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Intuitionistic fuzzy shortest hyperpath in a network

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ABSTRACT

Intuitionistic fuzzy graph theory is finding an increasing number of applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. Intuitionistic fuzzy models are becoming useful because of their aim in reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems. In this paper, a method for finding the shortest hyperpath in an intuitionistic fuzzy weighted hypergraph is proposed. An intuitionistic fuzzy number is converted into intuitionistic fuzzy scores. To find the intuitionistic fuzzy shortest hyperpath in the network, ranking is done using the scores and accuracy.

1. Introduction

A hypergraph is a common name for various combinatorial structures that generalize graphs. Besides, the most well-known undirected hypergraphs, a relevant role is played by directed hypergraphs which find applications in several areas of computer science and mathematics for representing implicative structures. Directed hypergraph is an extension of directed graphs, and have often been used in several areas such as a modeling and algorithmic tool. A technical as well as historical introduction to directed hypergraphs has been given by Gallo et al. [10]. A hyperpath in a hypergraph is a nontrivial extension of directed paths whose expressive power allows us to deal with more complex situations. One of the famous classical problems extended in the analysis of networks is the shortest path problem. Traditionally, the shortest path problem has the single objective to minimize total distance, or travel time, for traversing all the nodes in a directed graph. Nevertheless, due to the multiobjective nature of many transportation and routing problems, a single objective function is not sufficient to completely, characterize some real world problems. In a road network, the fastest path may be too costly or the cheapest path may be too long. Therefore, the decision makers must choose a solution among the paths. Any time a structure is represented by means of a hypergraph, it may be relevant to find hyperpaths that connect nodes or sets of nodes, or minimum hyperpaths, where the minimality is defined on the basis of a weight to hyperpath. Instead of assigning weights to hyperpath, we use intuitionistic fuzzy numbers for modeling the problem and finding intuitionistic fuzzy shortest hyperpath in a network. The fuzzy shortest path problem was first analyzed by Dubois and Prade [9] by assigning a fuzzy number instead of a real number, to each edge in the hypergraph. Intuitionistic fuzzy set (IFS), first introduced by Atanassov [4] has been found to be compatible to deal with vagueness. Ban [6], Burillo et al. [7], Wang [14] proposed definition of intuitionistic fuzzy number (IFN). The authors of this paper introduced the concept of Intuitionistic fuzzy

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hypergraph [13]. In this paper, we provide a new method namely, the score-based method for finding shortest hyperpaths in a network with intuitionistic fuzzy weights of hyperedges. The computation procedure of this method is simpler than other existing methods. We illustrate this point with a numerical example.

We have used standard definitions and terminologies in this paper. For other notations, terminologies and applications not mentioned in the paper, the readers are referred to [1–5,8].

2. Preliminaries

In this section, the definition of an intuitionistic fuzzy hypergraph, directed intuitionistic fuzzy hypergraph, triangular intuitionistic fuzzy number, score, ranking of intuitionistic fuzzy numbers are given. These are the basic concepts required for designing the algorithm to find the shortest hyperpath.

Definition 2.1. An intuitionistic fuzzy hypergraph (IFHG) H is an ordered pair $H=(V,E)$ where

1. $V=\{v_1,v_2,\dots,v_n\}$, a finite set of vertices.
2. $E=\{E_1,E_2,\dots,E_m\}$, a family of intuitionistic fuzzy subsets of V.
3. $E_j=\{(v_i,\mu_j(v_i),\nu_j(v_i))\colon\ \mu_j(v_i),\nu_j(v_i)\geqslant 0$ and $0\leqslant \mu_j(v_i)+\nu_j(v_i)\leqslant 1\}$, $j=1,2,\dots,m$.
4. $E_j\neq\phi$, $j=1,2,\dots,m$.
5. $\bigcup_j \operatorname{supp}(E_j)=V$, $j=1,2,\dots,m$.

