lite-cosmic-sim

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## lite-cosmic-sim

**lite-cosmic-sim** is a lightweight and modular framework for performing field simulations in cosmology. This framework was used for studying free-streaming of wave dark matter; see <a href="marxiv:xxxx.xxxx">arxiv:xxxx.xxxx</a> for the study and these <a href="marxiv:youtube">youtube videos</a> for visualization. The codebase contains several field equations on both CPU and GPU (CUDA), offering choices for numerical methods and simulation outputs.

#### 1.1 Overview

This codebase aims to be:

- 1. As fast as possible. Users should be able to write code that exhausts hardward potential within this framework.
- 2. Easily modifiable and extensible. Users should be able to focus on physics-relevant code, such as that for setting initial conditions or the field equation.

To achieve these goals, the framework is written in a modular structure. This allows users to easily switch between different initial conditions, field equations, output methods, and even between using CPUs or GPUs for computation. Users have to and only have to provide the low level implementation for the physics-relevant code. This means users have full control over optimization of core routines, and they are not limited to a specific set of provided features. This flexibility makes it easy for the user to test new ideas, which is useful in research.

## 1.2 Sample usage

The following code initializes a homogeneous Klein Gordon field with (initially) unit field strength and zero time derivative. Then the field is evolved from t=0 to t=10. Field and density spectra are saved to disk per unit time.

```
##
#include "param.hpp"
#include "initializer.hpp"
#include "equations.hpp"
#include "observer.hpp"

struct MyParam {
  long long int N = 256; // Number of lattice sites (per axis)
  double L = 256.0; // Box size
  double m = 1.0; // Field mass
  double f = 1.0; // The initial (homogeneous) field value
  double dt_f = 0.0; // The initial (homogeneous) field time derivative value
  double t_start = 0.0; // Start time of numerical integration
  double t_end = 10.0; // End time of numerical integration
  double t_interval = 1.0; // Interval between saving outputs
```

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```
int main() {
    using namespace Eigen;
    using namespace boost::numeric::odeint;

    typedef KleinGordonEquation Equation;
    typedef Eigen::VectorXd State;
    typedef WorkspaceGeneric<State> Workspace;

MyParam param;

Workspace workspace(param, homogeneous_field);

Equation eqn(workspace);

ConstIntervalObserver<Equation> observer("output/sample_equation/", param, eqn);
    auto stepper = runge_kutta4_classic<State>();
    integrate_const(stepper, eqn, workspace.state, param.t_start, param.t_end, 0.1, observer);
}
```

Here's a break down of the code:

- MyParam is a POD struct specifying parameters for the simulation. You may define your own struct to include new parameters (coupling strength, FRW universe parameters, time step size, etc), as long as it is a POD and contains lattice parameters N and L.
- Workspace is a type containing temporary variables for a simulation (e.g. the field). It is initialized with param and a callback homogeneous\_field, which sets the field to homogeneous value param.f and time derivative param.dt\_f. You can easily define your own callbacks (using lambdas) to set other sorts of initial conditions.
- Equation is the equation to be solved. Here it is the pre-defined KleinGordonEquation. You can of course write your own equations.
- ConstIntervalObserver<Equation> specifies how to save outputs during simulation. By default it saves spectra for field and density.
- stepper is the RK4 method provided by the boost odeint library. You can choose other methods (e.g. Euler, DOPRI5) in the library, or even write your own. The odeint library is responsible for the main numerical integration loop in this codebase.
- integrate\_const is a convenience function in the odeint library. It runs the simulation and saves results to "output/sample\_equation", as specified by observer.

## 1.3 How to get and build the project

Compiler requirement: a C++ compiler supporting C++20. (I used g++12.2.0.)

Required dependency: fftw3

Optional dependency: CUDA Toolkit

I also included header-only libraries Eigen 3.4.0 and boost 1.84 along with the codebase in the external directory.

Makefile is used for build system. I have tested compilation on Linux and MacOS systems. To compile the project:

• Download the project with (for example) git clone https://github.com/hypermania/lite-cosmic-sim.

- (If default settings don't work:) Modify the Makefile so that it knows where your fftw or CUDA include files / library files are.
- If you have CUDA Toolkit installed, simply run make -j.
- If you don't have CUDA Toolkit, run make -j disable-cuda=true. (I use compiler flags to comment out CUDA-dependent code. e.g. CudaComovingCurvatureEquationInFRW)

Note: If you have a CUDA compatible NVIDIA GPU, using CUDA is highly recommended. In our case, it produced more than 10 times speedup.

#### 1.4 Documentation

LaTeX version of documentation is in documentation.pdf. If you have doxygen, you can also build an html version by running doxygen doxygen.config.

### 1.5 Convenience utilities for visualizing output

Two Mathematica notebooks <code>spectra.nb</code>, <code>snapshots.nb</code> and a python <code>scriptplot\_util.py</code> are included for visualizing outputs from the program. By default, running the entire notebook / python script will read sample data from <code>output/Growth\_and\_FS</code> and produce spectra and snapshots. If you generate new outputs from the program, you just need to change <code>dir</code> or <code>project dir</code> variables to the new output directory.

## 1.6 Overview of implemented functionalities

Symbol	Description
generate_inhomogeneous_gaussian_↔	Function for initializing Gaussian random fields with
random_field	spatially inhomogeneous variances. This procedure is
	crucial for generating the initial conditions used in the
	paper.
KleinGordonEquationInFRW and	Klein Gordon equation that runs on CPU and GPU.
CudaKleinGordonEquationInFRW	Used in section 4.2.1 of paper.
ComovingCurvatureEquationInFRW,	A scalar field in the presence of external gravity that
CudaComovingCurvatureEquationInFRW	is consistent with some set of comoving curvature per-
and CudaApproximateComovingCurvatureEq	uturbations.FUsed in section 4.2.2 of paper.
CudaSqrtPotentialEquationInFRW	A scalar field with monodromy potential. Used in sec-
	tion 4.2.3 of paper.
CudaFixedCurvatureEquationInFRW	A scalar field in a fixed gravitational potential.
CudaLambdaEquationInFRW	A scalar field with lambda phi^4 interaction.

## 1.7 Notes on using CUDA

We do separate compilation of .cpp files and .cu files; .cu files are automatically compiled by nvcc, whereas .cpp files are compiled by the host compiler. We use the thrust library (included with CUDA Toolkit) extensively, with field state vectors having type thrust::device\_vector<double>. Initialization procedures usually prepare some profile on the CPU and then copy it to  $device\_vector<double>$  state in the workspace.

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A straightforward way to use CUDA for a simulation is to implement an Equation class with  $thrust \leftarrow :: device\_vector < double>$  as state vector. You will probably need to write your own CUDA kernels for that purpose. See equations\_cuda.cu for some examples. Don't worry about adapting CUDA with the numerical integrators (e.g. RK4); the files in  $src/odeint\_thrust$  will take care of that automatically.

## Implementing your own equation

Here we give an example of adding a field equation with  $\kappa \varphi^6$  interaction to the codebase.

$$\ddot{\varphi} - \nabla^2 \varphi + m^2 \varphi + \kappa \varphi^5 = 0$$

### 2.1 Adding the equation class

We use the boost odeint library for numerical integration. To use the library, we need to implement a new equation class. See this link for an example of odeint equation class. In our case, the equation class with  $\varphi^6$  looks like:

