Falling Together: Why Springs Can't See the Sun

Alexander Friedrich[1]

[1] *DLR Institute of Quantum Technologies, Wilhelm-Runge-Str. 10, 89081 Ulm, Germany*

(Dated: February 19, 2026)

A common claim holds that a sensor tilted relative to the local vertical can detect the direct gravitational pull of the Sun ($\sim 6 \times 10^{-3}$ m/s^2) or the Moon ($\sim 3.3 \times 10^{-5}$ m/s^2) as a time-varying signal. This is incorrect. We show, using only Newtonian point-mass mechanics, that the direct pull cancels exactly against the acceleration of the laboratory, leaving only the tidal residual — regardless of sensor orientation. The proof is given in an inertial frame where no terms are hidden, and the result is then transformed to the rotating laboratory frame.

I. INTRODUCTION

Can a spring-based accelerometer measure the Sun's direct gravitational pull on its test mass? This note shows that the answer is **no** — not because the pull is too small, but because the equivalence principle forbids it. The Sun pulls the sensor housing and the test mass by exactly the same amount; the spring connecting them cannot register a force that acts identically on both ends.

The difficulty in seeing this lies in the choice of reference frame. A laboratory on Earth's surface is non-inertial for two reasons:

1. **Earth rotates**, producing centrifugal and Coriolis pseudo-accelerations (~ 0.03 m/s^2 at the equator).

2. **Earth freely falls toward the Sun and Moon.** The sensor housing is rigidly attached to the Earth's crust; it shares the Earth's free-fall acceleration toward the Sun ($GM_S/R^2 \approx 5.93 \times 10^{-3}$ m/s^2) and Moon ($GM_M/D^2 \approx 3.32 \times 10^{-5}$ m/s^2) exactly. The test mass, connected only by a spring, is the one body free to deviate — but gravity accelerates it by the same amount, so the spring registers no difference.

The second point is the origin of the cancellation. When one writes equations of motion in the lab frame, the free-fall acceleration toward the Sun has already been subtracted from every term. A spring-based sensor physically implements this subtraction: both ends share the same gravitational acceleration, so the spring reads zero from that field. Only the tiny *gradient* of the field across the sensor — the tidal term — survives.

To make this cancellation explicit rather than hidden, we begin in a true inertial frame and carry every term (Sec. II). We then transform to the rotating laboratory (Sec. III), project onto an arbitrary sensor axis (Sec. IV), and compare the magnitudes of all surviving terms (Sec. V). Section VI places the result in the context of the equivalence principle.

II. THE PROOF

A. Setup

Four point masses in an inertial frame (Fig. 1). The test mass is connected to the sensor by a spring (or equivalent restoring mechanism). The sensor housing is rigidly attached to the Earth. The sensor reads the spring force $\mathbf{F}_{\text{spring}}$ — the non-gravitational force required to keep the test mass co-moving with the lab.

B. Test mass equation of motion

In the inertial frame, Newton's second law for the test mass:

$$m\,\ddot{\mathbf{x}} = -\frac{GM_E\,m}{|\mathbf{x}-\mathbf{R}_E|^3}(\mathbf{x}-\mathbf{R}_E) \;-\frac{GM_S\,m}{|\mathbf{x}-\mathbf{R}_S|^3}(\mathbf{x}-\mathbf{R}_S) \;-\frac{GM_M\,m}{|\mathbf{x}-\mathbf{R}_M|^3}(\mathbf{x}-\mathbf{R}_M) \;+\; \mathbf{F}_{\text{spring}} \tag{1}$$

Body	Mass	Position
Sun	M_S	$\mathbf{R}_S(t)$
Moon	M_M	$\mathbf{R}_M(t)$
Earth	M_E	$\mathbf{R}_E(t)$
Test mass	m	$\mathbf{x}(t)$

