Tree Path Labeling of Set Systems

A Generalization of Consecutive Ones Property

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as part of **M. S.** by Research
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An Illustration

Caveat

- A very simplistic example.
- Aims only to introduce the combinatorial problem of TPL.

A Study Group Housing problem

- A set of n students arrive for a summer course, say $\{a, b, c, d, e, f, g, h, i, j, k\}$, $n = 11$.

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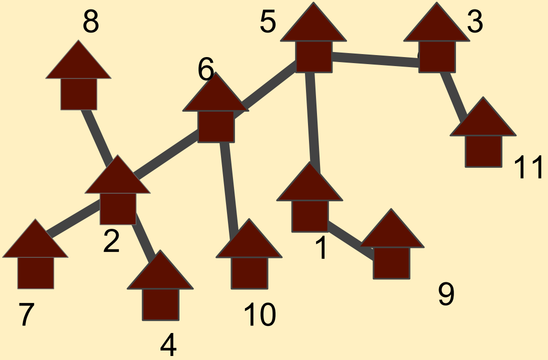
- There are n single occupancy apartments in the university campus for their accommodation.
- All these apartments are placed such that streets connecting them do not form loops

A Study Group Housing problem

The problem

How should the students be allocated apartments such that each study group has the least distance to travel for a discussion?

A Study Group Housing problem



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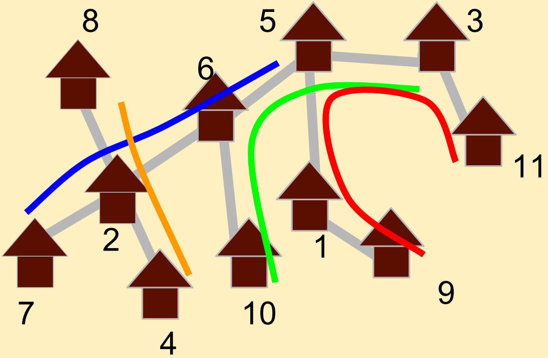

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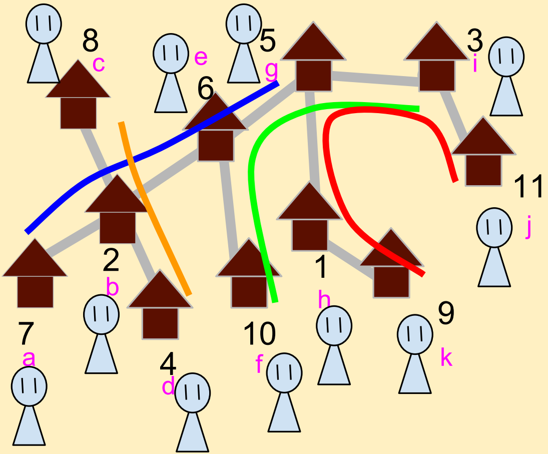

Tree Path Labeling of Set Systems

The combinatorial problem terminology

Terminology

- The set of study groups i.e. sets of students → SET SYSTEM / HYPERGRAPH
- The streets with apartments → TARGET TREE
- The route mapping to study groups → TREE PATH LABELING (TPL)
- The apartment allocation → PATH HYPERGRAPH ISOMORPHISM

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The combinatorial problem

Terminology [contd.]

There *exists* an apartment allocation that “fits” the route mapping

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Terminology [contd.]

There *exists* a hypergraph isomorphism that “fits” the TPL

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Terminology [contd.]

There *exists* a hypergraph isomorphism that "fits" the TPL
$\longrightarrow$ the TPL is FEASIBLE

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The combinatorial problem

Terminology [contd.]

There *exists* a hypergraph isomorphism that "fits" the TPL $\longrightarrow$ the TPL is FEASIBLE

There *exists* an apartment allocation that gives the optimal route mapping

Tree Path Labeling of Set Systems

The combinatorial problem

Terminology [contd.]

