

Optical stellar spectroscopy

II. Deriving Stellar Parameters

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Introduce myself

- Post-doc @BNU
- Ph.D. @NAOC
- Interests:
 - Stellar parameters
 - Spectroscopic binaries
 - Application of ML in spectroscopy
- Related Projects
 - LAMOST ($R \sim 2,000/7,500$)
 - SONG ($36,000 < R < 180,000$)
- Other data
 - MMT Hectospec
 - 2.16m HRS/BFOSC (Xinglong)
 - 2.4m HRS (Lijiang)
 - SDSS
 - SDSS/MaGNA
 - SDSS/APOGEE

Contents

1. Warm-up – some background knowledge about stellar spectra

2. Doppler effect – measuring stellar radial velocity

3. Teff, logg & [M/H] – fundamental stellar atmospheric parameters

4. $v \sin i$ profile*

5. Basics

1. Linux + Anaconda3 + Jupyter-notebook/Jupyter-Lab

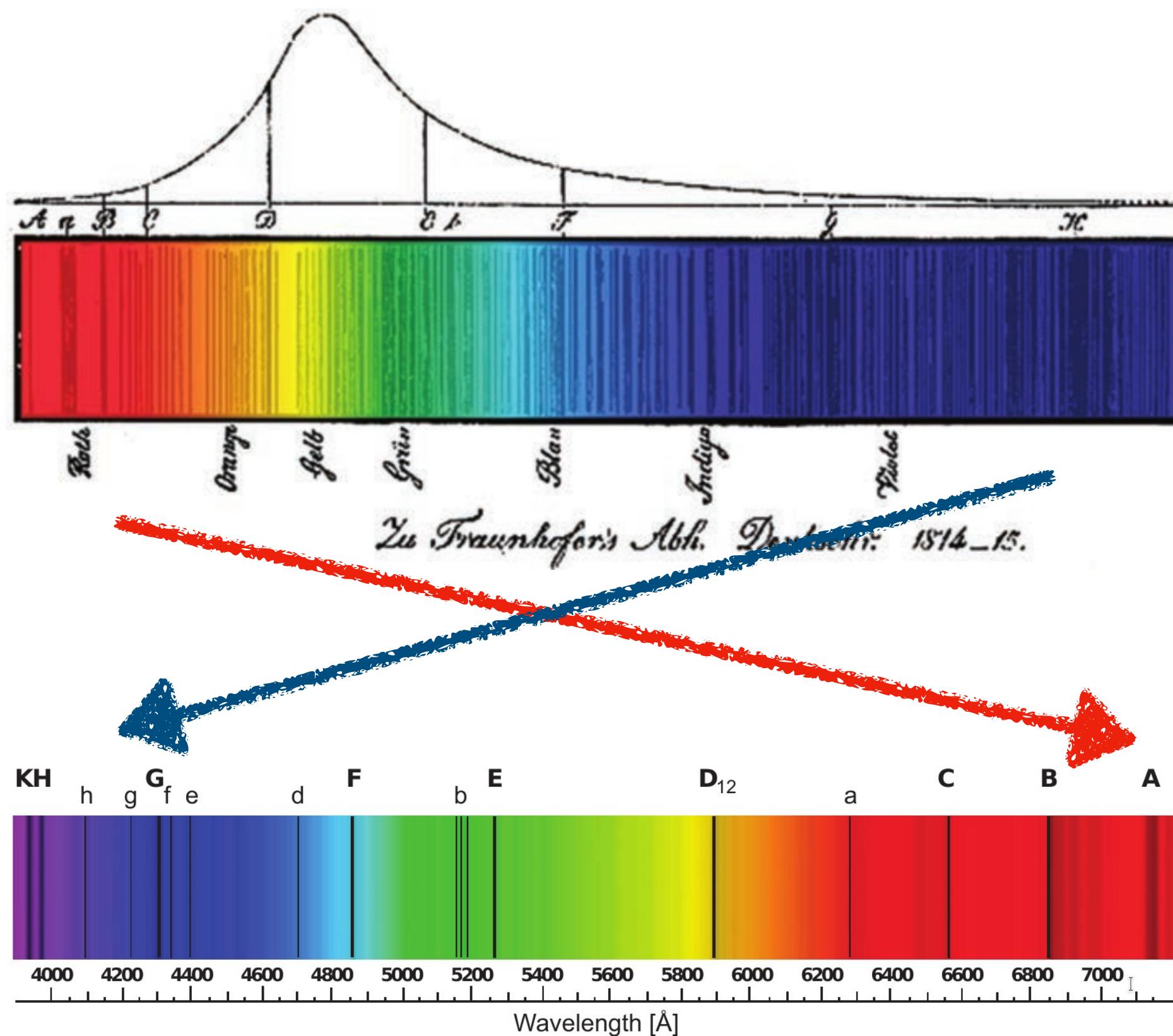
2. Numerical computing (cross-correlation, non-linear regression,...)

6. Link:

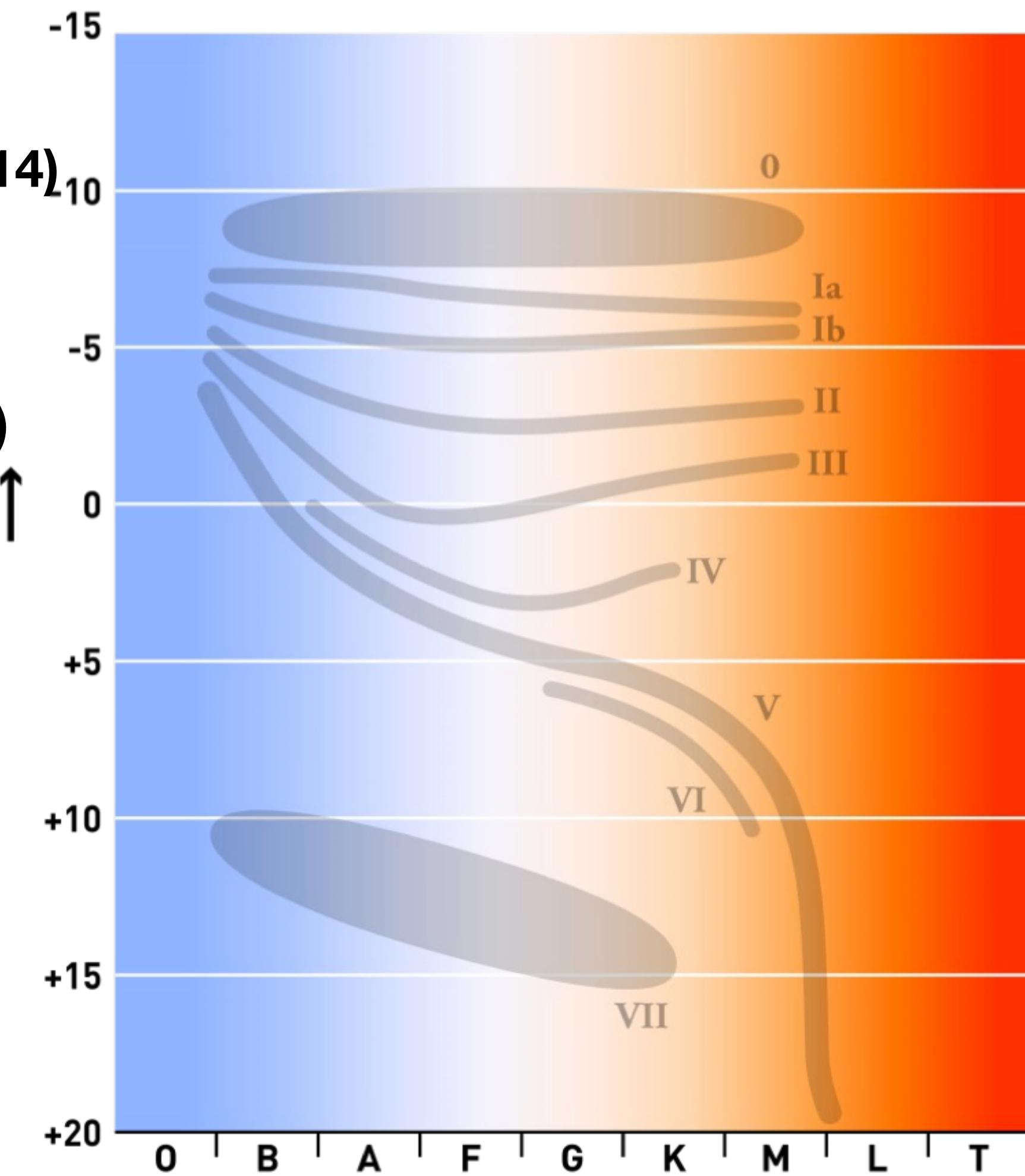
1. <https://github.com/hypergravity/spectroscopy>

1.0 History of stellar spectroscopy

Stellar spectroscopy over 200 years

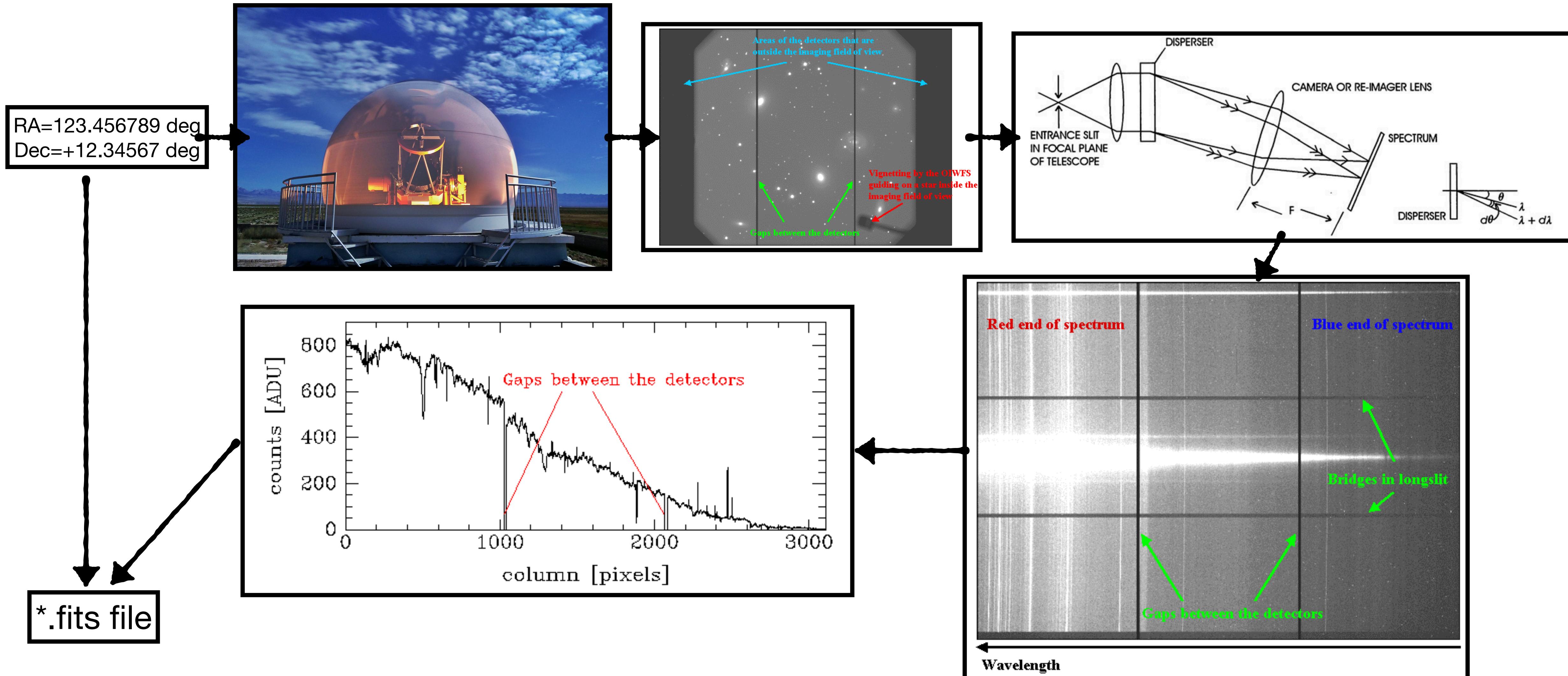


- **Joseph von Fraunhofer (1814)**
 - Solar spectrum
- **Annie Jump Cannon (1896-)**
 - Spectral type
 - OBAFGKM notation
- **Morgan & Keenan (~1943)**
 - Luminosity class
 - MK classification system



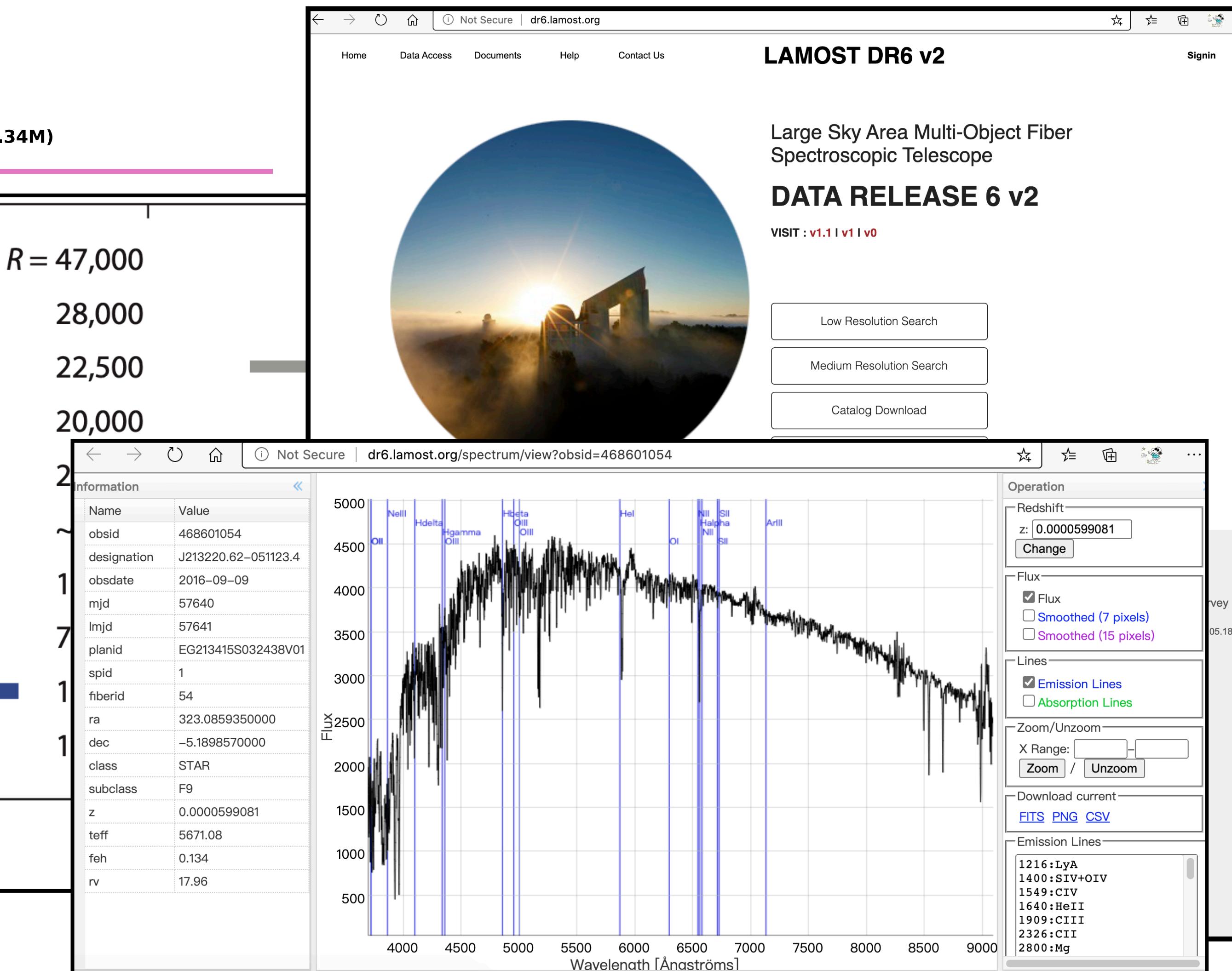
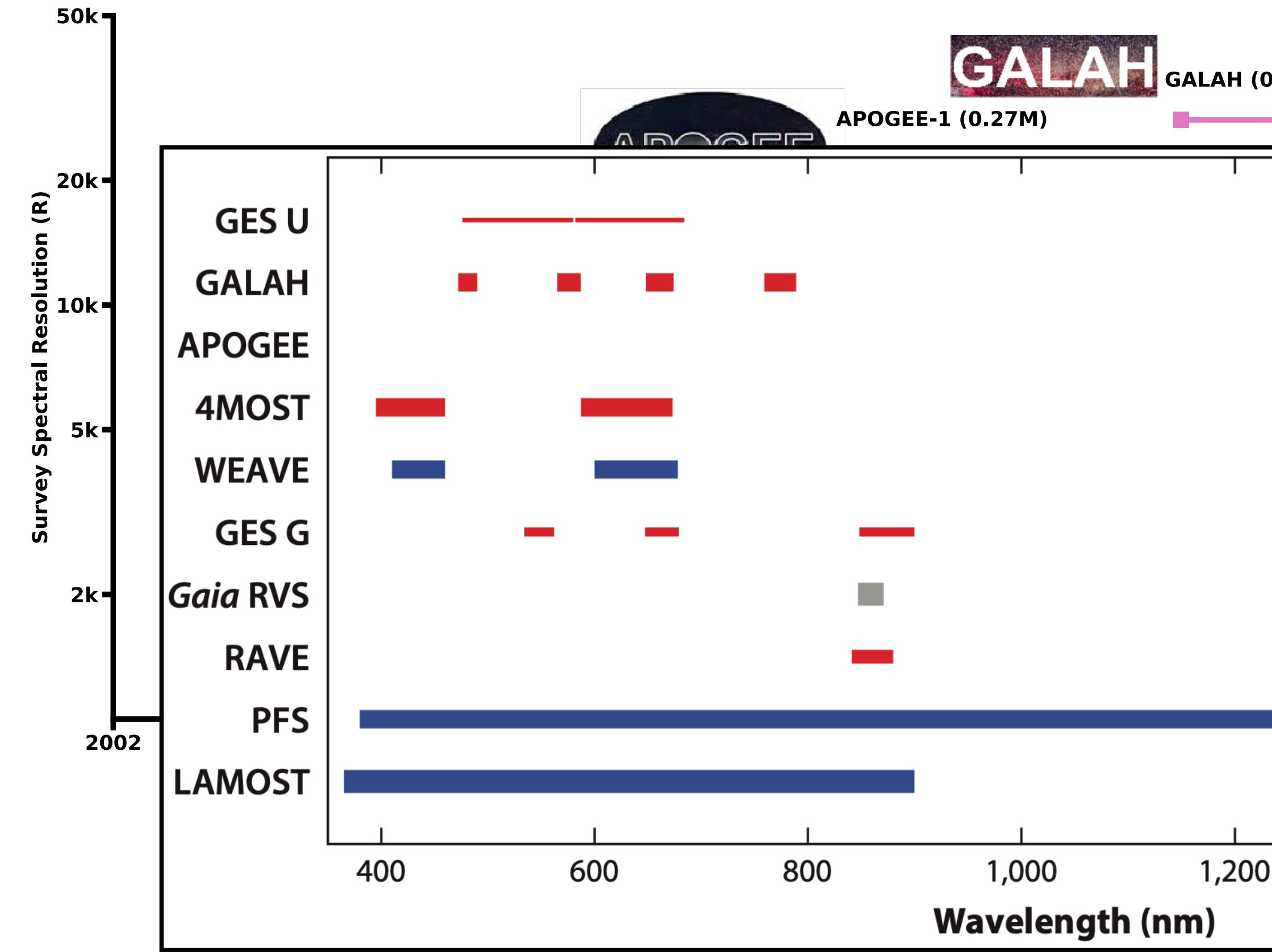
1.1 How to obtain stellar spectra?

A typical process of spectroscopic observation

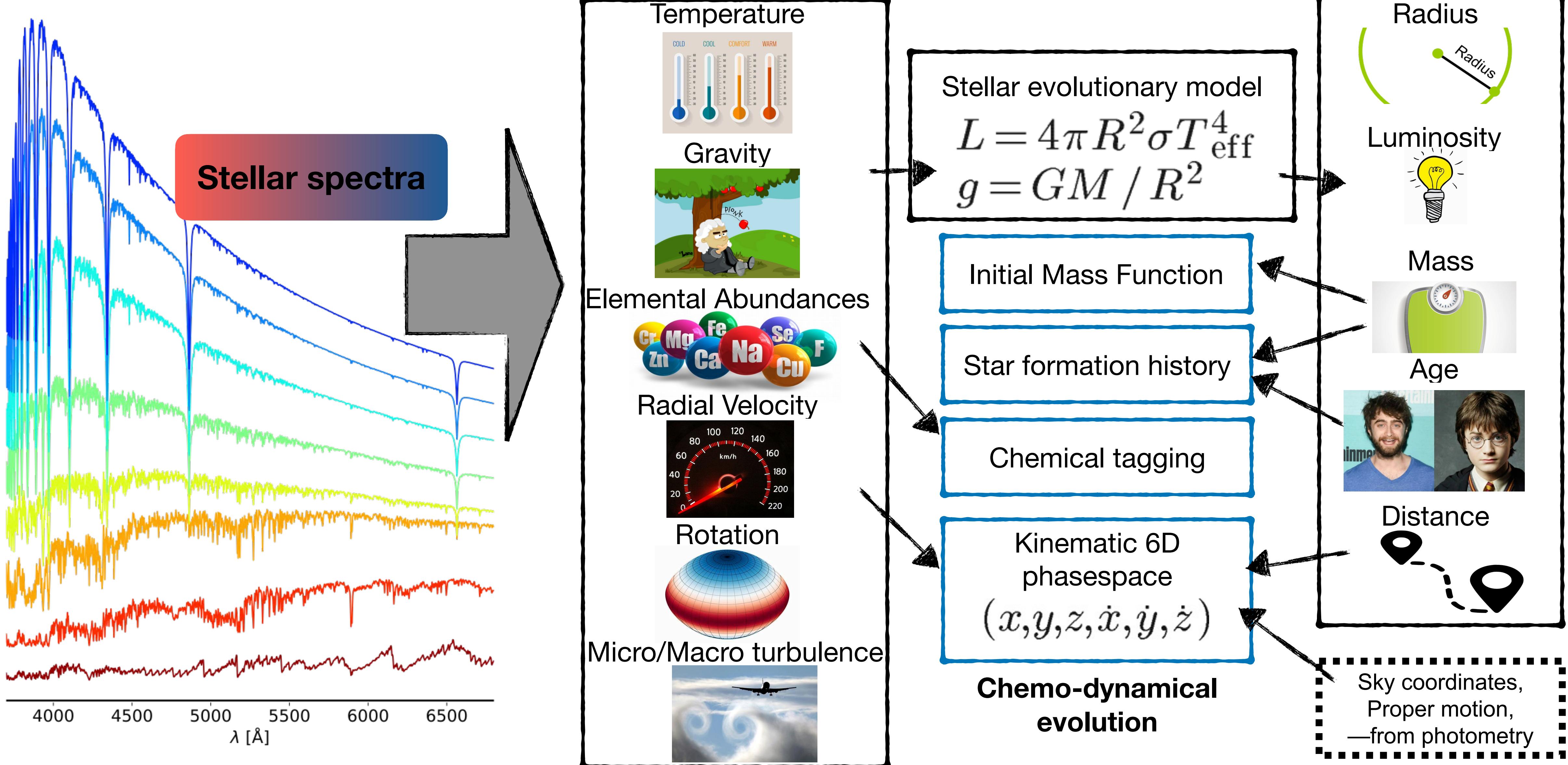


1.1 How to obtain stellar spectra?

Spectroscopic surveys, e.g., LAMOST, APOGEE, ...