Here, the edges E_j are IFSs. $\mu_j(x_i)$ and $\nu_j(x_i)$ denote the degrees of membership and non-membership of vertex v_i to edge E_j. Thus, the elements of the incidence matrix of IFHG are of the form $(a_{ij},\mu_j(x_i),\nu_j(x_i))$. The sets V and E are crisp sets.

Definition 2.2. A directed intuitionistic fuzzy hypergraph H is a pair (N,E) where N is a non-empty set of nodes and E is a set of intuitionistic fuzzy hyperarcs; an intuitionistic fuzzy hyperarc $e\in E$ is defined as a pair $(T(e),h(e))$, where $T(e)\subset N$, with $T(e)\neq\emptyset$, is its tail, and $h(e)\in N-T(e)$ is its head. A node s is said to be a source node in H if $h(e)\neq s$, for every $e\in E$. A node d is said to be a destination node in H if $d\neq T(e)$, for every $e\in E$.

Definition 2.3. Let X be a non-empty set and let $A=\{(x,\mu_A(x),\nu_A(x))/x\in X\}$ be an IFS, then the pair $(\mu_A(x),\nu_A(x))$ is called as an intuitionistic fuzzy number, denoted by $(\langle a,b,c\rangle,\langle e,f,g\rangle)$, where $\langle a,b,c\rangle\in F(I)$, $\langle e,f,g\rangle\in F(I)$, $I=[0,1]$, $0\leqslant c+g\leqslant 1$.

Definition 2.4. A triangular intuitionistic fuzzy number (TriIFN) A is denoted by $A=\{(\mu_A(x),\nu_A(x))\mid x\in X\}$, where $\mu_A(x)$ and $\nu_A(x)$ are triangular fuzzy numbers with $\nu_A(x)\leqslant\mu_A^c(x)$. So, a TriIFN A is given by $A=(\langle a,b,c\rangle,\langle e,f,g\rangle)$ with $(\langle e,f,g\rangle\leqslant\langle a,b,c\rangle^c)$. That is, either $e\geqslant b$ and $f\geqslant c$ or $f\leqslant a$ and $g\leqslant b$ are membership and non-membership fuzzy numbers of A. The diagrammatic representation of an intuitionistic fuzzy number $A=(\langle a,b,c\rangle,\langle e,f,g\rangle)$ with $e\geqslant b$ and $f\geqslant c$ is shown in Fig. 1.

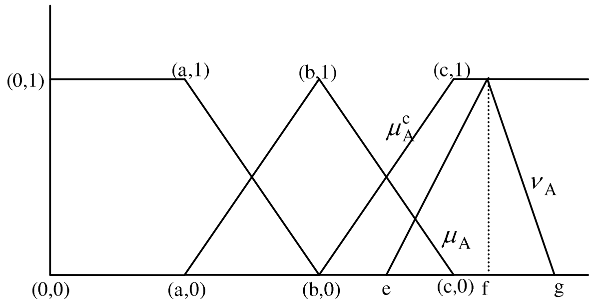

Fig. 1. Triangular intuitionistic fuzzy number $A=(\langle a,b,c\rangle,\langle e,f,g\rangle)$.

Definition 2.5. Let $A=(\langle a_1,b_1,c_1\rangle,\langle e_1,f_1,g_1\rangle)$ and $B=(\langle a_2,b_2,c_2\rangle,\langle e_2,f_2,g_2\rangle)$ be two TriIFNs. Then the addition of two TriIFN, denoted by $A+B$, is defined as $A+B=(\langle a_1+a_2,b_1+b_2,c_1+c_2\rangle,\langle e_1+e_2,f_1+f_2,g_1+g_2\rangle)$.

Definition 2.6. Let $\tilde{A}^I=\{(a,b,c),(e,f,g)\}$ be a TriIFN, then the score of $\tilde{A}^I$ is an IFS whose membership and non-membership values are given respectively as $S(\tilde{A}^{I\mu})=\frac{a+2b+c}{4}$ and $S(\tilde{A}^{I\nu})=\frac{e+2f+g}{4}$.

