Tree Path Labeling of Path Hypergraphs

A Generalization of Consecutive Ones Property

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as part of **M. S.** by Research
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25 Mar 2013

Motivation

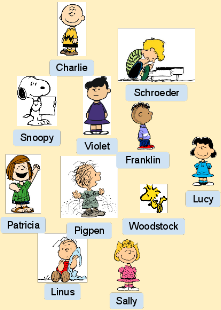
Combinatorial algorithms
$\downarrow$
Matrix reordering problems
$\downarrow$
Interval labeling [KKLV10] $\leftrightharpoons$ **Consecutive Ones Property** **[FG65, Tuc72, BL76, NS09]**
$\Downarrow$
Tree Path Labeling

An Illustration

An Illustration

of Tree Path Labeling problem

Study Group Accommodation problem



$$\begin{aligned}
\mathbb{B} &= \{\text{Ch}, \text{Sa}, \text{Fr}, \text{Sc}, \text{Lu}\} \\
\mathbb{T} &= \{\text{Pa}, \text{Pi}, \text{Vi}, \text{Ch}\} \\
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\end{aligned}$$

Students Study groups

Study Group Accommodation problem

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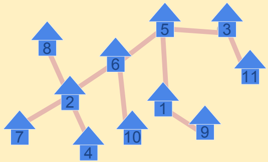
Study groups



Infinite Loop residential block

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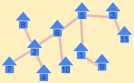
Study groups



Infinite Loop residential block

- A student may be in more than one study group but will be in at least one.
- There are equal number of single occupancy apartments in *Infinite Loop*.
- Streets connecting them do not form loops.

The problem

How should the students be allocated apartments such that students in each group should inhabit a (continuous) path?

Allocate Paths to Study Groups

tree path labeling

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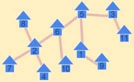
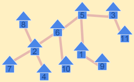


Study groups - $\mathbb{B}$, $\mathbb{T}$, $\mathbb{W}$, $\mathbb{F}$

Allocate Paths to Study Groups

tree path labeling



$\mathbb{B}$ = {Ch, Sa, Fr, Sc, Lu}
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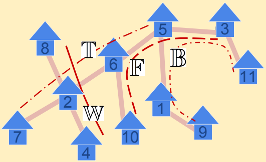
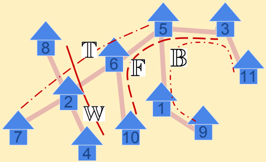


Study groups - $\mathbb{B}$, $\mathbb{T}$, $\mathbb{W}$, $\mathbb{F}$

Allocate Paths to Study Groups

tree path labeling - feasible?



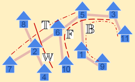
Is this feasible?

Allocate Apartments to Students

path graph isomorphism/feasibility bijection

Allocate Apartments to Students

path graph isomorphism/feasibility bijection



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Allocate Apartments to Students

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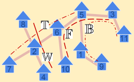

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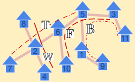
In this case, **is feasible**.

Allocate Apartments to Students

path graph isomorphism/feasibility bijection



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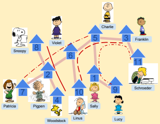

Basic terminology

a crash course on the TPL machinery

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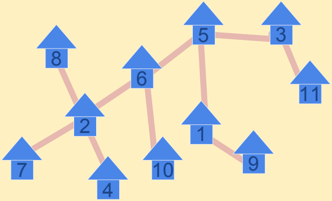
The set of study groups $\{\mathbb{B}, \mathbb{T}, \mathbb{W}, \mathbb{F}\} \rightarrow$ HYPERGRAPH

Basic terminology

a crash course on the TPL machinery



Infinite Loop residential block → TARGET TREE

Basic terminology

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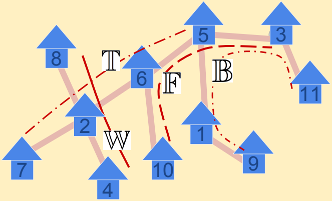

Study group path allocation → TREE PATH LABELING

Basic terminology

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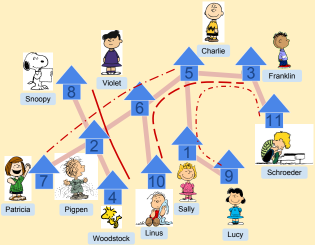

The apartment allocation → PATH HYPERGRAPH ISOMORPHISM

Basic terminology

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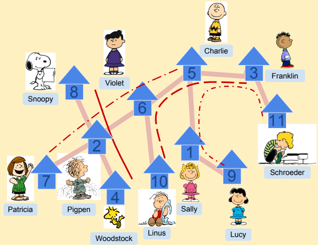

The apartment allocation → PATH HYPERGRAPH ISOMORPHISM

The problems studied

1. Compute Feasible Path Labeling

Computation of a feasible tree path labeling (FTPL) if any.