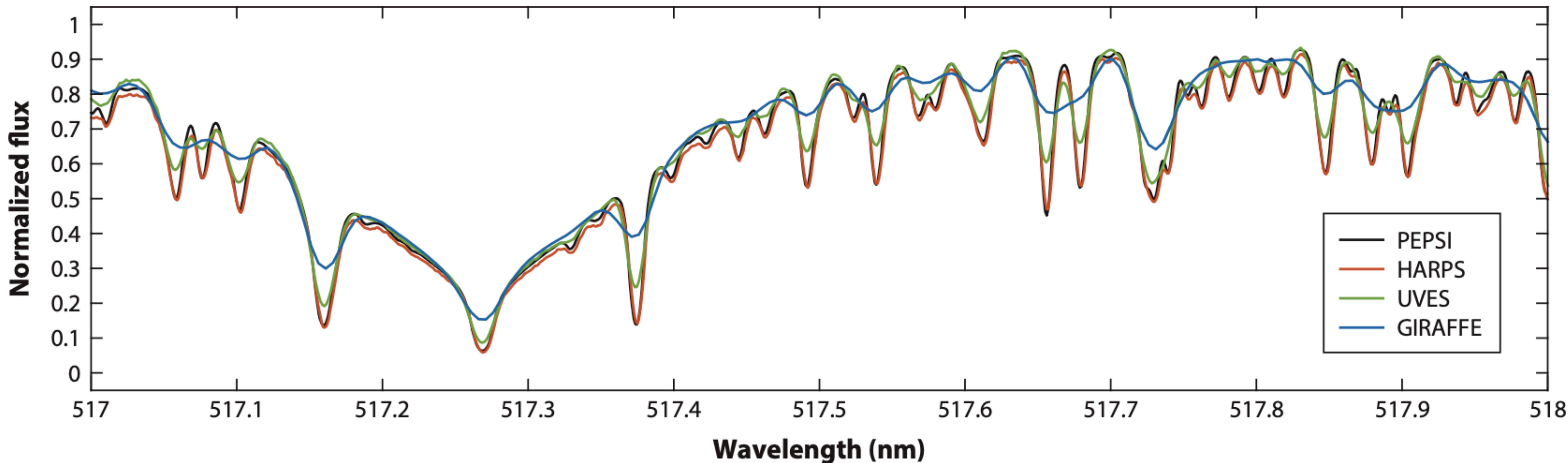


From stellar spectra to the Milky Way science



1.2 Spectral resolution R

Instrumental broadening / Line Spread Function (LSF)



Spectra for the *Gaia* benchmark star ϵ Eri near one of the Mg I b lines

1.2 Spectral resolution R

Solar spectrum at different R

laspec.qconv module

Teff=5800 logg=4.5

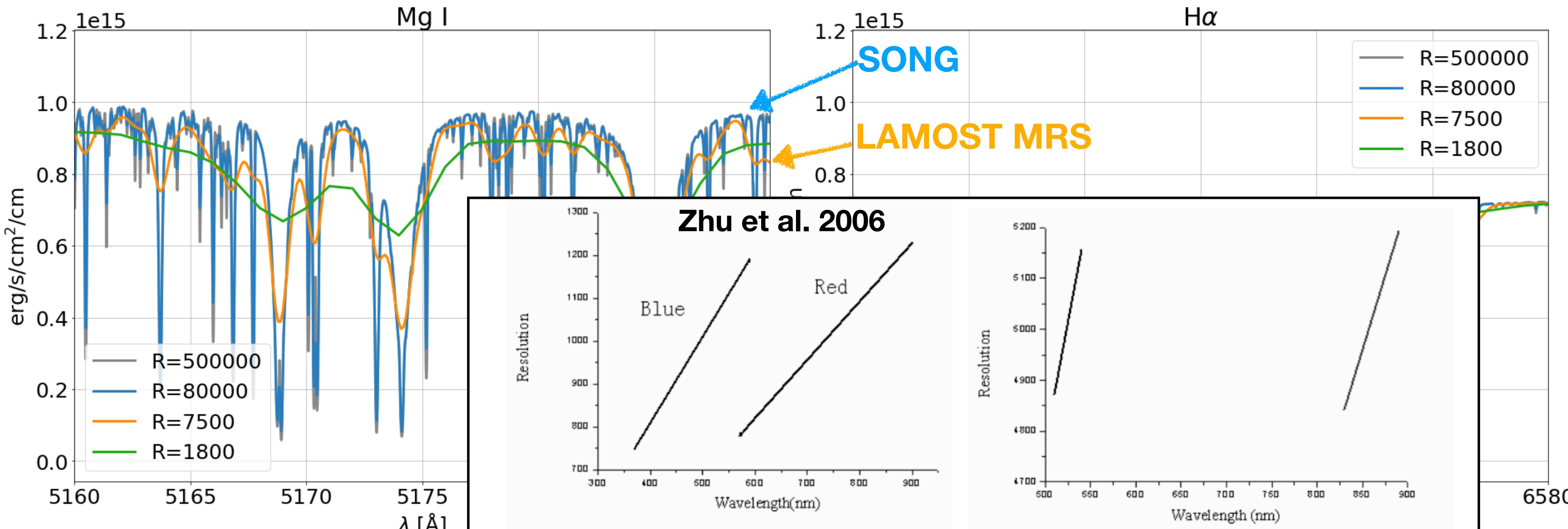
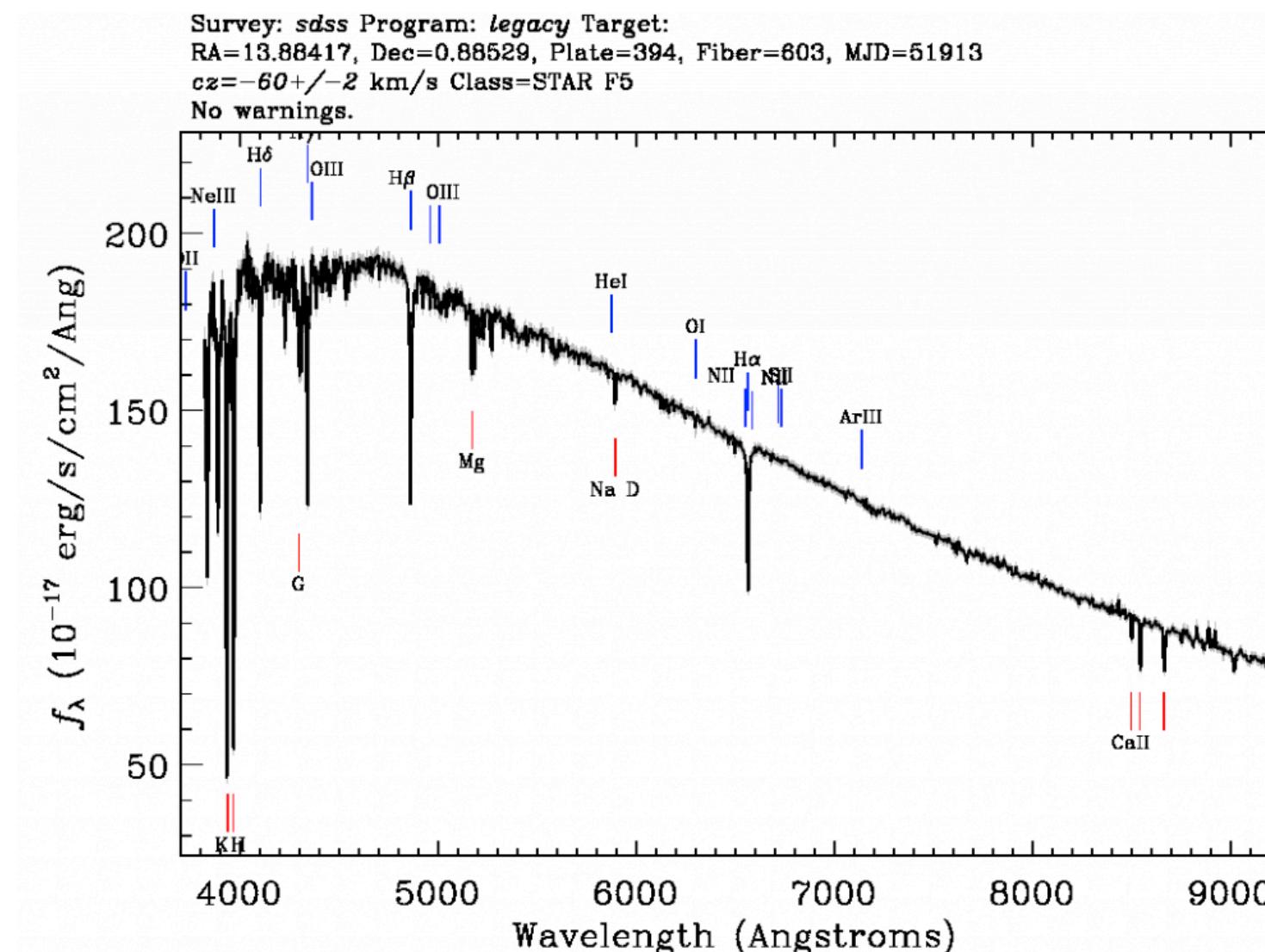


Fig. 2. Wavelength coverage with different resolution for LAMOST-LRS. Left figure shows the coverage of the low resolution configuration. Right figure shows the coverage of the medium resolution configuration. Resolution could be doubled by limiting the width of fiber slit.

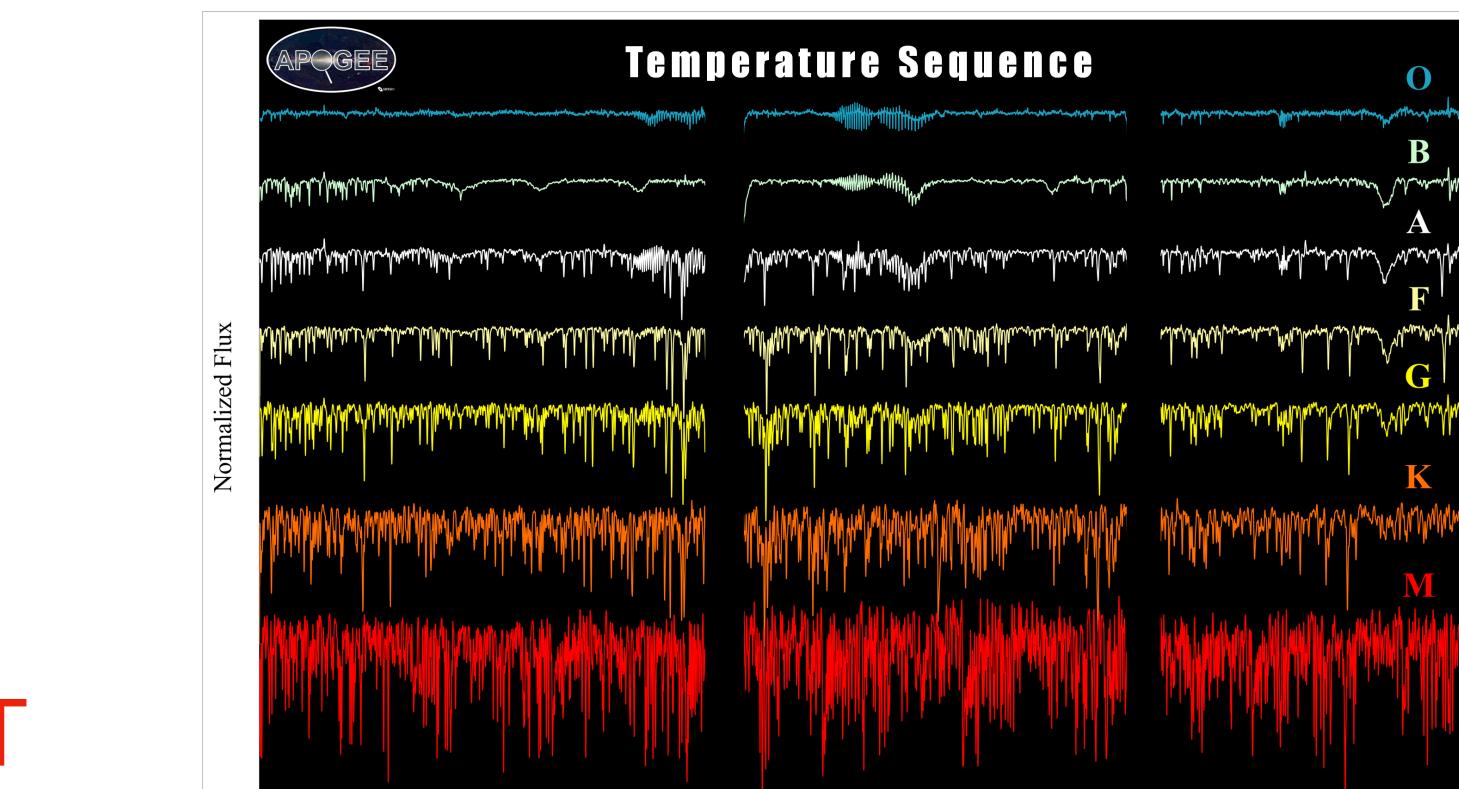
1.3 Atmospheric transmission



SDSS
LAMOST
GALAH
RAVE
Gaia-ESO
Gaia-RVS

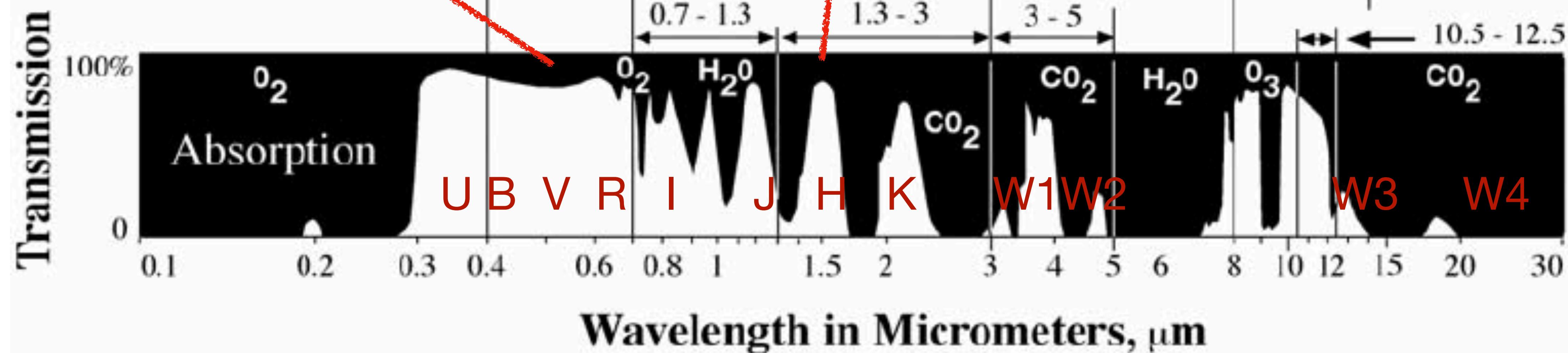
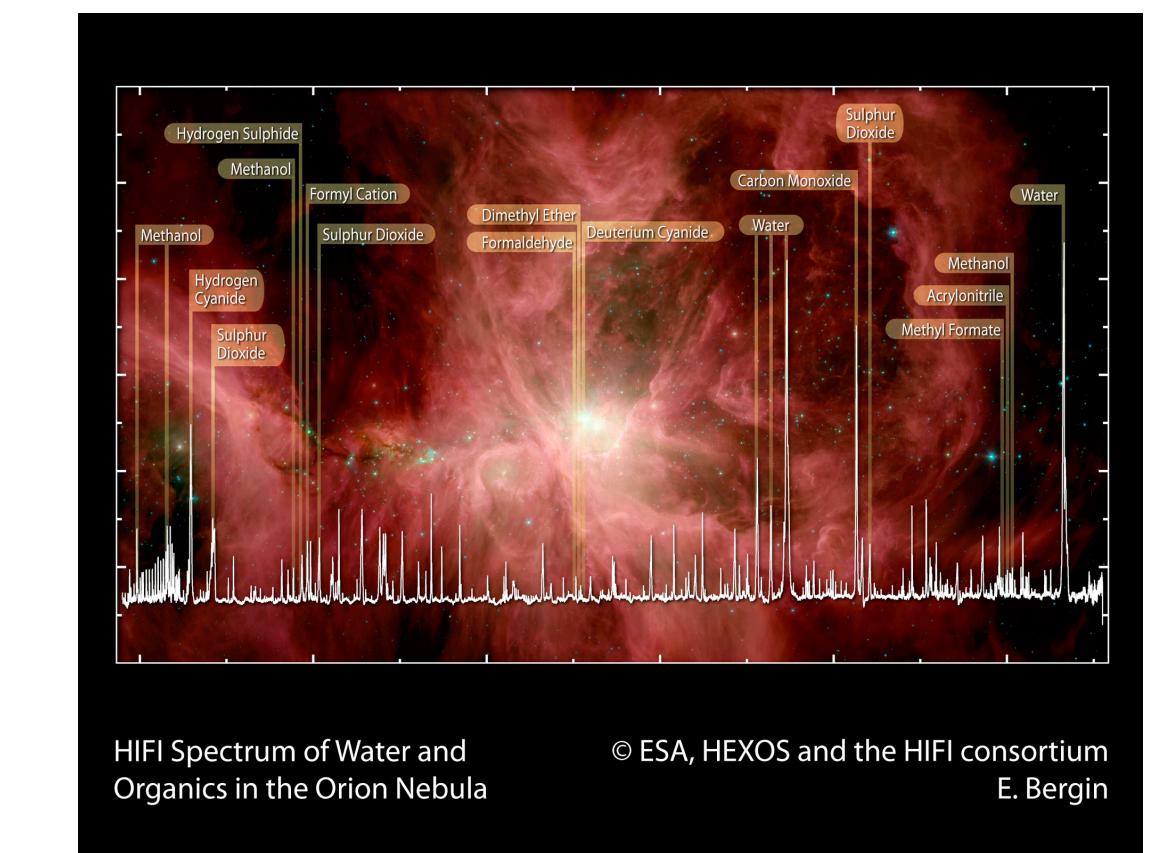
Reflective Infrared
0.7 - 3

Visible
0.4 - 0.7



Thermal Infrared
3 - 14

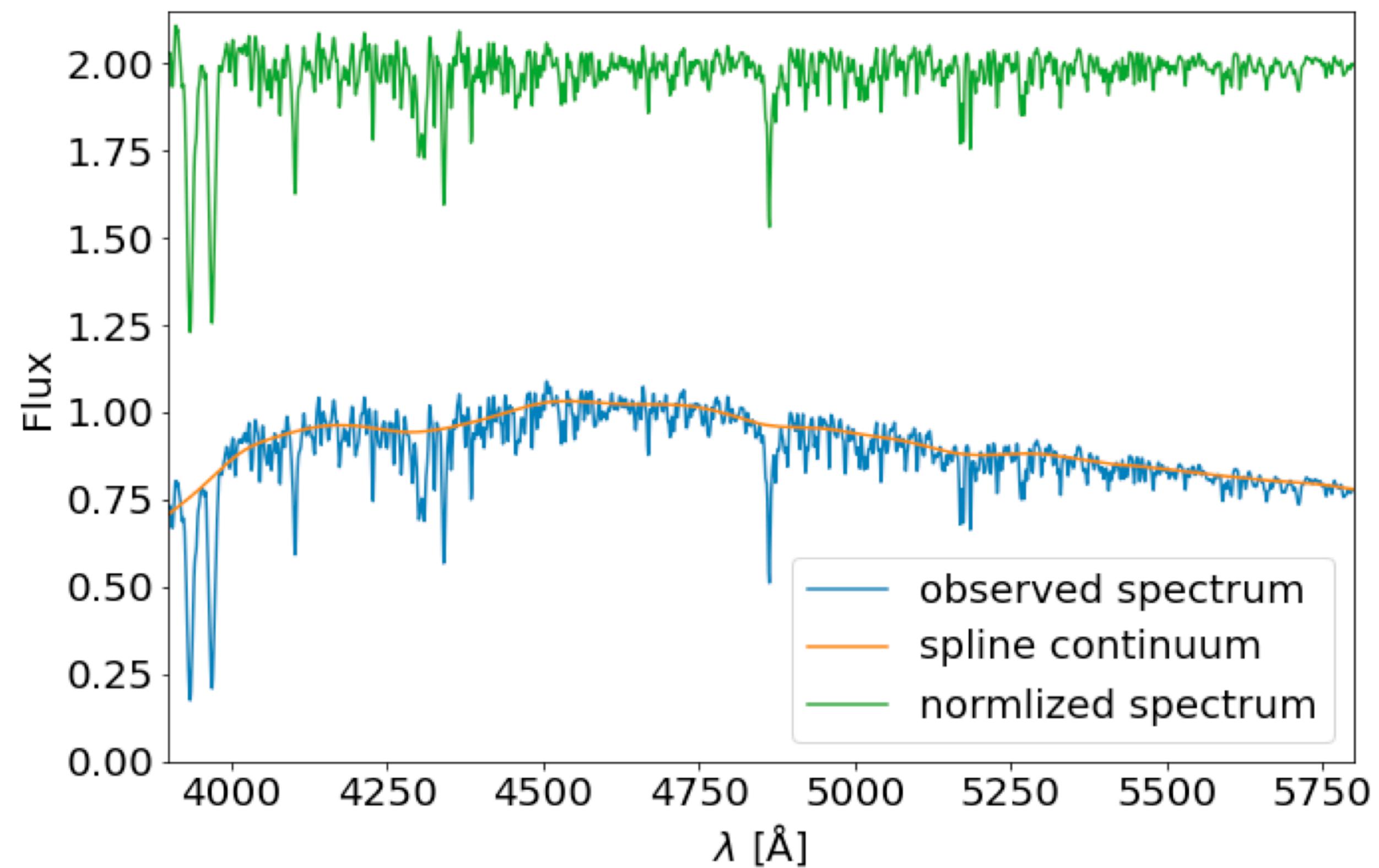
ISO
Sofia
Herschel/hifi



1.4 Pseudo-continuum normalization

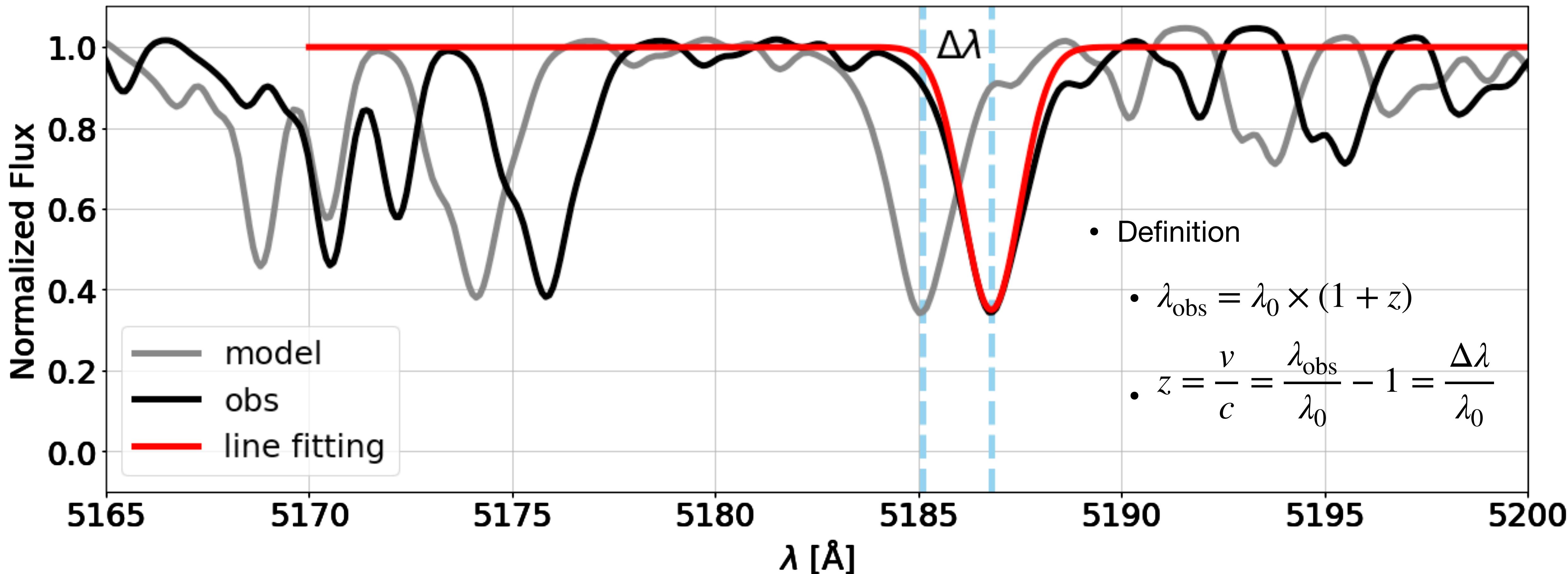
Place spectra at a standard scale

- Why?
 - Instrumental response curve
 - Interstellar extinction
 - Absolute flux calibration (>10%)
- `laspec.normalization` module
- An important source of uncertainty



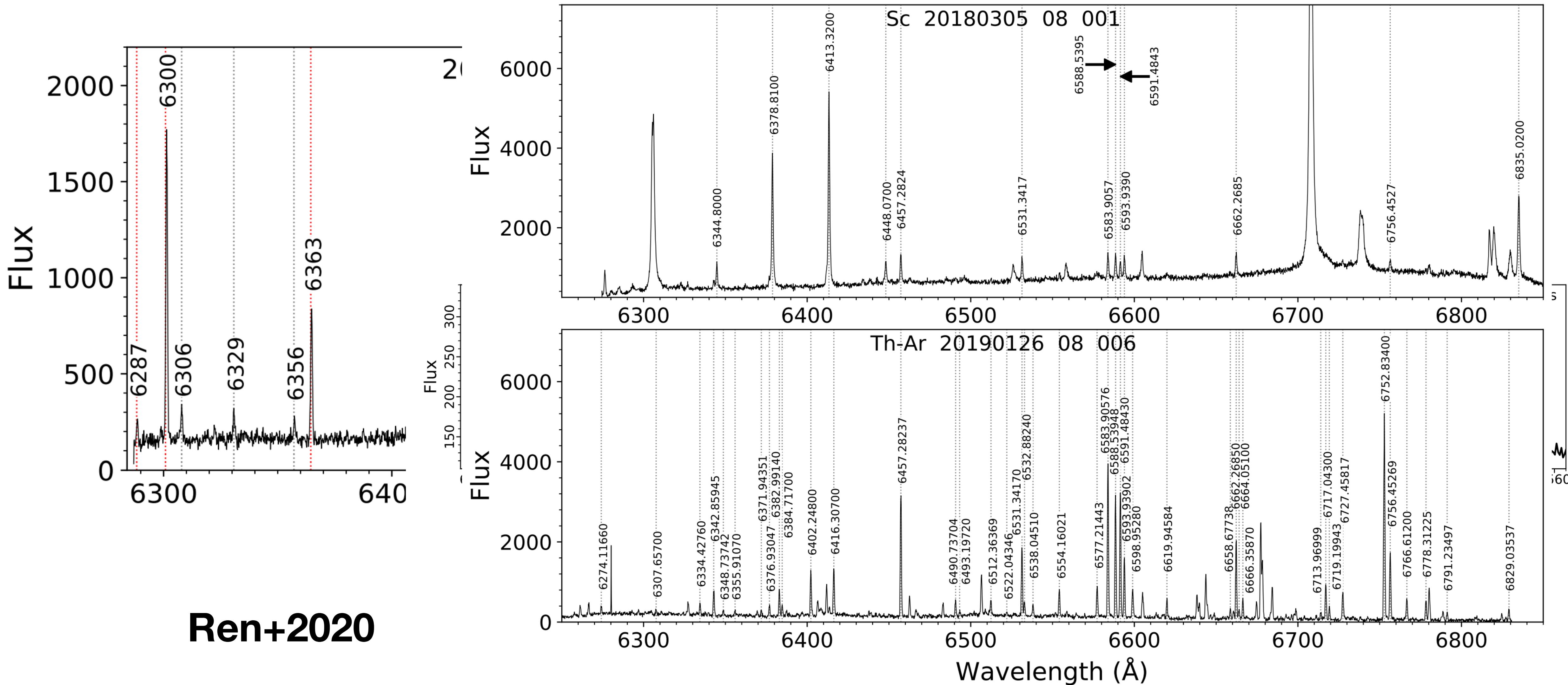
2. How to measure Doppler effect / Radial Velocity

Doppler effect & Spectral line fitting



2. How to measure Doppler effect / Radial Velocity

Doppler effect & Spectral line fitting



2. How to measure Doppler effect / Radial Velocity

Cross-correlation function (CCF, Tonry & Davis 1979)

- Find a proper template
 - (Teff, logg, [Fe/H],...)

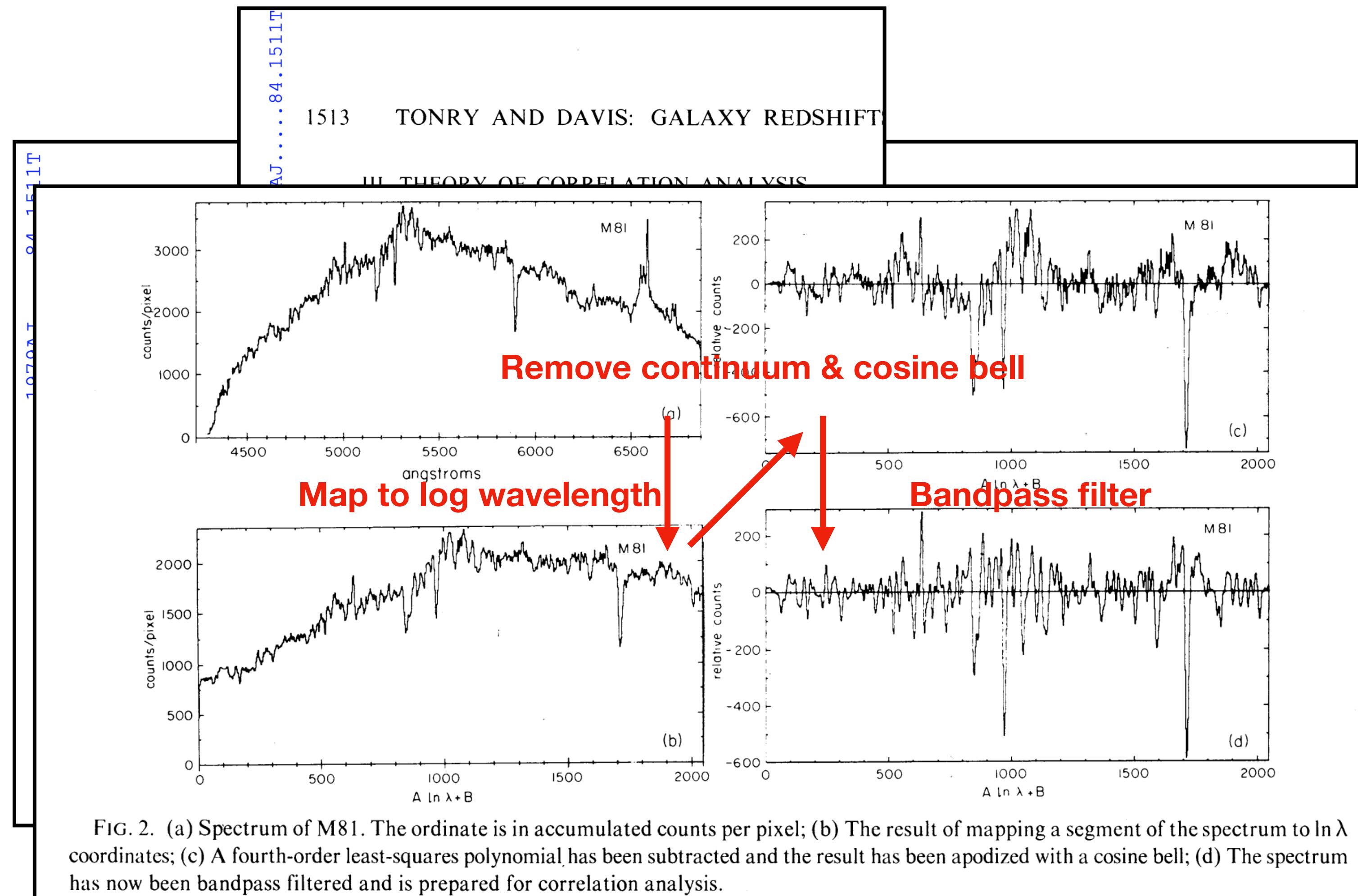
- Cross correlation function
 - Why CCF?

