Introduction to supersymmetry

(and some supergravity)

Sam Rogerson

February 3, 2011

Outline

What is supersymmetry

Why we need BSM physics

Constructing the details of SUSY

Models of supersymmetry

Reading

Really there is no one book that will have everything you need;

- A supersymmetry primer, S. P. Martin. arXiv:hep-ph/9709356
- Supersymmetry, supergravity and Particle Physics, H. P. Nilles Physical Reports 1977
- Dijet Searches for Supersymmetry at the LHC, arXiv:0806.1049v1
- Search for Supersymmetry in pp Collisions at 7 TeV in Events with Jets and Missing Transverse Energy, arXiv:1101.1628v1

Part I

What is supersymmetry

Symmetries and Physics

Definition

If a system is invariant under an operation, the operation is said to represent a symmetry of the system.

One can construct a Lagrangian (T-V) that embeds these symmetries and, using the Euler-Lagrange equations:

$$\frac{\partial L}{q_\alpha} - \frac{d}{dt}\frac{\partial L}{\dot{q}_\alpha} = 0,$$

derive the equations of motion for the system.

Charges and Conservation

Definition

Noether's Theorem: For each continuous symmetry of a system there is a corresponding conserved quantity

- Translational invariance: Linear Momentum
- Rotational invariance: Angular Momentum
- Time invariance: Energy

You can group these into a more compact notation:

$$\mathrm{Translations} \times \mathrm{SO}(3,1)$$

This is even more compactly known as the Poincaré group.

Symmetries in particle physics

The Standard Model embeds three gauge[1] symmetries:

$$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$$

These then predict various conservation laws:

- Colour charge (C)
- Weak Isospin (T)
- Weak hypercharge (Y)
- (Electric charge $Q = Y + T_3$)

[1] Invariant under some continuous group(s) of local transformations

Additional Symmetry?

Two distinct groups of particles:

- Bosons (spin $\in \mathbb{Z}$)
- Fermions (spin $\in (\mathbb{Z} \times \frac{1}{2})$)

In aiming for some unified theory we hope these would be indistinct. Posit the existence of symmetry that links fermions(f) and bosons(b)

$$\hat{O}\,|f\rangle = |b\rangle\,;\,\hat{O}\,|b\rangle = |f\rangle\,;$$

And our conserved quantity is particle number, rather than lepton/baryon numbers individually.

Imperial College London

Supersymmetry is...

Definition

A symmetry which relates the SM particles with spin S to a second set of particles with spin $S \pm \frac{1}{2}$.

This means for every SM particle there is a corresponding 'superpartner' with identical quantum numbers known as a sparticle.

Part II

Why we need BSM physics

Standard Model

Plenty of evidence in favour of the Standard Model:

- Predictions: Existence of W boson,Z boson, charm quark, top quark and the gluon.
- Electroweak precision measurements: Mass and width of W and Z bosons.

It is, however, not complete

What it doesn’t include

- Neutrino masses (Dirac/Majorana): can add terms, but not a direct result
- Gravity
- Dark Matter (‘orthogonal’ measurement)

What does it get “wrong”?

- Anomalous Magnetic Moment of the Muon: $(g-2)_\mu$
- Natural solution to W-scattering cross-section
- Higgs mass

There is also the problem of naturalness:

- Why do we need 19 parameters to describe the SM?
- Why do we have a hierarchy problem? (Fine tuned cancellations to Fermi's constant)

What is the hierarchy problem?

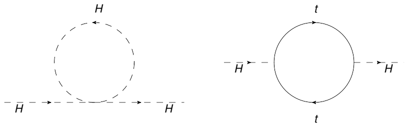
- Suppose SM valid up to GUT scale ($O(10^{15}\mathrm{GeV})$) or even the Planck scale ($O(10^{19}\mathrm{GeV})$)
- Weak interaction scale $\sim 100\mathrm{GeV}$
- If the inputs are chosen at these scales, the scalar mass terms in the Higgs sector must be chosen with an accuracy of 10^{-34} wrt the Planck mass.

