Optical stellar spectroscopy

II. Deriving Stellar Parameters

Bo Zhang (章 博)

Department of Astronomy, Beijing Normal University

bozhang@bnu.edu.cn

2020-11-06@PMO Nanjing

Introduce myself

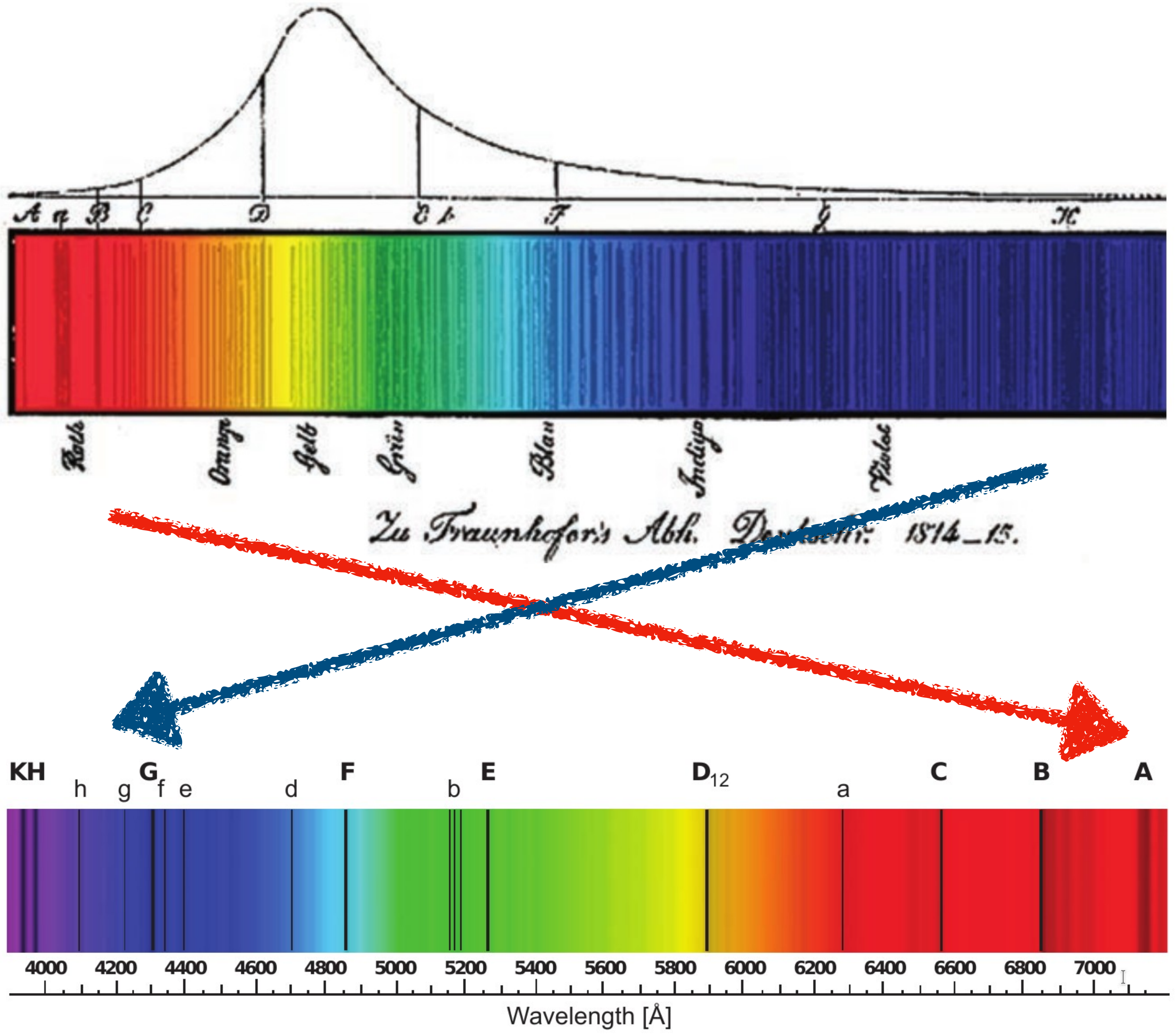
- Post-doc @BNU
- Ph.D. @NAOC
- Interests:
    - Stellar parameters
    - Spectroscopic binaries
    - Application of ML in spectroscopy
- Related Projects
    - LAMOST (R~2,000/7,500)
    - SONG ($36,000<R<180,000$)
- Other data
    - MMT Hectospec
    - 2.16m HRS/BFOSC (Xinglong)
    - 2.4m HRS (Lijiang)
    - SDSS
    - SDSS/MaGNA
    - SDSS/APOGEE

Contents

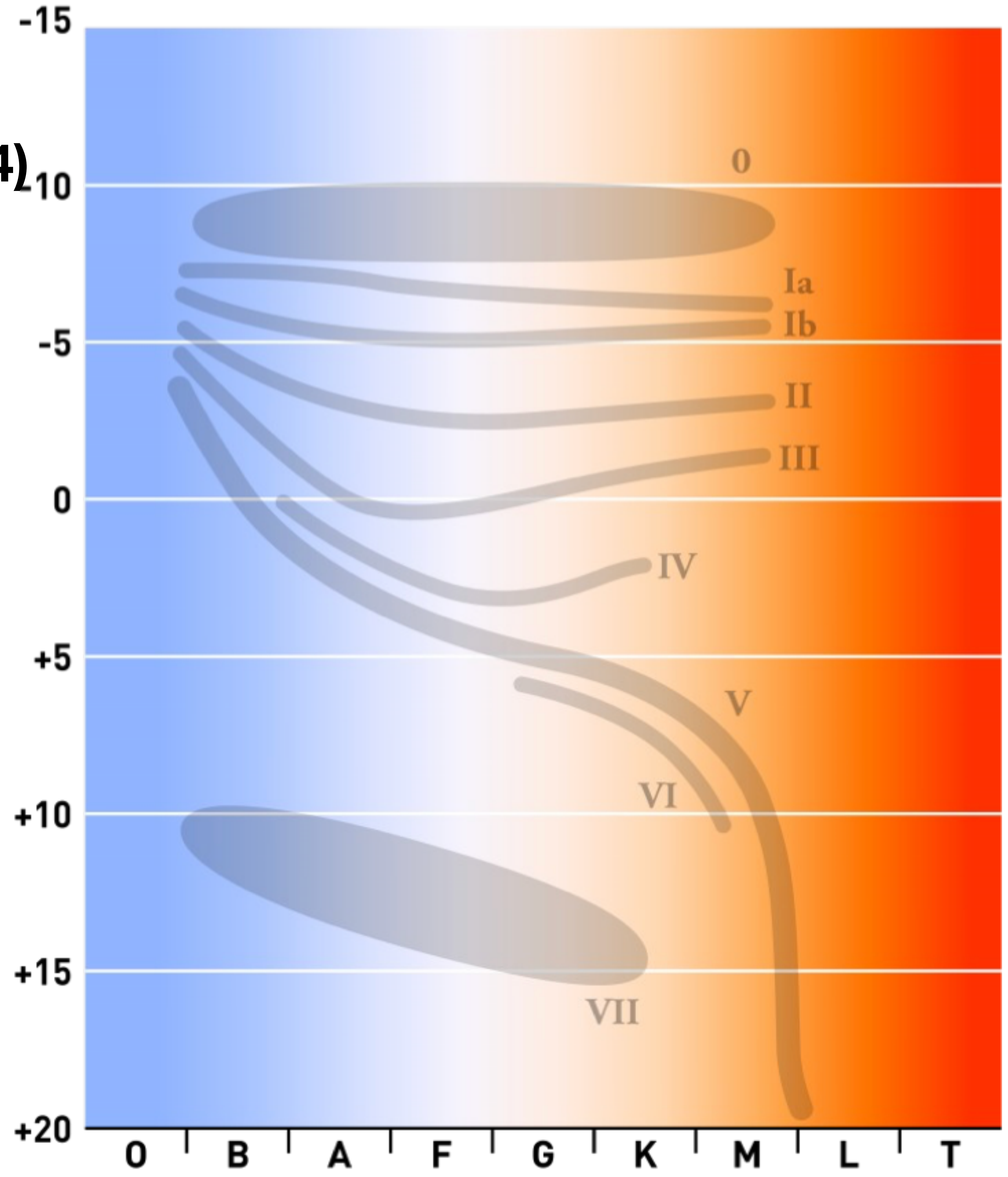
1. **Warm-up** — some background knowledge about stellar spectra
2. **Doppler effect** — measuring stellar radial velocity
3. **Teff, logg & [M/H]** — fundamental stellar atmospheric parameters
4. ***v* sin*i* profile***
5. **Basics**
    1. `Linux + Anaconda3 + Jupyter-notebook/Jupyter-Lab`
    2. `Numerical computing (cross-correlation, non-linear regression,…)`
6. **Link:**
    1. https://github.com/hypergravity/spectroscopy

1.0 History of stellar spectroscopy

Stellar spectroscopy over 200 years



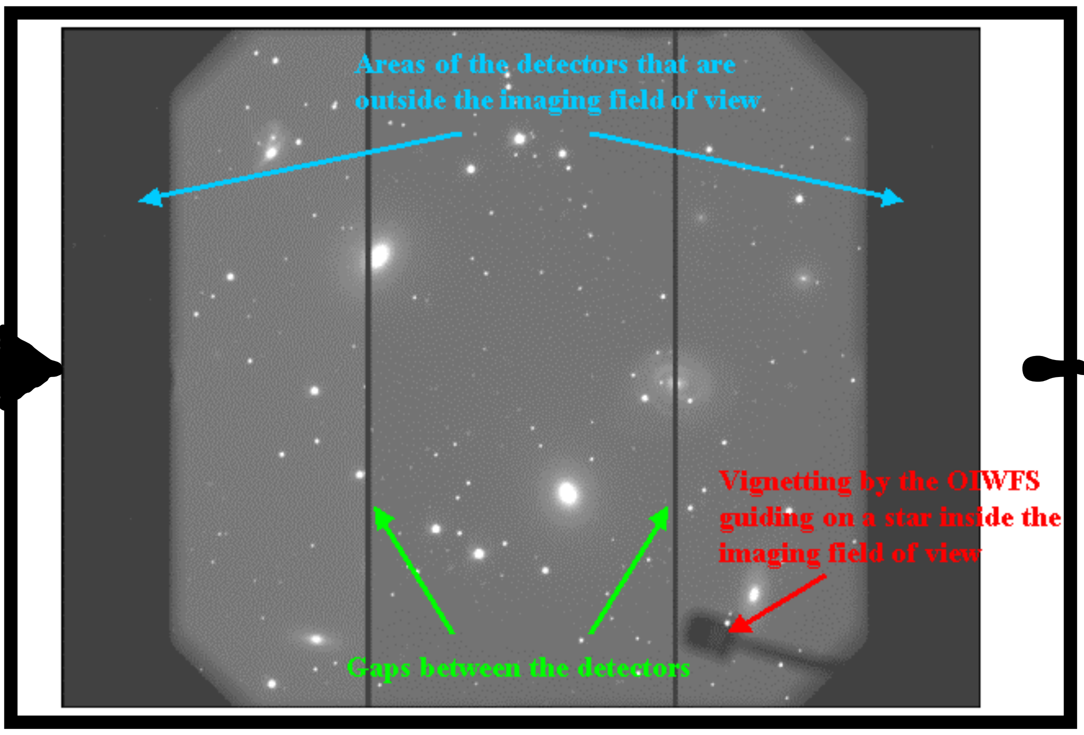
- **Joseph von Fraunhofer (1814)**
  - Solar spectrum
- **Annie Jump Cannon (1896-)**
  - Spectral type
  - OBAFGKM notation
- **Morgan & Keenan (~1943)**
  - Luminosity class
  - MK classification system

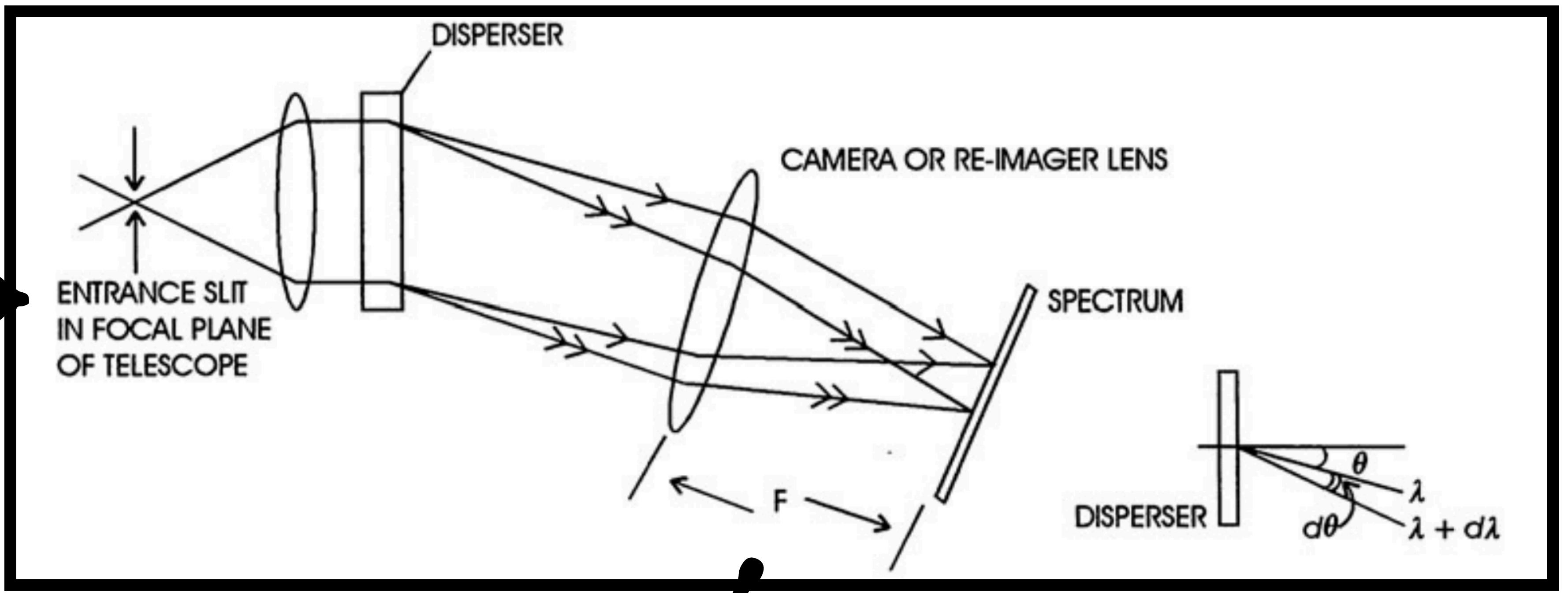


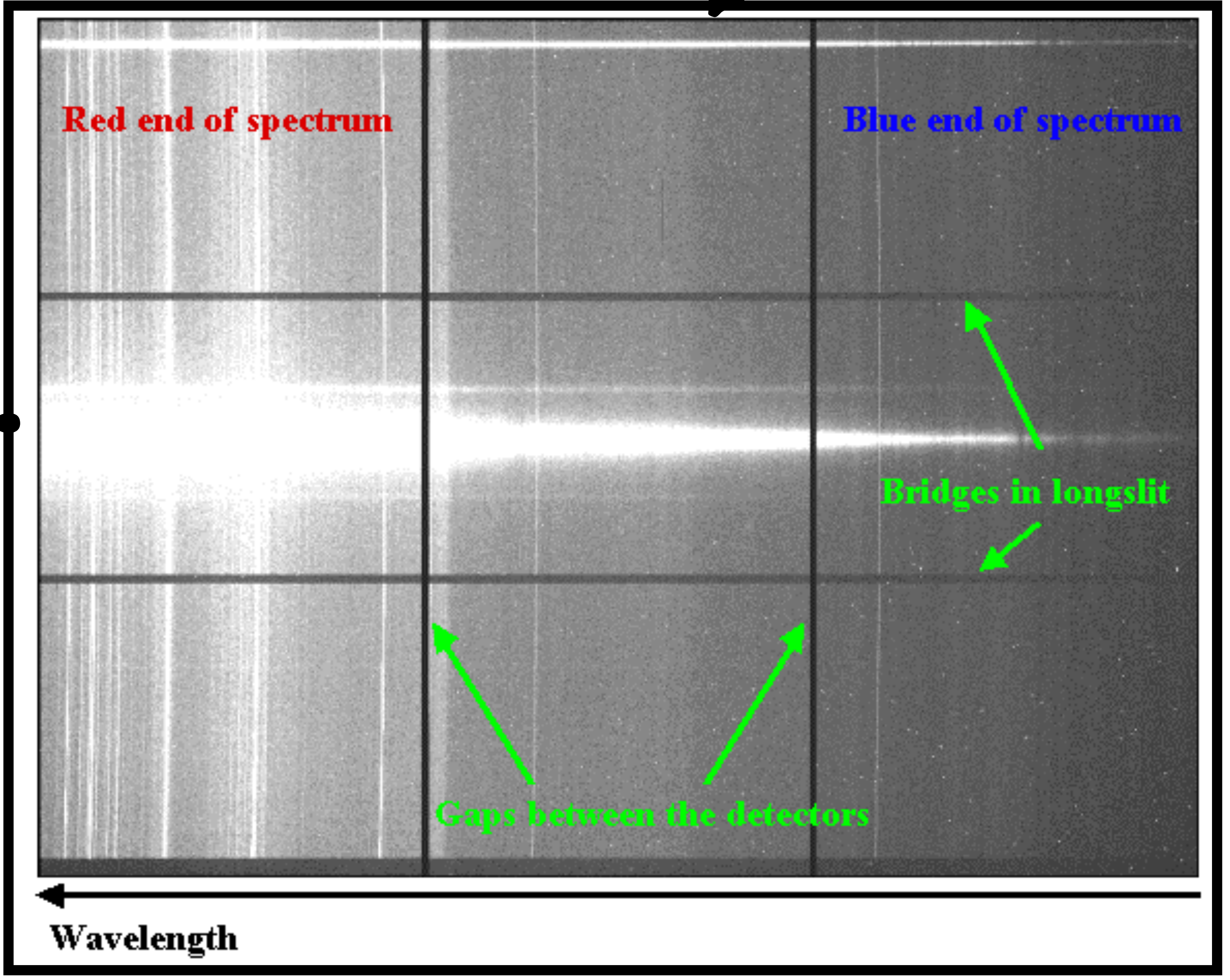
1.1 How to obtain stellar spectra?

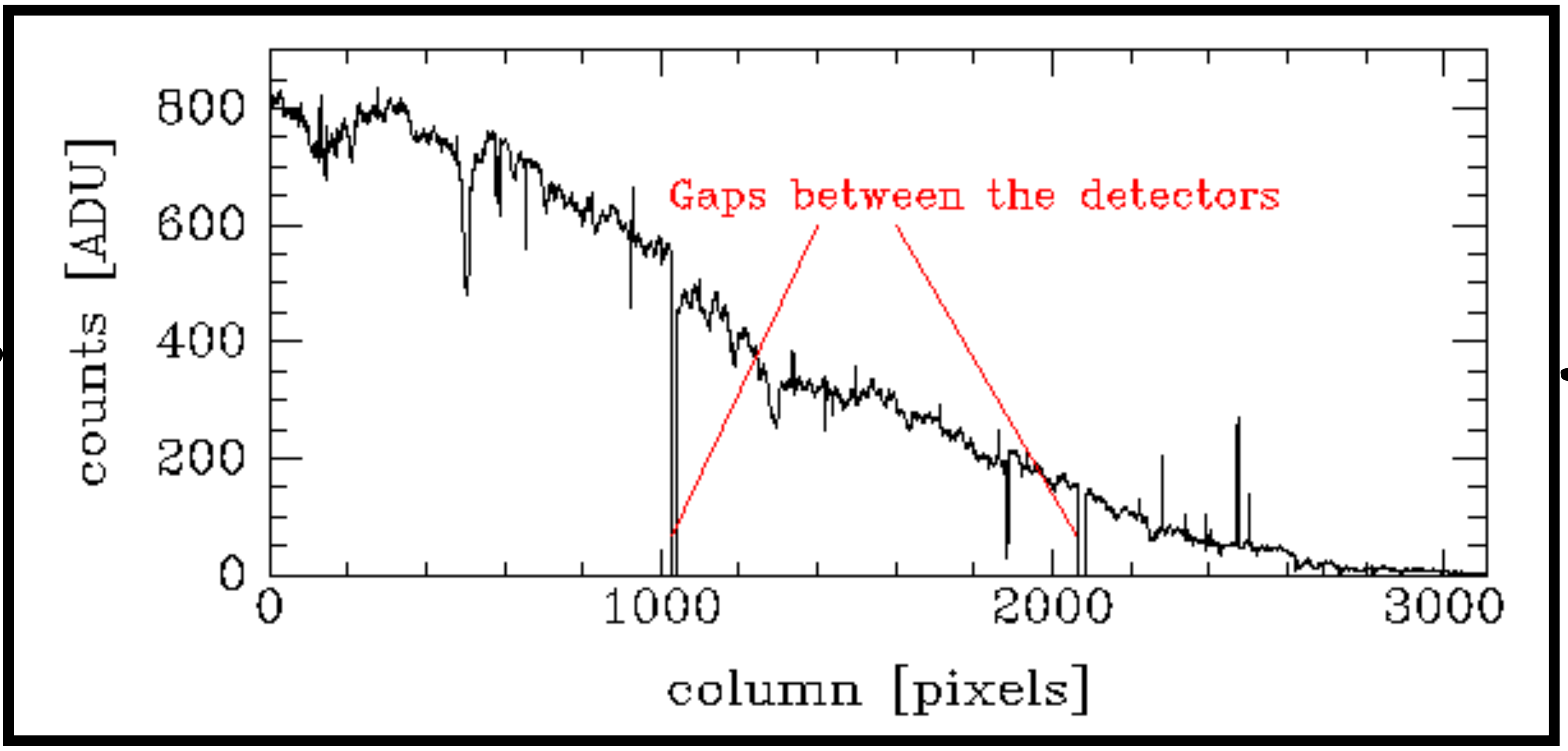
A typical process of spectroscopic observation

RA=123.456789 deg
Dec=+12.34567 deg







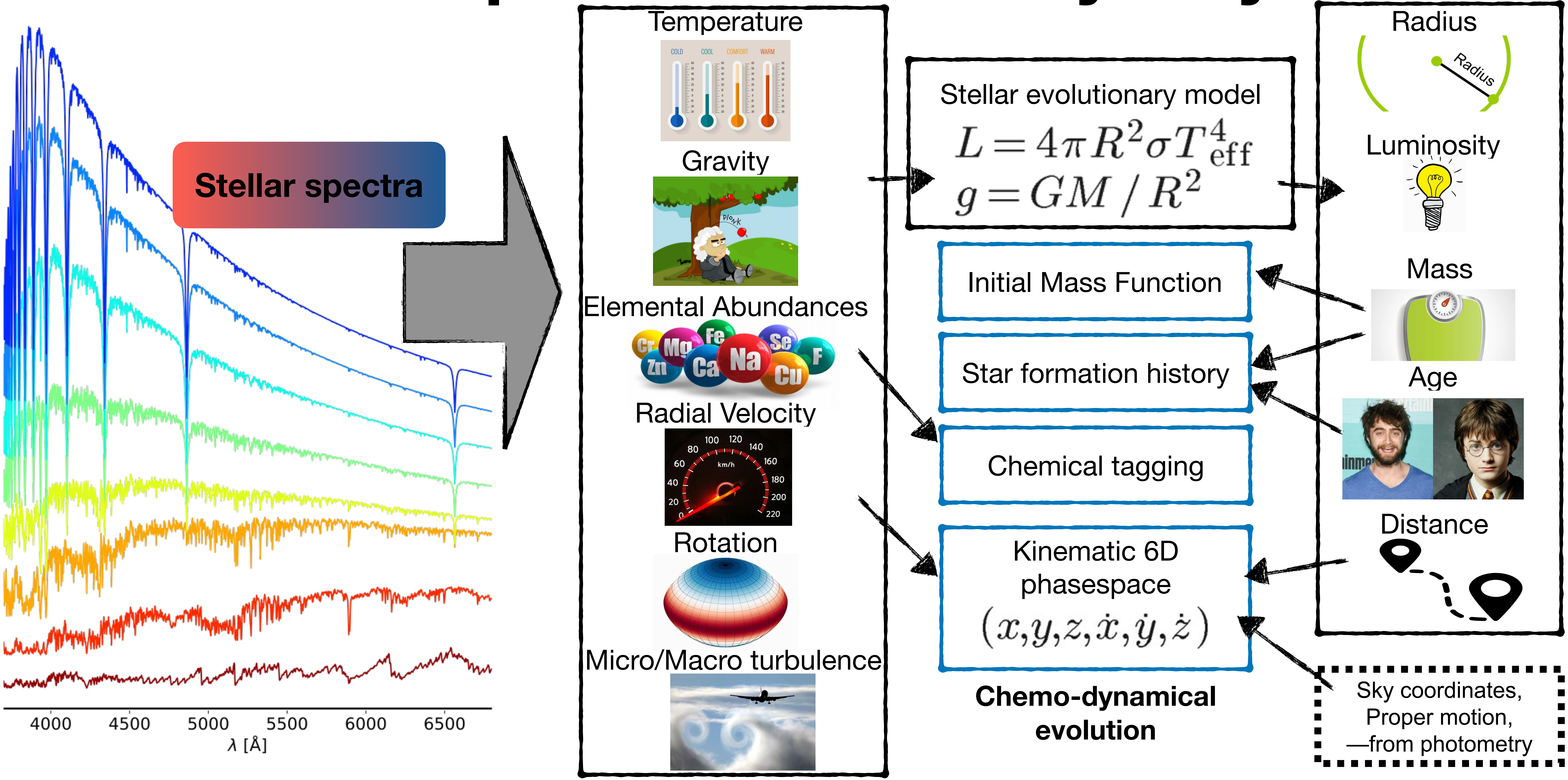


*.fits file

1.1 How to obtain stellar spectra?

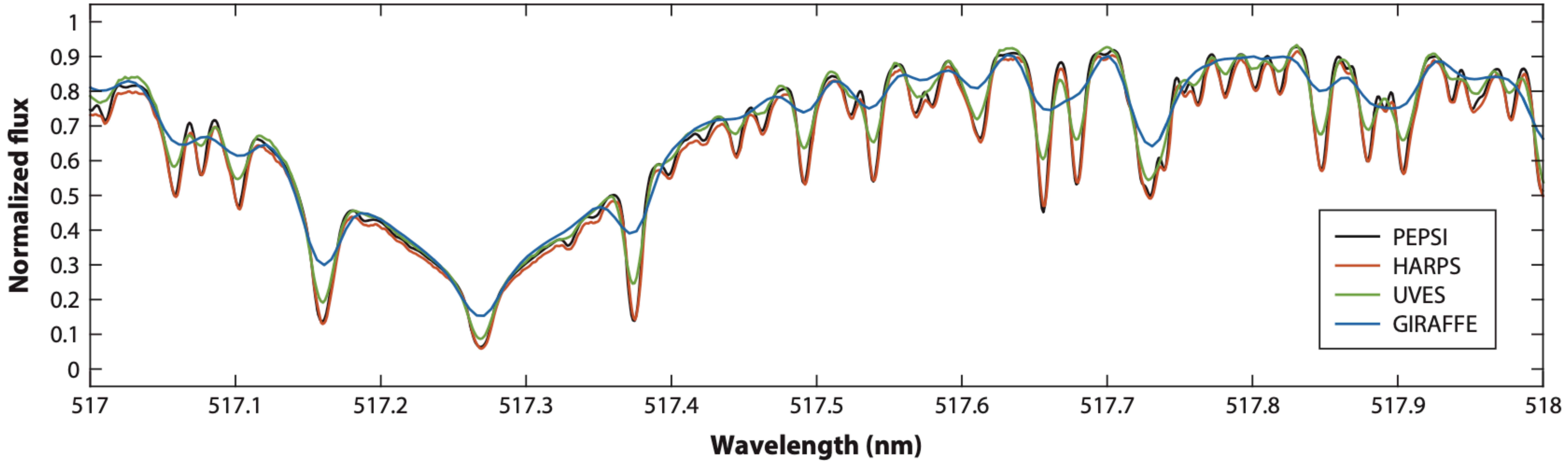
Spectroscopic surveys, e.g., LAMOST, APOGEE, …

