lite-cosmic-sim

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1 lite-cosmic-sim	1
1.1 Overview	1
1.2 Sample usage	1
1.3 How to get and build the project	2
1.4 Documentation	3
1.5 Convenience utilities for visualizing output	3
1.6 Overview of implemented functionalities	3
1.7 Notes on using CUDA	3
2 Writing your own equation	5
2.1 Adding the equation class	5
2.2 Adding the coupling parameter in workspace	6
2.3 Adding a new parameter	6
2.4 Add a function to compute energy density	6
2.5 Using CUDA	6
3 Concept Index	7
3.1 Concepts	7
4 Hierarchical Index	9
4.1 Class Hierarchy	9
5 Class Index	11
5.1 Class List	11
6 File Index	13
6.1 File List	13
7 Concept Documentation	15
7.1 HasFa Concept Reference	15
7.1.1 Concept definition	15
7.2 HasFRWParameters Concept Reference	15
7.2.1 Concept definition	15
7.3 HasLambda Concept Reference	15
7.3.1 Concept definition	15
7.4 HasLatticeParams Concept Reference	15
7.4.1 Concept definition	15
7.5 HasMass Concept Reference	16
7.5.1 Concept definition	16
7.6 HasPsiApproximationParameters Concept Reference	16
7.6.1 Concept definition	16
7.7 LatticeEquationConcept Concept Reference	16
7.7.1 Concept definition	16
8 Class Documentation	17

8.1 ComovingCurvatureEquationInFRW Struct Reference	17
8.1.1 Detailed Description	18
8.1.2 Member Function Documentation	18
8.1.2.1 compute_energy_density()	18
8.2 ConstIntervalObserver< Equation, save_field_spectrum, save_density_spectrum, save_density > Struct Template Reference	18
8.3 CudaApproximateComovingCurvatureEquationInFRW Struct Reference	19
8.4 CudaComovingCurvatureEquationInFRW Struct Reference	20
8.5 CudaFixedCurvatureEquationInFRW Struct Reference	20
8.6 CudaKleinGordonEquationInFRW Struct Reference	21
8.7 CudaLambdaEquationInFRW Struct Reference	21
8.8 CudaSqrtPotentialEquationInFRW Struct Reference	22
8.9 cufftWrapper Struct Reference	23
8.10 cufftWrapperBatchedD2Z Struct Reference	23
8.10.1 Detailed Description	24
8.11 cufftWrapperD2Z Struct Reference	24
8.11.1 Detailed Description	24
8.12 cufftWrapperNoBatching Struct Reference	24
8.13 empty Struct Reference	25
8.14 fftWrapperDispatcher< Vector > Struct Template Reference	25
8.15 fftWrapperDispatcher < Eigen::VectorXd > Struct Reference	25
8.16 fftWrapperDispatcher< thrust::device_vector< double >> Struct Reference	26
8.17 fftwWrapper Struct Reference	26
8.17.1 Detailed Description	26
8.18 KGParam Struct Reference	27
8.19 KleinGordonEquation Struct Reference	27
8.19.1 Detailed Description	27
8.19.2 Member Function Documentation	27
8.19.2.1 compute_energy_density()	27
8.19.2.2 operator()()	28
8.20 KleinGordonEquationInFRW Struct Reference	28
8.20.1 Detailed Description	29
8.20.2 Member Function Documentation	29
8.20.2.1 compute_energy_density()	29
$8.21 \ midpoint < State, \ Value, \ Deriv, \ Time, \ Algebra, \ Operations, \ Resizer > Class \ Template \ Reference . .$	29
8.22 MyParam Struct Reference	30
8.23 SampleParam Struct Reference	31
8.23.1 Detailed Description	31
8.23.2 Member Data Documentation	31
8.23.2.1 a1	31
8.23.2.2 H1	31
8.23.2.3 k_ast	31

	8.23.2.4 L	31
	8.23.2.5 lambda	32
	8.23.2.6 m	32
	8.23.2.7 N	32
	8.23.2.8 t1	32
	8.23.2.9 varphi_std_dev	32
	8.24 StaticEOSCosmology Struct Reference	32
	8.25 WKBSolutionForKleinGordonEquationInFRW Struct Reference	33
	8.26 WorkspaceGeneric < Vector > Struct Template Reference	33
	8.26.1 Detailed Description	34
9	File Documentation	35
	9.1 src/cuda_wrapper.cuh File Reference	35
	9.1.1 Detailed Description	36
	9.2 cuda_wrapper.cuh	36
	9.3 src/dispatcher.hpp File Reference	37
	9.3.1 Detailed Description	38
	9.4 dispatcher.hpp	38
	9.5 src/eigen_wrapper.hpp File Reference	39
	9.5.1 Detailed Description	39
	9.6 eigen_wrapper.hpp	39
	9.7 src/equations.hpp File Reference	39
	9.7.1 Detailed Description	40
	9.8 equations.hpp	40
	9.9 src/equations_cuda.cuh File Reference	41
	9.9.1 Detailed Description	41
	9.10 equations_cuda.cuh	42
	9.11 src/fdm3d.hpp File Reference	43
	9.11.1 Detailed Description	44
	9.11.2 Macro Definition Documentation	44
	9.11.2.1 PADDED_IDX_OF	44
	9.11.3 Function Documentation	44
	9.11.3.1 compute_cutoff_fouriers()	44
	9.11.3.2 compute_field_with_scaled_fourier_modes()	44
	9.11.3.3 compute_inverse_laplacian()	45
	9.11.3.4 compute_mode_power_spectrum()	45
	9.11.3.5 compute_power_spectrum()	47
	9.12 fdm3d.hpp	48
	9.13 src/fdm3d_cuda.cuh File Reference	48
	9.13.1 Detailed Description	49
	9.14 fdm3d_cuda.cuh	49
	9.15 src/fftw_wrapper.hpp File Reference	50

9.15.1 Detailed Description
9.16 fftw_wrapper.hpp
9.17 field_booster.hpp
9.18 src/initializer.hpp File Reference
9.18.1 Detailed Description
9.18.2 Variable Documentation
9.18.2.1 homogeneous_field
9.18.2.2 homogeneous_field_with_fluctuations
9.18.2.3 perturbed_grf
9.18.2.4 perturbed_grf_and_comoving_curvature_fft
9.18.2.5 perturbed_grf_without_saving_Psi
9.18.2.6 plane_wave
9.18.2.7 unperturbed_grf
9.18.2.8 unperturbed_grf_and_fixed_curvature
9.18.2.9 unperturbed_grf_with_background
9.18.2.10 unperturbed_grf_with_Psi
9.18.2.11 wave_packet
9.19 initializer.hpp
9.20 io.hpp
9.21 src/midpoint.hpp File Reference
9.21.1 Detailed Description
9.22 midpoint.hpp
9.23 src/observer.hpp File Reference
9.23.1 Detailed Description
9.24 observer.hpp
9.25 src/param.hpp File Reference
9.25.1 Detailed Description
9.26 param.hpp
9.27 src/physics.hpp File Reference
9.27.1 Detailed Description
9.28 physics.hpp
9.29 src/random_field.hpp File Reference
9.29.1 Detailed Description
9.29.2 Function Documentation
9.29.2.1 broken_power_law_given_amplitude_3d()
9.29.2.2 generate_inhomogeneous_gaussian_random_field() 6
9.29.2.3 power_law_with_cutoff_given_amplitude_3d() 6
9.29.2.4 scale_invariant_spectrum_3d()
9.30 random_field.hpp
9.31 special_function.hpp
9.32 src/utility.hpp File Reference
9.32.1 Detailed Description

Index	K	77
9.	.37 workspace.hpp	74
	9.36.1 Detailed Description	74
9.	.36 src/workspace.hpp File Reference	73
9.	.35 wkb.hpp	73
	9.34.1 Detailed Description	72
9.	.34 src/wkb.hpp File Reference	72
9.	.33 utility.hpp	71

lite-cosmic-sim

lite-cosmic-sim is a lightweight and modular framework for performing field simulations in cosmology. This framework was used for studying free-streaming of wave dark matter; see arxiv:xxxx.xxxx for the study and these youtube videos for visualization. The codebase contains several field equations on both CPU and GPU (CUDA), offering choices for numerical methods and simulation outputs.

1.1 Overview

This codebase aims to be:

- 1. As fast as possible. Users should be able to write code that exhausts hardward potential within this framework.
- 2. Easily modifiable and extensible. Users should be able to focus on physics-relevant code, such as that for setting initial conditions or the field equation.

To achieve these goals, the framework is written in a modular structure. This allows users to easily switch between different initial conditions, field equations, output methods, and even between using CPUs or GPUs for computation. Users have to and only have to provide the low level implementation for the physics-relevant code. This means users have full control over optimization of core routines, and they are not limited to a specific set of provided features. This flexibility makes it easy for the user to test new ideas, which is useful in research.

1.2 Sample usage

The following code initializes a homogeneous Klein Gordon field with (initially) unit field strength and zero time derivative. Then the field is evolved from t=0 to t=10. Field and density spectra are saved to disk per unit time.

```
##
#include "param.hpp"
#include "initializer.hpp"
#include "equations.hpp"
#include "observer.hpp"

struct MyParam {
  long long int N = 256; // Number of lattice sites (per axis)
  double L = 256.0; // Box size
  double m = 1.0; // Field mass
  double f = 1.0; // The initial (homogeneous) field value
  double dt_f = 0.0; // The initial (homogeneous) field time derivative value
  double t_start = 0.0; // Start time of numerical integration
  double t_end = 10.0; // End time of numerical integration
  double t_interval = 1.0; // Interval between saving outputs
```

2 lite-cosmic-sim

```
int main() {
    using namespace Eigen;
    using namespace boost::numeric::odeint;

    typedef KleinGordonEquation Equation;
    typedef Eigen::VectorXd State;
    typedef WorkspaceGeneric<State> Workspace;

MyParam param;

Workspace workspace(param, homogeneous_field);

Equation eqn(workspace);

ConstIntervalObserver<Equation> observer("output/sample_equation/", param, eqn);
    auto stepper = runge_kutta4_classic<State>();
    integrate_const(stepper, eqn, workspace.state, param.t_start, param.t_end, 0.1, observer);
}
```

Here's a break down of the code:

- MyParam is a POD struct specifying parameters for the simulation. You may define your own struct to include new parameters (coupling strength, FRW universe parameters, time step size, etc), as long as it is a POD and contains lattice parameters N and L.
- Workspace is a type containing temporary variables for a simulation (e.g. the field). It is initialized with param and a callback homogeneous_field, which sets the field to homogeneous value param.f and time derivative param.dt_f. You can easily define your own callbacks (using lambdas) to set other sorts of initial conditions.
- Equation is the equation to be solved. Here it is the pre-defined KleinGordonEquation. You can of course write your own equations.
- ConstIntervalObserver<Equation> specifies how to save outputs during simulation. By default it saves spectra for field and density.
- stepper is the RK4 method provided by the boost odeint library. You can choose other methods (e.g. Euler, DOPRI5) in the library, or even write your own. The odeint library is responsible for the main numerical integration loop in this codebase.
- integrate_const is a convenience function in the odeint library. It runs the simulation and saves results to "output/sample_equation", as specified by observer.

1.3 How to get and build the project

Compiler requirement: a C++ compiler supporting C++20. (I used g++12.2.0.)

Required dependency: fftw3

Optional dependency: CUDA Toolkit

I also included header-only libraries Eigen 3.4.0 and boost 1.84 along with the codebase in the external directory.

Makefile is used for build system. I have tested compilation on Linux and MacOS systems. To compile the project:

• Download the project with (for example) git clone https://github.com/hypermania/lite-cosmic-sim.

- (If default settings don't work:) Modify the Makefile so that it knows where your fftw or CUDA include files / library files are.
- If you have CUDA Toolkit installed, simply run make -j.
- If you don't have CUDA Toolkit, run make -j disable-cuda=true. (I use compiler flags to comment out CUDA-dependent code. e.g. CudaComovingCurvatureEquationInFRW)

Note: If you have a CUDA compatible NVIDIA GPU, using CUDA is highly recommended. In our case, it produced more than 10 times speedup.

1.4 Documentation

LaTeX version of documentation is in documentation.pdf. If you have doxygen, you can also build an html version by running doxygen doxygen.config.

1.5 Convenience utilities for visualizing output

Two Mathematica notebooks <code>spectra.nb</code>, <code>snapshots.nb</code> and a python <code>scriptplot_util.py</code> are included for visualizing outputs from the program. By default, running the entire notebook / python script will read sample data from <code>output/Growth_and_FS</code> and produce spectra and snapshots. If you generate new outputs from the program, you just need to change <code>dir</code> or <code>project dir</code> variables to the new output directory.

1.6 Overview of implemented functionalities

Symbol	Description
generate_inhomogeneous_gaussian_↔	Function for initializing Gaussian random fields with
random_field	spatially inhomogeneous variances. This procedure is
	crucial for generating the initial conditions used in the
	paper.
KleinGordonEquationInFRW and	Klein Gordon equation that runs on CPU and GPU.
CudaKleinGordonEquationInFRW	Used in section 4.2.1 of paper.
ComovingCurvatureEquationInFRW,	A scalar field in the presence of external gravity that
CudaComovingCurvatureEquationInFRW	is consistent with some set of comoving curvature per-
and CudaApproximateComovingCurvatureEq	uturbations.FUsed in section 4.2.2 of paper.
CudaSqrtPotentialEquationInFRW	A scalar field with monodromy potential. Used in sec-
	tion 4.2.3 of paper.
CudaFixedCurvatureEquationInFRW	A scalar field in a fixed gravitational potential.
CudaLambdaEquationInFRW	A scalar field with lambda phi^4 interaction.

1.7 Notes on using CUDA

We do separate compilation of .cpp files and .cu files; .cu files are automatically compiled by nvcc, whereas .cpp files are compiled by the host compiler. We use the thrust library (included with CUDA Toolkit) extensively, with field state vectors having type thrust::device_vector<double>. Initialization procedures usually prepare some profile on the CPU and then copy it to $device_vector<double>$ state in the workspace.

lite-cosmic-sim

A straightforward way to use CUDA for a simulation is to implement an Equation class with $thrust \leftarrow :: device_vector < double>$ as state vector. You will probably need to write your own CUDA kernels for that purpose. See equations_cuda.cu for some examples. Don't worry about adapting CUDA with the numerical integrators (e.g. RK4); the files in $src/odeint_thrust$ will take care of that automatically.

Writing your own equation

Here we give an example of adding a field equation with $\kappa \varphi^6$ interaction to the codebase.

$$\ddot{\varphi} - \nabla^2 \varphi + m^2 \varphi + \kappa \varphi^5 = 0$$

2.1 Adding the equation class

We use the boost odeint library for numerical integration. To use the library, we need to implement a new equation class. See this link for an example of odeint equation class. In our case, the equation class with φ^6 looks like:

```
struct KappaEquation {
  typedef Eigen::VectorXd Vector;
  typedef Vector State;
  typedef WorkspaceGeneric<State> Workspace;
  Workspace &workspace;

KappaEquation(Workspace &workspace_) : workspace(workspace_) {}
  void operator()(const State &dxdt, State &x, const double t);
};
```

In the first few lines, the types <code>Vector</code>, <code>State</code> and <code>Workspace</code> are defined. These definitions specify what state vector the equation is going to work with: if you want to use different state vector types (e.g. GPU device vector), you will need to define different equation classes. Here we use <code>Eigen::VectorXd</code>. The equation class also has a reference to a <code>workspace</code>, so that it has access to essential information for evolution (e.g. mass and coupling parameters).

The most important function here is the <code>operator()</code>. When this function is called, it computes the time derivative of the state vector x at time t, and stores it to dxdt. Implementing this function is the minimal requirement for a class to work with odeint. To do this, we can simply copy the implementation for <code>KleinGordonEquation</code> \leftarrow

Note the extra line kappa * pow (x (seqN (IDX_OF (N, a, b, 0), N)), 5) giving the $\kappa \varphi^5$ term in the equation.

2.2 Adding the coupling parameter in workspace

The code given above won't compile since workspace.kappa doesn't exist yet.

2.3 Adding a new parameter

You will need to define a new parameter struct that contains the new κ parameter.

2.4 Add a function to compute energy density

In order to save density spectrum, you would also want to implement a function to calculate energy density profile.

2.5 Using CUDA

You want your equation to work on GPU memory, so the state vector would be: typedef thrust::device_vector<double> Vector;

Your operator () should modify the state vector. To do that, the easiest way is to write your own CUDA kernel.

