

# Why a Tilted Sensor Cannot Measure Direct Gravitational Pull

## The Claim

A sensor tilted relative to the local vertical can detect the direct gravitational pull of the Sun ( $\sim 6 \times 10^{-3} \text{ m/s}^2$ ) or the Moon ( $\sim 3.3 \times 10^{-5} \text{ m/s}^2$ ) as a time-varying signal, because the projection of the gravitational acceleration onto the sensor axis changes as these bodies move across the sky.

This is incorrect. The derivation below shows, in a point-mass model with no approximations beyond Newtonian mechanics, that the direct pull of *every* external body cancels exactly against the corresponding acceleration of the laboratory, leaving only the tidal residual — regardless of sensor orientation.

## 1 Setup

Four point masses in an inertial frame:

Body	Mass	Position
Sun	$M_S$	$\mathbf{R}_S(t)$
Moon	$M_M$	$\mathbf{R}_M(t)$
Earth	$M_E$	$\mathbf{R}_E(t)$
Test mass	$m$	$\mathbf{x}(t)$

The test mass sits on Earth's surface and is connected to the sensor by a spring (or equivalent restoring mechanism). The sensor frame is rigidly attached to the Earth.

The sensor reads the **spring force**  $\mathbf{F}_{\text{spring}}$  — the non-gravitational force required to keep the test mass co-moving with the lab. This is the only quantity accessible to experiment; gravity itself is not directly measurable (equivalence principle).

## 2 Equation of Motion in the Inertial Frame

Newton's second law for the test mass:

$$m \ddot{\mathbf{x}} = \underbrace{-\frac{GM_E m}{|\mathbf{x} - \mathbf{R}_E|^3}(\mathbf{x} - \mathbf{R}_E)}_{\text{Earth}} - \underbrace{\frac{GM_S m}{|\mathbf{x} - \mathbf{R}_S|^3}(\mathbf{x} - \mathbf{R}_S)}_{\text{Sun}} - \underbrace{\frac{GM_M m}{|\mathbf{x} - \mathbf{R}_M|^3}(\mathbf{x} - \mathbf{R}_M)}_{\text{Moon}} + \mathbf{F}_{\text{spring}} \quad (1)$$

## 3 Equation of Motion for Earth's Center

Earth's center of mass is accelerated by both external bodies:

$$\ddot{\mathbf{R}}_E = -\frac{GM_S}{|\mathbf{R}_E - \mathbf{R}_S|^3}(\mathbf{R}_E - \mathbf{R}_S) - \frac{GM_M}{|\mathbf{R}_E - \mathbf{R}_M|^3}(\mathbf{R}_E - \mathbf{R}_M) \quad (2)$$

This is Earth's acceleration toward the Sun and Moon combined. Every object attached to the Earth — including the sensor housing, the mounting bracket, and the reference frame of the measurement — shares this acceleration.

## 4 Transform to the Earth-Centered Frame

Define the position of the test mass relative to Earth's center:

$$\mathbf{r} \equiv \mathbf{x} - \mathbf{R}_E \quad (3)$$

and the geocentric positions of the external bodies:

$$\mathbf{R} \equiv \mathbf{R}_S - \mathbf{R}_E, \quad \mathbf{D} \equiv \mathbf{R}_M - \mathbf{R}_E \quad (4)$$

The relative acceleration is  $\ddot{\mathbf{r}} = \ddot{\mathbf{x}} - \ddot{\mathbf{R}}_E$ . Substituting from Eqs. (1) and (2), each external body  $B$  produces a term of the form

$$-\frac{GM_B m}{|\mathbf{x} - \mathbf{R}_B|^3}(\mathbf{x} - \mathbf{R}_B) + \frac{GM_B m}{|\mathbf{R}_E - \mathbf{R}_B|^3}(\mathbf{R}_E - \mathbf{R}_B) \quad (5)$$

where  $B \in \{S, M\}$ . Rewriting in geocentric variables:

$$\boxed{m \ddot{\mathbf{r}} = -\frac{GM_E m}{|\mathbf{r}|^3} \mathbf{r} + GM_S m \left[ \frac{\mathbf{R} - \mathbf{r}}{|\mathbf{R} - \mathbf{r}|^3} - \frac{\mathbf{R}}{|\mathbf{R}|^3} \right] + GM_M m \left[ \frac{\mathbf{D} - \mathbf{r}}{|\mathbf{D} - \mathbf{r}|^3} - \frac{\mathbf{D}}{|\mathbf{D}|^3} \right] + \mathbf{F}_{\text{spring}}} \quad (6)$$