From stellar spectra to the Milky Way science



1.2 Spectral resolution *R*

Instrumental broadening / Line Spread Function (LSF)



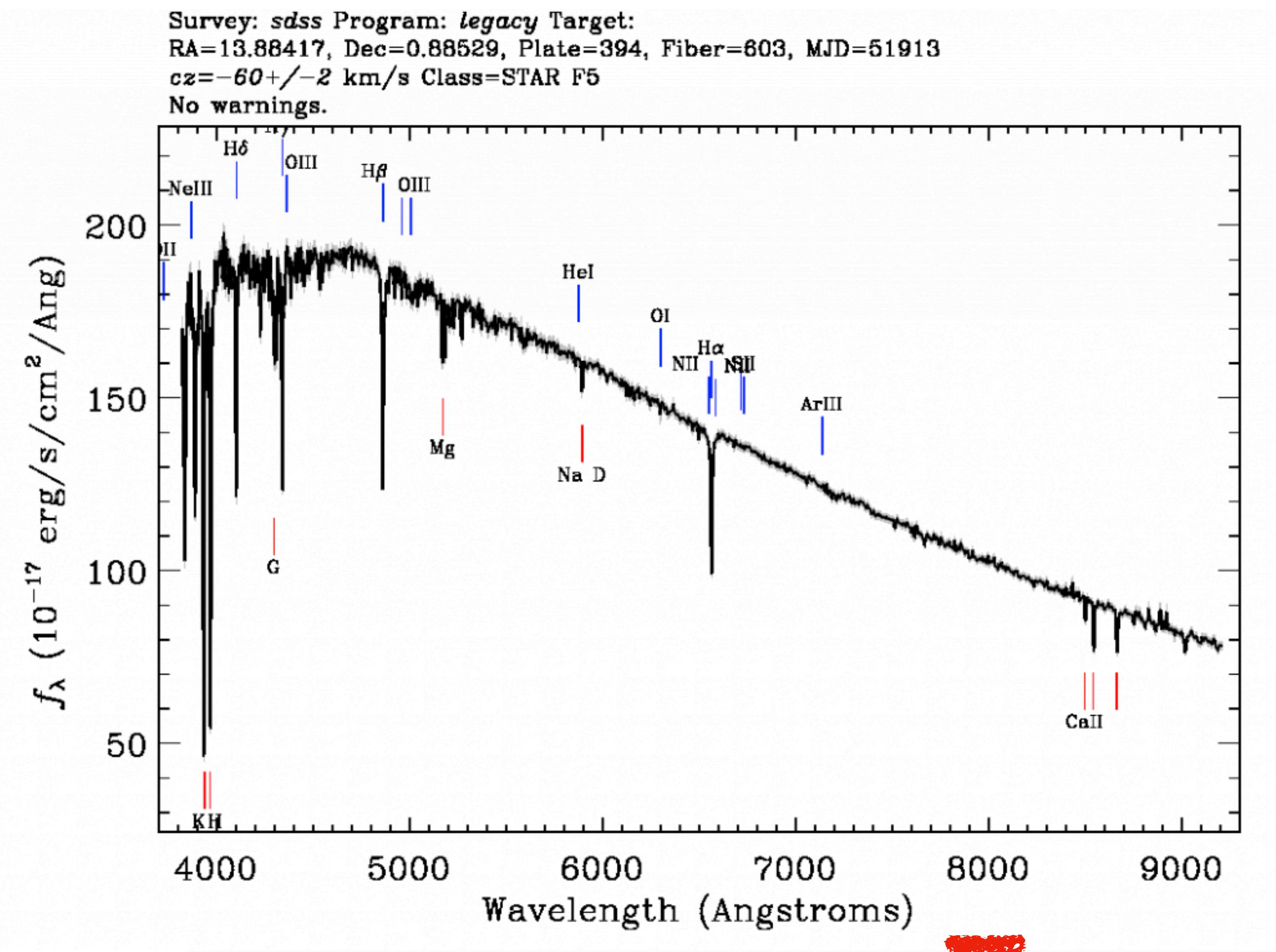
Spectra for the *Gaia* benchmark star ε Eri near one of the Mgi b lines

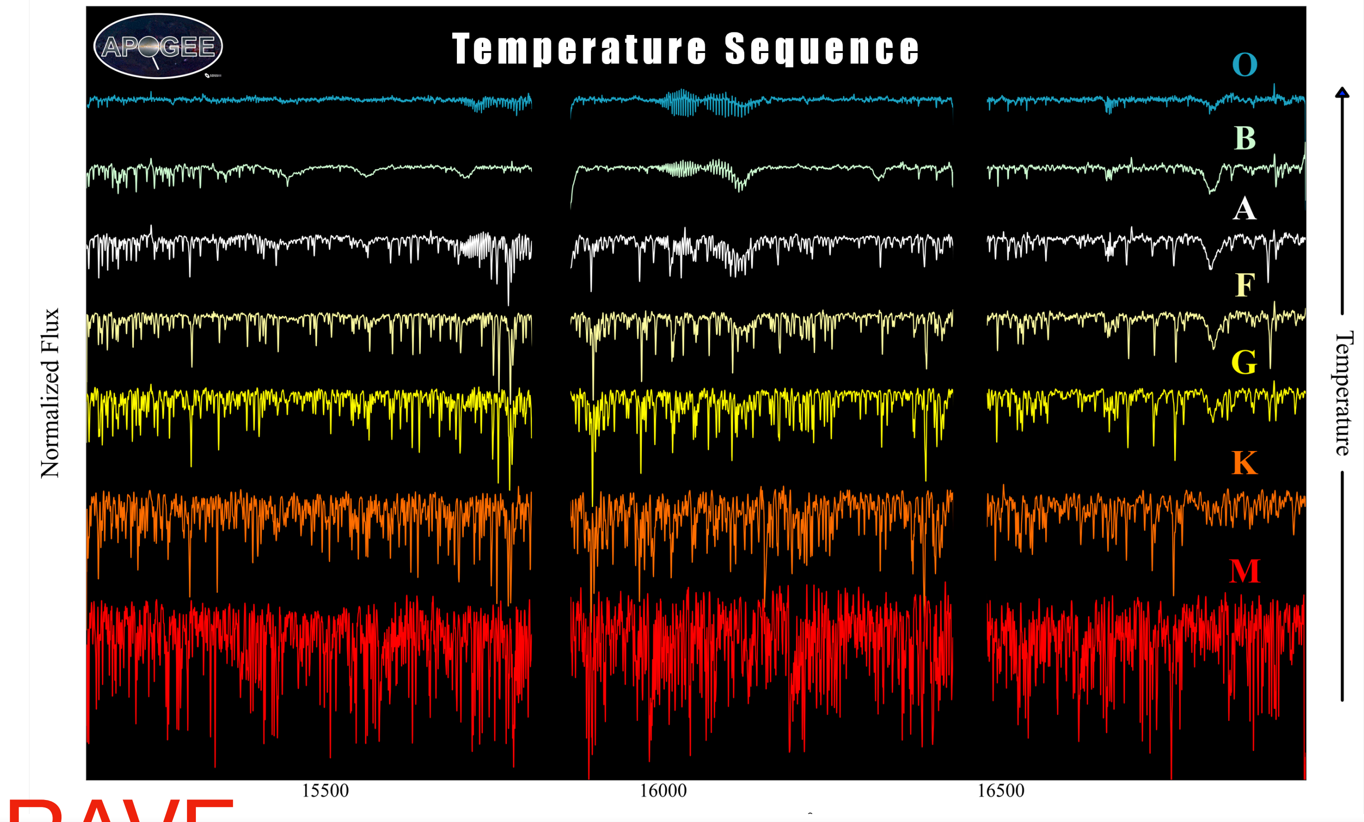
1.2 Spectral resolution *R* `laspec.qconv module`

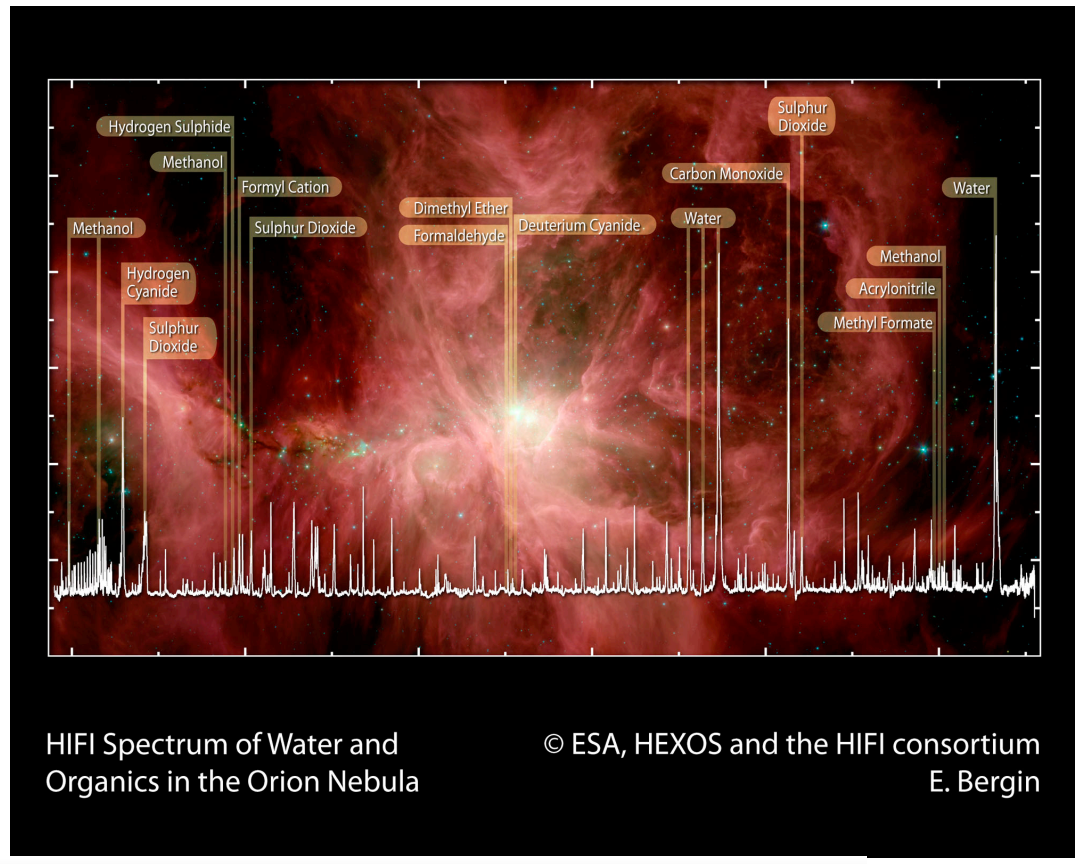
Solar spectrum at different *R* Teff=5800 logg=4.5

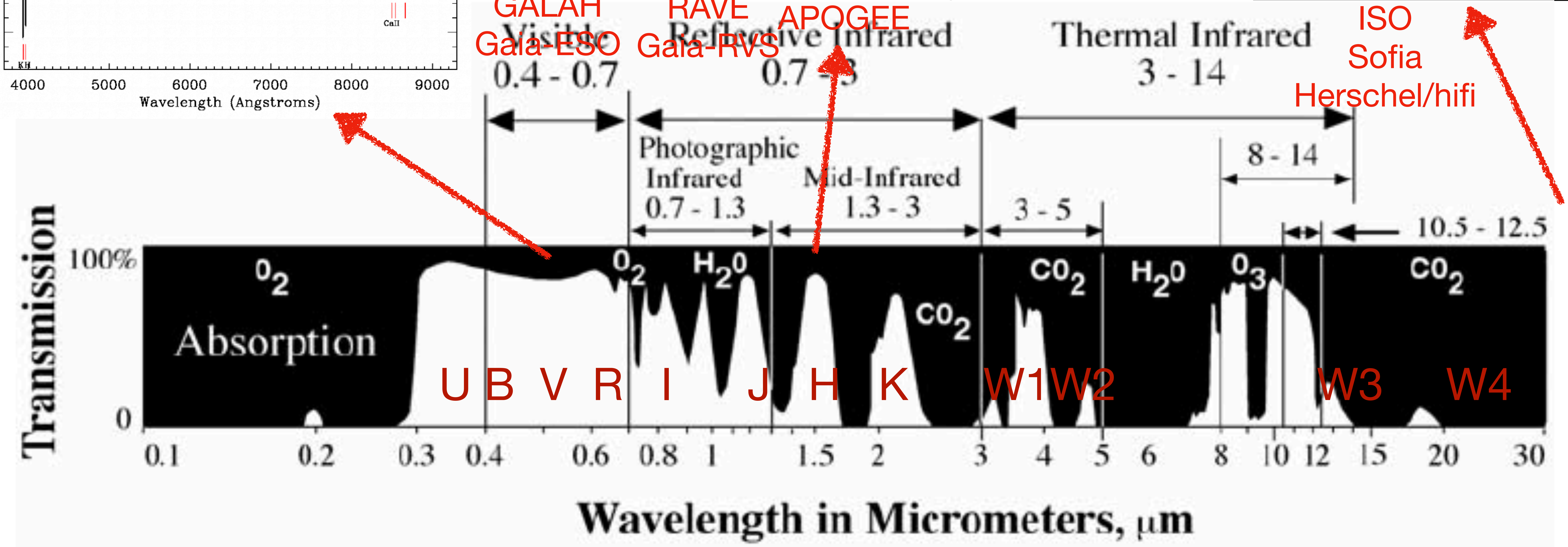
Fig. 2. Wavelength coverage with different resolution for LAMOST-LRS. Left figure shows the coverage of the low resolution configuration. Right figure shows the coverage of the medium resolution configuration. Resolution could be doubled by limiting the width of fiber slit.

1.3 Atmospheric transmission





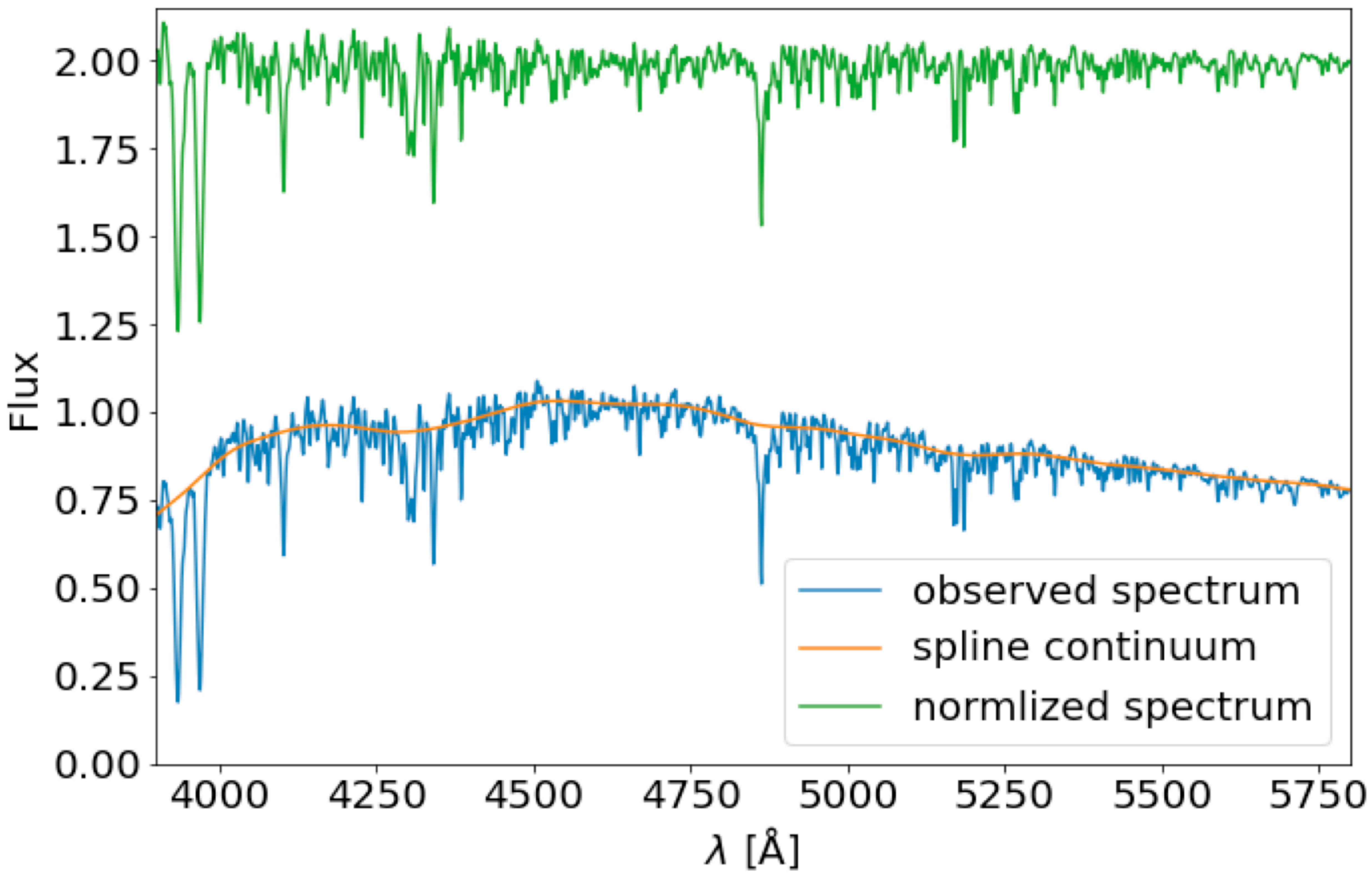




1.4 Pseudo-continuum normalization

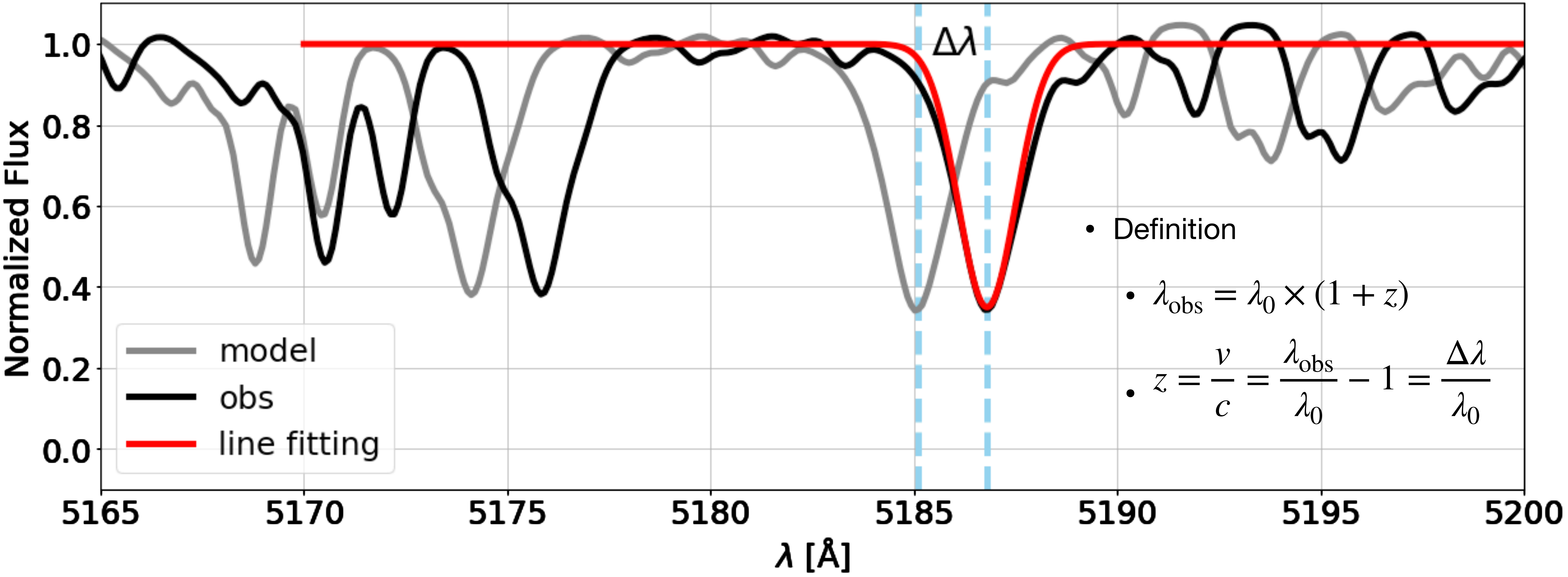
Place spectra at a standard scale

- Why?
  - Instrumental response curve
  - Interstellar extinction
  - Absolute flux calibration (>10%)
- `laspec.normalization` module
- An important source of uncertainty



2. How to measure Doppler effect / Radial Velocity

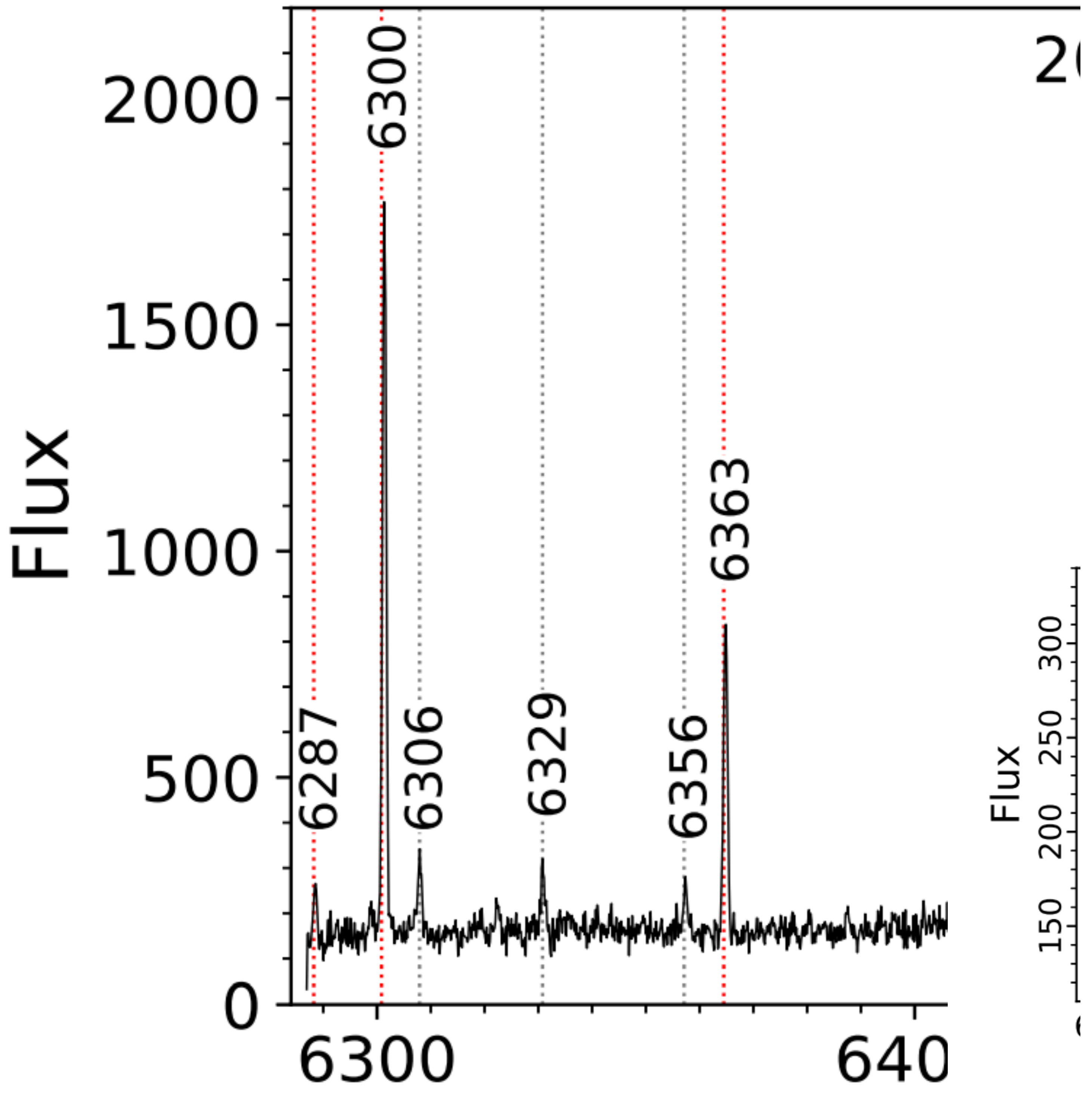
Doppler effect & Spectral line fitting

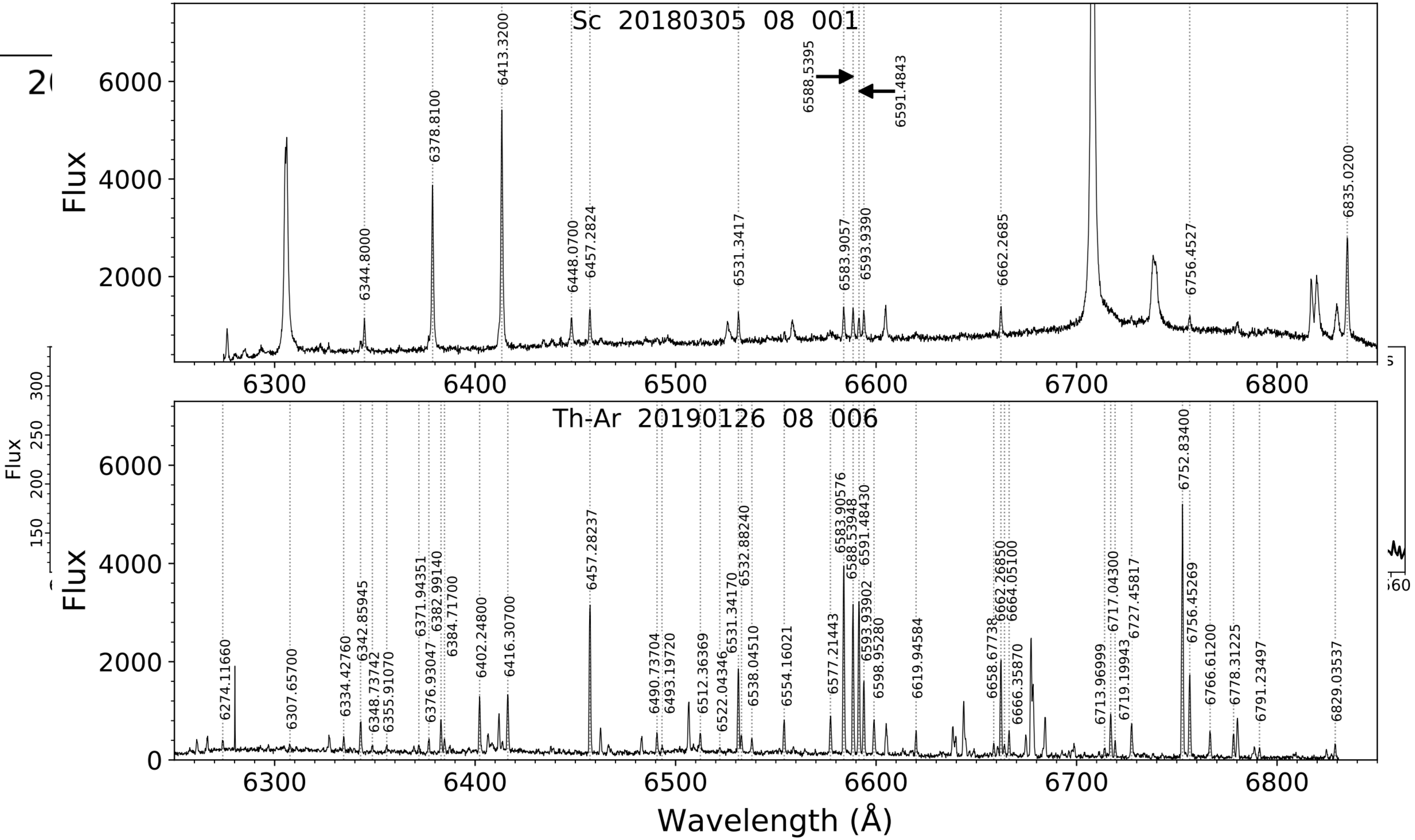


- Definition
  - $\lambda_{\mathrm{obs}} = \lambda_0 \times (1 + z)$
  - $z = \frac{v}{c} = \frac{\lambda_{\mathrm{obs}}}{\lambda_0} - 1 = \frac{\Delta\lambda}{\lambda_0}$

2. How to measure Doppler effect / Radial Velocity

Doppler effect & Spectral line fitting





Ren+2020

2. How to measure Doppler effect / Radial Velocity

Cross-correlation function (CCF, Tonry & Davis 1979)

- Find a proper template
  - (Teff, logg, [Fe/H],…)
- Cross correlation function
  - Why CCF?

