# Macroeconomic Theory

## Nova School of Business and Economics

Francesco Franco

T3 2026

# Simulation of an OLG in Dynare

## Professional macroeconomic modeling software

- Dynare is the main software used by macroeconomists at Central Banks to simulate and estimate macro models
- Mostly used for stochastic business cycle models (we will see it with the next model)
- Can be used to simulate (and estimate) deterministic long run models like the Ramsey (in discrete time) and the OLG
- We will learn how to create a .mod file that is then parsed by dynare

# Social Planner OLG Model

Starting with the mathematical formulation

Always start with your model

**Objective function:**

$$\boxed{U = \lambda_{-1}\beta u(c_{20}) + \sum_{t=0}^{\infty}\lambda_t\left(u(c_{1t}) + \beta u(c_{2t+1})\right)}$$

**Resource constraint (with multiplier $\kappa_t$):**

$$\boxed{k_{t-1} + z_t f(k_{t-1}) = (1+n)k_t + c_{1t} + (1+n)^{-1}c_{2t}}$$

# The .mod File

## Declaring variables

### Declare the endogenous variables:

Here consumption of the young, of the old and capital

```
var c1 c2 k;
```

### Declare the exogenous variables:

Here the $z_t$

```
varexo z;
```

# Parametric Assumptions

## Functional forms and parameter values

On the computer you need to assume functional forms. Use $u = \ln(c)$ and $f = zk^{\alpha}$. Also define $\psi = \frac{\lambda_t}{\lambda_{t-1}}$

### Declare the parameters:

```
parameters alpha bet n R psi rho;
T = 25;
alpha = 0.3;
rho = (1 + 0.04)^T - 1;
bet = 1/(1 + rho);
n = (1 + 0.01)^T - 1;
R = (1 + 0.02)^T -1;
psi = 1/(1 + R);
```

# Model

First order conditions

**The FOC conditions:**

$$\boxed{\frac{\lambda_t}{c_{1t}} = \kappa_t}$$

$$\boxed{\beta \frac{\lambda_{t-1}}{c_{2t}} = \kappa_t (1+n)^{-1}}$$

$$\boxed{\kappa_{t+1}\left(1 + z_{t+1}\alpha k_t^{\alpha-1}\right) = \kappa_t (1+n)}$$

# Model in the .mod

## Combining the FOCs

Combining the FOC we obtain the model dynamics:

$$\boxed{c_{2t} = \frac{\beta}{\psi}(1+n)c_{1t}}$$

$$\boxed{\frac{c_{1t+1}}{c_{1t}} = \frac{\psi}{(1+n)}\left(1 + z_{t+1}\alpha k_t^{\alpha-1}\right)}$$

$$\boxed{z_t k_{t-1}^{\alpha} = (1+n)k_t + c_{1t} + (1+n)^{-1}c_{2t}}$$



```
model;
```

## The Steady State

### Analytical solution

While the algorithm to solve the deterministic infinite horizon models looks for the global solution, it checks that the properties of the dynamic system around the end point are a saddle by linearizing the model around the steady state:

$$\boxed{k = \left(\frac{z\alpha\psi}{1+n-\psi}\right)^{\frac{1}{1-\alpha}}}$$

$$\boxed{c_2 = \frac{\beta}{\psi}(1+n)c_1}$$

And: $zk^{\alpha} - nk = \frac{\psi+\beta}{\psi}c_1$, so $c_1 = \frac{\psi}{\psi+\beta}(zk^{\alpha} - nk)$

# Overlapping Generations with Perpetual Youth

## Discrete time model

- Instead of finite lives: constant probability of death $\nu$
- Individuals are potentially infinitely lived but their life will come to an end
- Expected life is $\frac{1}{\nu}$ for every individual that has survived until then, independently of age — hence the name perpetual youth

**Expected utility:**

$$E_t\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right] = \sum_{t=0}^{\infty}(\beta(1-\nu))^t u(c_t)$$

# Individual Budget Constraint

## Assets and transfers

**Individual $i$ budget constraint:**

$$\boxed{a_{it+1} = (1 + r_t)\, a_{it} - c_{it} + w_t + z_{it}}$$

where $z_{it}$ is a transfer.

The transfer is present because individuals that die leave their assets behind and we need an assumption on how to redistribute them. For example they could leave them as bequests to the next generation or the government could seize them and redistribute them to the living.

# Annuity Markets

## Financial intermediation

An alternative is to introduce annuity markets that consist of financial intermediaries that make a payment during the life of an agent in exchange for receiving their wealth when they die. This is like a reverse mortgage.

### Profit of a financial intermediary:

$$\boxed{\pi(a_t) = \nu a_t - (1-\nu)\, z(a_t)}$$

### Perfect competition implies:

$$\boxed{z(a_t) = \tfrac{\nu}{(1-\nu)} a_t}$$

# Aggregation by Birth

Cohort-based analysis

## Aggregation should be as follows:

$$\boxed{A_{t,\tau} = a_{t,\tau} L_{t,\tau}}$$

$$\boxed{A_t = \sum_{\tau=-\infty}^{t} A_{t,\tau}}$$

where the $-\infty$ is there to include all cohorts, also those born in the distant past.

Using $\frac{L_{t,\tau}}{L_{t+\mathbf{1},\tau}} = \frac{1}{1-\nu}$, we get:

$$\frac{A_{t+\mathbf{1},\tau}}{(1-\nu)} = \left(1 + r_t + \frac{\nu}{(1-\nu)}\right) A_{t,\tau} - C_{t,\tau} + w_t L_{t,\tau}$$

# Aggregate Dynamics

## Final system

Divide by $L_t$ the aggregate equations and use $a_t = k_t$ in equilibrium together with the FOC of the firms to substitute for prices:

$$\boxed{\frac{k_{t+1}(1+n-\nu)}{(1-\nu)} = \left(1 + \frac{\nu}{(1-\nu)}\right) k_t - c_t + f\left(k_t\right)}$$

$$\boxed{c_t = \left(1 - \beta\left(1-\nu\right)\right)\left(1 + f'\left(k_t\right) + \frac{\nu}{(1-\nu)}\right)\left(k_t + \omega_t\right)}$$

where $\omega_t$ depends on the whole path of future $k_t$.

## Steady State

Perpetual youth model

**Use the fact that:**

$$\boxed{\omega = \left(f\left(k\right) - kf'\left(k\right)\right)\frac{1+f'(k)+\frac{\nu}{(1-\nu)}}{f'(k)+\frac{\nu}{(1-\nu)}}}$$

**Then:**

$$\boxed{c = f\left(k\right) - \frac{(n-\nu)}{(1-\nu)}k}$$

$$\boxed{c = \left(1-\beta\left(1-\nu\right)\right)\left(1+f'\left(k\right)+\frac{\nu}{(1-\nu)}\right)\left(k+\omega\right)}$$

# Conclusion

- ✓ The algebra is cumbersome, and not all the results are transparent
- ✓ Many difficulties were coming from the **discrete time** choice
- ✓ Dynare provides a powerful framework for simulating OLG models
- ✓ Perpetual youth models offer analytical tractability