Tree Path Labeling of Path Hypergraphs

A Generalization of Consecutive Ones Property

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as part of **M. S.** by Research
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1. **Introduction**
   - Motivation
   - An Illustration
2. **Results**
3. **Conclusion**
   - Application

Motivation

Combinatorial algorithms

$\downarrow$

Matrix reordering problems

$\downarrow$

Interval labeling [KKLV10] $\leftrightharpoons$ **Consecutive Ones Property** **[FG65, Tuc72, BL76, NS09]**

$\Downarrow$

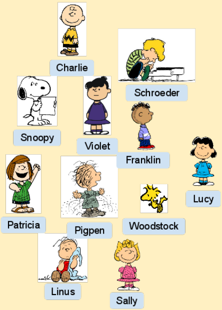
Tree Path Labeling

An Illustration

An Illustration

of Tree Path Labeling problem

Study Group Accommodation problem



Students

$$
\begin{aligned}
\mathbb{B} &= \{\text{Ch}, \text{Sa}, \text{Fr}, \text{Sc}, \text{Lu}\} \\
\mathbb{T} &= \{\text{Pa}, \text{Pi}, \text{Vi}, \text{Ch}\} \\
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\end{aligned}
$$

Study groups

Study Group Accommodation problem

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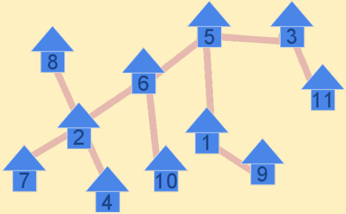
Study groups

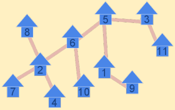


Infinite Loop residential block

Study Group Accommodation problem

$\mathbb{B}$ = {**Ch**, **Sa**, **Fr**, **Sc**, **Lu**}
$\mathbb{T}$ = {**Pa**, **Pi**, **Vi**, **Ch**}
$\mathbb{W}$ = {**Sn**, **Pi**, **Wo**}
$\mathbb{F}$ = {**Vi**, **Li**, **Ch**, **Fr**}

Study groups



Infinite Loop residential block

- A student may be in more than one study group but will be in at least one.
- There are equal number of single occupancy apartments in *Infinite Loop*.
- Streets connecting them do not form loops.

The problem

How should the students be allocated apartments such that students in each group should inhabit a (continuous) path?

Allocate Paths to Study Groups

tree path labeling

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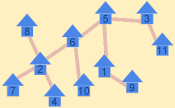
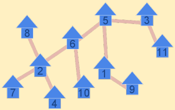


Study groups - $\mathbb{B}, \mathbb{T}, \mathbb{W}, \mathbb{F}$

Allocate Paths to Study Groups

tree path labeling



$\mathbb{B}$ = {Ch, Sa, Fr, Sc, Lu}
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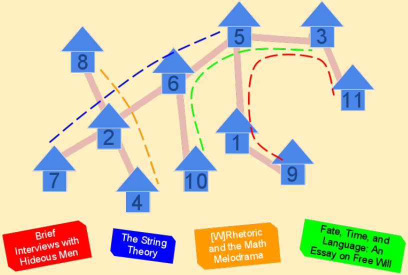
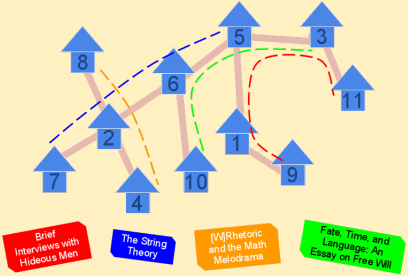


Study groups - $\mathbb{B}$, $\mathbb{T}$, $\mathbb{W}$, $\mathbb{F}$

Allocate Paths to Study Groups

tree path labeling - feasible?



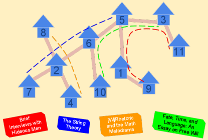
Is this feasible?

Allocate Apartments to Students

path graph isomorphism/feasibility bijection

Allocate Apartments to Students

path graph isomorphism/feasibility bijection



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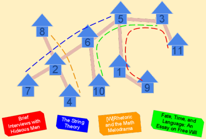
Allocate Apartments to Students

path graph isomorphism/feasibility bijection



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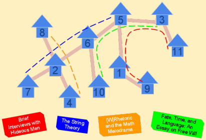
In this case, **is feasible**.