1. Remove continuum
2. Resample to log wavelength
3. Eval CCF using FFT*
4. Estimate RV

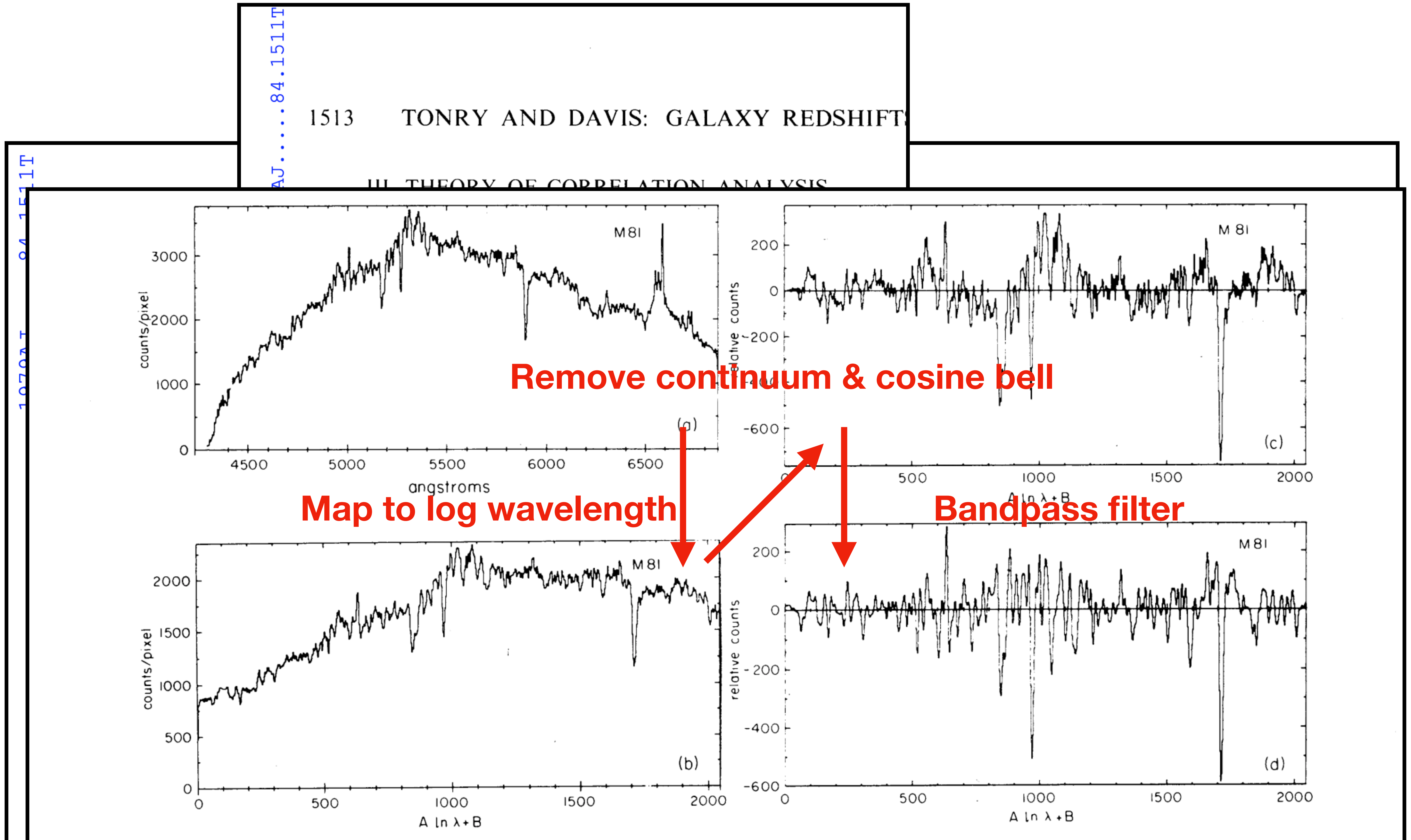
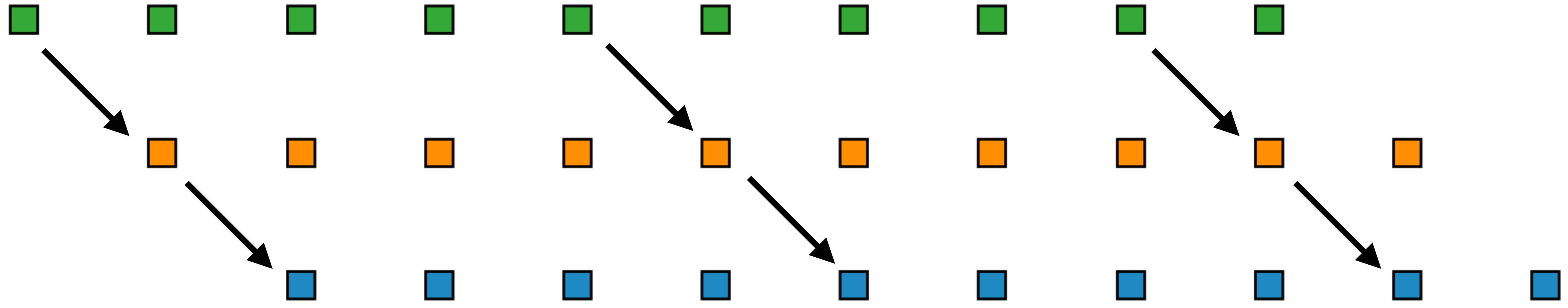


FIG. 2. (a) Spectrum of M81. The ordinate is in accumulated counts per pixel; (b) The result of mapping a segment of the spectrum to ln λ coordinates; (c) A fourth-order least-squares polynomial has been subtracted and the result has been apodized with a cosine bell; (d) The spectrum has now been bandpass filtered and is prepared for correlation analysis.

2. How to measure Doppler effect / Radial Velocity

Cross-correlation function (CCF, Tonry & Davis 1979)

- Why log wavelength $n = A \ln \lambda + B$



$$\Delta n = A \Delta \ln \lambda$$
$$= A \frac{\Delta \lambda}{\lambda}$$
$$= A z$$
$$= A \frac{v}{c}$$

III. THEORY OF CORRELATION ANALYSIS

a) Introduction

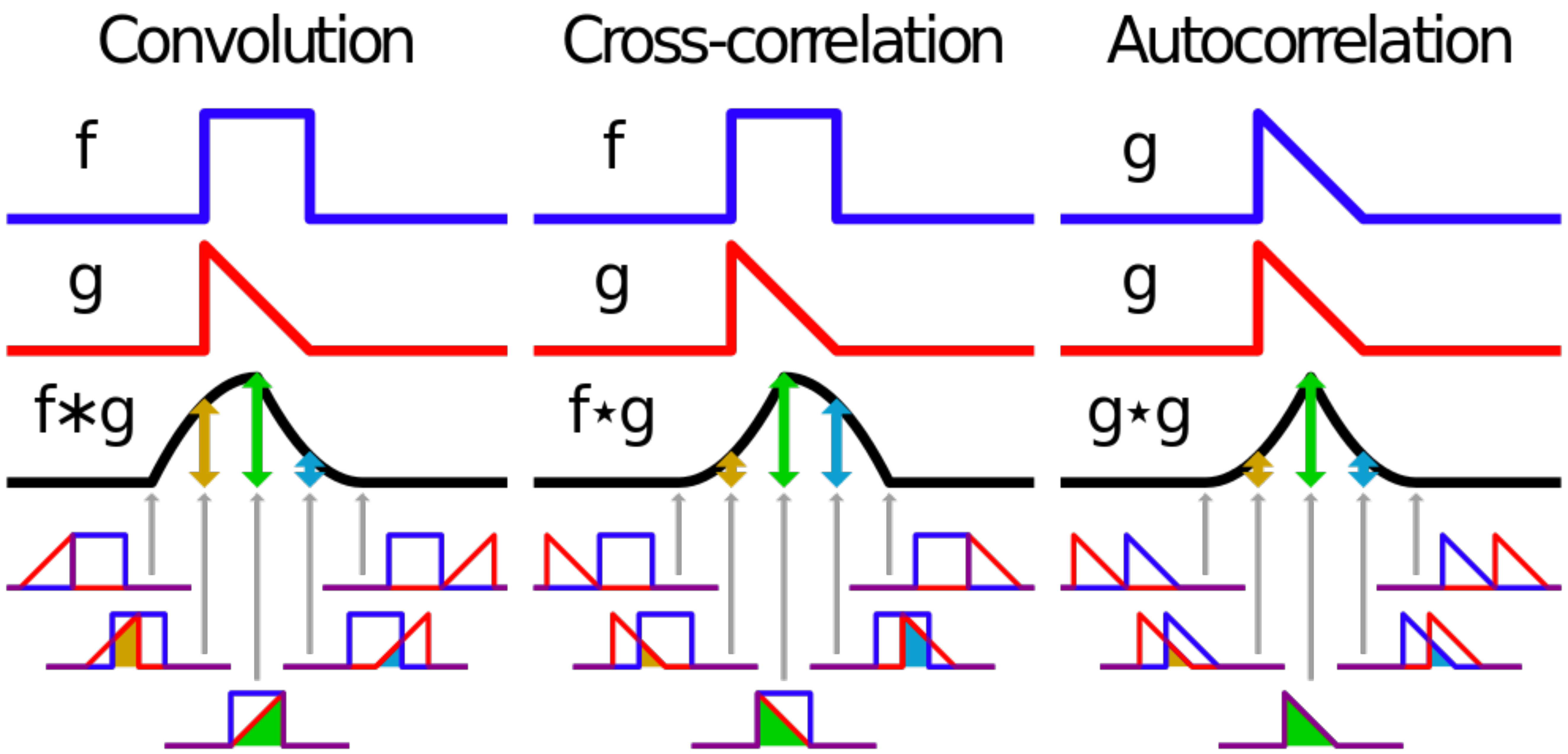
Let $g(n)$ be the spectrum of a galaxy whose redshift and velocity dispersion are to be found and let $t(n)$ be a template spectrum of zero redshift and instrumentally broadened stellar-line profiles. These spectra are discretely sampled into N bins, labelled by bin number n; the relationship between wavelength and bin number is

$$n = A \ln\lambda + B \qquad (1)$$

Because the spectra are binned linearly with $\ln \lambda$ a velocity redshift is a uniform linear shift. The spectra are assumed periodic with period N for the purposes of discrete Fourier transforms and correlation functions derived herefrom. In addition, the spectra are continuum subtracted, endmasked, and filtered to remove low-frequency spectral variations which arise from continuum variations and non-uniform photocathode sensitivity, and high-frequency noise components beyond the resolution. The endmasking assures that mismatch between the ends of the spectrum is removed.

2. How to measure Doppler effect / Radial Velocity

Cross-correlation function (a general scheme)



$$\mathrm{CCF}(\tau) = \int_{-\infty}^{+\infty} f(t)g(\tau + t)dt$$

$$\mathrm{CONV}(\tau) = \int_{-\infty}^{+\infty} f(t)g(\tau - t)dt$$

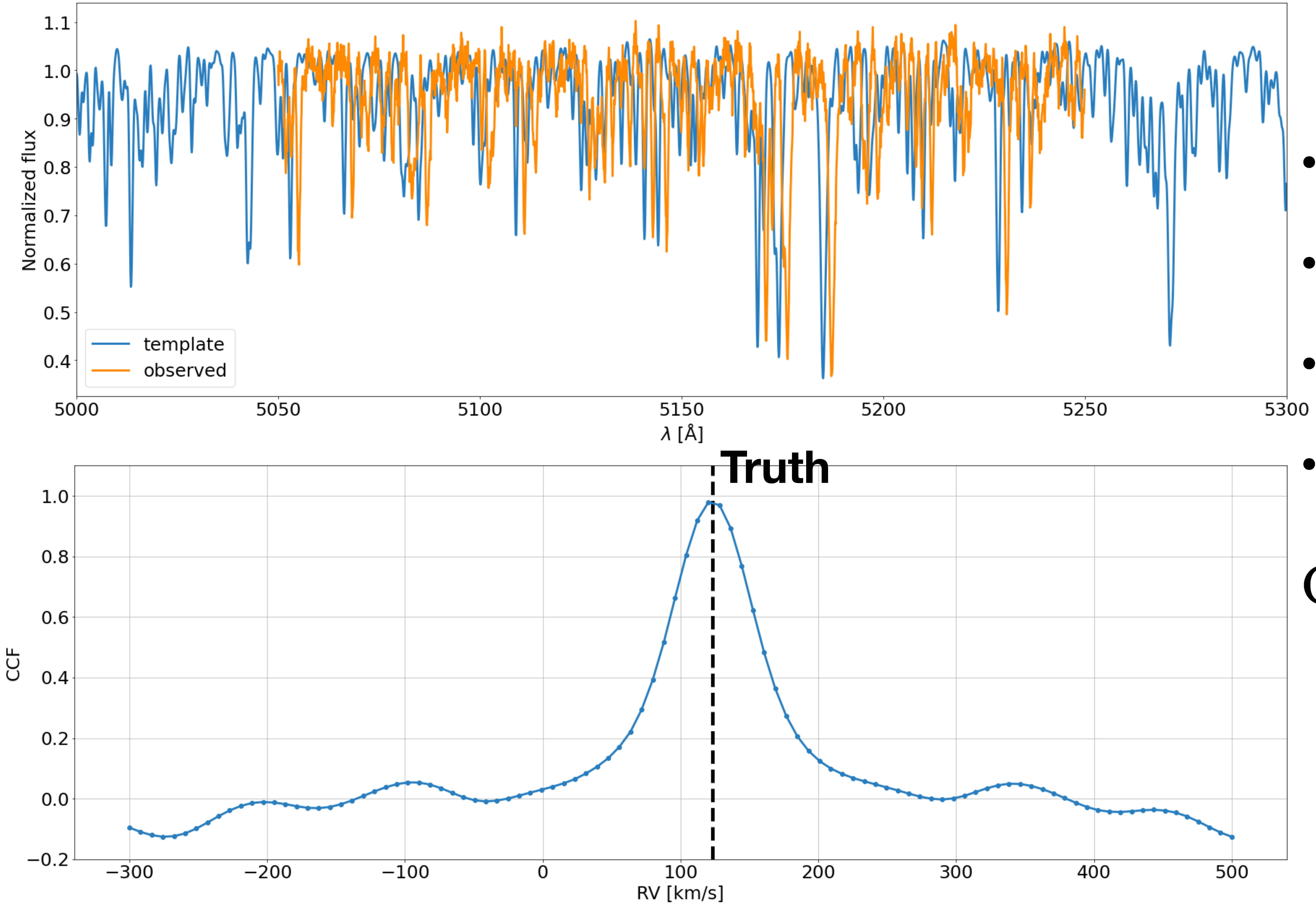
- Mean $\overline{\boldsymbol{X}} = \frac{1}{N}\sum_i X_i$
- Variance $\mathrm{Var}(\boldsymbol{X}) = \frac{1}{N}\sum_i (X_i - \overline{\boldsymbol{X}})^2$
- Covariance $\mathrm{Cov}(\boldsymbol{X}, \boldsymbol{Y}) = \frac{1}{N}\sum_i (X_i - \overline{\boldsymbol{X}})(Y_i - \overline{\boldsymbol{Y}})$
- Cross-correlation

$$\mathrm{CCF}(v|\boldsymbol{F}, \boldsymbol{G}) = \frac{\mathrm{Cov}(\boldsymbol{F}, \boldsymbol{G}(v))}{\sqrt{\mathrm{Var}(\boldsymbol{F})\mathrm{Var}(\boldsymbol{G}(v))}}$$

Linear Correlation coefficient

2. How to measure Doppler effect / Radial Velocity

Cross-correlation function (a general scheme)



- Mean $$\overline{\boldsymbol{X}} = \frac{1}{N}\sum_i X_i$$
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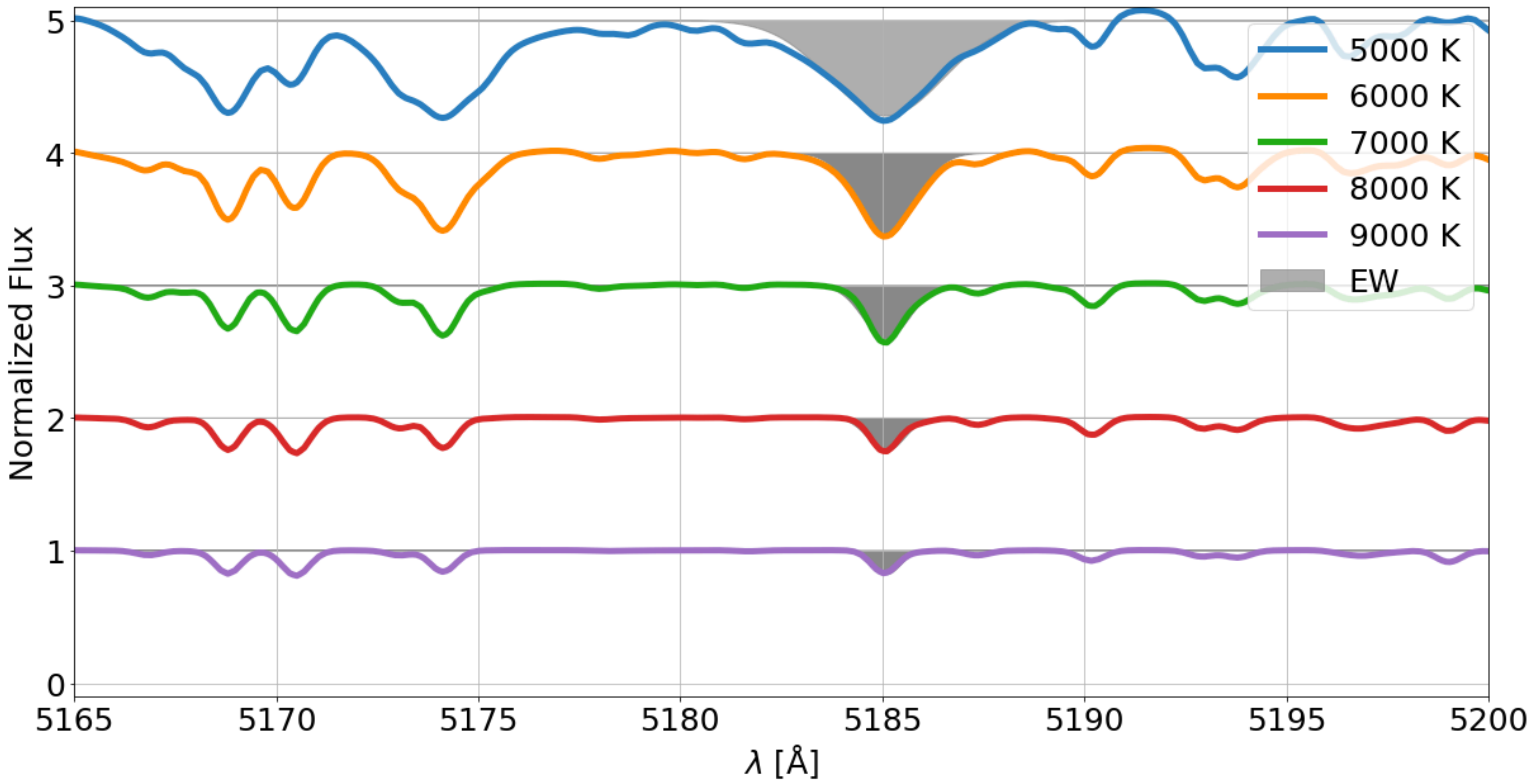
- Covariance $$\mathrm{Cov}(\boldsymbol{X}, \boldsymbol{Y}) = \frac{1}{N}\sum_i (X_i - \overline{\boldsymbol{X}})(Y_i - \overline{\boldsymbol{Y}})$$
- Cross-correlation

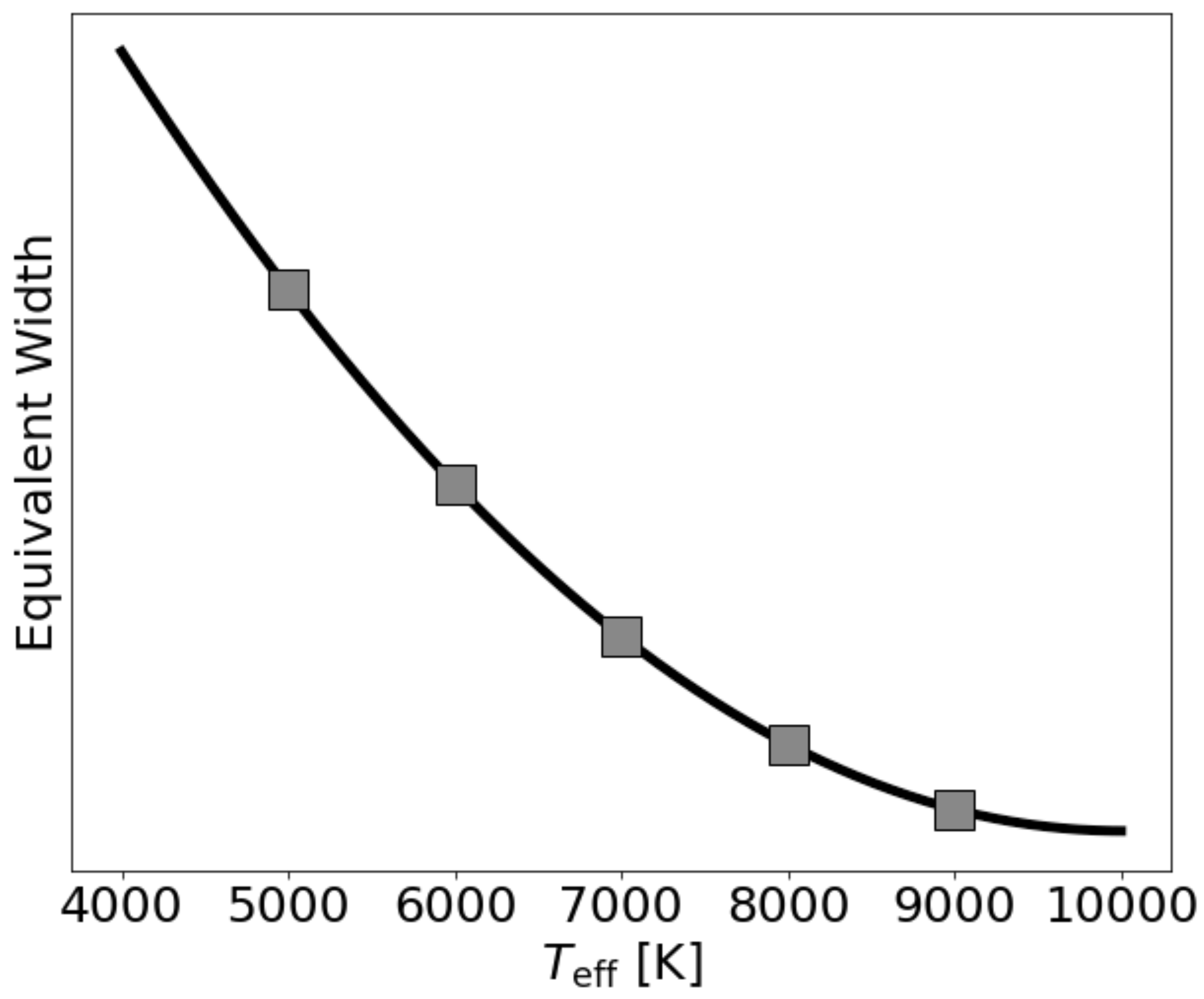
$$\mathrm{CCF}(v|\boldsymbol{F}, \boldsymbol{G}) = \frac{\mathrm{Cov}(\boldsymbol{F}, \boldsymbol{G}(v))}{\sqrt{\mathrm{Var}(\boldsymbol{F})\mathrm{Var}(\boldsymbol{G}(v))}}$$

Linear Correlation coefficient

3. How to derive Teff, logg & [M/H] from spectra

Equivalent widths of spectral lines





$$\mathrm{EW} = \int_{\lambda 1}^{\lambda 2} \frac{F_{\lambda,\mathrm{cont}} - F_{\lambda,\mathrm{obs}}}{F_{\lambda,\mathrm{cont}}} d\lambda$$

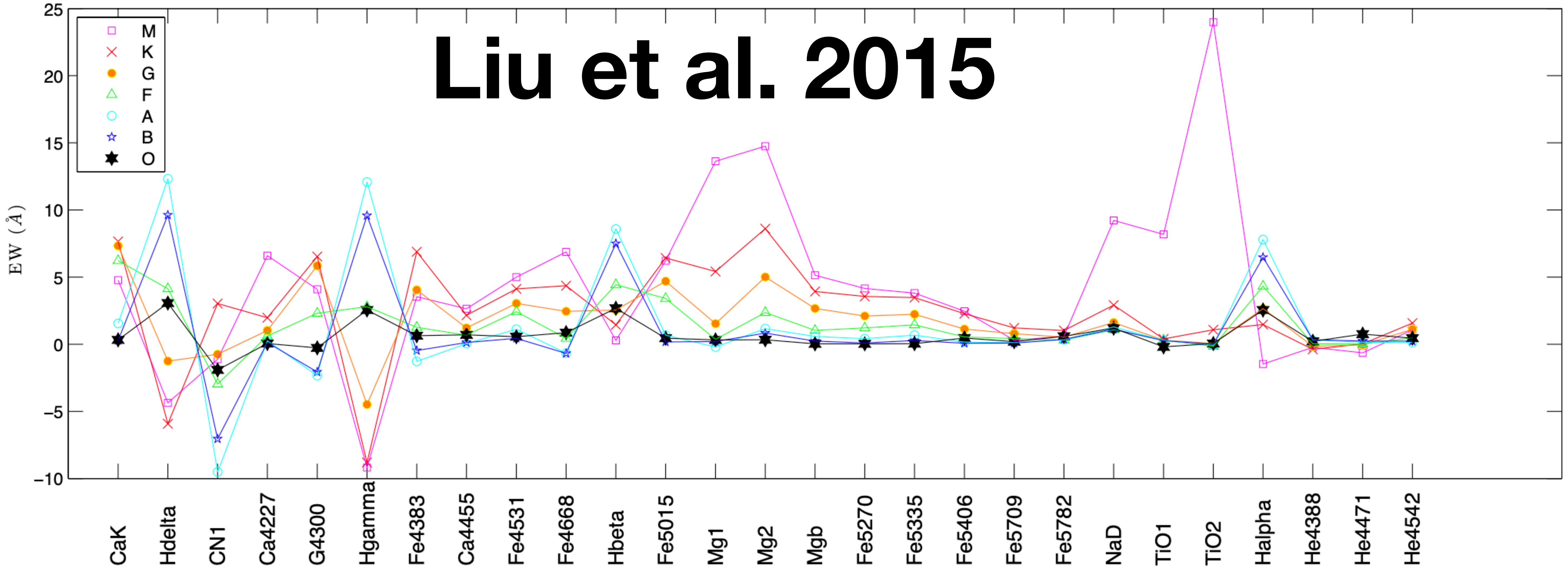
3. How to derive Teff, logg & [M/H] from spectra

