Electrical & Electronics Engineering Department
EE434 Biomedical Signal Processing Course - Project Report

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Performance Comparison of Different Filter Types for AWGN Channel

Abstract

This project aims to show performance differences between various filtering techniques such as Gaussian, Moving Average, Chebyshev, Butterworth, FIR (Finite Impulse Response), Kaiser and Hamming filters by comparing each of them in AWGN (Additive White Gaussian Noise) channel for an ECG (Electrocardiogram) Signal by Root Mean Square Error (RMSE). For secondary objective, there are also comparison between different filtering techniques by different approaches such as periodogram, Fast Fourier Transform, and phase finding. There are also cascaded filters to reduce noise at the output of the signal. There is also performance comparison between single filtered filters and double filtered signal by Root Mean Square Error. Results show how each filter performs against AWGN while saving frequency components between 0.05 and 150Hz which are meaningful for ECG signals using Root Mean Square Error and frequency spectrum which is provided by Fast Fourier Transform.

Introduction

The main goal of this study is determining which filtering technique gives the best results in AWGN channel for EGC signals. Determination process is done by comparison of Gaussian, Chebyshev, Moving Average, Butterworth, FIR, Kaiser, and Hamming filter outputs with original signals by Root Mean Square Error method. This study has very important role in choosing correct filters while there are AWGN in channel (environment). Because if a low performance filter has been chosen then the result would be misleading, and this leads to wrong diagnostics and wrong treatment. This problem could result in the death of patient.

Experimental Procedure

There are many filters and filtering techniques in signal processing industry some of them based on Infinite Impulse Response (IIR) like Butterworth, Chebyshev, and some of them based on Finite Impulse Response (FIR) like Gaussian, Moving Average, Kaiser, and Hamming. Filters in this project are realized by MATLAB using fir1(.), butter(.), cheby1(.), cheby2(.), filtfilt(.) functions. There are differences between filters. These differences between filters come from pass/stop widths, orders (number of peaks), highest/lowest amplitude levels in dB, or simply being FIR, IIR filter. Performance metric of this project is Root Mean Square Error values; the method which has lowest error value becomes most successive filtering method. This project also includes some different signal analyzing techniques such as Periodogram and Short-Time Fourier Transform, which allows us to see the signal in both frequency and time domains at the same time.

Analyzing Input ECG Signal

The input ECG signal is sampled at 1000 Hz and 2 second duration. The time domain representation of the ECG signal can be seen in Figure 1.

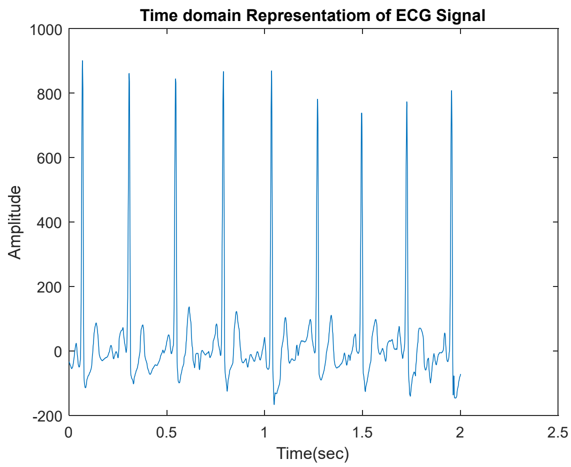

Figure 1 Input ECG Signal in Time Domain

Frequency domain representation of the input ECG signal shows that it has a non-linear phase, as can be seen in Figure 2. The frequency range of the ECG signal is from 0.05Hz to 150 Hz, as can be seen in Figure 2, frequency components are gathered between this frequency range.

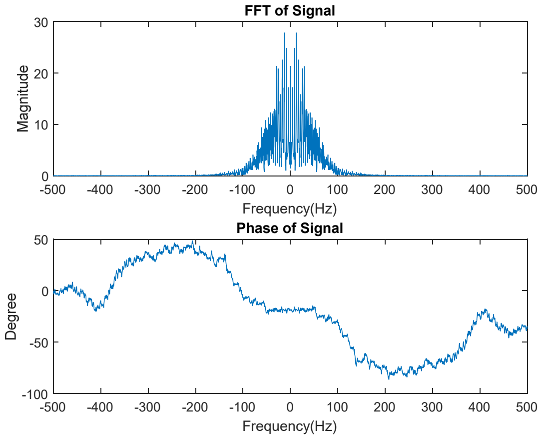

Figure 2 Input ECG Signal in Frequency Domain

Frequency changes in time can be seen with the spectrogram method. The results of this operation can be seen in Figure 3.