Allocate Apartments to Students

path graph isomorphism/feasibility bijection



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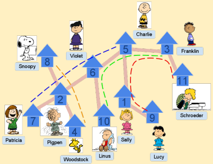

Basic terminology

a crash course on the TPL machinery

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The set of study groups $\{\mathbb{B}, \mathbb{T}, \mathbb{W}, \mathbb{F}\} \rightarrow$ HYPERGRAPH

Basic terminology

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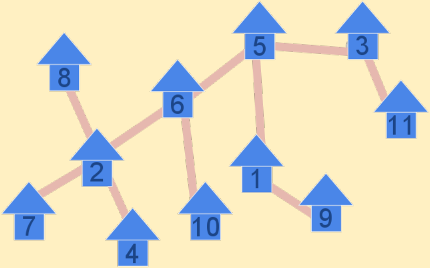
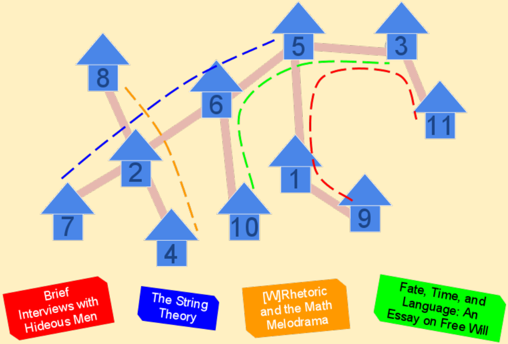


Infinite Loop residential block → Target tree

Basic terminology

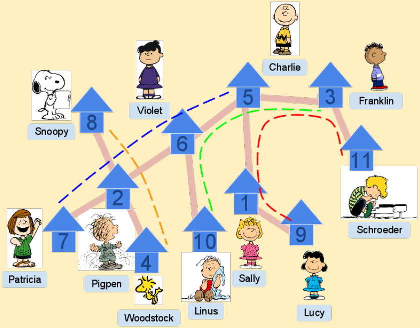
a crash course on the TPL machinery



Study group path allocation → TREE PATH LABELING

Basic terminology

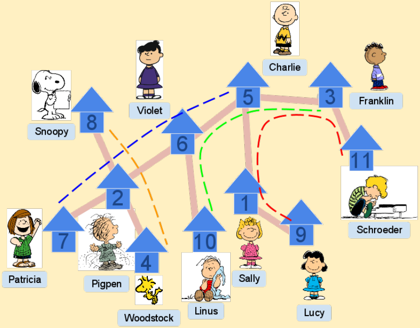
a crash course on the TPL machinery



The apartment allocation → PATH HYPERGRAPH ISOMORPHISM

Basic terminology

a crash course on the TPL machinery



The apartment allocation → PATH HYPERGRAPH ISOMORPHISM

The problems studied

1. Compute Feasible Path Labeling

Computation of a feasible tree path labeling (FTPL) if any.

2. Compute k-subdivided Star Path Labeling

Computation of an FTPL if any, if target tree is a k-subdivided star.

3. Feasible Tree Path Labeling

Characterization of an FTPL and finding the feasibility bijection/hypergraph isomorphism

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Characterization of an FTPL and finding the feasibility bijection/hypergraph isomorphism

3.

3.

Characterization

- Three way intersection cardinality preservation
- Filtering and pruning algorithm

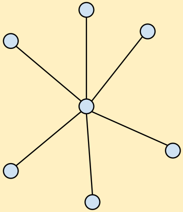
Special case

Interval assignment problem / COP

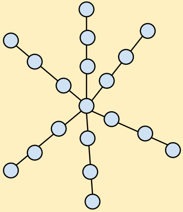
1. T is a path $\implies$ paths in T are intervals [a] [quick illustration]
2. Only pairwise intersection cardinality needs to be preserved $\implies$ ICPIA [NS09]
3. Higher level intersection cardinalities preserved by **Helly Property** – [Gol04]
4. *filter_1*,*filter_2* do not need the the **exit** conditions. [a] [is this cryptic?]

This problem is equivalent to Consecutive Ones Property of binary matrices [NS09]

2.

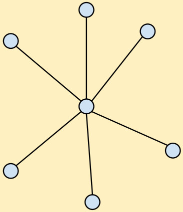


(a)



(b)

2.



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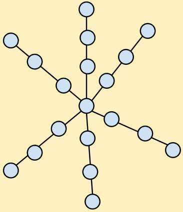

(b)

Compute TPL on k subdivided stars

- each rays of the k sub star are independent intervals when root is excluded.
- each ray is considered independently as interval assignment problem

Motivation

Combinatorial algorithms
$\downarrow$
Matrix reordering problems
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Interval labeling [KKLV10] $\leftrightharpoons$ **Consecutive Ones Property** **[FG65, Tuc72, BL76, NS09]**
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Interval labeling [KKLV10] $\leftrightharpoons$ **Consecutive Ones Property** **[FG65, Tuc72, BL76, NS09]** $\Longrightarrow$ Interval graph canonization [KKLV10]

Tree canonization [Lin92] $\downarrow$ Interval graph canonization [KKLV10]

Interval labeling [KKLV10] $\Downarrow$ **Tree Path Labeling**

Interval graph canonization [KKLV10] $\Downarrow$ Interval graph isomorphism [KKLV10]

Motivation

Combinatorial algorithms

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Interval labeling [KKLV10] ⇋ **Consecutive Ones Property** [FG65, Tuc72, BL76, NS09] ⟹ Interval graph canonization [KKLV10]

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Interval labeling [KKLV10] ⇓ **Tree Path Labeling** ⟹ Path graph canonization ⇓ Path graph isomorphism

Interval graph canonization [KKLV10] ⇓ Interval graph isomorphism [KKLV10]

Thank You

Q & A

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