ABSTRACT

Keywords: consecutive ones property, algorithmic graph theory, hypergraph isomorphism, interval labeling

Consecutive-ones property is a non-trivial combinatorial property of binary matrices that has been studied widely in the literature for over past 50 years. Detection of COP in a matrix is possible efficiently and there are several algorithms that achieve the same. This thesis documents the work done on an extension of COP. We extend COP from the equivalent interval assignment problem to what we call $Tree\ Path\ Labeling\ (TPL)$ problem. Inspired by the idea that COP is akin to assigning intervals to sets in a set system, we ask the question what if the assignments to sets are made not of intervals but of paths from any tree? While we show that TPL is in NP, we obtain a more efficient algorithm for a special case tree class called k-subdivided stars as well as for characterizing a feasible TPL. For extension of COP to arbitrary trees, we analyse the structure of a hypergraph (set system / columns of matrix) using the idea of prime submatrices and yield an algorithm for a feasible path labeling if one indeed exists. These new results rigorously prove the natural extension (to trees) of the ICPIA characterization as well as makes connections to graph isomorphism, namely path graph isomorphism.

Moreover, this thesis discusses the significance of COP and the evolution of existing data structures and algorithms used in detection of COP over the years – from PQ-trees to Intersection Cardinality Preserving Interval Assignment (ICPIA) data structure. We draw conclusions and connections of which some of them (for instance, PQR-tree and generalized PQ-tree) have not be observed before, to the best of our knowledge. This

includes the fact that COP can be tested in log space.