```
struct KappaEquation {
  typedef Eigen::VectorXd Vector;
  typedef Vector State;
  typedef WorkspaceGeneric<State> Workspace;
  Workspace &workspace;

KappaEquation(Workspace &workspace_) : workspace(workspace_) {}
  void operator() (const State &dxdt, State &x, const double t);
};
```

In the first few lines, the types <code>Vector</code>, <code>State</code> and <code>Workspace</code> are defined. These definitions specify what state vector the equation is going to work with: if you want to use different state vector types (e.g. GPU device vector), you will need to define different equation classes. Here we use <code>Eigen::VectorXd</code>. The equation class also has a reference to a <code>workspace</code>, so that it has access to essential information for evolution (e.g. mass and coupling parameters).

The most important function here is the operator (). When this function is called, it computes the time derivative of the state vector  $\mathbf x$  at time  $\mathbf t$ , and stores it to  $\mathtt{dxdt}$ . Implementing this function is the minimal requirement for a class to work with the odeint library. To do this, we can simply copy the implementation for  $\mathtt{KleinGordon} \leftarrow \mathtt{Equation}$ : operator () and add a  $\kappa \varphi^5$  term to it:

```
void KappaEquation::operator() (const State &x, State &dxdt, const double t)
{
  using namespace Eigen;
  const long long int N = workspace.N;
  const double L = workspace.L;
  const double m = workspace.m;
  const double kappa = workspace.kappa;
  const double inv_h_sqr = 1.0 / ((L / N) * (L / N));

for(long long int a = 0; a < N; ++a) {
    for(long long int b = 0; b < N; ++b) {
        dxdt(seqN(IDX_OF(N, a, b, 0), N)) = x(seqN(N*N*N+IDX_OF(N, a, b, 0), N));
        dxdt(seqN(N*N*N+IDX_OF(N, a, b, 0), N)) =
        (-1.0) * m * m * x(seqN(IDX_OF(N, a, b, 0), N)), 5)</pre>
```

Note the extra line kappa \* pow (x (seqN (IDX\_OF (N, a, b, 0), N)), 5) giving the  $\kappa \varphi^5$  term in the equation.

### 2.2 Adding the coupling parameter in workspace

The code given above won't compile yet since workspace.kappa doesn't exist. To make the code compile, add a new field in WorkspaceGeneric:

```
template<typename Vector>
struct WorkspaceGeneric {
    // Stuff
    double kappa;
    // Stuff
};
```

As a general paradigm, we put data (e.g. coupling parameters, temporary variables) needed to solve the equation in a <code>Workspace</code>. Note that different equations use the same <code>Workspace</code>, and the field names (e.g. <code>kappa</code>) can mean different things for different equations. You are responsible for ensuring that your modification on <code>Workspace</code> doesn't introduce bugs for other equations. To avoid accidently introducing a bug, you are advised to add new fields for new parameters / temporary objects.

## 2.3 Setting workspace. kappa from a parameter struct

Now suppose you define a new parameter class:

```
struct KappaParam {
    // The usual
    double kappa;
};
KappaParam param;
```

If you try calling the constructor Workspace(param, initializer), workspace.kappa would not be automatically set to param.kappa. To resolve this, add the following in workspace.hpp:

```
template<typename Param>
concept HasKappa = requires (Param param) { TYPE_REQUIREMENT(param.kappa, double) };

// ...

WorkspaceGeneric(const Param &param, auto &initializer) :
   N(param.N), L(param.L), fft_wrapper(param.N)

{
   if constexpr(HasKappa<Param>) { kappa = param.kappa; }
   // ...
}
```

This piece of code uses concept <code>HasKappa</code> to detect if <code>param.kappa</code> exists or not, and set <code>workspace.⇔kappa</code> to <code>param.kappa</code> in the case it exists. Having <code>KappaParam</code> is useful since it works with the utilities in <code>param.h</code>.

### 2.4 Add a function to compute energy density

In order to save density spectrum, you would also want to implement a function to calculate energy density profile. Again we can imitate the implementation for KleinGordonEquation:

```
struct KappaEquation {
    static Vector compute_energy_density(const Workspace &workspace, const double t);
KappaEquation::Vector KappaEquation::compute_energy_density(const Workspace &workspace, const double t)
  using namespace Eigen;
 const long long int N = workspace.N;
const double L = workspace.L;
 const double m = workspace.m;
 const double kappa = workspace.kappa;
  const double inv_h_sqr = 1.0 / ((L / N) * (L / N));
 VectorXd rho(workspace.state.size() / 2);
  for (long long int a = 0; a < N; ++a) {
    for (long long int b = 0; b < N; ++b) {
      rho(seqN(IDX_OF(N, a, b, 0), N)) = 0.5 *
    ( workspace.state(seqN(N*N*N+IDX_OF(N, a, b, 0), N)).cwiseAbs2()
      + m * m * workspace.state(seqN(IDX\_OF(N, a, b, 0), N)).cwiseAbs2()
      + (1.0 / 6.0) * kappa * pow(workspace.state(seqN(IDX_OF(N, a, b, 0), N)), 6)
      + 0.25 * inv h sgr *
      ( (workspace.state(seqN(IDX_OF(N, (a+1)%N, b, 0), N))
           workspace.state(seqN(IDX_OF(N, (a+N-1)%N, b, 0), N))).cwiseAbs2()
        + (workspace.state(seqN(IDX_OF(N, a, (b+1)%N, 0), N))
             workspace.state(seqN(IDX_OF(N, a, (b+N-1)%N, 0), N))).cwiseAbs2())
      rho(seqN(IDX_OF(N, a, b, 1), N-2)) += 0.5 * 0.25 * inv_h_sqr *
    (workspace.state(seqN(IDX_OF(N, a, b, 2), N-2))
     - workspace.state(seqN(IDX_OF(N, a, b, 0), N-2))).cwiseAbs2();
      rho(IDX_OF(N, a, b, 0)) += 0.5 * 0.25 * inv_h_sqr
    pow(workspace.state(IDX_OF(N, a, b, 1)) - workspace.state(IDX_OF(N, a, b, N-1)), 2);
    rho(IDX_OF(N, a, b, N-1)) += 0.5 * 0.25 * inv_h_sqr *
    pow(workspace.state(IDX\_OF(N, a, b, 0)) - workspace.state(IDX\_OF(N, a, b, N-2)), 2);
  return rho;
```

Note the extra  $\kappa \varphi^6/6$  term in the function above. Now ConstIntervalObserver knows how to compute the energy density for this theory.

## 2.5 Using CUDA

If you want your equation to run on GPU memory, then in KappaEquation the vector type should be set to: typedef thrust::device\_vector<double> Vector;

Here, thrust::device\_vector<double> is a class in the thrust library representing a double floating point array on the GPU. Much like std::vector<double>, the class thrust::device\_ $\hookleftarrow$  vector<double> takes care of GPU memory allocation / deallocation in an RAII manner, so that you don't have to call CUDA memory management API directly. See thrust documentation for more details.

Your operator() and compute\_energy\_density functions must now work on GPU device vectors. A straightforward way to do this is to write your own CUDA kernel. Here's an example on how to do it:

```
- \text{ kappa} \star \text{ x[IDX\_OF(N, a, b, c)]} \star \text{ x[IDX\_OF(N, a, b, c)]} \star \text{ x[IDX\_OF(N, a, b, c)]} \star \text{ x[IDX\_OF(N, a, b, c)]}
    * x[IDX_OF(N, a, b, c)]
+ inv_h_sqr * (-6.0 * x[IDX_OF(N, a, b, c)]
              + x[IDX_OF(N, (a+1)%N, b, c)]
              + x[IDX_OF(N, (a+N-1)%N, b, c)]
              + x[IDX_OF(N, a, (b+1)%N, c)]
+ x[IDX_OF(N, a, (b+N-1)%N, c)]
              + x[IDX_OF(N, a, b, (c+1)%N)]
              + x[IDX_OF(N, a, b, (c+N-1)%N)]);
void KappaEquation::operator()(const State &x, State &dxdt, const double t)
  const long long int N = workspace.N;
  const double L = workspace.L;
const double m = workspace.m;
  const double kappa = workspace.kappa;
  const double inv_h_sqr = 1.0 / ((L / N) * (L / N));
  dim3 threadsPerBlock((int)N, 1);
  dim3 numBlocks((int)N, (int)N);
  kappa_equation_kernel«<numBlocks, threadsPerBlock>»(thrust::raw_pointer_cast(x.data())),
      thrust::raw_pointer_cast(dxdt.data()), m, kappa, inv_h_sqr, N);
```

Here kappa\_equation\_kernel is the CUDA kernel, and the \_\_global\_\_ specifier means this function runs on the GPU. KappaEquation::operator() invokes the kernel via kappa\_equation\_ $\leftarrow$  kernel<<<numBlocks, threadsPerBlock>>>. Given the execution configuration threadsPer $\leftarrow$  Block and numBlocks, the function kappa\_equation\_kernel is executed once for each a,b,c, with 0 <= a,b,c < N. Depending on the kernel, different execution configurations could result in varying performance, or even introduce bugs. See the CUDA C programming guide for more details.

# **Concept Index**

## 3.1 Concepts

Here is a list of all documented concepts with brief descriptions:

HasFa	
HasFRWParameters	
HasLambda	
HasLatticeParams	
HasMass	
HasPsiApproximationParameters	
LatticeEquationConcept	

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# **Hierarchical Index**

## 4.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:

boost::numeric::odeint::algebra_stepper_base
midpoint < State, Value, Deriv, Time, Algebra, Operations, Resizer >
ComovingCurvatureEquationInFRW
$ConstIntervalObserver < Equation, save\_field\_spectrum, save\_density\_spectrum, save\_density > . \ . \ . \ . \ 200 cms (a) and a save\_density = 0.00 cms (b) and (c) are constituted as a save\_density = 0.00 cms (c) and (c) are constituted as a save\_density = 0.00 cms (c) are constit$
CudaApproximateComovingCurvatureEquationInFRW
CudaComovingCurvatureEquationInFRW
CudaFixedCurvatureEquationInFRW
CudaKleinGordonEquationInFRW
CudaLambdaEquationInFRW
CudaSqrtPotentialEquationInFRW
cufftWrapper
cufftWrapperBatchedD2Z
cufftWrapperD2Z
cufftWrapperNoBatching
empty
fftWrapperDispatcher < Vector >
fftWrapperDispatcher< Eigen::VectorXd >
fftWrapperDispatcher< thrust::device_vector< double >>
fftwWrapper
KGParam
KleinGordonEquation
KleinGordonEquationInFRW
MyParam
SampleParam
StaticEOSCosmology
WKBSolutionForKleinGordonEquationInFRW
WorkspaceGeneric < Vector >

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## **Class Index**

### 5.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

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## 6.1 File List

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## **Concept Documentation**

### 7.1 HasFa Concept Reference

#### 7.1.1 Concept definition