Concept Index

3.1 Concepts

Here is a list of all documented concepts with brief descriptions:

HasFa					 									 						- 1
HasFRWParameters					 									 						- 1
HasLambda					 									 						- 1
HasLatticeParams .					 									 						- 1
HasMass					 									 						- 1
HasPsiApproximationP	arai	me	ter	'S	 		 							 						- 1
LatticeEquationConcep	t				 		 							 						- 1

8 Concept Index

Hierarchical Index

4.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:

boost::numeric::odeint::algebra_stepper_base midpoint< State, Value, Deriv, Time, Algebra, Operations, Resizer >
ComovingCurvatureEquationInFRW
ConstIntervalObserver< Equation, save_field_spectrum, save_density_spectrum, save_density > 18
CudaApproximateComovingCurvatureEquationInFRW
CudaComovingCurvatureEquationInFRW
CudaLambdaEquationInFRW
CudaSqrtPotentialEquationInFRW
cufftWrapper
cufftWrapperBatchedD2Z
cufftWrapperD2Z 24
cufftWrapperNoBatching
empty
fftWrapperDispatcher < Vector >
fftWrapperDispatcher < Eigen::VectorXd >
lem:lem:lem:lem:lem:lem:lem:lem:lem:lem:
fftwWrapper
KGParam
KleinGordonEquation
KleinGordonEquationInFRW
MyParam
SampleParam
StaticEOSCosmology
WKBSolutionForKleinGordonEquationInFRW
WorkspaceGeneric < Vector >

10 Hierarchical Index

Class Index

5.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

ComovingCurvatureEquationInFRW	
Equation for free scalar field in FRW spacetime, including comoving metric perturbations (in	
radiation domination)	17
ConstIntervalObserver< Equation, save_field_spectrum, save_density_spectrum, save_density >	18
CudaApproximateComovingCurvatureEquationInFRW	19
CudaComovingCurvatureEquationInFRW	20
CudaFixedCurvatureEquationInFRW	20
CudaKleinGordonEquationInFRW	21
CudaLambdaEquationInFRW	21
CudaSqrtPotentialEquationInFRW	22
cufftWrapper	23
cufftWrapperBatchedD2Z	
Wrapper for 3D cufftPlanMany. Performs two double to complex double FFT for a N^3 grid \dots	23
cufftWrapperD2Z	
Wrapper for 3D cufftPlan3d. Performs double to complex double FFT for a N^3 grid	24
cufftWrapperNoBatching	24
empty	25
fftWrapperDispatcher < Vector >	25
fftWrapperDispatcher < Eigen::VectorXd >	25
fftWrapperDispatcher< thrust::device_vector< double >>	26
fftwWrapper	
Wrapper for various FFTW functions for a N^3 grid	26
KGParam	27
KleinGordonEquation	
The Klein Gordon equation, $\ddot{\varphi} - \nabla^2 \varphi + m^2 \varphi = 0$	27
KleinGordonEquationInFRW	
The Klein Gordon in FRW equation, $\ddot{\varphi}+3H\dot{\varphi}-\nabla^2\varphi/a^2+m^2\varphi=0$	28
midpoint < State, Value, Deriv, Time, Algebra, Operations, Resizer >	29
MyParam	30
SampleParam	
A sample parameter type specifying a lambda-phi-4 theory in an FRW background	31
StaticEOSCosmology	32
WKBSolutionForKleinGordonEquationInFRW	33
WorkspaceGeneric < Vector >	
A generic workspace for storing temporary	33

12 Class Index

File Index

6.1 File List

Here is a list of all documented files with brief descriptions:

src/cuda_wrapper.cuh	
Wrapper for CUDA Toolkit	35
src/dispatcher.hpp	
Automatically dispatching between using FFTW and CUFFT libraries	37
src/eigen_wrapper.hpp	
Wrap some Eigen functionalites	39
src/equations.hpp	
Header for field equations that runs on the CPU	39
src/equations_cuda.cuh	
Header for field equations that runs on the GPU	41
src/fdm3d.hpp	
Common procedures for manipulating / summarizing field configuration on a 3D lattice $\dots \dots$	43
src/fdm3d_cuda.cuh	
CUDA implementation for fdm3d.hpp. Common procedures for manipulating / summarizing field	
configuration on a 3D lattice	48
src/fftw_wrapper.hpp	
Wrapper for FFTW library	50
src/field_booster.hpp	51
src/initializer.hpp	
Snippets for initializing workpaces. (e.g. field initial conditions, etc)	51
src/io.hpp	59
src/midpoint.hpp	
Midpoint method implementation for odeint stepper	60
src/observer.hpp	
Implements "observers", which controls what gets saved during simulations	61
src/param.hpp	0.0
Utilities for managing simulations parameters	63
src/physics.hpp	0.0
Collection of repeatedly used physics formulas. (e.g. FRW cosmology related formulas)	66
src/random_field.hpp	0-
Utilities for generating Gaussian random fields of given spectrum and inhomogeneity	67
src/special_function.hpp	70
Utilities for debugging / profiling / pretty printing	71
src/wkb.hpp	/
Implementation of the WKB solution	72
src/workspace.hpp	12
A generic "workspace" class, containing parameters / data / tools used during simulations	73

14 File Index

Concept Documentation

7.1 HasFa Concept Reference

7.1.1 Concept definition

```
template<typename Param>
concept HasFa = requires (Param param) { TYPE_REQUIREMENT(param.f_a, double) }
```

7.2 HasFRWParameters Concept Reference

7.2.1 Concept definition

7.3 HasLambda Concept Reference

7.3.1 Concept definition

```
template<typename Param>
concept HasLambda = requires (Param param) { TYPE_REQUIREMENT(param.lambda, double) }
```

7.4 HasLatticeParams Concept Reference

7.4.1 Concept definition

```
template<typename Param>
concept HasLatticeParams = requires (Param param)
{ TYPE_REQUIREMENT(param.N, long long int)
    TYPE_REQUIREMENT(param.L, double) }
```

7.5 HasMass Concept Reference

7.5.1 Concept definition

```
template<typename Param>
concept HasMass = requires (Param param) { TYPE_REQUIREMENT(param.m, double) }
```

7.6 HasPsiApproximationParameters Concept Reference

7.6.1 Concept definition

```
template<typename Param>
concept HasPsiApproximationParameters = requires (Param param)
{ TYPE_REQUIREMENT(param.M, long long int) }
```

7.7 LatticeEquationConcept Concept Reference

7.7.1 Concept definition

```
template<typename Equation>
concept LatticeEquationConcept = requires (Equation eqn)
{
   eqn.workspace;
   eqn.compute_energy_density(eqn.workspace, 0.0);
}
```

Class Documentation

8.1 ComovingCurvatureEquationInFRW Struct Reference

Equation for free scalar field in FRW spacetime, including comoving metric perturbations (in radiation domination).

```
#include <equations.hpp>
```

Public Types

- typedef Eigen::VectorXd Vector
- · typedef Vector State
- typedef WorkspaceGeneric < State > Workspace

Public Member Functions

- ComovingCurvatureEquationInFRW (Workspace &workspace_)
- void operator() (const State &, State &, const double)

Static Public Member Functions

• static Vector compute_energy_density (Workspace &workspace, const double t)

Compute the energy density profile from the workspace.

Public Attributes

• Workspace & workspace

8.1.1 Detailed Description

Equation for free scalar field in FRW spacetime, including comoving metric perturbations (in radiation domination).

Equation is given by

$$\begin{split} \ddot{\varphi} + 3H\dot{\varphi} - e^{4\Psi} \frac{\nabla^2}{a^2} \varphi + e^{2\Psi} m^2 \varphi - 4\dot{\Psi}\dot{\varphi} &= 0 \\ \Psi_{\mathbf{k}}(t) &= 2\mathcal{R}_{\mathbf{k}} \frac{\sin\left(k\eta/\sqrt{3}\right) - (k\eta/\sqrt{3})\cos\left(k\eta/\sqrt{3}\right)}{(k\eta/\sqrt{3})^3} \\ \dot{\Psi}_{\mathbf{k}}(t) &= 2\mathcal{R}_{\mathbf{k}} H(t) \frac{3(k\eta/\sqrt{3})\cos\left(k\eta/\sqrt{3}\right) + ((k\eta/\sqrt{3})^2 - 3)\sin\left(k\eta/\sqrt{3}\right)}{(k\eta/\sqrt{3})^3} \\ \eta(t) &= \frac{(2H_i t)^{1/2}}{a_i H_i} \quad \text{is the conformal time} \end{split}$$

where $\mathcal{R}_{\mathbf{k}}$ is read from workspace variable R_fft. See equation (6.160) of Baumann's cosmology text-book. This implementation is not optimized. It was only used for verifying the GPU implementations CudaComovingCurvatureEquationInFRW and CudaApproximateComovingCurvatureEquationInFRW.

8.1.2 Member Function Documentation

8.1.2.1 compute_energy_density()

Compute the energy density profile from the workspace.

Parameters

in	workspace	The workspace for evaluating the energy density.
	t	The current time parameter.

Returns

A vector of size N^3 , giving the energy density profile $\rho=\frac{1}{2}(e^{-2\Psi}\dot{\varphi}^2+e^{2\Psi}(\nabla\varphi)^2/a(t)^2+m^2\varphi^2)$ on the lattice.

The documentation for this struct was generated from the following files:

- src/equations.hpp
- · src/equations.cpp

8.2 ConstIntervalObserver< Equation, save_field_spectrum, save_density_spectrum, save_density > Struct Template Reference

Public Types

- typedef Equation::Workspace Workspace
- typedef Workspace::State State
- · typedef State Vector

Public Member Functions

- template<typename Param >
 - ConstIntervalObserver (const std::string &dir_, const Param ¶m, Equation &eqn)
- ConstintervalObserver (const ConstintervalObserver &)=default
- void operator() (const State &x, double t)

Public Attributes

- Workspace & workspace
- int idx
- std::string dir
- double t_start
- double t end
- · double t_interval
- double t_last

The documentation for this struct was generated from the following file:

src/observer.hpp

8.3 CudaApproximateComovingCurvatureEquationInFRW Struct Reference

Public Types

- typedef thrust::device_vector< double > Vector
- typedef Vector State
- typedef WorkspaceGeneric< Vector > Workspace

Public Member Functions

- CudaApproximateComovingCurvatureEquationInFRW (Workspace &workspace_)
- void operator() (const State &, State &, const double)

Static Public Member Functions

• static Vector compute_energy_density (Workspace &workspace, const double t)

Public Attributes

• Workspace & workspace

The documentation for this struct was generated from the following files:

- src/equations_cuda.cuh
- src/equations_cuda.cu

8.4 CudaComovingCurvatureEquationInFRW Struct Reference

Public Types

- typedef thrust::device_vector< double > Vector
- typedef Vector State
- typedef WorkspaceGeneric< Vector > Workspace

Public Member Functions

- CudaComovingCurvatureEquationInFRW (Workspace &workspace_)
- void operator() (const State &, State &, const double)

Static Public Member Functions

• static Vector compute_energy_density (Workspace &workspace, const double t)

Public Attributes

Workspace & workspace

The documentation for this struct was generated from the following files:

- src/equations_cuda.cuh
- src/equations_cuda.cu

8.5 CudaFixedCurvatureEquationInFRW Struct Reference

Public Types

- typedef thrust::device_vector< double > Vector
- typedef Vector State
- typedef WorkspaceGeneric< Vector > Workspace

Public Member Functions

- CudaFixedCurvatureEquationInFRW (Workspace &workspace_)
- void operator() (const State &, State &, const double)

Static Public Member Functions

• static Vector compute_energy_density (const Workspace &workspace, const double t)

Public Attributes

Workspace & workspace

The documentation for this struct was generated from the following files:

- · src/equations_cuda.cuh
- src/equations_cuda.cu

8.6 CudaKleinGordonEquationInFRW Struct Reference

Public Types

- typedef thrust::device_vector< double > Vector
- · typedef Vector State
- typedef WorkspaceGeneric < Vector > Workspace

Public Member Functions

- CudaKleinGordonEquationInFRW (Workspace &workspace_)
- void operator() (const State &, State &, const double)

Static Public Member Functions

- static Vector compute_energy_density (const Workspace &workspace, const double t)
- static Vector compute_dot_energy_density (const Workspace &workspace, const double t)

Public Attributes

• Workspace & workspace

The documentation for this struct was generated from the following files:

- src/equations_cuda.cuh
- src/equations_cuda.cu

8.7 CudaLambdaEquationInFRW Struct Reference

Public Types

- $\bullet \ \ \text{typedef thrust::} \\ \text{device_vector} < \\ \text{double} > \\ \text{Vector}$
- typedef Vector State
- typedef WorkspaceGeneric< Vector > Workspace

Public Member Functions

- CudaLambdaEquationInFRW (Workspace &workspace_)
- void operator() (const State &, State &, const double)

Static Public Member Functions

• static Vector compute_energy_density (const Workspace &workspace, const double t)

Public Attributes

• Workspace & workspace

The documentation for this struct was generated from the following files:

- src/equations_cuda.cuh
- src/equations cuda.cu

8.8 CudaSqrtPotentialEquationInFRW Struct Reference

Public Types

- typedef thrust::device_vector< double > Vector
- typedef Vector State
- typedef WorkspaceGeneric< Vector > Workspace

Public Member Functions

- CudaSqrtPotentialEquationInFRW (Workspace &workspace)
- void operator() (const State &, State &, const double)

Static Public Member Functions

• static Vector compute_energy_density (const Workspace &workspace, const double t)

Public Attributes

• Workspace & workspace

The documentation for this struct was generated from the following files:

- · src/equations_cuda.cuh
- src/equations_cuda.cu

8.9 cufftWrapper Struct Reference

Public Member Functions

- cufftWrapper (int N)
- thrust::device vector< double > execute_d2z (thrust::device vector< double > &in)
- thrust::device_vector< double > execute_batched_d2z (thrust::device_vector< double > &in)
- thrust::device_vector< double > execute_z2d (thrust::device_vector< double > &in)
- cufftWrapper (const cufftWrapper &)=delete
- cufftWrapper & operator= (const cufftWrapper &)=delete
- cufftWrapper (cufftWrapper &&)=delete
- cufftWrapper & operator= (cufftWrapper &&)=delete

Public Attributes

- int N
- cufftHandle plan_d2z
- cufftHandle plan batched d2z
- cufftHandle plan_z2d
- thrust::device vector< double > work area

The documentation for this struct was generated from the following files:

- src/cuda_wrapper.cuh
- src/cuda_wrapper.cu

8.10 cufftWrapperBatchedD2Z Struct Reference

Wrapper for 3D cufftPlanMany. Performs two double to complex double FFT for a ${\cal N}^3$ grid.

Public Member Functions

- cufftWrapperBatchedD2Z (int N_)
- thrust::device vector< double > execute (thrust::device vector< double > &in)
- cufftWrapperBatchedD2Z (const cufftWrapperBatchedD2Z &)=delete
- cufftWrapperBatchedD2Z & operator= (const cufftWrapperBatchedD2Z &)=delete
- cufftWrapperBatchedD2Z (cufftWrapperBatchedD2Z &&)=delete
- cufftWrapperBatchedD2Z & operator= (cufftWrapperBatchedD2Z &&)=delete

Public Attributes

- int N
- cufftHandle plan

8.10.1 Detailed Description

Wrapper for 3D cufftPlanMany. Performs two double to complex double FFT for a N^3 grid.

The documentation for this struct was generated from the following files:

- src/cuda_wrapper.cuh
- src/cuda_wrapper.cu

8.11 cufftWrapperD2Z Struct Reference

Wrapper for 3D cufftPlan3d. Performs double to complex double FFT for a N^3 grid.

Public Member Functions

- cufftWrapperD2Z (int N_)
- thrust::device_vector< double > execute (thrust::device_vector< double > &in)
- cufftWrapperD2Z (const cufftWrapperD2Z &)=delete
- cufftWrapperD2Z & operator= (const cufftWrapperD2Z &)=delete
- cufftWrapperD2Z (cufftWrapperD2Z &&)=delete
- cufftWrapperD2Z & operator= (cufftWrapperD2Z &&)=delete

Public Attributes

- int N
- · cufftHandle plan

8.11.1 Detailed Description

Wrapper for 3D cufftPlan3d. Performs double to complex double FFT for a N^3 grid.

The documentation for this struct was generated from the following files:

- · src/cuda wrapper.cuh
- · src/cuda_wrapper.cu

8.12 cufftWrapperNoBatching Struct Reference

Public Member Functions

- cufftWrapperNoBatching (int N_)
- thrust::device_vector< double > execute_d2z (thrust::device_vector< double > &in)
- thrust::device_vector< double > execute_batched_d2z (thrust::device_vector< double > &in)
- thrust::device_vector< double > execute_z2d (thrust::device_vector< double > &in)
- void execute_inplace_z2d (thrust::device_vector< double > &inout)
- cufftWrapperNoBatching (const cufftWrapperNoBatching &)=delete
- cufftWrapperNoBatching & operator= (const cufftWrapperNoBatching &)=delete
- cufftWrapperNoBatching (cufftWrapperNoBatching &&)=delete
- cufftWrapperNoBatching & operator= (cufftWrapperNoBatching &&)=delete

Public Attributes

- int N
- cufftHandle plan_d2z
- · cufftHandle plan z2d
- thrust::device_vector< double > work_area

The documentation for this struct was generated from the following files:

- src/cuda_wrapper.cuh
- src/cuda_wrapper.cu

8.13 empty Struct Reference

The documentation for this struct was generated from the following file:

src/dispatcher.hpp

8.14 fftWrapperDispatcher < Vector > Struct Template Reference

Public Types

- · typedef empty D2Z
- · typedef empty BatchedD2Z
- typedef empty Generic

The documentation for this struct was generated from the following file:

src/dispatcher.hpp

8.15 fftWrapperDispatcher < Eigen::VectorXd > Struct Reference

Public Types

- typedef empty D2Z
- typedef empty BatchedD2Z
- typedef fftwWrapper Generic

The documentation for this struct was generated from the following file:

src/dispatcher.hpp

8.16 fftWrapperDispatcher< thrust::device_vector< double >> Struct Reference

Public Types

- typedef cufftWrapperD2Z D2Z
- typedef cufftWrapperBatchedD2Z BatchedD2Z
- typedef cufftWrapperNoBatching Generic

The documentation for this struct was generated from the following file:

· src/dispatcher.hpp

8.17 fftwWrapper Struct Reference

Wrapper for various FFTW functions for a N^3 grid.

```
#include <fftw_wrapper.hpp>
```

Public Member Functions

- fftwWrapper (int N)
- Eigen::VectorXd execute_d2z (Eigen::VectorXd &in)
- Eigen::VectorXd execute_batched_d2z (Eigen::VectorXd &in)
- Eigen::VectorXd execute_z2d (Eigen::VectorXd &in)
- void execute_z2d (Eigen::VectorXd &in, Eigen::VectorXd &out)
- void execute_inplace_z2d (Eigen::VectorXd &inout)
- fftwWrapper (const fftwWrapper &)=delete
- fftwWrapper & operator= (const fftwWrapper &)=delete
- fftwWrapper (fftwWrapper &&)=delete
- fftwWrapper & operator= (fftwWrapper &&)=delete

Public Attributes

- int N
- fftw_plan plan_d2z
- fftw_plan plan_z2d
- fftw_plan plan_inplace_z2d

8.17.1 Detailed Description

Wrapper for various FFTW functions for a N^3 grid.