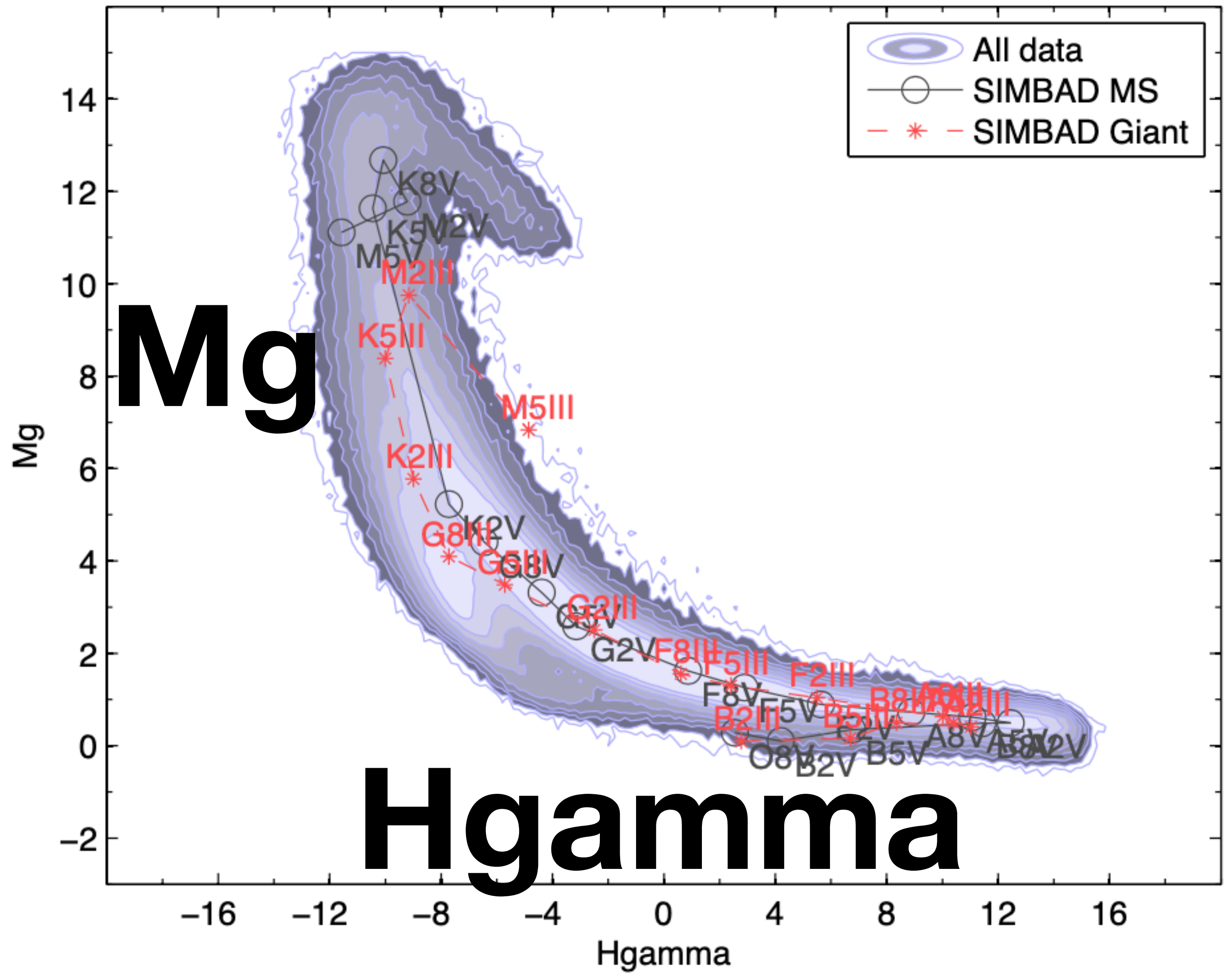
Equivalent widths of spectral lines

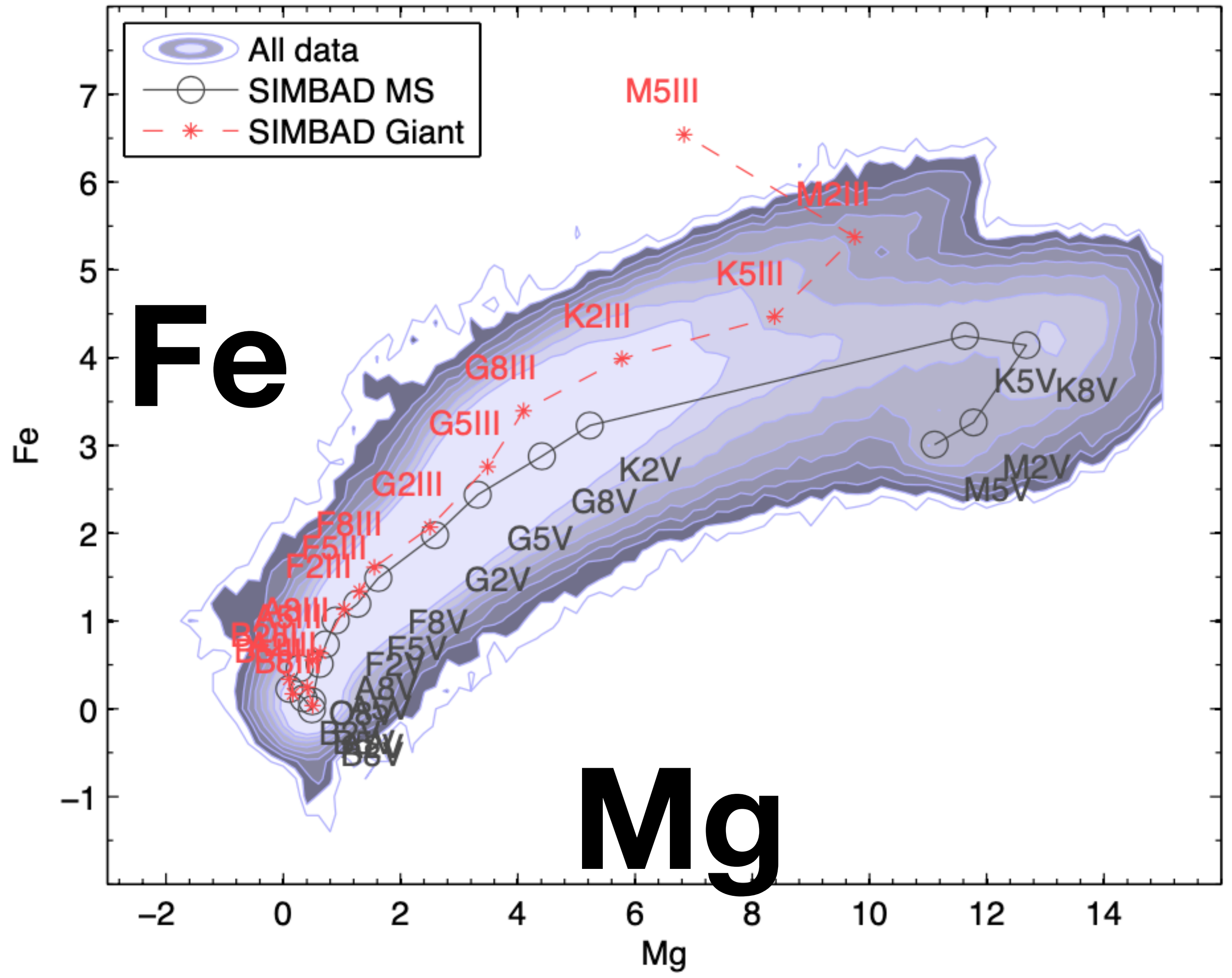
Table 2 Line Index Definitions

Name	Index Bandpass (Å)	Pseudocontinua (Å)
CaII K[a]	3927.7–3939.7	3903–3923 4000–4020
Hδ[b]	4083.50–4122.25	4041.60–4079.75 4128.50–4161.00
CN[c]	4143.375–4178.375	4081.375–4118.875 4245.375–4285.375
Ca4227[c]	4223.500–4236.000	4212.250–4221.000 4242.250–4252.250
G4300[c]	4282.625–4317.625	4267.625–4283.875 4320.125–4336.375
Hγ[b]	4319.75–4363.50	4283.50–4319.75 4367.25–4419.75
Fe4383[c]	4370.375–4421.625	4360.375–4371.625 4444.125–4456.625
He4388	4381–4399	4365–4380 4398–4408
Ca4455[c]	4453.375–4475.875	4447.125–4455.875 4478.375–4493.375
He4471	4462–4475	4450–4463 4485-4495
Fe4531[c]	4515.500–4560.500	4505.500–4515.500 4561.750–4580.500
He4542	4536–4548	4526–4536 4548–4558
Fe4668[c]	4635.250–4721.500	4612.750–4631.500 4744.000–4757.750
Hβ[b]	4847.875–4876.625	4827.875–4847.875 4876.625–4891.625
Fe5015[c]	4977.750–5054.000	4946.500–4977.750 5054.000–5065.250
Mg_1[c]	5069.125–5134.125	4895.125–4957.625 5301.125–5366.125
Mg_2[c]	5154.125–5196.625	4895.125–4957.625 5301.125–5366.125
Mg_b[c]	5160.125–5192.625	5142.625–5161.375 5191.375–5206.375
Fe5270[c]	5245.650–5285.650	5233.150–5248.150 5285.650–5318.150
Fe5335[c]	5312.125–5352.125	5304.625–5315.875 5353.375–5363.375
Fe5406[c]	5387.500–5415.000	5376.250–5387.500 5415.000–5425.000
Fe5709[c]	5698.375–5722.125	5674.625–5698.375 5724.625–5738.375
Fe5782[c]	5778.375–5798.375	5767.125–5777.125 5799.625–5813.375
NaD[c]	5878.625–5911.125	5862.375–5877.375 5923.875-5949.875
TiO_1[c]	5938.375–5995.875	5818.375–5850.875 6040.375–6105.375
TiO2[c]	6191.375–6273.875	6068.375–6143.375 6374.375–6416.875
Hα[d]	6548.00–6578.00	6420.00–6455.00 6600.00–6640.00

Notes: [a] Beers et al. (1999); [b] Worthey & Ottaviani (1997); [c] Worthey et al. (1994); [d] Cohen et al. (199



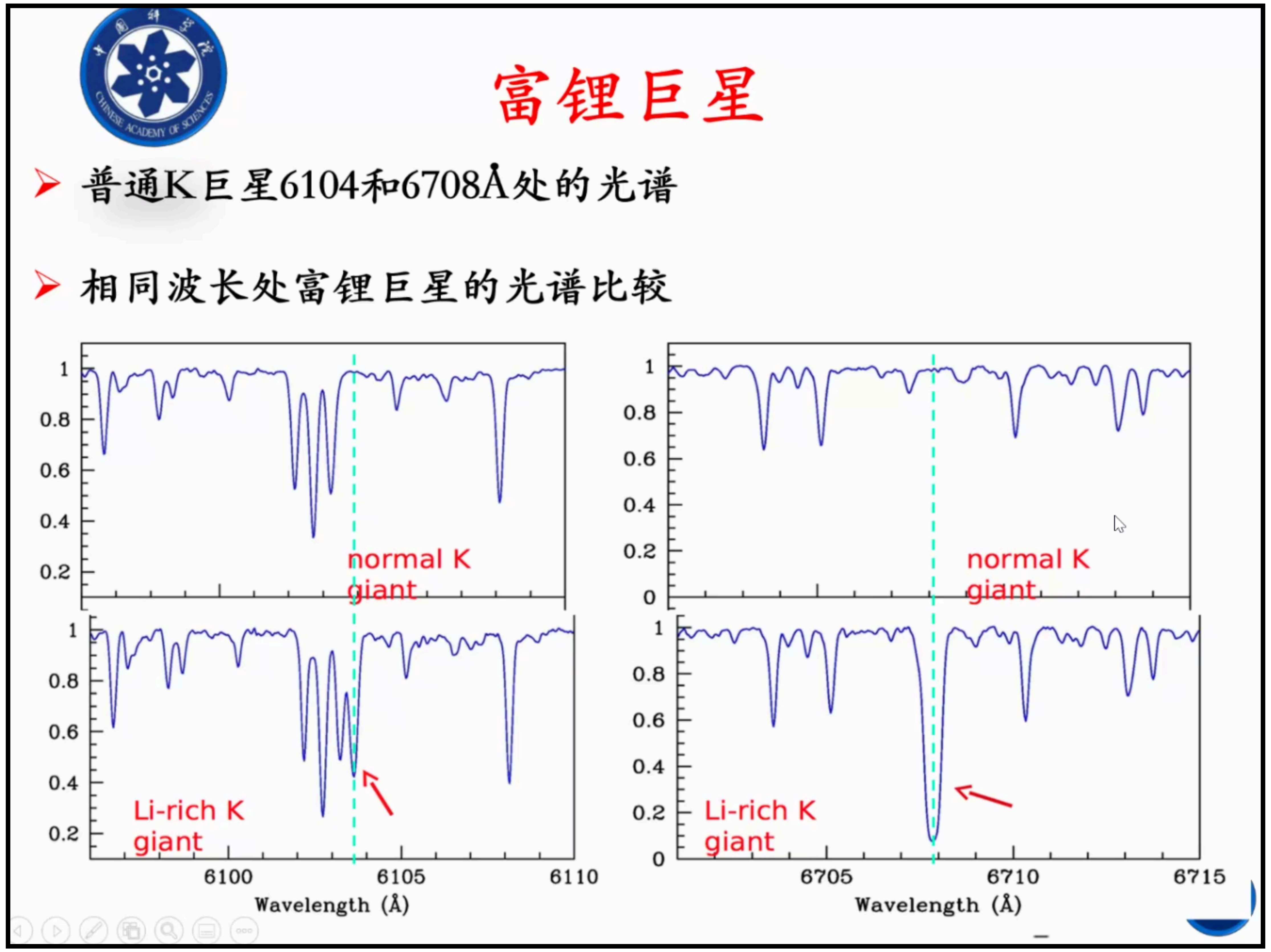


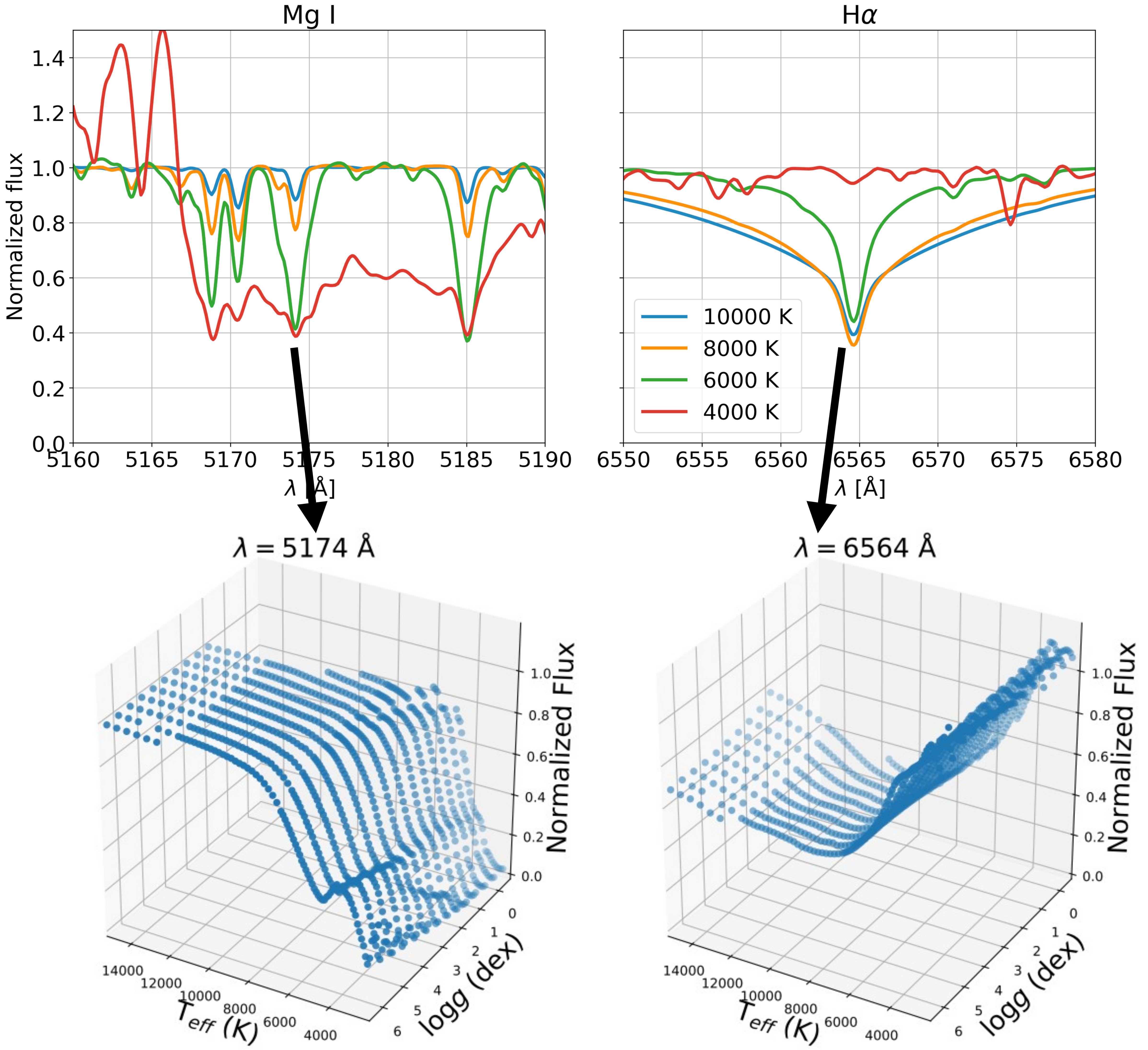


3. How to derive Teff, logg & [M/H] from spectra

Spectra as functions of Stellar parameters

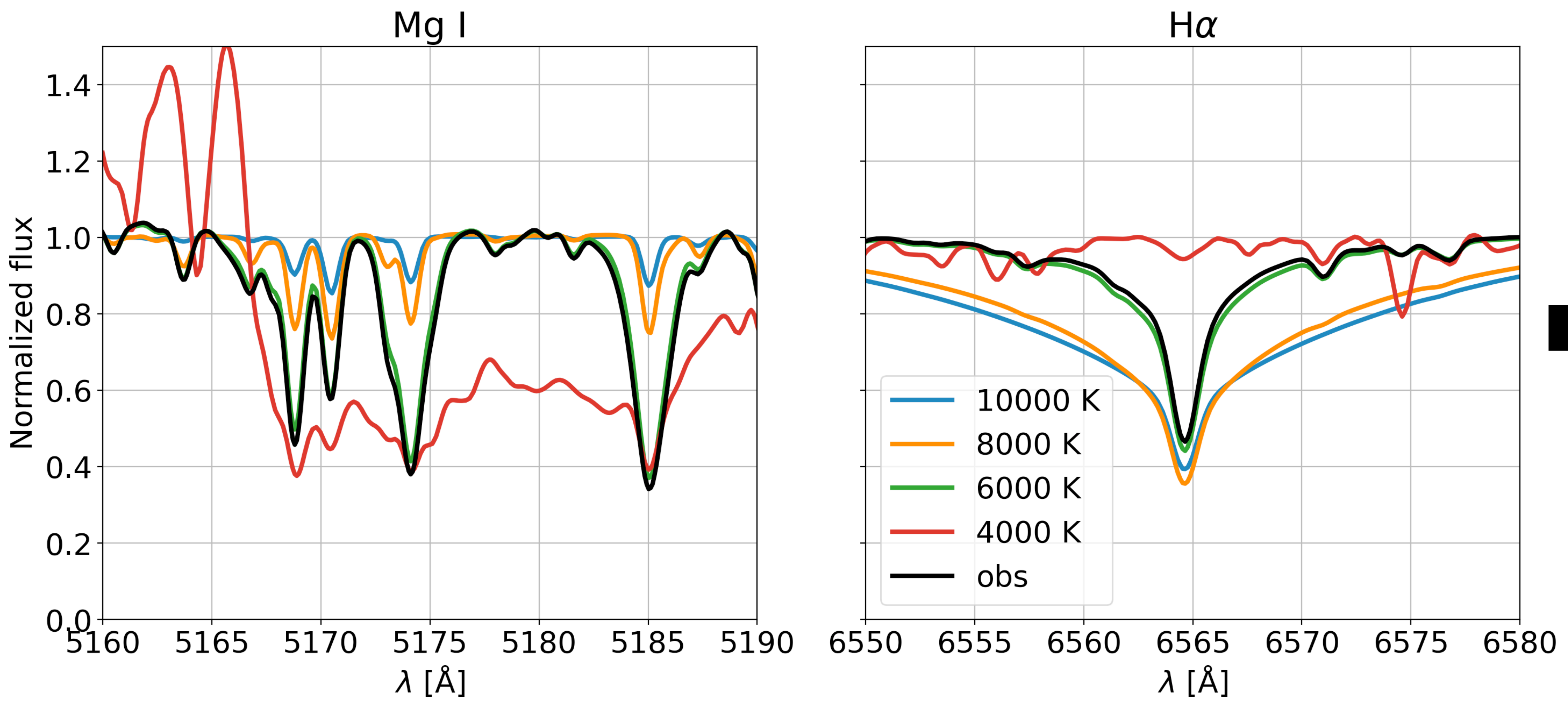
$$\boldsymbol{F}_\lambda = f_\lambda(T_{\rm eff}, \log g, [{\rm X/H}], v\sin i, v_{\rm mac}, v_{\rm mic})$$

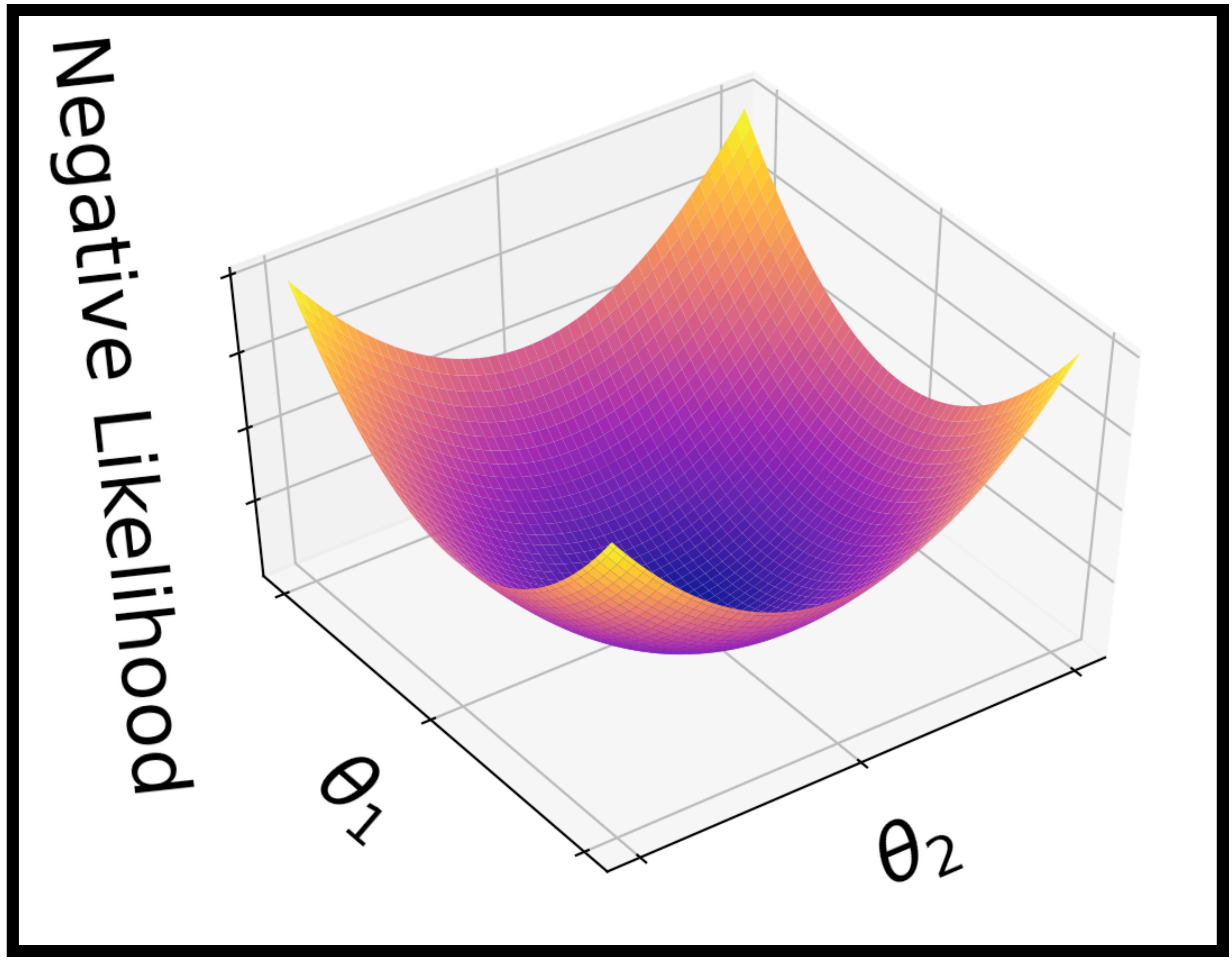




3. How to derive Teff, logg & [M/H] from spectra

Spectra as functions of Stellar parameters





$$\boldsymbol{F}_\lambda = f_\lambda(T_{\mathrm{eff}}, \log g, [\mathrm{X/H}], v \sin i, v_{\mathrm{mac}}, v_{\mathrm{mic}})$$

Define a likelihood, find theta by maximizing it: $$\ln p(\boldsymbol{\theta}|\boldsymbol{f}_{\mathrm{obs}}) = -\frac{1}{2}\sum_{j=1}^{n}\left\{\frac{[f_{j,\mathrm{obs}} - f_j(\boldsymbol{\theta})]^2}{\sigma_{j,\mathrm{obs}}^2 + \sigma_j(\boldsymbol{\theta})^2} + \ln\left[2\pi(\sigma_{j,\mathrm{obs}}^2 + \sigma_j(\boldsymbol{\theta})^2)\right]\right\}$$

3. How to derive Teff, logg & [M/H] from spectra

Bayesian perspective

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

3. How to derive Teff, logg & [M/H] from spectra

Bayesian perspective

$$\boldsymbol{F}_\lambda = f_\lambda(T_{\text{eff}}, \log g, [\text{X/H}], v\sin i, v_{\text{mac}}, v_{\text{mic}})$$

Likelihood(似然)

Prior(先验)

