Macroeconomic Theory

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The Supply Side

New Keynesian Phillips Curve

Remember we have derived the NKPC in terms of marginal cost:

$$\boxed{\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda\left(mc_t + \mu\right)}$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$, $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$

The marginal cost is:

$$\boxed{mc_t = (w_t - p_t) - (z_t - \alpha n_t + \log(1-\alpha))}$$

The Supply Side

Using labor supply and technology

We used the labor supply condition:

$$\boxed{w_t - p_t = \sigma c_t + \varphi n_t}$$

Combined with technology $n_t = \frac{1}{1-\alpha}(y_t - z_t)$ and goods market equilibrium to obtain:

$$\boxed{mc_t = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t - \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right) z_t - \log(1-\alpha)}$$

The Supply Side

Flexible prices and output gap

In the flexible prices equilibrium, real marginal cost is constant and equal to $-\mu$:

$$mc = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n - \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right) z_t - \log(1-\alpha)$$

where y_t^n is the natural level of output.

Combining with the NKPC, we get the dynamic IS curve:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

where $\widetilde{y}_t$ is the output gap.

Notes on the NKM Solution

The three-equation system

Consider the NKM model with:

Monetary Policy Rule:

$$\boxed{i_t = \rho + \phi_\pi \pi_t + v_t}$$

Dynamic IS Curve:

$$\boxed{\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)}$$

New Keynesian Phillips Curve:

Notes on the NKM Solution

Key definitions

- $\tilde{y}_t$ is the **output gap**
- r_t^n is the **natural rate** of real interest rate — the rate that implies zero output gap
- $r_t^n = \rho - \sigma(1 - \rho_z)\psi_{yz} z_t$

Household First Order Conditions

Classical model conditions

The first order conditions with respect to C_t and A_t are:

Labor Supply:

$$\boxed{w_t - p_t = \sigma c_t + \varphi n_t}$$

Euler Equation:

$$\boxed{c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)}$$

Notes on the NKM Solution

System reduction

Three endogenous variables: i_t, π_t, $\tilde{y}_t$
Two exogenous variables: z_t, v_t

Substitute the monetary policy rule into the IS curve:

$$\boxed{\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(\rho + \phi_\pi \pi_t + v_t - E_t\{\pi_{t+1}\} - r_t^n)}$$

Combined with the New Keynesian Phillips Curve:

$$\boxed{\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t}$$

Notes on the NKM Solution

Solution guess

Assume a unique solution where endogenous variables respond to state variables:

$$\boxed{\tilde{y}_t = \psi_{yu} u_t}$$

$$\boxed{\pi_t = \psi_{\pi u} u_t}$$

where $u_t = v_t - (r_t^n - \rho)$ is the difference between the two shocks.

We need to determine the unknown coefficients ψ_{yu}, $\psi_{\pi u}$ through guess and verify.

Notes on the NKM Solution

Simplified case: monetary shock only

For simplicity, consider $z_t = 0$ (no technology shock), so $r_t^n = \rho$ and $u_t = v_t$

Assume $v_t \sim N(0, 1)$ is i.i.d. Plugging our guess into the model:

$$\boxed{\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(\phi_\pi \pi_t + u_t - E_t\{\pi_{t+1}\})}$$

This becomes:

$$\boxed{\widetilde{y}_t = -\frac{1}{\sigma}(\phi_\pi \psi_{\pi u} + 1)u_t}$$

Notes on the NKM Solution

Using the NKPC

Using our guess in the NKPC:

$$\boxed{\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t}$$

Substituting the guess:

$$\boxed{\pi_t = \beta E_t\{\psi_{\pi u} u_{t+1}\} + \kappa \psi_{yu} u_t}$$

Since $E_t u_{t+1} = 0$:

$$\boxed{\pi_t = \kappa \psi_{yu} u_t}$$

Notes on the NKM Solution

Solving for coefficients

To be consistent with our guess, we need:

$$\boxed{\psi_{yu} = -\frac{1}{\sigma}(\phi_\pi \psi_{\pi u} + 1)}$$

$$\boxed{\psi_{\pi u} = \kappa \psi_{yu}}$$

Solving these two equations yields:

$$\boxed{\psi_{yu} = -\frac{1}{\sigma + \phi_\pi \kappa}}$$

Notes on the NKM Solution

Uniqueness condition

To ensure unique solution, check eigenvalues of matrix A_T are less than unity:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathrm{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathrm{B}_T u_t$$

where $u_t \equiv \widehat{r}_t^n - v_t$ and

$$\mathrm{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta\sigma \end{bmatrix} ; \mathrm{B}_\mathrm{T} \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

Notes on the NKM Solution

Jump variables intuition

The dynamics require both variables to have free initial values:

- After a shock, both output gap and inflation can take correct values ensuring unique solution
- Both variables are **jump variables** (like consumption in Ramsey model)
- Unlike capital in Ramsey (state variable), these have no predetermined values
- State variables require eigenvalue larger than unity in the system formulation

General Solution

Ensuring unique solution

How to solve and ensure unique solution (saddle property)?