```
template<typename Param>
concept HasFa = requires (Param param) { TYPE_REQUIREMENT(param.f_a, double) }
```

## 7.2 HasFRWParameters Concept Reference

#### 7.2.1 Concept definition

## 7.3 HasLambda Concept Reference

#### 7.3.1 Concept definition

```
template<typename Param>
concept HasLambda = requires (Param param) { TYPE_REQUIREMENT(param.lambda, double) }
```

## 7.4 HasLatticeParams Concept Reference

#### 7.4.1 Concept definition

```
template<typename Param>
concept HasLatticeParams = requires (Param param)
{ TYPE_REQUIREMENT(param.N, long long int)
    TYPE_REQUIREMENT(param.L, double) }
```

## 7.5 HasMass Concept Reference

#### 7.5.1 Concept definition

```
template<typename Param>
concept HasMass = requires (Param param) { TYPE_REQUIREMENT(param.m, double) }
```

## 7.6 HasPsiApproximationParameters Concept Reference

#### 7.6.1 Concept definition

```
template<typename Param>
concept HasPsiApproximationParameters = requires (Param param)
{ TYPE_REQUIREMENT(param.M, long long int) }
```

## 7.7 LatticeEquationConcept Concept Reference

#### 7.7.1 Concept definition

```
template<typename Equation>
concept LatticeEquationConcept = requires (Equation eqn)
{
    eqn.workspace;
    eqn.compute_energy_density(eqn.workspace, 0.0);
}
```

## **Class Documentation**

## 8.1 ComovingCurvatureEquationInFRW Struct Reference

Equation for free scalar field in FRW spacetime, including comoving metric perturbations (in radiation domination).