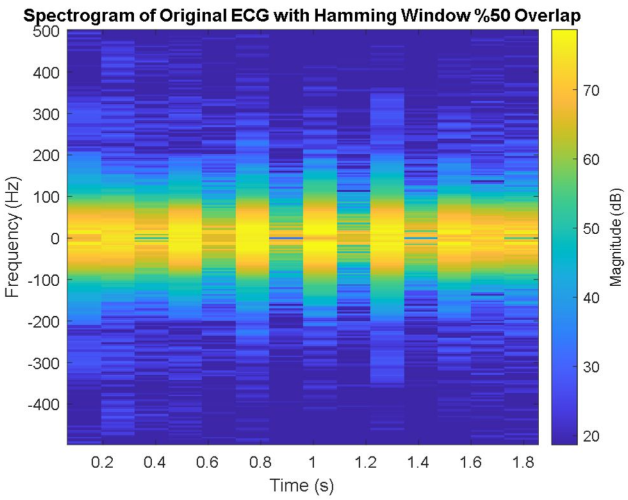

Figure 3 Spectrogram of Input ECG Signal

The periodogram of the input ECG allows to determine Power Spectral Density of the signal as the frequency changes. This process can be seen in Figure 4.

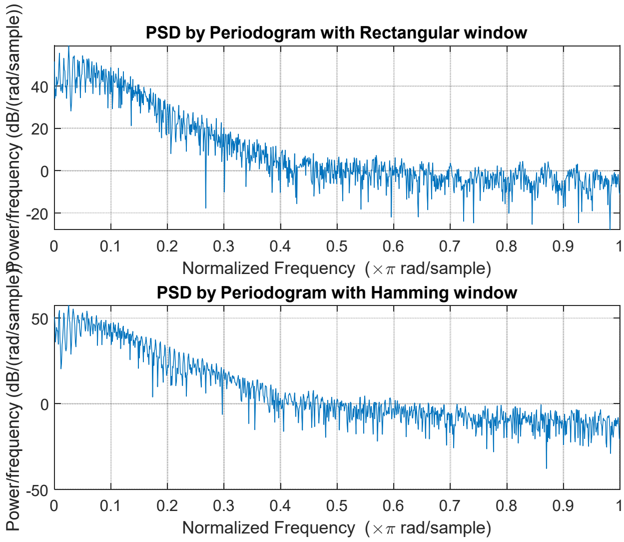

Figure 4 Periodogram of Input ECG Signal

This periodogram can be improved by Welch method which reduces ripples at PSD function and gives more clear results compared to periodogram by shortening the window size instead of using window size of length of the signal itself. This can be seen in Figure 5.

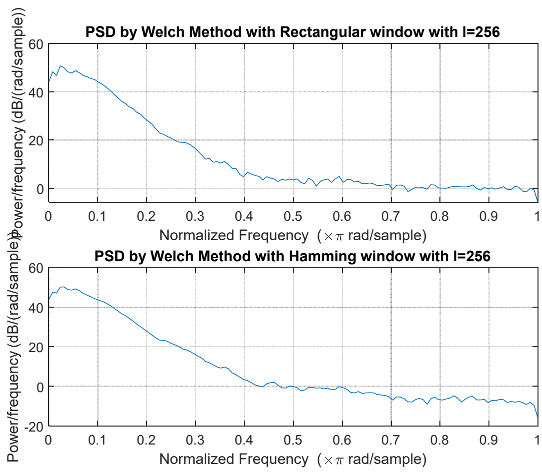

Figure 5 Periodogram of Input ECG Signal with using Welch Method

Gaussian Filter

Gaussian filter kernel is realized by array of [0.25,0.5,0.25]. Filter response in time and frequency domains are the same for gaussian filters, unlike box filters as can be seen in Figure 6. This feature makes a gaussian filter more realizable than a box filter since Gaussian has less oscillations than a box filter, so this implies that Gaussian filter is more stable than a box filter.