### What happened

The **direct pull** of each external body canceled independently. To see this explicitly, expand the bracket for a generic external body at geocentric position  $\mathbf{R}$  (with  $R = |\mathbf{R}|$  and  $\hat{\mathbf{R}} = \mathbf{R}/R$ ) in powers of the small parameter  $|\mathbf{r}|/R$ :

$$\frac{\mathbf{R} - \mathbf{r}}{|\mathbf{R} - \mathbf{r}|^3} - \frac{\mathbf{R}}{R^3} \quad (7)$$

Start with the inverse cube:

$$|\mathbf{R} - \mathbf{r}|^2 = R^2 \left( 1 - 2 \frac{\hat{\mathbf{R}} \cdot \mathbf{r}}{R} + \frac{r^2}{R^2} \right) \quad (8)$$

so, defining  $\epsilon = 2 \hat{\mathbf{R}} \cdot \mathbf{r}/R - r^2/R^2 = O(r/R)$ :

$$\frac{1}{|\mathbf{R} - \mathbf{r}|^3} = \frac{1}{R^3} (1 - \epsilon)^{-3/2} = \frac{1}{R^3} \left( 1 + \frac{3}{2} \epsilon + O(\epsilon^2) \right) = \frac{1}{R^3} \left( 1 + \frac{3 \hat{\mathbf{R}} \cdot \mathbf{r}}{R} + O\left(\frac{r^2}{R^2}\right) \right) \quad (9)$$

Multiply by  $(\mathbf{R} - \mathbf{r})$  and keep terms through first order in  $r/R$ :

$$\begin{aligned} \frac{\mathbf{R} - \mathbf{r}}{|\mathbf{R} - \mathbf{r}|^3} &= \frac{1}{R^3} \left( 1 + \frac{3 \hat{\mathbf{R}} \cdot \mathbf{r}}{R} \right) (\mathbf{R} - \mathbf{r}) + O\left(\frac{r^2}{R^4}\right) \\ &= \frac{1}{R^3} [\mathbf{R} + 3(\hat{\mathbf{R}} \cdot \mathbf{r}) \hat{\mathbf{R}} - \mathbf{r}] + O\left(\frac{r^2}{R^4}\right) \end{aligned} \quad (10)$$

where the cross term  $-3(\hat{\mathbf{R}} \cdot \mathbf{r}) \mathbf{r}/R$  is  $O(r^2/R)$  and has been absorbed into the remainder.

Now subtract  $\mathbf{R}/R^3$ :

$$\underbrace{\frac{\mathbf{R} - \mathbf{r}}{|\mathbf{R} - \mathbf{r}|^3} - \frac{\mathbf{R}}{R^3}}_{\text{Eq. (7)}} = \frac{1}{R^3} [\mathbf{R} + 3(\hat{\mathbf{R}} \cdot \mathbf{r}) \hat{\mathbf{R}} - \mathbf{r} - \mathbf{R}] + O\left(\frac{r^2}{R^4}\right) \quad (11)$$

The  $\mathbf{R}/R^3$  terms — which represent the **direct gravitational pull**  $GM/R^2$  in the direction  $\hat{\mathbf{R}}$  — cancel identically. What survives is the quadrupole (tidal) term:

$$\boxed{\mathbf{a}_{\text{tidal}} = \frac{GM}{R^3} [3(\hat{\mathbf{R}} \cdot \mathbf{r}) \hat{\mathbf{R}} - \mathbf{r}] + O\left(\frac{GM r^2}{R^4}\right)} \quad (12)$$

This is suppressed by a factor of  $r/R$  relative to the direct pull — the ratio of Earth's radius to the distance to the external body. For the Sun,  $r/R \approx 4.3 \times 10^{-5}$ ; for the Moon,  $r/D \approx 1.7 \times 10^{-2}$ .