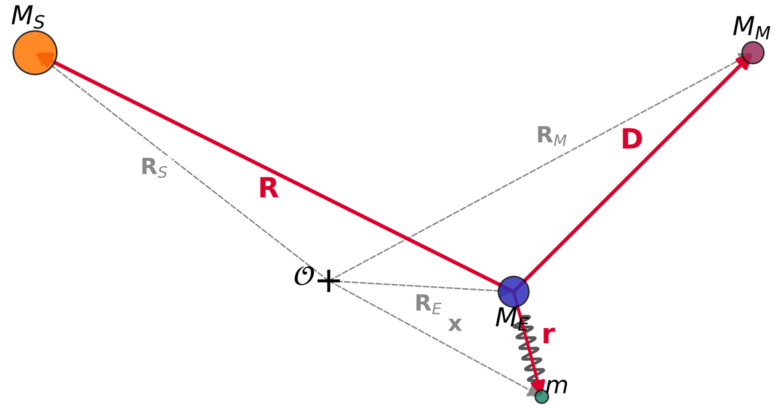

FIG. 1. Four-body setup. *Left*: masses and inertial-frame positions. *Right*: schematic geometry. Gray dashed arrows: position vectors $\mathbf{R}_S$, $\mathbf{R}_E$, $\mathbf{R}_M$, and $\mathbf{x}$ from the inertial origin. Red solid arrows: geocentric vectors $\mathbf{R} = \mathbf{R}_S - \mathbf{R}_E$, $\mathbf{D} = \mathbf{R}_M - \mathbf{R}_E$, and $\mathbf{r} = \mathbf{x} - \mathbf{R}_E$. The spring connects the test mass m to the Earth-fixed sensor housing.

C. Earth's equation of motion

Earth's center accelerates toward the Sun and Moon:

$$\ddot{\mathbf{R}}_E = -\frac{GM_S}{|\mathbf{R}_E - \mathbf{R}_S|^3}(\mathbf{R}_E - \mathbf{R}_S) \; - \; \frac{GM_M}{|\mathbf{R}_E - \mathbf{R}_M|^3}(\mathbf{R}_E - \mathbf{R}_M) \tag{2}$$

Every object rigidly attached to the Earth — including the sensor housing — shares this acceleration.

D. Subtract to get the relative motion

Define $\mathbf{r} \equiv \mathbf{x} - \mathbf{R}_E$ and the geocentric positions $\mathbf{R} \equiv \mathbf{R}_S - \mathbf{R}_E$, $\mathbf{D} \equiv \mathbf{R}_M - \mathbf{R}_E$. Then $\ddot{\mathbf{r}} = \ddot{\mathbf{x}} - \ddot{\mathbf{R}}_E$ gives:

$$m\,\ddot{\mathbf{r}} = -\frac{GM_E\,m}{|\mathbf{r}|^3}\mathbf{r} \; + \; GM_S\,m\left[\frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3} - \frac{\mathbf{R}}{R^3}\right] \; + \; GM_M\,m\left[\frac{\mathbf{D}-\mathbf{r}}{|\mathbf{D}-\mathbf{r}|^3} - \frac{\mathbf{D}}{D^3}\right] \; + \; \mathbf{F}_{\rm spring} \tag{3}$$

E. The cancellation

Each bracketed term has the form

$$\mathbf{f}(\mathbf{r}) = \frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3} - \frac{\mathbf{R}}{R^3} \tag{4}$$

This is the Sun's gravitational field at the test mass *minus* the field at Earth's center. At $\mathbf{r} = \mathbf{0}$:

$$\mathbf{f}(\mathbf{0}) = \frac{\mathbf{R}}{R^3} - \frac{\mathbf{R}}{R^3} = \mathbf{0} \qquad \text{(exactly)} \tag{5}$$

The direct pull — $GM_S\,\mathbf{R}/R^3$, the uniform field that accelerates the entire Earth at 5.93×10^{-3} m/s^2 — appears with equal magnitude and opposite sign in Eqs. (1) and (2), and cancels identically. No approximation is involved. The same holds for the Moon with $\mathbf{D}$ replacing $\mathbf{R}$.

What survives is $\mathbf{f}(\mathbf{r}) - \mathbf{f}(\mathbf{0})$: the *variation* of the gravitational field across the baseline $\mathbf{r}$ — the **tidal acceleration**. To leading order in r/R:

$$\mathbf{a}_{\rm tidal} = \frac{GM}{R^3}\Big[3\,(\mathbf{e_R}\cdot\mathbf{r})\,\mathbf{e_R} - \mathbf{r}\Big] + O\!\left(\frac{GM\,r^2}{R^4}\right) \tag{6}$$

This is suppressed by a factor r/R relative to the direct pull.