Definition 2.7. The accuracy of a TriIFN $\tilde{A}^I$ is defined as $Acc(A)=\frac{1}{2}(S(\tilde{A}^{I\mu}+S(\tilde{A}^{I\nu})))$.

3. Minimum arc length of an intuitionistic fuzzy hyperpath

In this section, as discussed earlier, the arc length in a hypernetwork is considered to be an TriIFN. The algorithm given in this section is based on [15]. Let L_i denotes arc length of the ith hyperpath.

Algorithm.

Step 1. Compute the lengths of all possible hyperpaths L_i for $i=1,2,3,\dots,n$, where $L_i=(\langle a_i',b_i',c_i'\rangle,\langle e_i',f_i',g_i'\rangle)$.

Step 2. Initialize $L_{\min}=(\langle a,b,c\rangle,\langle e,f,g\rangle)=L_1=(\langle a_1',b_1',c_1'\rangle,\langle e_1',f_1',g_1'\rangle)$.

Step 3. Set $i=2$.

Step 4. Compute the membership values $\langle a,b,c\rangle$ as

$$a=\min(a,a_i'),$$
$$b=\begin{cases} b, & \text{if } b\leqslant a_i',\\ \frac{bb_i'-aa_i'}{(b+b_i')-(a+a_i')}, & \text{if } b>a_i',\end{cases}$$
$$c=\min(c,b_i'),$$

and non-membership values $\langle e,f,g\rangle$ as

$$e=\min(e,e_i'),$$
$$f=\begin{cases} f, & \text{if } f\leqslant e_i',\\ \frac{ff_i'-ee_i'}{(f+f_i')-(e+e_i')}, & \text{if } f>e_i',\end{cases}$$
$$g=\min(g,f_i').$$

Step 5. Set $L_{\min} = (\langle a, b, c\rangle, \langle e, f, g\rangle)$ as calculated in step-4.
Step 6. $i = i + 1$.
Step 7. if $i < n + 1$, go to step 3, otherwise stop the procedure.

In computer science, the time complexity of an algorithm quantifies the amount of time taken by an algorithm to run as a function of the length of the string representing the input. This algorithm matches to travel salesman problem which is NP (Non-deterministic Polynomial-time) hard problem.

Example 3.1. Consider a network with a triangular intuitionistic fuzzy arc length shown in Fig. 2.

Step 1. There are six possible paths ($n = 6$) from source node 1 to destination node 8, as given below:
Path (1): $1 \to 2 \to 6 \to 8$, $L_1 = (\langle 10, 15, 18\rangle, \langle 13, 21, 25\rangle)$,
Path (2): $1 \to 3 \to 6 \to 8$, $L_2 = (\langle 13, 18, 23\rangle, \langle 18, 27, 32\rangle)$,
Path (3): $1 \to 3 \to 7 \to 8$, $L_3 = (\langle 16, 21, 26\rangle, \langle 22, 31, 36\rangle)$,
Path (4): $1 \to 4 \to 7 \to 8$, $L_4 = (\langle 5, 9, 13\rangle, \langle 12, 19, 24\rangle)$,
Path (5): $1 \to 5 \to 7 \to 8$, $L_5 = (\langle 6, 10, 13\rangle, \langle 15, 21, 26\rangle)$,
Path (6): $1 \to 5 \to 8$, $L_6 = (\langle 10, 14, 17\rangle, \langle 16, 20, 25\rangle)$.
Step 2. Initialize $L_{\min} = (\langle a, b, c\rangle, \langle e, f, g\rangle) = L_1 = (\langle a'_1, b'_1, c'_1\rangle, \langle e'_1, f'_1, g'_1\rangle) = (\langle 10, 15, 18\rangle, \langle 13, 21, 25\rangle)$.
Step 3. Initialize $i = 2$.
Step 4. Let $L_{\min} = (10, 15, 18)(13, 21, 25)$ and $L_2 = (\langle a'_2, b'_2, c'_2\rangle, \langle e'_2, f'_2, g'_2\rangle) = (13, 18, 23)(18, 27, 32)$. Compute the membership values $\langle a, b, c\rangle$ as