The documentation for this struct was generated from the following files:

- src/fftw_wrapper.hpp
- src/fftw_wrapper.cpp

8.18 KGParam Struct Reference

Public Attributes

- · long long int N
- double L
- · double m

The documentation for this struct was generated from the following file:

· src/field_booster.cpp

8.19 KleinGordonEquation Struct Reference

```
The Klein Gordon equation, \ddot{\varphi}-\nabla^2\varphi+m^2\varphi=0. #include <equations.hpp>
```

Public Types

- typedef Eigen::VectorXd Vector
- typedef Vector State
- typedef WorkspaceGeneric < State > Workspace

Public Member Functions

- KleinGordonEquation (Workspace &workspace_)
- void operator() (const State &, State &, const double)

The function called by odeint library.

Static Public Member Functions

static Vector compute_energy_density (const Workspace &workspace, const double t)
 Compute the energy density profile from the workspace.

Public Attributes

• Workspace & workspace

8.19.1 Detailed Description

The Klein Gordon equation, $\ddot{\varphi} - \nabla^2 \varphi + m^2 \varphi = 0$.

8.19.2 Member Function Documentation

8.19.2.1 compute_energy_density()

```
\label{lem:compute_energy_density} KleinGordonEquation::compute_energy_density ($$ const Workspace & workspace, $$ const double $t$ ) [static]
```

Compute the energy density profile from the workspace.

Parameters

in	workspace	The workspace for evaluating the energy density.
	t	The current time parameter.

Returns

A vector of size N^3 , giving the energy density profile $\rho=\frac{1}{2}(\dot{\varphi}^2+(\nabla\varphi)^2+m^2\varphi^2)$ on the lattice.

8.19.2.2 operator()()

The function called by odeint library.

Parameters

	in	X	The current state of the system.
	out	dxdt	The time derivative, dxdt of the system.
Ī		t	The current time parameter.

The documentation for this struct was generated from the following files:

- · src/equations.hpp
- · src/equations.cpp

8.20 KleinGordonEquationInFRW Struct Reference

```
The Klein Gordon in FRW equation, \ddot{\varphi}+3H\dot{\varphi}-\nabla^2\varphi/a^2+m^2\varphi=0.
```

```
#include <equations.hpp>
```

Public Types

- typedef Eigen::VectorXd Vector
- typedef Vector State
- typedef WorkspaceGeneric< State > Workspace

Public Member Functions

- KleinGordonEquationInFRW (Workspace &workspace_)
- void operator() (const State &, State &, const double)

Static Public Member Functions

static Vector compute_energy_density (const Workspace &workspace, const double t)
 Compute the energy density profile from the workspace.

Public Attributes

Workspace & workspace

8.20.1 Detailed Description

The Klein Gordon in FRW equation, $\ddot{\varphi}+3H\dot{\varphi}-\nabla^2\varphi/a^2+m^2\varphi=0.$

8.20.2 Member Function Documentation

8.20.2.1 compute_energy_density()

```
\label{lem:compute_energy_density} KleinGordonEquationInFRW::compute_energy_density ( \\ const Workspace & workspace, \\ const double $t$ ) [static]
```

Compute the energy density profile from the workspace.

Parameters

in	workspace	The workspace for evaluating the energy density.
	t	The current time parameter.

Returns

A vector of size N^3 , giving the energy density profile $\rho=\frac{1}{2}(\dot{\varphi}^2+(\nabla\varphi)^2/a(t)^2+m^2\varphi^2)$ given on the N^3 on the lattice.

The documentation for this struct was generated from the following files:

- src/equations.hpp
- · src/equations.cpp

8.21 midpoint< State, Value, Deriv, Time, Algebra, Operations, Resizer > Class Template Reference

Inheritance diagram for midpoint < State, Value, Deriv, Time, Algebra, Operations, Resizer >:

```
boost::numeric::odeint::algebra_stepper_base< typename boost::numeric::odeint::algebra_type, typename boost::numeric::odeint::operations_dispatcher< State >::operations_type > |
| midpoint< State, Value, Deriv, Time, Algebra, Operations, Resizer >
```

30 Class Documentation

Public Types

- typedef State state_type
- typedef State deriv type
- typedef Value value_type
- typedef Time time_type
- typedef unsigned short order_type
- typedef boost::numeric::odeint::stepper_tag stepper_category
- typedef boost::numeric::odeint::algebra_stepper_base< Algebra, Operations > algebra_stepper_base_←
 type
- typedef algebra_stepper_base_type::algebra_type algebra_type
- typedef algebra_stepper_base_type::operations_type operations_type

Public Member Functions

- template < class System > void do_step (System system, State &in, Time t, Time dt)
- bool resize_impl (const State &x)

Static Public Member Functions

• static order_type order (void)

The documentation for this class was generated from the following file:

• src/midpoint.hpp

8.22 MyParam Struct Reference

Public Attributes

- long long int N
- · double L
- · double m
- · double lambda
- double f a
- double k ast
- double k_Psi
- double varphi_std_dev
- · double Psi std dev
- double a1
- · double H1
- double t1
- double t start
- · double t end
- double t interval
- · double delta_t
- long long int M
- · double f
- · double delta varphi std dev
- double k_delta_varphi

The documentation for this struct was generated from the following file:

src/main.cpp

8.23 SampleParam Struct Reference

A sample parameter type specifying a lambda-phi-4 theory in an FRW background.

```
#include <param.hpp>
```

Public Attributes

- long long int N
- double L
- · double m
- · double lambda
- · double k ast
- double varphi_std_dev
- double a1
- · double H1
- double t1

8.23.1 Detailed Description

A sample parameter type specifying a lambda-phi-4 theory in an FRW background.

8.23.2 Member Data Documentation

```
8.23.2.1 a1
```

```
double SampleParam::a1  \label{eq:sampleParam}  the scale factor at time t_1
```

8.23.2.2 H1

```
double SampleParam::H1
```

the Hubble parameter at time t_1

8.23.2.3 k_ast

```
double SampleParam::k_ast
```

the wavenumber k_{st} for the peak of the field power spectrum

8.23.2.4 L

```
double SampleParam::L
```

the length of one side of the box (i.e. L=10.0 means the box has volume L^3)

32 Class Documentation

8.23.2.5 lambda

```
double SampleParam::lambda
```

quartic self-interaction of the scalar field (i.e. λ in $V(\varphi)=\frac{1}{2}m^2\varphi^2+\frac{1}{4}\lambda\varphi^4$)

8.23.2.6 m

```
double SampleParam::m
```

mass m of the scalar field

8.23.2.7 N

```
long long int SampleParam::N
```

the number of lattice points on one side of the box (i.e. N=256 means 256^3 lattice sites)

8.23.2.8 t1

```
double SampleParam::t1
```

coordinate time parameter t_1 (For radiation domination, $a(t)=a_1(1+2H_1(t-t_1))^{1/2}$, $H(t)=H_1(1+2H_1(t-t_1))^{-1}$.)

8.23.2.9 varphi_std_dev

```
double SampleParam::varphi_std_dev
```

the expected RMS value $\langle \varphi^2 \rangle$ for the field, averaged over the box

The documentation for this struct was generated from the following file:

• src/param.hpp

8.24 StaticEOSCosmology Struct Reference

Public Member Functions

- StaticEOSCosmology (const double a1_, const double H1_, const double t1_, const double p_)
- template<typename T >

StaticEOSCosmology (const T ¶m)

- double a (const double t) const
- double **H** (const double t) const
- double eta (const double t) const

Public Attributes

- double a1
- · double H1
- · double t1
- double **p**

The documentation for this struct was generated from the following file:

• src/physics.hpp

8.25 WKBSolutionForKleinGordonEquationInFRW Struct Reference

Public Types

- typedef Eigen::VectorXd Vector
- · typedef Vector State
- typedef WorkspaceGeneric < State > Workspace

Public Member Functions

- WKBSolutionForKleinGordonEquationInFRW (Workspace &workspace_, const double t_i_)
- Vector evaluate_at (const double t)

Public Attributes

- Workspace & workspace
- double t i
- Vector phi_ffts

The documentation for this struct was generated from the following files:

- src/wkb.hpp
- · src/wkb.cpp

8.26 WorkspaceGeneric < Vector > Struct Template Reference

A generic workspace for storing temporary.

```
#include <workspace.hpp>
```

Public Types

· typedef Vector State

34 Class Documentation

Public Member Functions

template < HasLattice Params Param >
 Workspace Generic (const Param & param, auto & initializer)

Public Attributes

- · long long int N
- double L
- double m
- StaticEOSCosmology cosmology {}
- · State state
- double lambda {0}
- double f_a {1.0}
- · Vector Psi
- · Vector dPsidt
- Vector Psi_fft
- Vector dPsidt_fft
- Vector R_fft
- std::vector< double > t_list
- fftWrapperDispatcher< Vector >::Generic fft wrapper
- bool Psi_approximation_initialized {false}
- · long long int M
- std::unique_ptr< typename fftWrapperDispatcher< Vector >::Generic > fft_wrapper_M_ptr
- Vector cutoff_R_fft

8.26.1 Detailed Description

```
template<typename Vector>
struct WorkspaceGeneric< Vector >
```

A generic workspace for storing temporary.

WorkspaceGeneric contains everything used during simulations, including the field state, gravitational potential, parameters, etc. It is initialized by a Param struct (containing just a few numbers) and an "initializer" (see initializer.hpp).

The lifetime of objects in the workspace are managed by us (this codebase), instead of external libraries (e.g. odeint).

The documentation for this struct was generated from the following file:

· src/workspace.hpp

Chapter 9

File Documentation

9.1 src/cuda_wrapper.cuh File Reference

Wrapper for CUDA Toolkit.

```
#include <iostream>
#include <Eigen/Dense>
#include <thrust/device_vector.h>
#include "cufft.h"
#include "cufftXt.h"
#include <cuda_runtime.h>
```

Classes

struct cufftWrapperD2Z

Wrapper for 3D cufftPlan3d. Performs double to complex double FFT for a N^3 grid.

struct cufftWrapperBatchedD2Z

Wrapper for 3D cufftPlanMany. Performs two double to complex double FFT for a N^3 grid.

- struct cufftWrapper
- struct cufftWrapperNoBatching

Typedefs

- typedef decltype(Eigen::VectorXd().begin()) eigen_iterator
- typedef decltype(thrust::device vector< double >().begin()) thrust iterator
- typedef thrust::detail::normal_iterator< thrust::device_ptr< const double >> thrust_const_iterator
- typedef Eigen::internal::pointer_based_stl_iterator< Eigen::Matrix< double, -1, 1 >> eigen_iterator_2

Functions

- void copy_vector (Eigen::VectorXd &out, const thrust::device_vector< double > &in)
- void show_gpu_memory_usage (void)

9.1.1 Detailed Description

Wrapper for CUDA Toolkit.

Author

Siyang Ling

9.2 cuda wrapper.cuh

```
00001
00006 #ifndef CUDA_WRAPPER_CUH
00007 #define CUDA_WRAPPER_CUH
00009 #include <iostream>
00010
00011 #include <Eigen/Dense>
00012
00013 #include <thrust/device_vector.h>
00014 // #include <thrust/host_vector.h>
00015 // #include <thrust/execution_policy.h>
00016 // #include <thrust/reduce.h>
00017 // #include <thrust/functional.h>
00018 // #include <thrust/fill.h>
00019 // #include <thrust/transform.h>
00021 #include "cufft.h"
00022 #include "cufftXt.h"
00023 #include <cuda_runtime.h>
00024
00025
00027 typedef decltype(Eigen::VectorXd().begin()) eigen_iterator;
00028 typedef decltype(thrust::device_vector<double>().begin()) thrust_iterator;
00029 typedef thrust::detail::normal_iterator<thrust::device_ptr<const double» thrust_const_iterator;
00030 typedef Eigen::internal::pointer_based_stl_iterator<Eigen::Matrix<double, -1, 1» eigen_iterator_2;
00031
00033 /*
00034 Explicit template instantiation declarations for the thrust library.
00035
       They are declared here so that they are instantiatiated in cuda_wrapper.cu (and compiled with nvcc),
        and don't get instantiated in other translation units.
00036
00037
       This is necessary since we want to call thrust functions in translation units compiled by other
      compilers (g++ / icpx).
00038 */
00039 extern template class thrust::device_vector<double>;
00040 extern template class thrust::device_ptr<double>;
00041 extern template thrust::device_ptr<double> thrust::for_each_n(const
      thrust::detail::execution_policy_base<thrust::cuda_cub::tag> &, thrust::device_ptr<double>, unsigned
      long, thrust::detail::device_generate_functor<thrust::detail::fill_functor<double»);</pre>
00042 extern template eigen_iterator thrust::copy(const
      thrust::detail::execution_policy_base<thrust::cuda_cub::cross_system<thrust::cuda_cub::tag,
      thrust::system::cpp::detail::tag> &, thrust_const_iterator, thrust_const_iterator, eigen_iterator);
00043
00044 extern template thrust_iterator thrust::copy(eigen_iterator, eigen_iterator, thrust_iterator);
00045 extern template eigen_iterator thrust::copy(thrust_iterator, thrust_iterator, eigen_iterator);
00047 //Eigen::VectorXd copy_vector(const thrust::device_vector<double> &in);
00048 void copy_vector(Eigen::VectorXd &out, const thrust::device_vector<double> &in);
00049 //void copy_vector(Eigen::VectorXd &out, const Eigen::VectorXd &in);
00050
00051
00052 void show_gpu_memory_usage(void);
00053
00057 struct cufftWrapperD2Z {
00058 int N:
       cufftHandle plan;
00059
       explicit cufftWrapperD2Z(int N_);
00060
         cufftWrapperD2Z();
00061
00062
       thrust::device_vector<double> execute(thrust::device_vector<double> &in);
00063
00064
        cufftWrapperD2Z(const cufftWrapperD2Z &) = delete;
00065
       cufftWrapperD2Z &operator=(const cufftWrapperD2Z &) = delete;
cufftWrapperD2Z (cufftWrapperD2Z &&) = delete;
00066
00067
       cufftWrapperD2Z &operator=(cufftWrapperD2Z &&) = delete;
00068 };
```

```
00069
00070
00075 struct cufftWrapperBatchedD2Z {
00076
       int N;
00077
        cufftHandle plan;
00078
       explicit cufftWrapperBatchedD2Z(int N_);
        ~cufftWrapperBatchedD2Z();
08000
       thrust::device_vector<double> execute(thrust::device_vector<double> &in);
00081
00082
       cufftWrapperBatchedD2Z(const cufftWrapperBatchedD2Z &) = delete;
00083
        cufftWrapperBatchedD2Z &operator=(const cufftWrapperBatchedD2Z &) = delete;
        cufftWrapperBatchedD2Z(cufftWrapperBatchedD2Z &&) = delete;
00084
       cufftWrapperBatchedD2Z &operator=(cufftWrapperBatchedD2Z &&) = delete;
00085
00086 };
00087
00088 /
        \brief Wrapper for various cufft functions for a f N^3 \f$ grid.
00089
00090
       Different cufft plans share the same work area so that GPU memory usage is minimized.
00091 */
00092 struct cufftWrapper {
00093
      int N;
00094
        cufftHandle plan_d2z;
00095
        cufftHandle plan_batched_d2z;
00096
        cufftHandle plan_z2d;
00097
       thrust::device_vector<double> work_area;
explicit cufftWrapper(int N_);
00098
00099
        ~cufftWrapper();
00100
00101
        thrust::device_vector<double> execute_d2z(thrust::device_vector<double> &in);
00102
       thrust::device_vector<double> execute_batched_d2z(thrust::device_vector<double> &in);
00103
       thrust::device_vector<double> execute_z2d(thrust::device_vector<double> &in);
00104
00105
        cufftWrapper(const cufftWrapper &) = delete;
00106
        cufftWrapper &operator=(const cufftWrapper &) = delete;
00107
        cufftWrapper(cufftWrapper &&) = delete;
00108
        cufftWrapper &operator=(cufftWrapper &&) = delete;
00109 };
00110
00111 /
00112
        \brief Wrapper for various cufft functions for a f N^3 \f$ grid.
00113
       Different cufft plans share the same work area so that GPU memory usage is minimized.
00114 */
00115 struct cufftWrapperNoBatching {
00116
       int N;
        cufftHandle plan_d2z;
00117
00118
        cufftHandle plan_z2d;
00119
        thrust::device_vector<double> work_area;
00120
        explicit cufftWrapperNoBatching(int N_);
00121
        ~cufftWrapperNoBatching();
00122
00123
        thrust::device_vector<double> execute_d2z(thrust::device_vector<double> &in);
00124
        thrust::device_vector<double> execute_batched_d2z(thrust::device_vector<double> &in);
00125
        thrust::device_vector<double> execute_z2d(thrust::device_vector<double> &in);
00126
       void execute_inplace_z2d(thrust::device_vector<double> &inout);
00127
00128
        cufftWrapperNoBatching(const cufftWrapperNoBatching &) = delete;
        cufftWrapperNoBatching &operator=(const cufftWrapperNoBatching &) = delete;
00130
        cufftWrapperNoBatching(cufftWrapperNoBatching &&) = delete;
       cufftWrapperNoBatching &operator=(cufftWrapperNoBatching &&) = delete;
00131
00132 };
00133
00134
00135 #endif
```

9.3 src/dispatcher.hpp File Reference

Automatically dispatching between using FFTW and CUFFT libraries.

```
#include "fftw_wrapper.hpp"
#include <thrust/device_vector.h>
#include "cuda_wrapper.cuh"
```

Classes

struct empty

- struct fftWrapperDispatcher< Vector >
- struct fftWrapperDispatcher< thrust::device_vector< double > >
- struct fftWrapperDispatcher< Eigen::VectorXd >

Macros

#define ALGORITHM_NAMESPACE thrust

9.3.1 Detailed Description

Automatically dispatching between using FFTW and CUFFT libraries.