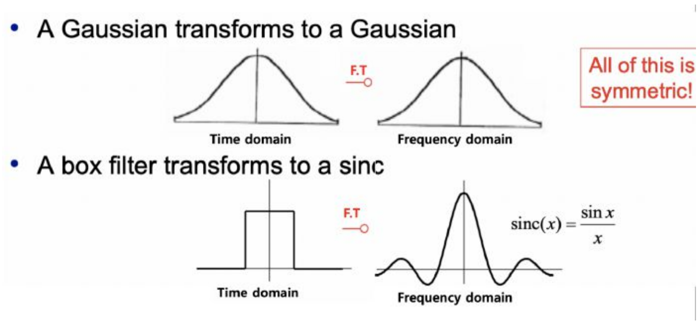

Figure 6 Gaussian vs Box Filter Time and Frequency Response [1]

Moving Average Filter

Moving Average filter is realized by an array of [0.25,0.25,0.25,0.25]. This filter type has different responses at time response and frequency response, as can be seen in Figure 7. It has a greater number of peaks than a gaussian filter and has the same magnitude for all 4 inputs (0.25) instead of different weights like a gaussian kernel.

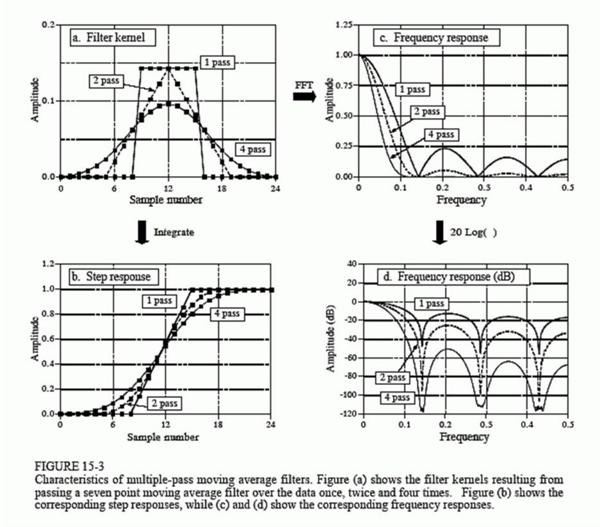

Figure 7 Moving Average Filter Time and Frequency Response [2]

Chebyshev Filter

Chebyshev filters are implemented by cheby1(.) and cheby2(.) on MATLAB functions. Since they are IIR filters, they have both zeros and poles, unlike FIR filters since they have only zeros. The frequency response of Chebyshev 1 and Chebyshev 2 filters is given below in Figure 8. They have differences at the passband and stopband. While Chebyshev 1 has oscillations at the passband and no oscillations in the stopband, Chebyshev 2 has no oscillations in the passband, unlike its stopband.

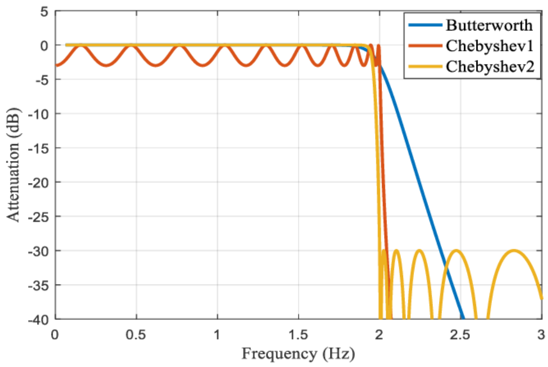

Figure 8 Chebyshev Type 1 and Type 2 Filter Time and Frequency Response [3]

Butterworth Filter

Butterworth filters are also IIR type filters, which means that they also have both zeros and poles like Chebyshev filters. They have no ripples at the passband, and stopband, but the width of the stopband is very large compared to Chebyshev Type 2 filters, as can be seen in Figure 8.

Kaiser Window

Kaiser window is a window that can be modified more than other windows, so it's better to control characteristics of the window than other windows by changing alpha values. The time and frequency domain representations can be seen in Figure 9.

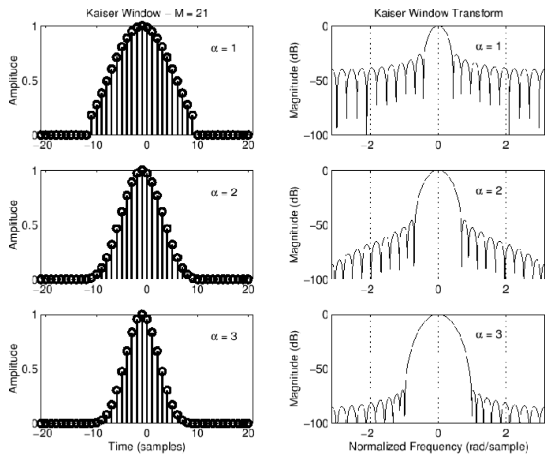

Figure 9 Kaiser Window Time and Frequency Response [4]

Hamming Window

Hamming window is a window that has better main lobe discrimination than Hann window but less descent rate than Hann window. If we compare Hamming window and rectangular window, we can see that they are similar, but the main lobe is narrower and more discriminated from the side lobes. Time and frequency representations of the hamming window can be seen in Figure 10.