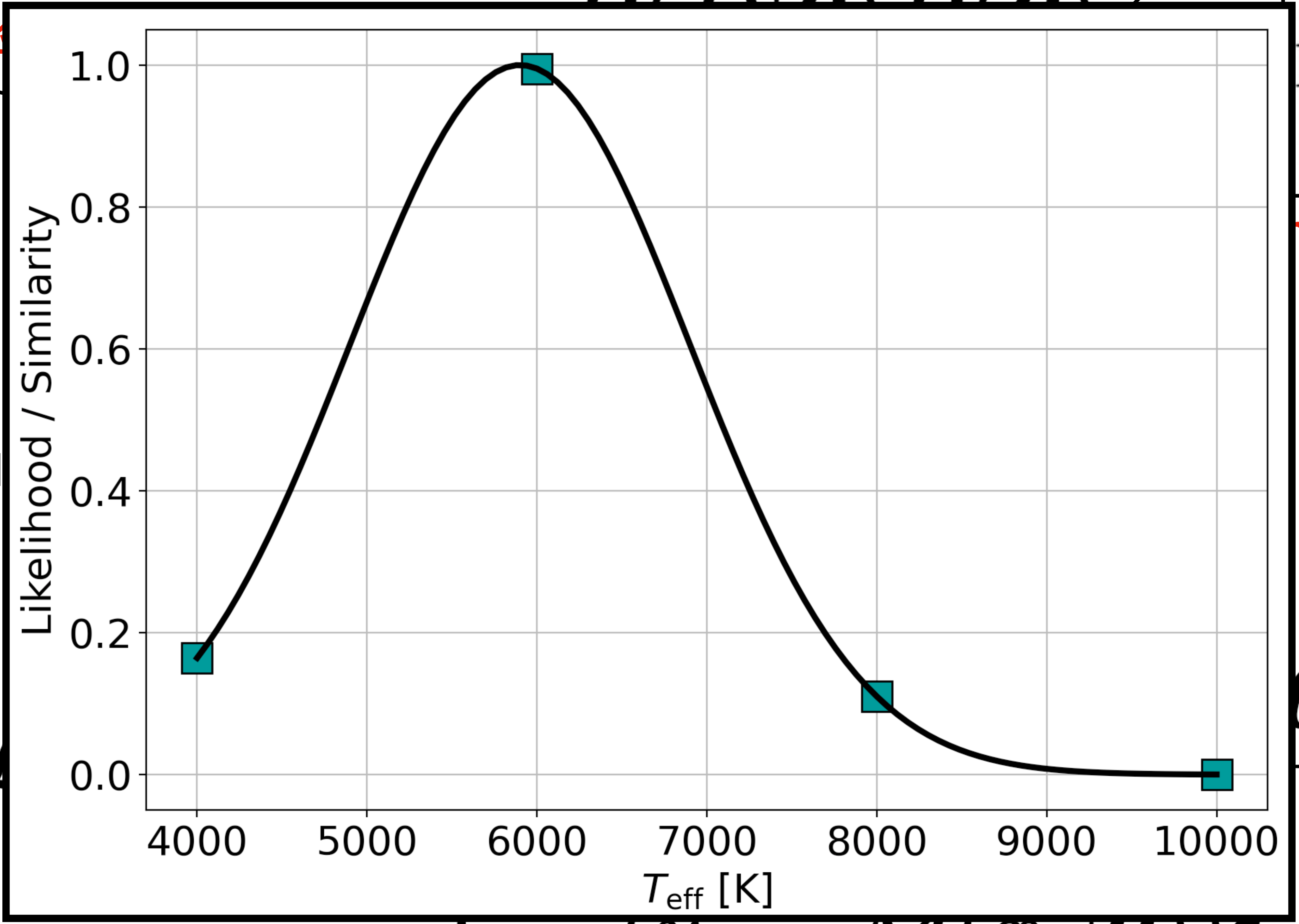
- Bayes' theorem

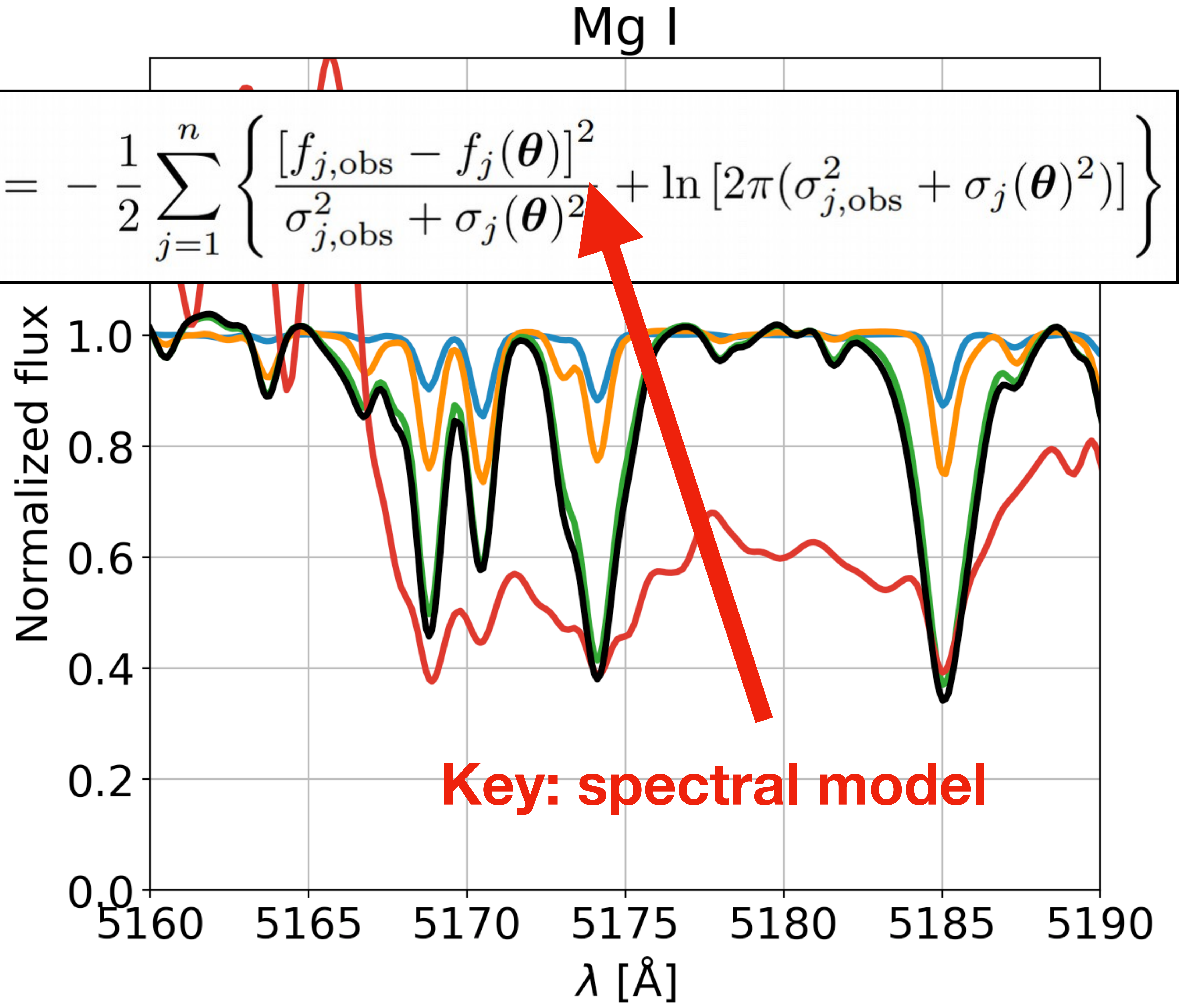
Posterior(后

nce(证据)

- G

$$\ln p(\boldsymbol{\theta}|\boldsymbol{f}_{\text{obs}}) = -\frac{1}{2}\sum_{j=1}^{n}\left\{\frac{[f_{j,\text{obs}} - f_j(\boldsymbol{\theta})]^2}{\sigma_{j,\text{obs}}^2 + \sigma_j(\boldsymbol{\theta})^2} + \ln\left[2\pi(\sigma_{j,\text{obs}}^2 + \sigma_j(\boldsymbol{\theta})^2)\right]\right\}$$





$$\ln \mathcal{L}(D|\boldsymbol{\theta}) \sim -\frac{1}{2}\Sigma_i \frac{(y_i - \mathcal{M}(x_i|\boldsymbol{\theta}))^2}{\sigma_i^2} + \text{const.}$$

$$\sim -\chi^2/2$$

Residual Sum of Squares (RSS, 残差平方和)

3. How to derive Teff, logg & [M/H] from spectra

Spectral model — Synthetic spectral libraries

- **TLUSTY** (OB-type, Lanz, T., & Hubeny, 2002, 2006)
  - http://tlusty.oca.eu/
- **ATLAS9** (FGK-type, Castelli & Kurucz 2003)
  - e.g., FERRE (Allende Prieto+2018)
  - http://cdsarc.u-strasbg.fr/viz-bin/qcat?J/A+A/618/A25
- **MARCS** (K/M-type, Gustafsson+2008)
  - https://marcs.astro.uu.se/
- **BT-Settl** (Allard 2014)
  - https://phoenix.ens-lyon.fr/Grids/BT-Settl/
- **PHOENIX** (Husser+2013)
  - https://phoenix.astro.physik.uni-goettingen.de/

很失望

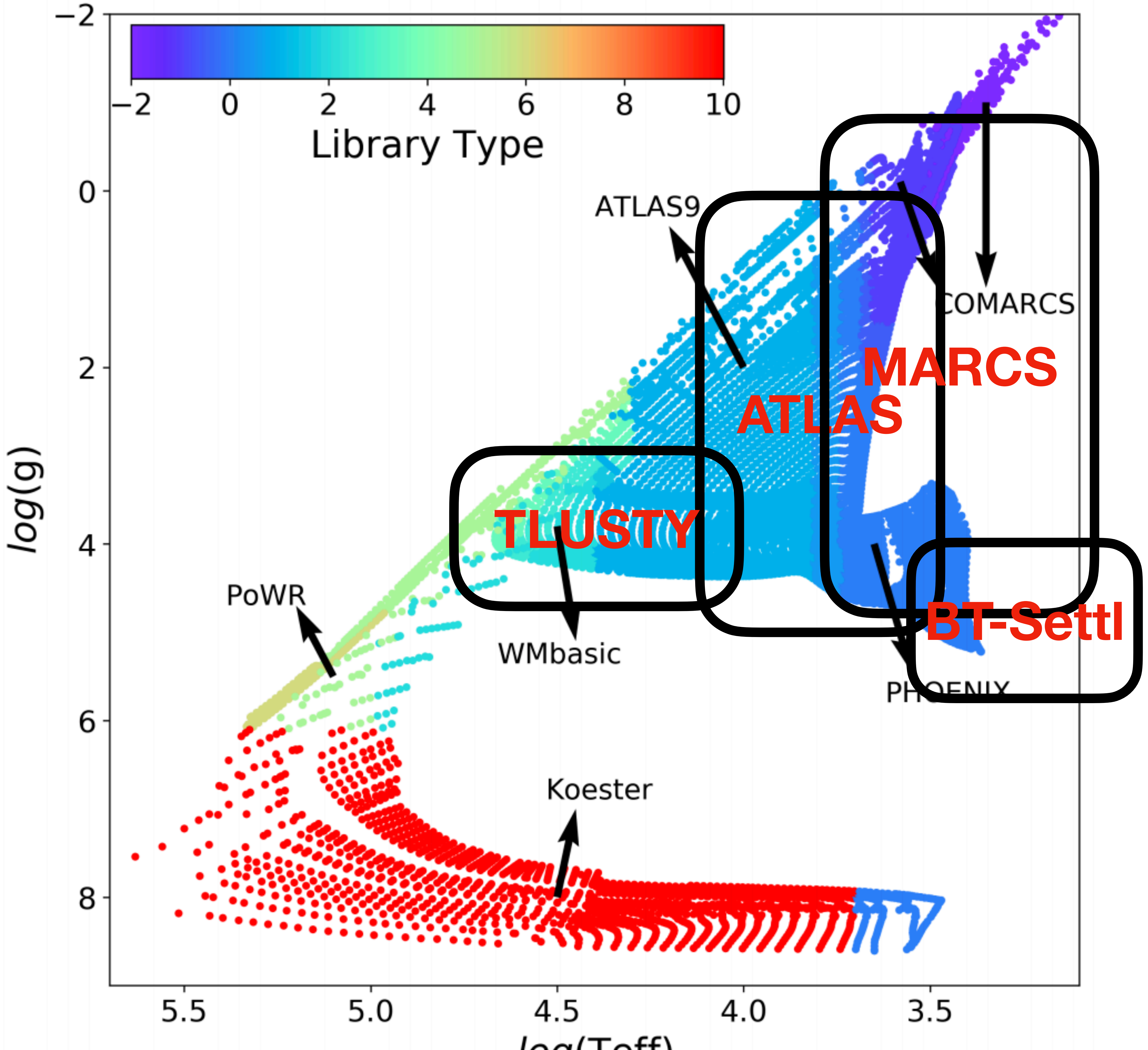
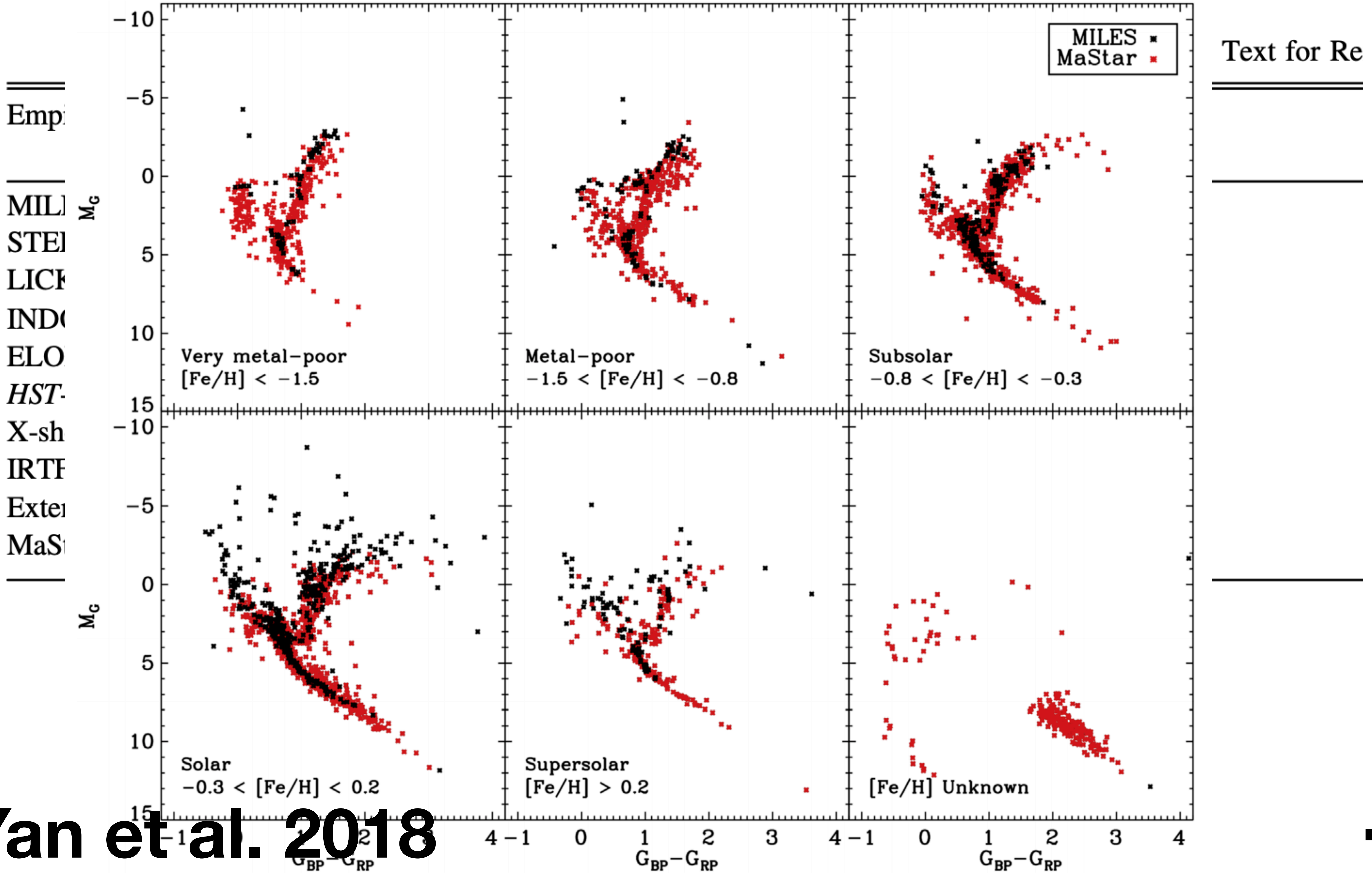
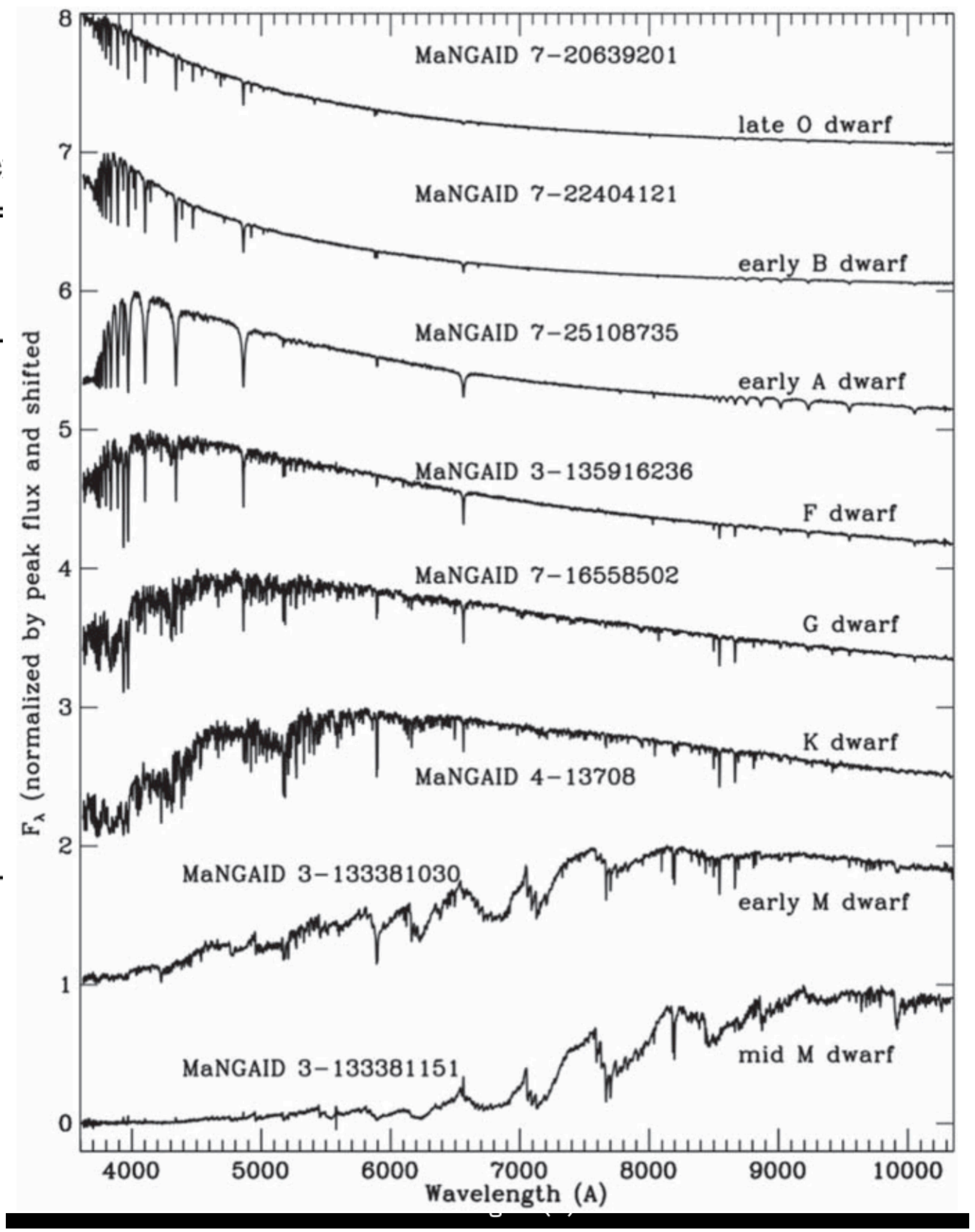


Figure from Chen+2019

3. How to derive Teff, logg & [M/H] from spectra

Spectral model — Empirical spectral libraries





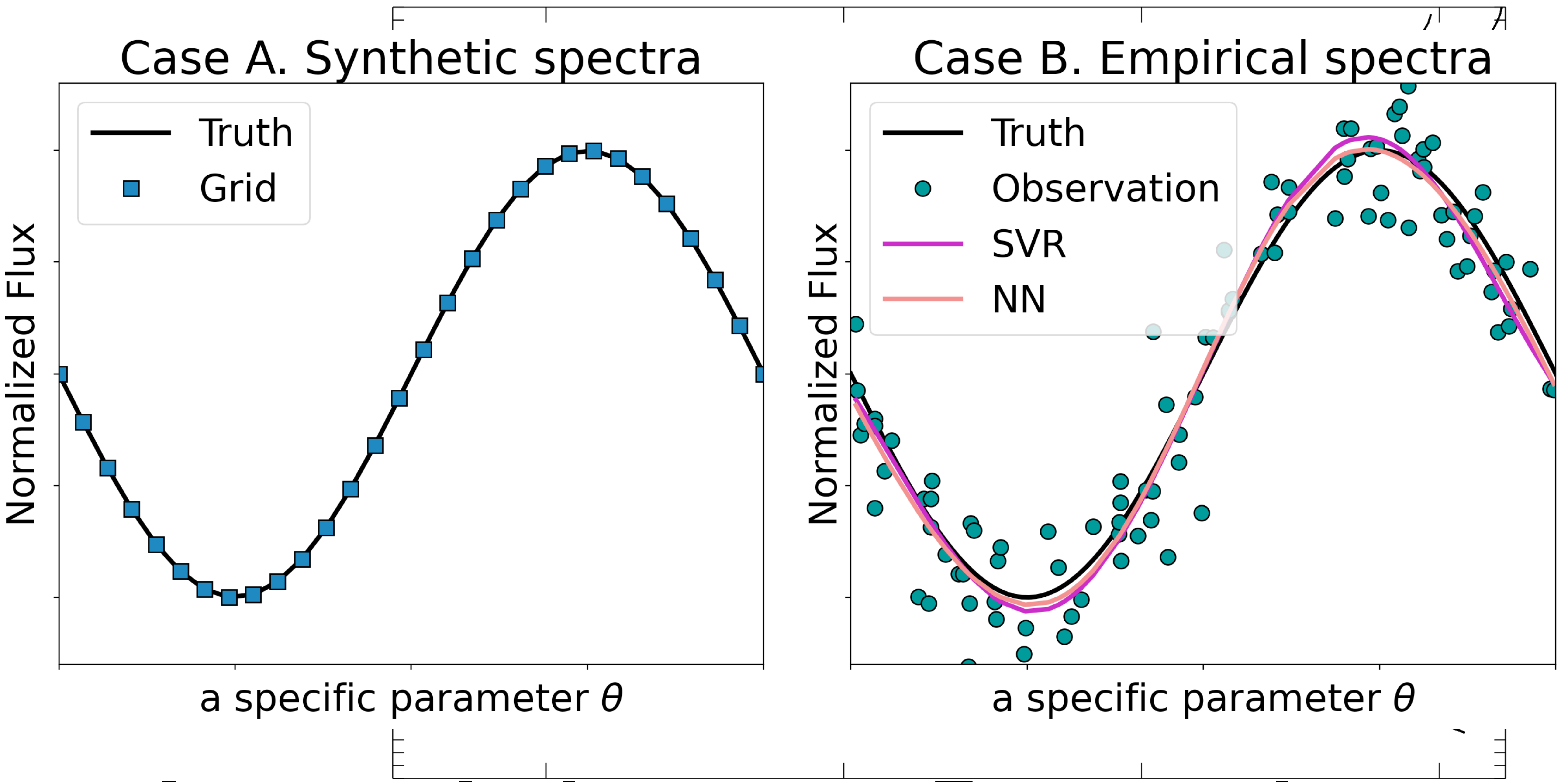
Yan et al. 2018

3. How to derive Teff, logg & [M/H] from spectra

How to build spectral models

C. Allende Prieto et al.: Model stellar spectra

- **Synthetic spectral grid**
  - Interpolation
  - Bayesian estimation (FERRE)
- **Empirical spectra**
  - **Polynomial**
    - Ulyss/LASP (Wu et al. 2010)
    - The Cannon (Ness et al. 2015)
  - **Neural-network**
    - ThePayne (Ting et al. 2019)
  - **Support Vector Regression**
    - SLAM (Zhang et al. 2020a)

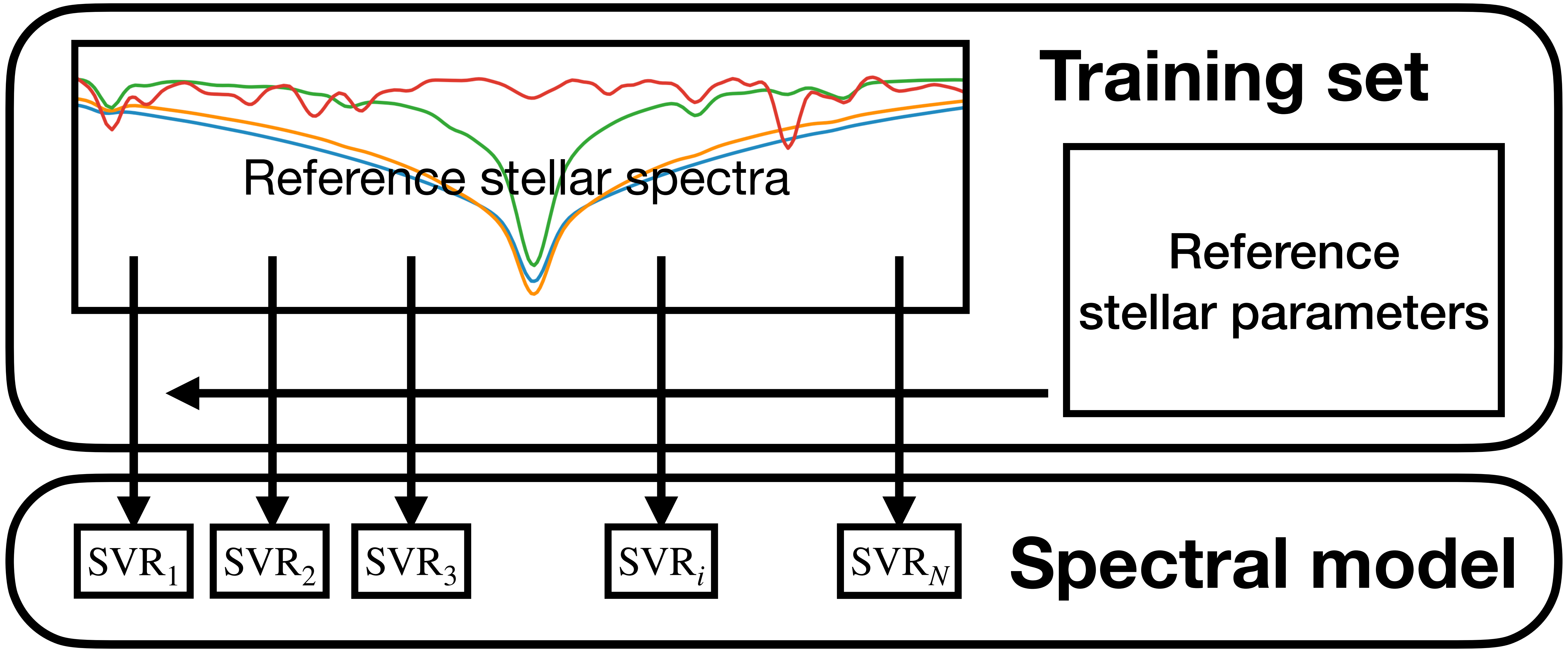


Interpolation **Regression**

3. How to derive Teff, logg & [M/H] from spectra

Training of SLAM

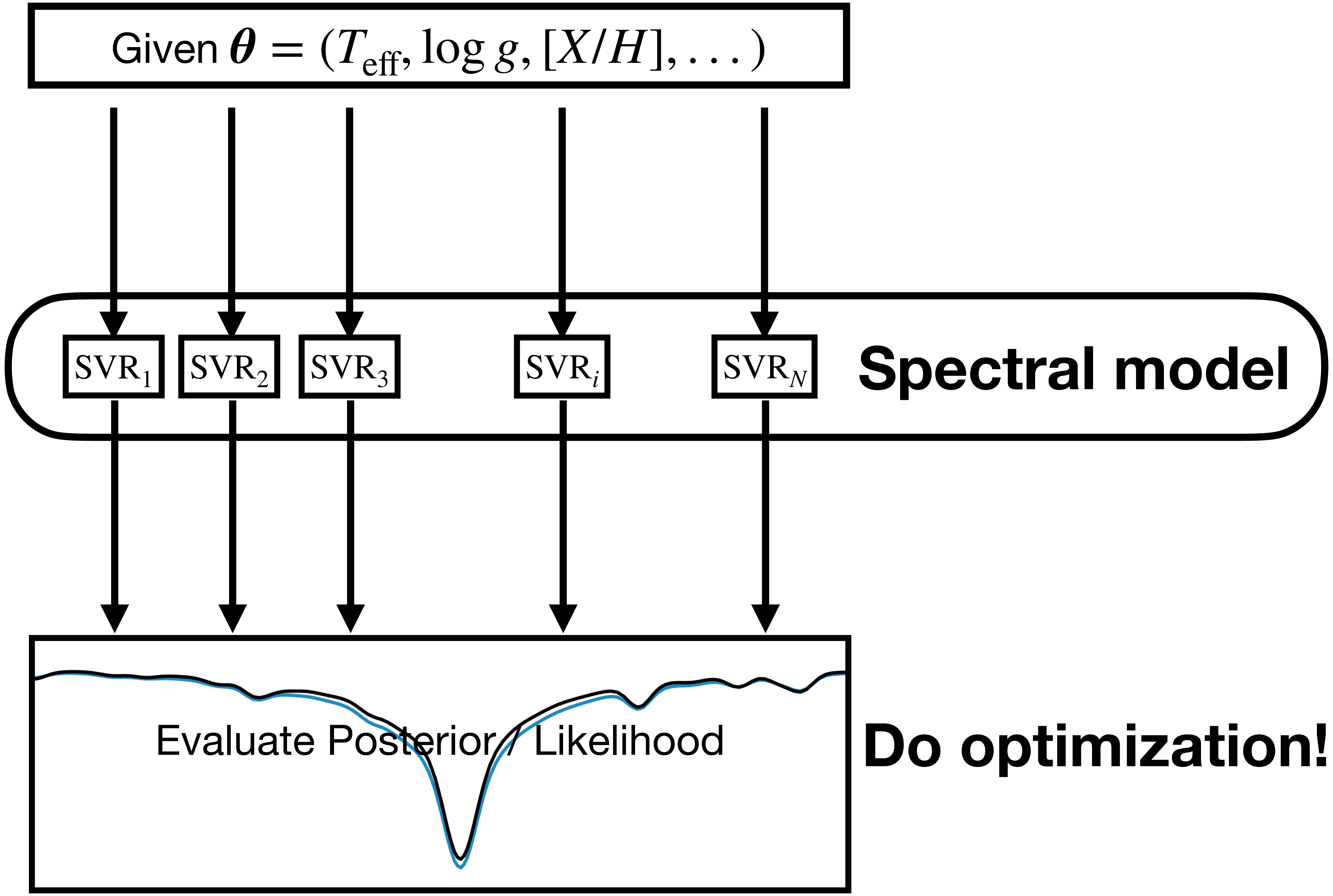
- **Synthetic spectral grid**
  - Interpolation
  - Bayesian estimation (FERRE)
- **Empirical spectra**
  - **Polynomial**
    - Ulyss/LASP (Wu et al. 2010)
    - The Cannon (Ness et al. 2015)
  - **Neural-network**
    - ThePayne (Ting et al. 2019)
  - **Support Vector Regression**
    - SLAM (Zhang et al. 2020a)



