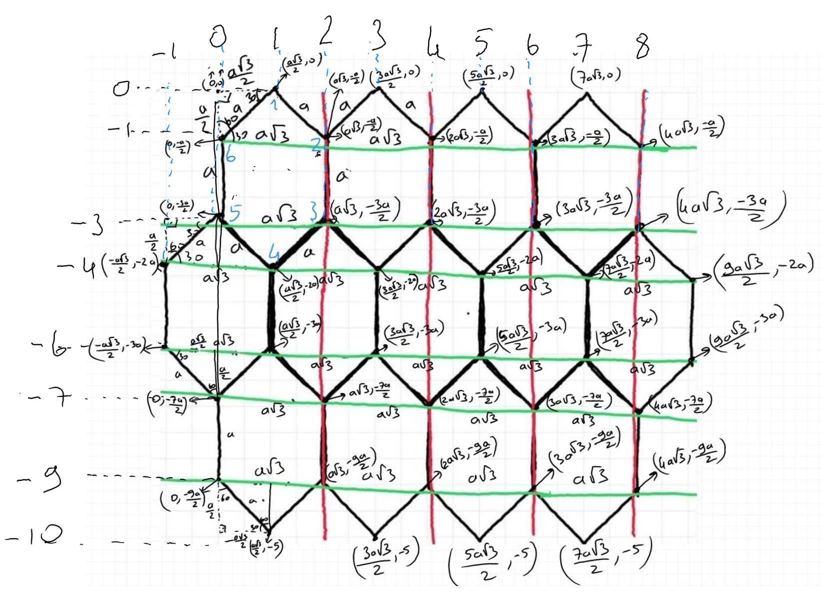
1) First way (Point precision)



For every point precision

Point$\left(x a\sqrt{3}, \frac{ya}{2}\right)$ For all x, For y equals sequace $\rightarrow$ $(0, -1, -3, -4, -6, -7, -9, -10, \ldots)$

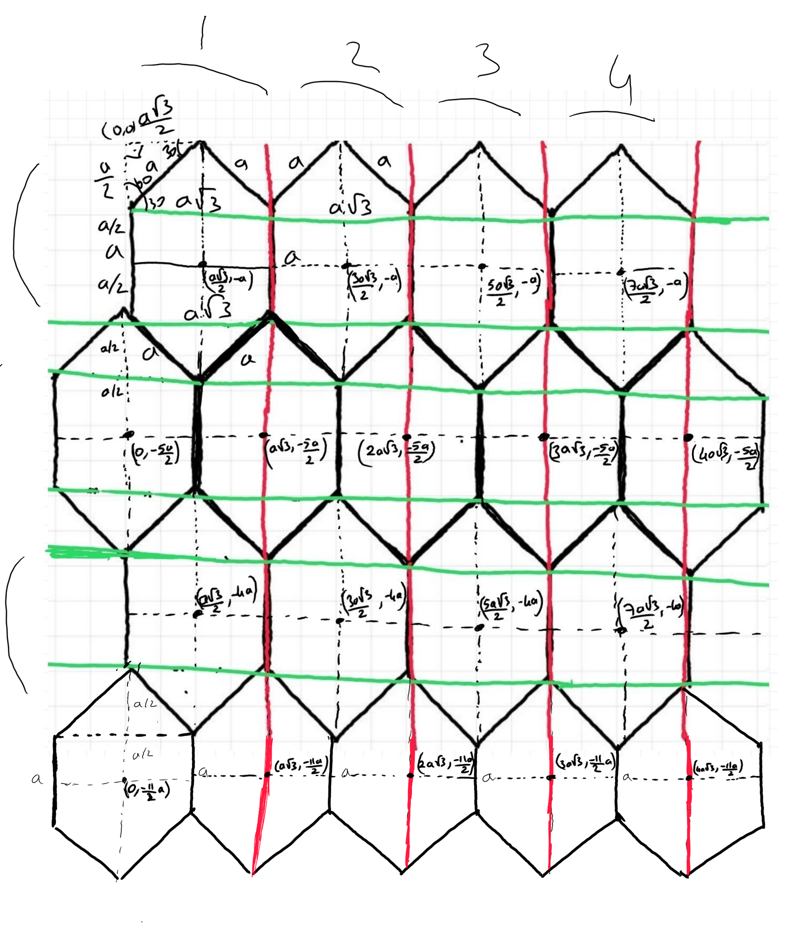
$-1 \quad +1 \quad -1 \quad +1 \quad -1 \quad +1$