```
#include <equations.hpp>
```

#### **Public Types**

- typedef Eigen::VectorXd Vector
- · typedef Vector State
- typedef WorkspaceGeneric < State > Workspace

#### **Public Member Functions**

- ComovingCurvatureEquationInFRW (Workspace &workspace\_)
- void operator() (const State &, State &, const double)

#### **Static Public Member Functions**

• static Vector compute\_energy\_density (Workspace &workspace, const double t)

Compute the energy density profile from the workspace.

#### **Public Attributes**

• Workspace & workspace

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#### 8.1.1 Detailed Description

Equation for free scalar field in FRW spacetime, including comoving metric perturbations (in radiation domination).

Equation is given by

$$\begin{split} \ddot{\varphi} + 3H\dot{\varphi} - e^{4\Psi} \frac{\nabla^2}{a^2} \varphi + e^{2\Psi} m^2 \varphi - 4\dot{\Psi}\dot{\varphi} &= 0 \\ \Psi_{\mathbf{k}}(t) = 2\mathcal{R}_{\mathbf{k}} \frac{\sin\left(k\eta/\sqrt{3}\right) - \left(k\eta/\sqrt{3}\right)\cos\left(k\eta/\sqrt{3}\right)}{\left(k\eta/\sqrt{3}\right)^3} \\ \dot{\Psi}_{\mathbf{k}}(t) = 2\mathcal{R}_{\mathbf{k}} H(t) \frac{3(k\eta/\sqrt{3})\cos\left(k\eta/\sqrt{3}\right) + \left((k\eta/\sqrt{3})^2 - 3\right)\sin\left(k\eta/\sqrt{3}\right)}{(k\eta/\sqrt{3})^3} \\ \eta(t) &= \frac{(2H_i t)^{1/2}}{a_i H_i} \quad \text{is the conformal time} \end{split}$$

where  $\mathcal{R}_{\mathbf{k}}$  is read from workspace variable R\_fft. See equation (6.160) of Baumann's cosmology text-book. This implementation is not optimized. It was only used for verifying the GPU implementations CudaComovingCurvatureEquationInFRW and CudaApproximateComovingCurvatureEquationInFRW.

#### 8.1.2 Member Function Documentation

#### 8.1.2.1 compute\_energy\_density()

Compute the energy density profile from the workspace.

#### **Parameters**

in	workspace	The workspace for evaluating the energy density.
	t	The current time parameter.

#### Returns

A vector of size  $N^3$ , giving the energy density profile  $\rho=\frac{1}{2}(e^{-2\Psi}\dot{\varphi}^2+e^{2\Psi}(\nabla\varphi)^2/a(t)^2+m^2\varphi^2)$  on the lattice.

The documentation for this struct was generated from the following files:

- src/equations.hpp
- · src/equations.cpp

# 8.2 ConstintervalObserver< Equation, save\_field\_spectrum, save density spectrum, save density > Struct Template Reference

#### **Public Types**

- typedef Equation::Workspace Workspace
- typedef Workspace::State State
- typedef State Vector

#### **Public Member Functions**

- template<typename Param >
  - ConstIntervalObserver (const std::string &dir\_, const Param &param, Equation &eqn)
- ConstintervalObserver (const ConstintervalObserver &)=default
- void operator() (const State &x, double t)

#### **Public Attributes**

- Workspace & workspace
- int idx
- std::string dir
- double t\_start
- double t end
- · double t\_interval
- double t\_last

The documentation for this struct was generated from the following file:

src/observer.hpp

# 8.3 CudaApproximateComovingCurvatureEquationInFRW Struct Reference

#### **Public Types**

- typedef thrust::device\_vector< double > Vector
- typedef Vector State
- typedef WorkspaceGeneric < Vector > Workspace

#### **Public Member Functions**

- CudaApproximateComovingCurvatureEquationInFRW (Workspace &workspace\_)
- void operator() (const State &, State &, const double)

#### **Static Public Member Functions**

• static Vector compute\_energy\_density (Workspace &workspace, const double t)

#### **Public Attributes**

• Workspace & workspace

The documentation for this struct was generated from the following files:

- src/equations\_cuda.cuh
- src/equations\_cuda.cu

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## 8.4 CudaComovingCurvatureEquationInFRW Struct Reference

#### **Public Types**

- typedef thrust::device\_vector< double > Vector
- typedef Vector State
- typedef WorkspaceGeneric< Vector > Workspace

#### **Public Member Functions**

- CudaComovingCurvatureEquationInFRW (Workspace &workspace\_)
- void operator() (const State &, State &, const double)

#### **Static Public Member Functions**

• static Vector compute\_energy\_density (Workspace &workspace, const double t)

#### **Public Attributes**

Workspace & workspace

The documentation for this struct was generated from the following files:

- src/equations\_cuda.cuh
- src/equations\_cuda.cu

## 8.5 CudaFixedCurvatureEquationInFRW Struct Reference

#### **Public Types**

- typedef thrust::device\_vector< double > Vector
- typedef Vector State
- typedef WorkspaceGeneric< Vector > Workspace

#### **Public Member Functions**

- CudaFixedCurvatureEquationInFRW (Workspace &workspace\_)
- void operator() (const State &, State &, const double)

#### **Static Public Member Functions**

• static Vector compute\_energy\_density (const Workspace &workspace, const double t)

#### **Public Attributes**

• Workspace & workspace

The documentation for this struct was generated from the following files:

- · src/equations\_cuda.cuh
- src/equations\_cuda.cu

## 8.6 CudaKleinGordonEquationInFRW Struct Reference

#### **Public Types**

- typedef thrust::device\_vector< double > Vector
- · typedef Vector State
- typedef WorkspaceGeneric < Vector > Workspace

#### **Public Member Functions**

- CudaKleinGordonEquationInFRW (Workspace &workspace\_)
- void operator() (const State &, State &, const double)

#### **Static Public Member Functions**

- static Vector compute\_energy\_density (const Workspace &workspace, const double t)
- static Vector compute\_dot\_energy\_density (const Workspace &workspace, const double t)

#### **Public Attributes**

• Workspace & workspace

The documentation for this struct was generated from the following files:

- src/equations\_cuda.cuh
- src/equations\_cuda.cu

## 8.7 CudaLambdaEquationInFRW Struct Reference

#### **Public Types**

- $\bullet \ \ typedef \ thrust:: device\_vector < \ double > \textbf{Vector}$
- typedef Vector State
- typedef WorkspaceGeneric< Vector > Workspace

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### **Public Member Functions**

- CudaLambdaEquationInFRW (Workspace &workspace\_)
- void operator() (const State &, State &, const double)

#### **Static Public Member Functions**

• static Vector compute\_energy\_density (const Workspace &workspace, const double t)

#### **Public Attributes**

• Workspace & workspace

The documentation for this struct was generated from the following files:

- src/equations\_cuda.cuh
- src/equations cuda.cu

### 8.8 CudaSqrtPotentialEquationInFRW Struct Reference

#### **Public Types**

- typedef thrust::device\_vector< double > Vector
- typedef Vector State
- typedef WorkspaceGeneric< Vector > Workspace

#### **Public Member Functions**

- CudaSqrtPotentialEquationInFRW (Workspace &workspace )
- void operator() (const State &, State &, const double)

#### **Static Public Member Functions**

• static Vector compute\_energy\_density (const Workspace &workspace, const double t)

#### **Public Attributes**

• Workspace & workspace

The documentation for this struct was generated from the following files:

- · src/equations\_cuda.cuh
- src/equations\_cuda.cu

### 8.9 cufftWrapper Struct Reference

Wrapper for various cufft functions for a  $N^3$  grid. Similar to fftwWrapper.

#### **Public Member Functions**

- cufftWrapper (int N\_)
- thrust::device\_vector< double > execute\_d2z (thrust::device\_vector< double > &in)
- thrust::device vector< double > execute batched d2z (thrust::device vector< double > &in)
- thrust::device\_vector< double > execute\_z2d (thrust::device\_vector< double > &in)
- cufftWrapper (const cufftWrapper &)=delete
- cufftWrapper & operator= (const cufftWrapper &)=delete
- cufftWrapper (cufftWrapper &&)=delete
- cufftWrapper & operator= (cufftWrapper &&)=delete

#### **Public Attributes**

- int N
- cufftHandle plan d2z
- cufftHandle plan\_batched\_d2z
- cufftHandle plan\_z2d
- thrust::device\_vector< double > work\_area

#### 8.9.1 Detailed Description

Wrapper for various cufft functions for a  $N^3$  grid. Similar to fftwWrapper.