W-scattering cross-section

One of the strongest indicators for something Higgs-like/BSM physics:

- Theory: the value explodes at some energy scale
- Violates perturbative unitarity
- Simplest solution: extra propagator = extra diagram $\rightarrow$ cancels divergences
- This propagator is a scalar $\rightarrow$ Higgs
- Good evidence for upper limit on Higgs mass

Divergent Higgs Mass



The one loop scalar(left) and fermion(right) corrections to the Higgs mass. When calculated we find the following contributions:

- dirac fermion $(-\lambda_f \bar{f} f)$: $\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2} \Lambda_{UV}^2 + \dots$
- $\mathbb{C}$-scalar$(-\lambda_S |H|^2 |S|^2)$:
  $\Delta m_H^2 = \frac{\lambda_S}{16\pi^2} \left[\Lambda_{UV}^2 - 2 m_S^2 \ln \left(\Lambda_{UV} / m_S \right) \right]$

Here Λ_{UV} is known as the 'ultraviolet' cutoff - it regulates the loop integral, i.e $M = \int_0^{\Lambda_{UV}} \dots$

Divergent Higgs Mass Part 2

These corrections imply two important things:

1. A quadratic dependence on Λ_{UV}
2. A quadratic dependence on m_S

This means it is both sensitive to the scale of 'new-physics', as well as the masses of the heaviest particles. Strangely enough the latter problem arises even if we have no direct coupling between the SM Higgs and the unknown "S".

We'll come back to this...

Why it could be SUSY

Phenomenologically While we haven't developed the details, yet, the extended particle spectrum gives hints at what it could provide,

- Corrections to $\Delta(g-2)_\mu$
- A candidate for dark matter.
- Cancellation terms for the Higgs mass
- A solution to the Hierarchy problem

Part III

Constructing the details of SUSY

Multiplets

In the standard model we arrange particles into the irriducible representations of the algebra called multiplets,

$$\begin{pmatrix} e_L \\ \nu_e \end{pmatrix}, \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

A similar approach is taken for SUSY; we form supermultiplets, simply an arrangement which contains both fermion and boson partner states

Multiplets Continued

If $|\Omega\rangle$ and $|\Omega'\rangle$ are both members of the same supermultiplet, then they are linear combinations of each other (with the SUSY operator $\hat{O}$ and $\hat{O}^\dagger$).
When constructing these multiplets we must ensure:

$$n_B = n_F$$

where n_B and n_F are the bosonic and fermionic degrees of freedom respectively (this can be shown).
The simplest arrangement: single weyl fermion (2 spin states, $n_F = 2$) and then 2 real scalars ($n_B = 1$ each)

More Multiplets

Note: $2 \times \mathbb{R}\text{-scalar} \equiv 1 \times \mathbb{C}\text{-scalar}$

The next simplest arrangement is a spin-1 vector boson ($n_B = 2$), with a massless Weyl fermion as its superpartner.

These two arrangements have names:

- spin-1/2 Weyl Fermion + $\mathbb{C}$-scalar: Chiral supermultiplet
- spin-1 Vector boson + massless Weyl fermion: Gauge supermultiplet

Imperial College London

Breaking SUSY

If SUSY were an exact symmetry of the Lagrangian, then we would have,

$$M_{\mathrm{SM}} = M_{\mathrm{SUSY}}$$

for all particles. If this were true, SUSY would be evident in experiments already. We therefore expect SUSY to be broken to some degree, i.e. the mass scale of the particles is not completely restricted. There is a much better reason than this...

What is a gauge anomaly

Definition

A gauge anomaly is any effect that breaks the gauge symmetry of the system, in our case the quantum field theory.

The Higgs Sector

Naively one would expect the following:

$$\text{Higgs } H \leftrightarrow \tilde{H} \text{ Higgsino}$$

There is a (two) problem(s) with this,

- Introduction of a gauge anomaly, namely Higgsino ($S = \pm 1/2$) has hypercharge $\mathrm{Y} = \frac{1}{2}$, so we no longer have $\sum \mathrm{Y} = 0$ (our $U(1)$ gauge symmetry has been broken)

The Higgs Sector Continued

Can remedy this gauge anomaly by introducing a second higgs doublet, labelling the two (unsuggestively) H_u and H_d.