3. How to derive Teff, logg & [M/H] from spectra

Prediction of SLAM

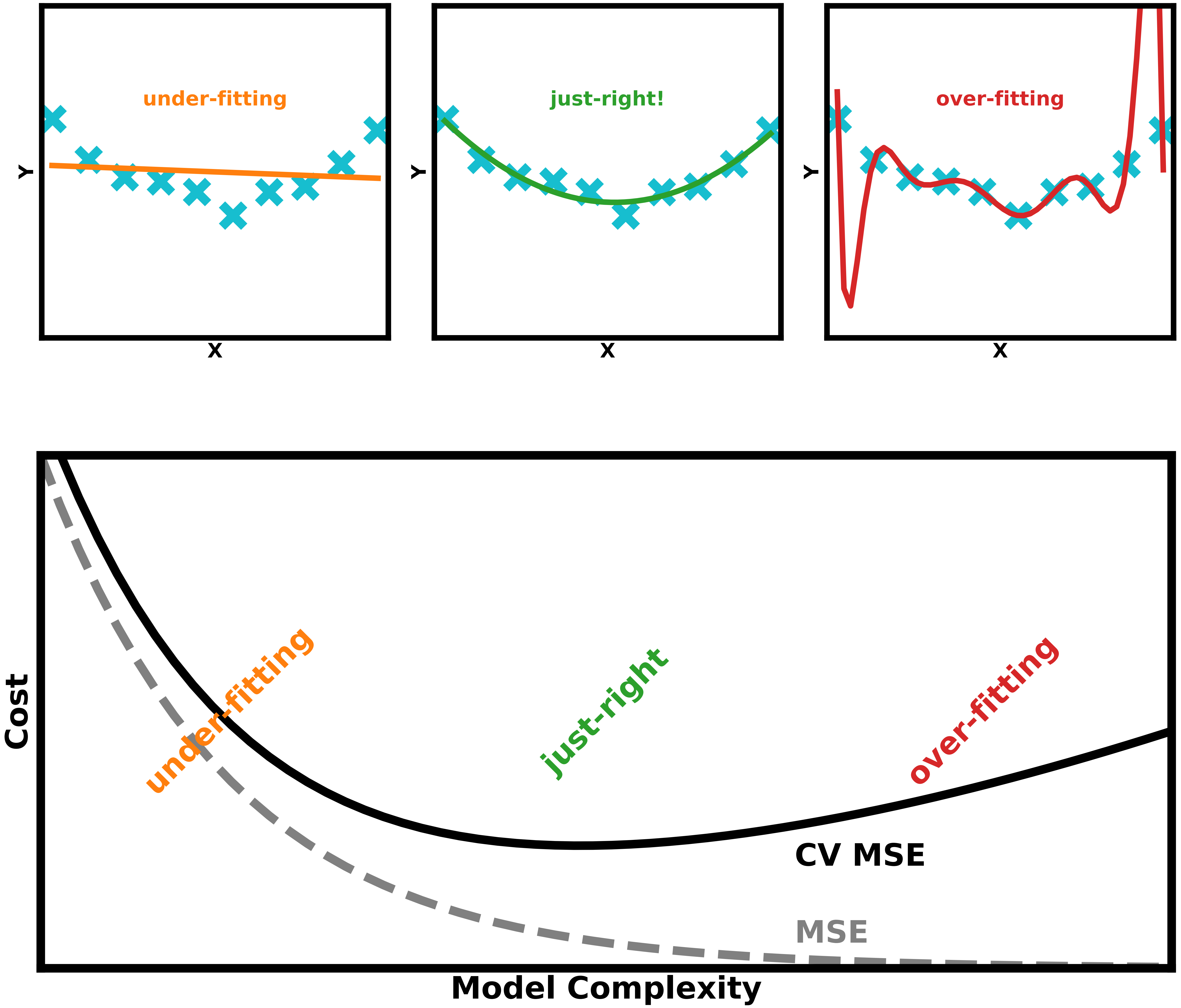
- **Synthetic spectral grid**
  - Interpolation
  - Bayesian estimation (FERRE)
- **Empirical spectra**
  - **Polynomial**
    - Ulyss/LASP (Wu et al. 2010)
    - The Cannon (Ness et al. 2015)
  - **Neural-network**
    - ThePayne (Ting et al. 2019)
  - **Support Vector Regression**
    - SLAM (Zhang et al. 2020a)



3. How to derive Teff, logg & [M/H] from spectra

Model complexity*

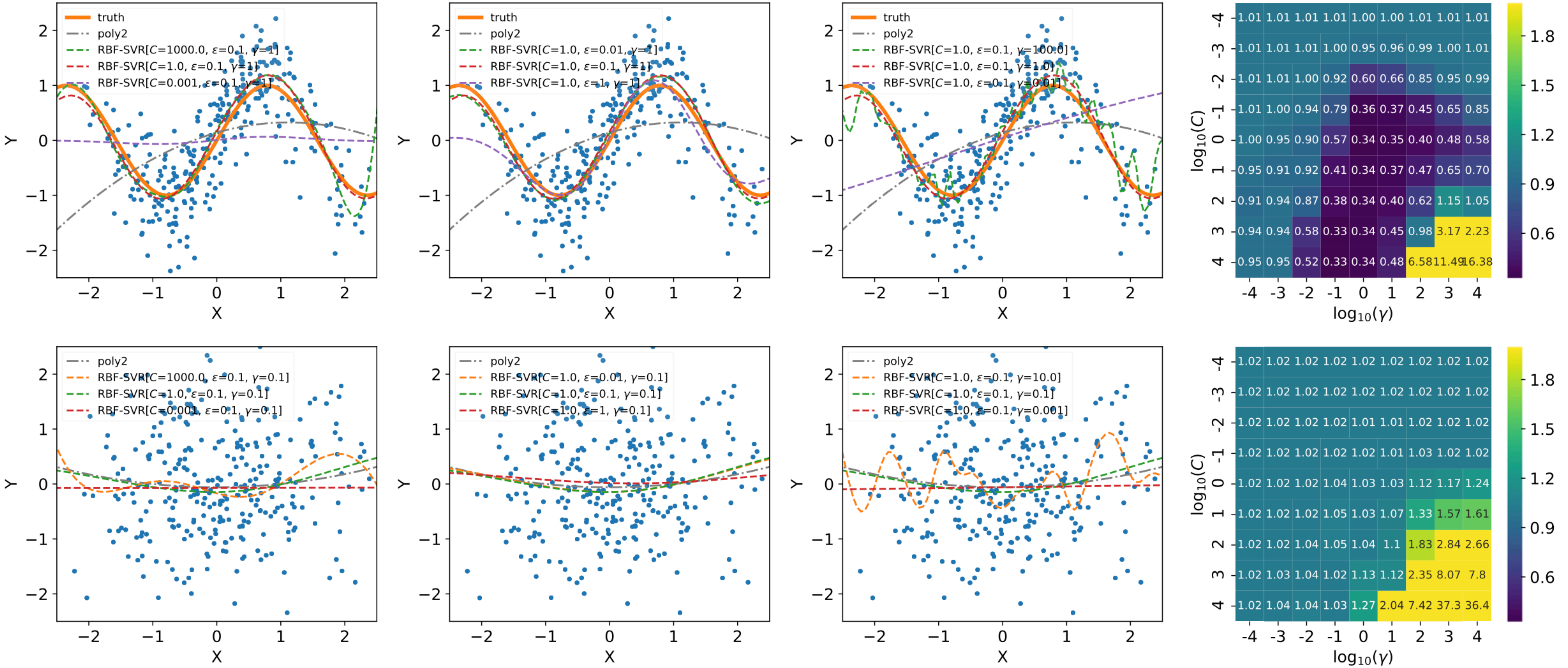
- **Polynomial**
    - order
- **NN**
    - n_layer
    - n_neuron
- **SVR**
    - C
    - eps
    - gamma
    - n_sv
- **Spline**
    - Smoothness



3. How to derive Teff, logg & [M/H] from spectra

Model complexity solution in SLAM*

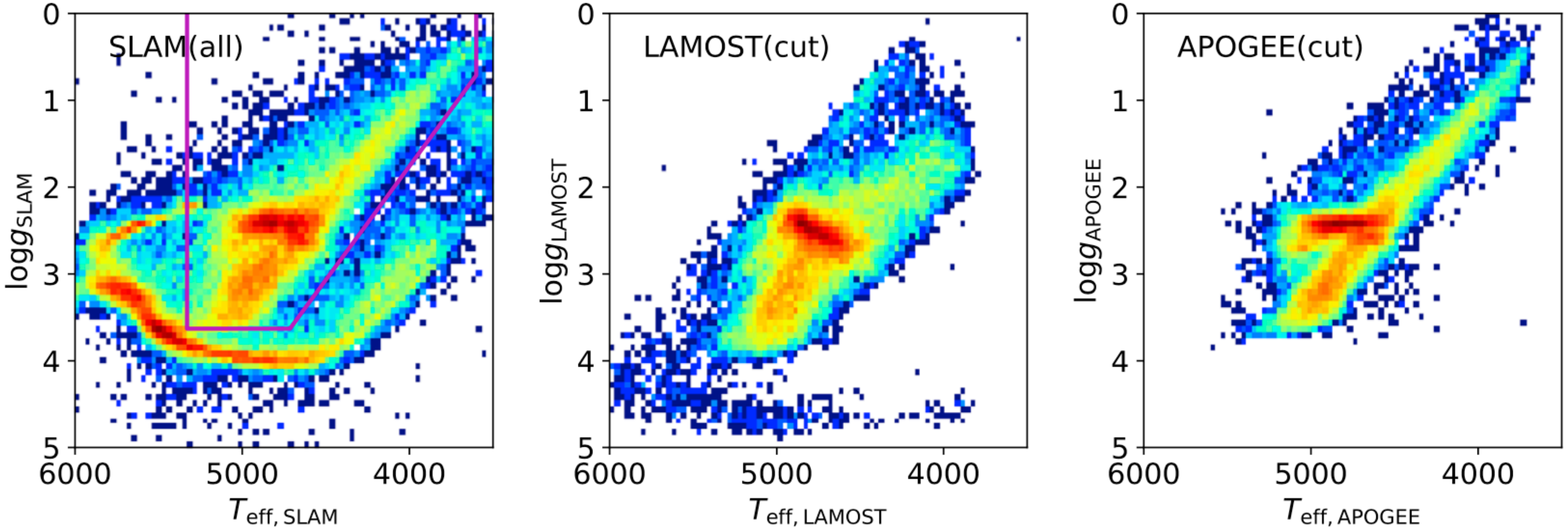
- **Polynomial**
  - order
- **NN**
  - n_layer
  - n_neuron
- **SVR**
  - C
  - eps
  - gamma
  - n_sv
- **Spline**
  - Smoothness

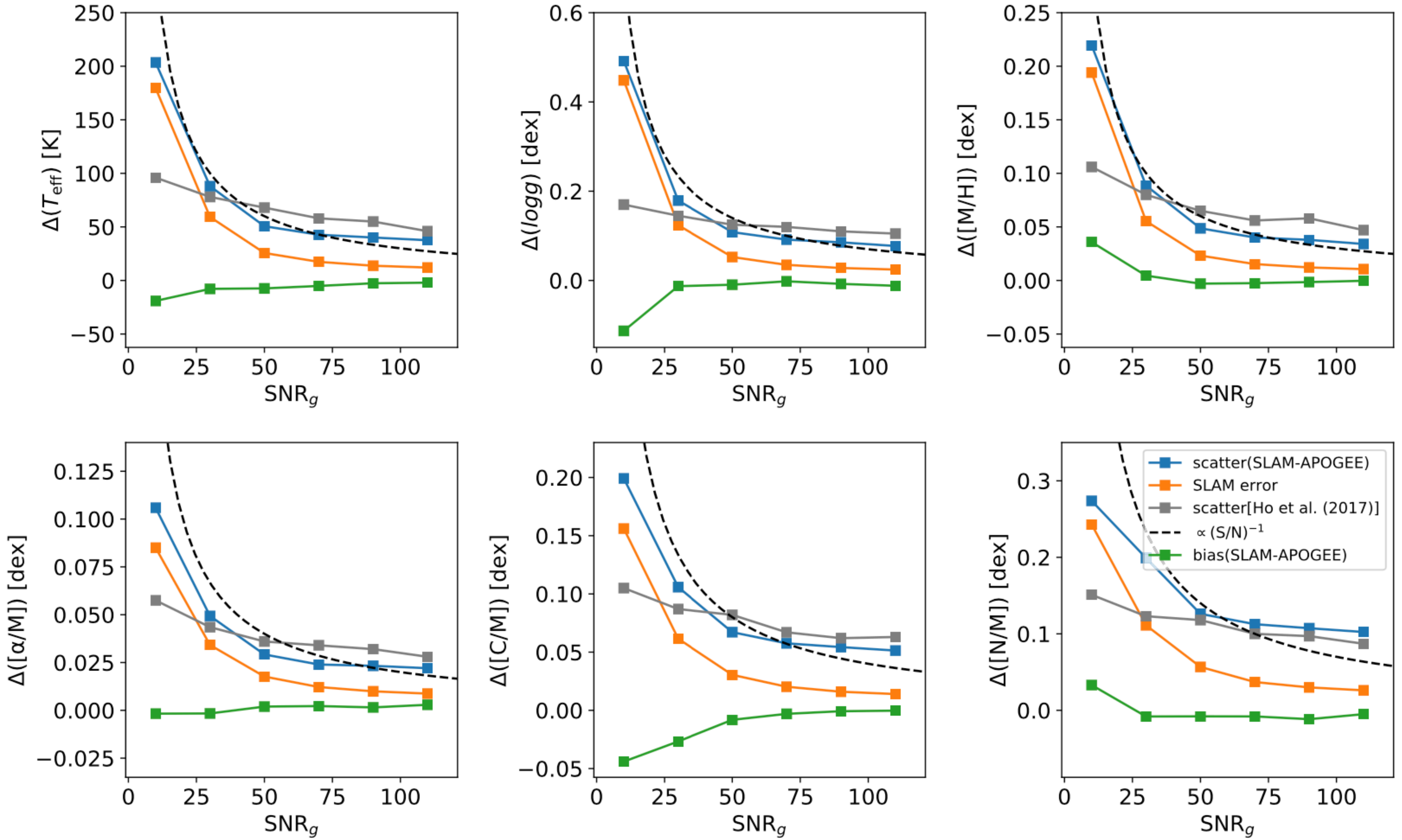


3. How to derive Teff, logg & [M/H] from spectra

Application of SLAM

- **Synthetic spectral grid**
  - Interpolation
  - Bayesian estimation (FERRE)
- **Empirical spectra**
  - **Polynomial**
    - Ulyss/LASP (Wu et al. 2010)
    - The Cannon (Ness et al. 2015)
  - **Neural-network**
    - ThePayne (Ting et al. 2019)
  - **Support Vector Regression**
    - SLAM (Zhang et al. 2020a)

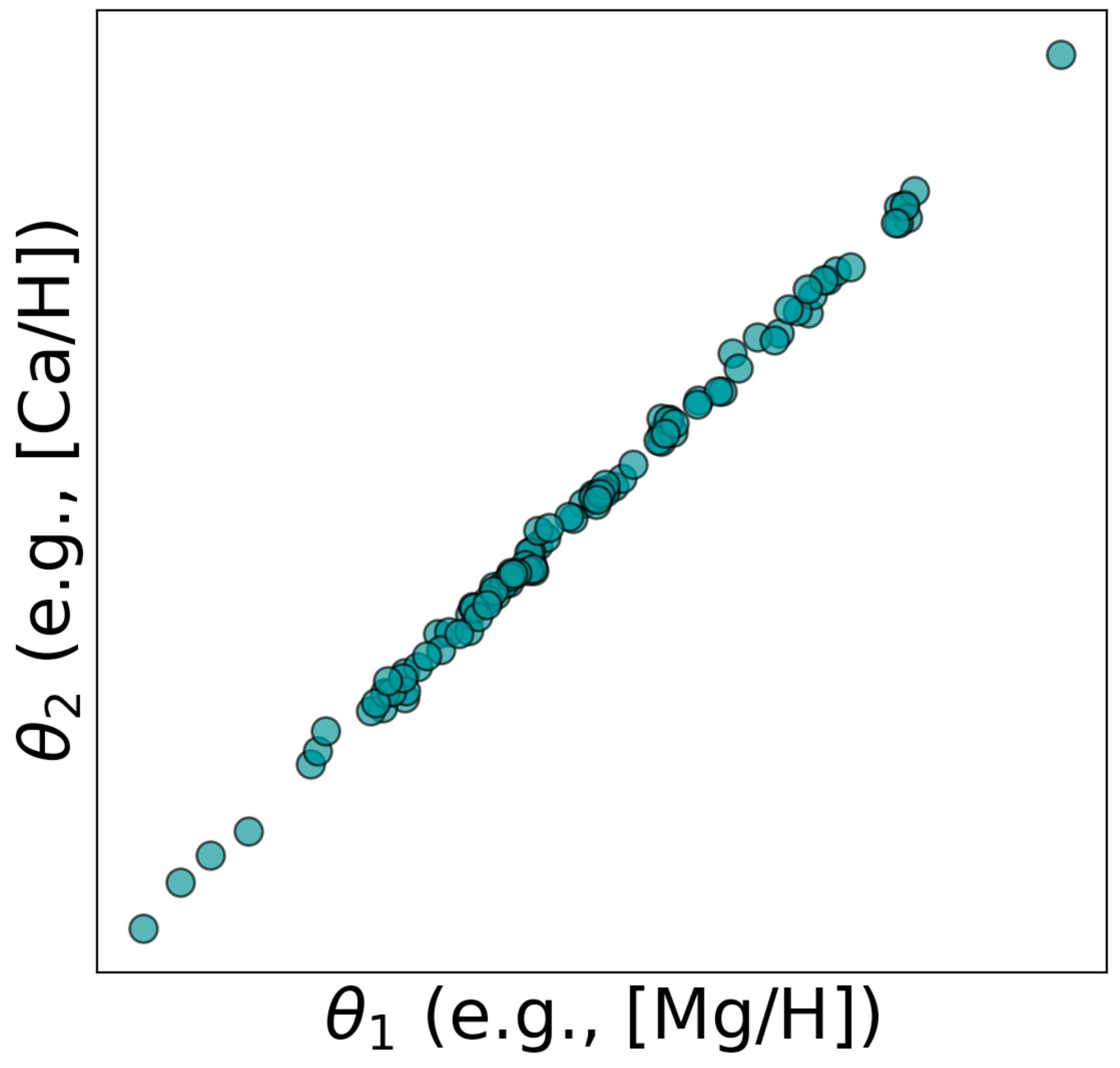


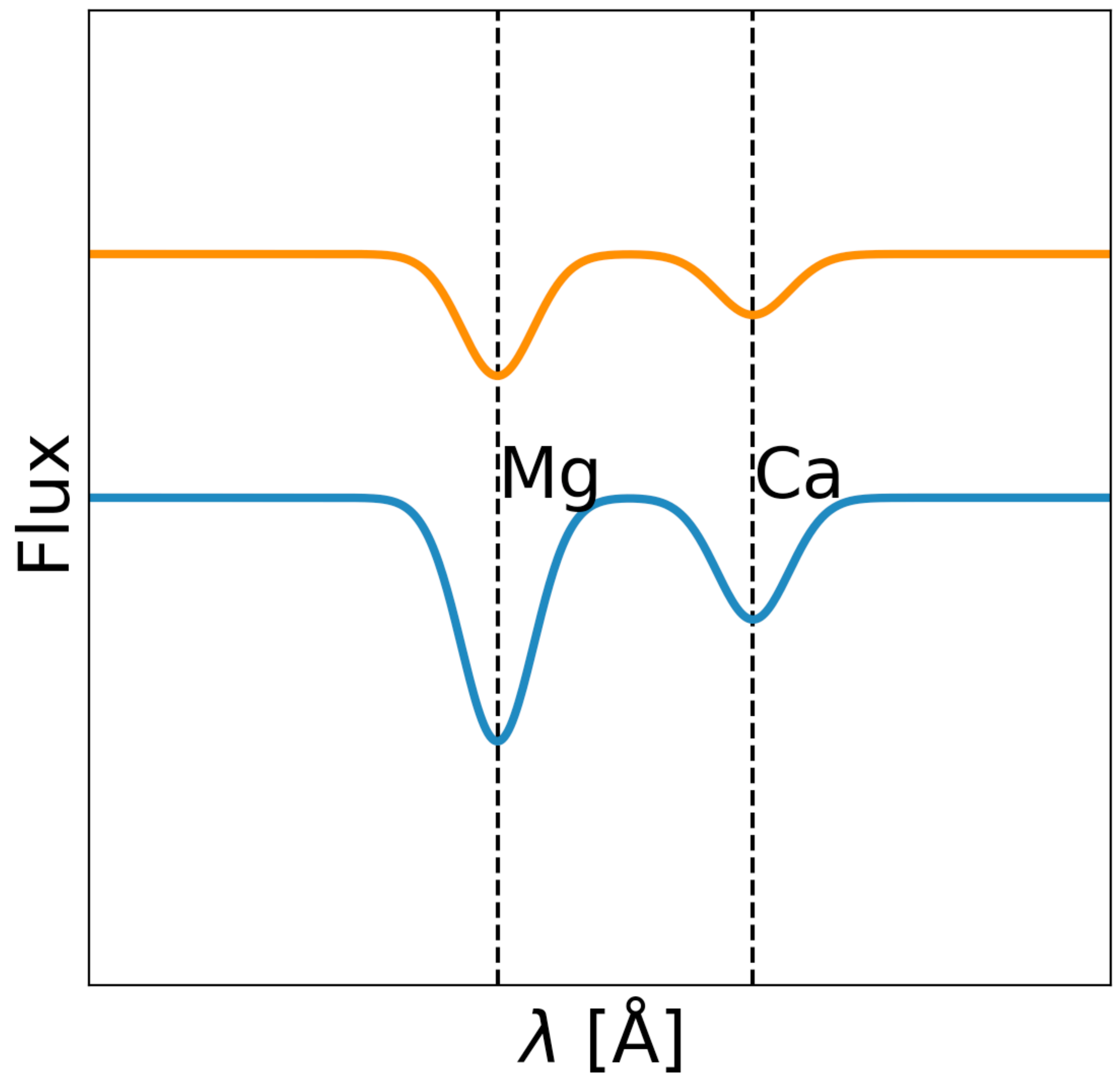


3. How to derive Teff, logg & [M/H] from spectra

Degeneracy between parameters*

- **The origin of degeneracy**
    - Non-uniform distribution of the training set
- **How to break?**
    - +model gradient?(The Payne)
-

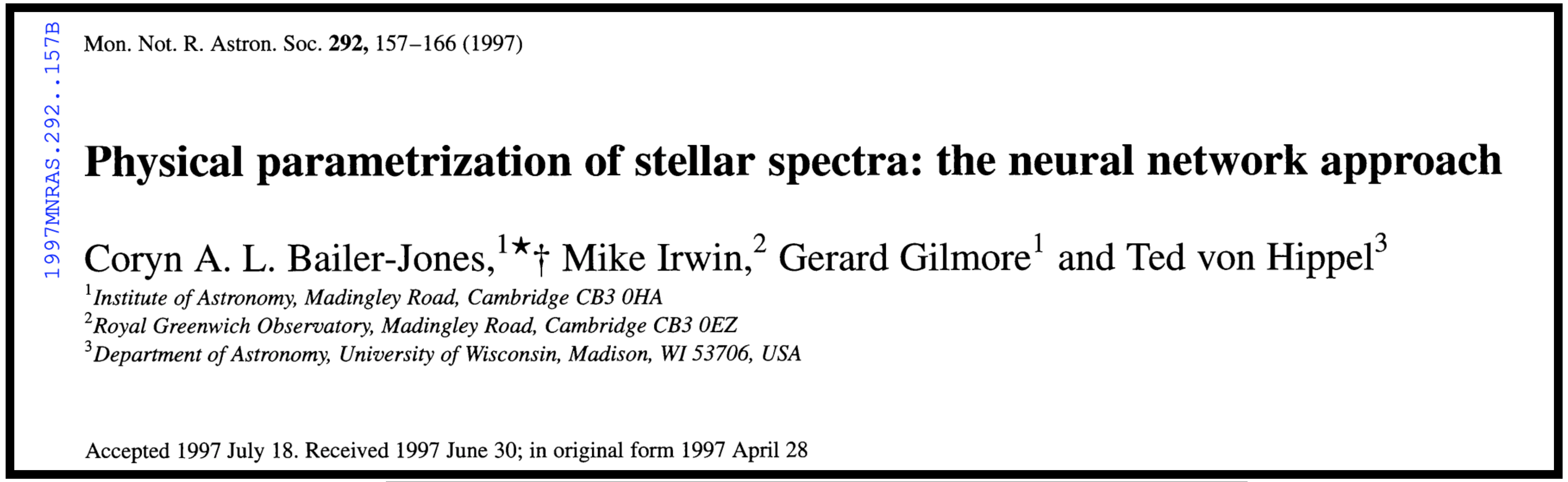




3. How to derive Teff, logg & [M/H] from spectra

Backward model — Neural-Networks

- **ANN**
  - Bailer-Jones et al. 1997
- **CNN**
  - AstroNet, Leung & Bovy 2018
  - SPCANet, Wang et al. 2020
  - Guiglion et al. 2020
- …
-



Mon. Not. R. Astron. Soc. **292,** 157–166 (1997)