1. Remove continuum

2. Resample to log wavelength

3. Eval CCF using FFT*

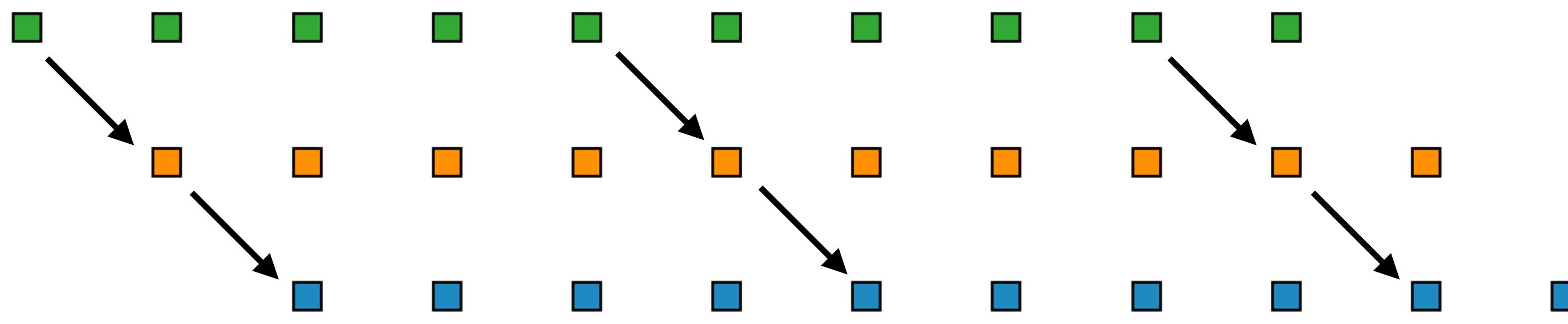
4. Estimate RV



2. How to measure Doppler effect / Radial Velocity

Cross-correlation function (CCF, Tonry & Davis 1979)

- Why log wavelength $n = A \ln \lambda + B$



$$\begin{aligned}\Delta n &= A \Delta \ln \lambda \\ &= A \frac{\Delta \lambda}{\lambda} \\ &= Az \\ &= A \frac{v}{c}\end{aligned}$$

1979AJ.....84.1511T

1513 TONRY AND DAVIS: GALAXY REDSHIFT

III. THEORY OF CORRELATION ANALYSIS
a) *Introduction*

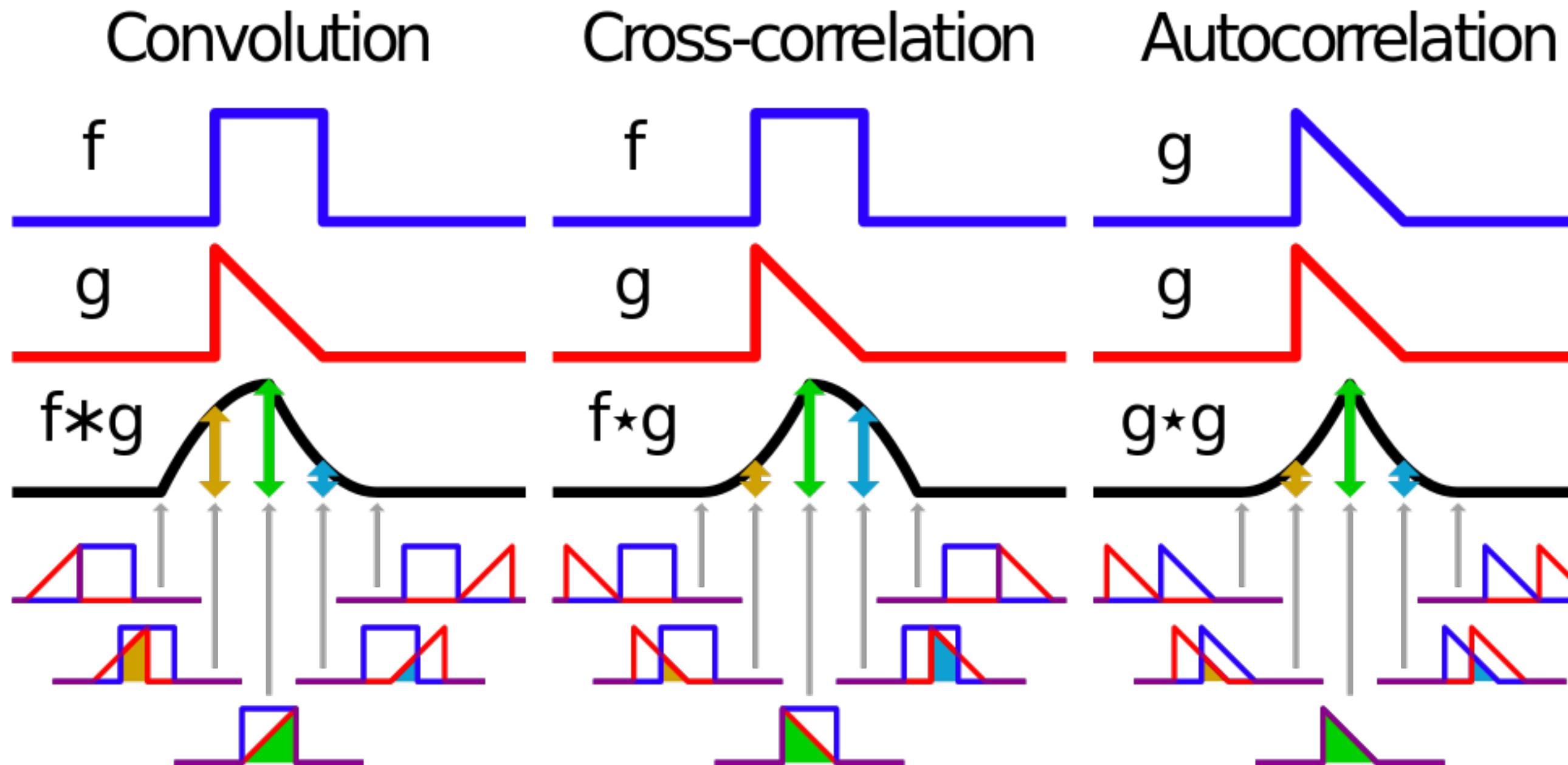
Let $g(n)$ be the spectrum of a galaxy whose redshift and velocity dispersion are to be found and let $t(n)$ be a template spectrum of zero redshift and instrumentally broadened stellar-line profiles. These spectra are discretely sampled into N bins, labelled by bin number n ; the relationship between wavelength and bin number is

$$n = A \ln \lambda + B \quad (1)$$

Because the spectra are binned linearly with $\ln \lambda$ a velocity redshift is a uniform linear shift. The spectra are assumed periodic with period N for the purposes of discrete Fourier transforms and correlation functions derived herefrom. In addition, the spectra are continuum subtracted, endmasked, and filtered to remove low-frequency spectral variations which arise from continuum variations and non-uniform photocathode sensitivity, and high-frequency noise components beyond the resolution. The endmasking assures that mismatch between the ends of the spectrum is removed.

2. How to measure Doppler effect / Radial Velocity

Cross-correlation function (a general scheme)



- Mean
- Variance
- Covariance
- Cross-correlation

$$\bar{\mathbf{X}} = \frac{1}{N} \sum_i X_i$$

$$\text{Var}(\mathbf{X}) = \frac{1}{N} \sum_i (X_i - \bar{\mathbf{X}})^2$$

$$\text{Cov}(\mathbf{X}, \mathbf{Y}) = \frac{1}{N} \sum_i (X_i - \bar{\mathbf{X}})(Y_i - \bar{\mathbf{Y}})$$

$$\text{CCF}(v|F, G) = \frac{\text{Cov}(F, G(v))}{\sqrt{\text{Var}(F)\text{Var}(G(v))}}$$

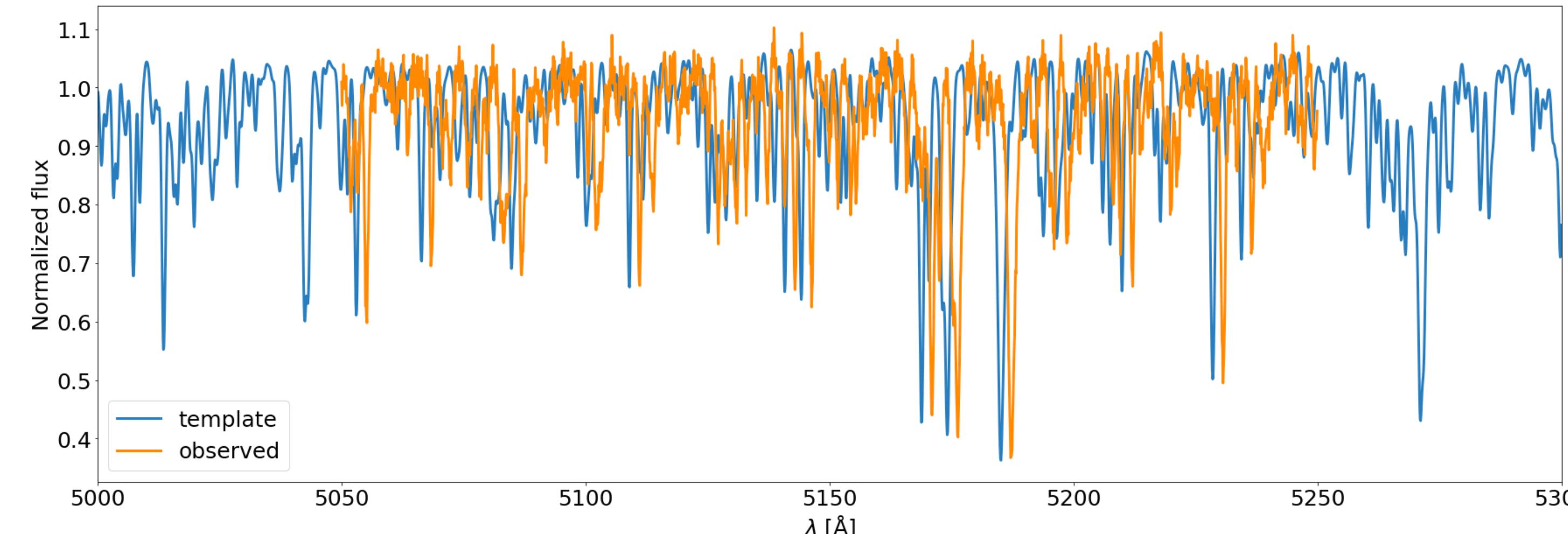
$$\text{CCF}(\tau) = \int_{-\infty}^{+\infty} f(t)g(\tau + t)dt$$

$$\text{CONV}(\tau) = \int_{-\infty}^{+\infty} f(t)g(\tau - t)dt$$

Linear Correlation coefficient

2. How to measure Doppler effect / Radial Velocity

Cross-correlation function (a general scheme)

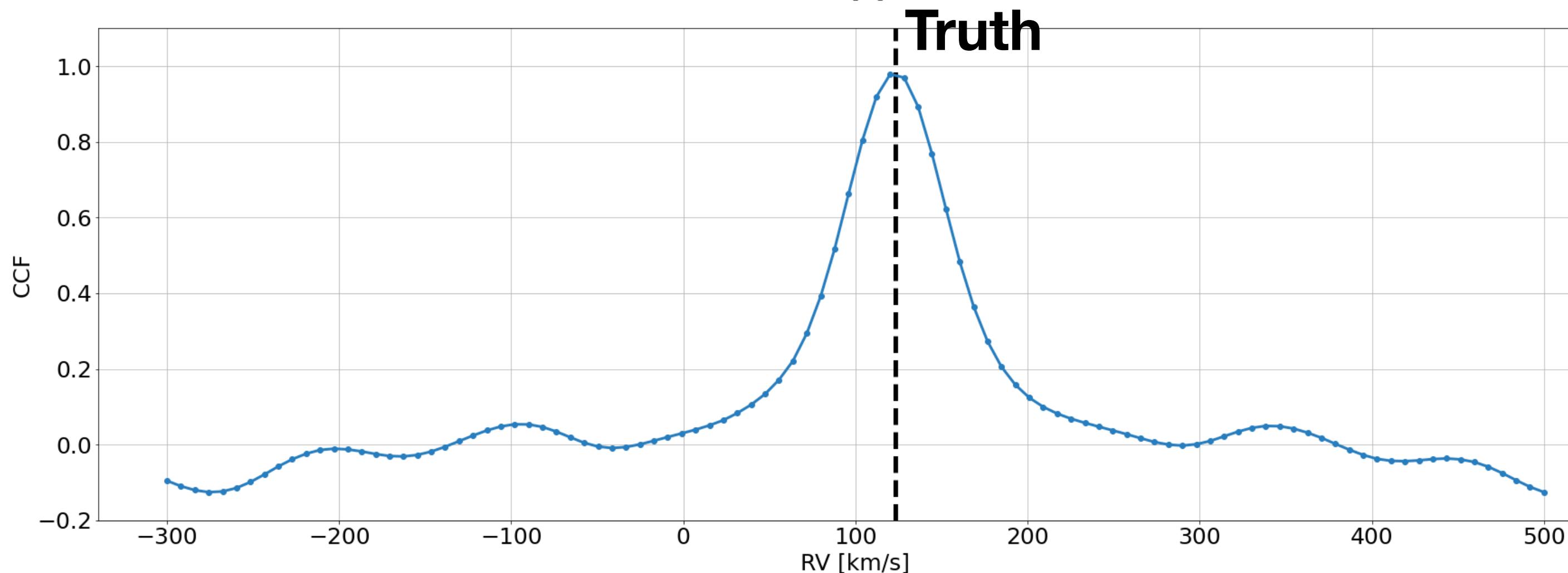


- Mean
- Variance
- Covariance
- Cross-correlation

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$$\text{Cov}(\mathbf{X}, \mathbf{Y}) = \frac{1}{N} \sum_i (X_i - \bar{\mathbf{X}})(Y_i - \bar{\mathbf{Y}})$$

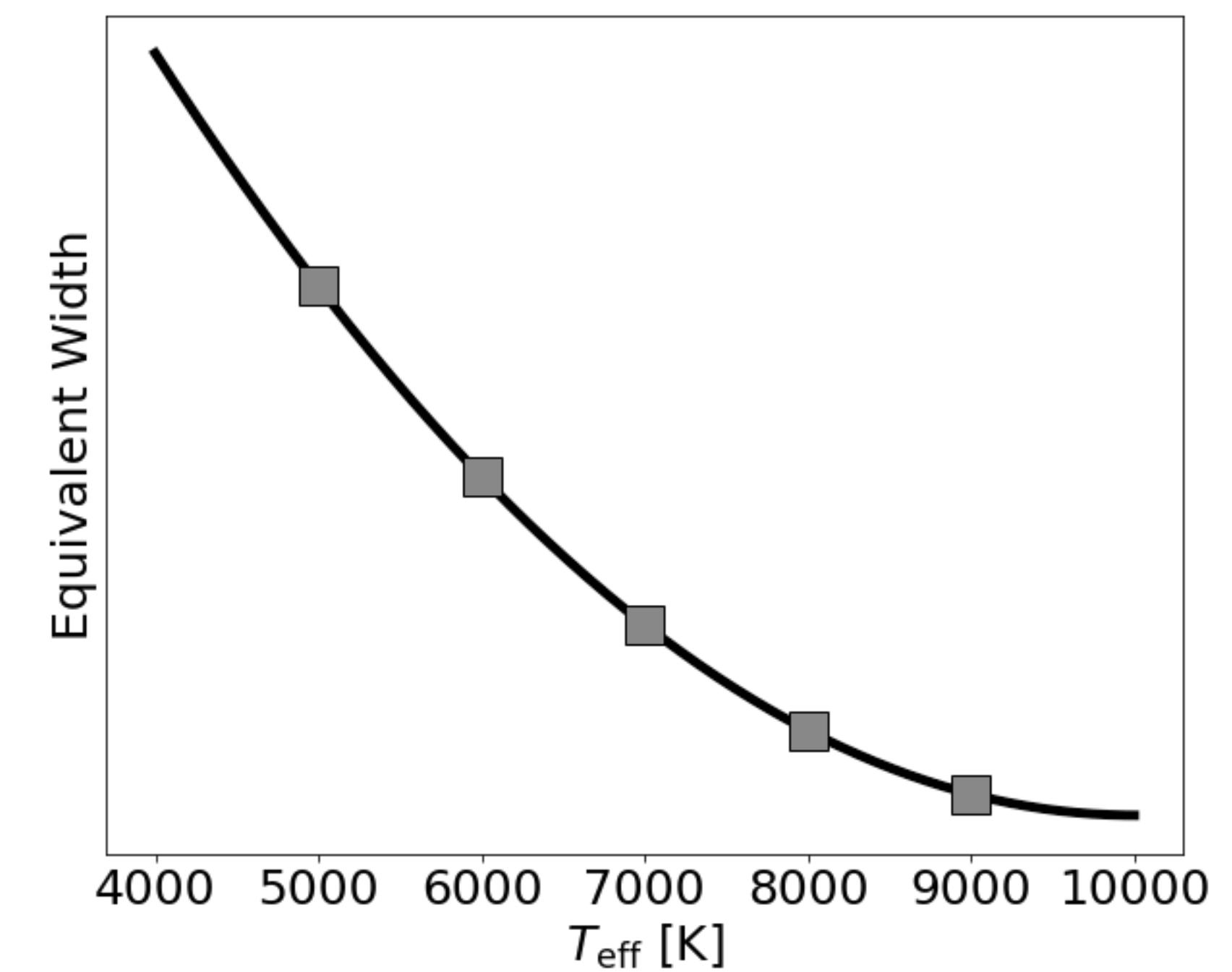
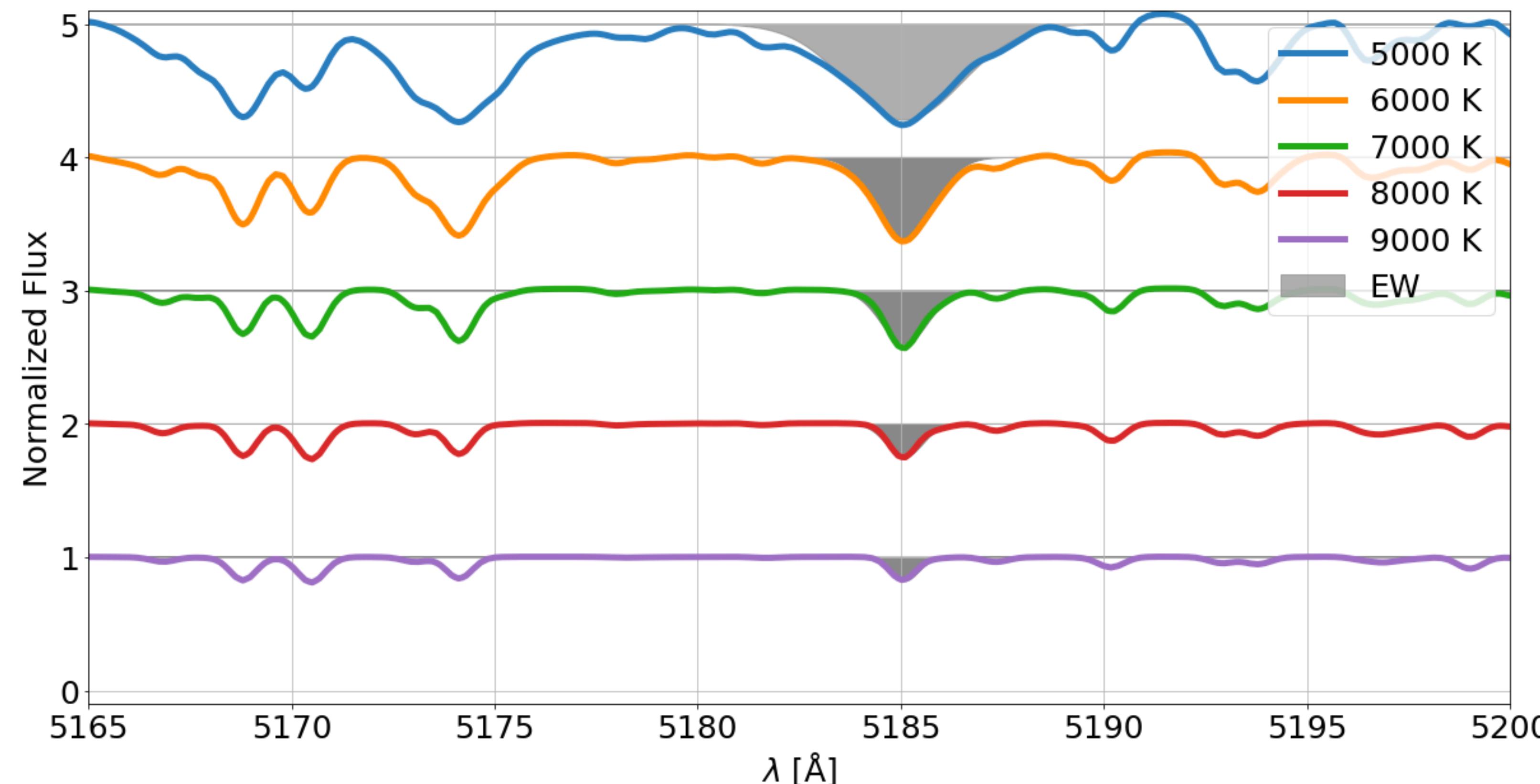


$$\text{CCF}(v | \mathbf{F}, \mathbf{G}) = \frac{\text{Cov}(\mathbf{F}, \mathbf{G}(v))}{\sqrt{\text{Var}(\mathbf{F})\text{Var}(\mathbf{G}(v))}}$$

Linear Correlation coefficient

3. How to derive Teff, logg & [M/H] from spectra

Equivalent widths of spectral lines



$$EW = \int_{\lambda_1}^{\lambda_2} \frac{F_{\lambda, \text{cont}} - F_{\lambda, \text{obs}}}{F_{\lambda, \text{cont}}} d\lambda$$

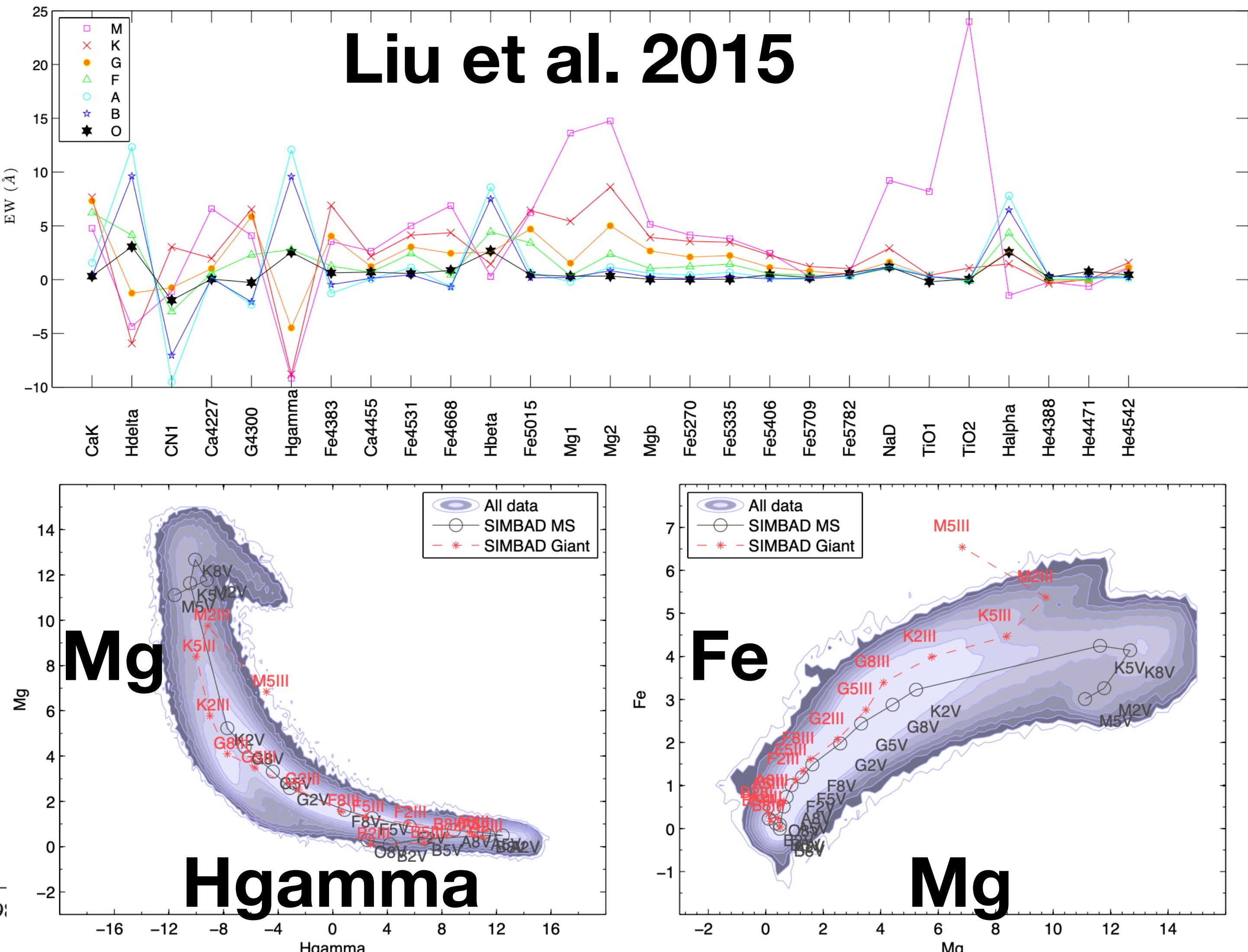
3. How to derive Teff, logg & [M/H] from spectra

