Macroeconomic Theory

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Perpetual Youth in Continuous Time

The Blanchard-Yaari model

An analytically tractable version of the perpetual youth model is the Blanchard-Yaari model of perpetual youth.

The key analytical trick:

- All stochastic events are i.i.d.
- In continuous time: they are drawn from a Poisson process

Poisson Process

Random events on the real line

The intuition on the real line:

- Events randomly distributed
- On average, there are ν points per unit length
- As time passes, move along the line and count the points

If $N(t)$ is the (random) number of events during interval of length t:

$$\boxed{E[N(t)] = \nu t}$$

For a short interval t, the probability of one event: νt.

Poisson Process

Probability distribution

The Poisson PDF:

$$\boxed{\Pr(N(t) = n) = \frac{(\nu t)^n}{n!} e^{-\nu t}}$$

Probability of no event:

$$\boxed{e^{-\nu t}}$$

This is the continuous time analogue of $(1-p)^t$ we had in the discrete time version of the model

Individual Problem

Expected utility with mortality risk

The expected utility of an individual born at τ:

$$\boxed{E_\tau \int_\tau^\infty e^{-\rho(t-\tau)} u(t,\tau) dt}$$

becomes:

$$\boxed{\int_\tau^\infty e^{-(\rho+\nu)(t-\tau)} u(t,\tau) dt}$$

Newborn households do not own any assets and we assume annuity markets:

Individual Problem

Budget constraint and optimality

Budget constraint:

$$\boxed{\dot{a}(t,\tau) = (r(t)+\nu)a(t,\tau) - c(t,\tau) + w(t)}$$

No-Ponzi-Game condition:

$$\lim_{z\to\infty} R(t,z)a(z,\tau) = 0$$

where $R(t,z) = e^{-\int_t^z (r(s)+\nu)ds}$

First-order condition (assuming $u = \ln(c)$):

$$\dot{c}(t,\tau)$$

Individual Problem

Integrating the budget constraint

Intertemporal budget constraint:

$$\int_t^{\infty} c(z,\tau)R(t,z)dz = a(t,\tau) + \omega(t,\tau)$$

Integrating the FOC:

$$c(t,\tau) = c(\tau,\tau)e^{\int_\tau^t (r(s)-\rho)ds}$$

Consumption function:

$$\boxed{c(t,\tau) = (r(t)+\nu)a(t,\tau) + \omega(t,\tau)}$$

Aggregation

Strong form of aggregation

Aggregation is much simpler relative to the discrete time model:

$$\boxed{\frac{\int_{-\infty}^{t} c(t,\tau)L(t,\tau)d\tau}{\int_{-\infty}^{t} L(t,\tau)d\tau} = c(t) = (\rho+\nu)a(t) + \omega(t)}$$

This is a **strong form of aggregation**: aggregate consumption behaves like individual consumption as if a single individual made the choice.

Dynamic System

System of equations

The equations that describe the system are:

Aggregate consumption:

$$c(t) = (\rho + \nu)a(t) + \omega(t)$$

Asset accumulation:

$$\dot{a}(t) = (r(t) + \nu - n)a(t) - c(t) + w(t)$$

Human wealth:

$$\omega(t) = \int_t^\infty w(z)R(t,z)dz$$

where this last term is annoying to work with.

Dynamic System

Simplifying the dynamics

Differentiate the consumption and ω functions with respect to time:

$$\dot{c}(t) = (\rho + \nu)(\dot{a}(t) + \dot{\omega}(t))$$

and

$$\dot{\omega}(t) = (r(t) + \nu)\omega(t) - w(t)$$

After combining and simplifying:

$$\boxed{\frac{\dot{c}(t)}{c(t)} = r(t) - \rho - (\rho + \nu)n\frac{a(t)}{c(t)}}$$

Aggregate Dynamic System

Using firm FOCs and market clearing

Consumption dynamics:

$$\boxed{\frac{\dot{c}(t)}{c(t)} = f'(k_t) - \rho - (\rho + \nu)n\frac{k(t)}{c(t)}}$$

Capital dynamics:

$$\boxed{\dot{k}(t) = f(k_t) - (n - \nu)k(t) - c(t)}$$

with k_0 given and transversality condition.

Steady State

Long-run equilibrium conditions

From consumption dynamics ($\dot{c} = 0$):

$$\boxed{c = \frac{(\rho + \nu)n}{f'(k) - \rho} k}$$

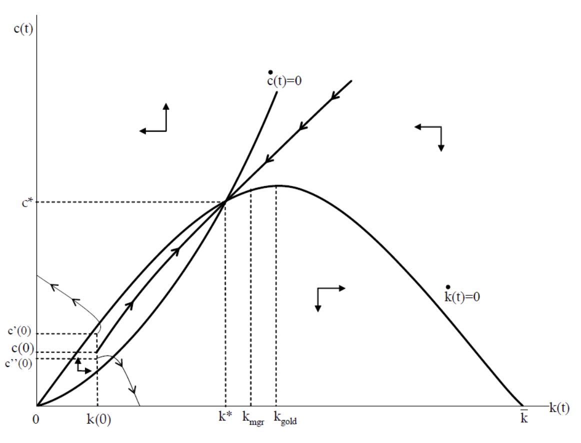
From capital dynamics ($\dot{k} = 0$):

$$\boxed{c = f(k) - (n - \nu)k}$$

Combining these two equations determines the steady state values of k and c.

Dynamics

Phase diagram



Life Cycle Extension

Declining income with age

Add declining income through the life of individuals:

$$e^{-\zeta(t-\tau)}w(t)$$

Now $\omega(t,\tau)$ depends on age. The new consumption dynamics:

$$\boxed{\frac{\dot{c}(t)}{c(t)} = f'(k_t) - \rho + \zeta - (\rho+\nu)(\zeta+n)\frac{k(t)}{c(t)}}$$

With $n > 0$ and $\zeta > 0$:

- ▷ You can have overaccumulation

Fiscal Policy

Government debt and intergenerational transfers

The government can issue debt at interest rate $r(t)$ and not $r(t) + \nu$.

Consider the case where the interest rate is positive (the other case makes the government's IBC irrelevant).

Tax reallocation:

- Decrease in taxes at t
- Increase in taxes at $t + s$
- Keep government spending G constant

The different interest rates at which government and individuals discount future taxes implies a change in consumption allocations: **taxes are partly shifted to future generations**.