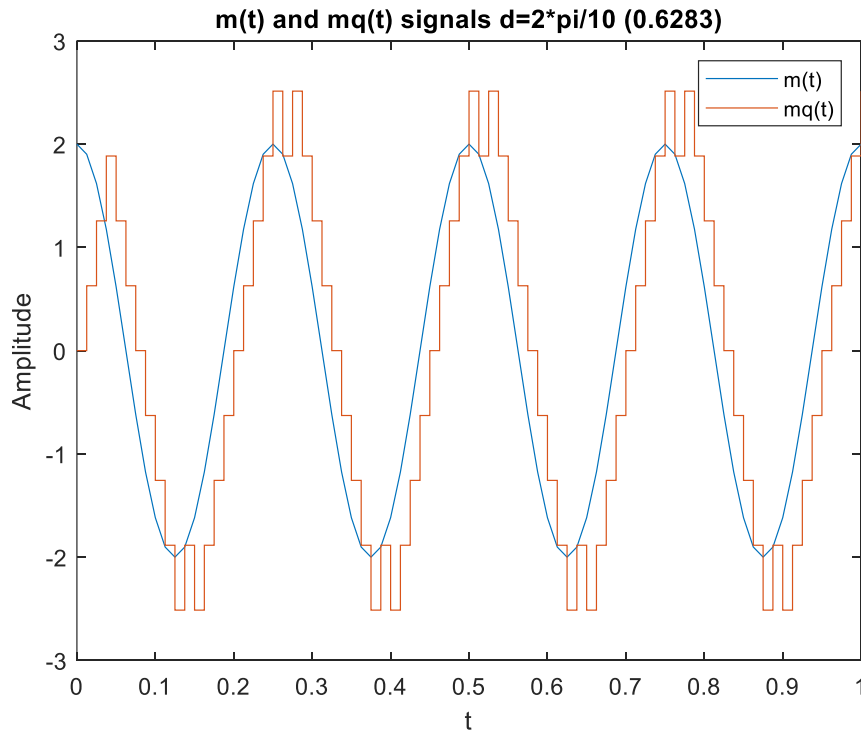


1.d

Minimum step size is calculated by taking derivative of our message signal and taking absolute value to get values all positive then we should take maximum value of the signal as a last step we should divide this value to sampling frequency. The required operations given below:

$$m = A_m \cos(2\pi f_m t)$$

$$m' = -A_m 2\pi f_m \sin(2\pi f_m t)$$

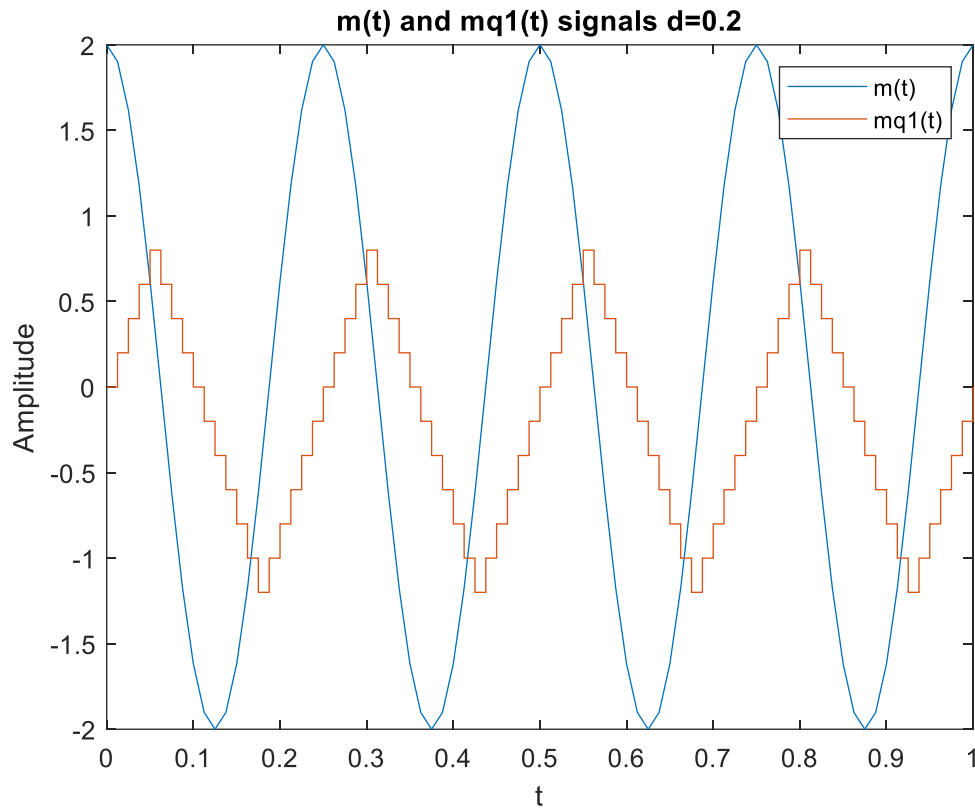
$$m' = -2 \cdot 2\pi \cdot 4 \sin(2\pi f_m t)$$

