Why a Tilted Sensor Cannot Measure Direct Gravitational Pull

The Claim

> A sensor tilted relative to the local vertical can detect the direct gravitational pull of the Sun ($\sim 6 \times 10^{-3}$ m/s^2) or the Moon ($\sim 3.3 \times 10^{-5}$ m/s^2) as a time-varying signal, because the projection of the gravitational acceleration onto the sensor axis changes as these bodies move across the sky.

This is incorrect. The direct pull cancels exactly against the acceleration of the laboratory, leaving only the tidal residual — regardless of sensor orientation. The proof requires nothing beyond Newton's laws applied to point masses.

1 Inertial and Non-Inertial Frames

An **inertial frame** is one in which Newton's second law $\mathbf{F} = m\,\mathbf{a}$ holds without correction. A body subject to no forces moves in a straight line at constant velocity.

A **non-inertial frame** is one that accelerates. In such a frame, Newton's laws require fictitious forces — centrifugal, Coriolis, etc. — to compensate for the acceleration of the frame itself.

The laboratory is non-inertial

A laboratory on Earth's surface is non-inertial for two reasons:

1. **Earth rotates**, producing centrifugal and Coriolis pseudo-accelerations (~ 0.03 m/s^2 at the equator).
2. **Earth freely falls toward the Sun and Moon.** The entire Earth — lab, sensor, test mass, and all — accelerates toward the Sun at $GM_S/R^2 \approx 5.93 \times 10^{-3}$ m/s^2 and toward the Moon at $GM_M/D^2 \approx 3.3 \times 10^{-5}$ m/s^2.

The second point is the origin of the cancellation. When you write equations in the lab frame, you have already subtracted these accelerations from every term. A sensor physically implements this subtraction: both ends of the spring accelerate identically in the Sun's field, so the spring reads zero from that field.

To see this without hidden subtractions, we work in a true inertial frame and carry every term explicitly.

2 The Proof

Setup

Four point masses in an inertial frame:

Body	Mass	Position
Sun	M_S	$\mathbf{R}_S(t)$
Moon	M_M	$\mathbf{R}_M(t)$
Earth	M_E	$\mathbf{R}_E(t)$
Test mass	m	$\mathbf{x}(t)$

The test mass is connected to the sensor by a spring (or equivalent restoring mechanism). The sensor housing is rigidly attached to the Earth. The sensor reads the spring force $\mathbf{F}_{\text{spring}}$ — the non-gravitational force required to keep the test mass co-moving with the lab.

Step 1: Test mass equation of motion

In the inertial frame, Newton's second law for the test mass:

$$m\,\ddot{\mathbf{x}} = -\frac{GM_E\,m}{|\mathbf{x}-\mathbf{R}_E|^3}(\mathbf{x}-\mathbf{R}_E) \;-\; \frac{GM_S\,m}{|\mathbf{x}-\mathbf{R}_S|^3}(\mathbf{x}-\mathbf{R}_S) \;-\; \frac{GM_M\,m}{|\mathbf{x}-\mathbf{R}_M|^3}(\mathbf{x}-\mathbf{R}_M) \;+\; \mathbf{F}_{\text{spring}} \tag{1}$$

Step 2: Earth's equation of motion

Earth's center accelerates toward the Sun and Moon:

$$\ddot{\mathbf{R}}_E = -\frac{GM_S}{|\mathbf{R}_E-\mathbf{R}_S|^3}(\mathbf{R}_E-\mathbf{R}_S) \;-\; \frac{GM_M}{|\mathbf{R}_E-\mathbf{R}_M|^3}(\mathbf{R}_E-\mathbf{R}_M) \tag{2}$$

Every object rigidly attached to the Earth — including the sensor housing — shares this acceleration.

Step 3: Subtract to get the relative motion

Define $\mathbf{r} \equiv \mathbf{x}-\mathbf{R}_E$ and the geocentric positions $\mathbf{R} \equiv \mathbf{R}_S-\mathbf{R}_E$, $\mathbf{D} \equiv \mathbf{R}_M-\mathbf{R}_E$. Then $\ddot{\mathbf{r}} = \ddot{\mathbf{x}}-\ddot{\mathbf{R}}_E$ gives:

$$\boxed{m\,\ddot{\mathbf{r}} = -\frac{GM_E\,m}{|\mathbf{r}|^3}\,\mathbf{r} \;+\; GM_S\,m\left[\frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3} - \frac{\mathbf{R}}{R^3}\right] \;+\; GM_M\,m\left[\frac{\mathbf{D}-\mathbf{r}}{|\mathbf{D}-\mathbf{r}|^3} - \frac{\mathbf{D}}{D^3}\right] \;+\; \mathbf{F}_{\text{spring}}} \tag{3}$$

The cancellation

Each bracketed term has the form

$$\mathbf{f}(\mathbf{r}) = \frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3} - \frac{\mathbf{R}}{R^3} \tag{4}$$

This is the Sun's gravitational field at the test mass *minus* the field at Earth's center. At $\mathbf{r}=\mathbf{0}$:

$$\mathbf{f}(\mathbf{0}) = \frac{\mathbf{R}}{R^3} - \frac{\mathbf{R}}{R^3} = \mathbf{0} \qquad \text{(exactly)} \tag{5}$$

The direct pull — $GM_S\,\mathbf{R}/R^3$, the uniform field that accelerates the entire Earth at 5.93×10^{-3} m/s^2 — appears with equal magnitude and opposite sign in Eqs. (1) and (2), and cancels identically. No approximation is involved. The same holds for the Moon with $\mathbf{D}$ replacing $\mathbf{R}$.