$$a = \min(a, a'_2) = \min(10, 13) = 10,$$

$$b = \left\{ \frac{(15 \times 18) - (13 \times 10)}{(15 + 18) - (13 + 10)} = 14 \quad \text{since } b > a'_2,\right.$$

$$c = \min(c, b'_2) = \min(18, 23) = 18,$$

and non-membership values $\langle e, f, g\rangle$ as

$$e = \min(e, e'_2) = \min(13, 18) = 13,$$

$$f = \left\{ \frac{(27 \times 21) - (13 \times 18)}{(27 + 21) - (13 + 18)} = 19.59 \quad \text{since } f > e'_2,\right.$$

$$g = \min(g, f'_2) = \min(25, 27) = 25.$$

Step 5. Set $L_{\min} = (\langle 10, 14, 18\rangle, \langle 13, 19.59, 25\rangle)$.
Step 6. $i = i + 1 = 3$.
Step 7. if $i < n + 1 (= 5)$, go to step 4.
Step 4. Let $L_{\min} = (10, 14, 18)(13, 19.59, 25)$ and $L_3 = (\langle a'_3, b'_3, c'_3\rangle), (\langle e'_3, f'_3, g'_3\rangle) = (16, 21, 26)(22, 31, 36)$. Compute the membership values $(\langle a, b, c\rangle)$ as

$$a = \min(a, a'_3) = \min(10, 16) = 10,$$

$$b = 14 \text{ since } b < a'_3,$$

$$c = \min(c, b'_3) = \min(18, 21) = 18,$$

and non-membership values $(\langle e, f, g\rangle)$ as

$$e = \min(e, e'_3) = \min(13, 22) = 13,$$

$$f = 19.59 \quad \text{since } f < e'_3,$$

$$g = \min(g, f'_3) = \min(25, 31) = 25.$$

Table 1

Path	Score	Accuracy	Rank
P_1	$\langle 14.5, 20\rangle$	17.25	4
P_2	$\langle 18, 26\rangle$	22	5
P_3	$\langle 21, 30\rangle$	25.5	6
P_4	$\langle 9, 18.5\rangle$	13.75	1
P_5	$\langle 9.75, 20.75\rangle$	15.25	2
P_6	$\langle 13.75, 20.25\rangle$	17	3

Step 5. Set $L_{\min} = (\langle 10, 14, 18\rangle, \langle 13, 19.59, 25\rangle)$. Repeat the procedure until $i = 6$.

Finally, we get the minimum of arc lengths of intuitionistic fuzzy hyperpath as

$$L_{\min} = \bigl(\langle 5, 7.65, 9\rangle, \langle 12, 15.55, 19\rangle\bigr).$$

4. An algorithm for searching the shortest hyperpath

The algorithm in Section 4 gives us the minimum arc length of the intuitionistic fuzzy hyperpath. But we aim at determining an intuitionistic fuzzy shortest hyperpath to traverse from source to destination. To achieve this, we have already existing methods, namely Similarity measure method [11] and Euclidean distance method [12]. Here, we propose a new method based on scores of IFNs, or simply score-based method. Among these three methods,for similarity and Euclidean methods, the arc length of intuitionistic fuzzy hyperpath must be known. As both the methods give the same result, we consider only Euclidean distance method for comparison.

4.1. Score-based method

In this section we find the intuitionistic fuzzy shortest hyperpath in the easiest way namely, score-based method and verify the result obtained from Euclidean method with the ranking using scores.

Step 1. Consider all possible paths from source node to destination node.
Step 2. Find the scores of the paths.
Step 3. Find their accuracy.
Step 4. Obtain the shortest hyperpath with the lowest accuracy.