$-1 \quad -2 \quad -1 \quad -2 \quad -1 \quad -2 \quad -1$

even odd odd even even odd odd even

(I did not find suitable sequence equation)

Second way (Center precision)



For odd numbers of Rows of hexagonal tesselation
The central point P_c at $\left(\underbrace{\frac{(2n-1)a\sqrt{3}}{2}}_{i}, \underbrace{\left(\frac{-3n+1}{2}\right)a}_{j}\right)$

For even numbers of Rows of hexagonal tesselation
The central point P_c at $\left(\underbrace{na\sqrt{3}}_{i}, \underbrace{\left(\frac{-3n+1}{2}\right)a}_{j}\right)$

1.a) For euclidion distances we can use $c = \sqrt{a^2+b^2}$ Formula while a is horizontal distances, b is vertical

- For odd number of rows of hexagonal tesselation

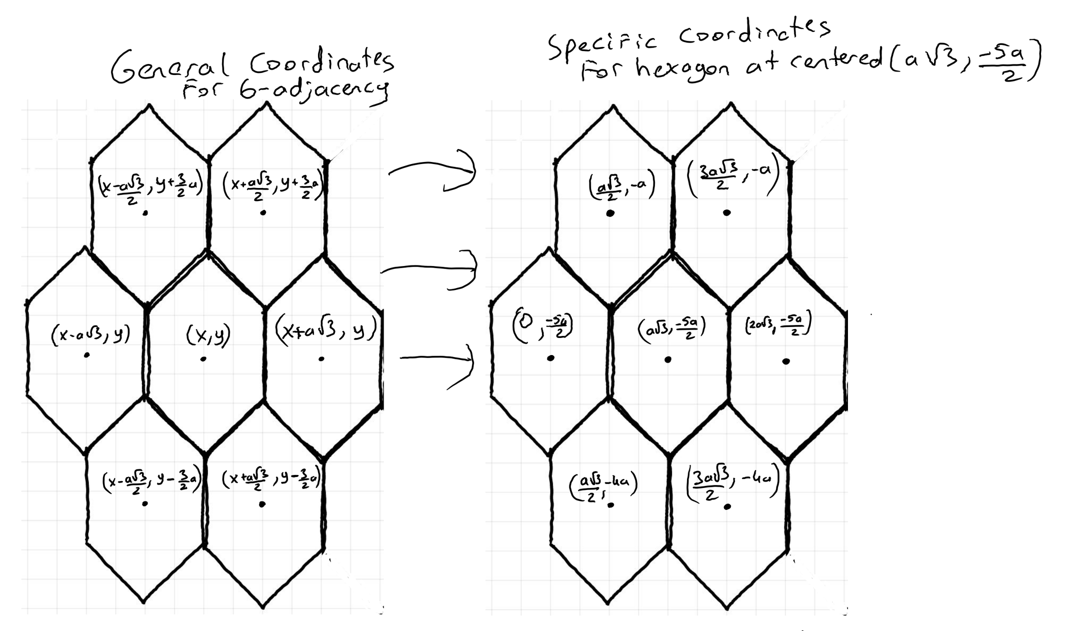
$$\text{Euclidion Distance} = \sqrt{\frac{(2\overset{n=|i_2-i_1|}{n}-1)^2 \cdot a^2 \cdot 3}{4} + \left(\frac{-3\overset{n=|j_2-j_1|}{n}+1}{2}\right)^2 a^2}$$

- For even number of rows of hexagonal tesselation

$$\text{Euclidion Distance} = \sqrt{3\overset{n=|i_2-i_1|}{n}^2 a^2 + \left(\frac{-3\overset{n=|j_2-j_1|}{n}+1}{2}\right)^2 a^2}$$

I have used second way (Center precision)

1.b) For 6-adjacency we should draw the figure below



Coordinates differences should be 1 but my approach is little different so the adjacent coordinate different can vary like $\frac{a\sqrt{3}}{2}$, $a\sqrt{3}$, $\frac{3}{2}a$. This approach can provide precise coordinates at 6-adjacency.