What survives is $\mathbf{f}(\mathbf{r})-\mathbf{f}(\mathbf{0})$: the *variation* of the gravitational field across the baseline $\mathbf{r}$ — the **tidal acceleration**. To leading order in r/R:

$$\boxed{\mathbf{a}_{\text{tidal}} = \frac{GM}{R^3}\Big[3\,(\mathbf{e}_{\mathbf{R}}\cdot\mathbf{r})\,\mathbf{e}_{\mathbf{R}} - \mathbf{r}\Big] + O\!\left(\frac{GM\,r^2}{R^4}\right)} \tag{6}$$

This is suppressed by a factor r/R relative to the direct pull.

Derivation of Eq. (6)

Expand $|\mathbf{R}-\mathbf{r}|^{-3}$ for $r \ll R$. Write

$$|\mathbf{R}-\mathbf{r}|^2 = R^2\left(1 - 2\,\frac{\mathbf{e_R}\cdot\mathbf{r}}{R} + \frac{r^2}{R^2}\right) \equiv R^2(1-\epsilon) \tag{7}$$

with $\epsilon = 2\,\mathbf{e_R}\cdot\mathbf{r}/R - r^2/R^2 = O(r/R)$. Then

$$\frac{1}{|\mathbf{R}-\mathbf{r}|^3} = \frac{1}{R^3}(1-\epsilon)^{-3/2} = \frac{1}{R^3}\left(1 + \frac{3\,\mathbf{e_R}\cdot\mathbf{r}}{R} + O\left(\frac{r^2}{R^2}\right)\right) \tag{8}$$

Multiply by $(\mathbf{R}-\mathbf{r})$ and keep terms through first order:

$$\frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3} = \frac{1}{R^3}\Big[\mathbf{R} + 3(\mathbf{e_R}\cdot\mathbf{r})\,\mathbf{e_R} - \mathbf{r}\Big] + O\left(\frac{r^2}{R^4}\right) \tag{9}$$

Subtract $\mathbf{R}/R^3$:

$$\mathbf{f}(\mathbf{r}) = \frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3} - \frac{\mathbf{R}}{R^3} = \frac{1}{R^3}\Big[3(\mathbf{e_R}\cdot\mathbf{r})\,\mathbf{e_R} - \mathbf{r}\Big] + O\left(\frac{r^2}{R^4}\right) \tag{10}$$

The $\mathbf{R}/R^3$ terms cancel identically — confirming that the direct pull drops out — and the leading surviving term is the tidal quadrupole, Eq. (6). $\square$

3 Transformation to the Laboratory Frame

Equation (3) is written in the non-rotating geocentric frame. The laboratory rotates with the Earth at angular velocity $\boldsymbol{\omega}$. This is a second source of non-inertiality (see Section 1).

Rotating-frame equation of motion

Let $\mathbf{r}'$ be the test mass position in the lab frame, related to its geocentric position by $\mathbf{r} = \mathcal{R}(t)\,\mathbf{r}'$ where $\mathcal{R}(t)$ is the time-dependent rotation matrix. Differentiating twice:

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}' + 2\,\boldsymbol{\omega}\times\dot{\mathbf{r}}' + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}') \tag{11}$$

(the Euler term $\dot{\boldsymbol{\omega}}\times\mathbf{r}'$ vanishes for uniform rotation). Substituting into Eq. (3) and expressing all vectors in the lab basis:

$$\boxed{\ddot{\mathbf{r}}' = \underbrace{-\frac{GM_E}{|\mathbf{r}'|^3}\,\mathbf{r}'}_{\text{Earth's gravity}} + \underbrace{\mathbf{a}'_{\text{tidal}}}_{\text{tidal}} \underbrace{-\,\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}')}_{\text{centrifugal}} \underbrace{-\,2\,\boldsymbol{\omega}\times\dot{\mathbf{r}}'}_{\text{Coriolis}} + \frac{\mathbf{F}'_{\text{spring}}}{m}} \tag{12}$$

The direct solar and lunar pulls do **not** reappear — they canceled in Eq. (3) before the frame change, and rotating the coordinate basis cannot restore a term that is already zero.