F. Derivation of Eq. (6)

Expand $|\mathbf{R}-\mathbf{r}|^{-3}$ for $r \ll R$. Write

$$|\mathbf{R}-\mathbf{r}|^2 = R^2\left(1 - 2\,\frac{\mathbf{e_R}\cdot\mathbf{r}}{R} + \frac{r^2}{R^2}\right) \equiv R^2(1-\epsilon) \tag{7}$$

with $\epsilon = 2\,\mathbf{e_R}\cdot\mathbf{r}/R - r^2/R^2 = O(r/R)$. Then

$$\frac{1}{|\mathbf{R}-\mathbf{r}|^3} = \frac{1}{R^3}(1-\epsilon)^{-3/2} = \frac{1}{R^3}\left(1 + \frac{3\,\mathbf{e_R}\cdot\mathbf{r}}{R} + O\!\left(\frac{r^2}{R^2}\right)\right) \tag{8}$$

Multiply by $(\mathbf{R}-\mathbf{r})$ and keep terms through first order:

$$\frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3} = \frac{1}{R^3}\Big[\mathbf{R} + 3(\mathbf{e_R}\cdot\mathbf{r})\,\mathbf{e_R} - \mathbf{r}\Big] + O\!\left(\frac{r^2}{R^4}\right) \tag{9}$$

Subtract $\mathbf{R}/R^3$:

$$\mathbf{f}(\mathbf{r}) = \frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3} - \frac{\mathbf{R}}{R^3} = \frac{1}{R^3}\Big[3(\mathbf{e_R}\cdot\mathbf{r})\,\mathbf{e_R} - \mathbf{r}\Big] + O\!\left(\frac{r^2}{R^4}\right) \tag{10}$$

The $\mathbf{R}/R^3$ terms cancel identically — confirming that the direct pull drops out — and the leading surviving term is the tidal quadrupole, Eq. (6). □

III. TRANSFORMATION TO THE LABORATORY FRAME

Equation (3) is written in the non-rotating geocentric frame. The laboratory rotates with the Earth at angular velocity $\boldsymbol{\omega}$. This is a second source of non-inertiality (see Sec. I).

A. Rotating-frame equation of motion

Let $\mathbf{r}'$ be the test mass position in the lab frame, related to its geocentric position by $\mathbf{r} = \mathcal{R}(t)\,\mathbf{r}'$ where $\mathcal{R}(t)$ is the time-dependent rotation matrix. For uniform rotation about axis $\hat{\mathbf{n}} = \boldsymbol{\omega}/\omega$ at angular rate $\omega = |\boldsymbol{\omega}|$, it takes the Rodrigues form

$$\mathcal{R}(t) = \mathbf{I}\cos\omega t + (1-\cos\omega t)\,\hat{\mathbf{n}}\,\hat{\mathbf{n}}^T + \sin\omega t\;[\hat{\mathbf{n}}]_\times \tag{11}$$

where $[\hat{\mathbf{n}}]_\times$ is the skew-symmetric matrix whose action on any vector $\mathbf{v}$ is $[\hat{\mathbf{n}}]_\times\mathbf{v} = \hat{\mathbf{n}}\times\mathbf{v}$ — i.e. the matrix representation of the cross product. Its time derivative satisfies

$$\dot{\mathcal{R}}(t) = [\boldsymbol{\omega}]_\times\,\mathcal{R}(t) \tag{12}$$

so that $\dot{\mathcal{R}}\,\mathbf{v} = \boldsymbol{\omega}\times(\mathcal{R}\,\mathbf{v})$ for every vector $\mathbf{v}$. Each time derivative of $\mathcal{R}$ thus generates a cross product with $\boldsymbol{\omega}$.