4.1.1. Example

Consider the intuitionistic fuzzy hypernetwork given in Fig. 2. The intuitionistic fuzzy shortest hyperpath in this hypernetwork is identified using score-based method. From Table 1, the path $P_4 : 1 \to 4 \to 7 \to 8$ with least accuracy has been identified as intuitionistic fuzzy shortest hyperpath.

4.2. Euclidean distance method

Step 1. Find out all possible hyperpaths from source node to destination node d and compute the arc lengths of corresponding hyperpath L_i, $i = 1, 2, 3, \ldots, n$.

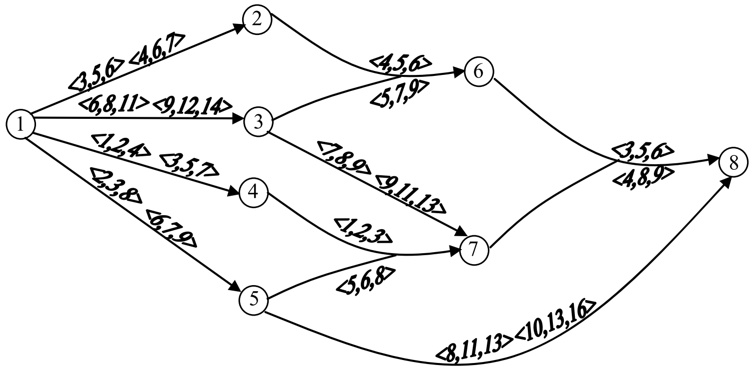

Fig. 2. Intuitionistic fuzzy hypernetwork.

Step 2. Compute $L_{\min}$ by using an intuitionistic fuzzy shortest hyperpath procedure.
Step 3. Find the Euclidean distance d_i for $i = 1, 2, 3, \ldots, n$ between all possible hyperpaths and $L_{\min}$.
Step 4. Decide the shortest hyperpath with the lowest Euclidean distance.

For the network given in Fig. 2, the algorithm is executed as follows:

Step 1.
Path (1): $1 \to 2 \to 6 \to 8$, $L_1 = (\langle 10, 15, 18\rangle, \langle 13, 21, 25\rangle)$,
Path (2): $1 \to 3 \to 6 \to 8$, $L_2 = (\langle 13, 18, 23\rangle, \langle 18, 27, 32\rangle)$,
Path (3): $1 \to 3 \to 7 \to 8$, $L_3 = (\langle 16, 21, 26\rangle, \langle 22, 31, 36\rangle)$,
Path (4): $1 \to 4 \to 7 \to 8$, $L_4 = (\langle 5, 9, 13\rangle, \langle 12, 19, 24\rangle)$,
Path (5): $1 \to 5 \to 7 \to 8$, $L_5 = (\langle 6, 10, 13\rangle, \langle 15, 21, 26\rangle)$,
Path (6): $1 \to 5 \to 8$, $L_6 = (\langle 10, 14, 17\rangle, \langle 16, 20, 25\rangle)$.
Step 2. $L_{\min} = (\langle 5, 7.65, 9\rangle, \langle 12, 15.55, 19\rangle)$.
Step 3. Euclidean distance between all hyperpath lengths P_i $(i = 1, 2, \ldots, 5)$ and $L_{\min}$:
$d(P_1, L_{\min}) = (\langle\sqrt{(10-5)^2 + (15-7.65)^2 + (18-9)^2}\rangle)$, $(\langle\sqrt{(13-12)^2 + (21-15.55)^2 + (25-19)^2}\rangle) = (\langle 12.65, 8.17\rangle)$,
$d(P_2, L_{\min}) = (\langle 19.16, 18.33\rangle)$,
$d(P_3, L_{\min}) = (\langle 24.25, 25.05\rangle)$,
$d(P_4, L_{\min}) = (\langle 4.22, 6.07\rangle)$,
$d(P_5, L_{\min}) = (\langle 4.74, 9.36\rangle)$,
$d(P_6, L_{\min}) = (\langle 11.37, 8.47\rangle)$.
Step 4. Decide the shortest hyperpath with the path having lowest membership and non-membership values by examining the Euclidean distance d between $L_{\min}$ and d_i for $i = 1, 2, \ldots, 6$. Hence it is concluded that Path (4): $1 \to 4 \to 7 \to 8$ has the least Euclidean distance.