Physical parametrization of stellar spectra: the neural network approach

Coryn A. L. Bailer-Jones,[1★†] Mike Irwin,[2] Gerard Gilmore[1] and Ted von Hippel[3]
[1]*Institute of Astronomy, Madingley Road, Cambridge CB3 0HA*
[2]*Royal Greenwich Observatory, Madingley Road, Cambridge CB3 0EZ*
[3]*Department of Astronomy, University of Wisconsin, Madison, WI 53706, USA*

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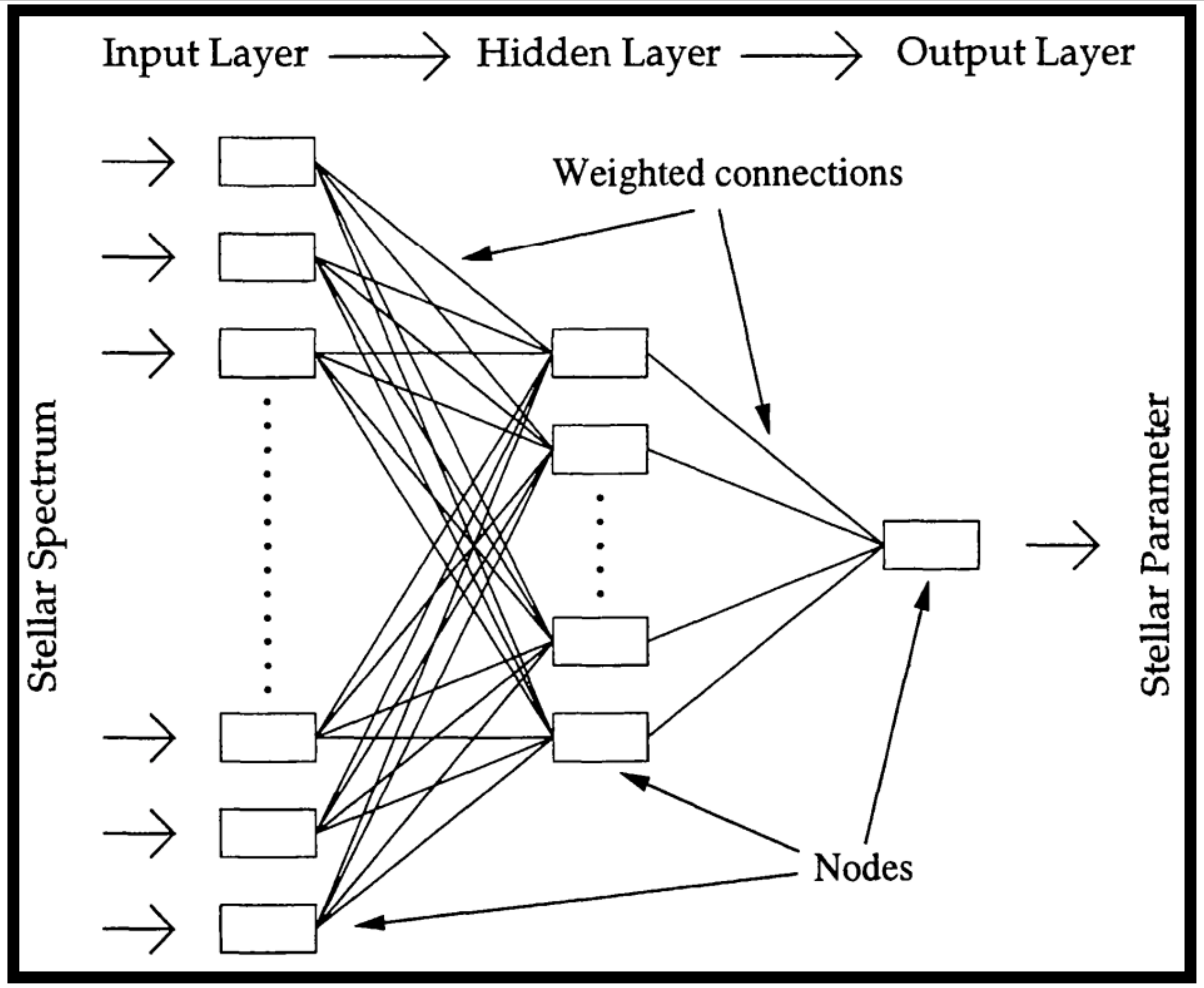
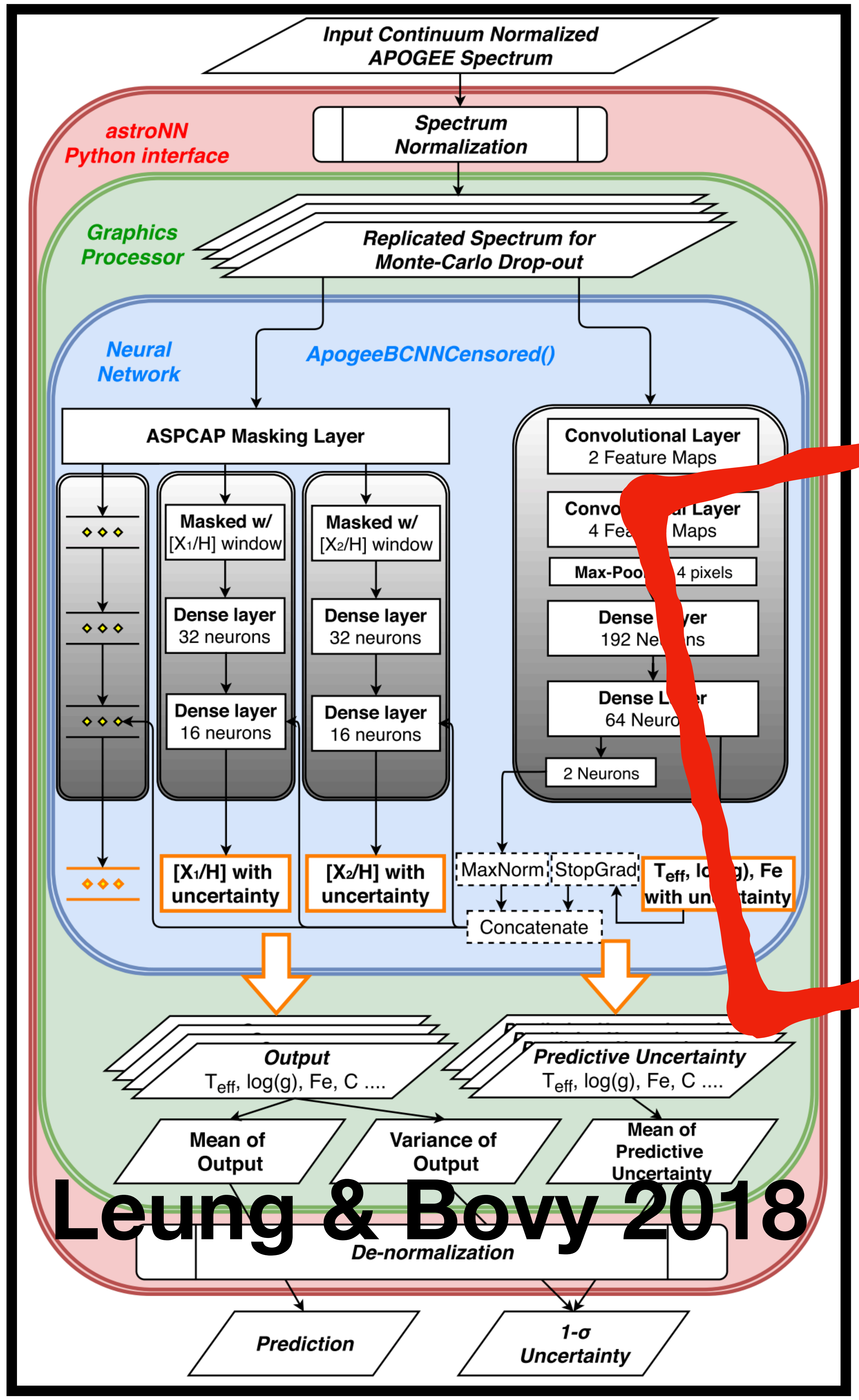
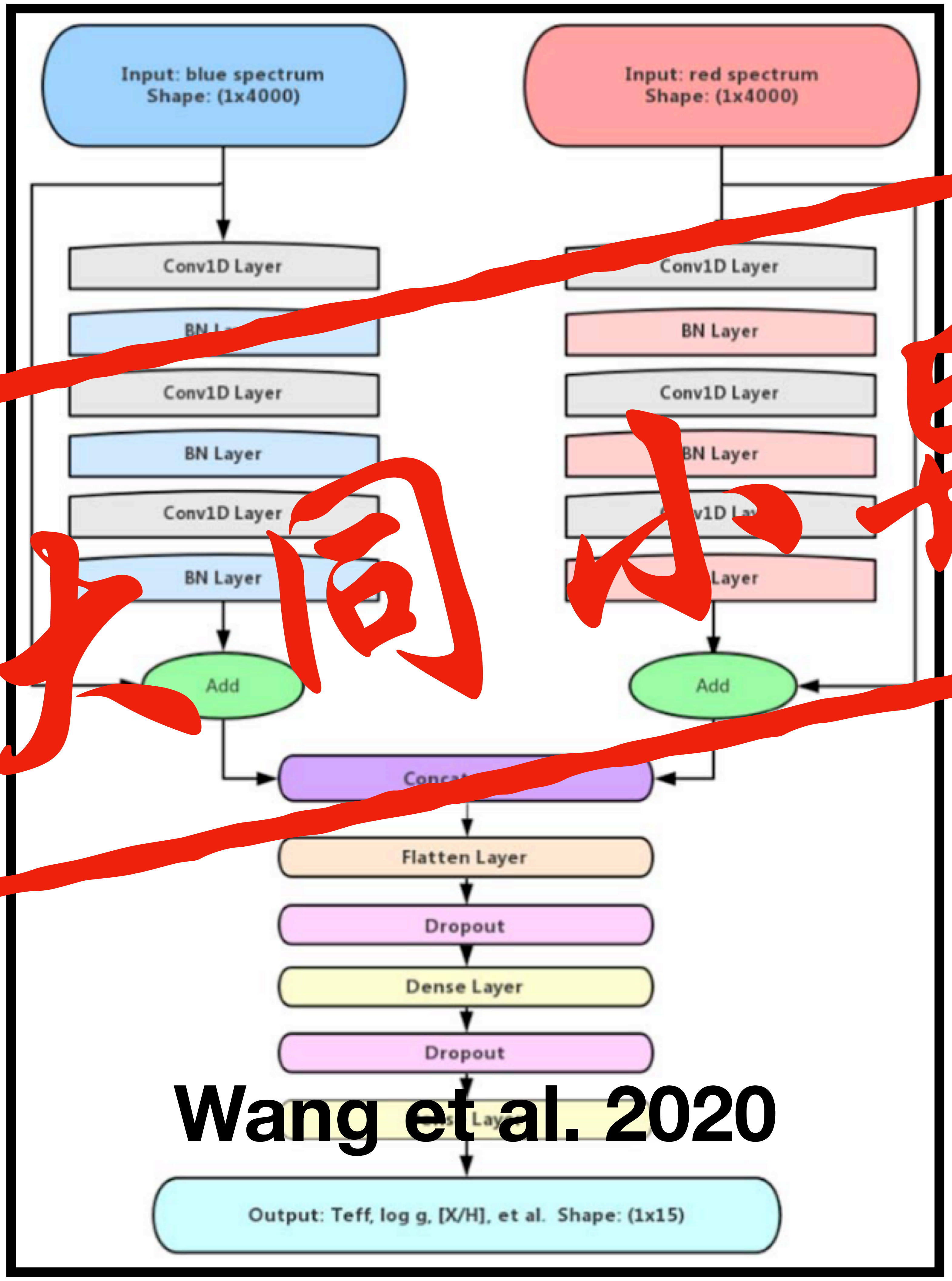


3. How to derive Teff, logg & [M/H] from spectra

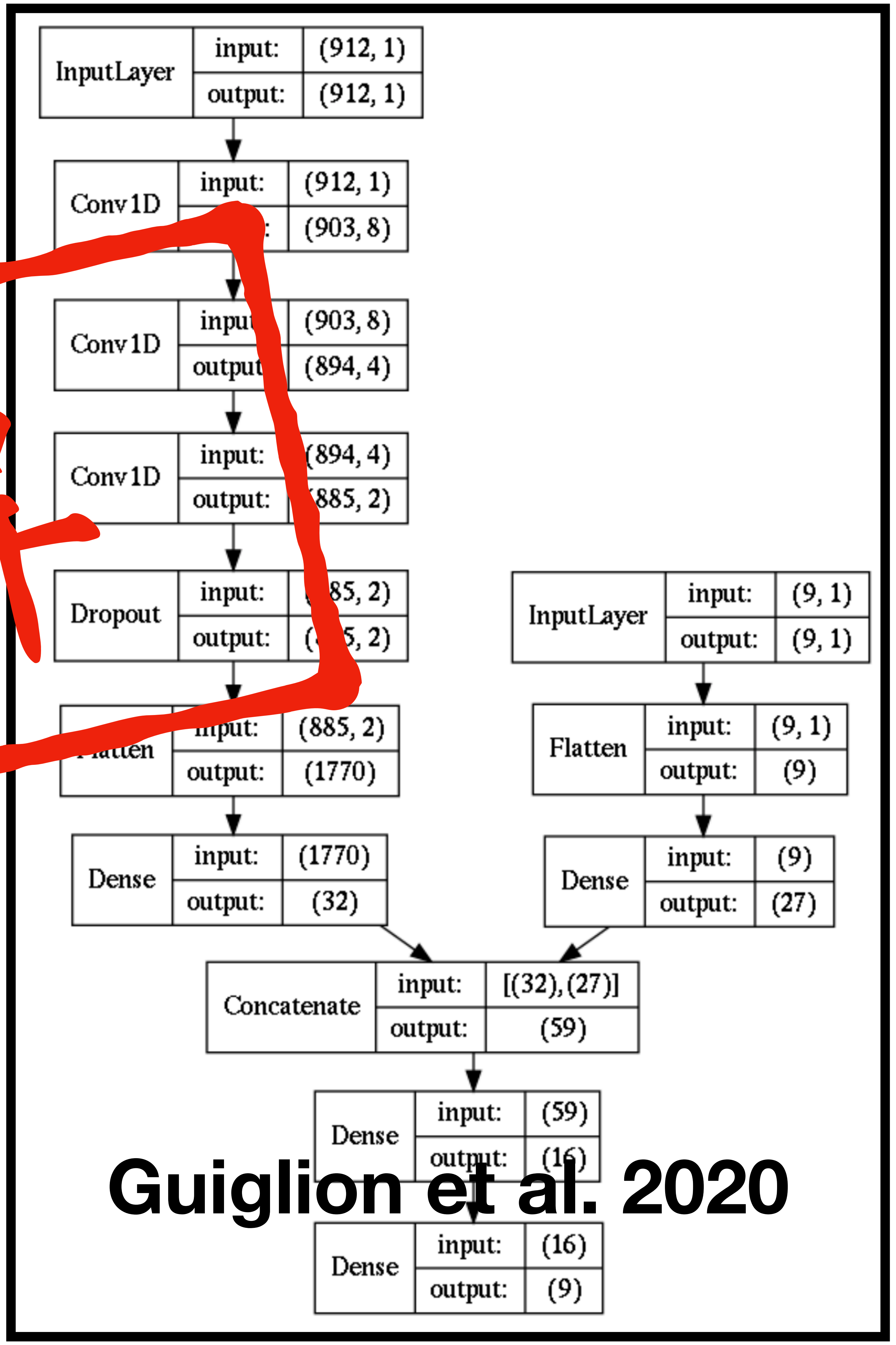
Backward model — Neural-Networks



Leung & Bovy 2018



Wang et al. 2020



Guiglion et al. 2020

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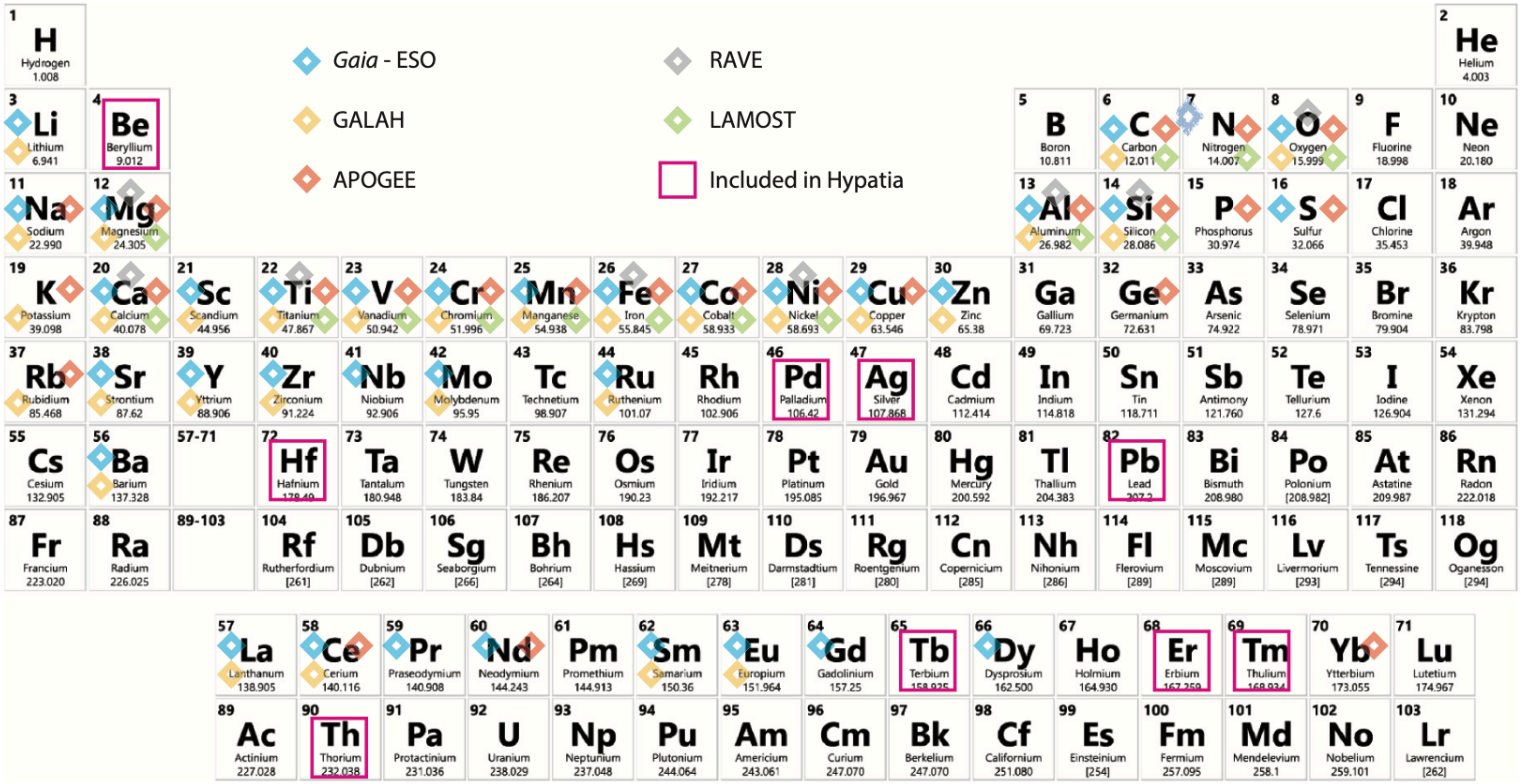
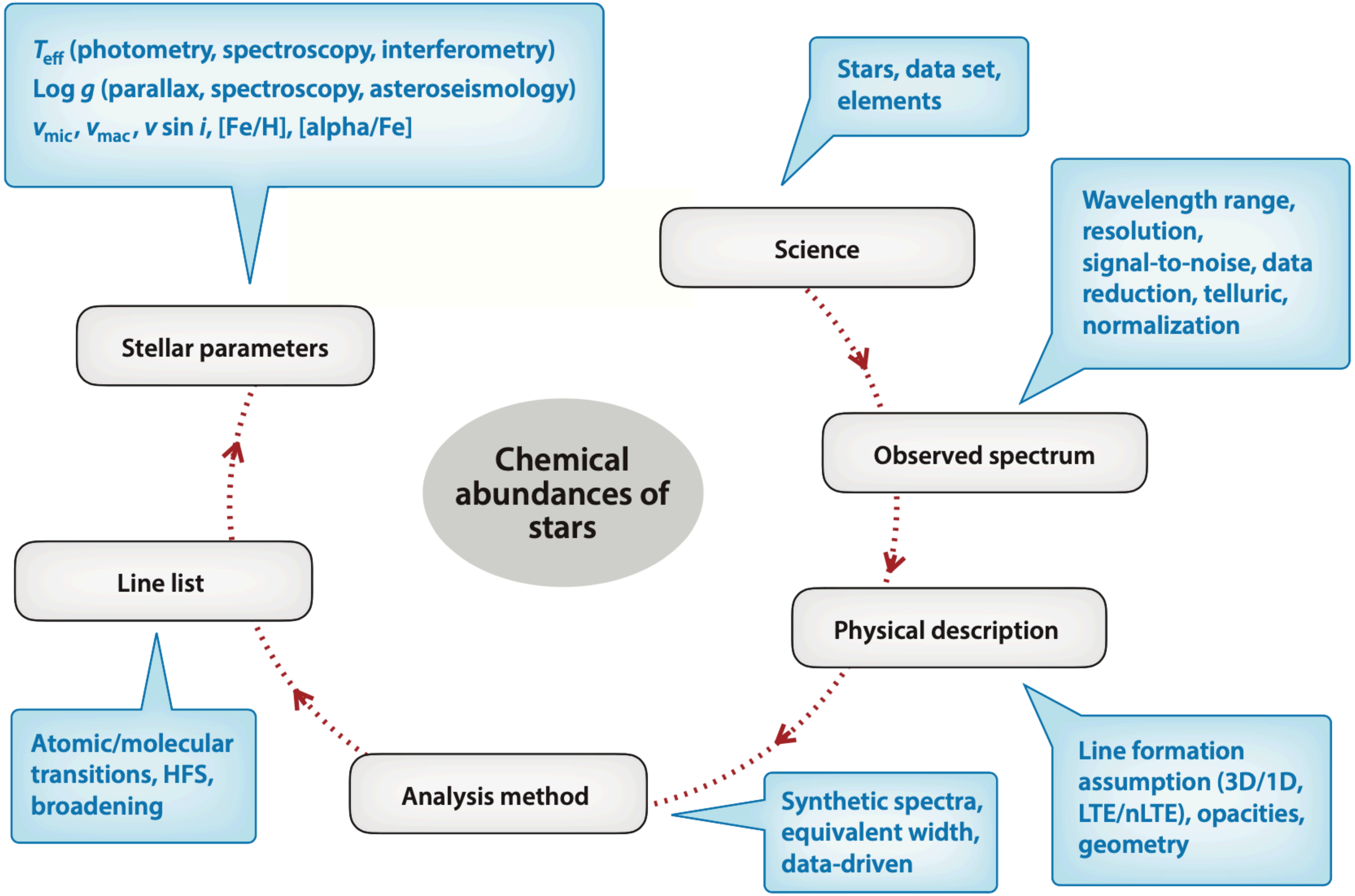
3. How to derive Teff, logg & [X/H] from spectra

More elemental abundances*



3. How to derive Teff, logg & [X/H] from spectra

Take a look back at the industrial stellar parameters



3. How to derive Teff, logg & [X/H] from spectra

Tools summrized by Jofre+2019

Material	Reference	Comment
	Spectral libraries	
SVO	**http://svo2.cab.inta-csic.es/theory/libtest/index.php**	Public libraries
Montes	**https://webs.ucm.es/info/Astrof/invest/actividad/spectra.html**	Compilation
	Model atmospheres	
MARCS	Gustafsson et al. (2008)	1D spherical geometry
ATLAS9	Castelli & Kurucz (2003)	1D plane-parallel geometry
STAGGER	Magic et al. (2013)	3D
CO5BOLD	Freytag et al. (2012)	3D
	Radiative transfer codes	
Turbospectrum	Plez et al. (1992)	LTE
MOOG	Sneden (1973)	LTE
SYNTHE	Kurucz (1993)	LTE
SPECTRUM	Gray & Corbally (1994)	LTE
DETAIL/SIU	e.g., Bergemann et al. (2012) via **http://nlte.mpia.de/**	Non-LTE
	Line lists	
VALD	Ryabchikova et al. (2015)	Literature compilation
NIST ASD	**https://www.nist.gov/pml/atomic-spectra-database**	Literature compilation
Sneden et al.	**https://www.as.utexas.edu/\~chris/lab.html**	Bibliography and molecular line lists
Linemake	**https://github.com/vmplacco/linemake**	Synthetic spectrum lists in MOOG style
ExoMol	Tennyson et al. (2016)	Very cool objects
BRASS	Laverick et al. (2018)	Centralization of sources
Barklem	Barklem et al. (2015)	Broadening cross-sections
Kurucz	Kurucz (2011)	Atomic data
VAMDC	**http://www.vamdc.eu**	Electronic infrastructure
	Grids of synthetic spectra	
AMBRE	de Laverny et al. (2012)	Optical high resolution
STAGGER	Chiavassa et al. (2018)	Ca II triplet centered
3D-non-LTE Balmer	Amarsi et al. (2018b)	Balmer lines centered
APOGEE	Mészáros et al. (2012)	IR
POLLUX	Palacios et al. (2010)	Database
	Automatic codes for the determination of abundances	
SME	Piskunov & Valenti (2017)	With non-LTE on the fly
iSpec	Blanco-Cuaresma et al. (2014a)	Python wrapper for various tools
FERRE	García Pérez et al. (2016)	Match models to data
GALA	Mucciarelli et al. (2013)	EW code
DOOp	Cantat-Gaudin et al. (2014)	Wrapper for EWs
ARES	Sousa et al. (2015)	Automatic EWs
The Cannon	Ness et al. (2015)	Label transfer from a training set
	Non-LTE abundance corrections	
INSPECT	**http://inspect-stars.com/**	Line-by-line corrections
MPIA	**http://nlte.mpia.de/**	Line-by-line corrections

4. v sini profile

Ref: D.F.Gray The Observation & Analysis of Stellar Photonsphere

Profile fitting for vsini 467

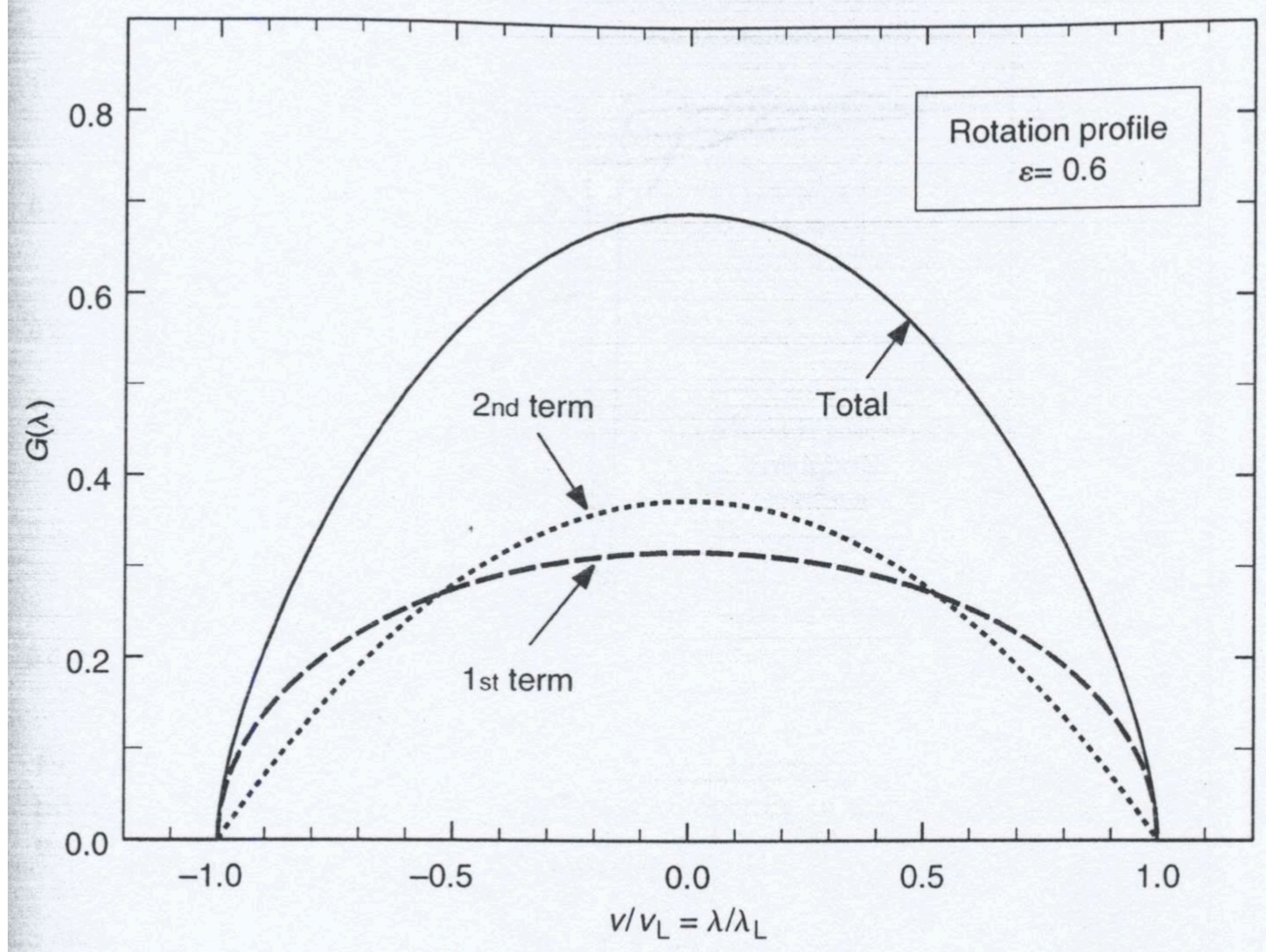


Fig. 18.5. The rotation profile, Eq. (18.14), is shown by the solid line for $\varepsilon = 0.6$ and labeled "total." It is the sum of the "1st term" and "2nd term" curves.

