Macroeconomic Theory

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Ramsey Model

Decentralized economy setup

The decentralized economy has the following features:

- 2 factor markets: labor with rental price $w(t)$ and capital services with rental price $r(t)$
- There is a debt market in which families can borrow and lend
- Many identical families can save in k or in bonds
- Many identical firms with CRS technology
- Perfect foresight

Ramsey Model

Family optimization problem

Each family decides how much to consume-save taking as given the sequence of prices by maximizing:

$$\boxed{U(s) = \int_s^{\infty} e^{-\rho(t-s)} u(c(t))dt}$$

subject to:

$$\boxed{c(t) + \dot{a}(t) + na(t) = w(t) + r(t)a(t)}$$

Ramsey Model

Firm optimization

Firms maximize profits. The first-order conditions are:

Capital rental rate:

$$\boxed{f'(k(t)) = r(t)}$$

Wage rate:

$$\boxed{f(k(t)) - k(t)f'(k(t)) = w(t)}$$

Ramsey Model

Decentralized equilibrium

Equilibrium concept:

Consider an arbitrary path for wages and rental rates. This sequence leads each family to determine consumption and wealth accumulation.

Private debt is in zero net supply, therefore wealth accumulation determines the capital stock sequence. In turn it determines the prices.

The market mechanism ensures that individual decisions aggregate to determine equilibrium prices and quantities.

Ramsey Model

The No-Ponzi-Game condition

There is an equilibrium we want to rule out: given the absence of borrowing constraints, agents can indebt to reach a level of consumption that commands a marginal utility of zero.

We need a condition that rules out this pathological case but allows for temporary indebtedness:

$$\boxed{\lim_{t\to\infty} a(t)e^{\left[-\int_0^t (r(\tau)-n)d\tau\right]} \geq 0}$$

Ramsey Model

Intertemporal budget constraint

Integrating the budget constraint from 0 to T and using the NPG condition, we get the intertemporal budget constraint (IBC):

$$\boxed{\int_0^\infty c(t) e^{-\int_0^t (r(\tau)-n)d\tau} dt = a_0 + h_0}$$

where:

$$\boxed{h_0 \equiv \int_0^\infty w(t) e^{-\int_0^t (r(\tau)-n)d\tau} dt}$$

Here h_0 represents human wealth — the present value of future labor income

Ramsey Model

First-order conditions

The necessary conditions for optimization are:

Marginal utility condition:

$$\boxed{u'(c(t)) = \lambda(t)}$$

Euler equation:

$$\boxed{\dot{\lambda}(t) = \lambda(t)\left[\rho + n - r(t)\right]}$$

Transversality condition:

Ramsey Model

Equilibrium dynamics

Using the first-order conditions of firms and the market clearing condition $a(t) = k(t)$, we obtain:

Capital accumulation:

$$\boxed{\dot{k}(t) = f(k(t)) - c(t) - nk(t)}$$

Consumption growth:

$$\boxed{\frac{\dot{c}(t)}{c(t)} = \sigma(c(t))\left[f'(k(t)) - \rho - n\right]}$$

Ramsey Model

Interest rate effects on consumption-saving

Consumption is a linear function of wealth, with a propensity β that depends on the expected path of interest rates.

An increase in interest rates has three effects:

1. **Substitution effect:** Makes consumption later more attractive and therefore increases saving
2. **Income effect:** Allows for higher consumption now and later, decreasing saving
3. **Wealth effect:** Changes human wealth h_0

Ramsey Model

Government with balanced budget

Assume government consumes resources $g(t)$ and pays for them with taxes $\tau(t)$ in a balanced budget every period.

Family budget constraint:

$$\boxed{c(t) + \dot{a}(t) + na(t) = w(t) + r(t)a(t) - \tau(t)}$$

Integrating and using the balanced budget condition:

$$\boxed{G_0 = \int_0^\infty \tau(t)R(t)dt}$$

Ramsey Model

Government with debt financing

Now assume debt financing with government budget constraint:

$$\boxed{\dot{b}(t) + nb(t) = g(t) - \tau(t) + r(t)b(t)}$$

Integrating and imposing NPG condition:

$$\boxed{b_0 + \int_0^\infty g(t)R(t)dt = \int_0^\infty \tau(t)R(t)dt}$$

This shows that the government need not run a balanced budget at every moment.

Ramsey Model

Ricardian equivalence

For families, now $a(t) = k(t) - b_p(t) + b(t)$. Using the same interest rate and integrating the budget constraint:

$$\boxed{\int_0^\infty c(t)R(t)dt = k_0 - b_{p0} + b_0 + h_0 - \int_0^\infty \tau(t)R(t)dt}$$

By substituting the government budget, we obtain the strong result:

> Only expenditure matters for allocations, not the method of financing (lump sum vs. debt). For a given path of g, the method of finance has no effect on resource allocation.

Dynamic Responses in Ramsey

Change in discount rate

Consider a decrease in the willingness to save due to an increase in the discount rate:

- In steady state, the MPK changes while the resource constraint does not
- New steady state has lower consumption and lower capital stock
- Can be analyzed using phase diagram techniques

Higher impatience leads to less capital accumulation and lower long-run consumption.

Dynamic Responses in Ramsey

Total factor productivity shock

Consider a production function with multiplicative productivity parameter $zf(k_t)$ (isomorphic to distortionary taxation).

Two scenarios to analyze:

- **Permanent productivity shock:** Unanticipated and permanent increase in z
- **Transitory productivity shock:** Unanticipated but temporary increase in z

Both can be analyzed using phase diagram techniques to understand the dynamic adjustment paths.