$$\max(\text{abs}(m')) = 16\pi$$

$$\max(\text{abs}(m'))/F_s = 16\pi/80 = \pi/5 = 0.6283$$

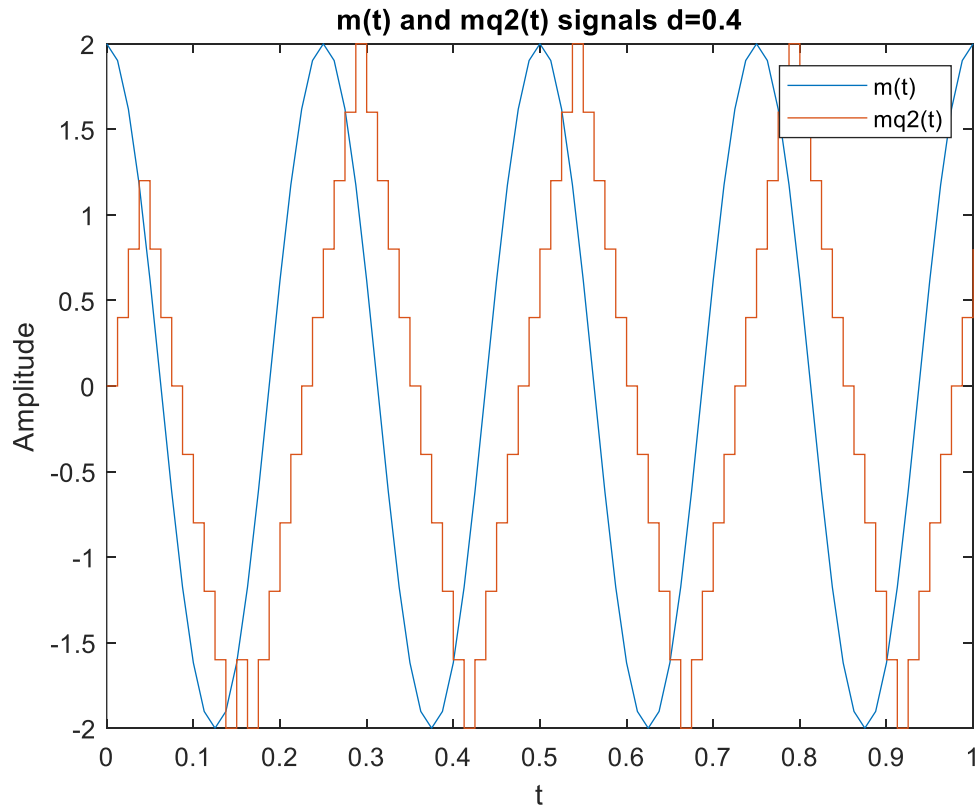
When we look at the shape it looks like it has phase difference, and this is caused by initialization of first element of $m_q(t)$ with 0. This happens for all the delta values next too. But we also see that there are ripples at both negative and positive peaks. This could be caused by the granular noise effect. This is because the slope at the peaks is 0 and its relatively low compared to our delta (0.6283). So, it ripples due to this effect.