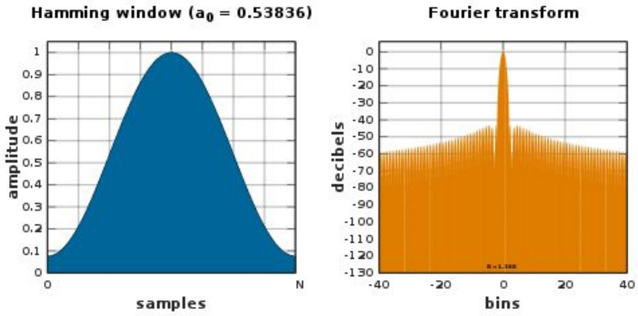

Figure 10 Hamming Window Time and Frequency Response [5]

Root Mean Square Error

Root Mean Square Error is a powerful tool to determine how two signals are differs from each other by comparing (taking difference) both signals at same iterations then take square of the results and takes mean of the results than taking square root of the result, the result can be obtained. The formula for the RMSE can be seen below, where N is sample number, x(t) is noisy signal, and y(t) is filtered signal.

$$RMSE = \sqrt{\frac{\sum_{t=1}^{N}\left(x(t) - y(t)\right)^2}{N}}$$

Results and Discussion

Best RMSE values for several SNR AWGN values can be found from Table 1 below.

- The best RMSE value is obtained by Moving Average followed by Hamming Filter for 0 dB AWGN
- The best RMSE value is obtained by Moving Average followed by Kaiser Filter for 5 dB AWGN
- The best RMSE value is obtained by Gaussian followed by Kaiser filter for 10 dB AWGN
- The best RMSE value is obtained by FIR followed by Kaiser filter for 15 dB AWGN
- The best RMSE value is obtained by Butterworth for 20 dB AWGN

Filter Type	Order/ Length	RMSE (0 dB AWGN)	RMSE (5 dB AWGN)	RMSE (10 dB AWGN)	RMSE (15 dB AWGN)	RMSE (20 dB AWGN)
Noisy Signal (No filter)	-	152.13	84.67	47.61	26.88	14.91
Gaussian Filter	9	78.90	44.66	24.69	15.15	9.32
Moving Average	4	63.25	37.38	22.02	16.91	13.47
FIR	9	72.91	41.29	23.14	13.76	8.61
Butterworth	9	80.18	45.46	25.35	14.79	8.40
Gaussian Hamming	10	59.78	36.49	22.85	19.24	16.54
Gaussian Kaiser	10	64.48	37.11	20.92	14.33	10.23
Moving Average Hamming	4/10	56.83	36.99	25.71	23.70	21.72
Moving Average Kaiser	4/10	59.38	35.69	21.87	17.86	15.08
FIR Hamming	9/10	61.08	35.81	21.00	15.87	12.62
FIR Kaiser	9/10	67.2	38.11	21.60	13.60	9.47
Butterworth Chebyshev 1	9/9	151.47	149.92	149.95	149.77	150.67
Butterworth Chebyshev 2	9/9	73.03	41.72	23.28	13.83	8.47

Table 1 RMSE values for different SNR AWGN

These results implies that when there are low SNR (0 dB) AWGN signals using Moving Average Filter is better choice to reduce noise and when there is high SNR (20 dB) AWGN signal using Butter Filter is better choice to reduce noise.

Time and Frequency Response of signals which are filtered best are given below:

The lowest RMSE obtained by Moving average filter followed by Hamming window, purple signal in Figure 11 while there are 0 dB SNR AWGN in channel.