Applying Eq. (12) to each body:

$$\mathbf{a}_{\text{tidal}}^{\text{Sun}} = GM_S \left[ \frac{\mathbf{R} - \mathbf{r}}{|\mathbf{R} - \mathbf{r}|^3} - \frac{\mathbf{R}}{R^3} \right] \approx \frac{GM_S}{R^3} [3(\hat{\mathbf{R}} \cdot \mathbf{r}) \hat{\mathbf{R}} - \mathbf{r}] \quad (13)$$

$$\mathbf{a}_{\text{tidal}}^{\text{Moon}} = GM_M \left[ \frac{\mathbf{D} - \mathbf{r}}{|\mathbf{D} - \mathbf{r}|^3} - \frac{\mathbf{D}}{D^3} \right] \approx \frac{GM_M}{D^3} [3(\hat{\mathbf{D}} \cdot \mathbf{r}) \hat{\mathbf{D}} - \mathbf{r}] \quad (14)$$

The structure of the cancellation is now transparent:

- The **zeroth-order** term ( $\mathbf{R}/R^3$ ) is the direct pull — the  $GM/R^2$  acceleration toward the external body. It appears identically in the expansion of  $(\mathbf{R} - \mathbf{r})/|\mathbf{R} - \mathbf{r}|^3$  and in the subtracted  $\mathbf{R}/R^3$ , so it cancels exactly (Eq. 11).
- The **first-order** term ( $\propto r/R^3$ ) is the tidal acceleration. It has the quadrupolar structure  $3(\hat{\mathbf{R}} \cdot \mathbf{r}) \hat{\mathbf{R}} - \mathbf{r}$ : stretching along the Earth–body axis, compression perpendicular to it, trace-free.
- The cancellation is **exact** in the full (unexpanded) bracket. The Taylor expansion is shown only to identify which term cancels and which survives — the boxed equation in (6) requires no expansion and is used in the code.

## 5 What the Sensor Reads

The test mass is stationary in the lab frame. Setting the acceleration to the centripetal term from Earth's rotation and solving for the spring force:

$$\mathbf{F}_{\text{spring}} = m \left[ \frac{GM_E}{|\mathbf{r}|^3} \mathbf{r} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - \mathbf{a}_{\text{tidal}}^{\text{Sun}} - \mathbf{a}_{\text{tidal}}^{\text{Moon}} \right] \quad (15)$$

The sensor projects this onto its measurement axis  $\hat{\mathbf{n}}$ :

$$g_{\text{measured}} = \frac{\mathbf{F}_{\text{spring}}}{m} \cdot \hat{\mathbf{n}} \quad (16)$$

**Neither the direct solar acceleration  $GM_S/R^2$  nor the direct lunar acceleration  $GM_M/D^2$  appears.** Both canceled in Eq. (6) as algebraic identities — no approximation was made, and the cancellation holds for any  $\hat{\mathbf{n}}$ .

## 6 Why Tilting Doesn't Help

The claim implicitly assumes that tilting the sensor changes the projection of a body's *direct* pull onto  $\hat{\mathbf{n}}$ :

$$g_{\text{claimed}}^{\text{Sun}} = \frac{GM_S}{R^2} (\hat{\mathbf{R}} \cdot \hat{\mathbf{n}}), \quad g_{\text{claimed}}^{\text{Moon}} = \frac{GM_M}{D^2} (\hat{\mathbf{D}} \cdot \hat{\mathbf{n}}) \quad \leftarrow \quad \text{these terms do not exist} \quad (17)$$

But each was canceled by the identical projection of Earth's acceleration toward that body onto the same axis. Tilting  $\hat{\mathbf{n}}$  rotates all projections equally, because the cancellation is **vectorial** — it holds component by component, in every direction simultaneously.