Author

Siyang Ling

9.4 dispatcher.hpp

```
00006 #ifndef DISPATCHER_HPP
00007 #define DISPATCHER_HPP
80000
00009 #include "fftw_wrapper.hpp"
00010
00011 #ifndef DISABLE_CUDA
00012 #include <thrust/device_vector.h>
00013 #include "cuda_wrapper.cuh"
00014 #define ALGORITHM_NAMESPACE thrust
00015 #else
00016 #define ALGORITHM_NAMESPACE std
00017 #endif
00018
00019
00020 // An empty placeholder object
00021 struct empty {};
00022
00023 // Dispatcher for fftWrapper* types
00024 template<typename Vector>
00025 struct fftWrapperDispatcher {
00026 typedef empty D2Z;
00027 typedef empty BatchedD2Z;
00028 typedef empty Generic;
00029 };
00030
00031 #ifndef DISABLE_CUDA
00032 template<>
00033 struct fftWrapperDispatcher<thrust::device_vector<double» {
00034 typedef cufftWrapperDZZ DZZ;
00035 typedef cufftWrapperBatchedDZZ BatchedDZZ;
00036
        //typedef cufftWrapper Generic;
00037
        typedef cufftWrapperNoBatching Generic;
00038 };
00039 #endif
00040
00041 template<>
00042 struct fftWrapperDispatcher<Eigen::VectorXd> {
00043 typedef empty D2Z;
00044 typedef empty BatchedD2Z;
00045 typedef fftwWrapper Generic;
00046 };
00047
00049 #endif
```

9.5 src/eigen wrapper.hpp File Reference

Wrap some Eigen functionalites.

```
#include <Eigen/Dense>
```

Functions

void copy_vector (Eigen::VectorXd &out, const Eigen::VectorXd &in)

9.5.1 Detailed Description

Wrap some Eigen functionalites.

Author

Siyang Ling

9.6 eigen_wrapper.hpp

Go to the documentation of this file.

```
00001
00006 #ifndef EIGEN_WRAPPER_HPP
00007 #define EIGEN_WRAPPER_HPP
00008
00009 #include <Eigen/Dense>
00010
00011 void copy_vector(Eigen::VectorXd &out, const Eigen::VectorXd &in);
00012
00013 #endif
```

9.7 src/equations.hpp File Reference

Header for field equations that runs on the CPU.

```
#include "Eigen/Dense"
#include <boost/numeric/odeint.hpp>
#include <boost/numeric/odeint/external/eigen/eigen.hpp>
#include "odeint_eigen/eigen_operations.hpp"
#include "workspace.hpp"
```

Classes

• struct KleinGordonEquation

```
The Klein Gordon equation, \ddot{\varphi} - \nabla^2 \varphi + m^2 \varphi = 0.
```

• struct KleinGordonEquationInFRW

```
The Klein Gordon in FRW equation, \ddot{\varphi} + 3H\dot{\varphi} - \nabla^2\varphi/a^2 + m^2\varphi = 0.
```

struct ComovingCurvatureEquationInFRW

Equation for free scalar field in FRW spacetime, including comoving metric perturbations (in radiation domination).

Concepts

concept LatticeEquationConcept

9.7.1 Detailed Description

Header for field equations that runs on the CPU.

Author

Siyang Ling

This is the header for field equations that are supposed to run on CPU. Equations declared here will be used by the odeint library via <code>operator()</code>. See <code>https://www.boost.org/doc/libs/1_85_ $\leftarrow 0/libs/numeric/odeint/doc/html/boost_numeric_odeint/getting_started/short_ <math display="inline">\leftarrow example.html$ for an example of odeint equation. Typically, <code>compute_energy_density</code> is also implemented for saving energy density spectrum.</code>

9.8 equations.hpp

```
00001
00011 #ifndef EQUATIONS HPP
00012 #define EQUATIONS_HPP
00014
00015 #include "Eigen/Dense"
00016
00017 #include <boost/numeric/odeint.hpp>
00018 #include <boost/numeric/odeint/external/eigen/eigen.hpp>
00020 #include "odeint_eigen/eigen_operations.hpp"
00021
00022 #include "workspace.hpp"
00023
00024 template<typename Equation>
00025 concept LatticeEquationConcept = requires (Equation eqn)
         //typename Equation::State;
00027
00028
        eqn.workspace;
00029
        eqn.compute_energy_density(eqn.workspace, 0.0);
00030
00031
00032
00036 struct KleinGordonEquation
00037
       typedef Eigen::VectorXd Vector;
00038
       typedef Vector State;
       typedef WorkspaceGeneric<State> Workspace;
00039
00040
       Workspace &workspace:
00041
00042
       KleinGordonEquation(Workspace &workspace_) : workspace(workspace_) {}
00043
00050
       void operator()(const State &, State &, const double);
00051
00058
       static Vector compute_energy_density(const Workspace &workspace, const double t);
00059 };
00060
00061
00065 struct KleinGordonEquationInFRW {
00066
       typedef Eigen::VectorXd Vector;
       typedef Vector State;
00067
00068
        typedef WorkspaceGeneric<State> Workspace;
00069
       Workspace &workspace;
00070
00071
       KleinGordonEquationInFRW(Workspace &workspace_) : workspace(workspace_) {}
00072
00073
       void operator()(const State &, State &, const double);
00074
00081
       static Vector compute_energy_density(const Workspace &workspace, const double t);
```

```
00082 };
00083
00084
00100 struct ComovingCurvatureEquationInFRW {
00101
       typedef Eigen::VectorXd Vector;
00102
       typedef Vector State:
        typedef WorkspaceGeneric<State> Workspace;
00103
00104
       Workspace &workspace;
00105
00106
       ComovingCurvatureEquationInFRW(Workspace &workspace_) : workspace(workspace_) {}
00107
00108
       void operator()(const State &, State &, const double);
00109
00116
       static Vector compute_energy_density(Workspace &workspace, const double t);
00117 };
00118
00119
00120
00121
00122 #endif
```

9.9 src/equations_cuda.cuh File Reference

Header for field equations that runs on the GPU.

```
#include "equations.hpp"
#include <thrust/device_vector.h>
#include "odeint_thrust/thrust.hpp"
```

Classes

- struct CudaKleinGordonEquationInFRW
- struct CudaLambdaEquationInFRW
- struct CudaSqrtPotentialEquationInFRW
- struct CudaFixedCurvatureEquationInFRW
- struct CudaComovingCurvatureEquationInFRW
- struct CudaApproximateComovingCurvatureEquationInFRW

9.9.1 Detailed Description

Header for field equations that runs on the GPU.

Author

Siyang Ling

This is the header for field equations that are supposed to run on GPU (via CUDA). Equations declared here will be used by the odeint library via operator(). See https://www.boost.org/doc/libs/1_85_c-0/libs/numeric/odeint/doc/html/boost_numeric_odeint/getting_started/short_c-example.html for an example of odeint equation. Typically, compute_energy_density is also implemented for saving energy density spectrum. Also see equations.hpp.

9.10 equations cuda.cuh

```
00001
00012 #ifndef EQUATIONS CUDA CUH
00013 #define EQUATIONS_CUDA_CUH
00014
00015 #include "equations.hpp"
00016
00017 #include <thrust/device_vector.h>
00018
00019 #include "odeint thrust/thrust.hpp"
00020
00021 struct CudaKleinGordonEquationInFRW {
00022
        typedef thrust::device_vector<double> Vector;
        typedef Vector State;
00023
        typedef WorkspaceGeneric<Vector> Workspace;
00024
00025
       Workspace &workspace;
00026
00027
        CudaKleinGordonEquationInFRW(Workspace &workspace_) : workspace(workspace_) {}
00028
00029
        void operator()(const State &, State &, const double);
00030
00031
        static Vector compute_energy_density(const Workspace &workspace, const double t);
00032
        static Vector compute_dot_energy_density(const Workspace &workspace, const double t);
00033 };
00034
00035
00036 struct CudaLambdaEquationInFRW {
        typedef thrust::device_vector<double> Vector;
00037
00038
        typedef Vector State;
        typedef WorkspaceGeneric<Vector> Workspace;
00039
00040
        Workspace &workspace;
00041
00042
        CudaLambdaEquationInFRW(Workspace &workspace_) : workspace(workspace_) {}
00043
00044
        void operator()(const State &, State &, const double);
00045
00046
        static Vector compute_energy_density(const Workspace &workspace, const double t);
00047 };
00048
00049
00050 struct CudaSqrtPotentialEquationInFRW {
00051
        typedef thrust::device_vector<double> Vector;
00052
        typedef Vector State;
00053
        typedef WorkspaceGeneric<Vector> Workspace;
00054
       Workspace &workspace;
00055
00056
        CudaSqrtPotentialEquationInFRW(Workspace &workspace_) : workspace(workspace_) {}
00057
00058
        void operator()(const State &, State &, const double);
00059
00060
        static Vector compute_energy_density(const Workspace &workspace, const double t);
00061 };
00062
00063
00064 struct CudaFixedCurvatureEquationInFRW {
00065
        typedef thrust::device_vector<double> Vector;
00066
        typedef Vector State;
00067
        typedef WorkspaceGeneric<Vector> Workspace;
00068
       Workspace &workspace:
00069
00070
        CudaFixedCurvatureEquationInFRW(Workspace &workspace_) : workspace(workspace_) {}
00071
00072
        void operator()(const State &, State &, const double);
00073
00074
       static Vector compute_energy_density(const Workspace &workspace, const double t);
00075 };
00076
00077
00078 struct CudaComovingCurvatureEquationInFRW {
00079
        typedef thrust::device_vector<double> Vector;
08000
        typedef Vector State;
        typedef WorkspaceGeneric<Vector> Workspace;
00081
00082
       Workspace &workspace:
00083
00084
        CudaComovingCurvatureEquationInFRW(Workspace &workspace_) : workspace(workspace_) {}
00085
00086
        void operator()(const State &, State &, const double);
00087
00088
        static Vector compute_energy_density(Workspace &workspace, const double t);
00089 };
00090
00092 struct CudaApproximateComovingCurvatureEquationInFRW {
```

```
typedef thrust::device_vector<double> Vector;
00094
        typedef Vector State;
00095
        typedef WorkspaceGeneric<Vector> Workspace;
00096
       Workspace &workspace;
00097
00098
        CudaApproximateComovingCurvatureEquationInFRW(Workspace &workspace) : workspace(workspace) {}
00099
00100
        void operator() (const State &, State &, const double);
00101
00102
        static Vector compute_energy_density(Workspace &workspace, const double t);
00103 };
00104
00105
00106 // Explicit template instantiation declaration for the thrust library.
00107 extern template double thrust::reduce(const
      thrust::detail::execution_policy_base<thrust::cuda_cub::tag> &, thrust_const_iterator,
      thrust_const_iterator, double, boost::numeric::odeint::detail::maximum<double>);
00108
00109 // Deprecated function for testing CUDA kernels.
00111 void compute_deriv_test(const Eigen::VectorXd &in, Eigen::VectorXd &out,
00112
                  const double m, const double lambda,
00113
                  const double a_t, const double H_t, const double inv_ah_sqr,
00114
                 const long long int N);
00115 */
00116 /*
00117 void kernel_test(const thrust::device_vector<double> &R_fft, thrust::device_vector<double> &Psi,
     thrust::device_vector<double> &dPsidt,
00118
              const long long int N, const double L, const double m,
00119
              const double a_t, const double H_t, const double eta_t, const double inv_ah_sqr,
00120
              const double t, fftWrapperDispatcher<thrust::device_vector<double»::Generic &fft_wrapper);</pre>
00121 */
00122 #endif
```

9.11 src/fdm3d.hpp File Reference

Common procedures for manipulating / summarizing field configuration on a 3D lattice.

```
#include "Eigen/Dense"
#include "dispatcher.hpp"
```

Macros

• #define IDX_OF(N, i, j, k) ((N)*(N)*(i) + (N)*(j) + (k))

Give the index of a lattice point, assuming row major ordering in (i,j,k).

#define PADDED_IDX_OF(N, i, j, k) ((N)*2*((N)/2+1)*(i) + 2*((N)/2+1)*(j) + (k))

Give the index of a lattice point, assuming that the array is in FFTW padded format.

Functions

Eigen::VectorXd compute_power_spectrum (const long long int N, Eigen::VectorXd &f, fftWrapperDispatcher
 Eigen::VectorXd >::Generic &fft_wrapper)

Sum Fourier mode power of a field over directions.

• Eigen::VectorXd compute_mode_power_spectrum (const long long int N, const double L, const double m, const double a_t, Eigen::VectorXd &state, fftWrapperDispatcher< Eigen::VectorXd >::Generic &fft_wrapper)

Sum Fourier mode power of a field over directions, along with the power in time derivatives.

• Eigen::VectorXd compute_inverse_laplacian (const long long int N, const double L, Eigen::VectorXd &f, fftWrapperDispatcher< Eigen::VectorXd >::Generic &fft_wrapper)

Compute the inverse Laplacian of a field. AKA solve the Poisson equation.

 Eigen::VectorXd compute_field_with_scaled_fourier_modes (const long long int N, const double L, Eigen::VectorXd &f, std::function< double(const double)> kernel, fftWrapperDispatcher< Eigen::VectorXd >::Generic &fft_wrapper)

Scale each Fourier mode of a field by a kernel, returning the new field.

• Eigen::VectorXd compute_cutoff_fouriers (const long long int N, const long long int M, Eigen::VectorXd &fft)

Downsample a Fourier transform on a N³ grid so that it looks like a Fourier transform on a M³ grid.

9.11.1 Detailed Description

Common procedures for manipulating / summarizing field configuration on a 3D lattice.

Author

Siyang Ling

9.11.2 Macro Definition Documentation

9.11.2.1 PADDED_IDX_OF

Give the index of a lattice point, assuming that the array is in FFTW padded format.

9.11.3 Function Documentation

9.11.3.1 compute_cutoff_fouriers()

Downsample a Fourier transform on a ${\cal N}^3$ grid so that it looks like a Fourier transform on a ${\cal M}^3$ grid.

Parameters

Ν	Number of lattice points (of full grid).
М	Number of lattice points (of downsampled grid).
fft	The DFT of a real field. Should be a vector of size $2N^2(N/2+1)$.

Returns

The downsampled DFT the input DFT. Should be a vector of size $2M^2(M/2+1)$.

9.11.3.2 compute_field_with_scaled_fourier_modes()

```
const double L,
Eigen::VectorXd & f,
std::function< double(const double) > kernel,
fftWrapperDispatcher< Eigen::VectorXd >::Generic & fft_wrapper )
```

Scale each Fourier mode of a field by a kernel, returning the new field.

Parameters

N	Number of lattice points.
L	Box size.
f	The field on a 3D lattice. Should be a vector of size ${\cal N}^3$ with row major ordering. See IDX_OF.
kernel	A function K determining how the Fourier modes are scaled.
fft_wrapper	A fftwWrapper initialized to do Fourier transforms on grid size N .

Returns

The field with $f_{\mathbf{k}} \mapsto K(k) f_{\mathbf{k}}$, where K is given by kernel.

9.11.3.3 compute_inverse_laplacian()

Compute the inverse Laplacian of a field. AKA solve the Poisson equation.

Parameters

N	Number of lattice points.
L	Box size.
f	The field on a 3D lattice. Should be a vector of size \mathbb{N}^3 with row major ordering. See IDX_OF.
fft_wrapper	A fftwWrapper initialized to do Fourier transforms on grid size N .

Returns

The solution to the Poisson equation with RHS f, namely $\nabla^{-2}f$. The output have zero homogeneous mode regardless of whether f has one.

9.11.3.4 compute_mode_power_spectrum()

```
Eigen::VectorXd & state,
fftWrapperDispatcher< Eigen::VectorXd >::Generic & fft_wrapper )
```

Sum Fourier mode power of a field over directions, along with the power in time derivatives.

Parameters

N	Number of lattice points.
L	Box size.
m	Mass m of (free) scalar field.
a_t	Current scale factor.
state	The state $(\varphi,\dot{\varphi})$ of a scalar field on a 3D lattice. Should be a vector of size $2N^3$, with the first half (first N^3 indices) containing φ , and the second half containing $\dot{\varphi}$.
fft_wrapper	A fftwWrapper initialized to do Fourier transforms on grid size N .

Returns

A vector of size $3(N/2)^2 + 1$, with its s index containing the power in Fourier modes with wavenumber $\sqrt{s}k_{\rm IR}$. Specifically:

output[s] =
$$\sum_{i^2+j^2+k^2=s} |\tilde{\varphi}_{i,j,k}|^2 + \frac{|\dot{\tilde{\varphi}}_{i,j,k}|^2}{\omega_k^2}$$
$$\omega_k^2 = m^2 + sk_{\rm IR}^2/a^2(t)$$

Here, $\tilde{\varphi}_{a,b,c}$ and $\dot{\tilde{\varphi}}_{a,b,c}$ are the DFT's, (i,j,k) labels a site on the reciprocal lattice, and $-N/2+1 \leq i,j,k \leq N/2$; see <code>https://garrettgoon.com/gaussian-fields/</code> for details on this convention. Also see compute_power_spectrum.