1.g.d.1



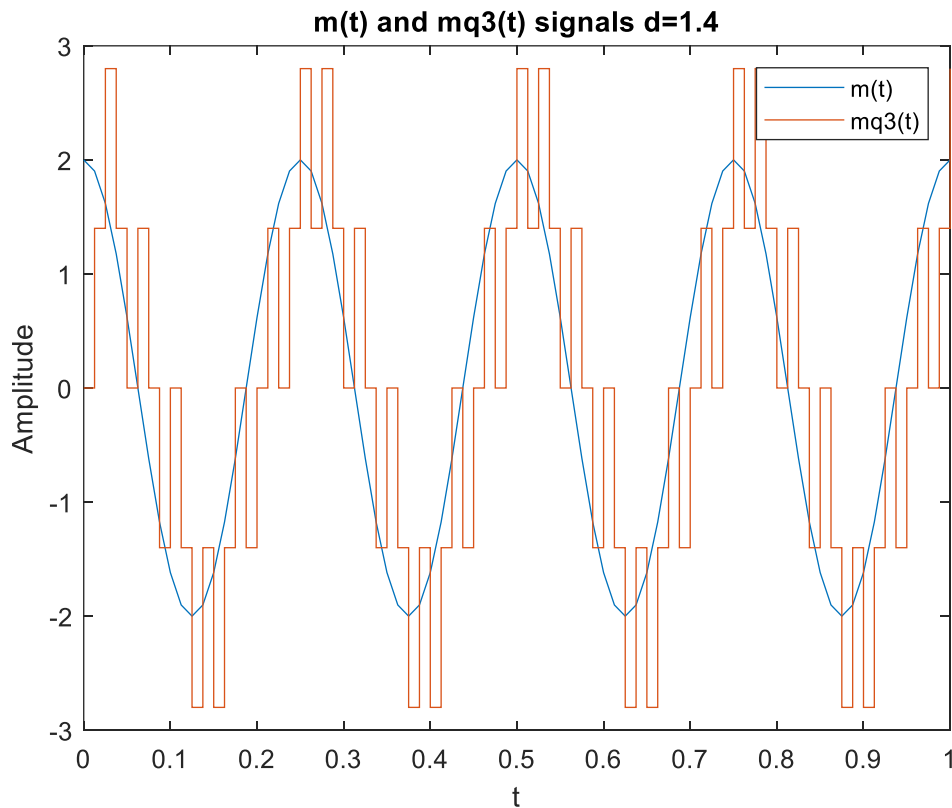
This graph also has some phase difference due to same reason before about initialization. The peaks cannot reach where they should reached. This because the delta values is so little (0.2) per sample the required time for reaching peak value from 0 is much higher than 1 cycle of our signal. So mq cannot catch up with amplitude of the message signal instead of it reaches 0.8 at maximum. Slope overlap distortions has been occurred result of this process.

1.g.d.2



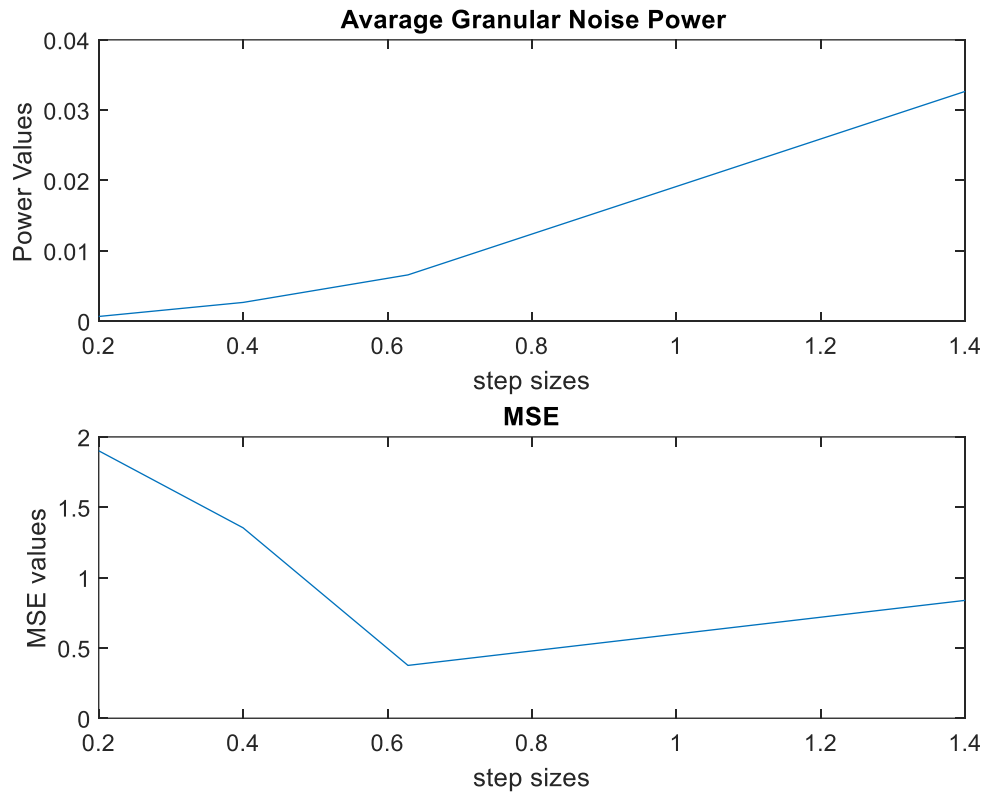
We can see this quantized message signal has also phase difference like others because of initialization. This time amplitudes have been matched after several values. This is possible because of the delta value (0.4) is enough for this signal to keep up with the amplitude. 5 sample time ($5 \cdot 0.0125$) required for reach peak value of the signal (2) from zero point. There are also some overlap distortions due to not satisfy minimum delta requirements. The 0 slope points at peaks does not affect this quantized signal because it did not reach the slope value at the time instant which message signal reached the peak value.