Equivalent widths of spectral lines

Table 2 Line Index Definitions

Name	Index Bandpass (Å)	Pseudocontinua (Å)
CaII K ^a	3927.7–3939.7	3903–3923 4000–4020
H δ ^b	4083.50–4122.25	4041.60–4079.75 4128.50–4161.00
CN ^c	4143.375–4178.375	4081.375–4118.875 4245.375–4285.375
Ca4227 ^c	4223.500–4236.000	4212.250–4221.000 4242.250–4252.250
G4300 ^c	4282.625–4317.625	4267.625–4283.875 4320.125–4336.375
H γ ^b	4319.75–4363.50	4283.50–4319.75 4367.25–4419.75
Fe4383 ^c	4370.375–4421.625	4360.375–4371.625 4444.125–4456.625
He4388	4381–4399	4365–4380 4398–4408
Ca4455 ^c	4453.375–4475.875	4447.125–4455.875 4478.375–4493.375
He4471	4462–4475	4450–4463 4485–4495
Fe4531 ^c	4515.500–4560.500	4505.500–4515.500 4561.750–4580.500
He4542	4536–4548	4526–4536 4548–4558
Fe4668 ^c	4635.250–4721.500	4612.750–4631.500 4744.000–4757.750
H β ^b	4847.875–4876.625	4827.875–4847.875 4876.625–4891.625
Fe5015 ^c	4977.750–5054.000	4946.500–4977.750 5054.000–5065.250
Mg ₁ ^c	5069.125–5134.125	4895.125–4957.625 5301.125–5366.125
Mg ₂ ^c	5154.125–5196.625	4895.125–4957.625 5301.125–5366.125
Mg _b ^c	5160.125–5192.625	5142.625–5161.375 5191.375–5206.375
Fe5270 ^c	5245.650–5285.650	5233.150–5248.150 5285.650–5318.150
Fe5335 ^c	5312.125–5352.125	5304.625–5315.875 5353.375–5363.375
Fe5406 ^c	5387.500–5415.000	5376.250–5387.500 5415.000–5425.000
Fe5709 ^c	5698.375–5722.125	5674.625–5698.375 5724.625–5738.375
Fe5782 ^c	5778.375–5798.375	5767.125–5777.125 5799.625–5813.375
NaD ^c	5878.625–5911.125	5862.375–5877.375 5923.875–5949.875
TiO ₁ ^c	5938.375–5995.875	5818.375–5850.875 6040.375–6105.375
TiO ₂ ^c	6191.375–6273.875	6068.375–6143.375 6374.375–6416.875
H α ^d	6548.00–6578.00	6420.00–6455.00 6600.00–6640.00

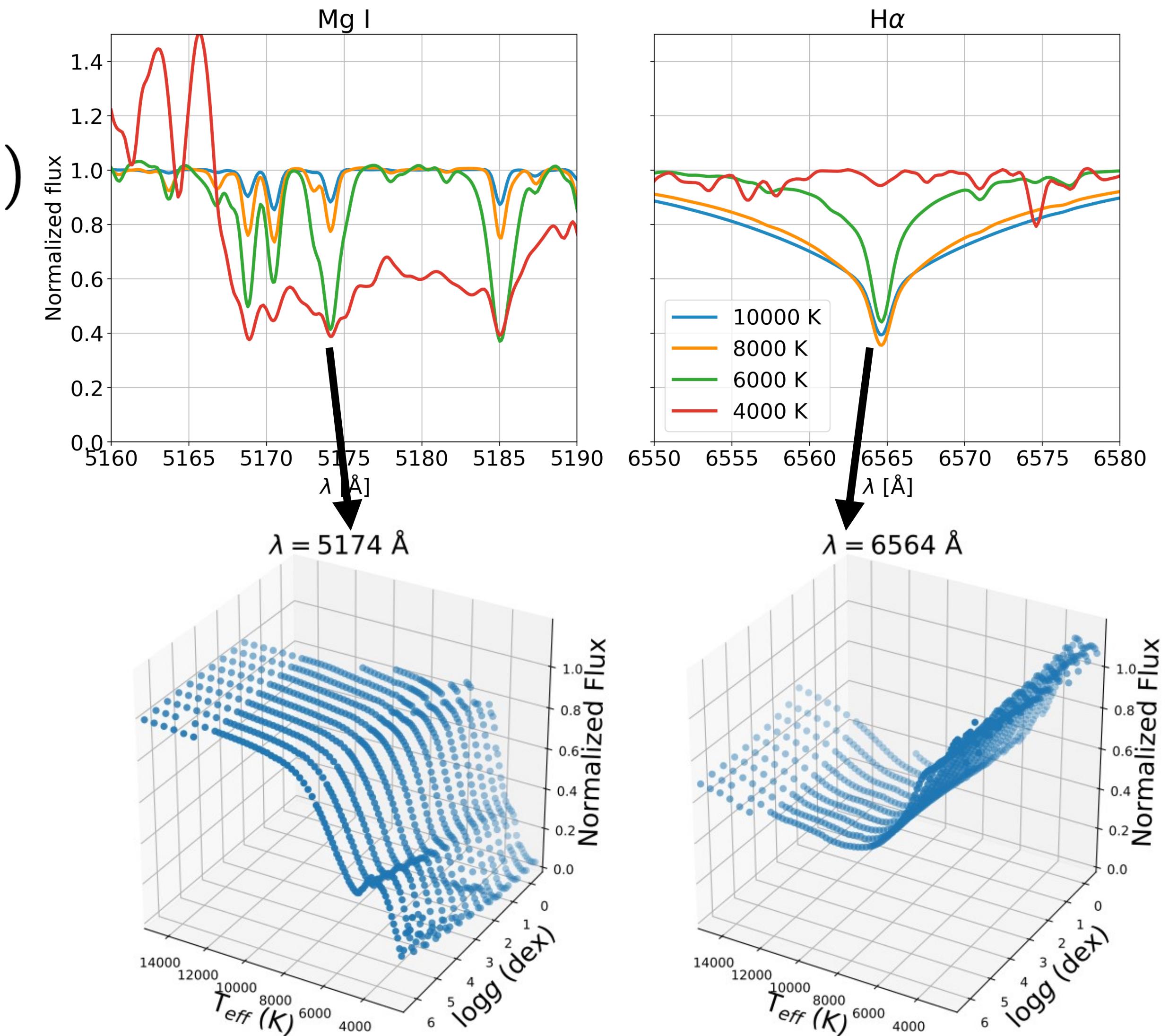
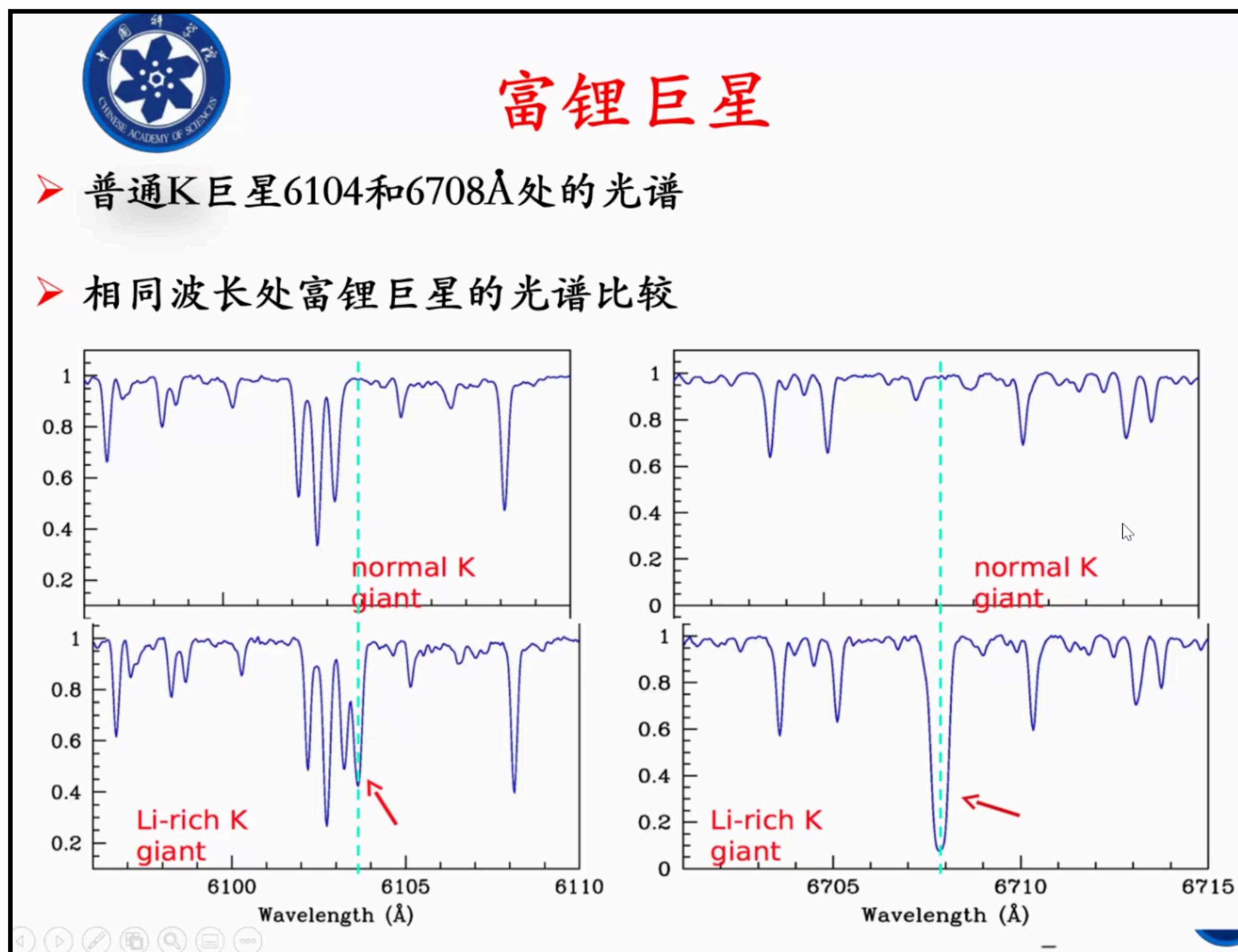
Notes: ^a Beers et al. (1999); ^b Worley & Ottaviani (1997); ^c Worley et al. (1994); ^d Cohen et al. (1999)



3. How to derive Teff, log g & [M/H] from spectra

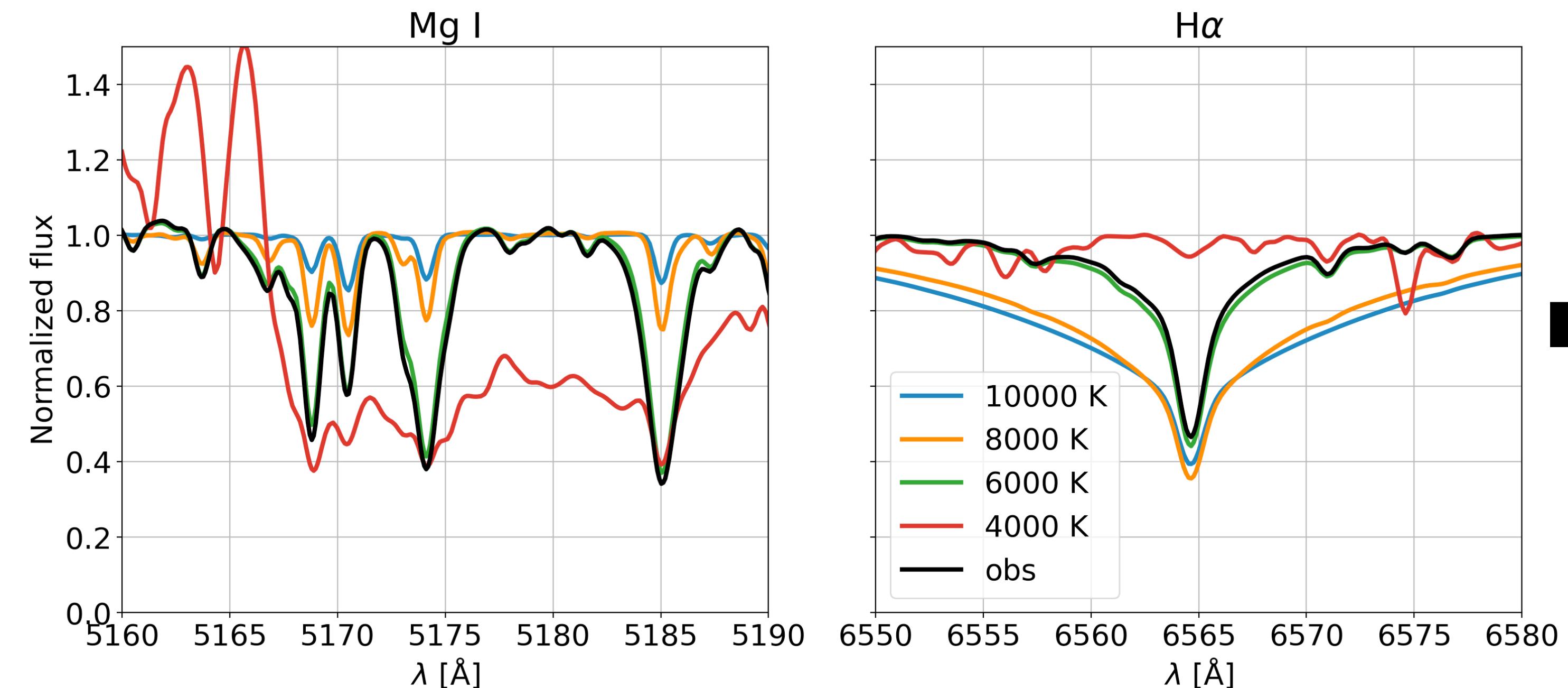
Spectra as functions of Stellar parameters

$$F_\lambda = f_\lambda(T_{\text{eff}}, \log g, [\text{X}/\text{H}], v \sin i, v_{\text{mac}}, v_{\text{mic}})$$



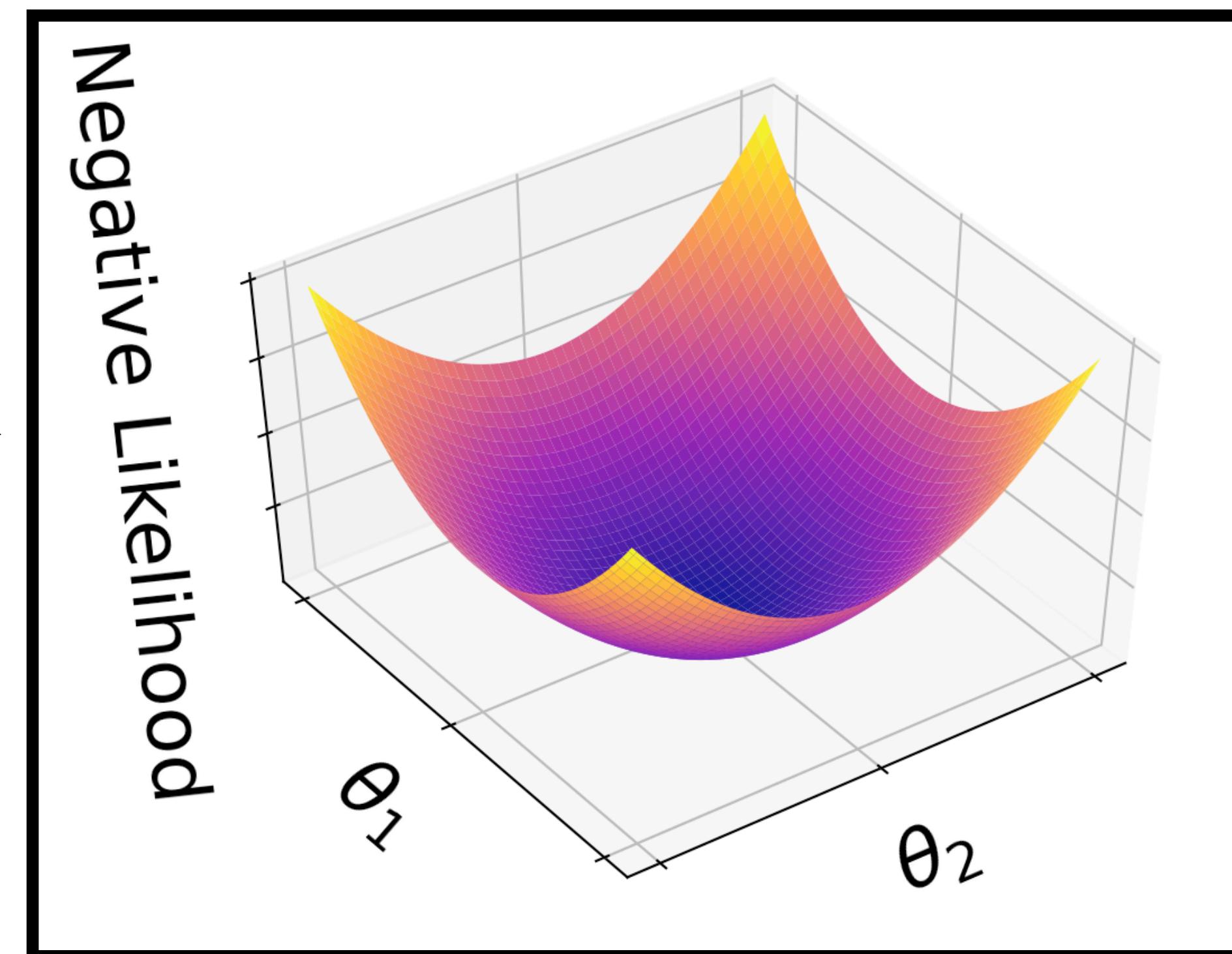
3. How to derive Teff, logg & [M/H] from spectra

Spectra as functions of Stellar parameters



$$F_\lambda = f_\lambda(T_{\text{eff}}, \log g, [\text{X}/\text{H}], v \sin i, v_{\text{mac}}, v_{\text{mic}})$$

Define a likelihood, find theta by maximizing it:

$$\ln p(\boldsymbol{\theta} | f_{\text{obs}}) = -\frac{1}{2} \sum_{j=1}^n \left\{ \frac{[f_{j,\text{obs}} - f_j(\boldsymbol{\theta})]^2}{\sigma_{j,\text{obs}}^2 + \sigma_j(\boldsymbol{\theta})^2} + \ln [2\pi(\sigma_{j,\text{obs}}^2 + \sigma_j(\boldsymbol{\theta})^2)] \right\}$$


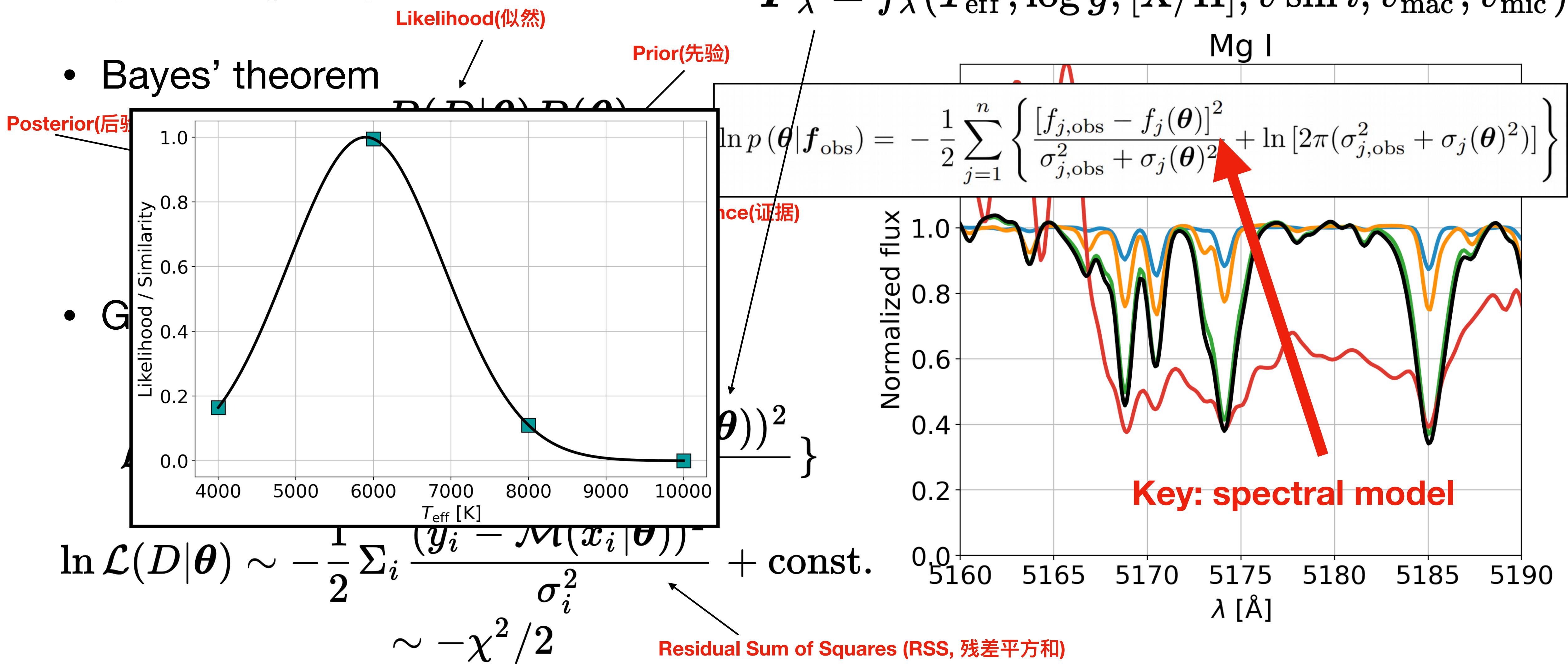
3. How to derive Teff, logg & [M/H] from spectra