9.11.3.5 compute_power_spectrum()

Sum Fourier mode power of a field over directions.

Parameters

N	Number of lattice points.
f	The field on a 3D lattice. Should be a vector of size N^3 with row major ordering. See IDX_OF.
fft_wrapper	A fftwWrapper initialized to do Fourier transforms on grid size N .

Returns

A vector of size $3(N/2)^2+1$, with its s index containing the power in Fourier modes with wavenumber $\sqrt{s}k_{\rm IR}$. Specifically:

output[s] =
$$\sum_{i^2+j^2+k^2=s} |\tilde{f}_{i,j,k}|^2$$

 $\tilde{f}_{i,j,k} = \sum_{a,b,c} e^{-2\pi i(a,b,c).(i,j,k)/N} f_{a,b,c}$

Here, \tilde{f} is the DFT of f, (i,j,k) labels a site on the reciprocal lattice, and $-N/2+1 \le i,j,k \le N/2$. See https://garrettgoon.com/gaussian-fields/ for details on this convention.

9.12 fdm3d.hpp

Go to the documentation of this file.

```
00006 #ifndef FDM3D_HPP
00007 #define FDM3D_HPP
00008
00009 #include "Eigen/Dense"
00010 #include "dispatcher.hpp"
00011
00016 #define IDX_OF(N, i, j, k) ((N)*(N)*(i) + (N)*(j) + (k))
00017
00018
00024 #define PADDED IDX OF(N, i, j, k) ((N) \star 2 \star ((N)/2+1) \star (i) + 2 \star ((N)/2+1) \star (j) + (k))
00025
00041 Eigen::VectorXd compute_power_spectrum(const long long int N,
00042
                              Eigen:: VectorXd &f,
00043
                              fftWrapperDispatcher<Eigen::VectorXd>::Generic &fft_wrapper);
00044
00065 Eigen:: VectorXd compute mode power spectrum(const long long int N, const double L, const double m,
     const double a_t,
00066
                               Eigen::VectorXd &state,
00067
                               fftWrapperDispatcher<Eigen::VectorXd>::Generic &fft_wrapper);
00068
00069 // Eigen::VectorXd compute_mode_power_spectrum(const long long int N, const double L, const double m,
00070 //
                              Eigen:: VectorXd &state,
00071 //
                               fftWrapperDispatcher<Eigen::VectorXd>::Generic &fft_wrapper);
00072
00082 Eigen::VectorXd compute_inverse_laplacian(const long long int N, const double L,
00083
                             Eigen::VectorXd &f,
00084
                             fftWrapperDispatcher<Eigen::VectorXd>::Generic &fft_wrapper);
00085
00086
00096 Eigen::VectorXd compute_field_with_scaled_fourier_modes(const long long int N, const double L,
00097
                                   Eigen:: VectorXd &f,
00098
                                   std::function<double(const double) > kernel,
00099
                                   fftWrapperDispatcher<Eigen::VectorXd>::Generic &fft_wrapper);
00100
00101
00109 Eigen::VectorXd compute_cutoff_fouriers(const long long int N, const long long int M,
00110
                          Eigen::VectorXd &fft);
00111
00112
00113 // Deprecated
00114 // Eigen::VectorXd compute_power_spectrum(const long long int N, Eigen::VectorXd &phi);
00115 // Eigen::VectorXd compute_fourier(const long long int N, const double L, Eigen::VectorXd &phi);
00116 // Eigen::VectorXd compute_laplacian(const long long int N, const double L, const Eigen::VectorXd &f);
00117
00118
00119
00120 #endif
```

9.13 src/fdm3d_cuda.cuh File Reference

CUDA implementation for fdm3d.hpp. Common procedures for manipulating / summarizing field configuration on a 3D lattice.

```
#include <thrust/device_vector.h>
#include <thrust/reduce.h>
#include <thrust/functional.h>
#include "cuda_wrapper.cuh"
#include "dispatcher.hpp"
#include "fdm3d.hpp"
```

Functions

 thrust::device_vector< double > compute_mode_power_spectrum (const long long int N, const double L, const double m, const double a_t, thrust::device_vector< double > &state, fftWrapperDispatcher
 thrust::device_vector< double > >::Generic &fft_wrapper) 9.14 fdm3d cuda.cuh 49

thrust::device_vector< double > compute_power_spectrum (const long long int N, thrust::device_vector< double > &f, fftWrapperDispatcher< thrust::device vector< double > >::Generic &fft wrapper)

- thrust::device_vector< double > compute_laplacian (const long long int N, const double L, thrust::device_vector< double > &f)
- thrust::device_vector< double > compute_inverse_laplacian (const long long int N, const double L, thrust::device_vector< double > &f, fftWrapperDispatcher< thrust::device_vector< double > >::Generic &fft_wrapper)
- thrust::device_vector< double > compute_cutoff_fouriers (const long long int N, const long long int M, const thrust::device vector< double > &fft)
- void compute_inverse_laplacian_test (const long long int N, const double L, thrust::device_vector< double > &fft)

9.13.1 Detailed Description

CUDA implementation for fdm3d.hpp. Common procedures for manipulating / summarizing field configuration on a 3D lattice.

Author

Siyang Ling

9.14 fdm3d_cuda.cuh

```
00001
00006 #ifndef FDM3D_CUDA_CUH
00007 #define FDM3D_CUDA_CUH
80000
00009 #include <thrust/device_vector.h>
00010 #include <thrust/reduce.h>
00011 #include <thrust/functional.h>
00013 //#include "odeint_thrust/thrust.hpp"
00014 #include "cuda_wrapper.cuh"
00015 #include "dispatcher.hpp"
00016
00017 #include "fdm3d.hpp"
00018
00019 thrust::device_vector<double> compute_mode_power_spectrum(const long long int N, const double L, const
              double m, const double a_t,
00020
                                                                                         thrust::device_vector<double> &state,
00021
                                                                                         fft \verb|WrapperDispatcher| < thrust:: device\_vector| < double >> :: Generic | fitting the content of the conten
               &fft_wrapper);
00022
00023 thrust::device_vector<double> compute_power_spectrum(const long long int N,
00024
                                                                                      thrust::device_vector<double> &f,
00025
                                                                                      fftWrapperDispatcher<thrust::device_vector<double>::Generic
               &fft_wrapper);
00026
00027 thrust::device vector<double> compute laplacian(const long long int N, const double L,
                                                                          thrust::device_vector<double> &f);
00030 thrust::device_vector<double> compute_inverse_laplacian(const long long int N, const double L,
00031
                                                                                    thrust::device_vector<double> &f,
00032
                                                                                    fftWrapperDispatcher<thrust::device_vector<double»::Generic &fft_wrapper);</pre>
00033
00034 thrust::device_vector<double> compute_cutoff_fouriers(const long long int N, const long long int M,
                                                                                         const thrust::device_vector<double> &fft);
00037 void compute_inverse_laplacian_test(const long long int N, const double L,
00038
                                                             thrust::device_vector<double> &fft);
00039 #endif
```

9.15 src/fftw_wrapper.hpp File Reference

Wrapper for FFTW library.

```
#include <iostream>
#include <Eigen/Dense>
#include <fftw3.h>
```

Classes

struct fftwWrapper

Wrapper for various FFTW functions for a N^3 grid.

9.15.1 Detailed Description

Wrapper for FFTW library.

Author

Siyang Ling

9.16 fftw_wrapper.hpp

```
00001
00006 #ifndef FFTW_WRAPPER_HPP
00007 #define FFTW_WRAPPER_HPP
80000
00009 #include <iostream>
00010
00011 #include <Eigen/Dense>
00012 #include <fftw3.h>
00014
00018 struct fftwWrapper {
00019 int N;
00020 fftw_plan plan_d2z;
        fftw_plan plan_z2d;
fftw_plan plan_inplace_z2d;
00021
00023
        explicit fftwWrapper(int N_);
00024
        ~fftwWrapper();
00025
00026
        Eigen::VectorXd execute_d2z(Eigen::VectorXd &in);
00027
        Eigen::VectorXd execute_batched_d2z(Eigen::VectorXd &in);
00028
        Eigen::VectorXd execute_z2d(Eigen::VectorXd &in);
00029
        void execute_z2d(Eigen::VectorXd &in, Eigen::VectorXd &out);
00030
        void execute_inplace_z2d(Eigen::VectorXd &inout);
00031
00032
        fftwWrapper(const fftwWrapper &) = delete;
        fftwWrapper &operator=(const fftwWrapper &) = delete;
fftwWrapper(fftwWrapper &&) = delete;
00033
00035
        fftwWrapper &operator=(fftwWrapper &&) = delete;
00036 };
00037
00038
00039 #endif
```

9.17 field booster.hpp 51

9.17 field booster.hpp

```
00001 /*
       Tools related to boosting (adding velocity to) fields.
00002
00003 */
00004 #ifndef FIELD_BOOSTER_HPP
00005 #define FIELD_BOOSTER_HPP
00006
00007 #include "Eigen/Dense"
80000
00009 void add_phase_to_state(Eigen::VectorXd &state, const Eigen::VectorXd &phase);
00010
00011 void boost_klein_gordon_field(Eigen::VectorXd &varphi, Eigen::VectorXd &dt_varphi, const
     Eigen::VectorXd &theta,
00012
                        const long long int N, const double L, const double m);
00013
00014
00015 #endif
```

9.18 src/initializer.hpp File Reference

Snippets for initializing workpaces. (e.g. field initial conditions, etc)

```
#include "fdm3d.hpp"
#include "random_field.hpp"
#include "fftw_wrapper.hpp"
#include "special_function.hpp"
#include <thrust/device_vector.h>
#include "cuda_wrapper.cuh"
```

Macros

#define ALGORITHM_NAMESPACE thrust

Variables

· auto unperturbed grf

Initialize a field and its derivative from a white noise power spectrum with cutoff k_ast.

auto unperturbed_grf_with_background

Initialize a field and its derivative from a white noise power spectrum with cutoff k_ast, plus homogeneous background.

· auto perturbed grf

Setup a scalar field with inhomogeneous Gaussian random initial conditions.

auto perturbed_grf_without_saving_Psi

Same as perturbed_grf, but does not store Psi in workspace to save memory.

auto unperturbed_grf_with_Psi

Same as unperturbed_grf, but with an extra scale-invariant Psi.

auto unperturbed_grf_and_fixed_curvature

Initialize a homogeneous Gaussian random field and some scale invariant curvature perturbation.

auto perturbed_grf_and_comoving_curvature_fft

Initialize an inhomogeneous Gaussian random field and the fft of some scale invariant comoving curvature perturba-

· auto homogeneous_field

Initialize a homogeneous field with amplitude f and time derivative dt_f. For testing the numerical code.

· auto homogeneous_field_with_fluctuations

Initialize a homogeneous field with amplitude f, plus scale-invariant perturbations (resembling quantum fluctutations).

· auto plane_wave

Plane wave initial condition. For testing the numerical code.

· auto wave packet

Wave packet initial condition.

9.18.1 Detailed Description

Snippets for initializing workpaces. (e.g. field initial conditions, etc)

Author

Siyang Ling

Contains a collection of lambda functions used to initialize the workspace for simulation. Each lambda function initializer is meant to be passed to a workspace constructor Workspace (param, initializer). The use of lambda's makes it easy to switch between param types and workspace types. Moreover, in order for easy switching between CPU code and GPU code, the initializers compute everything on CPU first, and then decide whether the results should be copied to CPU or GPU memory.

9.18.2 Variable Documentation

9.18.2.1 homogeneous_field

```
Initial value:
=
[] (const auto param, auto &workspace) {
    const long long int N = param.N;
    Eigen::VectorXd varphi = Eigen::VectorXd::Constant(N*N*N, param.f);
    Eigen::VectorXd dt_varphi = Eigen::VectorXd::Constant(N*N*N, param.dt_f);

    auto &state = workspace.state;
    state.resize(varphi.size() + dt_varphi.size());
    ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
    ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
```

Initialize a homogeneous field with amplitude f and time derivative dt_f. For testing the numerical code.

9.18.2.2 homogeneous_field_with_fluctuations

```
auto homogeneous_field_with_fluctuations [inline]
```

Initial value:

```
[](const auto param, auto &workspace) {
  const long long int N = param.N;
  Eigen::VectorXd varphi = Eigen::VectorXd::Constant(N*N*N, param.f);
  Eigen::VectorXd dt_varphi = Eigen::VectorXd::Constant(N*N*N, 0.0);

Spectrum P_delta_varphi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L,
        param.delta_varphi_std_dev, param.k_delta_varphi, -3);
  Eigen::VectorXd delta_varphi = generate_gaussian_random_field(param.N, param.L, P_delta_varphi);
  varphi += delta_varphi;

auto &state = workspace.state;
  state.resize(varphi.size() + dt_varphi.size());
  ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
  ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
}
```

Initialize a homogeneous field with amplitude f, plus scale-invariant perturbations (resembling quantum fluctutations).

9.18.2.3 perturbed_grf

```
auto perturbed_grf [inline]
Initial value:
  [](const auto param, auto &workspace) {
    Spectrum P_Psi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.Psi_std_dev,
      param.k_Psi, -3);
    Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
      param.k_ast, 0);
    Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
    Eigen::VectorXd Psi = generate_gaussian_random_field(param.N, param.L, P_Psi);
    Eigen::VectorXd varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L, Psi, P_f);
    Eigen::VectorXd dt_varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L, Psi, P_dtf);
    auto &state = workspace.state;
    state.resize(varphi.size() + dt_varphi.size());
    workspace.Psi.resize(Psi.size());
    ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
    ALGORITHM_NAMESPACE::copy(Psi.begin(), Psi.end(), workspace.Psi.begin());
```

Setup a scalar field with inhomogeneous Gaussian random initial conditions.

Initialize a field and its derivative from a white noise power spectrum with cutoff k_ast, but with a large scale perturbation specified by Psi. Psi is initialized from a scale-invariant power spectrum with cutoff k_Psi.

9.18.2.4 perturbed grf and comoving curvature fft

```
auto perturbed_grf_and_comoving_curvature_fft [inline]
```

Initialize an inhomogeneous Gaussian random field and the fft of some scale invariant comoving curvature perturbation.

This is the procedure used for section 4.2.2 of the paper.

9.18.2.5 perturbed_grf_without_saving_Psi

```
auto perturbed_grf_without_saving_Psi [inline]
```

Initial value:

Same as perturbed_grf, but does not store Psi in workspace to save memory.

9.18.2.6 plane_wave

```
Initial value:

[] (const auto param, auto &workspace) {
    const long long int N = param.N;
    Eigen::VectorXd varphi(N*N*N);
    for(int a = 0; a < N; ++a) {
        for(int b = 0; b < N; ++b) {
        for(int c = 0; c < N; ++c) {
            varphi(IDX_OF(N, a, b, c)) = cos(2 * std::numbers::pi * c / N);
        }
        }
    }
    Eigen::VectorXd dt_varphi = Eigen::VectorXd::Constant(N*N*N, 0);
    auto &state = workspace.state;
    state.resize(varphi.size() + dt_varphi.size());
    ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
    ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
}</pre>
```

Plane wave initial condition. For testing the numerical code.

9.18.2.7 unperturbed grf

```
Initial value:

[] (const auto param, auto &workspace) {
    Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
    param.k_ast, 0);
    Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
    Eigen::VectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f);
    Eigen::VectorXd dt_varphi = generate_gaussian_random_field(param.N, param.L, P_dtf);
    auto &state = workspace.state;
    state.resize(varphi.size() + dt_varphi.size());
    ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin() + varphi.size());
}
```

Initialize a field and its derivative from a white noise power spectrum with cutoff k_ast.

9.18.2.8 unperturbed_grf_and_fixed_curvature

```
auto unperturbed_grf_and_fixed_curvature [inline]
```

Initial value:

Initialize a homogeneous Gaussian random field and some scale invariant curvature perturbation.

9.18.2.9 unperturbed_grf_with_background

```
auto unperturbed_grf_with_background [inline]
```

Initial value:

```
[](const auto param, auto &workspace) {
    Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
    param.k_ast, 0);
    Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
    Eigen::VectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f);
    varphi.array() += param.varphi_mean;
    Eigen::VectorXd dt_varphi = generate_gaussian_random_field(param.N, param.L, P_dtf);

auto &state = workspace.state;
    state.resize(varphi.size() + dt_varphi.size());
    ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
}
```

Initialize a field and its derivative from a white noise power spectrum with cutoff k_ast, plus homogeneous background.

9.18.2.10 unperturbed grf with Psi

```
auto unperturbed_grf_with_Psi [inline]
```

Initial value:

Same as unperturbed grf, but with an extra scale-invariant Psi.

9.18.2.11 wave_packet

```
Initial value:

[] (const auto param, auto &workspace) {
    const long long int N = param.N;
    Eigen::VectorXd varphi(N*N*N);
    Eigen::VectorXd dt_varphi(N*N*N);
    Eigen::VectorXd Psi(N*N*N);

for(int a = 0; a < N; ++a) {
    for(int b = 0; b < N; ++b) {
    for(int c = 0; c < N; ++c) {
        double dist_to_center = sqrt(std::pow(std::min((double)a, (double)std::abs(N-a)), 2) + (b - N/3) * (b - N/3) + (c - N/3) * (c - N/3)) * (param.L / param.N);
    varphi(IDX_OF(N, a, b, c)) = exp(- dist_to_center * dist_to_center / 40.0);
    dt_varphi(IDX_OF(N, a, b, c)) = 0;

Psi(IDX_OF(N, a, b, c)) = - param.Psi_std_dev * cos(2 * std::numbers::pi * c / N);
}</pre>
```

```
auto &state = workspace.state;
state.resize(varphi.size() + dt_varphi.size());
ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
workspace.Psi.resize(Psi.size());
ALGORITHM_NAMESPACE::copy(Psi.begin(), Psi.end(), workspace.Psi.begin());
}
```

Wave packet initial condition.