2)

a	b	c	d	p
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

$L(\overline{P}) = i*N+J$

$L(\overline{P}) = L(d)$

$L(\overline{P}) = L(c)$

$L(d) = L(c)$

$L(P) = L(c)$ $\quad L(d) = L(c)$

$L(\overline{P}) = L(b)$

$L(d) = L(b)$

$L(P) = L(b)$ $\quad L(d) = L(b)$

$L(c) = L(b)$

$L(P) = L(b)$ $\quad L(c) = L(b)$

$L(d) = L(c) = L(b)$

$L(p) = L(b)$ $\quad L(d) = L(c) = L(b)$

$L(\overline{p}) = L(a)$

$L(d) = L(a)$

$L(P) = L(a)$ $\quad L(d) = L(a)$

$L(c) = L(a)$

$L(P) = L(a)$ $\quad L(c) = L(a)$

$L(d) = L(c) = L(a)$

$L(P) = L(a)$ $\quad L(d) = L(c) = L(a)$

$L(b) = L(a)$

$L(p) = L(a)$ $\quad L(b) = L(a)$

$L(d) = L(b) = L(a)$

$L(p) = L(a)$ $\quad L(d) = L(b) = L(a)$

$L(c) = L(b) = L(a)$

$L(p) = L(a)$ $\quad L(c) = L(b) = L(a)$

$L(d) = L(c) = L(b) = L(a)$

$L(P) = L(a)$ $\quad L(d) = L(c) = L(b) = L(a)$

Example

	a	b	c
	1	1	0
	0	1	
	d	p	

$\Rightarrow L(p) = L(a) = 1$

$L(b) = L(a) = 1$

3) $A = \frac{s^2 n}{4\tan\left(\frac{180}{n}\right)}$

n = number of side

s = length of each side

($\frac{180}{n}$ is in degrees)

$P = s.n$ $\quad$ If we increase n our shape would be more similar to the circle

$|P|^2 = s^2 n^2$

$$\frac{|P|^2}{A} = \frac{\cancel{s^2} n^{\cancel{2}}}{\frac{\cancel{s^2} \cancel{n}}{4\tan\left(\frac{180}{n}\right)}} = 4 \cdot n \cdot \tan\left(\frac{180}{n}\right)$$

$$\lim_{n\to\infty} 4n\tan\left(\frac{180}{n}\right) \Rightarrow (\infty \times 0)$$

$$= \frac{4\tan\left(\frac{180}{n}\right)}{\left(\frac{1}{n}\right)} \Rightarrow \left(\frac{0}{0}\right)$$

$\Downarrow$ L'hopital

$$\lim_{n\to\infty} \frac{-180 \cdot 4 \cdot n^{-2}\left(1+\tan^2\left(\frac{180}{n}\right)\right)}{-n^{-2}}$$

$$= 180 \cdot 4 \quad (180 \text{ in degrees})$$

$$\lim_{n\to\infty} \frac{|P|^2}{A} = \boxed{4\pi}$$

For a finite number of n

$\frac{|P|^2}{A}$ would be greater than 4π

4-a)
If we want to closing space between (connect) two points firstly we should create line shaped structuring element. Then we should apply dilution then erosion operations respectively.
Dilution makes pixel value as maximum pixel value in neighbor pixels. Erotion makes pixel value as minimum pixel value in neighbor pixels.
The minimum length of line shaped structuring element should be greater or equal to distance between two points d. The pixels between these two points which are 0 (black) would be never 1 (white) if we choose length less than distance d. Because they never would be neihbor pixels due to short length of structure element.
To explain this situation I have used MATLAB. The source code has been given below:

```
clc; clear; close all; r=6;
n=0;
 x=zeros(256,256);
%% for 2 points (line)
SE = strel('line',r,0);
x(128,125)=1; x(128,131)=1;
X=mat2gray(x); figure;
imshow(X);title('Original Image(256x256)'); figure; imshow(SE.Neighborhood);
title('Neighborhood of Struction Element(Circular)');
closeBW = imclose(X,SE);
figure; imshow(closeBW);
title('Image after Closing operation(Dilution+Erosion)'); %% for
4 points (square)
SEsq = strel('disk',r);
x(134,125)=1;
x(134,131)=1;
```