468 *Stellar rotation*

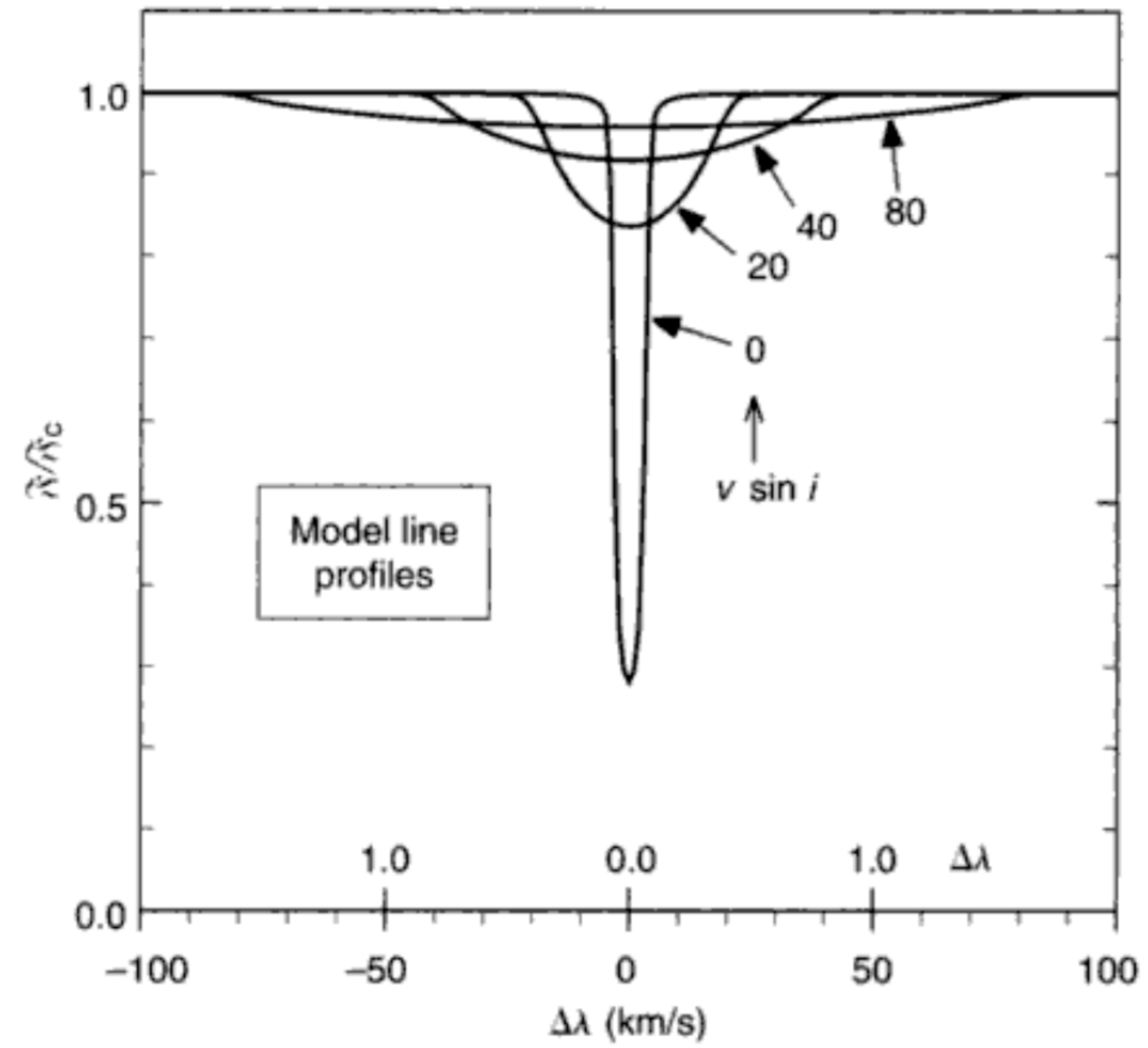


Fig. 18.6. Computed profiles illustrate the broadening effect of retation. The profiles are labeled with $v\sin i$ in km/s. The equivalent width is conserved.

Fourier analysis for large vsini 471

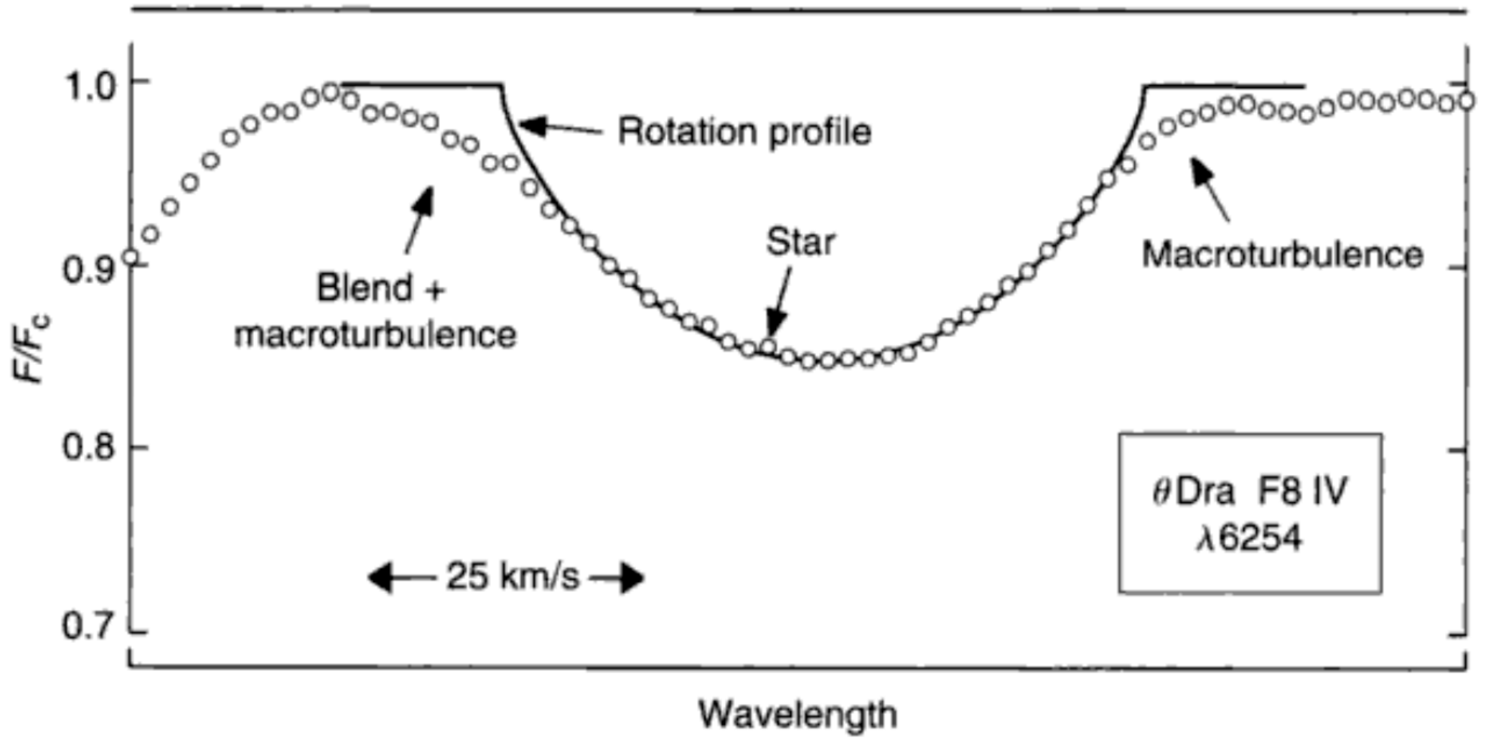


Fig. 18.10. An observed profile of θ Dra is compared to the classic rotation profile. The rotation profile shows sharp corners where it joins the continuum. The high Fourier frequencies needed to form these corners are filtered away by other broadeners, in this case macroturbulence of several km/s. Data from the Elginfield Observatory.

$$G(\Delta\lambda) = G(v) = \frac{2(1-\epsilon)[1-(v_z/v_L)^2]^{1/2} + \frac{1}{2}\pi\epsilon[1-(v_z/v_L)^2]}{\pi v_L(1-\epsilon/3)}$$

4. v sini profile

Ref: D.F.Gray The Observation & Analysis of Stellar Photonsphere

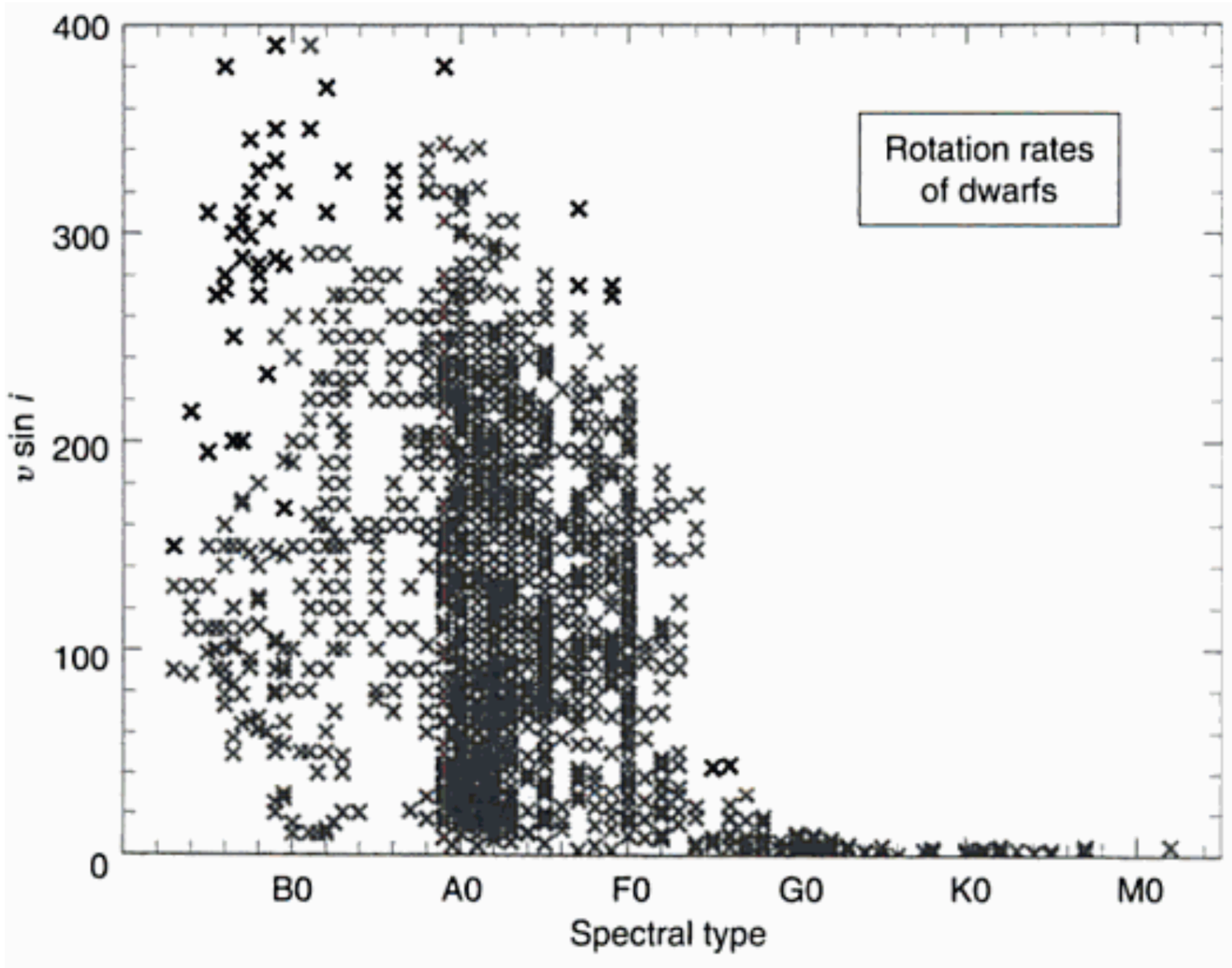


Fig. 18.21. Rapid rotation is normal for hot stars; slow rotation is seen for cool stars. The transition occurs near F5. Data from: Slettebak *et al.* (1975), Conti and Ebbets (1977), Soderblom (1982), Gray (1984b), Halbedel (1996), Penny (1996), Fekel (1997, 2003), and Royer *et al.* (2002b).

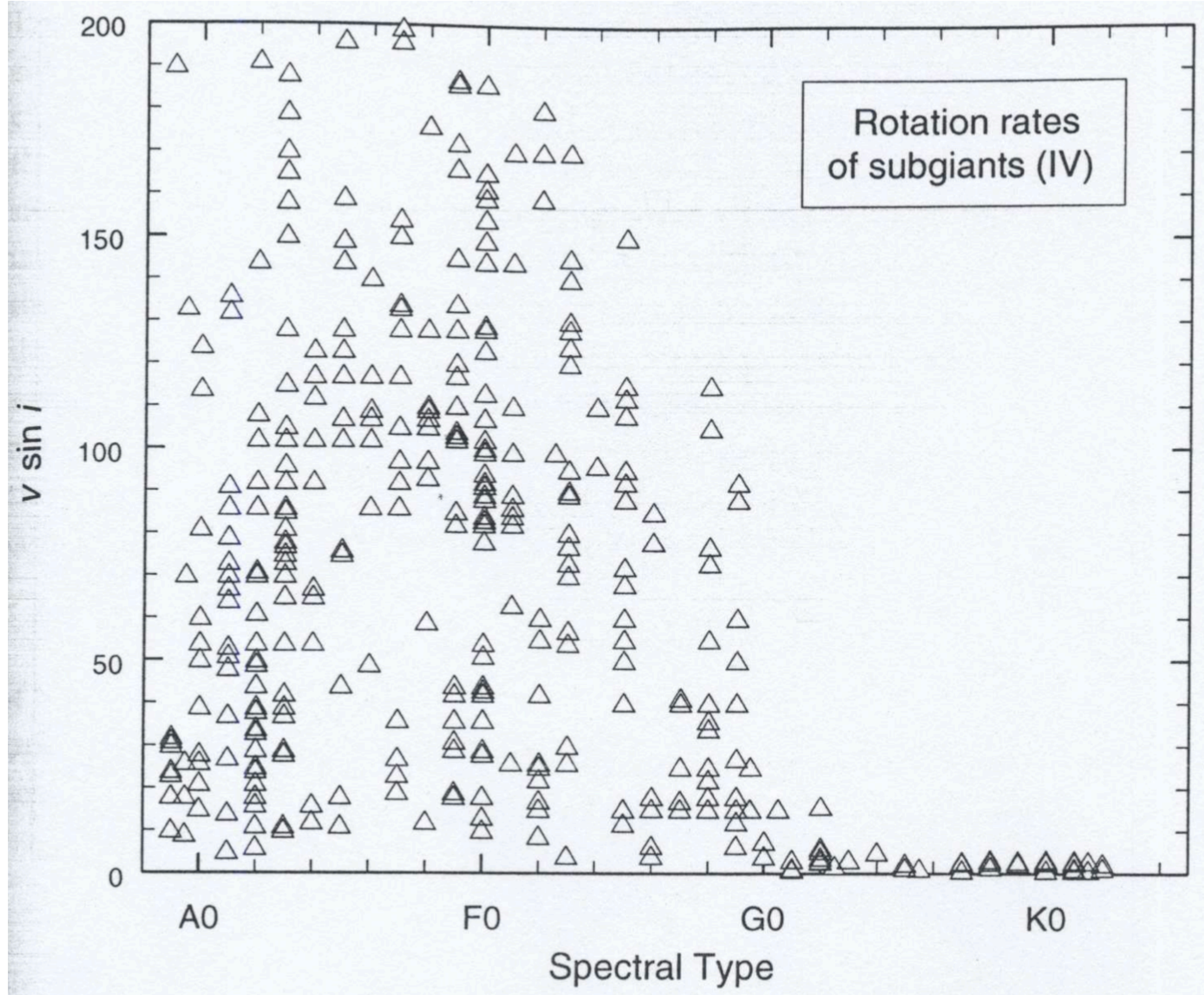


Fig. 18.23. Subgiants show a wide range of rotation rates for stars hotter than G0 IV, but slow rotation for cooler stars. Data from: Uesugi and Fukuda (1982), Gray and Nagar (1985), Fekel (1997, 2003), and Royer *et al.* (2002b).

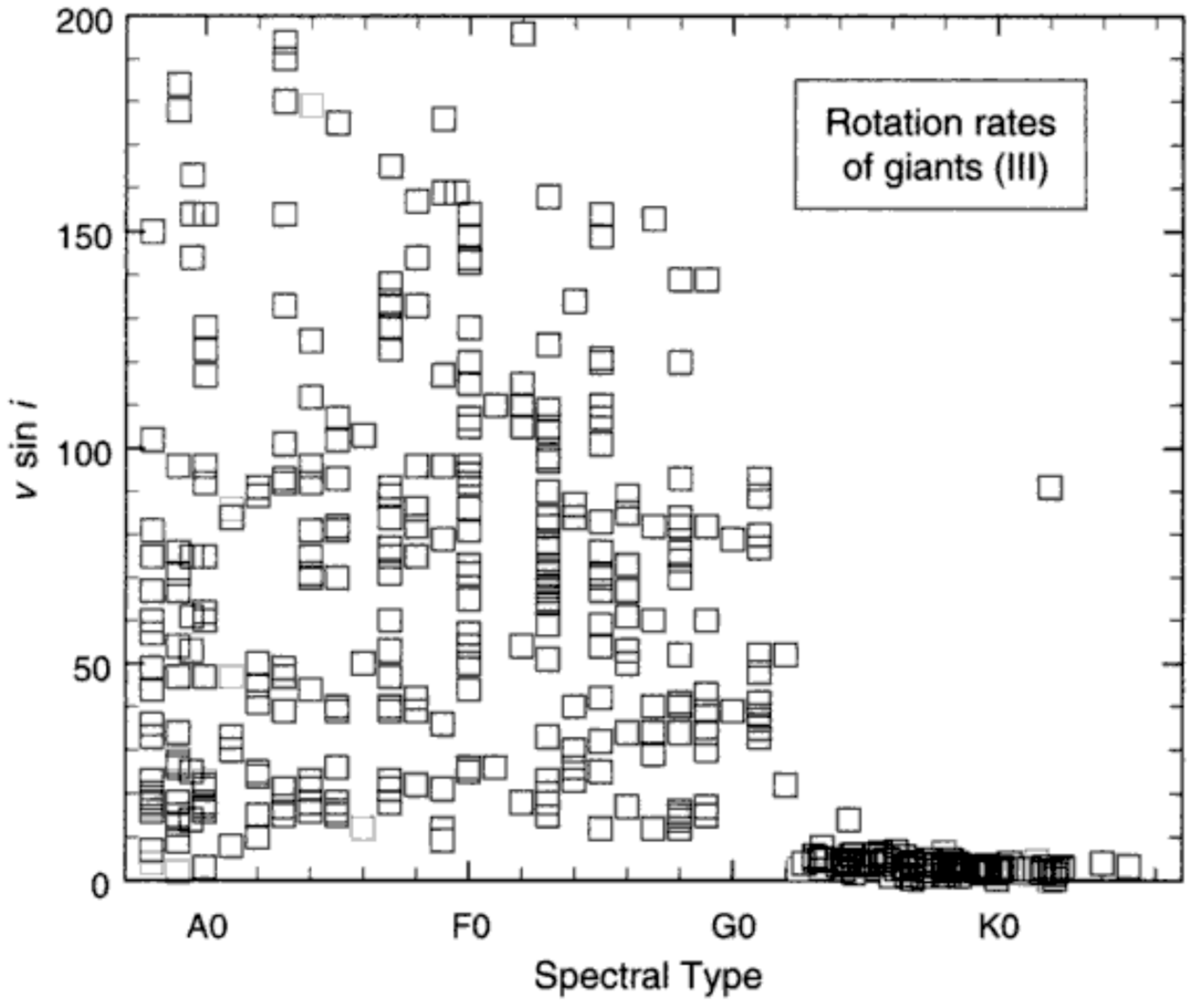


Fig. 18.24. Giants mimic the lower luminosity classes, but the drop in rotation occurs at G2 III. From G2 III to K2 III, rotation is a single-valued function of spectral type as given by Eq. (18.25). Data from: Alschuler (1975), Gray (1982c, 1989a), Hoffleit and Jaschek (1982), Royer *et al.* (2002b), and Fekel (2003).

$$G(\Delta\lambda) = G(v) = \frac{2(1-\epsilon)[1-(v_z/v_L)^2]^{1/2} + \frac{1}{2}\pi\epsilon[1-(v_z/v_L)^2]}{\pi v_L(1-\epsilon/3)}$$

4. v sini profile

Ref: D.F.Gray The Observation & Analysis of Stellar Photonsphere

Linear limb-darkening coefficients

Disk integration mechanics 437

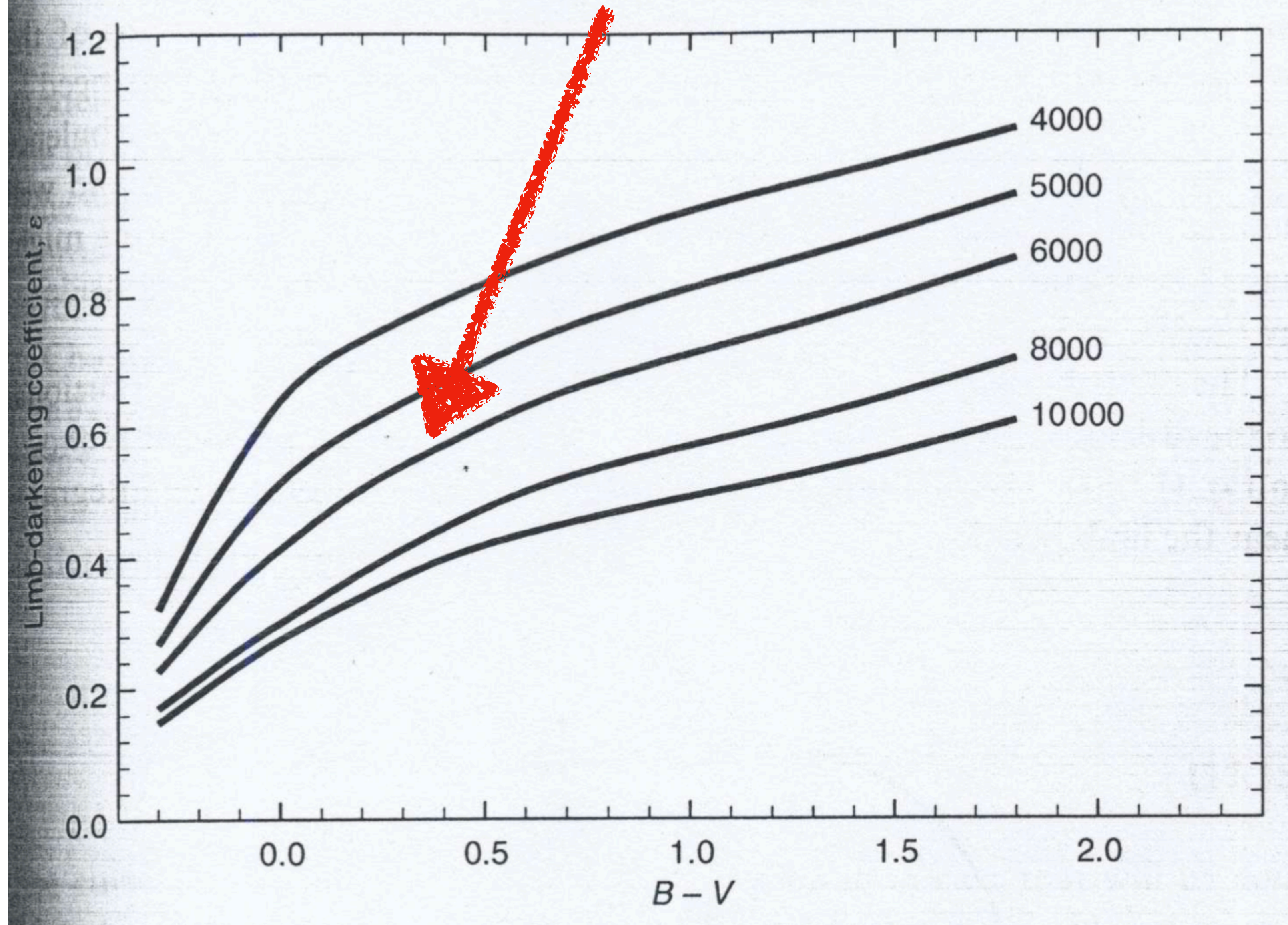


Fig. 17.6. Linear limb-darkening coefficients, ε in Eq. (17.11), are shown here as a function of $B-V$ color index of the star for a selection of wavelengths. Both coordinates depend slightly on metallicity and surface gravity. Expressions more precise than Eq. (17.11) can be used, but the basic uncertainty in temperature distribution limits what can be gained.

$$G(\Delta\lambda) = G(v) = \frac{2(1-\epsilon)[1-(v_z/v_L)^2]^{1/2} + \frac{1}{2}\pi\epsilon[1-(v_z/v_L)^2]}{\pi v_L(1-\epsilon/3)}$$

Summary

- **History of stellar spectroscopy**
- **How to measure RV**
  - Line profile fitting
  - Cross-correlation function (CCF)
- **How to measure atmospheric parameters**
  - EW-parameter relation
  - Forward model — Synthetic model / ThePayne/SLAM
  - Backward model — ANN/CNN

Thanks

Practice makes perfect

Link: https://github.com/hypergravity/spectroscopy

- **Practice**
  - **CCF: 试利用模板光谱(wave_temp, flux_temp)估计观测光谱(wave_obs, flux_obs)的视向速度及其误差**
  - **SLAM: 运行代码，找出流程错误，并改正（小错误，改正不会超过10行代码）**
- **Think about**
  - How to estimate vsini?
  - How to estimate more elemental abundances?
  - Can multi-band photometry / parallax help?
- **Q&A…**