Differentiating $\mathbf{r} = \mathcal{R}\,\mathbf{r}'$ twice using Eq. (12):

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}' + 2\,\boldsymbol{\omega}\times\dot{\mathbf{r}}' + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}') \tag{13}$$

(the Euler term $\dot{\boldsymbol{\omega}}\times\mathbf{r}'$ vanishes for uniform rotation). Note that $\boldsymbol{\omega}$ carries no prime: $\mathcal{R}(t)$ is a rotation *about* $\boldsymbol{\omega}$, so $\mathcal{R}^{-1}\boldsymbol{\omega} = \boldsymbol{\omega}$ — the rotation vector is invariant under the rotation it generates. Substituting into Eq. (3) and expressing all vectors in the lab basis:

$$\ddot{\mathbf{r}}' = \underbrace{-\frac{GM_E}{|\mathbf{r}'|^3}\,\mathbf{r}'}_{\text{Earth's gravity}} + \underbrace{\mathbf{a}'_{\text{tidal}}}_{\text{tidal}}\;\underbrace{-\,\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}')}_{\text{centrifugal}}\;\underbrace{-\,2\,\boldsymbol{\omega}\times\dot{\mathbf{r}}'}_{\text{Coriolis}} + \frac{\mathbf{F}'_{\text{spring}}}{m} \tag{14}$$

The direct solar and lunar pulls do **not** reappear — they canceled in Eq. (3) before the frame change, and rotating the coordinate basis cannot restore a term that is already zero.

B. What the sensor reads

The sensor housing is held in place by the laboratory floor, which provides whatever normal force $\mathbf{N}$ is required to prevent acceleration. This constraint force acts on the housing, not on the test mass, and therefore does not appear in Eq. (14). Its effect enters through the spring: the housing end of the spring follows the Earth's motion, so the spring force $\mathbf{F}'_{\text{spring}}$ adjusts to keep the test mass co-moving.

The spring force is thus entirely *caused by* the constraint. The floor holds the housing fixed; the spring transmits that constraint to the test mass. This is the equivalence principle in action: if the floor were removed and the entire apparatus fell freely, housing and test mass would share the same gravitational acceleration. The spring would relax and $\mathbf{F}'_{\text{spring}} \to \mathbf{0}$ — precisely because no constraint force distinguishes the housing from the test mass. A spring-based sensor can only measure accelerations that *differ* between its two ends, i.e. tidal effects and non-gravitational forces.

The test mass is thus at rest in the lab: $\dot{\mathbf{r}}' = \ddot{\mathbf{r}}' = \mathbf{0}$. The Coriolis term vanishes. Setting the left-hand side of Eq. (14) to zero determines the spring force — the quantity the sensor reads:

$$\frac{\mathbf{F}'_{\text{spring}}}{m} = \frac{GM_E}{|\mathbf{r}'|^3}\,\mathbf{r}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') - \mathbf{a}'_{\text{tidal}} \tag{15}$$

The sensor projects this onto its measurement axis $\mathbf{e_n}$:

$$g_{\text{measured}} = \frac{\mathbf{F}'_{\text{spring}}}{m} \cdot \mathbf{e_n} \tag{16}$$

The three contributions are:

- $GM_E\,\mathbf{r}'/|\mathbf{r}'|^3$: Earth's gravity (the dominant, static term).
- $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') = -\omega^2 \mathbf{r}'_\perp$: the centrifugal reduction (objects weigh less at the equator by $\sim 0.3\%$).
- $-\mathbf{a}'_{\text{tidal}}$: the tidal perturbation from Sun and Moon, of order 10^{-7} m/s^2.

The direct pull GM_S/R^2 and GM_M/D^2 appear nowhere. This holds for any $\mathbf{e_n}$.

C. Size of each term

Table I lists every acceleration that enters Eq. (15), together with the two direct pulls that cancel before reaching it.

TABLE I. Magnitude of each acceleration term at Earth's surface, ordered by size. The grayed-out rows — the direct gravitational pulls — cancel exactly (Sec. II) and do not appear in the sensor reading [Eq. (15)]. The last column gives the ratio of each term to the solar tidal residual (5.0×10^{-7} m/s^2); for the canceled rows it is the ratio of direct pull to corresponding tidal residual (11,700× for the Sun, 30× for the Moon).