9.19 initializer.hpp

```
00001
00012 #ifndef INITIALIZER HPP
00013 #define INITIALIZER_HPP
00015 #include "fdm3d.hpp'
00016 #include "random_field.hpp"
00017 #include "fftw_wrapper.hpp"
00018 #include "special_function.hpp"
00019 //#include "dispatcher.hpp"
00020 //#include "field_booster.hpp"
00021 //#include "param.hpp"
00022 //#include "physics.hpp"
00023
00024 #ifndef DISABLE CUDA
00025 #include <thrust/device_vector.h>
00026 #include "cuda_wrapper.cuh"
00027 #define ALGORITHM_NAMESPACE thrust
00028 #else
00029 #define ALGORITHM_NAMESPACE std
00030 #endif
00031
00033 inline auto unperturbed grf =
        [](const auto param, auto &workspace) {
           Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
      param.k_ast, 0);
00036
           Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
00037
           Eigen::VectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f); // Initial ULDM
      field
00038
           Eigen::VectorXd dt_varphi = generate_gaussian_random_field(param.N, param.L, P_dtf); // Initial
      ULDM field time derivative
00039
           auto &state = workspace.state;
00040
00041
           state.resize(varphi.size() + dt_varphi.size());
           // thrust::copy handles both copies between Eigen::VectorXd and copies from Eigen::VectorXd to
00042
      thrust::device vector<double>
00043
          ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00044
           ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00045
00046
00047
00049 inline auto unperturbed grf with background =
        [](const auto param, auto &workspace) {
           Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
      param.k_ast, 0);
00052
           Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
00053
           Eigen::VectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f);
           refer::vectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f);
varphi.array() += param.varphi_mean;
Eigen::VectorXd dt_varphi = generate_gaussian_random_field(param.N, param.L, P_dtf);
00054
00055
00056
00057
           auto &state = workspace.state;
00058
           state.resize(varphi.size() + dt_varphi.size());
           ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00059
00060
           ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00061
         };
00062
00063
00071 inline auto perturbed_grf =
00072
        [](const auto param, auto &workspace) {
           Spectrum P_Psi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.Psi_std_dev,
00073
      param.k_Psi, -3);
00074
          Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
      param.k_ast, 0);
00075
           Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
```

9.19 initializer.hpp 57

```
00076
           Eigen::VectorXd Psi = generate_gaussian_random_field(param.N, param.L, P_Psi);
           Eigen::VectorXd varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L, Psi, P_f);
00077
00078
           Eigen::VectorXd dt_varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L, Psi,
      P_dtf);
00079
08000
           auto &state = workspace.state;
           state.resize(varphi.size() + dt_varphi.size());
00082
           workspace.Psi.resize(Psi.size());
00083
           ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
          ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size()); ALGORITHM_NAMESPACE::copy(Psi.begin(), Psi.end(), workspace.Psi.begin());
00084
00085
00086
00087
           //std::cout « boost::typeindex::type id runtime(workspace.Psi).pretty name() « '\n';
00088
00089
00090
00092 inline auto perturbed_grf_without_saving_Psi =
00093
        [](const auto param, auto &workspace) {
          Spectrum P_Psi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.Psi_std_dev,
00094
      param.k_Psi, -3);
00095
          Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
      param.k_ast, 0);
00096
           Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
           Eigen::VectorXd Psi = generate_gaussian_random_field(param.N, param.L, P_Psi);
Eigen::VectorXd varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L, Psi, P_f);
00097
00098
           Eigen::VectorXd dt_varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L, Psi,
00099
      P_dtf);
00100
00101
           auto &state = workspace.state;
00102
           state.resize(varphi.size() + dt_varphi.size());
00103
           ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00104
          ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00105
00106
00107
00109 inline auto unperturbed_grf_with_Psi =
         [](const auto param, auto &workspace)
00110
          Spectrum P_Psi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.Psi_std_dev,
00111
      param.k_Psi, -3);
00112
          Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
      param.k_ast, 0);
00113
           Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
           Eigen::VectorXd Psi = generate_gaussian_random_field(param.N, param.L, P_Psi);
Eigen::VectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f);
00114
00115
00116
           Eigen::VectorXd dt_varphi = generate_gaussian_random_field(param.N, param.L, P_dtf);
00117
00118
           auto &state = workspace.state;
00119
           state.resize(varphi.size() + dt_varphi.size());
00120
           workspace.Psi.resize(Psi.size());
00121
           ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00122
           ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
           ALGORITHM_NAMESPACE::copy(Psi.begin(), Psi.end(), workspace.Psi.begin());
00123
00124
00125
00126
00128 inline auto unperturbed grf and fixed curvature =
        [] (const auto param, auto &workspace)
00130
           Spectrum P_Psi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.Psi_std_dev,
      param.k_Psi, -3);
00131
          Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
      param.k_ast, 0);
00132
           Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
00133
           Eigen::VectorXd Psi = generate_gaussian_random_field(param.N, param.L, P_Psi);
           Eigen::VectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f);
00134
00135
           Eigen::VectorXd dt_varphi = generate_gaussian_random_field(param.N, param.L, P_dtf);
00136
00137
           auto &state = workspace.state;
00138
           state.resize(varphi.size() + dt_varphi.size());
00139
           workspace.Psi.resize(Psi.size());
00140
           ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00141
           ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00142
           ALGORITHM_NAMESPACE::copy(Psi.begin(), Psi.end(), workspace.Psi.begin());
00143
        };
00144
00150 inline auto perturbed_grf_and_comoving_curvature_fft =
00151
        [] (const auto param, auto &workspace) {
00152
           using namespace std::numbers;
00153
00154
           // Generate comoving curvature perturbation
00155
           double eta_i = workspace.cosmology.eta(param.t_start);
           double A_s = (-576 * pow(pi, 6) * pow(eta_i, 6) * pow(param.Psi_std_dev, 2)) / (-81 * pow(param.L, 4) * (pow(param.L, 2) + 2 * pow(pi, 2) * pow(eta_i, 2)) +
00156
00157
00158
00159
              (81 * pow(param.L, 5) - 54 * pow(param.L, 3) * pow(pi, 2) * pow(eta_i, 2) +
           48 * param.L * pow(pi, 4) * pow(eta_i, 4)) * cos((4 * pi * eta_i) / (sqrt(3) * param.L)) + 256 * pow(pi, 6) * pow(eta_i, 6) * Ci_pade_approximant_12_12((4 * pi * eta_i) / (sqrt(3) *
00160
00161
00162
```

```
param.L)) +
           4 * sqrt(3) * param.L * pi * eta_i *
00163
             (27 * pow(param.L, 4) + 6 * pow(param.L, 2) * pow(pi, 2) * pow(eta_i, 2) -
00164
          16 * pow(pi, 4) * pow(eta_i, 4)) * sin((4 * pi * eta_i) / (sqrt(3) * param.L)));
00165
00166
00167
          Spectrum P R = scale invariant spectrum 3d(param.N, param.L, A s);
00168
00169
          // Manual cutoff for P_R at around horizon. The effect of imposing this cutoff is negligible.
00170
          // Spectrum P_R_with_cutoff = [P_R] (double k) { return k <= 0.5 ? P_R(k) : 0.0; };
00171
00172
          Eigen::VectorXd R = generate_gaussian_random_field(param.N, param.L, P_R);
          // std::cout « "A_s = " « A_s « '\n';
00173
00174
00175
          // Calculate initial gravitational potential Psi.
00176
          // Convention for potentials: \mathbb{R}_k = (3 / 2) \text{Psi}_k for superhorizon.
     auto kernel = [eta_i] (double k) {
    return k == 0.0 ? 0.0 : (6 * sqrt(3) * (-((k * eta_i * cos((k * eta_i) / sqrt(3))) /
sqrt(3)) + sin((k * eta_i) / sqrt(3)))) / (pow(k, 3) * pow(eta_i, 3));
00177
00178
00179
                };
00180
           auto fft_wrapper = fftwWrapper(param.N);
          Eigen::VectorXd Psi = compute_field_with_scaled_fourier_modes(param.N, param.L, R, kernel,
00181
      fft_wrapper);
00182
          // Calculate \ \pi^2, \dot{\pi^6}^2 perturbations as a multiple of Psi. // See Eqn (3.17) of paper.
00183
00184
          // There is an extra factor of 0.5 in front since "generate_inhomogeneous_gaussian_random_field"
00185
      use \exp(2\Psi) \sim 1 + 2 \Psi for variance perturbation convention.
          double v = param.k_ast / (param.al * param.m);
double alpha_varphi_sqr = 0.5 * (- 3 * pow(4*pow(v,2)+5, 2)) / (12*pow(v,4) + 50*pow(v,2) + 50);
00186
00187
          double alpha_dot_varphi_sqr = 0.5 * (25 - 20*pow(v,2)) / (12*pow(v,4) + 50*pow(v,2) + 50);
00188
00189
00190
          Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
      param.k_ast, 0);
          Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
00191
00192
          Eigen::VectorXd varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L,
      alpha_varphi_sqr * Psi, P_f);
    Eigen::VectorXd dt_varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L,
alpha_dot_varphi_sqr * Psi, P_dtf);
00193
00194
00195
           auto &state = workspace.state;
00196
          state.resize(varphi.size() + dt_varphi.size());
          ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00197
00198
          ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00199
00200
           // Save the comoving curvature perturbation for reference
00201
00202
            decltype(workspace.state) R_dvec(R.size());
00203
            ALGORITHM_NAMESPACE::copy(R.begin(), R.end(), R_dvec.begin());
00204
             workspace.R_fft = workspace.fft_wrapper.execute_d2z(R_dvec);
00205
00206
        };
00207
00208
00209
00214 inline auto homogeneous_field =
00215
        [](const auto param, auto &workspace) {
          const long long int N = param.N;
00216
          Eigen::VectorXd varphi = Eigen::VectorXd::Constant(N*N*N, param.f);
00217
00218
          Eigen::VectorXd dt_varphi = Eigen::VectorXd::Constant(N*N*N, param.dt_f);
00219
00220
          auto &state = workspace.state;
00221
          state.resize(varphi.size() + dt_varphi.size());
00222
          ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00223
          ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00224
00225
00226
00230 inline auto homogeneous field with fluctuations =
00231
        [](const auto param, auto &workspace) {
00232
          const long long int N = param.N;
00233
          Eigen::VectorXd varphi = Eigen::VectorXd::Constant(N*N*N, param.f);
00234
          Eigen::VectorXd dt_varphi = Eigen::VectorXd::Constant(N*N*N, 0.0);
00235
          Spectrum P_delta_varphi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L,
00236
Eigen::VectorXd delta_varphi = generate_gaussian_random_field(param.N, param.L, P_delta_varphi);
00238
          varphi += delta_varphi;
00239
          auto &state = workspace.state;
00240
00241
          state.resize(varphi.size() + dt varphi.size());
          ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00242
          ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00243
00244
00245
00246
00251 inline auto plane wave =
00252
        [] (const auto param, auto &workspace) {
```

9.20 io.hpp 59

```
const long long int N = param.N;
00254
          Eigen::VectorXd varphi(N*N*N);
00255
          for (int a = 0; a < N; ++a) {
           for(int b = 0; b < N; ++b) {
00256
          for(int c = 0; c < N; ++c){
  varphi(IDX_OF(N, a, b, c)) = cos(2 * std::numbers::pi * c / N);</pre>
00257
00258
00259
00260
00261
          }
00262
          Eigen::VectorXd dt_varphi = Eigen::VectorXd::Constant(N*N*N, 0);
00263
00264
00265
          auto &state = workspace.state;
00266
          state.resize(varphi.size() + dt_varphi.size());
00267
          ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00268
          ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00269
00270
00275 inline auto wave_packet =
00276
       [](const auto param, auto &workspace) {
00277
          const long long int N = param.N;
00278
          Eigen::VectorXd varphi(\bar{N*N*N});
00279
          Eigen::VectorXd dt_varphi(N*N*N);
00280
          Eigen::VectorXd Psi(N*N*N);
00281
          for (int a = 0; a < N; ++a) {
00282
00283
            for (int b = 0; b < N; ++b) {
00284
          for (int c = 0; c < N; ++c) {
     double dist_to_center = sqrt(std::pow(std::min((double)a, (double)std::abs(N-a)), 2) + (b - N/3)  
* (b - N/3) + (c - N/3) * (c - N/3)) * (param.L / param.N);
00285
            varphi(IDX_OF(N, a, b, c)) = exp(- dist_to_center * dist_to_center / 40.0);
00286
00287
            dt_varphi(IDX_OF(N, a, b, c)) = 0;
00288
            param.L / 3.0 / 3.0)));
    Psi(IDX_OF(N, a, b, c)) = - param.Psi_std_dev * cos(2 * std::numbers::pi * c / N);
00289
00290
00291
           }
00292
          }
00293
00294
          auto &state = workspace.state;
          state.resize(varphi.size() + dt_varphi.size());
00295
          ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00296
00297
          ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00298
00299
          workspace.Psi.resize(Psi.size());
00300
          ALGORITHM_NAMESPACE::copy(Psi.begin(), Psi.end(), workspace.Psi.begin());
00301
        };
00302
00303
00304 #endif
```

9.20 io.hpp

```
00001 #ifndef IO_HPP
00002 #define IO_HPP
00003 #include <cstdlib>
00004 #include <iostream>
00005 #include <fstream>
00006 #include <string>
00007 #include <vector>
00008 #include <iomanip>
00009
00010 #include <Eigen/Dense>
00011
00012
00013 std::vector<double> load_vector_from_file(std::string filename);
00014 void write_vector_to_file(std::vector<double> vector, std::string filename);
00015 void write_data_to_file(const char *buf, ssize_t size, std::string filename);
00016
00017 void write_VectorXd_to_file(const Eigen::VectorXd &vector, std::string filename);
00018 void write_VectorXd_to_filename_template(const Eigen::VectorXd &vector, const std::string
      format_string, const int idx);
00019 Eigen::VectorXd load_VectorXd_from_file(const std::string &filename);
00020
00021
00022 #endif
```

9.21 src/midpoint.hpp File Reference

Midpoint method implementation for odeint stepper.

```
#include <boost/numeric/odeint/algebra/default_operations.hpp>
#include <boost/numeric/odeint/algebra/algebra_dispatcher.hpp>
#include <boost/numeric/odeint/util/state_wrapper.hpp>
#include <boost/numeric/odeint/util/is_resizeable.hpp>
#include <boost/numeric/odeint/util/is_resizeable.hpp>
#include <boost/numeric/odeint/util/resizer.hpp>
#include "cuda_wrapper.cuh"
```

Classes

class midpoint < State, Value, Deriv, Time, Algebra, Operations, Resizer >

9.21.1 Detailed Description

Midpoint method implementation for odeint stepper.