What the sensor reads

The test mass sits at rest in the lab: $\dot{\mathbf{r}}' = \ddot{\mathbf{r}}' = \mathbf{0}$. The Coriolis term vanishes. Setting the left-hand side of Eq. (12) to zero:

$$\frac{\mathbf{F}'_{\text{spring}}}{m} = \frac{GM_E}{|\mathbf{r}'|^3}\,\mathbf{r}' + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}') - \mathbf{a}'_{\text{tidal}} \tag{13}$$

The sensor projects this onto its measurement axis $\mathbf{e_n}$:

$$g_{\text{measured}} = \frac{\mathbf{F}'_{\text{spring}}}{m} \cdot \mathbf{e_n} \tag{14}$$

The three contributions are:

- $GM_E\,\mathbf{r}'/|\mathbf{r}'|^3$: Earth's gravity (the dominant, static term).
- $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') = -\omega^2 \mathbf{r}'_\perp$: the centrifugal reduction (objects weigh less at the equator by $\sim 0.3\%$).
- $-\mathbf{a}'_{\text{tidal}}$: the tidal perturbation from Sun and Moon, of order 10^{-7} m/s^2.

The direct pull GM_S/R^2 and GM_M/D^2 appear nowhere. This holds for any $\mathbf{e_n}$.

Size of each term

Term	Expression	Magnitude
Earth's gravity	$GM_E/r_\oplus^2$	9.81 m/s^2
Centrifugal (equator)	$\omega^2 r_\oplus$	3.4×10^{-2} m/s^2
Lunar tide	$GM_M\,r_\oplus/D^3$	1.1×10^{-6} m/s^2
Solar tide	$GM_S\,r_\oplus/R^3$	5.1×10^{-7} m/s^2
Solar direct pull (canceled)	GM_S/R^2	5.93×10^{-3} m/s^2

The sensor reads a superposition of the first four terms. The fifth — the direct pull — has been subtracted out by the physics of the measurement: the lab accelerates with the Earth, and the spring cannot see it.

4 Why Sensor Orientation Is Irrelevant

The cancellation in Eq. (5) is **vectorial**: the direct pull vanishes as a three-component vector, not as a particular scalar projection. Projecting onto any measurement axis $\mathbf{e_n}$:

$$\mathbf{e_n} \cdot \mathbf{f}(\mathbf{0}) = \mathbf{e_n} \cdot \mathbf{0} = 0 \qquad \text{for all } \mathbf{e_n} \tag{15}$$

Tilting the sensor changes $\mathbf{e_n}$ but cannot make zero non-zero. The spring force along any axis contains only the tidal terms from Eq. (6).

Physically: the sensor housing and the test mass both accelerate at GM_S/R^2 toward the Sun. The spring connecting them measures only the *difference* between the accelerations of its two endpoints. A uniform field produces no difference, regardless of the spring's orientation.

Sweeping the sensor orientation

Parameterise the measurement axis by zenith angle θ and azimuth φ:

$$\mathbf{e_n}(\theta, \varphi) = \sin\theta\cos\varphi\,\mathbf{e_E} + \sin\theta\sin\varphi\,\mathbf{e_N} + \cos\theta\,\mathbf{e_U} \tag{16}$$

where $\mathbf{e_E}$, $\mathbf{e_N}$, $\mathbf{e_U}$ are the local East–North–Up unit vectors.

From Eq. (13), the measured acceleration as a function of orientation is

$$g(\theta, \varphi) = g_0 \cos\theta \;-\; a'_E \sin\theta\cos\varphi \;-\; a'_N \sin\theta\sin\varphi \;-\; a'_U \cos\theta \tag{17}$$

where $g_0 \equiv GM_E/r_\oplus^2 - \omega^2 r_\perp$ is the static effective gravity, and a'_E, a'_N, a'_U are the East, North, and Up components of the tidal acceleration.

Sweeping θ and φ traces out the full angular dependence. If the direct pull GM_S/R^2 were present, it would contribute a term $\propto \sin\theta\cos(\varphi-\varphi_\odot)$ with amplitude $\sim 6\times10^{-3}$ m/s^2. Instead, the horizontal terms carry amplitudes $a'_E, a'_N \sim 10^{-7}$ m/s^2 — the tidal values — and the vertical tidal correction a'_U is of the same order.

An angular scan therefore confirms the cancellation directly: one measures $g_0\cos\theta$ plus tidal corrections of order 10^{-7} m/s^2, with no 10^{-3} m/s^2 sinusoidal component at any orientation.

5 Numerical Scale

Body	Direct pull (cancels)	Tidal residual (observable)	Ratio
Sun	5.93×10^{-3} m/s^2	5.1×10^{-7} m/s^2	11,700×
Moon	3.32×10^{-5} m/s^2	1.1×10^{-6} m/s^2	30×

The Sun's direct pull is 180× stronger than the Moon's, but its tidal effect is 2.2× *weaker* ($1/R^3$ vs. $1/R^2$ scaling). This inversion is the hallmark of tidal physics, and a direct consequence of the cancellation.

6 What IS Observable

Tilting the sensor accesses different components of the *tidal* field, not the direct pull:

- A vertical sensor sees the vertical tidal component ($\sim 5\times10^{-7}$ m/s^2, semi-diurnal).
- A horizontal sensor picks up horizontal tidal components (diurnal).
- Three non-coplanar sensors reconstruct the full tidal tensor.

These effects are real and measurable, but four orders of magnitude smaller than the claimed signal.