```
See https://docs.nvidia.com/cuda/cufft/index.html.
```

The documentation for this struct was generated from the following files:

- src/cuda\_wrapper.cuh
- src/cuda\_wrapper.cu

## 8.10 cufftWrapperBatchedD2Z Struct Reference

#### **Public Member Functions**

- cufftWrapperBatchedD2Z (int N )
- thrust::device\_vector< double > execute (thrust::device\_vector< double > &in)
- cufftWrapperBatchedD2Z (const cufftWrapperBatchedD2Z &)=delete
- cufftWrapperBatchedD2Z & operator= (const cufftWrapperBatchedD2Z &)=delete
- cufftWrapperBatchedD2Z (cufftWrapperBatchedD2Z &&)=delete
- cufftWrapperBatchedD2Z & operator= (cufftWrapperBatchedD2Z &&)=delete

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#### **Public Attributes**

- int N
- · cufftHandle plan

The documentation for this struct was generated from the following files:

- · src/cuda wrapper.cuh
- · src/cuda wrapper.cu

### 8.11 cufftWrapperD2Z Struct Reference

#### **Public Member Functions**

- cufftWrapperD2Z (int N\_)
- thrust::device\_vector< double > execute (thrust::device\_vector< double > &in)
- cufftWrapperD2Z (const cufftWrapperD2Z &)=delete
- cufftWrapperD2Z & operator= (const cufftWrapperD2Z &)=delete
- cufftWrapperD2Z (cufftWrapperD2Z &&)=delete
- cufftWrapperD2Z & operator= (cufftWrapperD2Z &&)=delete

#### **Public Attributes**

- int N
- · cufftHandle plan

The documentation for this struct was generated from the following files:

- · src/cuda\_wrapper.cuh
- · src/cuda\_wrapper.cu

## 8.12 cufftWrapperNoBatching Struct Reference

Wrapper for various cufft functions for a  $N^3$  grid. Similar to fftwWrapper.

#### **Public Member Functions**

- cufftWrapperNoBatching (int N )
- thrust::device\_vector< double > execute\_d2z (thrust::device\_vector< double > &in)
- thrust::device\_vector< double > execute\_batched\_d2z (thrust::device\_vector< double > &in)
- thrust::device\_vector< double > execute\_z2d (thrust::device\_vector< double > &in)
- void execute\_inplace\_z2d (thrust::device\_vector< double > &inout)
- cufftWrapperNoBatching (const cufftWrapperNoBatching &)=delete
- cufftWrapperNoBatching & operator= (const cufftWrapperNoBatching &)=delete
- cufftWrapperNoBatching (cufftWrapperNoBatching &&)=delete
- cufftWrapperNoBatching & operator= (cufftWrapperNoBatching &&)=delete

#### **Public Attributes**

- int N
- cufftHandle plan d2z
- cufftHandle plan\_z2d
- thrust::device\_vector< double > work\_area

#### 8.12.1 Detailed Description

Wrapper for various cufft functions for a  $N^3$  grid. Similar to fftwWrapper.

Uses less GPU memory than cufftWrapper. See  $https://docs.nvidia.com/cuda/cufft/index. \leftarrow html$ 

The documentation for this struct was generated from the following files:

- src/cuda\_wrapper.cuh
- · src/cuda\_wrapper.cu

### 8.13 empty Struct Reference

The documentation for this struct was generated from the following file:

· src/dispatcher.hpp

### 8.14 fftWrapperDispatcher < Vector > Struct Template Reference

#### **Public Types**

- typedef empty D2Z
- typedef empty BatchedD2Z
- typedef empty Generic

The documentation for this struct was generated from the following file:

· src/dispatcher.hpp

## 8.15 fftWrapperDispatcher < Eigen::VectorXd > Struct Reference

#### **Public Types**

- typedef empty D2Z
- typedef empty BatchedD2Z
- typedef fftwWrapper Generic

The documentation for this struct was generated from the following file:

src/dispatcher.hpp

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# 8.16 fftWrapperDispatcher< thrust::device\_vector< double >> Struct Reference

#### **Public Types**

- typedef cufftWrapperD2Z D2Z
- typedef cufftWrapperBatchedD2Z BatchedD2Z
- typedef cufftWrapperNoBatching Generic

The documentation for this struct was generated from the following file:

· src/dispatcher.hpp

### 8.17 fftwWrapper Struct Reference

Wrapper for various FFTW functions for a  $N^3$  grid.

```
#include <fftw_wrapper.hpp>
```

#### **Public Member Functions**

- fftwWrapper (int N )
- Eigen::VectorXd execute d2z (Eigen::VectorXd &in)

(Double floating point) Real to complex transform.

Eigen::VectorXd execute\_batched\_d2z (Eigen::VectorXd &in)

(Double floating point) Real to complex transform.

• Eigen::VectorXd execute\_z2d (Eigen::VectorXd &in)

(Double floating point) Complex to real transform.

void execute\_z2d (Eigen::VectorXd &in, Eigen::VectorXd &out)

No-return version of the complex to real transform.

void execute inplace z2d (Eigen::VectorXd &inout)

In-place version of the complex to real transform.

- fftwWrapper (const fftwWrapper &)=delete
- fftwWrapper & operator= (const fftwWrapper &)=delete
- fftwWrapper (fftwWrapper &&)=delete
- fftwWrapper & operator= (fftwWrapper &&)=delete

#### **Public Attributes**

- · int N
- fftw\_plan plan\_d2z
- fftw plan plan z2d
- fftw\_plan plan\_inplace\_z2d

### 8.17.1 Detailed Description

Wrapper for various FFTW functions for a  $N^3$  grid.

### 8.17.2 Constructor & Destructor Documentation

### 8.17.2.1 fftwWrapper()

Constructor for grid size N.

### 8.17.3 Member Function Documentation

### 8.17.3.1 execute\_batched\_d2z()

(Double floating point) Real to complex transform.

### **Parameters**

```
in A real vector of size 2N^3.
```

### Returns

A real vector of size  $4N^2(N/2+1)$  (or a complex vector of size  $2N^2(N/2+1)$ ), containing discrete Fourier transforms of input. The first  $2N^2(N/2+1)$  entries of the output are the DFT of the first  $N^3$  entries of the input, and similar for the rest.

### 8.17.3.2 execute\_d2z()

(Double floating point) Real to complex transform.

### **Parameters**