To make the argument concrete: the sensor housing accelerates at  $\ddot{\mathbf{R}}_E$  (toward both the Sun and Moon). The test mass, if released, would also accelerate at  $\ddot{\mathbf{R}}_E$  (plus the tidal corrections). The spring connecting them sees only the difference — which is the sum of the two tidal terms. This is true regardless of the spring's orientation.

## 7 Numerical Comparison

### Sun

Quantity	Formula	Value
Direct pull	$GM_S/R^2$	$5.93 \times 10^{-3} \text{ m/s}^2$
Earth's acceleration toward Sun (cancels)	same	$5.93 \times 10^{-3} \text{ m/s}^2$
<b>Tidal residual</b>	$\sim 2 GM_S r_{\oplus}/R^3$	<b><math>5.1 \times 10^{-7} \text{ m/s}^2</math></b>

$$\frac{a_{\text{tidal}}^{\text{Sun}}}{a_{\text{direct}}^{\text{Sun}}} = \frac{2 r_{\oplus}}{R_{\text{Sun}}} = \frac{2 \times 6.371 \times 10^6}{1.496 \times 10^{11}} \approx 8.5 \times 10^{-5} \quad (18)$$

The direct pull is **11,700**× larger than the measurable tidal signal.

### Moon

Quantity	Formula	Value
Direct pull	$GM_M/D^2$	$3.32 \times 10^{-5} \text{ m/s}^2$
Earth's acceleration toward Moon (cancels)	same	$3.32 \times 10^{-5} \text{ m/s}^2$
<b>Tidal residual</b>	$\sim 2 GM_M r_{\oplus}/D^3$	<b><math>1.1 \times 10^{-6} \text{ m/s}^2</math></b>

$$\frac{a_{\text{tidal}}^{\text{Moon}}}{a_{\text{direct}}^{\text{Moon}}} = \frac{2 r_{\oplus}}{D_{\text{Moon}}} = \frac{2 \times 6.371 \times 10^6}{3.844 \times 10^8} \approx 3.3 \times 10^{-2} \quad (19)$$

The direct pull is **30**× larger than the measurable tidal signal. The Moon's ratio is much less extreme than the Sun's because the Moon is only  $\sim 60$  Earth radii away, so the tidal approximation is coarser — but the cancellation is still exact.

### Summary

Body	Direct pull	Tidal residual	Ratio	Suppression by
Sun	$5.93 \times 10^{-3} \text{ m/s}^2$	$5.1 \times 10^{-7} \text{ m/s}^2$	11,700×	$R_{\text{Sun}}/2r_{\oplus}$
Moon	$3.32 \times 10^{-5} \text{ m/s}^2$	$1.1 \times 10^{-6} \text{ m/s}^2$	30×	$D_{\text{Moon}}/2r_{\oplus}$

Note the inversion: the Sun's direct pull is 180× stronger than the Moon's, but its tidal effect is 2.2× *weaker*. This is the  $1/R^3$  vs  $1/R^2$  scaling — the hallmark of a tidal interaction, and a direct consequence of the cancellation derived above.

## 8 The Point-Mass Model Proves the Cancellation

The crucial point: the cancellation above used **only** Newton’s laws applied to point masses. No general relativity, no equivalence principle, no appeal to “free fall” as a concept — just subtracting the equation of motion of Earth’s center from the equation of motion of the test mass.

For each external body  $B$ , the term  $GM_B/|\mathbf{R}'_B|^2$  appears with the same sign, same magnitude, and same direction in both equations, so it drops out identically.

Anyone who computes  $GM_S/R^2 \approx 6 \times 10^{-3} \text{ m/s}^2$  or  $GM_M/D^2 \approx 3.3 \times 10^{-5} \text{ m/s}^2$  and projects it onto a sensor axis is computing a quantity that is real (those bodies do pull the test mass that hard) but **unobservable** — because the sensor, the lab, and the entire Earth are being pulled equally hard in the same direction. The spring between the test mass and the sensor frame cannot detect a uniform acceleration field.

