The following are a few observations mentioned in the review document with my responses/changes.

1. Observation: (in Page 1, second last paragraph) **The result on characterization of TPL in the thesis is a variant of the 1978 JCT(B) result due to Fournier who showed that two hypergraphs $H_1 = (V_1, X)$ and $H_2 = (V_2, Y)$ such that $|X| = |Y| = m$ are isomorphic iff there is a bijection ϕ on $I = \{1, \dots, m\}$ s.t. for every $L \subset I$, $|\bigcap_{i \in L} X_i| = |\bigcap_{j=\phi(i), i \in L} Y_j|$**

Response: This is an important observation and was not cited in the thesis. Also [Fou80] generalizes [BR72] and [FG65] by characterizing the isomorphism of two hypergraphs by means of equicardinality of certain edge intersections and the exclusion of certain pairs of subhypergraphs. TPL characterization is a special case of this with one of the hypergraphs having hyperedges that are paths from a tree and the characterization only uses edge intersections of at most 3 hyperedges. This has been added in the Conclusions chapter Page 72 under *Graph isomorphism and logspace canonization*.

2. Observation: (in Page 2, first paragraph) **A k -subdivided tree is a tree with a single central vertex with a number of paths each of k edges emanating from it.**

Response: k -subdivided as defined in this thesis (Section 3.2.4, Page 43) is such that each of the aforementioned paths are of length " $k + 1$ " edges since each such path has k nodes apart from the central node and the leaf node, i.e. $k + 2$ nodes. No change in the thesis.

2.2 Comments

Responses to the comments in review document Page 2 items 0 to 7.

- 0 Comment: **In thesis Page 7, Definition 1.3.7: After $x_m \leq a$, you need to add $x_m \neq a$.**

Response: True, earlier definition did not accomodate the fact $x_m \leq x_m$. Suggested change made in Page 7 to the definiton of Poset.

1. Comment: **In thesis Page 7, Definition 1.3.8: E is a set of "unordered" pairs. You need to add the word "unordered"**

Response: Suggested change made in to definition of Undirected Graph Page 7 Definition 1.3.8 (1).

2. Comment: **In thesis Page 8, A tree is a *connected* and acyclic graph**

Response: The word "connected" has been added to the definition of Tree in Page 7 Definition 1.3.8 (3).

3. Comment: **In thesis Page 8, Definition 1.3.10 you need to add that one is looking at intervals of integers. Also define isomorphism between graphs for the sake of completeness.**

Response: Definition of Interval graph has been changed to say "a set of intervals (of integers)" in Page 8, Definition 1.3.10. Graph isomorphism definition has been added in Page 7, Definition 1.3.8(4) as follows: A **graph isomorphism** between two graphs G_1 and G_2 is a bijection $\phi : V(G_1) \rightarrow V(G_2)$ such that $u, v \in V(G_1)$ are adjacent in G_1 if and only if $\phi(u), \phi(v)$ are adjacent in G_2 . If such a bijection exists, it is said that G_1 is **isomorphic** to G_2 , denoted by $G_1 \cong G_2$.

4. Comment: **In thesis Page 8, Definition 1.3.10 definition of path graph: You need to emphasise that \mathcal{P} forms a set system**

Response: This change has been made - "a set system of paths \mathcal{P} " in Page 8 Definition 1.3.10(2).

5. Comment: **In thesis Page 12, third para of Section 1.5: you need to write it more precisely and carefully conveying the idea. As it is it is not clear what you are saying.**

Response: The problem COMPUTE FEASIBLE TREE PATH LABELING and its result have been described formally and precisely in Chapter 3 (New

results). The aforementioned paragraph describes the same informally. These informal sentences have been footnoted with the corresponding definitions and theorems from Chapter 3. This change is in Page 12.

6. Comment: **In thesis Page 28, Definition 7 (of binary operator /): it is not defined precisely. What do you mean by “ b is a new element added..”?**

Response: This was in thesis Page 26. A/B is created by removing the elements of second operand set B from the first operand set A and introducing a new element *not* in the universe to A to represent set B . This is a set operation mentioned in [MM96] (they use “ B ” to denote the new element to represent set B which is also confusing) which is eventually used to decompose permutation of elements in A to permutation of elements in A/B and permutation of elements in B . Perhaps it would be clearer if b is replaced by x_B . This change has been made in Page 26 Definition 2.2.4 (7) and other mentions of this operation “/” Page 27 item (iii) and h has been replaced with x_H in Corollary 2.2.8. There was also a typo in Page 27 paragraph under Corollary 2.2.8 item (ii) where “ $U \setminus H$ ” was written instead of U/H . This has also been corrected.

7. Comment: **In thesis Page 39 Definition 2.3.1: the definition of $A_{\mathcal{F}}$ is not clear. Where is the dependence on \mathcal{F}**

Response: This was in thesis Page 36. It is true, there is no dependence in \mathcal{F} . It is only dependent on the universe U of \mathcal{F} . All occurrences of $A_{\mathcal{F}}$ has been changed to $A_{\mathcal{U}}$ in Page 36.

3 Response to Reviewer 2

References

- [BR72] C Berge and R Rado. Note on isomorphic hypergraphs and some extensions of whitney’s theorem to families of sets. *Journal of Combinatorial Theory, Series B*, 13(3):226 – 241, 1972.
- [FG65] D. R. Fulkerson and O. A. Gross. Incidence matrices and interval graphs. *Pac. J. Math.*, 15:835–855, 1965.
- [Fou80] J.-C. Fournier. Isomorphismes d’hypergraphes par intersections équicardinales d’arêtes et configurations exclues. *J. Comb. Theory, Ser. B*, 29(3):321–327, 1980.
- [MM96] J. Meidanis and E. G. Munuera. A theory for the consecutive ones property. In *Proc. of the III South American Workshop on String Processing*, volume 88, pages 194–202, Recife, Brazil, 1996.