Author

Siyang Ling

9.22 midpoint.hpp

```
00001
00006 #ifndef MIDPOINT HPP
00007 #define MIDPOINT_HPP
80000
00009
00010
00011 //#include <boost/numeric/odeint/stepper/base/explicit_stepper_base.hpp>
00012 //#include <boost/numeric/odeint/algebra/range_algebra.hpp>
00013 #include <boost/numeric/odeint/algebra/default_operations.hpp>
00014 #include <boost/numeric/odeint/algebra/algebra_dispatcher.hpp>
00015 #include <boost/numeric/odeint/algebra/operations_dispatcher.hpp>
00017 #include <boost/numeric/odeint/util/state_wrapper.hpp>
00018 #include <boost/numeric/odeint/util/is_resizeable.hpp>
00019 #include <boost/numeric/odeint/util/resizer.hpp>
00020
00021 #ifndef DISABLE_CUDA
00022 #include "cuda_wrapper.cuh"
00023 #endif
00024
00025 template<
00026
       class State,
00027
       class Value = double,
       class Deriv = State,
00028
00029
        class Time = Value,
00030
       class Algebra = typename boost::numeric::odeint::algebra_dispatcher<State>::algebra_type,
00031
       class Operations = typename boost::numeric::odeint::operations_dispatcher<State>::operations_type,
00032
       class Resizer = boost::numeric::odeint::initially_resizer //boost::numeric::odeint::always_resizer
00033
00034 class midpoint : public boost::numeric::odeint::algebra_stepper_base<Algebra, Operations>
00035 {
00037 typedef State state_type;
00038 typedef State '
00036 public :
       typedef Value value_type;
00039
00040 typedef Time time_type;
```

```
typedef unsigned short order_type;
00042
              typedef boost::numeric::odeint::stepper_tag stepper_category;
00043
00044
              typedef boost::numeric::odeint::algebra_stepper_base<Algebra, Operations> algebra_stepper_base_type;
00045
              {\tt typedef\ typename\ algebra\_stepper\_base\_type:: algebra\_type\ algebra\_type;}
00046
              typedef typename algebra_stepper_base_type::operations_type operations_type;
00048
              static order_type order(void) { return 2; }
00049
00050
              midpoint(){}
00051
00052
              template<class System>
00053
              void do_step(System system, State &in, Time t, Time dt)
00054
00055
                  static const Value val1 = static_cast<Value>(1);
                 const Time dh = dt / static_cast<Value>(2);
const Time th = t + dh;
00056
00057
00058
00059
                  //m_resizer.adjust_size(in, boost::numeric::odeint::detail::bind(&stepper_type::template
          resize_impl<State>, boost::numeric::odeint::detail::ref(*this), boost::numeric::odeint::detail::_l));
00060
                 m_resizer.adjust_size(in, [&](const auto &arg) { return resize_impl(arg); });
00061
00062
                  typename boost::numeric::odeint::unwrap_reference<System>::type &sys = system;
00063
00064
                  sys(in, deriv_tmp.m_v, t);
00065
                  algebra_stepper_base_type::m_algebra.for_each3(state_tmp.m_v, in, deriv_tmp.m_v,
                                                             typename operations_type::template scale_sum2<Value, Time>(val1, dh));
00066
00067
00068
                  sys(state_tmp.m_v, deriv_tmp.m_v, th);
00069
                  \verb|algebra_stepper_base_type::m_algebra.for_each3(state_tmp.m_v, in, deriv_tmp.m_v, deriv_tmp
00070
                                                             typename operations_type::template scale_sum2<Value, Time>(val1, dt));
00071
00072
                  in.swap(state_tmp.m_v);
00073
                  // Release memory
00074
00075
                  //m_resizer.adjust_size(State(), [&](const auto &arg){ return resize_impl(arg); });
00076
                  // deriv tmp.m v.clear();
                 // State().swap(deriv_tmp.m_v);
00078
                  // state_tmp.m_v.clear();
00079
                  // State().swap(state_tmp.m_v);
00080
00081
00082
              // template<class StateType>
00083
              // void adjust_size(const StateType &x)
00084
00085
                        resize_impl(x);
00086
00087
00088
              bool resize_impl(const State &x)
00089
00090
                 bool resized = false;
                  resized |= boost::numeric::odeint::adjust_size_by_resizeability(deriv_tmp, x, typename
00091
          boost::numeric::odeint::is_resizeable<State>::type());
00092
                 resized |= boost::numeric::odeint::adjust_size_by_resizeability(state_tmp, x, typename
         boost::numeric::odeint::is_resizeable<State>::type());
00093
                 return resized;
00094
00095
00096 private:
00097
              Resizer m_resizer;
00098
00099
              boost::numeric::odeint::state_wrapper<State> deriv_tmp;
00100
             boost::numeric::odeint::state_wrapper<State> state_tmp;
00101 };
00102
00103
00104
00105 #endif
```

9.23 src/observer.hpp File Reference

Implements "observers", which controls what gets saved during simulations.

```
#include <cstdlib>
#include <iostream>
#include <string>
#include <type_traits>
#include "Eigen/Dense"
```

```
#include <boost/numeric/odeint.hpp>
#include <boost/numeric/odeint/external/eigen/eigen.hpp>
#include "odeint_eigen/eigen_operations.hpp"
#include "eigen_wrapper.hpp"
#include "fdm3d.hpp"
#include "io.hpp"
#include "physics.hpp"
#include "workspace.hpp"
#include "cuda_wrapper.cuh"
#include "fdm3d_cuda.cuh"
```

Classes

struct ConstIntervalObserver< Equation, save field spectrum, save density spectrum, save density >

9.23.1 Detailed Description

Implements "observers", which controls what gets saved during simulations.

Author

Siyang Ling

9.24 observer.hpp

```
00007 #ifndef OBSERVER HPP
00008 #define OBSERVER_HPP
00009
00010 #include <cstdlib>
00011 #include <iostream>
00012 #include <string>
00013 #include <type_traits>
00014
00015 #include "Eigen/Dense"
00016
00017 #include <boost/numeric/odeint.hpp>
00018 #include <boost/numeric/odeint/external/eigen/eigen.hpp>
00020 #include "odeint_eigen/eigen_operations.hpp"
00021
00022 #include "eigen_wrapper.hpp"
00023 #include "fdm3d.hpp"
00024 #include "io.hpp"
00025 #include "physics.hpp"
00026 #include "workspace.hpp"
00027
00028 #ifndef DISABLE CUDA
00029 #include "cuda_wrapper.cuh"
00030 #include "fdm3d_cuda.cuh"
00031 #endif
00032
00033
00034 template<typename Equation,
00035 bool save_field_spectrum = true,
            bool save_density_spectrum = true,
00037
           bool save_density = false>
00038 struct ConstIntervalObserver {
00039 typedef typename Equation::Workspace Workspace;
00040 typedef typename Workspace::State State;
00041 typedef State Vector;
00042 Workspace &workspace;
00043 int idx;
        typedef State Vector;
```

```
00044
       std::string dir;
00045
        double t_start;
00046
        double t_end;
00047
       double t_interval;
00048
       double t last;
00049
00050
        template<typename Param>
00051
        ConstIntervalObserver(const std::string &dir_, const Param &param, Equation &eqn) :
00052
        workspace(eqn.workspace), idx(0), dir(dir_),
00053
          t_start(param.t_start), t_end(param.t_end), t_interval(param.t_interval), t_last(param.t_start) { }
00054
00055
       ConstIntervalObserver(const ConstIntervalObserver &) = default;
00056
00057
        void operator()(const State &x, double t)
00058
00059
          if(t >= t_last + t_interval || t == t_end || t == t_start) {
            const long long int N = workspace.N;
const double L = workspace.L;
const double m = workspace.m;
00060
00061
00062
00063
            const double a_t = workspace.cosmology.a(t);
00064
00065
            if constexpr(save_field_spectrum) {
00066
           Vector varphi_plus_spectrum = compute_mode_power_spectrum(N, L, m, a_t, workspace.state,
     workspace.fft_wrapper);
00067
            Eigen::VectorXd varphi_plus_spectrum_out(varphi_plus_spectrum.size());
00068
            copy_vector(varphi_plus_spectrum_out, varphi_plus_spectrum);
00069
            write_VectorXd_to_filename_template(varphi_plus_spectrum_out, dir +
     "varphi_plus_spectrum_%d.dat", idx);
00070
00071
00072
            if constexpr(save_density_spectrum) {
Vector rho = Equation::compute_energy_density(workspace, t);
00073
00074
            Vector rho_spectrum = compute_power_spectrum(N, rho, workspace.fft_wrapper);
00075
            Eigen::VectorXd rho_spectrum_out(rho_spectrum.size());
00076
            copy_vector(rho_spectrum_out, rho_spectrum);
            write_VectorXd_to_filename_template(rho_spectrum_out, dir + "rho_spectrum_%d.dat", idx);
00077
00078
00079
08000
            if constexpr(save_density) {
00081
            Vector rho = Equation::compute_energy_density(workspace, t);
00082
            Eigen::VectorXd rho_copy(rho.size());
            copy_vector(rho_copy, rho);
Eigen::VectorXd rho_slice = rho_copy.head(N*N); // Save the density for a = 0 slice.
00083
00084
00085
            density overaged over a axis.
00086
00087
            write_VectorXd_to_filename_template(rho_slice, dir + "rho_slice_%d.dat", idx);
00088
            write_VectorXd_to_filename_template(rho_axis_average, dir + "rho_axis_average_%d.dat", idx);
00089
00090
00091
            workspace.t_list.push_back(t);
00092
00093
            ++idx;
00094
       }
00095
00096 };
00097
00098
00099
00100 #endif
```

9.25 src/param.hpp File Reference

Utilities for managing simulations parameters.

```
#include "utility.hpp"
#include "boost/pfr.hpp"
#include "boost/type_index.hpp"
#include <fstream>
#include <string>
```

Classes

struct SampleParam

A sample parameter type specifying a lambda-phi-4 theory in an FRW background.

Functions

```
    template < typename T > void print_param (const T &param)
        Pretty prints a parameter struct T.
    template < typename T > void save_param_names (const std::string &filename)
        Save the member names of parameter struct T to filename.
    template < typename T > void save_param_Mathematica_formats (const std::string &filename)
        Save the member types of parameter struct T to filename. Type names are in Mathematica convention.
    template < typename T > void save_param_for_Mathematica (const T &param, const std::string &dir)
        Save member names, types and values of param to directory dir.
    template < typename T > void save_param_types (const std::string &filename)
```

9.25.1 Detailed Description

Utilities for managing simulations parameters.

Author

Siyang Ling

This header file contains utilities for pretty-printing and saving parameters of a simulation. By convention, we collect all parameters in a (trivial, standard layout) struct containing double's or long long int's. (e.g. SampleParam) The utilities here are generic for different parameter structs; you can define your own new type containing new parameters, and use the utilities here as usual. Typically, we use these utilities to export a struct along with some meta-information, so that external code (Mathematica / Python) can also use the parameters.

9.26 param.hpp

```
00014 #ifndef PARAM_HPP
00015 #define PARAM_HPP
00016
00017 #include "utility.hpp"
00017 #Include defility.hpp
00018 #include "boost/pfr.hpp"
00019 #include "boost/type_index.hpp"
00020 #include <fstream>
00021 #include <string>
00022
00026 struct SampleParam {
00027
         long long int N;
00028
         double L;
00029
         double m;
00030
         double lambda;
00031
         double k_ast;
00032
         double varphi_std_dev;
00033
         double a1:
         double H1;
00034
00035
         double t1;
00036 };
00037
00041 template<typename T>
00042 void print_param(const T &param) {
00043 auto names = boost::pfr::names_as_array<T>();
00044 auto func = [&](const auto &field, std::size_t i) {
                std::cout « names[i] « ": " « field
00045
```

9.26 param.hpp 65

```
00046
                     « " (" « boost::typeindex::type_id_runtime(field) « ")\n";
00047
00048
        // std::cout « line_separator_with_description("The parameters for the simulation") « ' \n';
00049
        // boost::pfr::for_each_field(param, func);
       /// std::cout « line_separator_with_description() « '\n';
auto c = [$](){ boost::pfr::for_each_field(param, func); };
00050
00051
        run_and_print("The parameters for the simulation", c);
00052
00053 }
00054
00058 template<typename T>
00059 void save_param_names(const std::string &filename) {
00060 std::ofstream outstream(filename);
00061
        auto names = boost::pfr::names_as_array<T>();
       for(auto name : names) {
00062
00063
         outstream « name « '\n';
00064
00065 }
00066
00067 /
00068 // Compiles with Intel icpx, but doesn't compile with gcc due to "Explicit template specialization
      cannot have a storage class"
00069 template<typename T> std::string_view Mathematica_format;
00070
00071 template<> constexpr static std::string_view Mathematica_format<double> = "Real64";
00072
00073 template<> constexpr static std::string_view Mathematica_format<long long int> = "Integer64";
00074 */
00075
00076 /
00077 // Compiles with gcc, fails at link stage with Intel icpx due to multiple definitions
00078 template<typename T> std::string_view Mathematica_format;
00080 template<> constexpr std::string_view Mathematica_format<double> = "Real64";
00081
00082 template<> constexpr std::string_view Mathematica_format<long long int> = "Integer64";
00083 */
00084
00085 namespace {
00086 template<typename T> std::string_view Mathematica_format;
00087
00088 template<> constexpr std::string_view Mathematica_format<double> = "Real64";
00089
00090 template<> constexpr std::string_view Mathematica_format<long long int> = "Integer64";
00091 }
00092
00096 template<typename T>
00097 void save_param_Mathematica_formats(const std::string &filename) {
00098 std::ofstream outstream(filename);
       auto func = [&] (const auto &field) {
00099
             typedef std::remove_const_t<std::remove_reference_t<decltype(field)» type_of_field;
00100
00101
              outstream « Mathematica_format<type_of_field> « '
00102
00103
       boost::pfr::for_each_field(T(), func);
00104 }
00105
00109 template<typename T>
00110 static void save_param(const T &param, const std::string &filename){
       std::ofstream outstream(filename, std::ios::binary);
00111
00112
       if (outstream.is_open()) {
00113
         outstream.write((const char *)&param, sizeof(T));
       }
00114
00115 }
00116
00120 template<typename T>
00121 void save_param_for_Mathematica(const T &param, const std::string &dir) {
00122
       save_param_names<T>(dir + "paramNames.txt");
00123
        save_param_Mathematica_formats<T>(dir + "paramTypes.txt");
       save_param<T>(param, dir + "param.dat");
00124
00125 }
00126
00127
00128 template<typename T>
00129 void save_param_types(const std::string &filename) { 00130    std::ofstream outstream(filename);
       auto func = [&] (const auto &field) {
00131
00132
              outstream « boost::typeindex::type_id_runtime(field) « '\n';
00133
00134
       boost::pfr::for_each_field(T(), func);
00135 }
00136
00137
00138
00139
00140 #endif
```

9.27 src/physics.hpp File Reference

Collection of repeatedly used physics formulas. (e.g. FRW cosmology related formulas)

```
#include <cmath>
```

Classes

struct StaticEOSCosmology

9.27.1 Detailed Description

Collection of repeatedly used physics formulas. (e.g. FRW cosmology related formulas)

Author

Siyang Ling

9.28 physics.hpp

Go to the documentation of this file.

```
00001
00006 #ifndef PHYSICS_HPP
00007 #define PHYSICS_HPP
00009 #include <cmath>
00010 //#include "param.hpp"
00011
00012 struct StaticEOSCosmology {
00013
       double a1;
00014
        double H1;
00015
        double t1;
00016
       double p;
00017
       00018
00019
00020
00021
        // The default constructor from a param assumes radiation domination
00022
        template<typename T>
00023
        StaticEOSCosmology(const T &param)
00024
          : a1(param.a1), H1(param.H1), t1(param.t1), p(1.0) {}
00025
00026
        StaticEOSCosmology(void)
00027
          : a1(1.0), H1(0), t1(0), p(1.0) {}
00028
00029
        double a(const double t) const {
        return a1 * pow(1 + (1 + 1 / p) * H1 * (t - t1), p / (1 + p));
}
00030
00031
        double H(const double t) const {
  return H1 * pow(1 + (1 + 1 / p) * H1 * (t - t1), -1);
00032
00033
00034
00035
        // We use convention etal = p / (a1 * H1).
        double eta(const double t) const {    //return eta1 + (p / (a1 * H1)) * (-1 + pow(1 + (1 + 1 / p) * H1 * (t - t1), 1 / (1 + p)));    return (p / (a1 * H1)) * pow(1 + (1 + 1 / p) * H1 * (t - t1), 1 / (1 + p));
00036
00037
00038
00039
00040 };
00041
00042
00043 #endif
```

9.29 src/random field.hpp File Reference

Utilities for generating Gaussian random fields of given spectrum and inhomogeneity.

```
#include "Eigen/Dense"
#include <functional>
#include <random>
#include <vector>
```

Typedefs

typedef std::function< double(const double)> Spectrum

Typedef for spectrum P(k). Given momentum k, the spectrum should return P(k).

Functions

- void RandomNormal::set_generator_seed (std::mt19937::result_type seed)
- std::mt19937 RandomNormal::get_generator_from_device ()
- double RandomNormal::generate_random_normal()
- Spectrum power_law_with_cutoff_given_amplitude_3d (const long long int N, const double L, const double sigma, const double k_ast, const double alpha)

 k^{α} power law spectrum with a sharp cutoff at k_* .

• Spectrum broken_power_law_given_amplitude_3d (const long long int N, const double L, const double sigma, const double k_ast, const double alpha, const double beta)

Broken power law spectrum with the break at k_* .

• Spectrum scale_invariant_spectrum_3d (const long long int N, const double L, const double As)

 k^{α} power law spectrum with a sharp cutoff at k_* .

• Spectrum to_deriv_spectrum (const double m, const Spectrum &P_f)

```
Given spectrum P_{\varphi}, return a new spectrum given by P_{\dot{\varphi}}(k) = (k^2 + m^2)P_{\varphi}(k).
```

• Spectrum to_deriv_spectrum (const double m, const double a, const Spectrum &P_f)

Given spectrum P_{φ} , return a new spectrum given by $P_{\dot{\varphi}}(k) = (k^2/a^2 + m^2)P_{\varphi}(k)$.

Eigen::VectorXd generate_gaussian_random_field (const long long int N, const double L, const Spectrum &P)

Special case of generate_inhomogeneous_gaussian_random_field.

Eigen::VectorXd generate_inhomogeneous_gaussian_random_field (const long long int N, const double L, const Eigen::VectorXd &Psi, const Spectrum &P)

Generate an inhomogeneous 3D real Gaussian random field from spectral data P(k).

9.29.1 Detailed Description

Utilities for generating Gaussian random fields of given spectrum and inhomogeneity.

Author

Siyang Ling

This file contains utilities for generating Gaussian random fields (GRF), including some example spectra and a function for generating field realizations from a spectra. See function generate_inhomogeneous_gaussian_random_field for details.

9.29.2 Function Documentation

9.29.2.1 broken_power_law_given_amplitude_3d()

Broken power law spectrum with the break at k_* .

Parameters

N	Number of lattice points.
L	Box size.
sigma	Standard deviation σ of generated function f .
k_ast	The break k_* .
alpha	Power law index α .
beta	Power law index β .

Returns

The spectrum P, which can be called to get P(k).

The spectrum is given by

$$P(0) = 0$$

$$P(k) = P(k_0)(k/k_0)^{\alpha} \text{ for } k < k_0$$

$$P(k) = P(k_0)(k/k_0)^{\beta} \text{ for } k > k_0$$

$$\overline{f^2} = \sigma^2$$

9.29.2.2 generate inhomogeneous gaussian random field()

Generate an inhomogeneous 3D real Gaussian random field from spectral data P(k).

Parameters

Ν	Number of lattice points.	
L	Box size.	
Psi	The inhomogeneity function ψ , given in terms of values on the lattice (of size N^3)	
Р	The spectrum P .	

Returns

The generated GRF, as values on the lattice (of size N^3).

Generate an inhomogeneous Gaussian random field f, such that the spectrum of f is P, and the variance of the field has inhomogeneity like $\langle f^2(x) \rangle \approx \overline{f^2} e^{2\psi(x)}$. See section 3.2 of paper for details of this procedure.