There *exists* a hypergraph isomorphism that "fits" the TPL
$\longrightarrow$ the TPL is FEASIBLE

There *exists* a hypergraph isomorphism that gives paths/adjacent vertices in tree

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The combinatorial problem

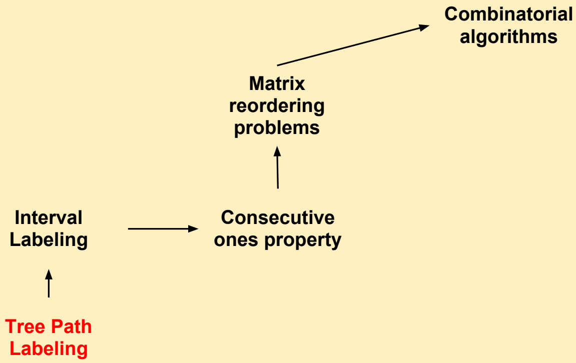
Terminology [contd.]

There *exists* a hypergraph isomorphism that “fits” the TPL
$\longrightarrow$ the TPL is FEASIBLE

There *exists* a hypergraph isomorphism that gives paths/adjacent vertices in tree
$\longrightarrow$ the hypergraph is a PATH HYPERGRAPH

Consecutive Ones → Path Labeling

The motivation



Tree path labeling of path hypergraphs

The two problems

1

Characterization of a *feasible TPL* and finding the certificate for feasibility i.e. *hypergraph isomorphism*

2

Computation of a *feasible TPL* if any

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Characterization of feasible TPL

Given

i. a set system or hypergraph $\mathcal{F}$,
ii. a feasible TPL $\ell : \mathcal{F} \to \mathcal{P}$ where $\mathcal{P}$ is a path system from tree T and $supp(\mathcal{P}) = V(T)$,

what is the hypergraph isomorphism

$$\phi : supp(\mathcal{F}) \to supp(\mathcal{P})$$

such that the induced labeling $\ell_\phi = \ell$?

2

Computing a feasible TPL

Given hypergraph $\mathcal{F}$ with certain properties and a k-subdivided star T, can we find a feasible TPL ℓ to T?

1

Characterization of feasible TPL

The characterization

ICPPL + a filtering algorithm

a: [TBD Write the theorem]

The characterization

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2

Computing a feasible TPL

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Special case

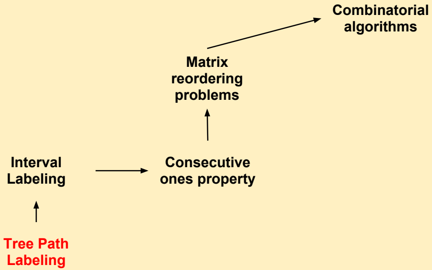
Interval assignment problem / COP

1. T is a path $\implies$ paths in T are intervals [a: quick illustration]
2. Only pairwise intersection cardinality needs to be preserved $\implies$ ICPIA [NS09]
3. Higher level intersection cardinalities preserved by **Helly Property** – [Gol04]
4. *filter_1*,*filter_2* do not need the the **exit** conditions. [a: is this cryptic?]

This problem is equivalent to Consecutive Ones Property of binary matrices [NS09]

Path Labeling → Graph Isomorphism

Application



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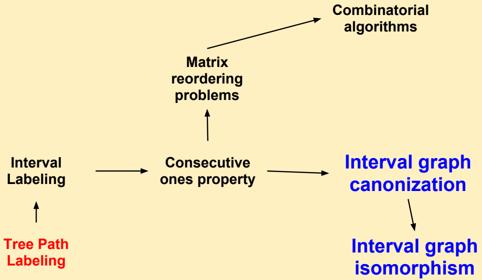

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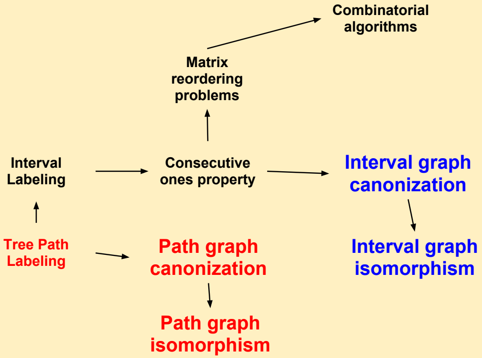

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