Bayesian perspective

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

3. How to derive Teff, logg & [M/H] from spectra

Bayesian perspective

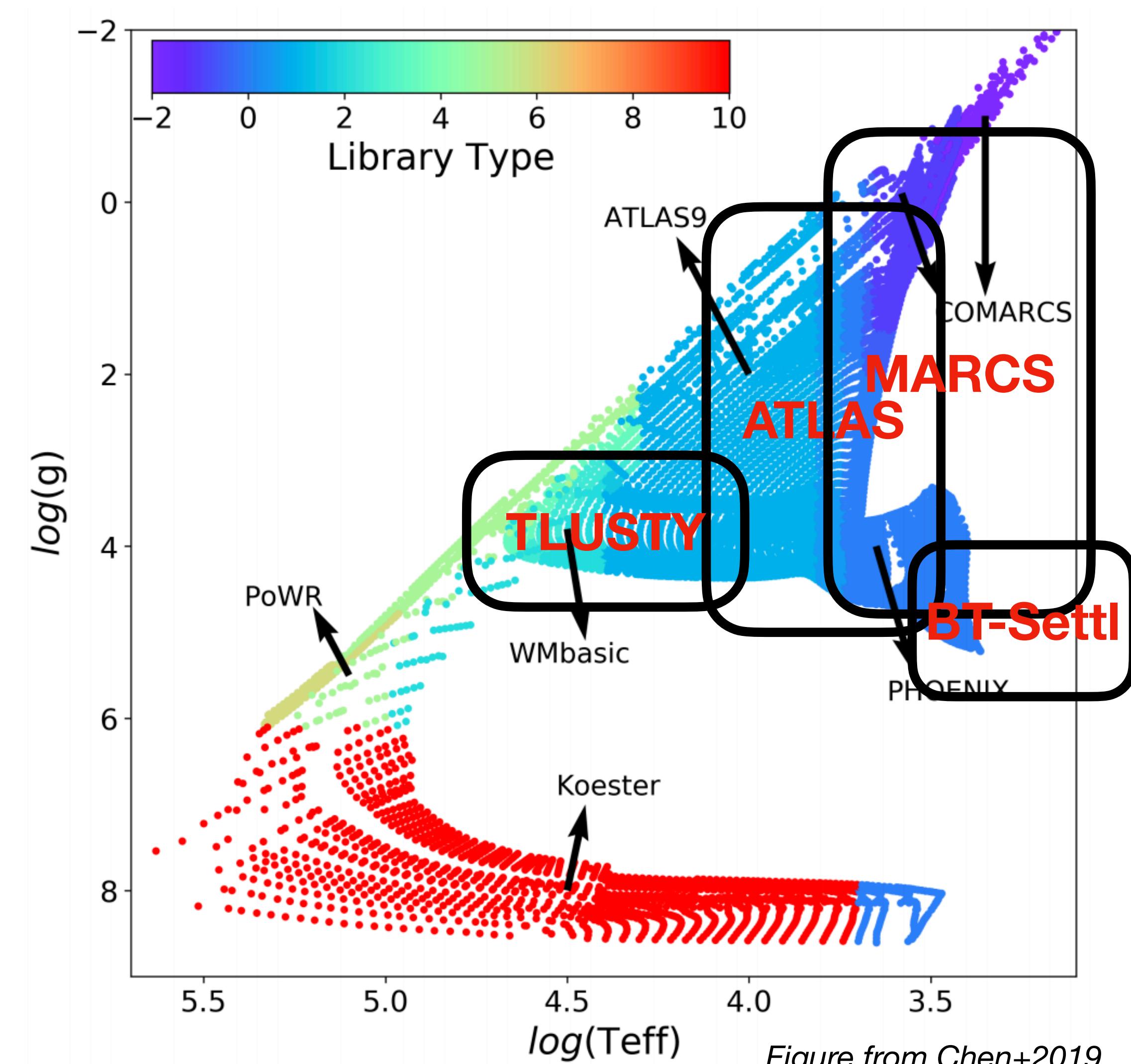


3. How to derive Teff, logg & [M/H] from spectra

Spectral model – Synthetic spectral libraries

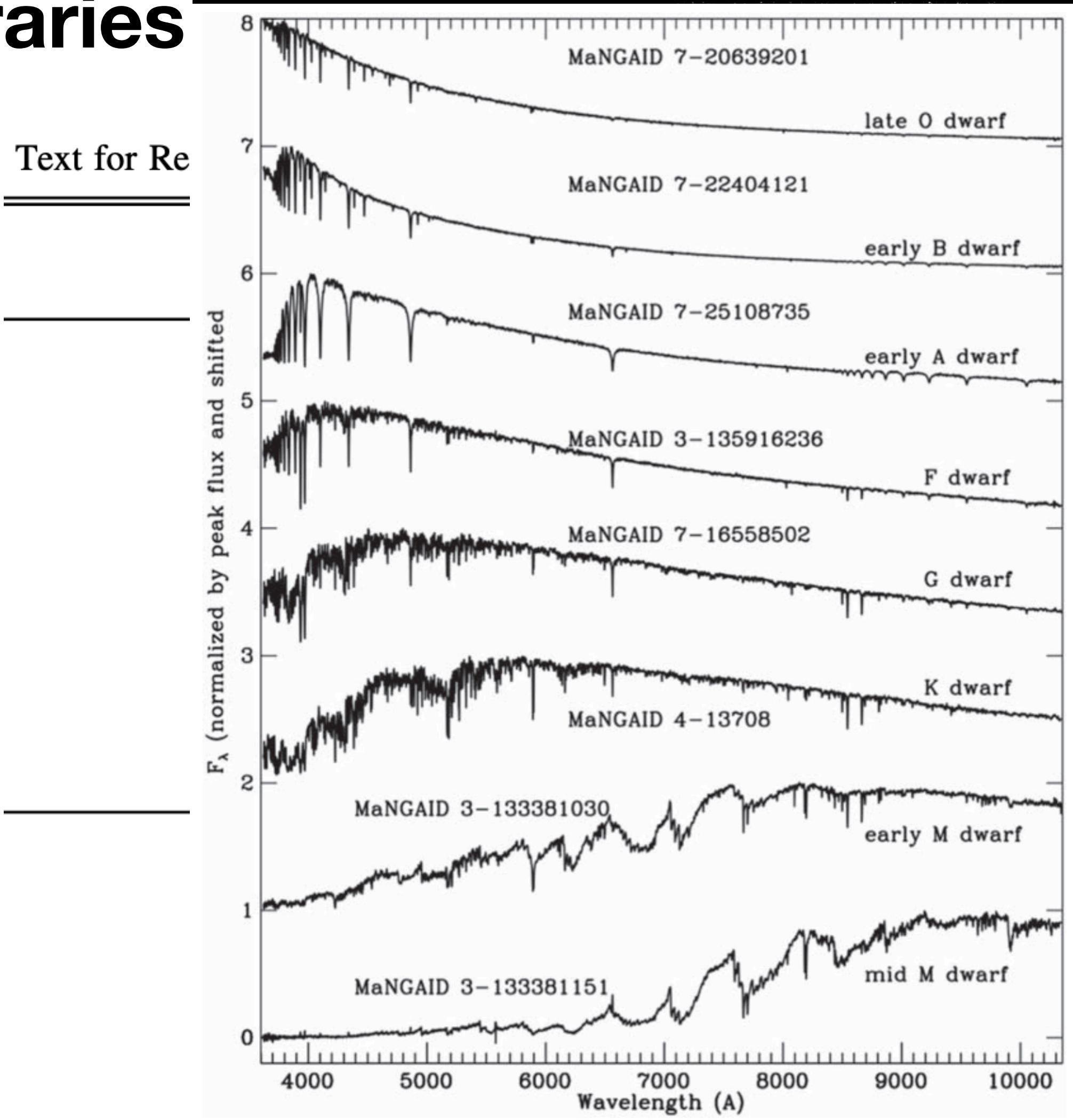
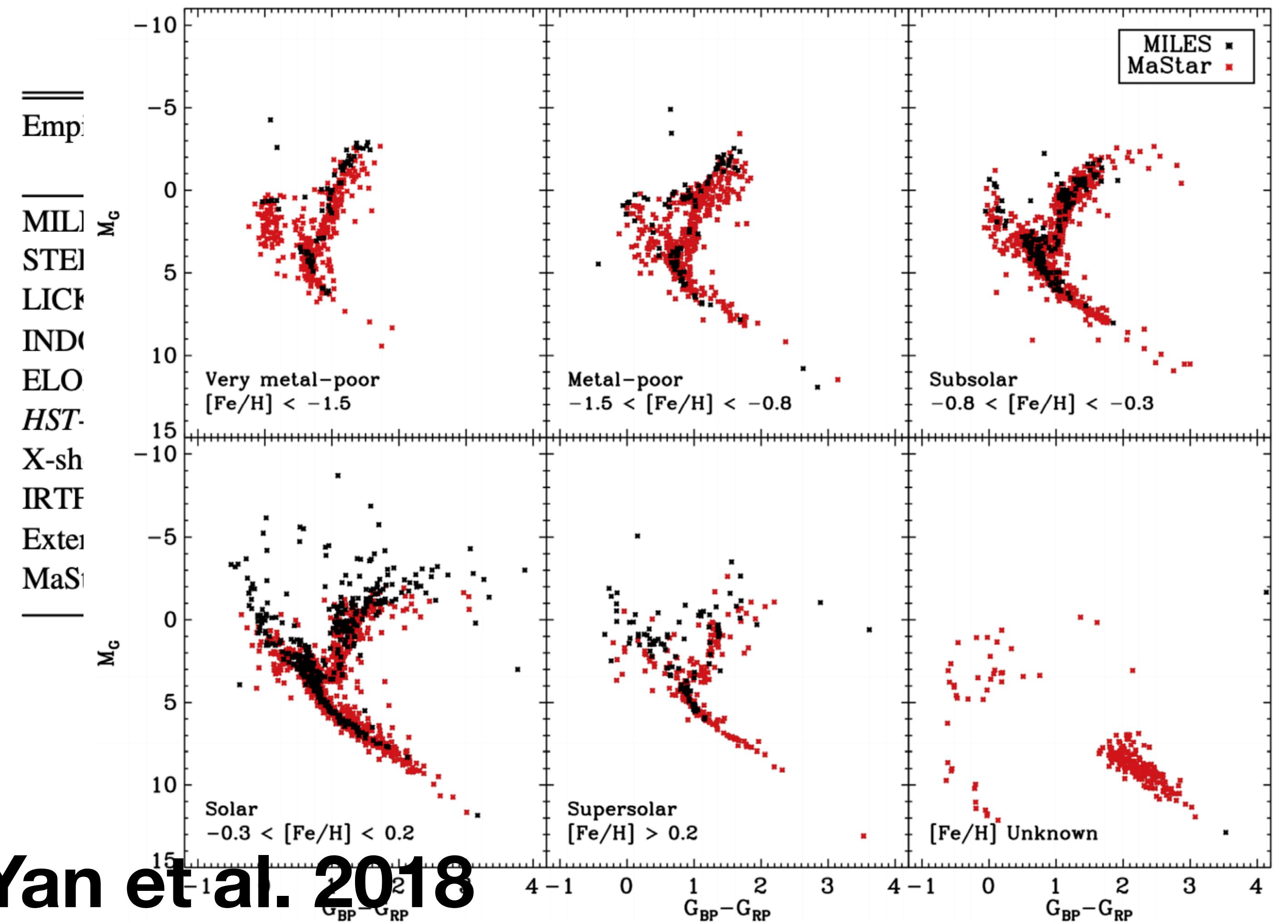
- **TLUSTY** (OB-type, Lanz, T., & Hubeny, 2002, 2006)
 - <http://tlusty.oca.eu/>
- **ATLAS9** (FGK-type, Castelli & Kurucz 2003)
 - e.g., FERRE (Allende Prieto+2018)
 - <http://cdsarc.u-strasbg.fr/viz-bin/qcat?J/A+A/618/A25>
- **MARCS** (K/M-type, Gustafsson+2008)
 - <https://marcs.astro.uu.se/>
- **BT-Settl** (Allard 2014)
 - <https://phoenix.ens-lyon.fr/Grids/BT-Settl/>
- **PHOENIX** (Husser 2015)
 - <https://phoenix.astro.physik.uni-goettingen.de/>

很失望



3. How to derive Teff, logg & [M/H] from spectra

Spectral model – Empirical spectral libraries

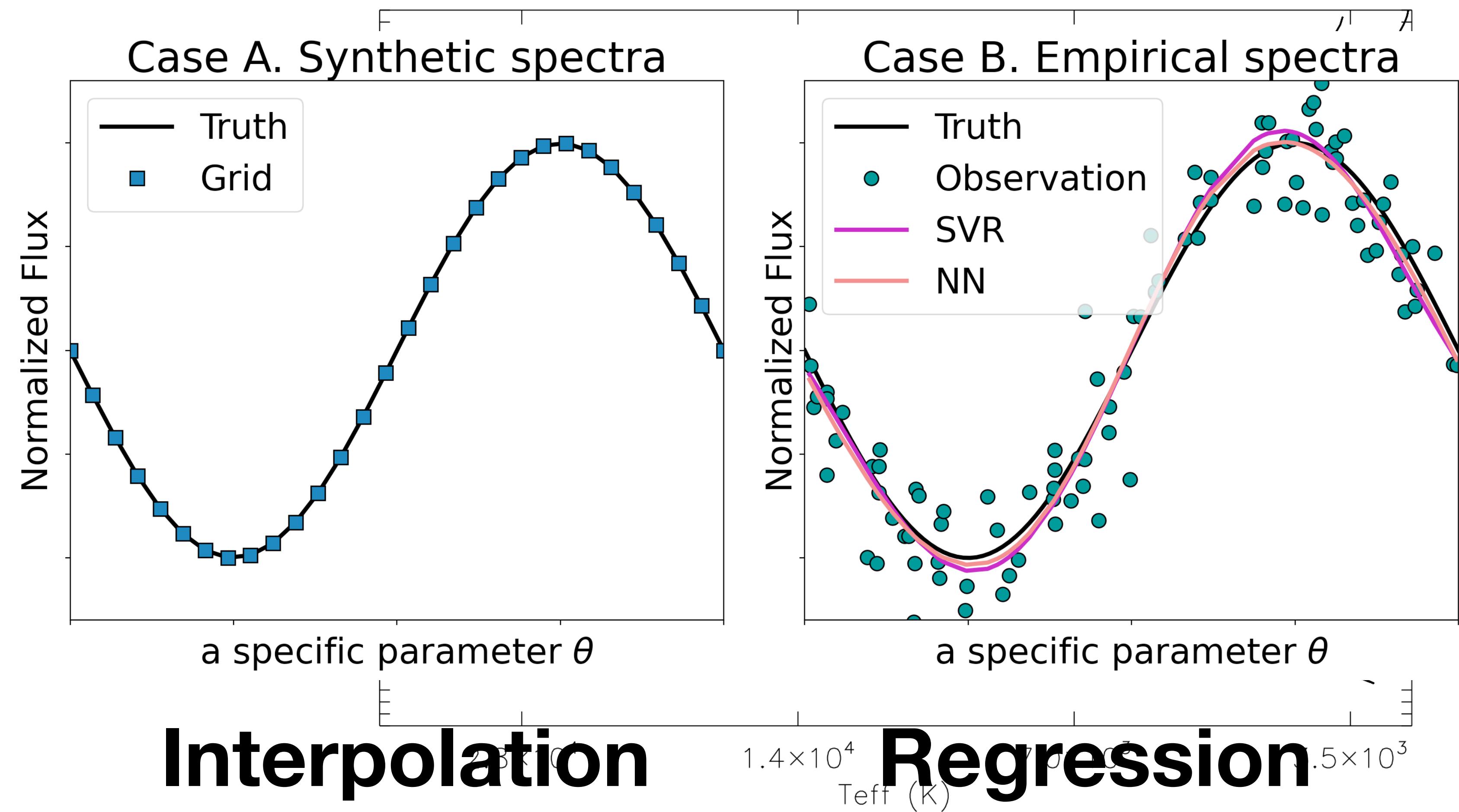


3. How to derive Teff, logg & [M/H] from spectra

How to build spectral models

C. Allende Prieto et al.: Model stellar spectra

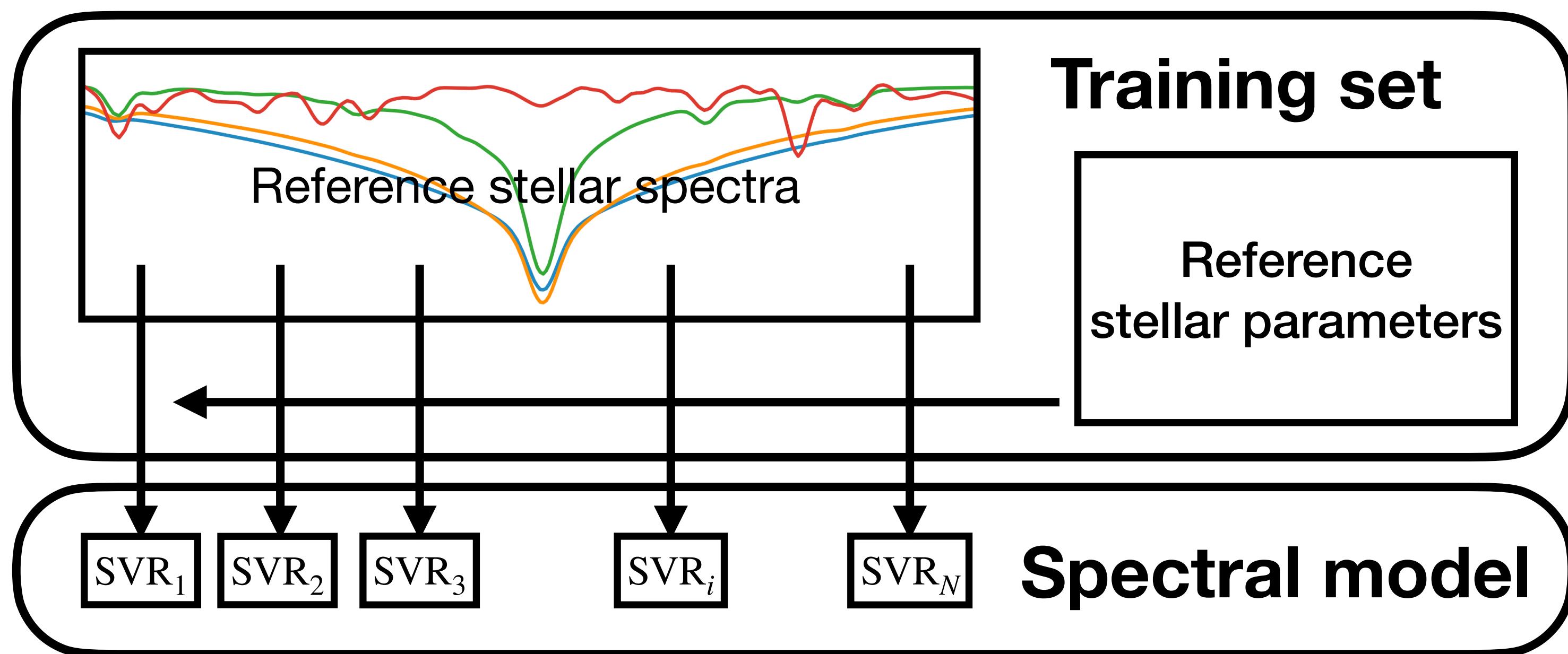
- **Synthetic spectral grid**
 - Interpolation
 - Bayesian estimation (FERRE)
- **Empirical spectra**
 - **Polynomial**
 - Ulyss/LASP (Wu et al. 2010)
 - The Cannon (Ness et al. 2015)
 - **Neural-network**
 - ThePayne (Ting et al. 2019)
 - **Support Vector Regression**
 - SLAM (Zhang et al. 2020a)



3. How to derive Teff, logg & [M/H] from spectra

Training of SLAM

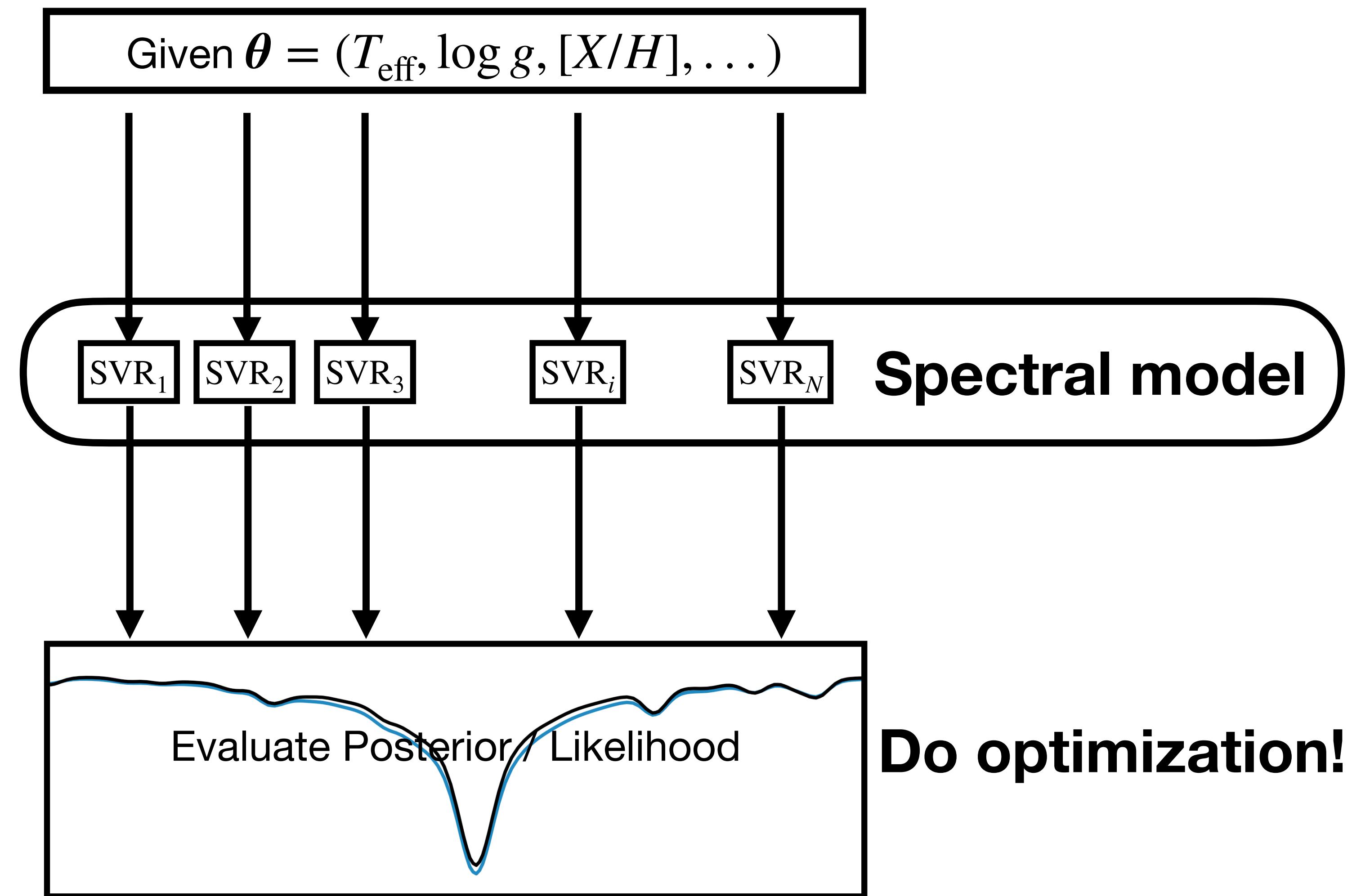
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3. How to derive Teff, logg & [M/H] from spectra

Prediction of SLAM

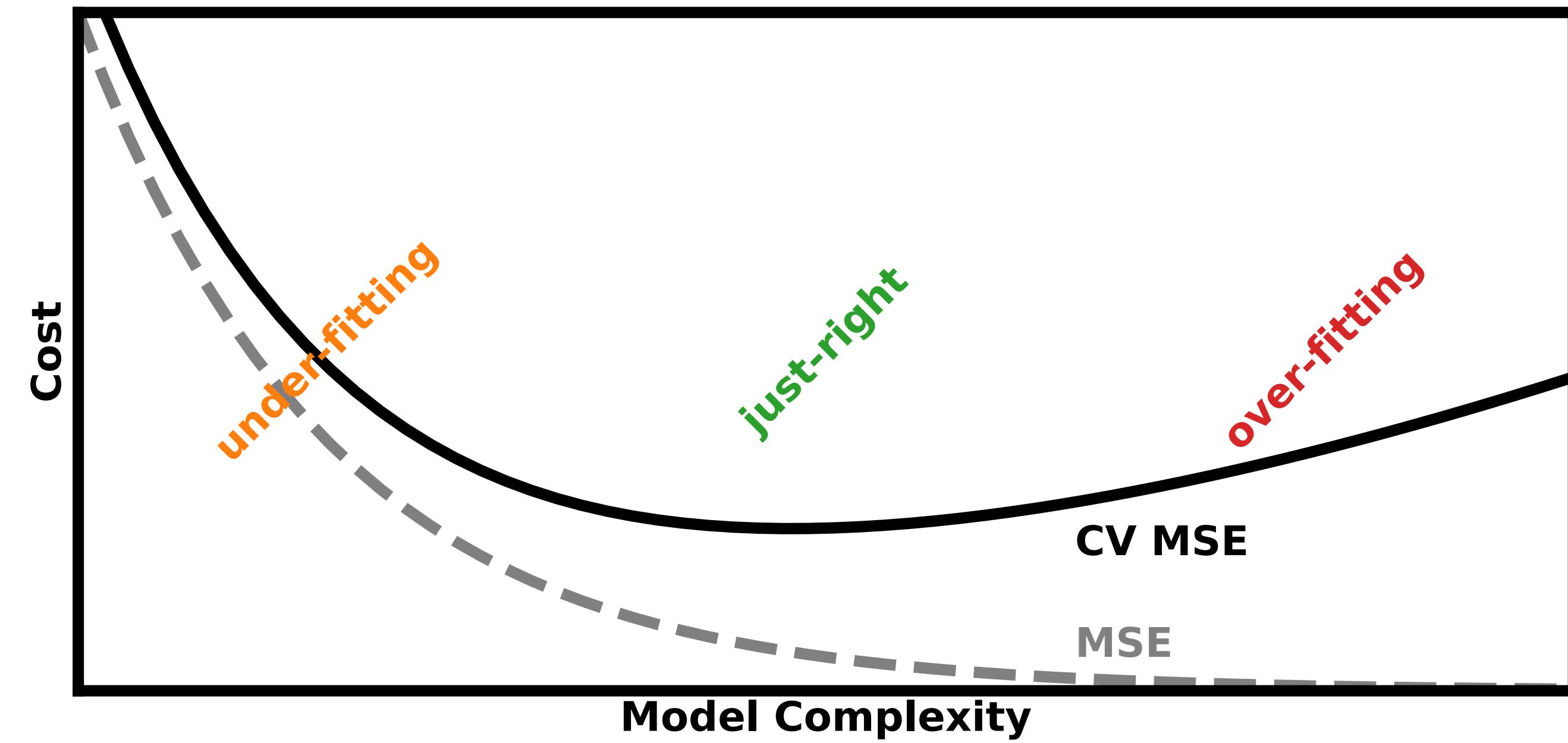
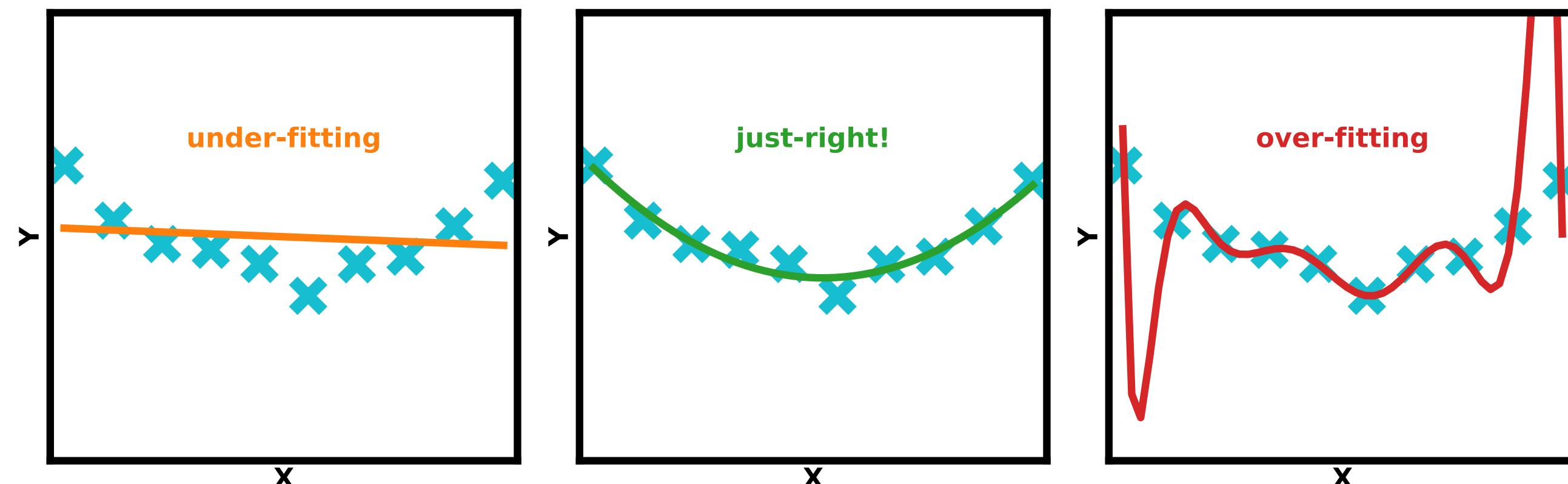
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3. How to derive Teff, logg & [M/H] from spectra