Term	Expression	Magnitude (m/s^2)	Daily variation (m/s^2)	Ratio
Earth's gravity	$GM_E/r_\oplus^2$	9.82	static	2×10^7
Centrifugal (λ=48.4)	$\omega^2 r_\oplus \cos^2\lambda$	$3.4 \times 10^{-2} \cos^2\lambda$	static	$7 \times 10^4 \cos^2\lambda$
Solar direct pull (canceled)	GM_S/R^2	5.93×10^{-3}	—	11,700×
Lunar direct pull (canceled)	GM_M/D^2	3.32×10^{-5}	—	30×
Lunar tide	$GM_M\,r_\oplus/D^3$	1.1×10^{-6}	$\lesssim 2 \times 10^{-6}$	—
Solar tide	$GM_S\,r_\oplus/R^3$	5.0×10^{-7}	$\lesssim 8 \times 10^{-7}$	—

The hierarchy is striking. Earth's gravity dominates at ~ 10 m/s^2. The centrifugal correction is four orders of magnitude smaller ($\sim 10^{-2}$ m/s^2), yet still measurable — it is the reason objects weigh less at the equator than at the poles. The tidal accelerations are smaller still, at 10^{-6}–10^{-7} m/s^2, but are routinely detected by superconducting gravimeters and satellite missions such as GRACE.

The canceled direct pulls occupy a revealing intermediate scale. The Sun's direct pull (5.93×10^{-3} m/s^2) is 180× stronger than the Moon's (3.32×10^{-5} m/s^2), yet its tidal effect is 2.2× *weaker* ($1/R^3$ vs. $1/R^2$ scaling). This inversion — the Sun pulls harder but tidally disturbs less — is a direct signature of the cancellation. If direct pulls were observable, the Sun would dominate the signal by two orders of magnitude. Instead, the Moon dominates the tidal signal, exactly as observed.

The sensor reads a superposition of the four non-grayed terms in Table I. The direct pulls have been subtracted out by the physics of the measurement: the lab accelerates with the Earth, and the spring cannot see it.

IV. WHY SENSOR ORIENTATION IS IRRELEVANT

The cancellation in Eq. (5) is **vectorial**: the direct pull vanishes as a three-component vector, not as a particular scalar projection. Projecting onto any measurement axis $\mathbf{e_n}$:

$$\mathbf{e_n} \cdot \mathbf{f}(\mathbf{0}) = \mathbf{e_n} \cdot \mathbf{0} = 0 \qquad \text{for all } \mathbf{e_n} \tag{17}$$

Tilting the sensor changes $\mathbf{e_n}$ but cannot make zero non-zero. The spring force along any axis contains only the tidal terms from Eq. (6).

Physically: the sensor housing and the test mass both accelerate at GM_S/R^2 toward the Sun. The spring connecting them measures only the *difference* between the accelerations of its two endpoints. A uniform field produces no difference, regardless of the spring's orientation.

A. Sweeping the sensor orientation

Parameterise the measurement axis by zenith angle θ and azimuth φ:

$$\mathbf{e_n}(\theta, \varphi) = \sin\theta\cos\varphi\,\mathbf{e_E} + \sin\theta\sin\varphi\,\mathbf{e_N} + \cos\theta\,\mathbf{e_U} \tag{18}$$

where $\mathbf{e_E}$, $\mathbf{e_N}$, $\mathbf{e_U}$ are the local East–North–Up unit vectors defined relative to the *geodetic* vertical. Projecting Eq. (15) onto $\mathbf{e_n}(\theta, \varphi)$ and writing each term explicitly:

$$\begin{aligned} g(\theta, \varphi) &= \underbrace{\frac{GM_E}{r_\oplus^2}\cos\theta \;+\; \left[\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}')\right] \cdot \mathbf{e_n}}_{\text{gravity + centrifugal}} \;+\; \underbrace{(-2\boldsymbol{\omega} \times \dot{\mathbf{r}}') \cdot \mathbf{e_n}}_{=0\ \text{(static)}} \;-\; \mathbf{a}'_{\text{tidal}} \cdot \mathbf{e_n} \\ &= g_0\cos\theta \;-\; a'_E\sin\theta\cos\varphi \;-\; a'_N\sin\theta\sin\varphi \;-\; a'_U\cos\theta \end{aligned} \tag{19}$$

In the first line, the gravitational and centrifugal projections combine into a single $\cos\theta$ term because both point along $\mathbf{e_U}$, and the Coriolis term vanishes for a stationary test mass ($\dot{\mathbf{r}}' = \mathbf{0}$). The result defines the effective gravity $g_0 \equiv GM_E/r_\oplus^2 - \omega^2 r_\oplus \cos^2\lambda$ (where λ is the geodetic latitude), absorbing the centrifugal correction into the amplitude.