2. Compute k-subdivided Star Path Labeling

Computation of an FTPL if any, if target tree is a k-subdivided star.

3. Feasible Tree Path Labeling

Characterization of an FTPL and finding the feasibility bijection/hypergraph isomorphism

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The problems studied

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3.

3.

Characterization

- Three way intersection cardinality preservation
- Filtering and pruning algorithm

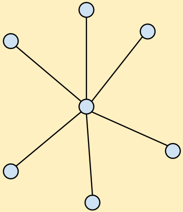
Special case

Interval assignment problem / COP

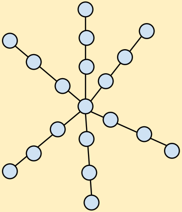
1. T is a path $\implies$ paths in T are intervals
2. Only pairwise intersection cardinality needs to be preserved $\implies$ ICPIA [NS09]
3. Higher level intersection cardinalities preserved by **Helly Property** – [Gol04]
4. *filter_1*,*filter_2* do not need the the **exit** conditions.

This problem is equivalent to Consecutive Ones Property of binary matrices [NS09]

2.

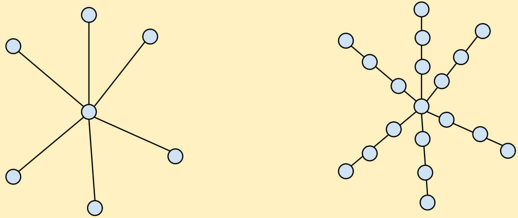


(a)



(b)

2.



(a) (b)

Compute TPL on k subdivided stars

- each rays of the k sub star are independent intervals when root is excluded.
- each ray is considered independently as interval assignment problem

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Combinatorial algorithms

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Matrix reordering problems

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Tree Path Labeling

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		Matrix reordering problems		Tree canonization [Lin92]
		$\downarrow$		$\downarrow$
Interval labeling [KKLV10]	$\leftrightharpoons$	**Consecutive Ones Property** **[FG65, Tuc72, BL76, NS09]**	$\Longrightarrow$	Interval graph canonization [KKLV10]
$\Downarrow$				$\Downarrow$
Tree Path Labeling				Interval graph isomorphism [KKLV10]

Motivation

Combinatorial algorithms
↓
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$\Downarrow$				$\Downarrow$
Tree Path Labeling	$\Longmapsto$	Path graph canonization		Interval graph isomorphism [KKLV10]
		$\Downarrow$		
		Path graph isomorphism		

Thank You

Q & A

Kellogg S. Booth and George S. Lueker.
Testing for the consecutive ones property, interval graphs, and graph planarity using *PQ*-tree algorithms.
Journal of Computer and System Sciences, 13(3):335–379, December 1976.

D. R. Fulkerson and O. A. Gross.
Incidence matrices and interval graphs.
Pac. J. Math., 15:835–855, 1965.

Martin Charles Golumbic.
Algorithmic graph theory and perfect graphs, volume 57 of *Annals of Discrete Mathematics*.
Elsevier Science B.V., 2004.
Second Edition.

Johannes Köbler, Sebastian Kuhnert, Bastian Laubner, and Oleg Verbitsky.
Interval graphs: Canonical representation in logspace.
Electronic Colloquium on Computational Complexity (ECCC), 17:43, 2010.

Steven Lindell.
A logspace algorithm for tree canonization (extended abstract).
In *STOC*, pages 400–404. ACM, 1992.

N. S. Narayanaswamy and R. Subashini.
A new characterization of matrices with the consecutive ones property.
Discrete Applied Mathematics, 157(18):3721–3727, 2009.

Alan Tucker.
A structure theorem for the consecutive 1's property.
J. Comb. Theory Series B, 12:153–162, 1972.