Model complexity*

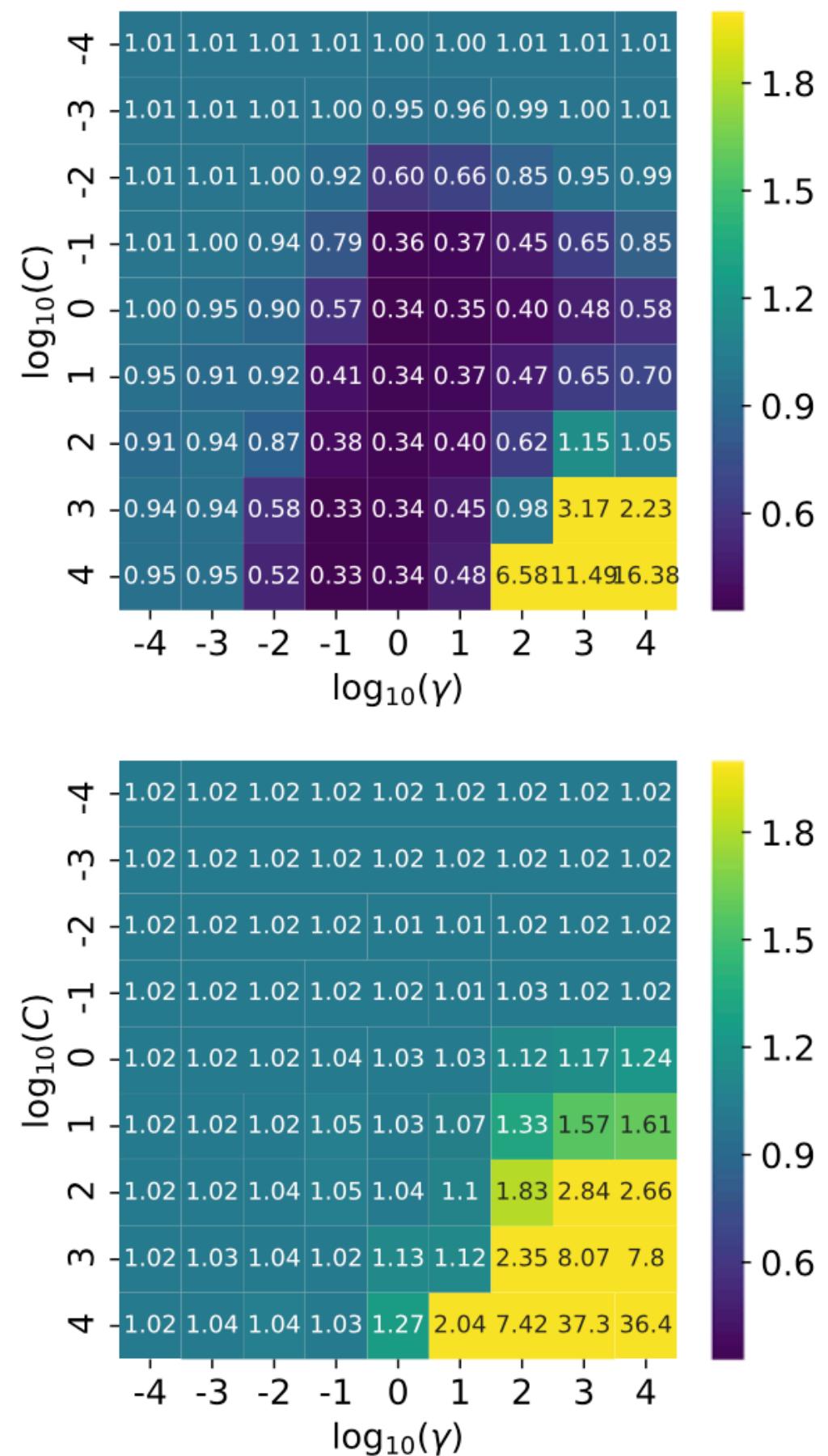
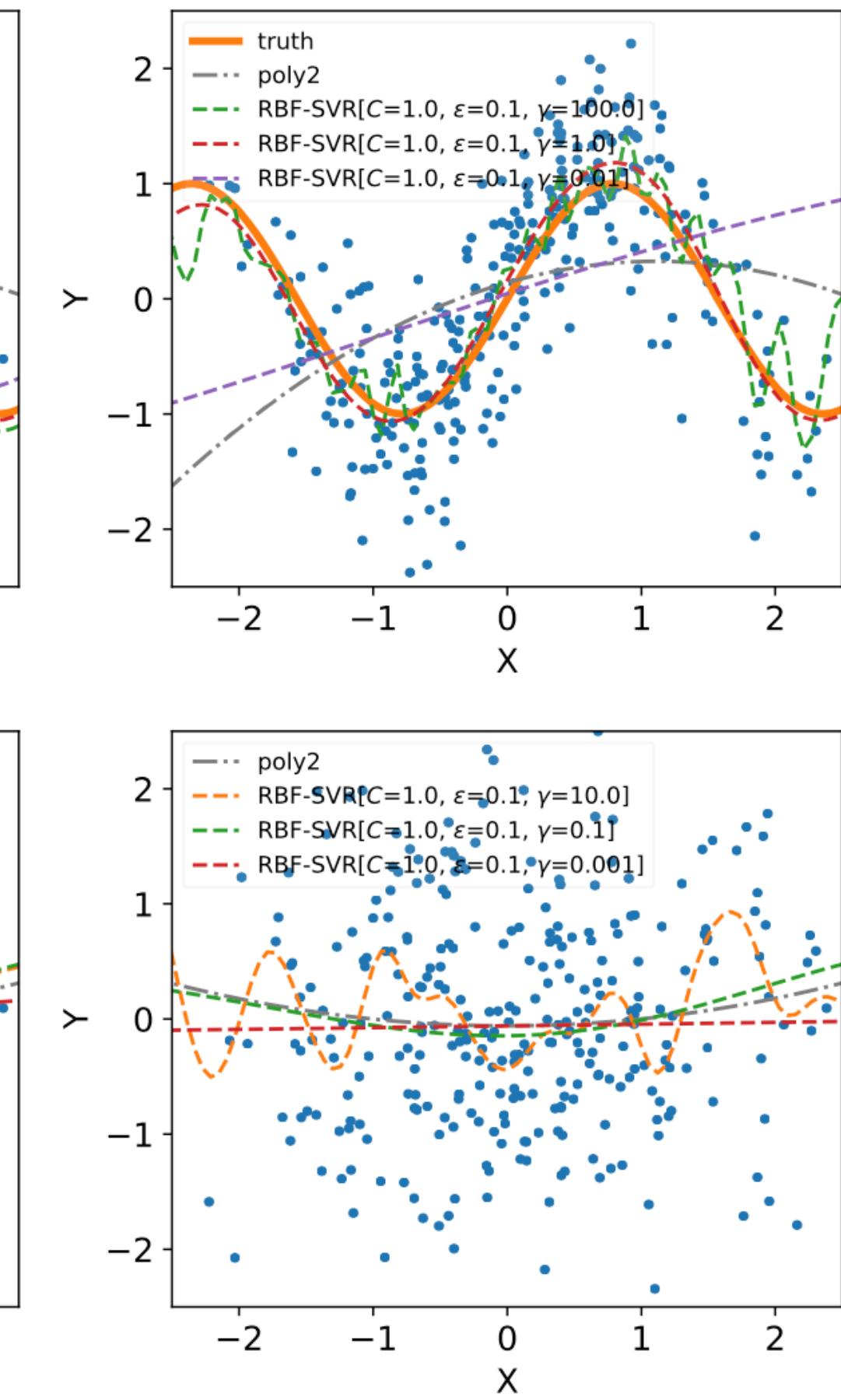
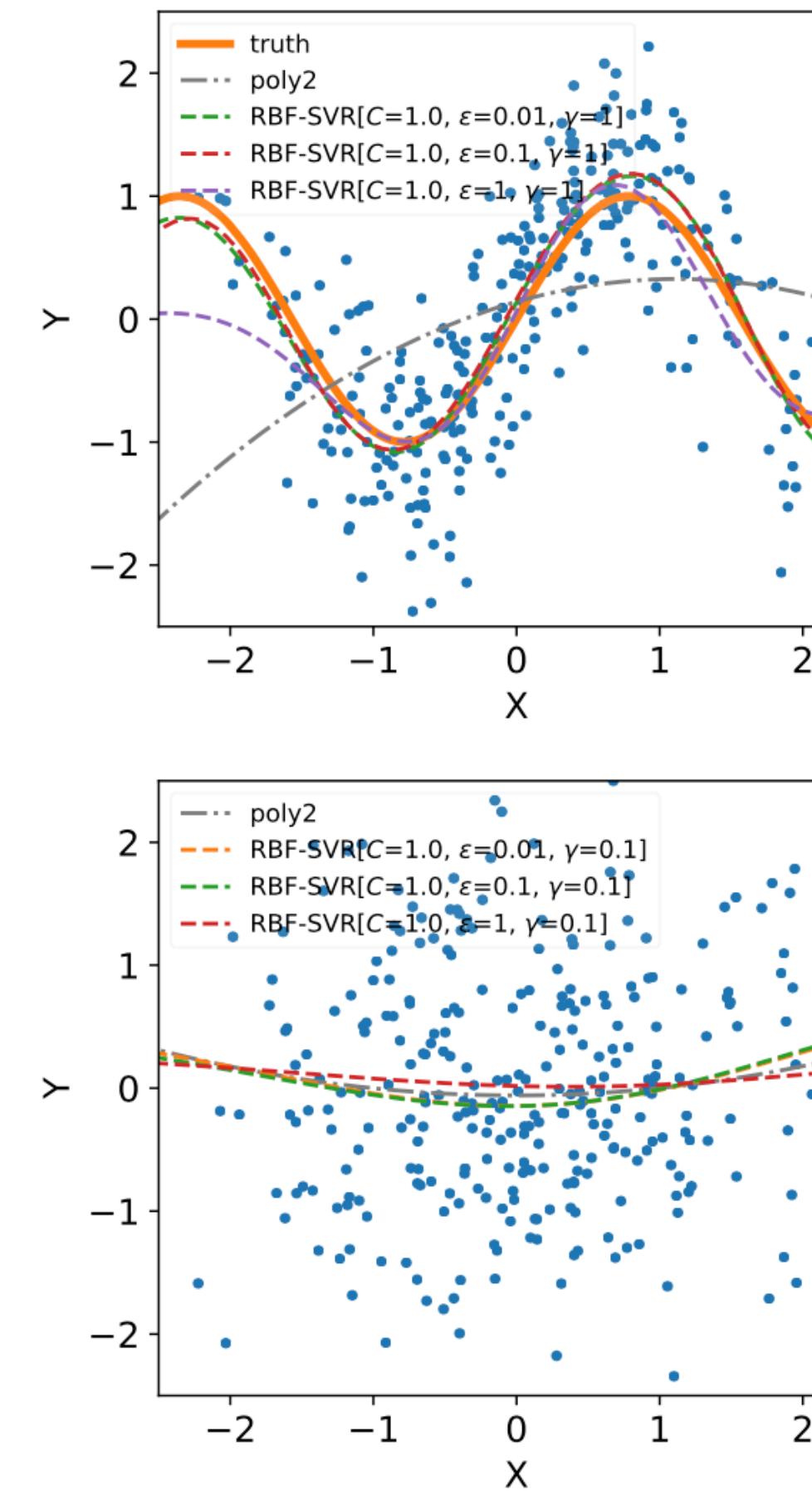
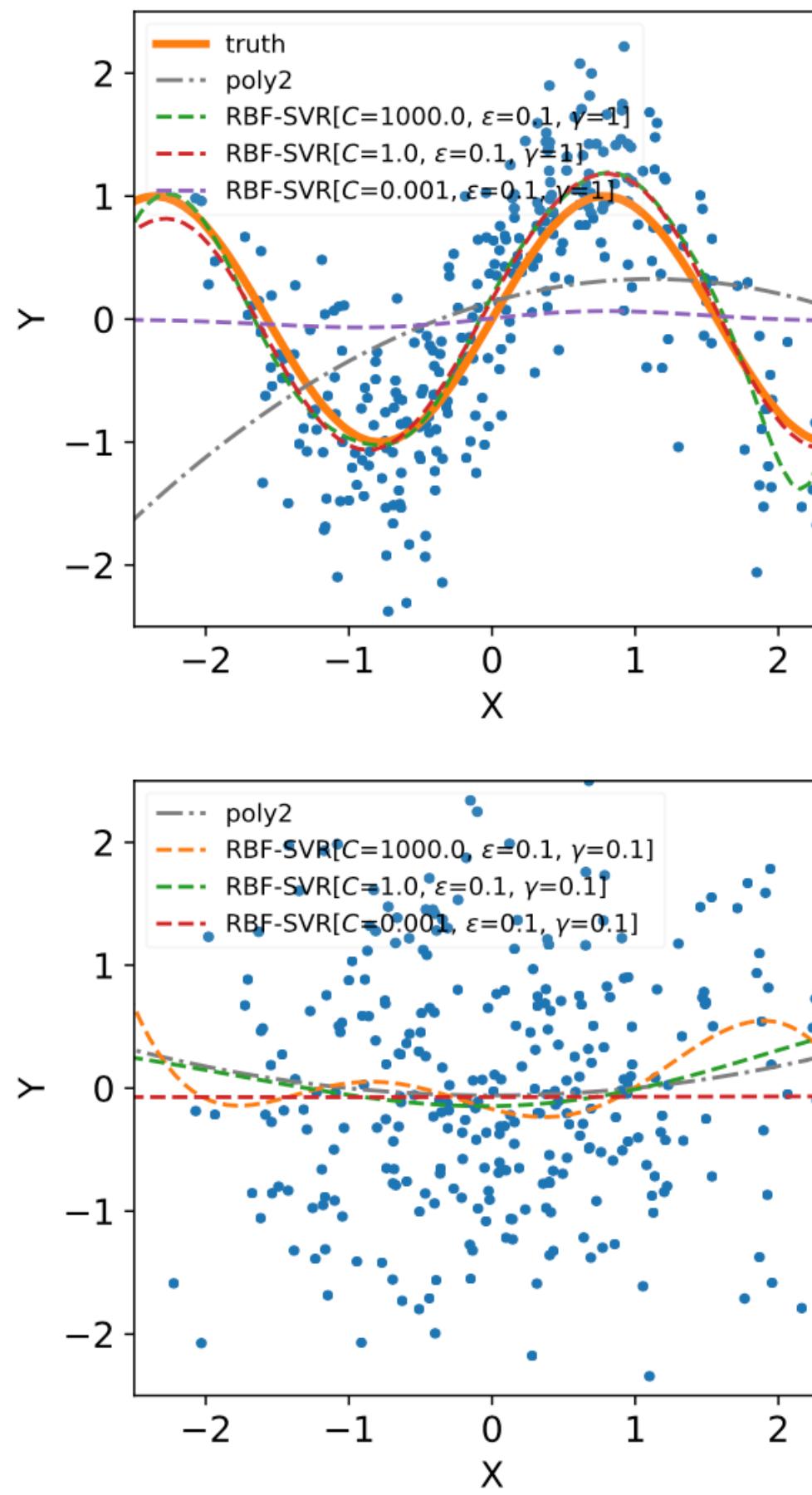
- Polynomial
 - order
- NN
 - n_layer
 - n_neuron
- SVR
 - C
 - eps
 - gamma
 - n_sv
- Spline
 - Smoothness



3. How to derive Teff, logg & [M/H] from spectra

Model complexity solution in SLAM*

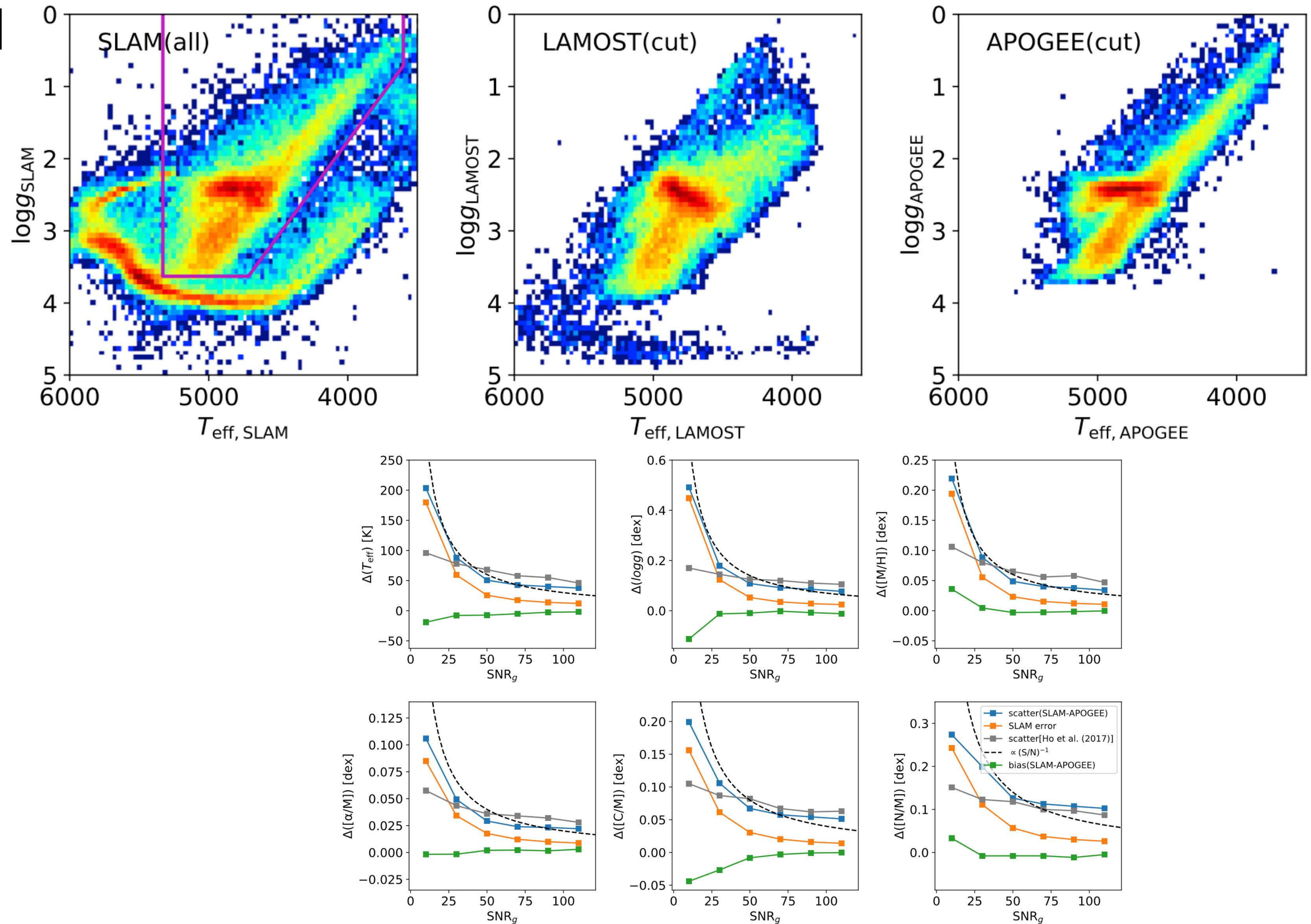
- **Polynomial**
 - order
- **NN**
 - n_layer
 - n_neuron
- **SVR**
 - C
 - eps
 - gamma
 - n_SV
- **Spline**
 - Smoothness



3. How to derive Teff, logg & [M/H] from spectra

Application of SLAM

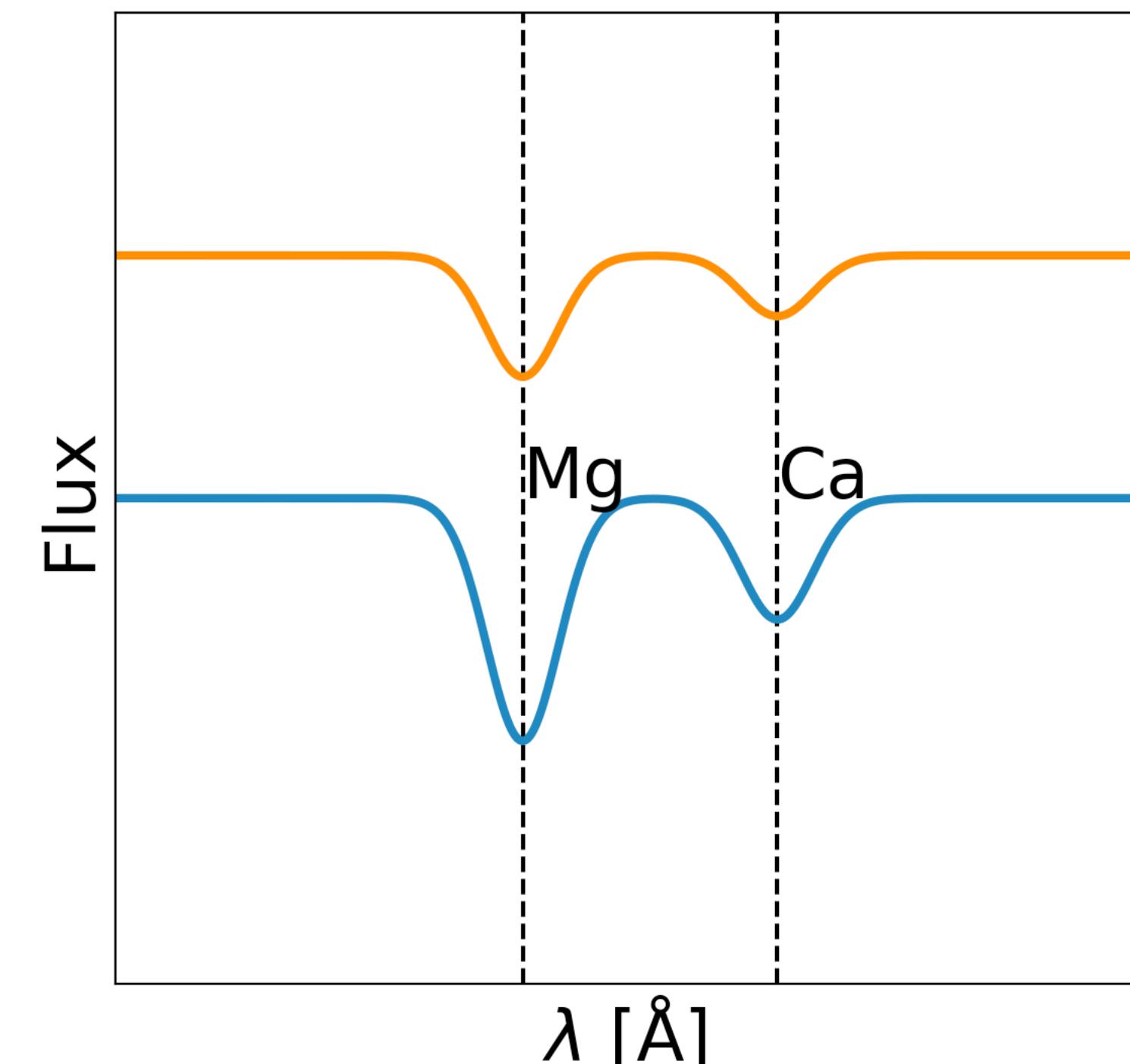
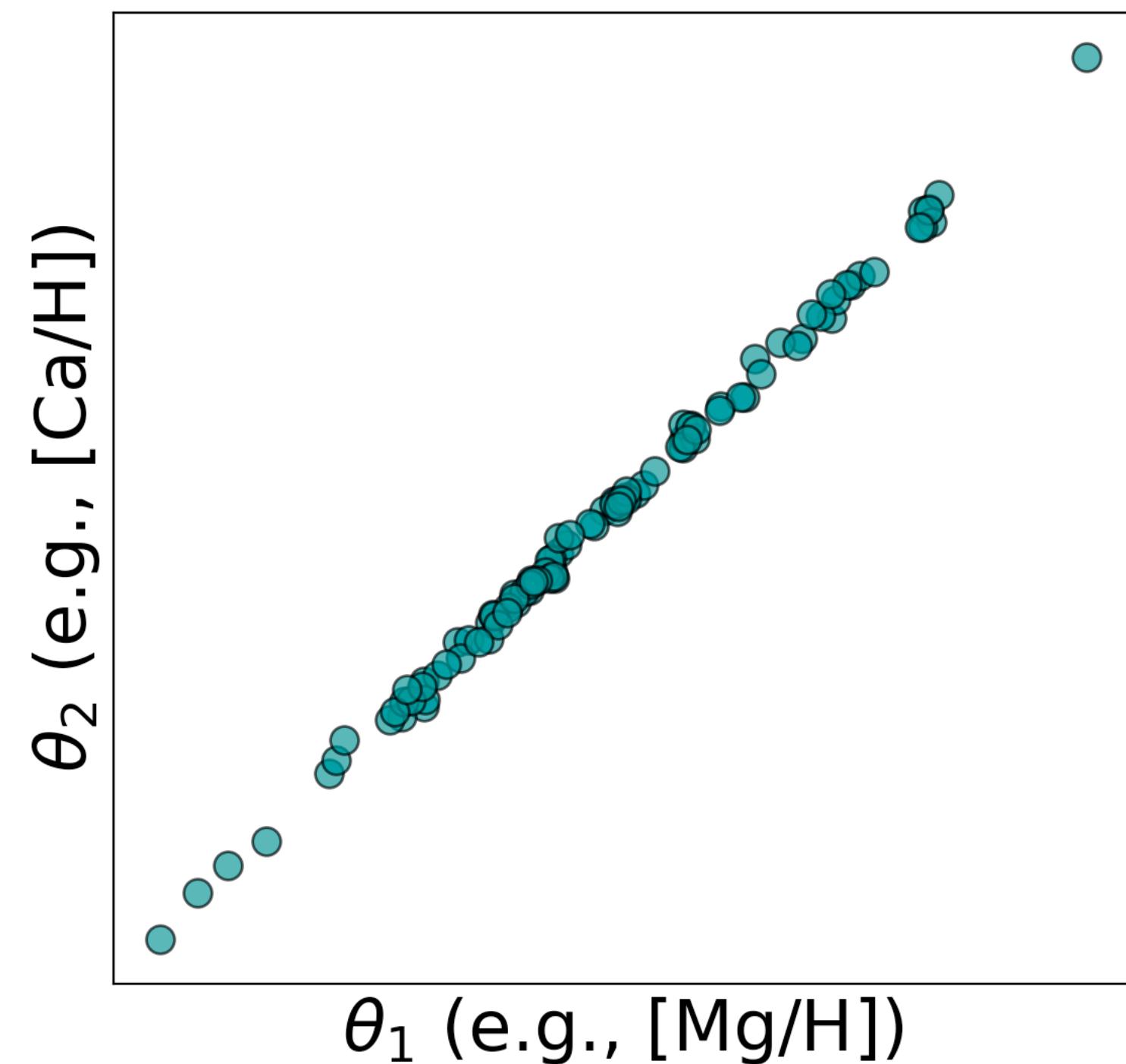
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 - Support Vector Regression
 - SLAM (Zhang et al. 2020a)



3. How to derive Teff, logg & [M/H] from spectra

Degeneracy between parameters*

- **The origin of degeneracy**
 - Non-uniform distribution of the training set
- **How to break?**
 - +model gradient?(The Payne)
-

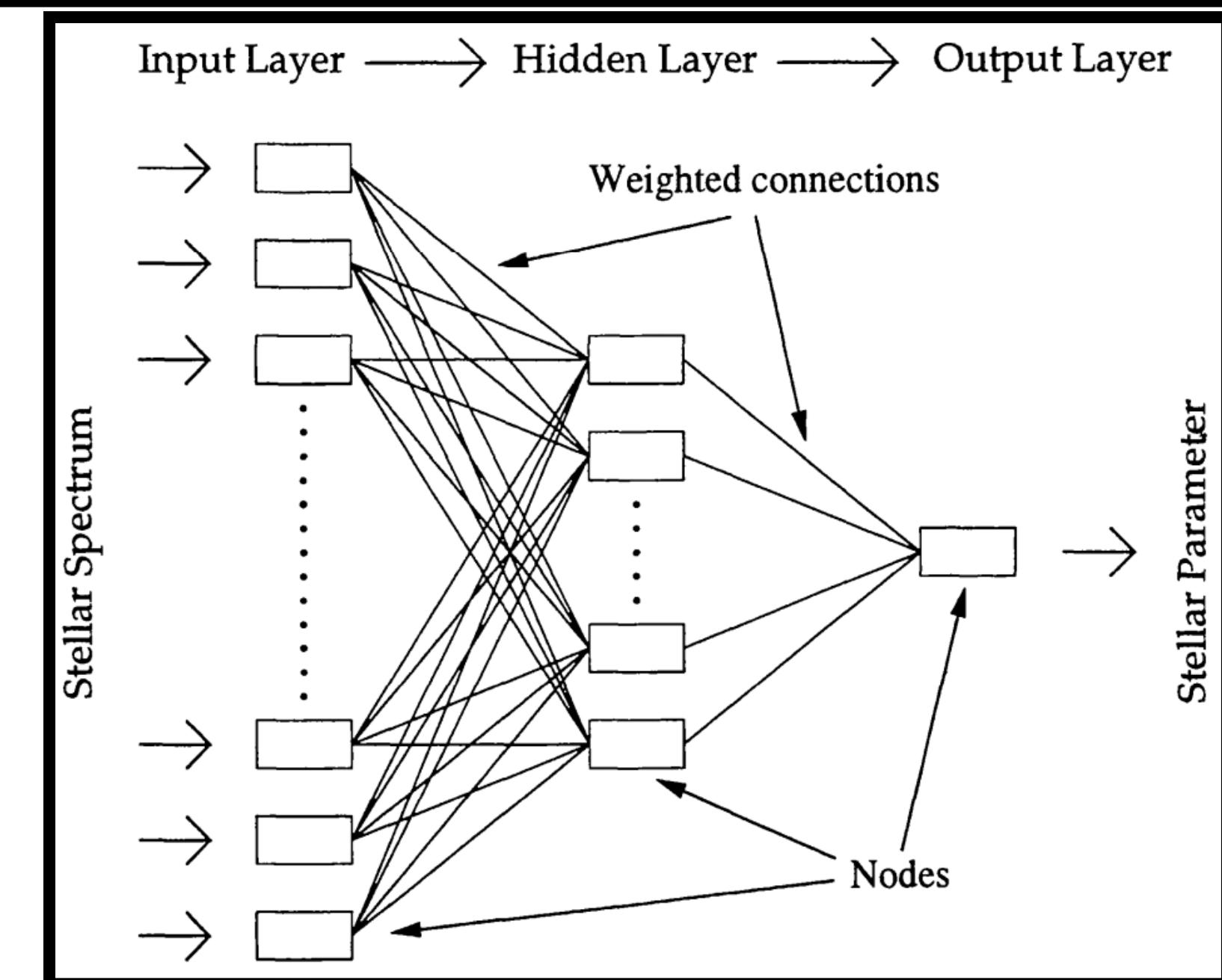
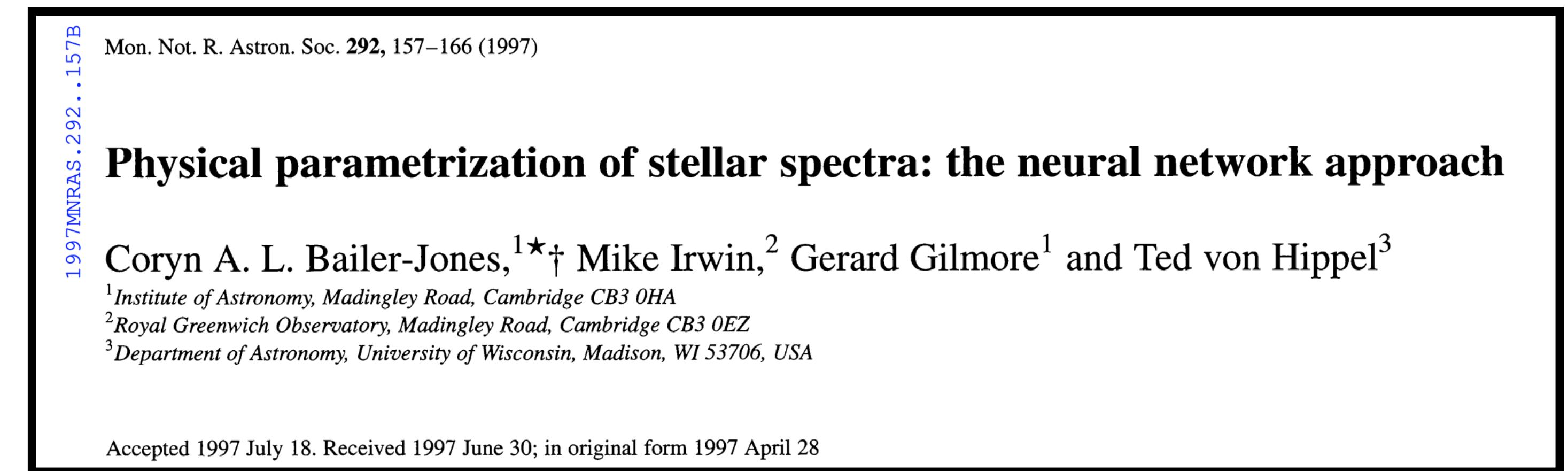