```
in A real vector of size N^3.
```

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#### Returns

A real vector of size  $2N^2(N/2+1)$  (or a complex vector of size  $N^2(N/2+1)$ ), containing the discrete Fourier transform of input.

#### 8.17.3.3 execute inplace z2d()

In-place version of the complex to real transform.

### **Parameters**

inout

A real vector of size  $2N^2(N/2+1)$  (or a complex vector of size  $N^2(N/2+1)$ ). After the function call the data in <code>inout</code> is changed its inverse DFT. The vector still has size  $2N^2(N/2+1)$ , but only  $N^3$  of the entries are meaningful. The entries are in FFTW padded format.

#### Note

Make sure to access the elements inside with PADDED\_IDX\_OF (instead of IDX\_OF). See <a href="https://www.fftw.org/fftw3\_doc/Multi\_002dDimensional-DFTs-of-Real-Data.html">https://www.fftw.org/fftw3\_doc/Multi\_002dDimensional-DFTs-of-Real-Data.html</a> for details of the padded format.

### 8.17.3.4 execute\_z2d() [1/2]

(Double floating point) Complex to real transform.

#### **Parameters**

```
in A real vector of size 2N^2(N/2+1) (or a complex vector of size N^2(N/2+1)).
```

### Returns

A real vector of size  $N^3$ , containing the inverse discrete Fourier transform of input.

### Note

This function destroys the information in the input in. See FFTW's documentation  $https://www. \leftarrow fftw.org/fftw3_doc/Planner-Flags.html.$ 

### 8.17.3.5 execute\_z2d() [2/2]

No-return version of the complex to real transform.

Note

This version is useful if you want to reuse the same memory location for the output; doing this can reduce unnecessary memory allocation / deallocation, saving lots of time. Like the other version, this function destroys the data in input in.

The documentation for this struct was generated from the following files:

- src/fftw\_wrapper.hpp
- src/fftw\_wrapper.cpp

### 8.18 KGParam Struct Reference

### **Public Attributes**

- long long int N
- double L
- double m

The documentation for this struct was generated from the following file:

· src/field\_booster.cpp

### 8.19 KleinGordonEquation Struct Reference

The Klein Gordon equation,  $\ddot{\varphi} - \nabla^2 \varphi + m^2 \varphi = 0$ .

#include <equations.hpp>

### **Public Types**

- typedef Eigen::VectorXd Vector
- typedef Vector State
- typedef WorkspaceGeneric < State > Workspace

#### **Public Member Functions**

- KleinGordonEquation (Workspace &workspace\_)
- void operator() (const State &, State &, const double)

The function called by odeint library.

### **Static Public Member Functions**

• static Vector compute\_energy\_density (const Workspace &workspace, const double t)

Compute the energy density profile from the workspace.

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### **Public Attributes**

• Workspace & workspace

### 8.19.1 Detailed Description

The Klein Gordon equation,  $\ddot{\varphi} - \nabla^2 \varphi + m^2 \varphi = 0$ .

### 8.19.2 Member Function Documentation

### 8.19.2.1 compute\_energy\_density()

Compute the energy density profile from the workspace.

#### **Parameters**

in	workspace	The workspace for evaluating the energy density.
	t	The current time parameter.

### Returns

A vector of size  $N^3$ , giving the energy density profile  $\rho=\frac{1}{2}(\dot{\varphi}^2+(\nabla\varphi)^2+m^2\varphi^2$  on the lattice.

### 8.19.2.2 operator()()

The function called by odeint library.

### **Parameters**

in	Х	The current state of the system.
out	dxdt	The time derivative, dxdt of the system.
	t	The current time parameter.

The documentation for this struct was generated from the following files:

- · src/equations.hpp
- · src/equations.cpp

### 8.20 KleinGordonEquationInFRW Struct Reference

The Klein Gordon in FRW equation,  $\ddot{\varphi}+3H\dot{\varphi}-\nabla^2\varphi/a^2+m^2\varphi=0.$ 

```
#include <equations.hpp>
```

### **Public Types**

- typedef Eigen::VectorXd Vector
- typedef Vector State
- typedef WorkspaceGeneric < State > Workspace

### **Public Member Functions**

- KleinGordonEquationInFRW (Workspace &workspace\_)
- void operator() (const State &, State &, const double)

### **Static Public Member Functions**

• static Vector compute\_energy\_density (const Workspace &workspace, const double t)

Compute the energy density profile from the workspace.

### **Public Attributes**

• Workspace & workspace

### 8.20.1 Detailed Description

The Klein Gordon in FRW equation,  $\ddot{\varphi}+3H\dot{\varphi}-\nabla^2\varphi/a^2+m^2\varphi=0.$ 

### 8.20.2 Member Function Documentation

### 8.20.2.1 compute\_energy\_density()

```
\label{lem:const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const_const
```

Compute the energy density profile from the workspace.

### Parameters

in	workspace	The workspace for evaluating the energy density.
	t	The current time parameter.

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#### Returns

A vector of size  $N^3$ , giving the energy density profile  $\rho=\frac{1}{2}(\dot{\varphi}^2+(\nabla\varphi)^2/a(t)^2+m^2\varphi^2)$  given on the  $N^3$  on the lattice.

The documentation for this struct was generated from the following files:

- src/equations.hpp
- · src/equations.cpp

# 8.21 midpoint < State, Value, Deriv, Time, Algebra, Operations, Resizer > Class Template Reference

Inheritance diagram for midpoint < State, Value, Deriv, Time, Algebra, Operations, Resizer >:

### **Public Types**

- typedef State state\_type
- typedef State deriv type
- typedef Value value\_type
- typedef Time time\_type
- typedef unsigned short order\_type
- typedef boost::numeric::odeint::stepper\_tag stepper\_category
- typedef boost::numeric::odeint::algebra\_stepper\_base< Algebra, Operations > algebra\_stepper\_base\_←
   type
- typedef algebra\_stepper\_base\_type::algebra\_type algebra\_type
- typedef algebra\_stepper\_base\_type::operations\_type operations\_type

#### **Public Member Functions**

template<class System > void do\_step (System system, State &in, Time t, Time dt)
 bool resize\_impl (const State &x)

### **Static Public Member Functions**

• static order\_type order (void)

The documentation for this class was generated from the following file:

• src/midpoint.hpp

### 8.22 MyParam Struct Reference

### **Public Attributes**

- long long int N
- double L
- double **m**
- · double lambda
- · double f a
- double k\_ast
- double k\_Psi
- double varphi\_std\_dev
- double Psi\_std\_dev
- double a1
- · double H1
- double t1
- double t\_start
- double t\_end
- · double t\_interval
- double delta\_t
- long long int M
- double f
- double delta\_varphi\_std\_dev
- double k\_delta\_varphi

The documentation for this struct was generated from the following file:

• src/main.cpp

### 8.23 SampleParam Struct Reference

A sample parameter type specifying a lambda-phi-4 theory in an FRW background.