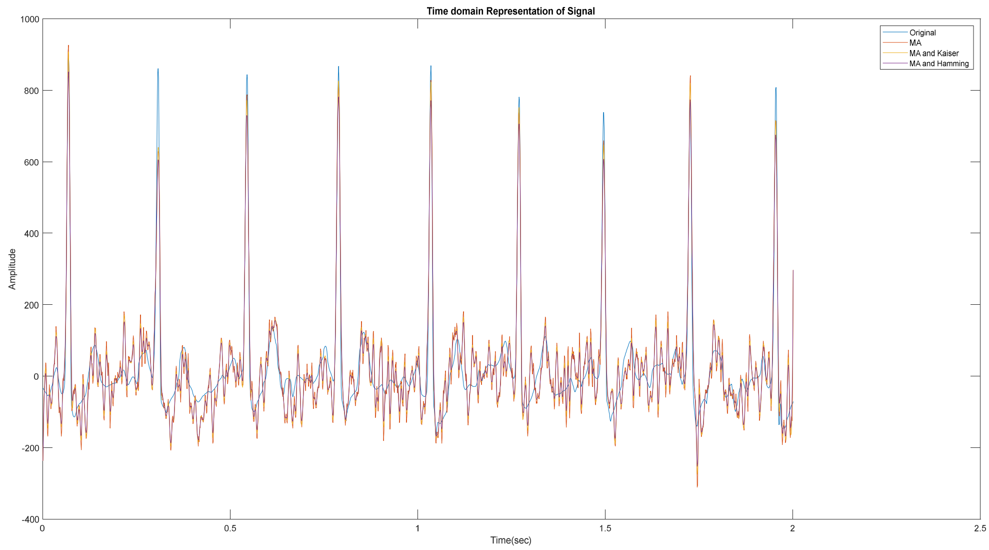

Figure 11 Time Response of Moving Average Filtered Signals for 0 dB AWGN

The frequency and phase responses of MA-filtered signals for 0 dB AWGN can be seen in Figure 12.

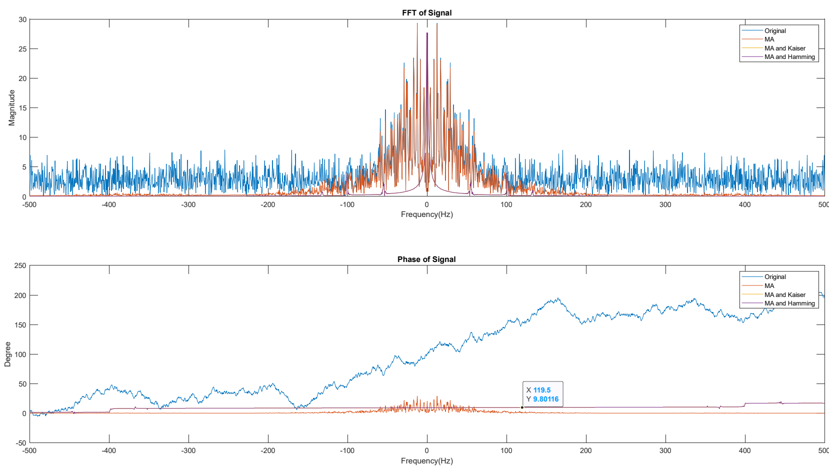

Figure 12 Frequency and Phase Responses of Moving Average Filtered Signals for 0 dB AWGN

The phases are nearly zero; however, most of the frequency components are lost in the 0.05-150 Hz range; even RMSE is lowest. So, the Moving Average Filtered Signal should be chosen even if it has more RMSE value, it also saves important frequencies for ECG signals.
The lowest RMSE obtained by Moving Average followed by Kaiser Filter, yellow signal in Figure 13 while there are 5 dB SNR AWGN in channel.

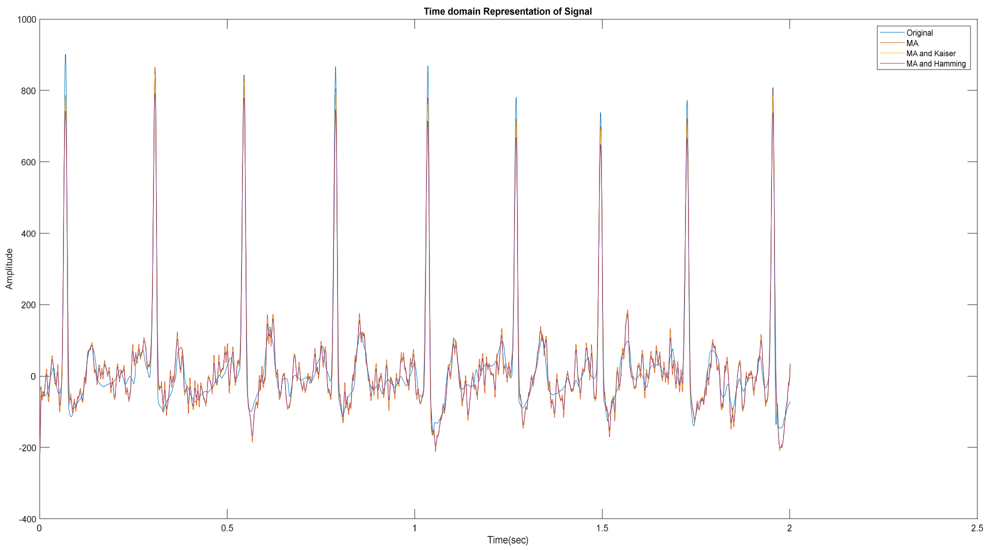

Figure 13 Time Response of Moving Average Filtered Signals for 5 dB AWGN

The frequency and phase responses of MA-filtered signals for 5 dB AWGN can be seen in Figure 14.