As we can see from the code above the matrix(256x256) has been filled with zeros(blacks). Then two points has been decleared at P1=(128,125), P2=(128,131). The pixel difference between two points is 6 and its also equals to d. When we gave r as equal to d (6).
4-b)
Same explaining works for 4 point too. We should use radius of circular structure element as 6(r=d) or greater too. Because while radius less than 6 (take 5 for example) when the upper right point P3(134,131) is located just rightmost side of the circle the other point on the same line P4(134,125) would located outside of the circle. So these two points could not be neighbors and eventually they stay not connected. The points are connected when we choose the r adequate(6) as we can see below:

Original Image(256x256)

Neighborhood of Struction Element(Line)

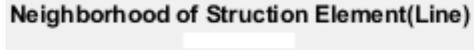
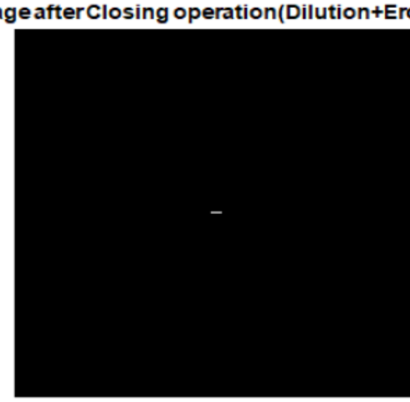
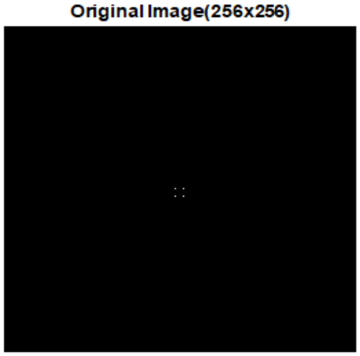


Image after Closing operation(Dilution+Erosion)



Original Image(256x256)



Neighborhood of Struction Element(Circular)

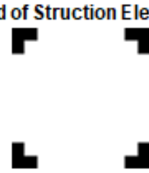
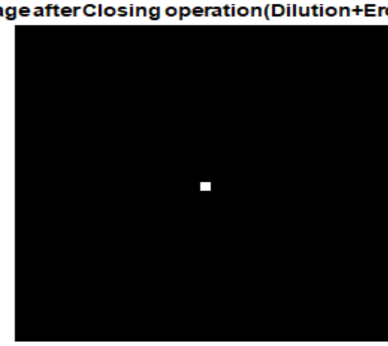


Image after Closing operation(Dilution+Erosion)



5)

3-D Forward Affine Transformation Matrices

identity matrix $\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Scaling $\rightarrow \begin{bmatrix} cx & 0 & 0 \\ 0 & cy & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rotation $\rightarrow \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Translation $\rightarrow \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$

Horizontal Shear $\rightarrow \begin{bmatrix} 1 & sh & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Vertical shear $\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ sv & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Since we need to rotate 45° clockwise → 315 degrees $\left(\frac{7\pi}{4}\right)$
then scale the image from 4:3 to 16:9
We should use Rotation and Scaling matrices

Rotation Matrix can be also written as below
Since we work on 2-D images R_m becomes

$$R_M = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$

This operation is Rotation about origin but we need to rotate an image around image center to do that we can modify our points by shifting our points by half of number of Rows and columns respectively.