We now trace each contribution through this projection.

Earth's gravity and centrifugal ($g_0\cos\theta$). The effective gravity $\mathbf{g}_{\text{eff}} = g_0\,\mathbf{e_U}$ points purely along the geodetic vertical by definition. Its projection onto the sensor axis is $g_0\cos\theta$: it reads the full $g_0 \approx 9.81$ m/s^2 when the sensor is vertical ($\theta = 0$), vanishes when horizontal ($\theta = 90$), and reverses when inverted ($\theta = 180$). The centrifugal correction modifies g_0 by at most 0.3% (latitude-dependent) but introduces no separate angular signature — it is entirely absorbed into the amplitude of the $\cos\theta$ term. There is no "rotational residual" in the sweep.

Solar and lunar tides (a'_E, a'_N, a'_U). The tidal accelerations from Eq. (6) decompose into East, North, and Up components. The horizontal components a'_E and a'_N enter through $\sin\theta\cos\varphi$ and $\sin\theta\sin\varphi$: they vanish for a vertical sensor and are maximized for a horizontal one pointed in the appropriate direction. The vertical component a'_U enters as $\cos\theta$, superimposed on the much larger $g_0\cos\theta$. Representative combined amplitudes (from Sec. III): solar tidal $\sim 5.0 \times 10^{-7}$ m/s^2, lunar tidal $\sim 1.1 \times 10^{-6}$ m/s^2. These are the only time-varying signals in the sweep.

Direct solar pull (canceled). Were the direct pull GM_S/R^2 not canceled by Earth's free fall, it would project as $a_\odot \sin\theta\cos(\varphi - \varphi_\odot)$ with amplitude $a_\odot \approx 5.93 \times 10^{-3}$ m/s^2 — four orders of magnitude above the tidal terms. As shown in Fig. 2, no such signal exists at any orientation. An angular scan therefore confirms the cancellation directly: one measures $g_0\cos\theta$ plus tidal corrections of order 10^{-7} m/s^2, with no 10^{-3} m/s^2 sinusoidal component.

V. WHAT IS OBSERVABLE

Given that the direct pulls cancel, what *does* a tilted sensor actually measure? From Eq. (19), the sensor reading decomposes into three physically distinct contributions. Each responds differently to changes in orientation and time.