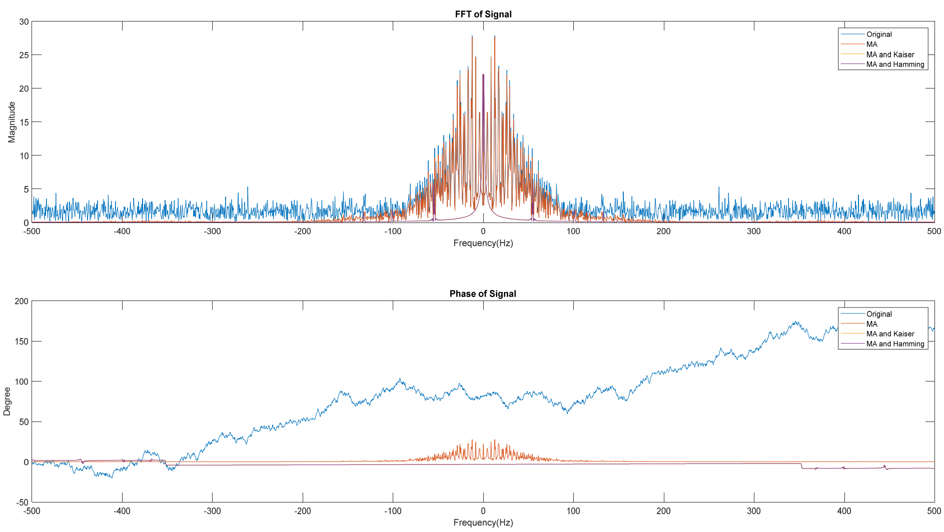

Figure 14 Frequency and Phase Responses of Moving Average Filtered Signals for 5 dB AWGN

The comment for 0 dB SNR AWGN is also valid for 5 dB SNR AWGN. Most of the significant frequency components are eliminated with 2-cascaded filters, so, the moving average filtered signal should be chosen (orange signal) in Figure 14.
The lowest RMSE obtained by Gaussian followed by Kaiser window, yellow signal in Figure 15 while there are 10 dB SNR AWGN in channel.

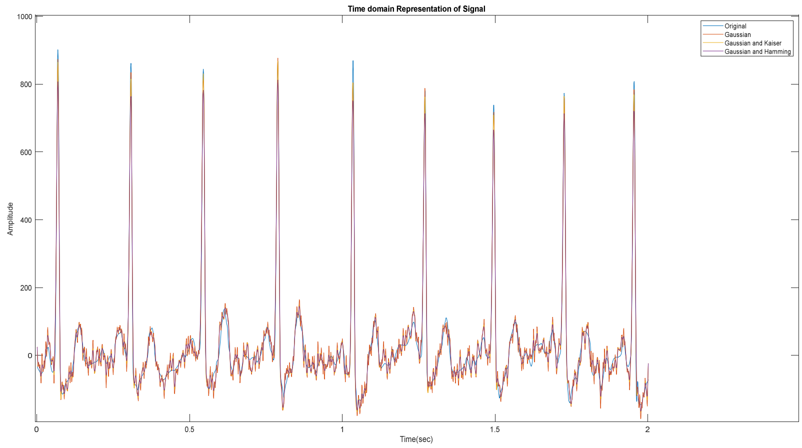

Figure 15 Time Response of Gaussian Filtered Signals for 10 dB AWGN

The frequency and phase responses of Gaussian filtered signals for 10 dB-AWGN can be seen in Figure 16 as yellow(purple).