Number of Rows defined as NR
Number of Columns defined as NC

$$R_m = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x - \frac{NC}{2} \\ y - \frac{NR}{2} \end{bmatrix} = \begin{bmatrix} \left(x-\frac{NC}{2}\right)\cos\theta - \left(y-\frac{NR}{2}\right)\sin\theta \\ \left(x-\frac{NC}{2}\right)\sin\theta + \left(y-\frac{NR}{2}\right)\cos\theta \end{bmatrix}$$

But we should shift back the points by same amount how we shifted before which were $\frac{NR}{2}$ and $\frac{NC}{2}$ The Result will be

$$\text{Rotated image Matrix} = \begin{bmatrix} \left(x-\frac{NC}{2}\right)\cos\theta - \left(y-\frac{NR}{2}\right)\sin\theta + \frac{NC}{2} \\ \left(x-\frac{NC}{2}\right)\sin\theta + \left(y-\frac{NR}{2}\right)\cos\theta + \frac{NR}{2} \end{bmatrix}$$

Then we should scale this matrix from 4:3 to 16:9 which is square of (4:3) so we should use the matrix below.

$$\text{Scale Matrix} = \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix}$$

where $s_1 = \left(x-\frac{NC}{2}\right)\cos\theta - \left(y-\frac{NR}{2}\right)\sin\theta + \frac{NC}{2}$

$s_2 = \left(x-\frac{NC}{2}\right)\sin\theta + \left(y-\frac{NR}{2}\right)\cos\theta + \frac{NR}{2}$

If we multiply Scale matrix with our coordinates we get the result above

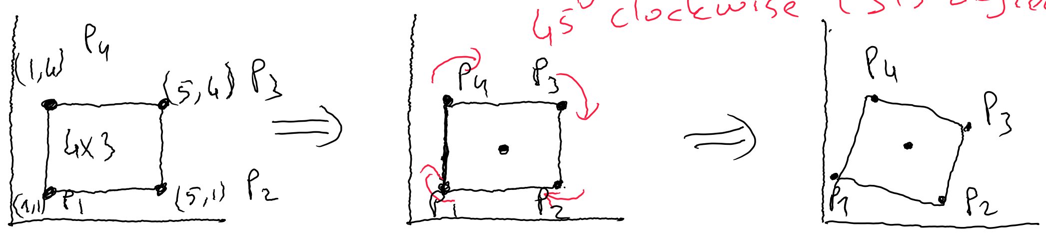
$$\boxed{\text{Rotated and Scaled Matrix} = \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} s_1^2 \\ s_2^2 \end{bmatrix} = \begin{bmatrix} \left(\left(x-\frac{NC}{2}\right)\cos\theta - \left(y-\frac{NR}{2}\right)\sin\theta + \frac{NC}{2}\right)^2 \\ \left(\left(x-\frac{NC}{2}\right)\sin\theta + \left(y-\frac{NR}{2}\right)\cos\theta + \frac{NR}{2}\right)^2 \end{bmatrix}}$$

Example:

$\frac{NR}{2} = 3/2 = 1.5$

$\frac{NC}{2} = 4/2 = 2$

45° clockwise (315 degree Counter clock)



$x=1$, $y=1$:
$(-1)\cos(315) - (-0.5)\sin(315) + 2 \rightarrow 0.94$
$(-1)\sin(315) + (-0.5)\cos(315) + 1.5 \rightarrow 1.85$
$(\underbrace{0.94}_{x}, \underbrace{1.85}_{y}) \rightarrow P_1 \rightarrow$ x decreased, y increased

$x=5$, $y=1$:
$(3)\cos(315) - (-0.5)\sin(315) + 2 \rightarrow 3.76$
$(3)\sin(315) - (-0.5)\cos(315) + 1.5 \rightarrow -0.97$
$(3.76, -0.97) \rightarrow P_2 \rightarrow$ x decreased, y decreased

$x=5$, $y=4$:
$(3)\cos(315) - (2.5)\sin 315 + 2 \rightarrow 5.88$
$(3)\sin(315) - (2.5)\cos(315) + 1.5 \rightarrow 1.14$
$(\underbrace{5.88}_{x}, \underbrace{1.14}_{y}) \rightarrow P_3 \rightarrow$ x increased, y decreased

$x=1$, $y=4$:
$(-1)\cos 315 - (2.5)\sin 315 + 2 \rightarrow 3.06$
$(-1)\sin 315 - (2.5)\cos(315) + 1.5 \rightarrow 3.97$
$(\underbrace{3.06}_{x}, \underbrace{3.97}_{y}) \rightarrow P_4 \rightarrow$ x increased, y (stays same)

