Macroeconomic Theory

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The New Keynesian Model

New ingredients to the classical model

Building upon classical models with key modifications:

- **Sticky prices** (staggered price setting): real effects of nominal interest rate
- **Monopolistic competition**: price setters rather than price takers

These ingredients break the classical dichotomy between real and nominal variables

Household Problem

Many goods with CES preferences

Representative agent maximizes:

$$\boxed{\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]}$$

where consumption is a CES bundle:

$$\boxed{C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}}$$

subject to budget constraint and no Ponzi condition

Intratemporal Demand

Cost minimization across goods

Households solve the cost minimization problem:

$$\min_{C_t(i)} \int_0^1 P_t(i) C_t(i) di$$

This yields the price deflator:

$$P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$$

and individual good demand:

Household Optimality

First order conditions

The key first-order conditions remain as in the classical model:

Labor supply:

$$w_t - p_t = \sigma c_t + \varphi n_t$$

Euler equation:

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$

Where lowercase variables denote log deviations from steady state

Firms as Price Setters

Monopolistic competition

Each firm faces household demand:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

Production technology:

$$\boxed{Y_t(i) = Z_t N_t(i)^{1-\alpha}}$$

where $z_t = \ln(Z_t)$ is an aggregate technology shock

Flexible Prices Benchmark

Profit maximization

With flexible prices, firms maximize:

$$\Pi_t(i) = P_t(i)Y_t(i) - W_t N_t(i)$$

This gives the optimal price:

$$\boxed{P_t(i) = \frac{\epsilon}{\epsilon-1}\frac{W_t}{Z_t N_t(i)^{-\alpha}(1-\alpha)}}$$

where $\frac{\epsilon}{\epsilon-1}$ is the markup M

Monopolistic competition decreases employment and output relative to perfect competition

Sticky Prices

Calvo (1983) model

Key assumptions:

- Probability of price adjustment: $1-\theta$ (independent across firms)
- $\theta \in [0,1]$ is the index of price stickiness
- $\frac{1}{1-\theta}$ is the average price duration
- Adjusting firms choose the same optimal price P_t^*

Price level evolution:

$$\boxed{P_t = \left[\theta(P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}$$

Optimal Pricing Decision

Forward-looking price setting

Firms choose their price taking into account future periods:

$$\boxed{\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left(P_t^* Y_{t+k|t} - W_{t+k} N_{t+k|k} \right) \right\}}$$

subject to technology and demand constraints

where $Q_{t,t+k} \equiv \beta^k (U_{c,t+k}/U_{c,t})(P_t/P_{t+k})$ is the stochastic discount factor

Optimal Price Condition

First order condition

The optimality condition is:

$$\boxed{\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(P_t^* - \mathcal{M} \Psi_{t+k|t} \right) \right\} = 0}$$

where:

- $\Psi_{t+k|t}$ is the nominal marginal cost
- $\mathcal{M} = \frac{\epsilon}{\epsilon - 1}$ is the markup

Log-linearized around zero inflation steady state:

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \psi_{t+k|t}$$

Marginal Cost

Individual versus average

Individual firm marginal cost:

$$\psi_{t+k|t} = w_{t+k} - (z_{t+k} - \alpha n_{t+k|t} + \log(1-\alpha))$$

Economy's average real marginal cost:

$$\psi_{t+k} = w_{t+k} - (z_{t+k} - \alpha n_{t+k} + \log(1-\alpha))$$

The relationship between them:

$$\boxed{\psi_{t+k|t} = \psi_{t+k} - \tfrac{\alpha\epsilon}{1-\alpha}\left(p_t^* - p_{t+k}\right)}$$

Phillips Curve Derivation

Combining optimality conditions

Combining the price setting condition with marginal cost relation and using:

$$\pi_t = p_t - p_{t-1} = (1-\theta)\,(p_t^* - p_{t-1})$$

We obtain the New Keynesian Phillips Curve:

$$\boxed{\pi_t = \lambda\,(\psi_t - p_t + \mu) + \beta E_t\{\pi_{t+1}\}}$$

where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$ and $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$

Equilibrium Conditions

Market clearing

Goods market clearing:

$$Y_t(i) = C_t(i)$$

Aggregating gives:

$$y_t = c_t$$

Labor market clearing:

$$N_t = \int_0^1 N_t(i)di = \int_0^1 \left(\frac{Y_t(i)}{Z_t}\right)^{\frac{1}{1-\alpha}} di$$

This implies:

$$n_t = \frac{1}{1-\alpha}(y_t - z_t) + d_t$$

Marginal Cost and Output Gap

Linking real variables

Using labor supply and technology:

$$mc_t = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t - \left(1 + \frac{\varphi + \alpha}{1 - \alpha}\right) z_t - \log(1 - \alpha)$$

The natural level of output (flexible prices):

$$y_t^n = \psi_y + \psi_{yz} z_t$$

Therefore, the marginal cost gap is:

$$\boxed{(mc_t - mc^n) = \left(\sigma + \tfrac{\varphi+\alpha}{1-\alpha}\right)(y_t - y_t^n)}$$

New Keynesian Phillips Curve

Final form

Substituting the marginal cost gap:

$$\boxed{\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t}$$

where:

- $\tilde{y}_t = y_t - y_t^n$ is the output gap
- $\kappa = \lambda \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right)$

This is the first equation of the New Keynesian model

Dynamic IS Curve

Consumption-saving condition

From the Euler equation and goods market equilibrium:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$

In terms of output gap:

$$\boxed{\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \tfrac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)}$$

where r_t^n is the natural rate of interest (Wicksellian):

$$r_t^n = \rho + \sigma E_t[y_{t+1}^n - y_t^n]$$

Policy Block

Monetary policy rule

To close the model, we need a rule for the nominal interest rate.

Interest rate rule:

$$\boxed{i_t = \rho + \phi_\pi \pi_t + v_t}$$

where v_t is a monetary policy shock

We have approximated the large microfounded nonlinear model to a 3-equation linear system

Complete Model

The three-equation New Keynesian system

Dynamic IS:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

New Keynesian Phillips Curve:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

Monetary Policy Rule:

$$i_t = \rho + \phi_\pi \pi_t + v_t$$

Uniqueness condition: $\kappa(\phi_\pi - 1) > 0$ (Taylor principle)