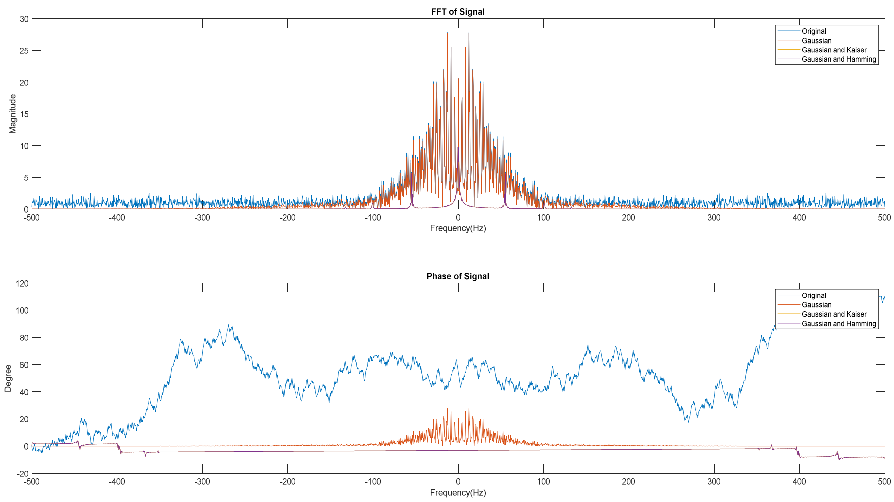

Figure 16 Frequency and Phase Responses of Gaussian Filtered Signals for 10 dB AWGN

As can be seen in Figure 16, most of the significant frequency components are lost just like moving average followed by any window(kaiser/hamming). So, picking a filter with gaussian followed by kaiser is not a good option for denoising process. Instead of using two concatenated filters, usage of only a gaussian filter would be better; even resultant phases of this filter oscillate at low frequencies while gaussian followed by kaiser has zero phase at significant frequencies (0.05-150 Hz).

The lowest RMSE obtained by FIR followed by Kaiser window, purple signal in Figure 17 while there are 15 dB SNR AWGN in channel.

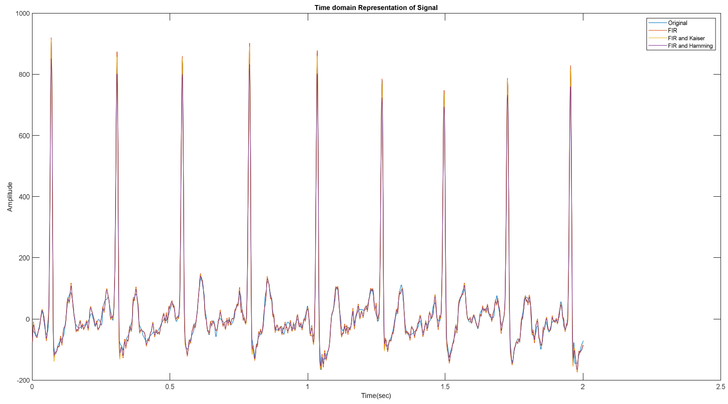

Figure 17 Time Response of FIR Filtered Signals for 15 dB AWGN

The frequency and phase responses of FIR-filtered signals for 15 dB AWGN can be seen in Figure 18 as yellow(purple).

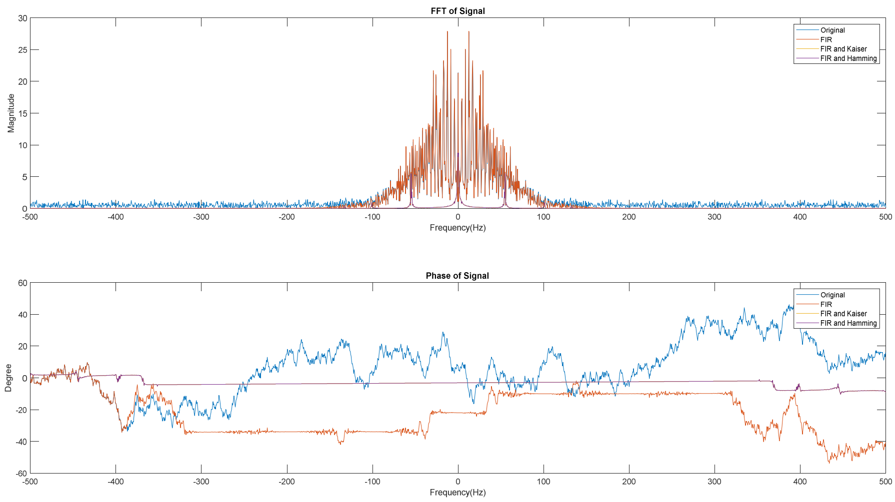

Figure 18 Frequency and Phase Responses of FIR Filtered Signals for 15 dB AWGN

As can be seen in Figure 18, most frequency components are eliminated when used filter is FIR followed by Kaiser window as yellow(purple) signal. So, this filter cannot be used to save

significant frequency components. Instead of using this filter configuration using only FIR would be better to save these significant frequency components; even their phase responses are nonlinear while FIR followed by Kaiser window has zero phases, which is the ideal case.
The lowest RMSE obtained by Butterworth, purple signal in Figure 19 while there are 20 dB SNR AWGN in channel.