## 9 Why the Cancellation Works: Universality of Free Fall

One might object: the test mass  $m$  and the Earth  $M_E$  have very different masses, so why should the Sun’s effect cancel between them?

The answer is that gravitational **force** is proportional to the accelerated mass ( $F = GM_S m/R^2$ ), so the **acceleration** is mass-independent:

$$a = \frac{F}{m} = \frac{GM_S}{R^2} \quad (20)$$

This is the equality of inertial and gravitational mass ( $m_{\text{inert}} = m_{\text{grav}}$ ), built into Newton’s law of gravitation. The test mass and Earth’s center experience the same gravitational acceleration from the Sun (to leading order in  $r/R$ ), despite their mass ratio of  $\sim 10^{-25}$ .

This is **not** Newton’s third law (action = reaction between two interacting bodies). It is a deeper property: two *different* bodies in the *same* external field accelerate identically. The spring connecting them therefore reads zero from the external field — only the spatial *variation* of the field (the tidal gradient) survives.

If any material violated this equality — if some substance fell faster than others in the Sun’s field — the direct pull *would* be measurable as a composition-dependent residual. This is precisely what Eötvös-type experiments test, achieving constraints at the  $10^{-15}$  level. Their null results confirm that the cancellation is exact to extraordinary precision, and that the tidal residual is all that remains.

## 10 Why Converting to Frequency Doesn’t Help

One might attempt to circumvent the cancellation by converting the acceleration measurement into a frequency measurement: use an optomechanical oscillator (a mirror on a spring, coupled to a cavity) whose resonance frequency depends on  $g$ , and beat it against a reference cavity whose frequency depends only on its length. If the Sun’s pull shifts  $\omega_{\text{mech}}$  but not  $f_{\text{cav}}$ , the beat note should reveal the direct pull.

This fails because the cancellation occurs *before* the readout — at the level of what forces exist between the test mass and its mount.

### The optomechanical oscillator

Consider a mirror of mass  $m$  on a spring of constant  $k$ , anchored to the lab ceiling. In an inertial frame, equilibrium requires:

$$k x_{\text{eq}} = m (g_{\text{mass}} - a_{\text{anchor}}) \quad (21)$$

where  $g_{\text{mass}}$  is the total gravitational acceleration at the mirror, and  $a_{\text{anchor}}$  is the acceleration of the anchor point (which co-moves with the Earth). From Section 4:

$$g_{\text{mass}} - a_{\text{anchor}} = g_{\text{Earth}} - \omega^2 r_{\perp} + a_{\text{tidal}}^{\text{Sun}} + a_{\text{tidal}}^{\text{Moon}} \quad (22)$$

The Sun’s direct pull appears in both  $g_{\text{mass}}$  and  $a_{\text{anchor}}$  and cancels. The equilibrium position — and therefore the oscillation frequency  $\omega_{\text{mech}} \propto \sqrt{g_{\text{eff}}/L}$  — contains only the tidal part of the solar and lunar fields.

### The reference cavity

A Fabry–Pérot reference cavity has resonance frequency  $f = mc/(2L)$ . The spacer length  $L$  is set by electromagnetic bond forces. The Sun’s uniform gravitational field accelerates every atom in the spacer equally — no differential stress, no deformation. Only the tidal gradient across the cavity length  $L$  would deform it:

$$\frac{\delta L}{L} \sim \frac{GM_S}{R^3} \frac{L}{E/\rho} \sim 10^{-26} \quad (23)$$

which is unmeasurable.

### The beat note

The beat frequency  $\Delta f = f_{\text{mech}} - f_{\text{cav}}$  therefore contains: tidal effects from the oscillator, minus negligible tidal deformation of the cavity. The direct pull  $GM_S/R^2$  appears in neither channel.