Rewrite the system:

$$\boxed{\begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} = \mathrm{A}_D \begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} + \mathrm{B}_D u_t}$$

Decouple dynamics using decomposition:

$$\boxed{\mathrm{A}_D = C \Lambda C^{-1}}$$

General Solution

System transformation

The system becomes:

$$\boxed{\begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} = C\Lambda C^{-1} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} + \mathrm{B}_D u_t}$$

Premultiply by C^{-1}:

$$\boxed{C^{-1} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} = \Lambda C^{-1} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} + C^{-1}\mathrm{B}_D u_t}$$

General Solution

Forward iteration

Rearranging: $X_t = \Lambda^{-1} E_t X_{t+1} - \Lambda^{-1} D u_t$

Using Law of Iterated Expectations $E_t E_{t+1} = E_t$:

$$\boxed{X_t = \sum_{i=1}^{\infty} \left(\Lambda^{-1}\right)^i E_t \left[X_{t+i} - D u_{t+i}\right] - \Lambda^{-1} D u_t}$$

Since $E_t \left[D u_{t+i}\right] = 0$ for $i > 0$ and both eigenvalues greater than 1:

$$\boxed{X_t = -\Lambda^{-1} D u_t}$$

Transforming back to original system:

Optimal Price Condition

Basic formulation

The optimal price condition gives:

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left((\psi_{t+k}+\mu) - \frac{\alpha\epsilon}{1-\alpha}(p_t^* - p_{t+k})\right)$$

which can be rewritten as:

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(\Theta\left(\psi_{t+k} - p_{t+k} + \mu\right) + p_{t+k}\right)$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$

Optimal Price Condition

Recursive form

The recursive form is:

$$\boxed{p_t^* = (1-\beta\theta)\left(\Theta\left(\psi_t - p_t + \mu\right) + p_t\right) + \beta\theta E_t p_{t+1}^*}$$

Subtracting p_{t-1}:

$$\boxed{p_t^* - p_{t-1} = (1-\beta\theta)\left(\Theta\left(\psi_t - p_t + \mu\right) + p_t\right) - p_{t-1} + \beta\theta E_t p_{t+1}^*}$$

Using $\pi_t = p_t - p_{t-1} = (1-\theta)\left(p_t^* - p_{t-1}\right)$ to get the NKPC:

$$\boxed{\pi_t = \lambda\left(\psi_t - p_t + \mu\right) + \beta E_t\{\pi_{t+1}\}}$$

NK-Phillips Curve

Final form

Replacing in the NKPC:

$$\boxed{\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda\left(mc_t + \mu\right)}$$

where:

- $mc_t = \psi_t - p_t$ is the marginal cost
- $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$
- $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$

(Solve forward to get the forward-looking structure)

Equilibrium

Goods market clearing

Clearing in the continuum of goods markets:

$$\boxed{Y_t(i) = C_t(i)}$$

Aggregating:

$$\boxed{Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} = C_t}$$

In log-linear form:

Labor Market Clearing

Aggregation across firms

Labor market clearing:

$$\boxed{N_t = \int_0^1 N_t(i)di}$$

Using firm labor demand:

$$\boxed{N_t = \int_0^1 \left(\frac{Y_t(i)}{Z_t}\right)^{\frac{1}{1-\alpha}} di}$$

This gives:

Labor Market Clearing

Labor supply and marginal cost

The labor supply is:

$$\boxed{w_t - p_t = \sigma c_t + \varphi n_t}$$

Using labor demand and technology:

$$\boxed{mc_t = (w_t - p_t) - (z_t - \alpha n_t + \log(1-\alpha))}$$

Substituting:

$$mc_t = (\sigma y_t + \varphi n_t) - (z_t - \alpha n_t + \log(1-\alpha))$$

Natural Level of Output and Markup Gap

Flexible prices benchmark

The natural level of output prevails with flexible prices (constant markup):

$$mc = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n - \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right) z_t - \log(1-\alpha)$$

The marginal cost gap is therefore:

$$\left(mc_{t|t} - mc\right) = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) (y_t - y_t^n)$$

The New Keynesian Phillips Curve

Final specification

Substituting the marginal cost gap gives the NKPC:

$$\boxed{\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(y_t - y_t^n)}$$

More compactly:

$$\boxed{\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t}$$

This is the first equation of our NK model.

The C-S Condition (Dynamic IS)

From Euler equation to output gap

The Euler condition plus goods market equilibrium gives:

$$\boxed{y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)}$$

Rewritten in terms of output gap:

$$\boxed{\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)}$$

where $r_t^n = \rho - \sigma(1-\rho_z)\psi_{yz}z_t$ is the **natural rate of interest** (Wicksellian)

The C-S Condition (Dynamic IS)

Definition of natural rate

Definition of r_t^n:

$$\boxed{y_t^n = E_t\left[y_{t+1}^n\right] - \frac{1}{\sigma}\left(r_t^n - \rho\right)}$$

Therefore:

$$\boxed{r_t^n = \rho + \sigma E_t\left[y_{t+1}^n - y_t^n\right]}$$

Using the solution $y_t^n = \psi_y + \psi_{yz} z_t$ and $z_t = \rho_z z_{t-1} + \epsilon_t^z$

Express the Dynamic IS in terms of $\tilde{y} = y - y^n$

The Policy Block

Completing the model

To determine the nominal side of the economy, we need an equation for the nominal interest rate.

We need a rule that gives **determinacy** of equilibrium.

> Without a monetary policy rule, the nominal variables would be indeterminate in this model.

Interest Rate Rule

Simple Taylor rule

Consider the following rule:

$$\boxed{i_t = \rho + \phi_\pi \pi_t + v_t}$$

where v_t is a monetary policy shock.

The complete model is now:

$$\boxed{\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)}$$

$$\boxed{\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t}$$

Dynamics

Matrix representation

The system is:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathsf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathsf{B}_T u_t$$

where $u_t \equiv \widehat{r}_t^n - v_t$ and:

$$\mathsf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta\sigma \end{bmatrix} ; \mathsf{B}_\mathsf{T} \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$