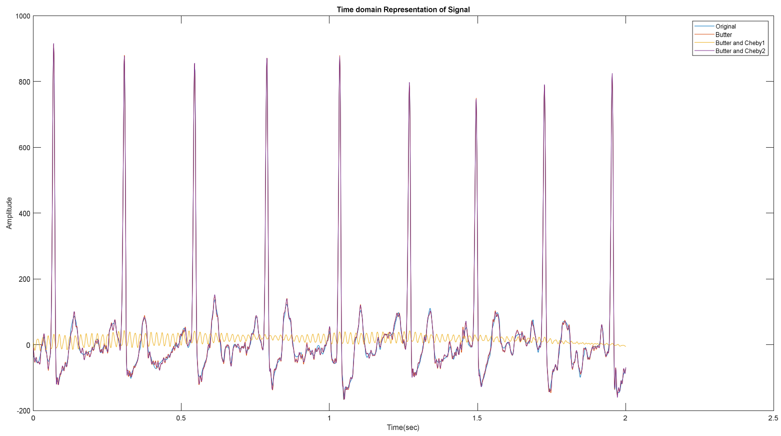

Figure 19 Time Response of Butterworth Filtered Signals for 20 dB AWGN

The frequency and phase responses of Butterworth-filtered signals for 20 dB AWGN can be seen in Figure 20 as orange.

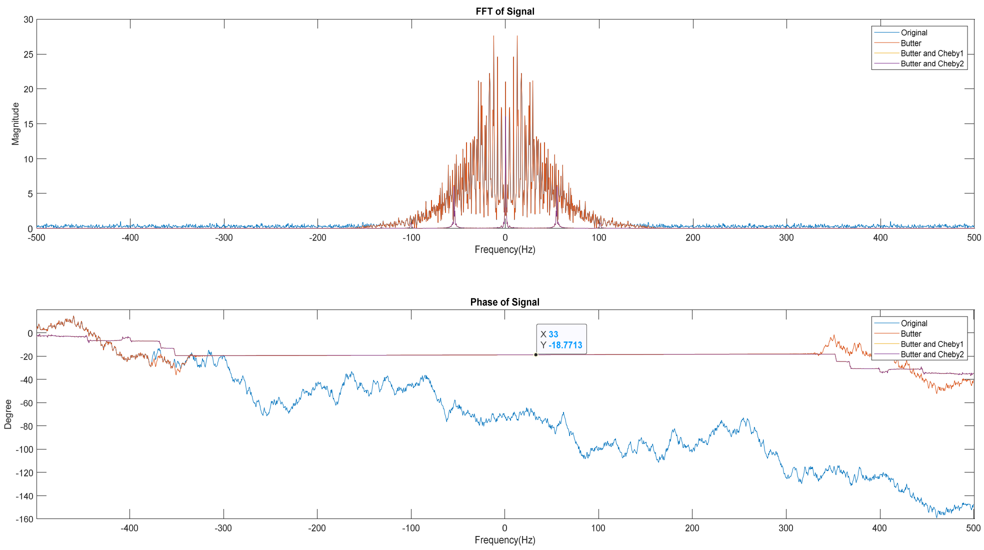

Figure 20 Frequency and Phase Responses of Butterworth Filtered Signals for 20 dB AWGN

As can be seen in Figure 20, the orange signal is the most similar signal to the original ECG signal. This visual expression was also proved by having the lowest RMSE value. The phase is also constant, so this is a good result. The significant frequencies have been saved so this is the truly best result for the 20 dB AWGN channel.

More graphs and results can be obtained by running the MATLAB code, which is in the same file as this report is.

Conclusion

Main purpose was to analyze performance of various filtering techniques by comparing RMSE values. Different SNR values for AWGN channels require different optimum filtering techniques. Optimum techniques obtained by choosing lowest RMSE value in all cases. In low SNR AWGN channels moving average is best option in general, but as SNR increases the optimum filtering technique becomes Gaussian, FIR and Butterworth consequently. Even if filters have the lowest RMSE value among all the filters, this does not imply that it is the best option to use. Because there are cases the resultant signal has lowest RMSE value but also eliminated important frequencies for ECG signals. The best option changes with priorities of purpose. This project's priority was saving these components, so optimal filtering techniques were determined by this priority. In the end, this project reached its goals.

References

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