5. Conclusions

Several methods have been reported in literature to deal with the intuitionistic fuzzy shortest hyperpath problems. In our method, similarity measure or Euclidean distance has been defined for TrIFNs, which helps in identifying the shortest hyperpath between intuitionistic fuzzy arc length and intuitionistic fuzzy shortest hyperpath. We have presented a new method, namely, the score-based method for finding shortest hyperpaths in a network with intuitionistic fuzzy weights for hyperedges without defining similarity measure and Euclidean distance. The scores of TrIFNs and ranking of the paths based on lowest accuracy help the decision maker to identify the preferable intuitionistic fuzzy shortest hyperpath. A natural extension of this research work is application of intuitionistic fuzzy digraphs in the area of soft computing, including neural networks, decision-making, and geographical information systems.

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References

[1] M. Akram, W.A. Dudek, Intuitionistic fuzzy hypergraphs with applications, Information Sciences 218 (2013) 182–193.
[2] M. Akram, B. Davvaz, Strong intuitionistic fuzzy graphs, Filomat 26 (2012) 177–196.
[3] M. Akram, M.G. Karunambigai, O.K. Kalaivani, Some metric aspects of intuitionistic fuzzy graphs, World Applied Sciences Journal 17 (2012) 1789–1801.
[4] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.
[5] K.T. Atanassov, Intuitionistic Fuzzy Sets: Theory and Applications, Studies in Fuzziness and Soft Computing, Physica-Verlag, Heidelberg, New York, 2012.
[6] A.I. Ban, Nearest interval approximation of an intuitionistic fuzzy number, in: Bernd Reusch (Ed.), Computational Intelligence, Theory and Applications, Springer-Verlag, Berlin, Heidelberg, 2006, pp. 229–240.
[7] P. Burillo, H. Bustince, V. Mohedano, Some definition of intuitionistic fuzzy number, in: Fuzzy Based Expert Systems, Fuzzy Bulgarian Enthusiasts, Sofia, Bulgaria, 1994, pp. 28–30.
[8] T.-N. Chuang, J.-Y. Kung, The fuzzy shortest path length and the corresponding shortest path in a network, Computers and Operations Research 32 (6) (2005) 1409–1428.
[9] D. Dubois, H. Prade, Fuzzy Sets and Systems: Theory and Applications, Academic Press, New York, 1980.
[10] G. Gallo, G. Longo, S. Nguyen, S. Pallottino, Directed hypergraphs and applications, Discrete Applied Mathematics 40 (1993) 177–201.
[11] G. Geetharamani, P. Jayagowri, Using similarity degree approach for shortest path in intuitionistic fuzzy network, in: IEEE International Conference on Computing, Communication and Applications (ICCCA), 22–24 Feb. 2012, pp. 151–156.

[12] A. Nagoorgani, M.M. Jabbarulla, On searching intuitionistic fuzzy shortest path in a network, Applied Mathematical Sciences 69 (4) (2010) 3447–3454.
[13] S. Thilagavathi, R. Parvathi, M.G. Karunambigai, Intuitionistic fuzzy hypergraphs, Cybernetics and Information Technologies 9 (2) (2009) 46–53.
[14] X. Wang, Fuzzy number intuitionistic fuzzy arithmetic aggregation operators, International Journal of Fuzzy systems 10 (2) (2008) 104–111.
[15] A.K. Yadav, B.R. Biswas, On searching fuzzy shortest path in a network, International Journal of Recent Trends in Engineering 2 (3) (2009) 16–18.