3. How to derive Teff, logg & [M/H] from spectra

Backward model – Neural-Networks

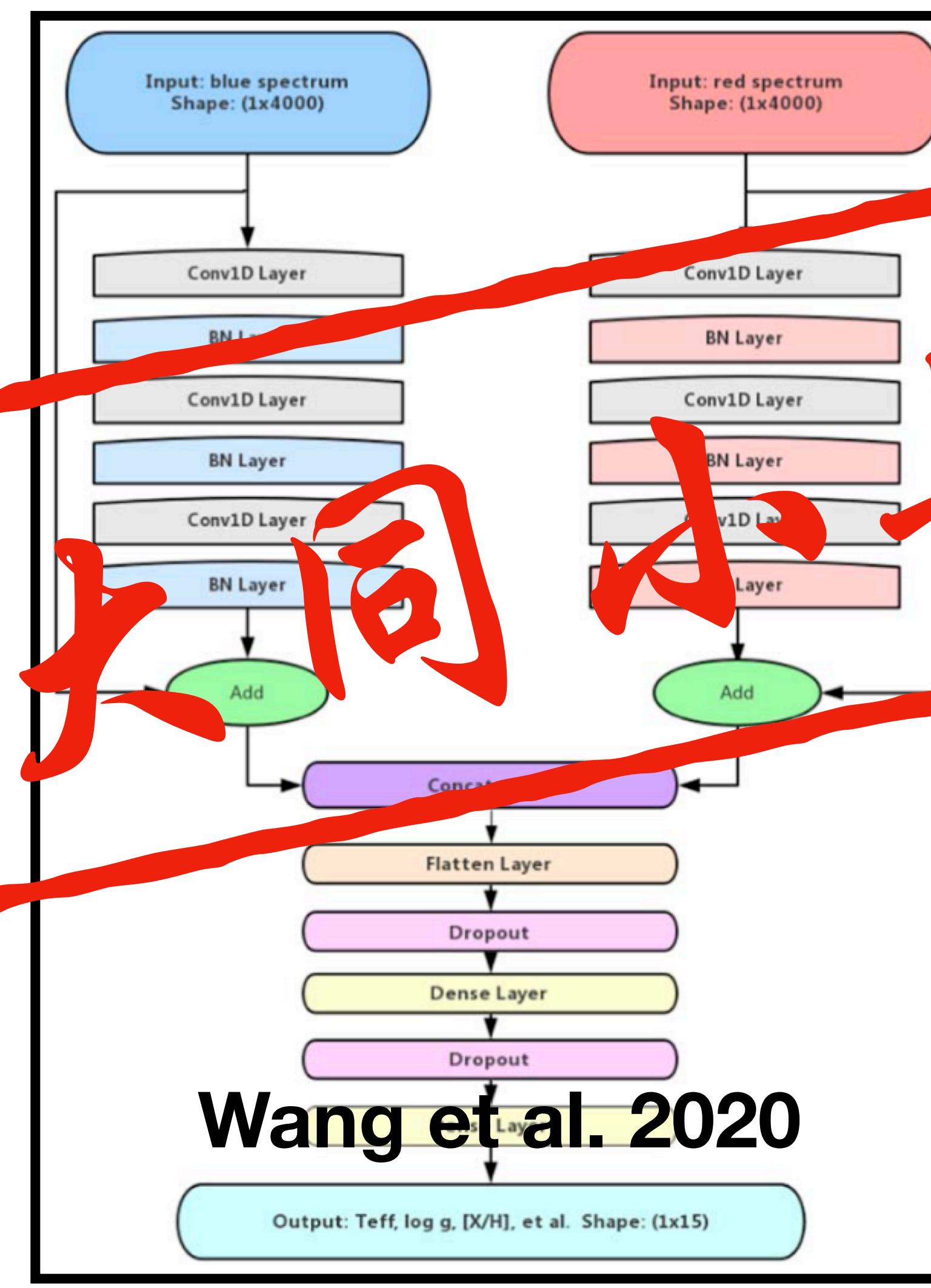
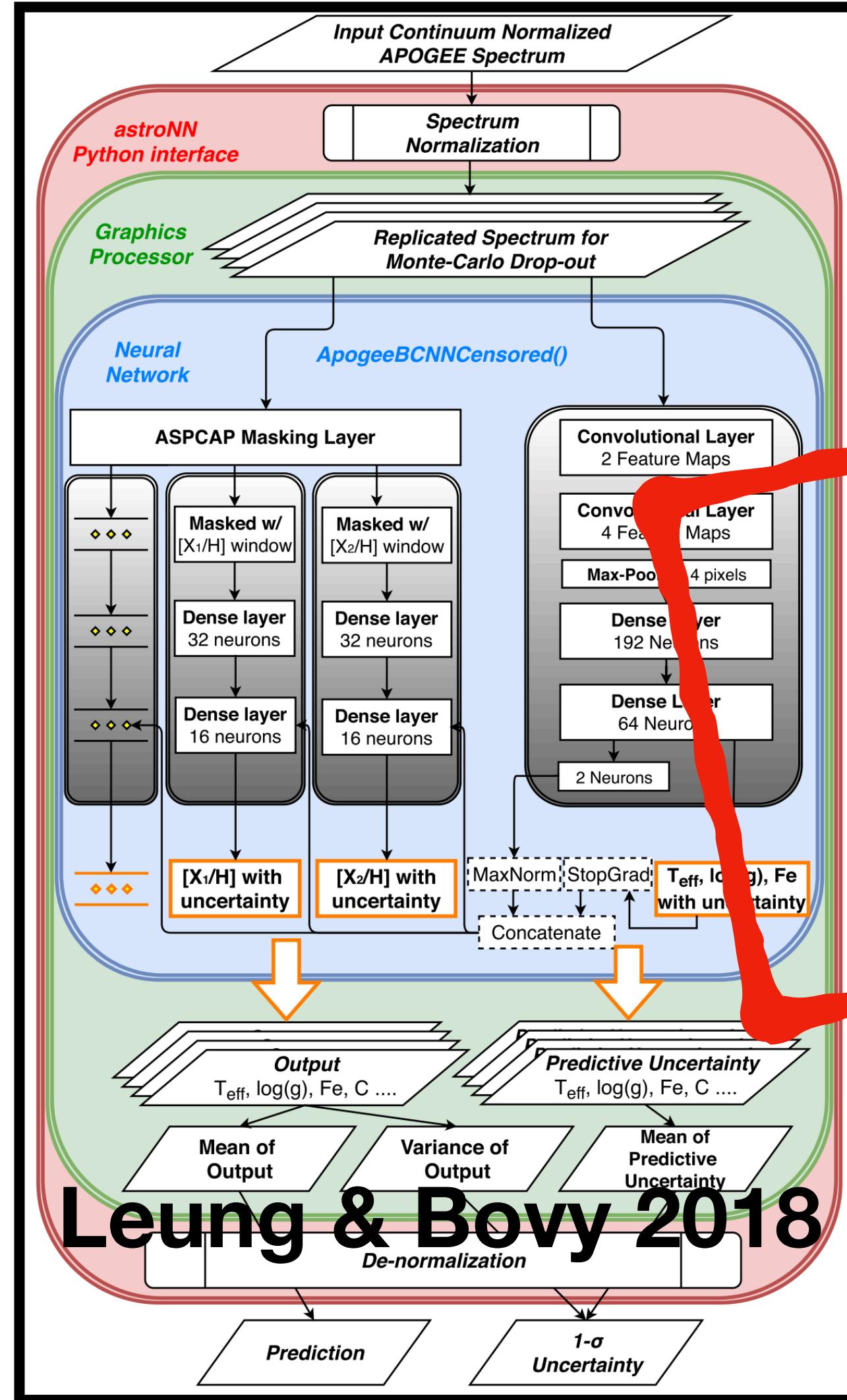
- ANN
 - Bailer-Jones et al. 1997

- CNN
 - AstroNet, Leung & Bovy 2018
 - SPCANet, Wang et al. 2020
 - Guiglion et al. 2020
- ...

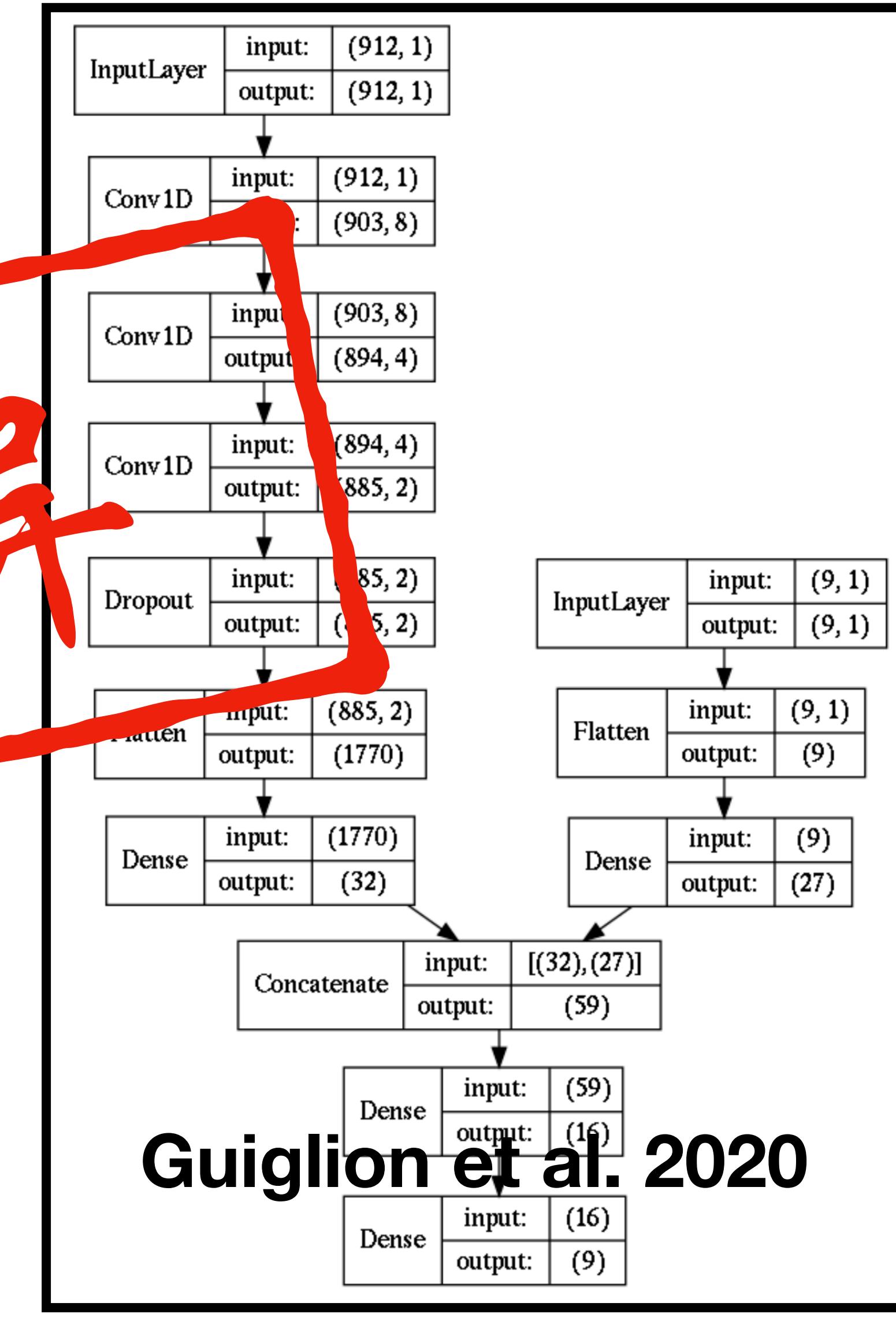


3. How to derive Teff, logg & [M/H] from spectra

Backward model – Neural-Networks



Wang et al. 2020



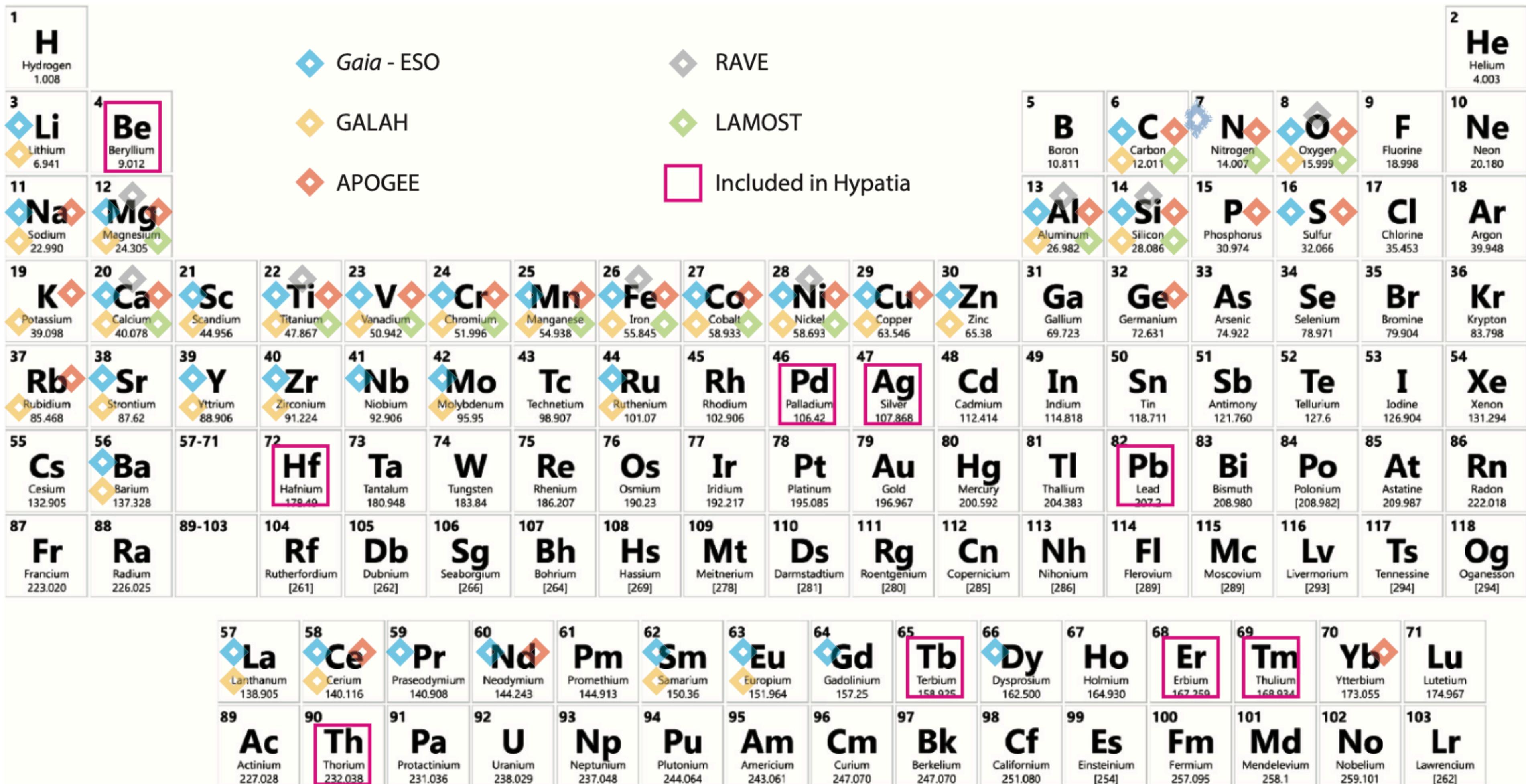
Guiglion et al. 2020

Leung & Bovy 2018

大同小异

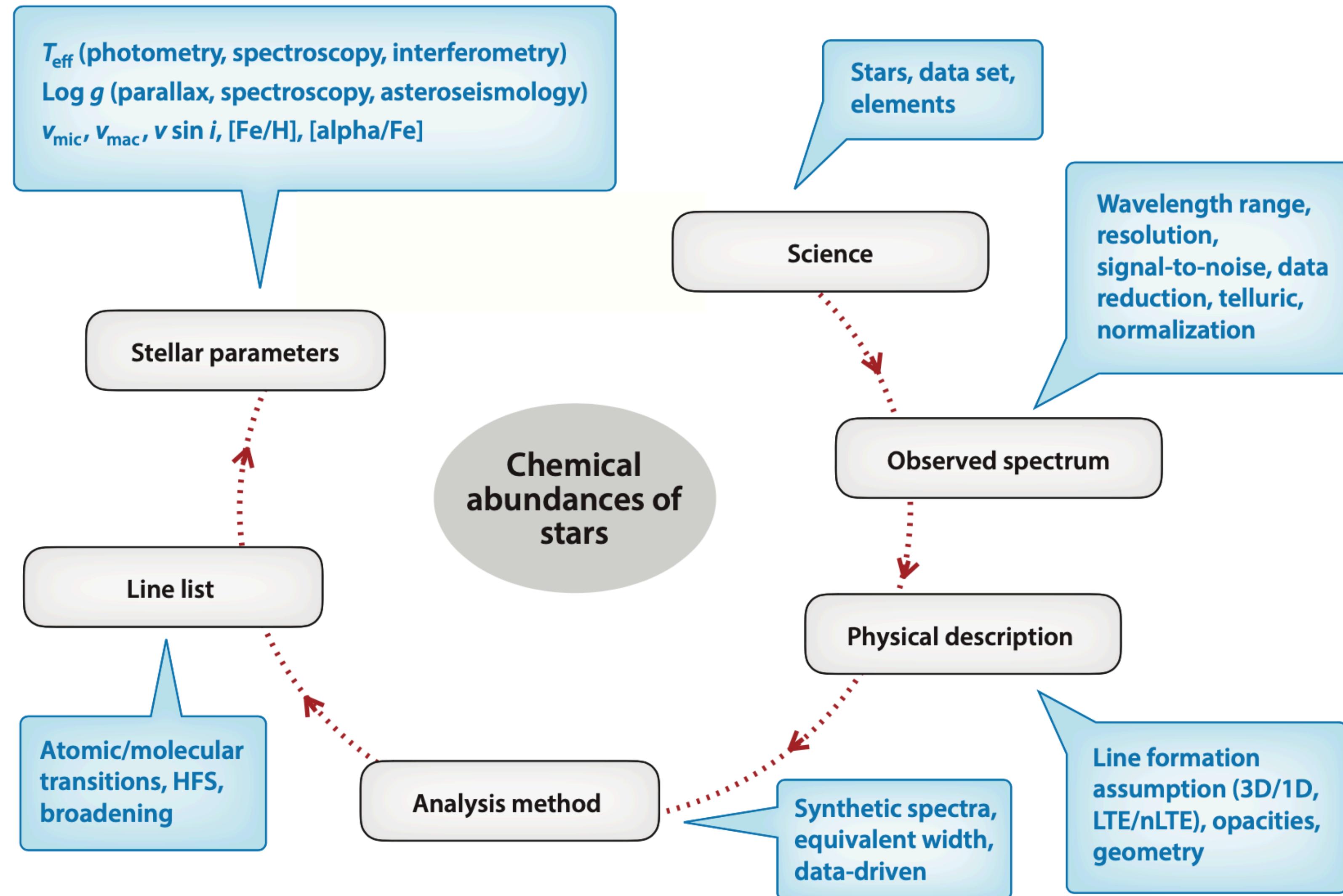
3. How to derive Teff, logg & [X/H] from spectra

More elemental abundances*



3. How to derive Teff, logg & [X/H] from spectra

Take a look back at the industrial stellar parameters



3. How to derive Teff, logg & [M/H] from spectra

Tools summarized by Jofre+2019

Material	Reference	Comment
SVO Montes	Spectral libraries http://svo2.cab.inta-csic.es/theory/libtest/index.php https://webs.ucm.es/info/Astrof/invest/actividad/spectra.html	Public libraries Compilation
MARCS ATLAS9 STAGGER CO5BOLD	Model atmospheres Gustafsson et al. (2008) Castelli & Kurucz (2003) Magic et al. (2013) Freytag et al. (2012)	1D spherical geometry 1D plane-parallel geometry 3D 3D
Turbospectrum MOOG SYNTHE SPECTRUM DETAIL/SIU	Radiative transfer codes Plez et al. (1992) Sneden (1973) Kurucz (1993) Gray & Corbally (1994) e.g., Bergemann et al. (2012) via http://nlte.mpia.de/	LTE LTE LTE LTE Non-LTE
VALD NIST ASD Sneden et al. Linemake ExoMol BRASS Barklem Kurucz VAMDC	Line lists Ryabchikova et al. (2015) https://www.nist.gov/pml/atomic-spectra-database https://www.as.utexas.edu/\~chris/lab.html https://github.com/vmplacco/linemake Tennyson et al. (2016) Laverick et al. (2018) Barklem et al. (2015) Kurucz (2011) http://www.vamdc.eu	Literature compilation Literature compilation Bibliography and molecular line lists Synthetic spectrum lists in MOOG style Very cool objects Centralization of sources Broadening cross-sections Atomic data Electronic infrastructure
AMBRE STAGGER 3D-non-LTE Balmer APOGEE POLLUX	Grids of synthetic spectra de Laverny et al. (2012) Chiavassa et al. (2018) Amarsi et al. (2018b) Mészáros et al. (2012) Palacios et al. (2010)	Optical high resolution CaII triplet centered Balmer lines centered IR Database
SME iSpec FERRE GALA DOOp ARES The Cannon	Automatic codes for the determination of abundances Piskunov & Valenti (2017) Blanco-Cuaresma et al. (2014a) García Pérez et al. (2016) Mucciarelli et al. (2013) Cantat-Gaudin et al. (2014) Sousa et al. (2015) Ness et al. (2015)	With non-LTE on the fly Python wrapper for various tools Match models to data EW code Wrapper for EWs Automatic EWs Label transfer from a training set
INSPECT MPIA	Non-LTE abundance corrections http://inspect-stars.com/ http://nlte.mpia.de/	Line-by-line corrections Line-by-line corrections

4. $v \sin i$ profile

Ref: D.F.Gray The Observation & Analysis of Stellar Photosphere

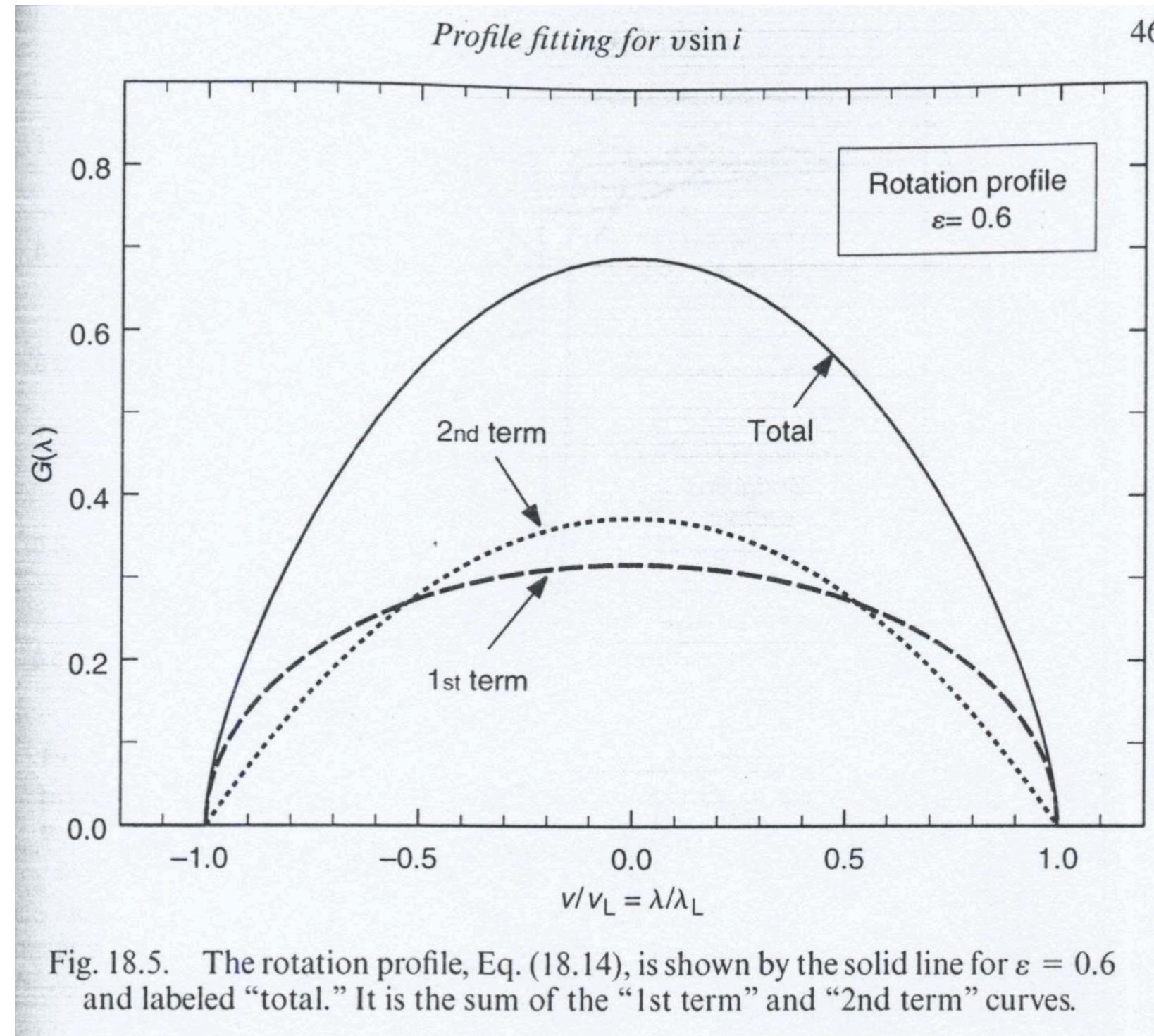


Fig. 18.5. The rotation profile, Eq. (18.14), is shown by the solid line for $\epsilon = 0.6$ and labeled "total." It is the sum of the "1st term" and "2nd term" curves.