After Rotation we need to scale it from 4:3 to 16:9

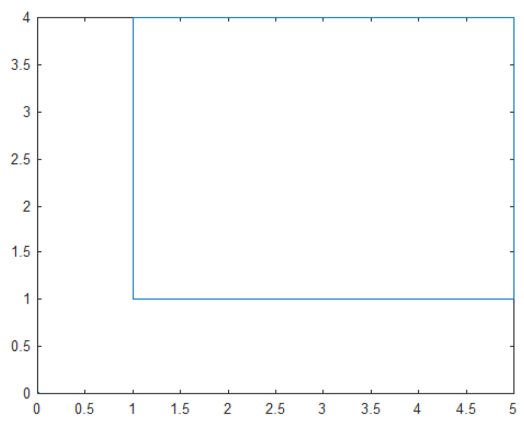
$\left.\begin{matrix} x_1 = 1 \\ y_1 = 1 \end{matrix}\right) \rightarrow \begin{matrix} x = 0.94 \\ y = 1.85 \end{matrix} \xrightarrow[s^2]{\text{Scaling}} \begin{matrix} x_1^2 = 0.8836 \\ y_1^2 = 3.4225 \end{matrix}$

$\left.\begin{matrix} x_2 = 5 \\ y_2 = 1 \end{matrix}\right) \rightarrow \begin{matrix} x = 3.76 \\ y = -0.97 \end{matrix} \xrightarrow[s^2]{\text{Scaling}} \begin{matrix} x_2^2 = 14.13 \\ y_2^2 = 0.9409 \end{matrix}$

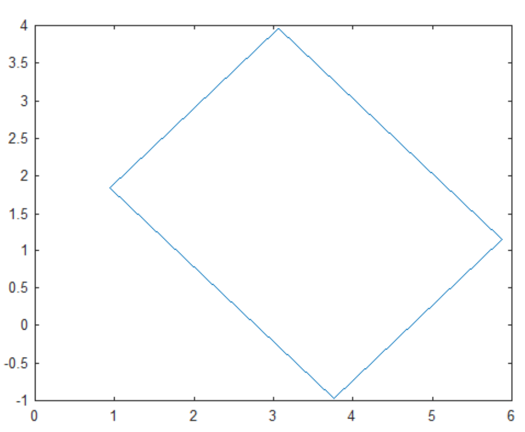
$\left.\begin{matrix} x_3 = 5 \\ y_3 = 4 \end{matrix}\right) \rightarrow \begin{matrix} x = 5.88 \\ y = 1.14 \end{matrix} \xrightarrow[s^2]{\text{Scaling}} \begin{matrix} x_3^2 = 34.57 \\ y_3^2 = 1.2996 \end{matrix}$

$\left.\begin{matrix} x_4 = 1 \\ y_4 = 4 \end{matrix}\right) \rightarrow \begin{matrix} x = 3.06 \\ y = 3.97 \end{matrix} \xrightarrow[s^2]{\text{Scaling}} \begin{matrix} x_4^2 = 9.3636 \\ y_4^2 = 15.7609 \end{matrix}$

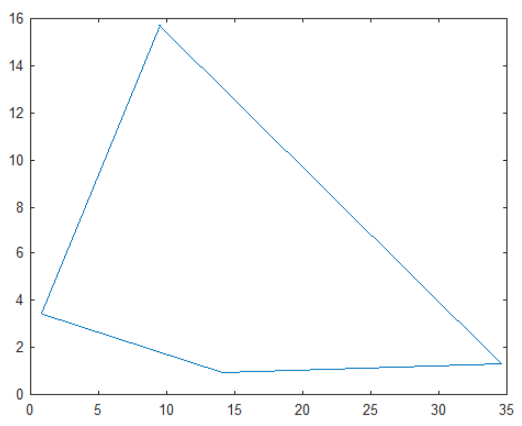
The Resultant shapes has been given below
(From Matlab)



Original Shape



45° clockwise Rotated



45° clockwise Rotated + Scaled From 4:3 to 16:9