- Opposite hypercharge (fixes the anomaly)
- H_u now couples to up-type fermions (u, c, t, ℓ)
- H_d couples to down-type fermions (d, s, b, ν_ℓ)

Supersymmetry is...

A symmetry that generates a second set of particles matching those in the SM but with masses at a higher scale.

- Each boson(fermion) has a corresponding (fermion)boson superpartner
- A second Higgs doublet is included, resulting in 5 physical Higgs bosons

Part IV

Models of supersymmetry

Minimal Supersymmetric Standard Model

So far we have constructed what is known as the Minimal Supersymmetric Standard Model (MSSM)

- super-partners for each SM particle
- softly-broken (mass differences, but no quadratic divergences)

This was motivated by the SM seeming “unnatural”,

- Number of parameters (19)
- Energy scale (M_{pl} vs. M_W)

Imperial College London

Not so natural...

Unfortunately we have only really solved one of these problems,

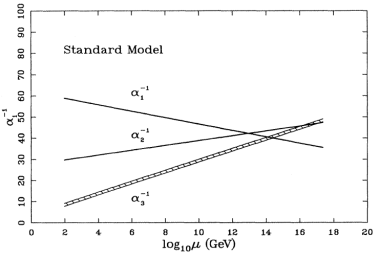
- Energy scale almost
- Parameters

To fully specify all the masses and couplings in the MSSM we need 108 free parameters.
Solution: Choose some ‘natural‘ boundary condition to alleviate this problem.

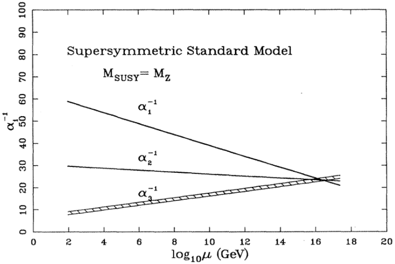
Constrained MSSM

Motivating unification,

Standard model



MSSM



CMSSM: A GUT scale condition

We now impose another unifying constraint:

- Scalar masses unify $= m_0$
- Gaugino masses unify $= m_{1/2}$
- Trilinear couplings unify $= A_0$

This almost gives us a complete description of the physics, just need to describe the Higgs sector,

- Two higgs fields - $\phi_{H_u}/\phi_{H_d} = \tan(\beta)$

CMSSM: dragging it out...

While phenomenologically not the most interesting:

- Simple unification collapses parameter space
- Forces ratio between the gauge couplings

This last point is important for the phenomenology. Think about this decay:

$$\tilde{g} \to \tilde{q} \to \tilde{\chi}_1^0$$

If the couplings are fixed, then the mass ratios are also fixed.

$$6 : 4 : 1$$

NUHM

While there is “motivation” for the CMSSM, the phenomenology is not wide ranging. NUHM is an attempt to remedy that, and introduce a degree of freedom:

- NUHM1 $m_{H_u} = m_{H_d} \neq m_0$
- NUHM2 $m_{H_u} \neq m_{H_d} \neq m_0$

So, gravity?

Briefly touched on SUSY-breaking

- Choose a scale at which SUSY-breaks down M_s
- Allows you to calculate M_W, the scale of EWSB
- Usually expect $M_s \sim 100\mathrm{GeV}$
- Actually possible to have $M_s \sim 10^{11}\mathrm{GeV}$
- Can still recover M_W and "reasonable" mass splittings

(actually can lead to preferred behavior)

Supergravity

Recall $M_p \sim 10^{19}\mathrm{GeV}$.

- Can no longer ignore gravity
- All particles participate in gravitational interactions
- Mass splitting of supermultiplets of order $M_s^2/M_p \sim 10^3\mathrm{GeV}$

This means that we have a mass-splitting of order the weak scale, this mass splitting includes the gravitino with

$$m_{3/2} = M_s^2/M_p \tag{1}$$

This actually provides some interesting phenomenology (next lecture)

Summary

- SUSY adds a new set of particles on top of the SM group
- Remedies many theoretical and phenomenological problems with the SM
- Is verifiable - interesting phenomenology - tomorrow :)