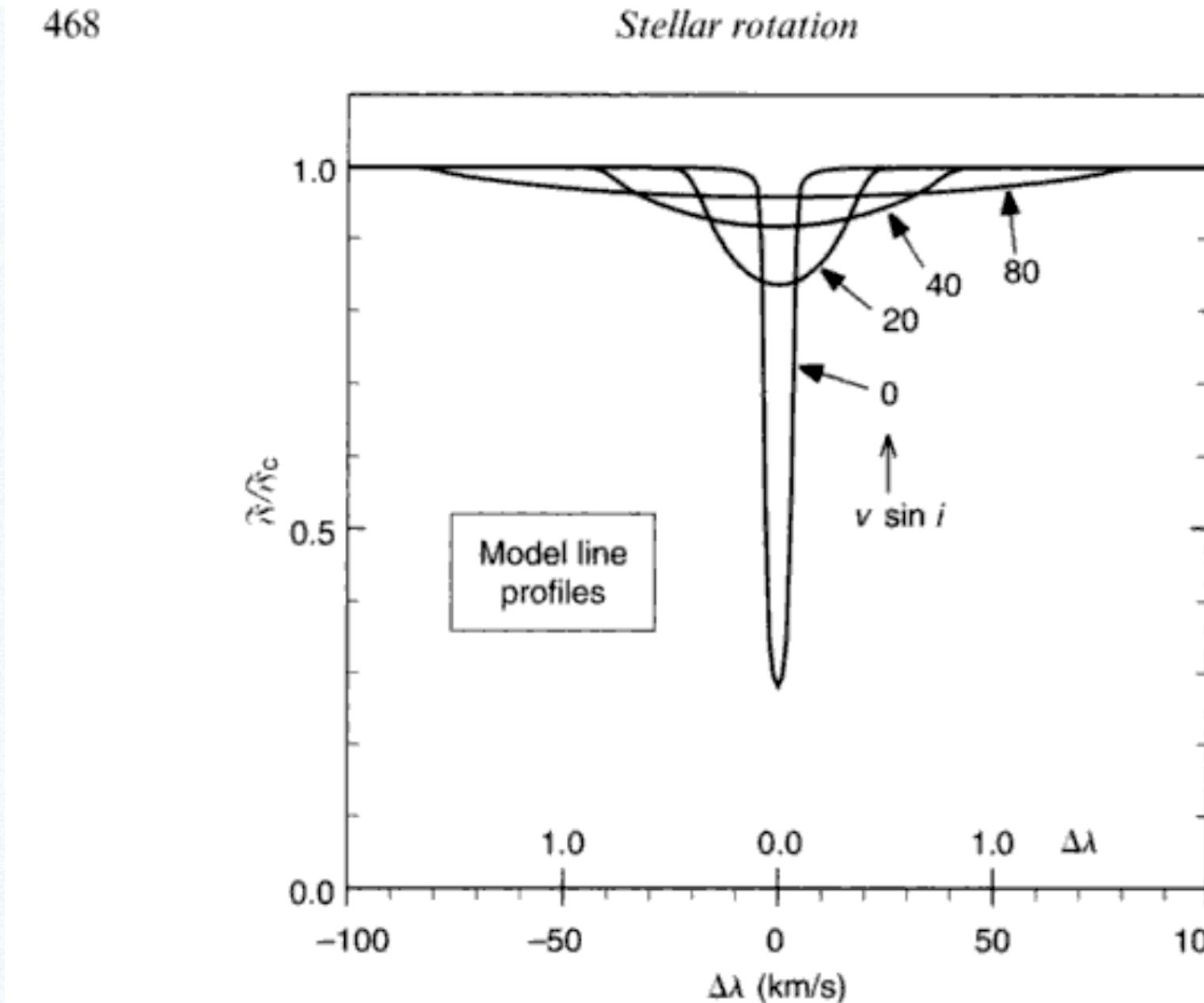


Fig. 18.6. Computed profiles illustrate the broadening effect of rotation. The profiles are labeled with $v \sin i$ in km/s. The equivalent width is conserved.

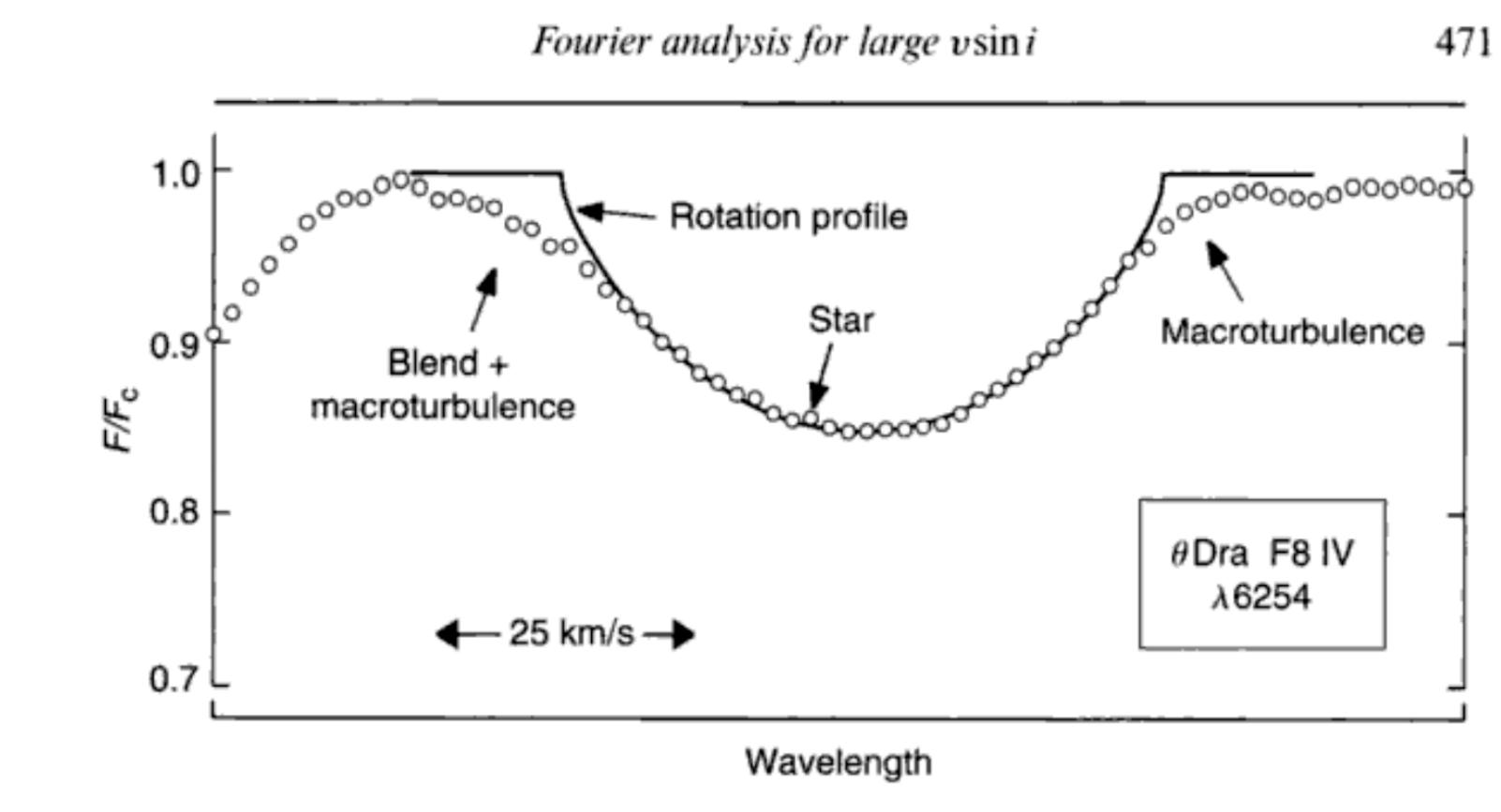


Fig. 18.10. An observed profile of θ Dra is compared to the classic rotation profile. The rotation profile shows sharp corners where it joins the continuum. The high Fourier frequencies needed to form these corners are filtered away by other broadeners, in this case macroturbulence of several km/s. Data from the Elginfield Observatory.

$$G(\Delta\lambda) = G(v) = \frac{2(1 - \epsilon)[1 - (v_z/v_L)^2]^{1/2} + \frac{1}{2}\pi\epsilon[1 - (v_z/v_L)^2]}{\pi v_L(1 - \epsilon/3)}$$

4. $v \sin i$ profile

Ref: D.F.Gray The Observation & Analysis of Stellar Photosphere

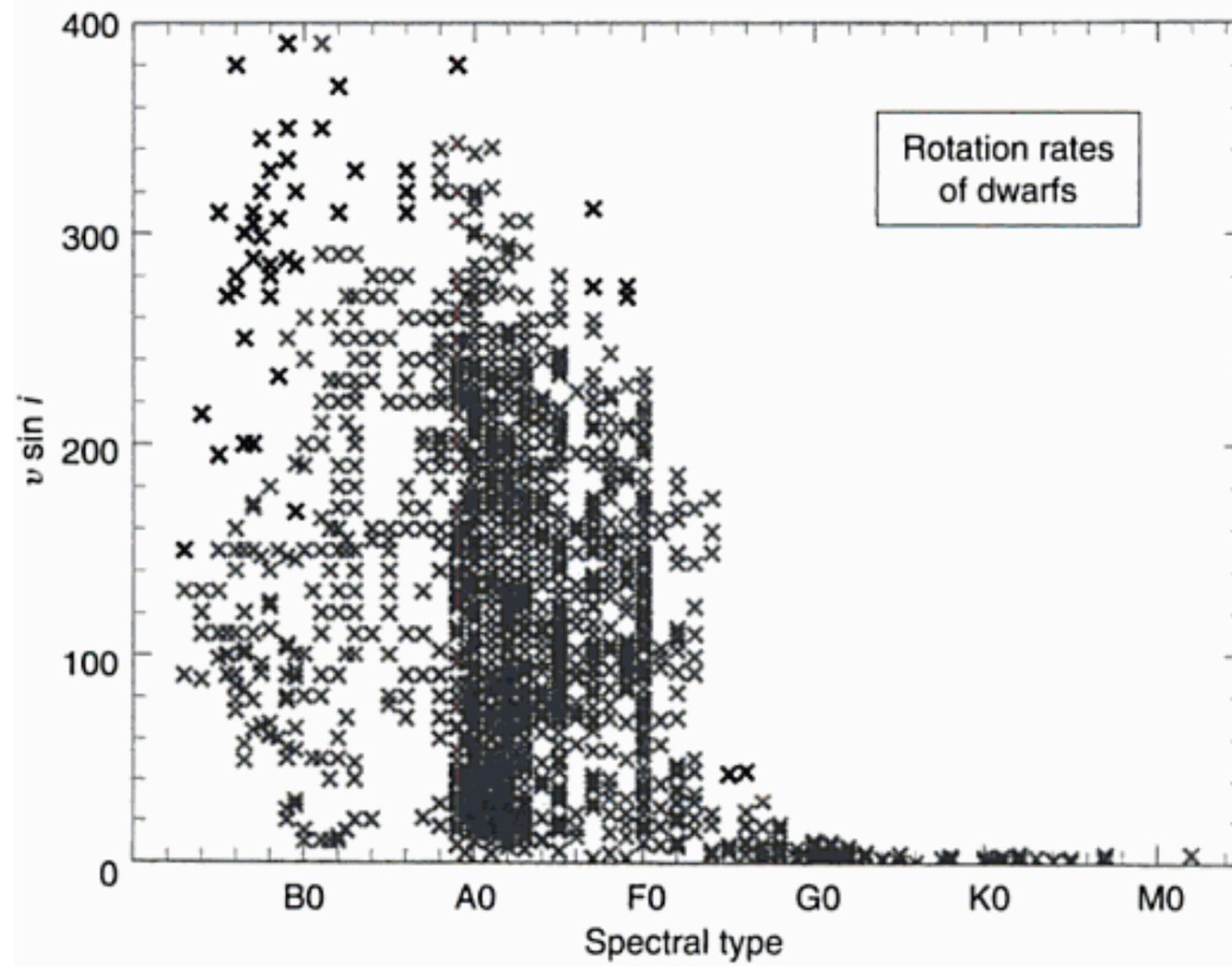


Fig. 18.21. Rapid rotation is normal for hot stars; slow rotation is seen for cool stars. The transition occurs near F5. Data from: Slettebak *et al.* (1975), Conti and Ebbets (1977), Soderblom (1982), Gray (1984b), Halbedel (1996), Penny (1996), Fekel (1997, 2003), and Royer *et al.* (2002b).

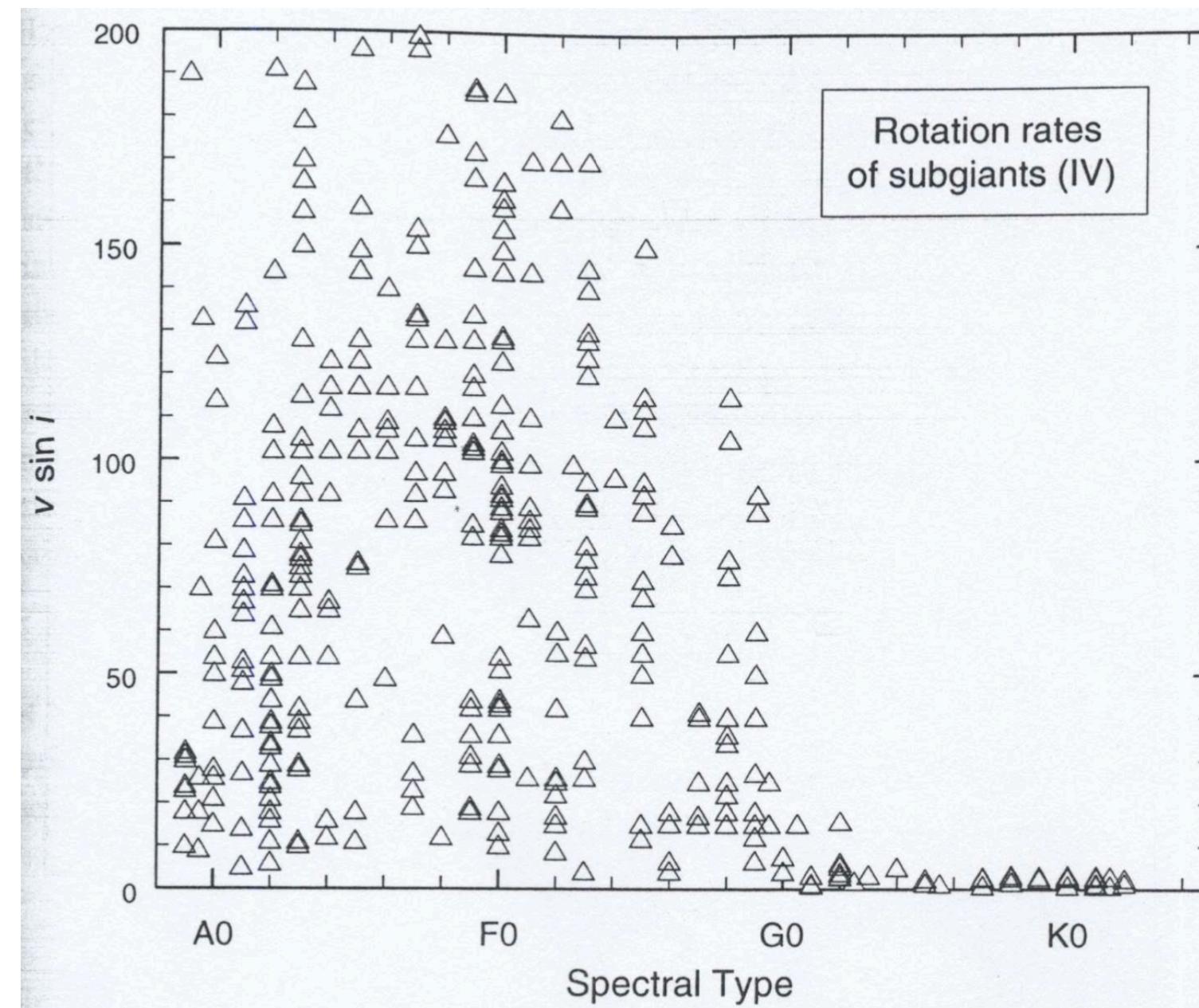


Fig. 18.23. Subgiants show a wide range of rotation rates for stars hotter than G0 IV, but slow rotation for cooler stars. Data from: Uesugi and Fukuda (1982), Gray and Nagar (1985), Fekel (1997, 2003), and Royer *et al.* (2002b).

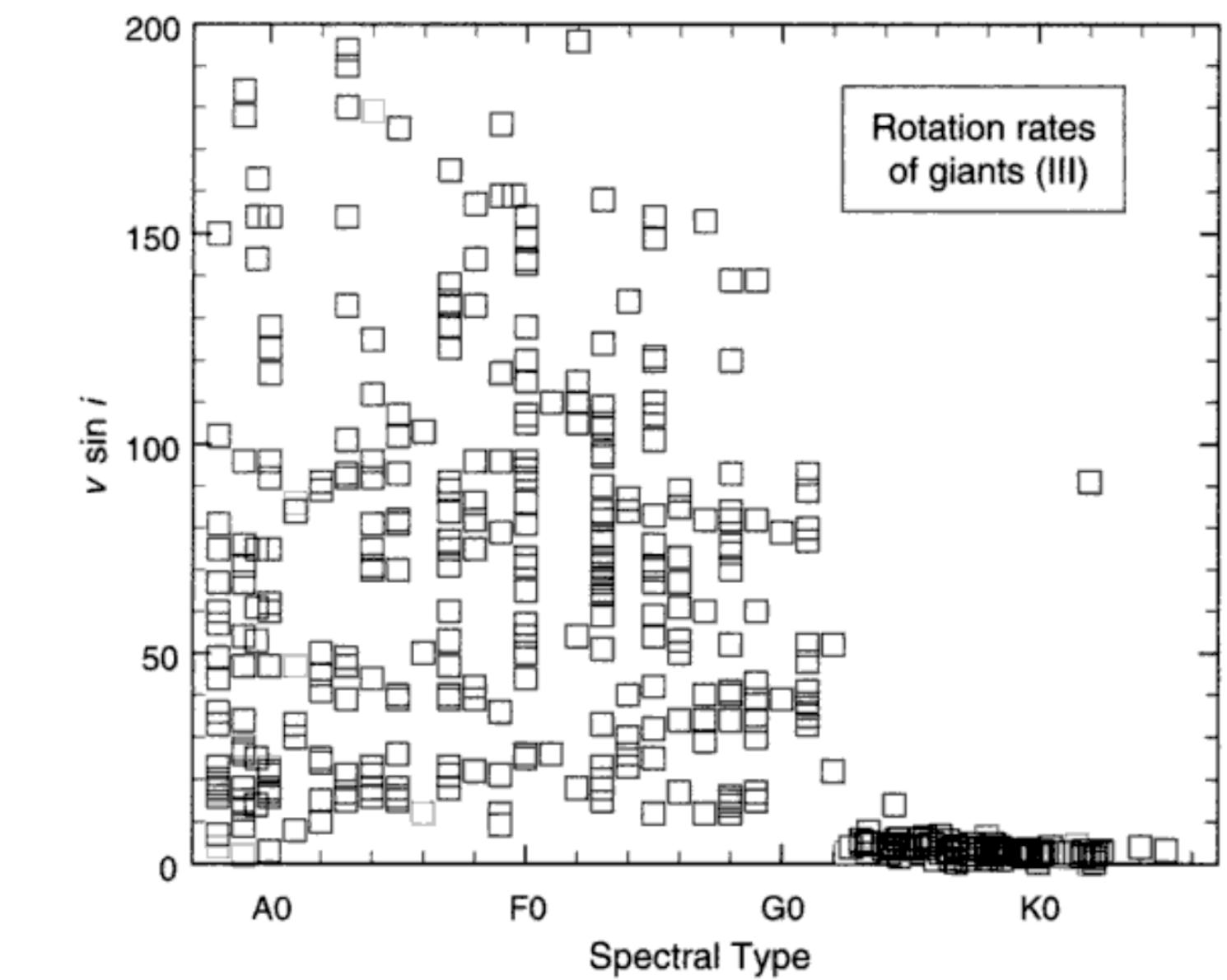


Fig. 18.24. Giants mimic the lower luminosity classes, but the drop in rotation occurs at G2 III. From G2 III to K2 III, rotation is a single-valued function of spectral type as given by Eq. (18.25). Data from: Alschuler (1975), Gray (1982c, 1989a), Hoffleit and Jaschek (1982), Royer *et al.* (2002b), and Fekel (2003).

$$G(\Delta\lambda) = G(v) = \frac{2(1 - \epsilon)[1 - (v_z/v_L)^2]^{1/2} + \frac{1}{2}\pi\epsilon[1 - (v_z/v_L)^2]}{\pi v_L(1 - \epsilon/3)}$$

-

Ref: D.F.Gray The Observation & Analysis of Stellar Photosphere

Linear limb-darkening coefficients

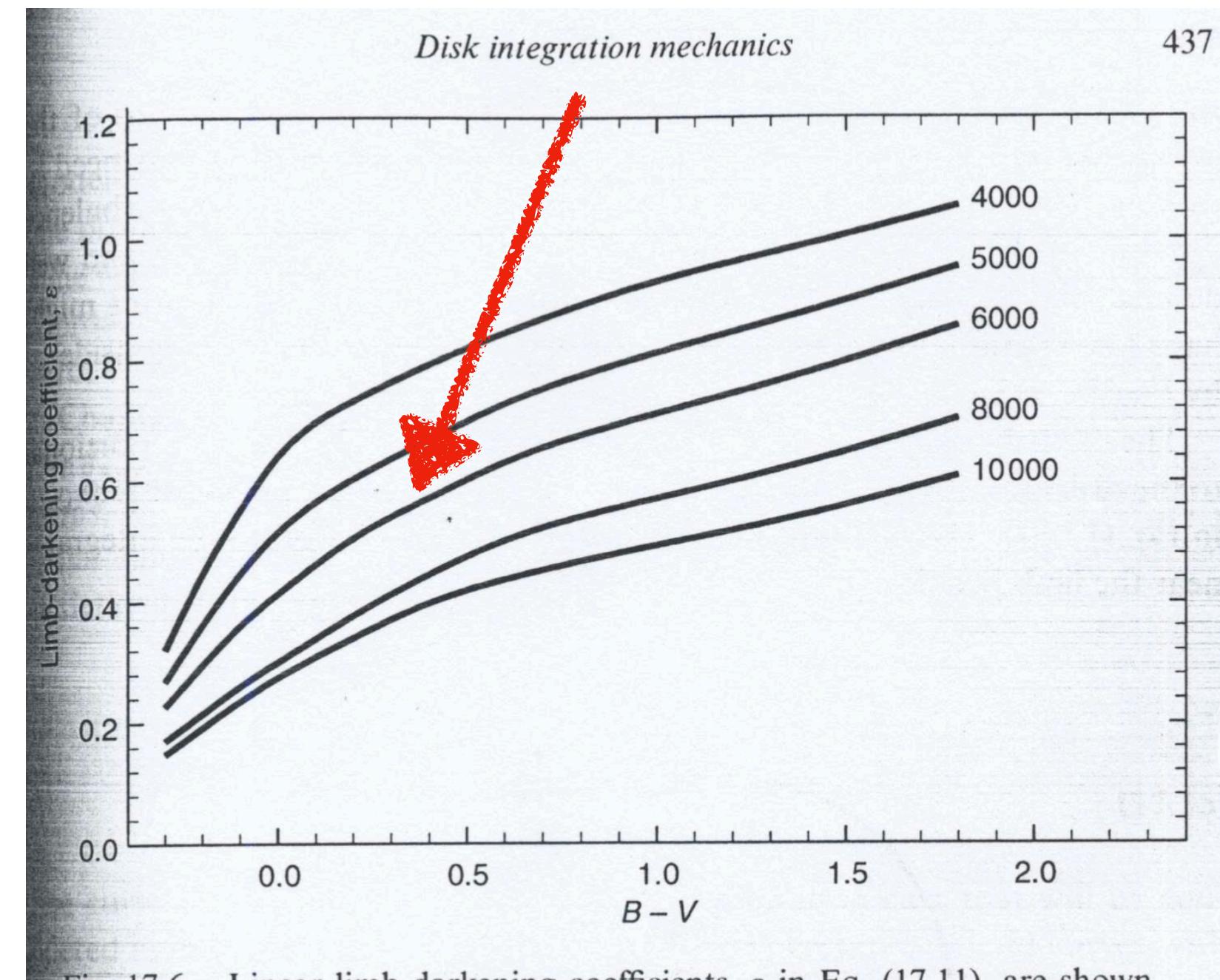


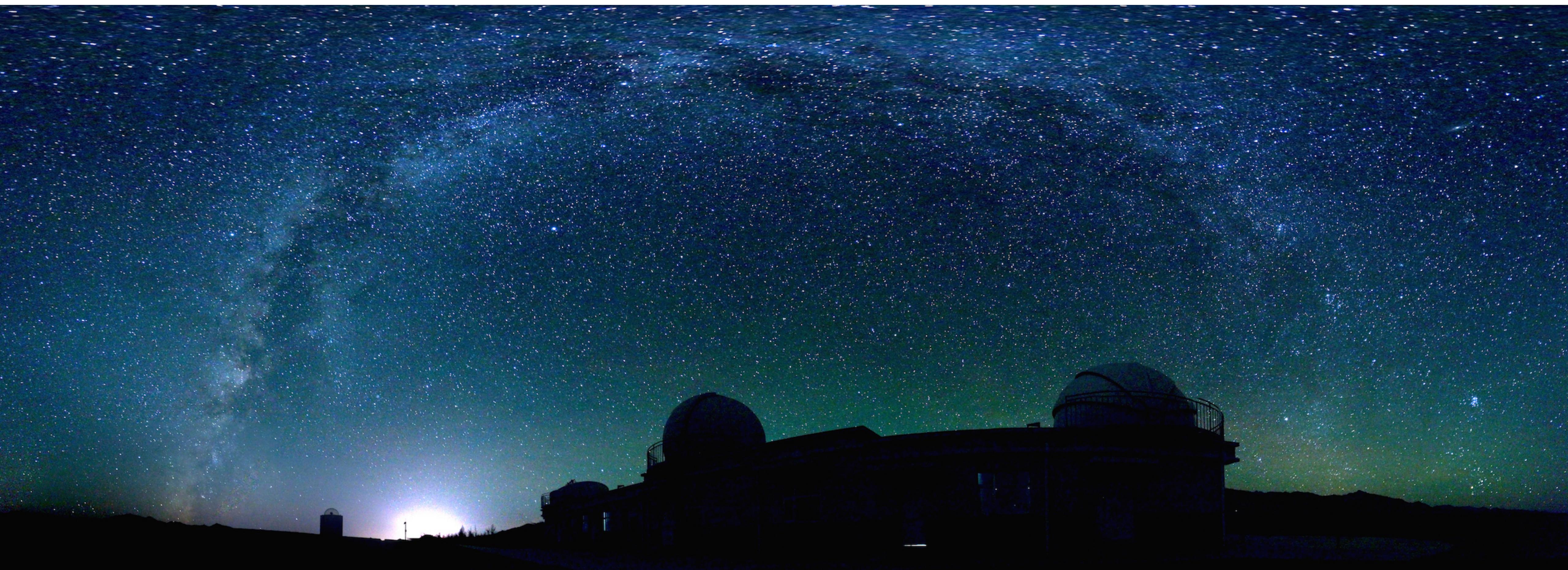
Fig. 17.6. Linear limb-darkening coefficients, ϵ in Eq. (17.11), are shown here as a function of $B - V$ color index of the star for a selection of wavelengths. Both coordinates depend slightly on metallicity and surface gravity. Expressions more precise than Eq. (17.11) can be used, but the basic uncertainty in temperature distribution limits what can be gained.

$$G(\Delta\lambda) = G(v) = \frac{2(1 - \epsilon)[1 - (v_z/v_L)^2]^{1/2} + \frac{1}{2}\pi\epsilon[1 - (v_z/v_L)^2]}{\pi v_L(1 - \epsilon/3)}$$

Summary

- **History of stellar spectroscopy**
- **How to measure RV**
 - Line profile fitting
 - Cross-correlation function (CCF)
- **How to measure atmospheric parameters**
 - EW-parameter relation
 - Forward model – Synthetic model / ThePayne/SLAM
 - Backward model – ANN/CNN

Thanks



Practice makes perfect

<https://github.com/hypergravity/spectroscopy>

- Practice
 - CCF: 试利用模板光谱(wave_temp, flux_temp)估计假光谱(wave_obs, flux_obs)的视向速度及其误差
 - SLAM: 运行代码，找出流程错误，并改正（小错误，改正不会超过10行代码）
- Think about
 - How to estimate $v_{\text{sin}i}$?
 - How to estimate more elemental abundances?
 - Can multi-band photometry / parallax help?
- Q&A...