9.29.2.3 power_law_with_cutoff_given_amplitude_3d()

 k^{α} power law spectrum with a sharp cutoff at k_* .

Parameters

N	Number of lattice points.
L	Box size.
sigma	Standard deviation σ of generated function f .
k_ast	Cutoff k_* .
alpha	Power law index α .

Returns

The spectrum P, which can be called to get P(k).

The spectrum is given by

$$P(0) = 0$$

$$P(k) = P(k_0)(k/k_0)^{\alpha} \text{ for } k < k_0$$

$$\overline{f^2} = \sigma^2$$

9.29.2.4 scale_invariant_spectrum_3d()

 k^{α} power law spectrum with a sharp cutoff at k_{*} .

Parameters

Ν	Number of lattice points.	
L	Box size.	
As	The height of the spectrum A_s .	

Returns

The spectrum P, which can be called to get P(k).

The spectrum is given by

$$P(0) = 0$$

$$P(k) = A_s$$

9.30 random field.hpp

```
Go to the documentation of this file.
```

```
00010 #ifndef RANDOM_FIELD_HPP
00011 #define RANDOM_FIELD_HPP
00012
00013 #include "Eigen/Dense"
00014
00015 #include <functional>
00016 #include <random>
00017 #include <vector>
00018
00019
00020 // A self-initializing random number generator for standard normal distribution
00021 namespace RandomNormal
00023
        void set_generator_seed(std::mt19937::result_type seed);
00024
        std::mt19937 get_generator_from_device();
00025
       double generate_random_normal();
00026 }
00027
00031 typedef std::function<double(const double)> Spectrum;
00032
00033 // Typical spectra.
00034
00051 Spectrum power_law_with_cutoff_given_amplitude_3d(const long long int N, const double L, const double
      sigma, const double k_ast, const double alpha);
00052
00071 Spectrum broken_power_law_given_amplitude_3d(const long long int N, const double L, const double
      sigma, const double k_ast, const double alpha, const double beta);
00072
00086 Spectrum scale invariant spectrum 3d(const long long int N, const double L, const double As);
00087
00091 Spectrum to_deriv_spectrum(const double m, const Spectrum &P_f);
00096 Spectrum to_deriv_spectrum(const double m, const double a, const Spectrum &P_f);
00097
00101 Eigen::VectorXd generate_gaussian_random_field(const long long int N, const double L, const Spectrum
00102
00115 Eigen::VectorXd generate_inhomogeneous_gaussian_random_field(const long long int N, const double L,
      const Eigen::VectorXd &Psi, const Spectrum &P);
00116
00117
00118
00119
00120 #endif
```

9.31 special_function.hpp

```
00001 #ifndef SPECIAL_FUNCTION_HPP
00002 #define SPECIAL_FUNCTION_HPP
00003
00004 // Pade approximant for Si(x), with m=15, n=12
00005 inline double Si_pade_approximant_15_12(double x) {
00006 using namespace std;
                        0.045439340981633 * pow(x, 3) + 0.0011545722575101668 * pow(x, 5) -
         return (x -
80000
                    0.000014101853682133025 * pow(x, 7) + 9.432808094387131e-8 * pow(x, 9) -
00009
                    3.5320197899716837e-10 * pow(x, 11) + 7.08240282274876e-13 * pow(x, 13) -
00010
                   6.053382120104225e-16 * pow(x, 15)) /
                   (1. + 0.010116214573922555 * pow(x, 2) + 0.000049917511616975513 * pow(x, 4) + 1.556549863087456e-7 * pow(x, 6) + 3.280675710557897e-10 * pow(x, 8) + 4.5049097575386586e-13 * pow(x, 10) + 3.211070511937122e-16 * pow(x, 12));
00011
00012
00013
00014 }
```

9.32 src/utility.hpp File Reference

Utilities for debugging / profiling / pretty printing.

```
#include <iostream>
#include <iomanip>
#include <chrono>
#include <filesystem>
```

Functions

template<typename Callable > void profile_function (long long int repeat, Callable &&c)

9.32.1 Detailed Description

Utilities for debugging / profiling / pretty printing.

Author

Siyang Ling

9.33 utility.hpp

Go to the documentation of this file.

```
00006 #ifndef UTILITY_HPP
00007 #define UTILITY_HPP
80000
00009 #include <iostream>
00010 #include <iomanip>
00011 #include <chrono>
00012 #include <filesystem>
00013
00014 // Pretty print functions
00015 inline static std::string line_separator_with_description(const std::string &description) {
00016    std::string result(80, '=');
00017    const int length = description.length() + 2;
00018    result.replace(80 / 2 - length / 2, length, " " + description + " ");
00019
00020 }
00021
00022
00023 inline static std::string line_separator_with_description(void) {
00024 std::string result(80, '=');
```

```
return result;
00026 }
00027
00028
00029 template<typename Callable>
00030 static void run_and_print(const std::string &description, const Callable &c) {
        std::cout « line_separator_with_description(description) « '\n';
00032
00033
        std::cout « line_separator_with_description() « '\n';
00034 }
00035
00036
00037 template<typename Callable>
00038 static void run_and_measure_time(const std::string &description, const Callable &c) {
00039
        std::cout « line_separator_with_description(description) « '\n';
00040
        auto time_start = std::chrono::system_clock::now();
00041
00042
        auto time end = std::chrono::system clock::now();
        std::chrono::duration<double> time_diff = time_end - time_start;
00044
        std::cout « std::fixed « std::setprecision(9) « std::left;
00045
        std::cout « std::setw(9) « "time spent = " « time_diff.count() « " s" « '\n';
00046
        std::cout « line_separator_with_description() « '\n';
00047 }
00048
00049
00050 static void prepare_directory_for_output(const std::string &dir) {
00051
        const std::filesystem::path dir_path(dir);
00052
        std::error_code ec;
00053
        std::cout « line_separator_with_description("Preparing directory for output") « '\n';
        std::cout « "Saving results in directory: " « dir « '\n';
00054
        std::filesystem::create_directories(dir_path, ec);
std::cout « "ErrorCode = " « ec.message() « '\n';
00055
00056
00057
        std::cout « line_separator_with_description() « '\n';
00058 }
00059
00060
00061 // Simple profiler for a big task, taking many cycles
00062 // Note that the function call incurs some time cost, so this is not totally accurate
00063 template<typename Callable>
00064 inline void profile_function(long long int repeat, Callable &&c) {
        auto time_start = std::chrono::system_clock::now();
for(long long int i = 0; i < repeat; ++i) {</pre>
00065
00066
00067
          c();
00068
00069
        std::cout « line_separator_with_description("Profiling a callable") « '\n';
00070
        auto time_end = std::chrono::system_clock::now();
00071
        std::chrono::duration<double> time_diff = time_end - time_start;
        std::cout « std::fixed « std::setprecision(9) « std::left;
std::cout « std::setw(9) « "total time spent = " « time_diff.count() « " s" « '\n';
std::cout « std::setw(9) « "time spent per iteration = " « time_diff.count() / repeat « " s" « '\n';
00072
00073
00074
00075
        std::cout « line_separator_with_description() « '\n';
00076 }
00077
00078
00079
00080 #endif
```

9.34 src/wkb.hpp File Reference

Implementation of the WKB solution.

```
#include "Eigen/Dense"
#include "workspace.hpp"
```

Classes

struct WKBSolutionForKleinGordonEquationInFRW

9.34.1 Detailed Description

Implementation of the WKB solution.

Used to extend an existing field profile to a later time.

9.35 wkb.hpp 73

9.35 wkb.hpp

Go to the documentation of this file.

```
00001
00007 #ifndef WKB HPP
00008 #define WKB_HPP
00009
00010 #include "Eigen/Dense"
00011 #include "workspace.hpp"
00012
00013 struct WKBSolutionForKleinGordonEquationInFRW {
00014
00015
       typedef Eigen::VectorXd Vector;
       typedef Vector State;
00017
       typedef WorkspaceGeneric<State> Workspace;
00018
00019
       Workspace &workspace;
00020
       double t_i;
       Vector phi_ffts;
00021
00022
00023
       WKBSolutionForKleinGordonEquationInFRW(Workspace &workspace_, const double t_i_);
00024
00025
       Vector evaluate_at(const double t);
00026
00027 };
00028
00029 #endif
```

9.36 src/workspace.hpp File Reference

A generic "workspace" class, containing parameters / data / tools used during simulations.

```
#include <memory>
#include "param.hpp"
#include "physics.hpp"
#include "fftw_wrapper.hpp"
#include "dispatcher.hpp"
```

Classes

struct WorkspaceGeneric< Vector >

A generic workspace for storing temporary.

Concepts

- concept HasLatticeParams
- concept HasMass
- · concept HasLambda
- concept HasFa
- concept HasFRWParameters
- concept HasPsiApproximationParameters

Macros

#define TYPE_REQUIREMENT(value, type) {std::remove_cvref_t<decltype((value))>()} -> std::same_as<type>;

9.36.1 Detailed Description

A generic "workspace" class, containing parameters / data / tools used during simulations.

Author

Siyang Ling

9.37 workspace.hpp

Go to the documentation of this file.

```
00001
00006 #ifndef WORKSPACE_HPP
00007 #define WORKSPACE_HPP
80000
00009 #include <memory>
00010
00011 #include "param.hpp"
00012 #include "physics.hpp"
00013 #include "fftw_wrapper.hpp"
00014 #include "dispatcher.hpp
00015
00016 #define TYPE_REQUIREMENT(value, type) {std::remove_cvref_t<decltype((value))>()} ->
     std::same_as<type>;
00017
00018
00019
00020 template<typename Param>
00021 concept HasLatticeParams = requires (Param param)
00022 { TYPE_REQUIREMENT(param.N, long long int)
00023
          TYPE REQUIREMENT (param.L, double) };
00024
00025 template<typename Param>
00026 concept HasMass = requires (Param param) { TYPE_REQUIREMENT(param.m, double) };
00027
00028 template<typename Param>
00029 concept HasLambda = requires (Param param) { TYPE_REQUIREMENT(param.lambda, double) };
00030
00031 template<typename Param>
00032 concept HasFa = requires (Param param) { TYPE_REQUIREMENT(param.f_a, double) };
00033
00034 template<typename Param>
00035 concept HasFRWParameters = requires (Param param)
00036 { TYPE_REQUIREMENT(param.a1, double)
00037 TYPE_REQUIREMENT(param.H1, double)
00038
          TYPE_REQUIREMENT(param.t1, double) };
00039
00040 template<typename Param>
00041 concept HasPsiApproximationParameters = requires (Param param)
00042
       { TYPE_REQUIREMENT(param.M, long long int) };
00043
00044
00053 template<typename Vector>
00054 struct WorkspaceGeneric {
00055
       typedef Vector State;
00056
        long long int N;
       double L;
00057
00058
        double m;
00059
        StaticEOSCosmology cosmology{};
00060
        State state;
00061
        double lambda{0};
00062
        double f_a{1.0};
00063
        Vector Psi;
        Vector dPsidt;
00064
00065
        Vector Psi_fft;
        Vector dPsidt_fft;
00066
00067
        Vector R fft:
00068
        std::vector<double> t list;
        typename fftWrapperDispatcher<Vector>::Generic fft_wrapper;
00069
00070
00071
        bool Psi_approximation_initialized{false};
00072
00073
        std::unique_ptr<typename fftWrapperDispatcher<Vector>::Generic> fft_wrapper_M_ptr;
00074
        Vector cutoff_R_fft;
00075
00076
        template<HasLatticeParams Param>
       WorkspaceGeneric(const Param &param, auto &initializer) :
```

9.37 workspace.hpp 75

```
00078
       N(param.N), L(param.L), fft_wrapper(param.N)
00079
       00080
00081
00082
00083
00085
00086
       assert(N >= M); }
00087
00088 }
       initializer(param, *this);
00089 };
00090
00091
00092
00093
00094 #endif
```

Index

```
а1
                                                     generate_inhomogeneous_gaussian_random_field
    SampleParam, 31
                                                          random field.hpp, 68
broken_power_law_given_amplitude_3d
                                                     H1
    random field.hpp, 68
                                                          SampleParam, 31
                                                     HasFa, 15
ComovingCurvatureEquationInFRW, 17
                                                     HasFRWParameters, 15
    compute_energy_density, 18
                                                     HasLambda, 15
compute cutoff fouriers
                                                     HasLatticeParams, 15
    fdm3d.hpp, 44
                                                     HasMass, 16
compute_energy_density
                                                     HasPsiApproximationParameters, 16
    ComovingCurvatureEquationInFRW, 18
                                                     homogeneous field
    KleinGordonEquation, 27
                                                          initializer.hpp, 52
    KleinGordonEquationInFRW, 29
                                                     homogeneous field with fluctuations
compute_field_with_scaled_fourier_modes
                                                          initializer.hpp, 52
    fdm3d.hpp, 44
compute inverse_laplacian
                                                     initializer.hpp
    fdm3d.hpp, 45
                                                          homogeneous field, 52
compute_mode_power_spectrum
                                                          homogeneous field with fluctuations, 52
    fdm3d.hpp, 45
                                                          perturbed grf, 52
compute_power_spectrum
                                                          perturbed_grf_and_comoving_curvature_fft, 53
    fdm3d.hpp, 47
                                                          perturbed_grf_without_saving_Psi, 53
ConstIntervalObserver< Equation, save_field_spectrum,
                                                          plane wave, 53
         save density spectrum, save density >, 18
                                                          unperturbed grf, 54
CudaApproximateComovingCurvatureEquationInFRW,
                                                          unperturbed grf and fixed curvature, 54
                                                          unperturbed_grf_with_background, 54
         19
CudaComovingCurvatureEquationInFRW, 20
                                                          unperturbed_grf_with_Psi, 55
CudaFixedCurvatureEquationInFRW, 20
                                                          wave packet, 55
CudaKleinGordonEquationInFRW, 21
                                                     k ast
CudaLambdaEquationInFRW, 21
                                                          SampleParam, 31
CudaSqrtPotentialEquationInFRW, 22
                                                     KGParam, 27
cufftWrapper, 23
                                                     KleinGordonEquation, 27
cufftWrapperBatchedD2Z, 23
cufftWrapperD2Z, 24
                                                          compute_energy_density, 27
                                                          operator(), 28
cufftWrapperNoBatching, 24
                                                     KleinGordonEquationInFRW, 28
empty, 25
                                                          compute energy density, 29
fdm3d.hpp
                                                     L
    compute cutoff fouriers, 44
                                                          SampleParam, 31
    compute field with scaled fourier modes, 44
                                                     lambda
                                                          SampleParam, 31
    compute inverse laplacian, 45
                                                     LatticeEquationConcept, 16
    compute_mode_power_spectrum, 45
    compute_power_spectrum, 47
                                                     lite-cosmic-sim, 1
    PADDED IDX OF, 44
                                                     m
fftWrapperDispatcher< Eigen::VectorXd >, 25
                                                          SampleParam, 32
fftWrapperDispatcher< thrust::device_vector< double
                                                     midpoint < State, Value, Deriv, Time, Algebra, Opera-
         >>, 26
                                                              tions, Resizer >, 29
fftWrapperDispatcher < Vector >, 25
                                                     MyParam, 30
fftwWrapper, 26
```

78 INDEX

N	t1			
SampleParam, 32	SampleParam, 32			
Campor aram, 02	oumpier drain, oz			
operator()	unperturbed_grf			
KleinGordonEquation, 28	initializer.hpp, 54			
	unperturbed_grf_and_fixed_curvature			
PADDED_IDX_OF	initializer.hpp, 54			
fdm3d.hpp, 44	unperturbed_grf_with_background			
perturbed_grf	initializer.hpp, 54			
initializer.hpp, 52	unperturbed_grf_with_Psi			
perturbed_grf_and_comoving_curvature_fft	initializer.hpp, 55			
initializer.hpp, 53	- FF)			
perturbed_grf_without_saving_Psi	varphi_std_dev			
initializer.hpp, 53	SampleParam, 32			
plane_wave	•			
initializer.hpp, 53	wave_packet			
power_law_with_cutoff_given_amplitude_3d	initializer.hpp, 55			
random_field.hpp, 69	WKBSolutionForKleinGordonEquationInFRW, 33			
_ 117	WorkspaceGeneric< Vector >, 33			
random_field.hpp	Writing your own equation, 5			
broken_power_law_given_amplitude_3d, 68				
generate_inhomogeneous_gaussian_random_field,				
68				
power_law_with_cutoff_given_amplitude_3d, 69				
scale_invariant_spectrum_3d, 69				
SampleParam, 31				
a1, 31				
H1, 31				
k_ast, 31				
L, 31				
lambda, 31				
m, <mark>32</mark>				
N, 32				
t1, 32				
varphi_std_dev, 32				
scale_invariant_spectrum_3d				
random field.hpp, 69				
src/cuda_wrapper.cuh, 35, 36				
src/dispatcher.hpp, 37, 38				
src/eigen_wrapper.hpp, 39				
src/equations.hpp, 39, 40				
src/equations_cuda.cuh, 41, 42				
src/fdm3d.hpp, 43, 48				
src/fdm3d_cuda.cuh, 48, 49				
src/fftw_wrapper.hpp, 50				
src/field_booster.hpp, 51				
src/initializer.hpp, 51, 56				
src/io.hpp, 59				
src/midpoint.hpp, 60				
src/observer.hpp, 61, 62				
src/param.hpp, 63, 64				
src/physics.hpp, 66				
src/random_field.hpp, 67, 70				
src/special_function.hpp, 70				
src/utility.hpp, 71				
src/wkb.hpp, 72, 73				
src/workspace.hpp, 73, 74				
StaticEOSCosmology, 32				