Macroeconomic Theory

Francesco Franco

T3 2026

Perpetual Youth in Continuous Time

The Blanchard-Yaari model

An analytically tractable version of the perpetual youth model is the Blanchard-Yaari model of perpetual youth.

The key analytical trick:

- All stochastic events are i.i.d.
- In continuous time: they are drawn from a Poisson process

Poisson Process

Random events on the real line

The intuition on the real line:

- Events randomly distributed
- On average, there are ν points per unit length
- As time passes, move along the line and count the points

If $N(t)$ is the (random) number of events during interval of length t:

$$\boxed{E[N(t)] = \nu t}$$

For a short interval t, the probability of one event: νt.

Poisson Process

Probability distributions

The Poisson PDF:

$$\boxed{\Pr(N(t) = n) = \frac{(\nu t)^n}{n!} e^{-\nu t}}$$

So the probability of no event is:

$$\boxed{e^{-\nu t}}$$

This is the continuous time analogue of $(1 - p)^t$ we had in the previous version of the model.

Individual Problem

Expected utility with mortality risk

The expected utility of an individual born at τ:

$$E_\tau \int_\tau^\infty e^{-\rho(t-\tau)} u(t,\tau)dt$$

becomes:

$$\int_\tau^\infty e^{-(\rho+\nu)(t-\tau)} u(t,\tau)dt$$

Newborn households do not own assets and we assume annuity markets:

Individual Problem

Budget constraint and optimization

The budget constraint:

$$\dot{a}(t,\tau) = (r(t)+\nu)a(t,\tau) - c(t,\tau) + w(t)$$

The No-Ponzi-Game condition with $R(t,z) = e^{-\int_t^z (r(s)+\nu)ds}$:

$$\lim_{z\to\infty} R(t,z)a(z,\tau) = 0$$

The first-order condition assuming $u = \ln(c)$:

Individual Problem

Integrated budget constraint and consumption rule

Integrate the budget constraint:

$$\int_t^\infty c(z,\tau)R(t,z)dz = a(t,\tau) + \omega(t,\tau)$$

Integrate the first-order condition:

$$c(t,\tau) = c(\tau,\tau)e^{\int_\tau^t (r(s)-\rho)ds}$$

This gives the consumption rule:

Aggregation

Strong aggregation result

Aggregation is much simpler relative to the discrete time model:

$$\boxed{\frac{\int_{-\infty}^{t} c(t,\tau)L(t,\tau)d\tau}{\int_{-\infty}^{t} L(t,\tau)d\tau} = c(t) = (\rho+\nu)(a(t)+\omega(t))}$$

This is a **strong form of aggregation**: aggregate consumption behaves like individual consumption as if a single individual made the choice. Will not be the case in terms of dynamics.

Dynamic System

The fundamental equations

The equations that describe the system are:

Consumption rule:

$$\boxed{c(t) = (\rho + \nu)(a(t) + \omega(t))}$$

Asset accumulation:

$$\boxed{\dot{a}(t) = (r(t) + \nu - n)a(t) - c(t) + w(t)}$$

Human wealth:

Dynamic System

Deriving the consumption dynamics

Differentiate the consumption and ω functions with respect to time:

$$\boxed{\dot{c}(t) = (\rho + \nu)(\dot{a}(t) + \dot{\omega}(t))}$$

and:

$$\boxed{\dot{\omega}(t) = (r(t) + \nu)\omega(t) - w(t)}$$

Dynamic System

Aggregate consumption dynamics

After combining the previous equations and using market clearing:

$$\boxed{\frac{\dot{c}(t)}{c(t)} = f'(k_t) - \rho - (\rho + \nu) n \frac{k(t)}{c(t)}}$$

$$\boxed{\dot{k}(t) = f(k_t) - (n - \nu) k(t) - c(t)}$$

with k_0 given and appropriate transversality conditions.

You can see that the aggregate consumption dynamic behavior is **different from the indi-**

Steady State

Equilibrium conditions

In steady state:

$$\boxed{c = \frac{(\rho+\nu)n}{f'(k)-\rho}k}$$

and:

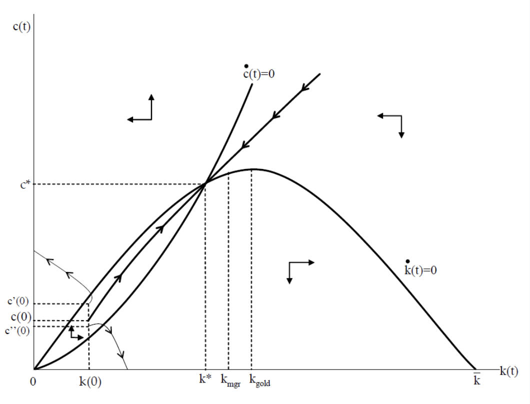
$$\boxed{c = f(k) - (n-\nu)k}$$

You can show there is a unique k in steady state and a unique level of consumption.

Notice that $f'(k) > \rho$ which always gives you under-accumulation relative to the

Dynamics

Phase diagram



Life Cycle

Declining income through life

Add declining income through the life of individuals:

$$\boxed{e^{-\zeta(t-\tau)}w(t)}$$

Now $\omega(t,\tau)$ depends on age. The new aggregate consumption dynamics:

$$\boxed{\frac{\dot{c}(t)}{c(t)} = f'(k_t) - \rho + \zeta - (\rho+\nu)(\zeta+n)\frac{k(t)}{c(t)}}$$

With $n > 0$ and $\zeta > 0$ you can have over-accumulation. You can also have a negative real interest rate in steady state equilibrium.

Fiscal Policy

Government debt and intergenerational redistribution

The government can issue debt at the interest rate $r(t)$ and not $r(t) + \nu$.

Consider the case where the interest rate is positive (the other is interesting but makes the IBC of the government irrelevant).

Consider a reallocation of taxes with a decrease in t associated with an increase in $t + s$ keeping constant G.

The different interest rates at which government and individuals discount future taxes implies a change in the allocations of consumption: **taxes are partly shifted to future generations**.