1.g.d.3



We can clearly see that quantized signal has some phase difference due to our message signal this can be explained by initialization which is stated before. Also we have some ripples near to the peaks there are no overlapping slope distortion but there are huge amount of granular noise due to high difference between delta(1.4) and slope of peaks (0). High valued ripples can be explained by the difference between message signal and quantized signal is positive (message signal value is higher than quantized message signal) so that even if quantized signal has amplitude as 1.4 at the first sampling time instance(0.0125) the message signal value is higher than quantized so it increments the approximation by 1.4 again and it reaches 2.8. There are also ripples at signal values becomes 0 which indicates also maximum slope. At this time instants message signal values becomes more than quantized message signal values so it increments by 1.4 again and reaches 1.4 from 0 again. Then it returns 0 again and this is actually error (unnecessary incrementation).

1.h



The detailed comment for 1.h has been given in Report Questions Answers for Q3

Report Questions Answers:

Q1: Formula for avoiding slope overload is given below:

$$\frac{\Delta}{Ts} \geq \max \left(\text{abs} \left(\frac{d(m(t))}{dt} \right) \right)$$

This is required to catching up with the message signal slope. If this condition is not satisfied the modulated signal (Delta Modulation) fall behind of the modulating signal (for values at each time instant). Then resulting signal would be very different from our message signal like the graph for $\Delta = 0.2$. There are slope overload because our minimum delta value is $2 \cdot \pi \cdot A_m \cdot f_m / F_s$ which is $2 \cdot \pi \cdot 2 \cdot 4 / 80 = \pi / 5$ which is nearly equal to 0.62.

Q2: For delta values of 0.2, 0.4 there are slope distortion on modulated signals. Because when delta at these values the minimum requirement of delta is not satisfied ($\Delta \geq 0.628$) So approximation could not chase changes in message signal values. There are granular noises for delta values such that delta equals to 1.4. The local slopes of our original message signal are very low compared to delta. Slope of our cosine signal is decreasing while we are getting to peak points so at peak points local slope becomes 0 while our delta is 1.4.

Q3: Granular noise is inversely related with slope overload distortion by magnitude of delta which is step size. When step-size is lower than minimum required value there would be slope overload distortion and while it's becoming much higher than slope of the signal at time instance there would be granular noises. MSE is decreasing while we are increasing step size until our delta min value which is minimum required step size value for delta. Then when we got to the last step size which is 1.4 the MSE value increased unlike the previous decreasing trend of MSE values which is expected because there are granular noises when we increase step size too much so that at some values of the signal the slope is becoming much lower than our step size and this situation cause some unwanted ripples at corresponding values for quantized message.

Q4: We can use adaptive step size method for delta modulation (ADM) which provides us to arrange step size due to slopes of the signal. The basic principle of this method is given like further:

- While we are having high changing rate (high slope) the step size is increased
- While we are having more static signal part (low slope) the step size is decreased

Q5: We are having 1-bit stream (1 or 0) at the end of the delta modulation. ('Encode' array in our code)