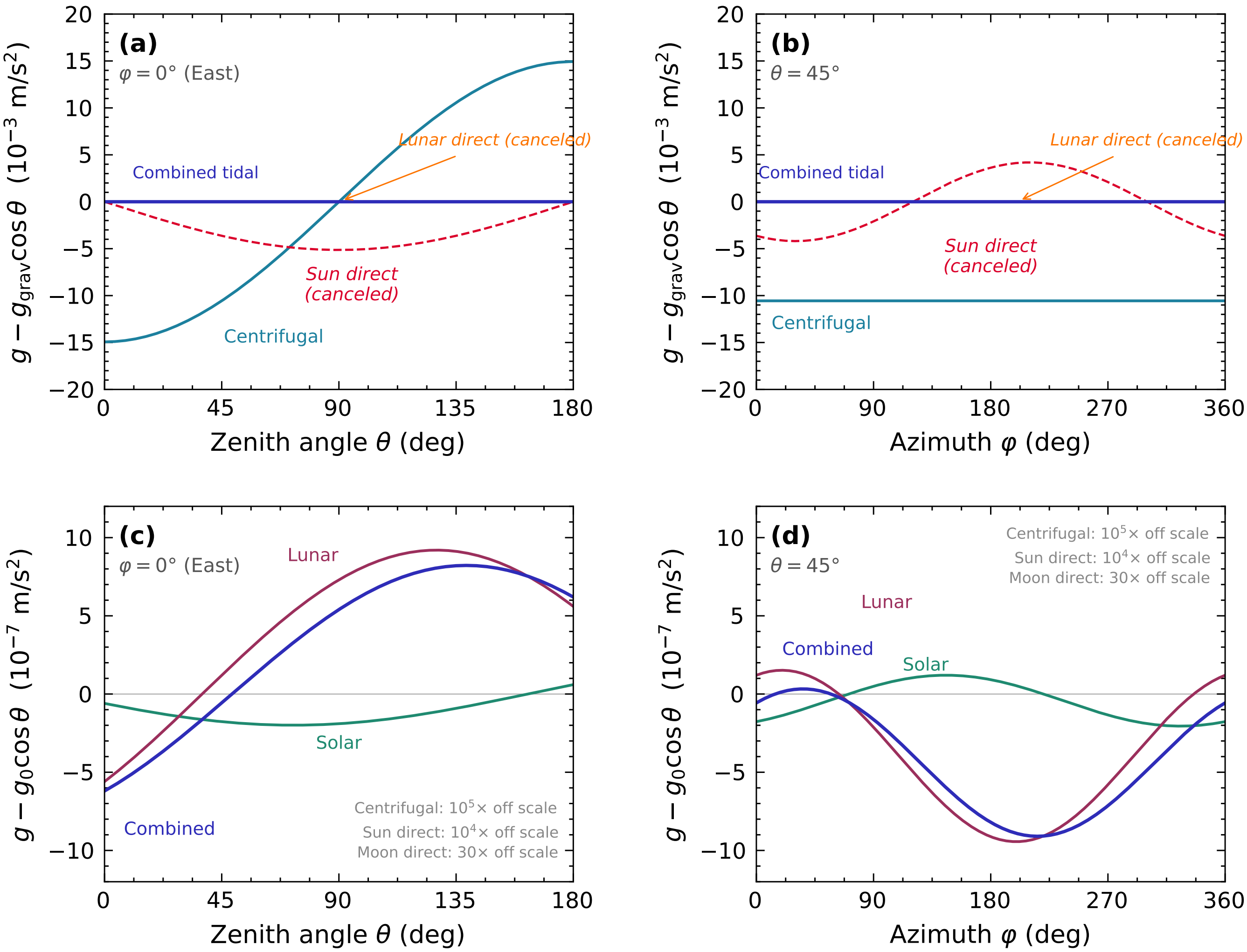

FIG. 2. Residual acceleration $g - g_{\rm grav}\cos\theta$ as a function of sensor orientation, where $g_{\rm grav}$ is the pure gravitational acceleration (without centrifugal). Left column: sweep over zenith angle θ at fixed azimuth $\varphi = 0$ (East). Right column: sweep over azimuth φ at fixed $\theta = 45$. **Top row** (a, b): 10^{-3} m/s^2 scale. Solid blue: centrifugal correction ($\omega^2 r_\oplus \cos^2\lambda \approx 1.7 \times 10^{-2}$ m/s^2 at Ulm, $\lambda = 48.4$); red dashed: hypothetical Sun direct pull (5.93×10^{-3} m/s^2); orange dashed: hypothetical Moon direct pull (3.3×10^{-5} m/s^2); blue solid: combined tidal residual (indistinguishable from zero at this scale). The centrifugal term is the largest non-gravitational contribution and exceeds even the (canceled) Sun direct pull. **Bottom row** (c, d): 10^{-7} m/s^2 scale, zoomed to the actual tidal signals. Teal: solar tidal; magenta: lunar tidal; blue: combined. The centrifugal ($10^5\times$), canceled direct Sun ($10^4\times$), and Moon ($30\times$) pulls are all off this scale. No orientation reveals the claimed 10^{-3} m/s^2 signal.

A. Static baseline: gravity and centrifugal

The dominant term $g_0 \cos\theta$ is the projection of the effective gravity onto the measurement axis. It is entirely static — independent of time, the positions of Sun and Moon, or anything else in the sky. Tilting the sensor from vertical ($\theta = 0$) to horizontal ($\theta = 90$) sweeps this term from $g_0 \approx 9.81$ m/s^2 through zero.

Embedded within g_0 is the centrifugal correction, which at Ulm amounts to $\omega^2 r_\oplus \cos^2\lambda \approx 1.5 \times 10^{-2}$ m/s^2. It projects as $1.5 \times 10^{-2} \cos\theta$ m/s^2 and is visible in Fig. 2 (panels a, b) as the solid blue curve. This is the largest non-gravitational contribution the sensor can detect — it exceeds even the (canceled) solar direct pull by a factor of 2.5. Unlike the tidal terms, however, it carries no time dependence: it is a fixed geometric projection that shifts the baseline reading as the sensor is tilted.