Converting acceleration to frequency does not change *what physical quantity* is being measured. The spring (or pendulum, or optomechanical cavity) connects two objects — test mass and housing — that share the same orbital acceleration. It can only measure the differential force between its endpoints, regardless of whether the readout is a displacement, a voltage, or a frequency.

## 11 What Frequency Measurements *Can* Detect

While no local mechanical measurement can access the direct pull, the gravitational **potential** (as opposed to the force) can be detected through a fundamentally different channel: the gravitational redshift.

An atomic clock’s tick rate depends on the spacetime metric:

$$\frac{d\tau}{dt} = \sqrt{-g_{00}} \approx 1 + \frac{\Phi}{c^2} \quad (24)$$

where  $\Phi$  is the gravitational potential. This is not a force between two co-accelerating parts — it is a property of spacetime itself.

However, the equivalence principle guarantees that *all* local physics is shifted equally by the potential. A single clock in the Sun’s field has nothing local to compare against. To detect the potential, one needs a **non-local** reference — a frequency source outside the freely-falling frame.

### Clock vs. distant reference (pulsar)

The Sun’s potential at Earth is  $\Phi_{\odot} = -GM_S/R \approx -8.9 \times 10^8 \text{ m}^2/\text{s}^2$ . Because Earth’s orbit is elliptical ( $e \approx 0.0167$ ), this varies annually:

$$\frac{\Delta f}{f} = \frac{GM_S}{c^2} \left( \frac{1}{R_{\text{peri}}} - \frac{1}{R_{\text{aph}}} \right) \approx 3.3 \times 10^{-10} \quad (25)$$

This is the **full, direct** solar potential — not tidal, not suppressed by  $r_{\oplus}/R$ . Pulsar timing arrays detect exactly this effect as the “Einstein delay,” with an annual amplitude of  $\sim 1.66$  ms.

## Two terrestrial clocks

Two clocks at different locations on Earth see different solar potentials, but the difference is only the tidal potential:

$$\frac{\delta f}{f} \sim \frac{GM_S r_\oplus^2}{R^3 c^2} \sim 10^{-17} \quad (26)$$

Modern optical lattice clocks ( $\Delta f/f \sim 10^{-18}$ ) are just reaching this regime.

## Hierarchy of measurements

Method	Measures	Sun contribution	Magnitude
Accelerometer	$-\nabla\Phi$ (force)	tidal only	$5 \times 10^{-7} \text{ m/s}^2$
Optomech. vs. ref. cavity	$-\nabla\Phi$ (force as frequency)	tidal only	$5 \times 10^{-7} \text{ m/s}^2$
Two local clocks	$\Delta\Phi$ (potential difference)	tidal potential	$\Delta f/f \sim 10^{-17}$
Clock vs. pulsar	$\Phi$ (absolute potential)	<b>full direct</b>	$\Delta f/f \sim 3 \times 10^{-10}$

The dividing line: any instrument where two parts are co-accelerating sees the cancellation. To escape it, one end of the measurement must be outside the freely-falling frame — a non-local comparison of proper time flow rates at different locations in the potential.

## 12 What IS Observable by Tilting

Tilting the sensor does reveal genuinely interesting physics, but at the tidal scale ( $\sim 10^{-7} \text{ m/s}^2$ ), not at the direct-pull scale ( $\sim 10^{-3} \text{ m/s}^2$ ):

- **Horizontal tidal acceleration.** A vertical sensor sees only the vertical component of  $\mathbf{a}_{\text{tidal}}$ . A tilted sensor picks up horizontal components, which have comparable magnitude but different time dependence.
- **Angular separation of static and tidal signals.** Normal gravity is purely vertical (projects as  $g_0 \cos \theta$ ), while the tidal vector has East/North/Up components with independent angular signatures. Scanning the sensor azimuth at  $\theta = 90^\circ$  gives a pure tidal signal with zero static background.
- **Reconstruction of the full tidal vector.** Three non-coplanar measurements at one instant determine all four unknowns ( $g_0, a_E, a_N, a_U$ ) without a time series.

These effects are real, measurable, and interesting — but they are four orders of magnitude smaller than the claimed signal.