B. Time-varying signals: tidal components

The only orientation-dependent terms that vary in time are the tidal projections $a'_E \sin\theta\cos\varphi$, $a'_N \sin\theta\sin\varphi$, and $a'_U \cos\theta$. These encode the three components of the tidal acceleration vector in the geodetic frame:

- A vertical sensor ($\theta = 0$) measures a'_U, the vertical tidal component ($\sim 5.0 \times 10^{-7}$ m/s^2, semi-diurnal with ~ 12 h period).
- A horizontal sensor ($\theta = 90$) pointing East ($\varphi = 0$) measures a'_E; pointing North ($\varphi = 90$) it measures a'_N. These horizontal components are diurnal (~ 24 h period).
- Three non-coplanar sensors reconstruct the full tidal acceleration vector and, in principle, the tidal tensor.

The lunar tidal signal ($\sim 1.1 \times 10^{-6}$ m/s^2) is roughly twice the solar ($\sim 5.0 \times 10^{-7}$ m/s^2), consistent with the $1/R^3$ scaling that favors the nearby Moon over the distant Sun. Superconducting gravimeters routinely resolve these signals; they are the standard observable in terrestrial tidal gravimetry.

C. What is absent

Conspicuously absent from Eq. (19) are two terms that would dominate the reading if direct gravitational pulls were not canceled:

- A **solar** term of order 5.93×10^{-3} m/s^2, projecting as $a_\odot \sin\theta\cos(\varphi - \varphi_\odot)$ with a diurnal period (~ 24 h). This would exceed the actual tidal signals by four orders of magnitude.
- A **lunar** term of order 3.32×10^{-5} m/s^2, projecting as $a_M \sin\theta\cos(\varphi - \varphi_M)$ with a period of ~ 24.8 h. Smaller than the solar term by a factor of 180, but still $30\times$ larger than the lunar tidal signal it would accompany.

Figure 2 confirms their absence directly: the top panels show both hypothetical direct pulls as dashed curves that dwarf the actual tidal residual (blue, indistinguishable from zero at this scale). The bottom panels zoom into the 10^{-7} m/s^2 scale where the real tidal signals live — orders of magnitude below either claimed effect.

All orientation-dependent signals the sensor can detect — centrifugal and tidal — are real and well understood. None is a direct gravitational pull.

VI. THE EQUIVALENCE PRINCIPLE

The cancellation derived in Sec. II is not a calculational coincidence. It is the equivalence principle.

Einstein's key insight was that a uniform gravitational field is locally indistinguishable from an accelerating reference frame. No experiment confined to a sufficiently small laboratory can determine whether the laboratory is at rest in a gravitational field or accelerating through empty space. This means that a freely falling observer cannot detect the uniform component of any gravitational field — only its *gradient* (the tidal part) is locally measurable.

The Earth, together with the sensor, the laboratory, and the observer, is in free fall in the Sun's gravitational field. The entire system accelerates at GM_S/R^2 toward the Sun. A spring-based sensor measures the force difference between its two endpoints. Because both endpoints share the same free-fall acceleration, the spring is blind to the Sun's uniform pull. Only the *non-uniformity* of the field across the sensor — the tidal term, of order $GM_S\, r_\oplus/R^3$ — can produce a differential signal.

In the language of general relativity, the Earth follows a geodesic in the Sun's spacetime. Along a geodesic, no proper acceleration is felt — the "force of gravity" is replaced by the geometry of spacetime. What remains observable is geodesic deviation: nearby geodesics converge or diverge due to spacetime curvature. This is the tidal acceleration of Eq. (6), encoded in the Riemann tensor. The direct pull GM_S/R^2 has no counterpart in this description; it is an artifact of the Newtonian decomposition into "gravitational force" and "inertial motion," a decomposition that the equivalence principle declares unphysical.

This is why no adjustment of sensor orientation, no change of measurement axis, and no increase in sensitivity can reveal a 5.93×10^{-3} m/s^2 signal from the Sun. The signal does not exist at the sensor because the equivalence principle forbids it. What the sensor can measure — and what is measured routinely — are tidal effects at 10^{-7}–10^{-6} m/s^2, four orders of magnitude below the direct pull. The proof in Sec. II is the Newtonian statement of this principle; the equivalence principle elevates it from a property of gravity to a law of nature.