```
#include <param.hpp>
```

### **Public Attributes**

- long long int N
- double L
- · double m
- · double lambda
- double k ast
- double varphi\_std\_dev
- double a1
- double H1
- double t1

### 8.23.1 Detailed Description

A sample parameter type specifying a lambda-phi-4 theory in an FRW background.

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### 8.23.2 Member Data Documentation

#### 8.23.2.1 a1

```
double SampleParam::a1
```

the scale factor at time  $t_1$ 

### 8.23.2.2 H1

```
double SampleParam::H1
```

the Hubble parameter at time  $t_1$ 

### 8.23.2.3 k ast

```
double SampleParam::k_ast
```

the wavenumber  $k_{st}$  for the peak of the field power spectrum

### 8.23.2.4 L

```
double SampleParam::L
```

the length of one side of the box (i.e.  $L=10.0\ \mathrm{means}$  the box has volume  $L^3$  )

### 8.23.2.5 lambda

```
double SampleParam::lambda
```

quartic self-interaction of the scalar field (i.e.  $\lambda$  in  $V(\varphi)=\frac{1}{2}m^2\varphi^2+\frac{1}{4}\lambda\varphi^4$  )

### 8.23.2.6 m

```
double SampleParam::m
```

mass m of the scalar field

### 8.23.2.7 N

```
long long int SampleParam::N
```

the number of lattice points on one side of the box (i.e.  $N=256~{\rm means}~256^3$  lattice sites)

#### 8.23.2.8 t1

```
double SampleParam::t1
```

coordinate time parameter  $t_1$  (For radiation domination,  $a(t) = a_1(1 + 2H_1(t - t_1))^{1/2}$ ,  $H(t) = H_1(1 + 2H_1(t - t_1))^{-1}$ .)

### 8.23.2.9 varphi\_std\_dev

```
double SampleParam::varphi_std_dev
```

the expected RMS value  $\langle \varphi^2 \rangle$  for the field, averaged over the box

The documentation for this struct was generated from the following file:

src/param.hpp

### 8.24 StaticEOSCosmology Struct Reference

A convenience class used to calculate FRW related quantities for constant EOS spacetimes.

```
#include <physics.hpp>
```

### **Public Member Functions**

- StaticEOSCosmology (const double a1\_, const double H1\_, const double t1\_, const double p\_)
- template<typename T >

StaticEOSCosmology (const T &param)

This constructor assumes radiation domination.

StaticEOSCosmology (void)

The default constructor gives Minkowski spacetime.

• double a (const double t) const

Returns the scale factor at coordinate time t.

• double **H** (const double t) const

Returns the Hubble parameter at coordinate time t.

• double eta (const double t) const

Returns the conformal time  $\eta$  at coordinate time t.

### **Public Attributes**

- double a1
- double H1
- double t1
- double p

### 8.24.1 Detailed Description

A convenience class used to calculate FRW related quantities for constant EOS spacetimes.

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### 8.24.2 Member Function Documentation

### 8.24.2.1 eta()

Returns the conformal time  $\eta$  at coordinate time t.

Converts coordinate time t to conformal time  $\eta$ . The conversion assumes the convention  $a=a_1(\eta/\eta_1)^p$ , where  $\eta_1=p/(a_1H_1)$ . In this convention,  $\eta_1$  and  $t_1$  are at the same physical time. The conversion formula is

$$\eta = \frac{p}{a_1 H_1} \left( 1 + (1 + 1/p) H_1(t - t_1) \right)^{1/(1+p)}$$

.

### 8.24.3 Member Data Documentation

### 8.24.3.1 a1

```
double StaticEOSCosmology::a1
```

The scale factor at  $t_1$ .

### 8.24.3.2 H1

```
double StaticEOSCosmology::H1
```

The Hubble parameter at  $t_1$ .

### 8.24.3.3 p

```
double StaticEOSCosmology::p
```

Power-law between scale factor and conformal time,  $a \propto \eta^p$ . In terms of EOS w, we have  $p = \frac{2}{1+3w}$ .

#### 8.24.3.4 t1

```
double StaticEOSCosmology::t1
```

A pivot coordinate time  $t_1$ .

The documentation for this struct was generated from the following file:

· src/physics.hpp

### 8.25 WKBSolutionForKleinGordonEquationInFRW Struct Reference

### **Public Types**

- typedef Eigen::VectorXd Vector
- typedef Vector State
- $\bullet \ \ type def \ Work space Generic < State > \textbf{Work space}$

### **Public Member Functions**

- WKBSolutionForKleinGordonEquationInFRW (Workspace &workspace\_, const double t\_i\_)
- Vector evaluate\_at (const double t)

### **Public Attributes**

- Workspace & workspace
- double t i
- Vector phi\_ffts

The documentation for this struct was generated from the following files:

- src/wkb.hpp
- · src/wkb.cpp

### 8.26 WorkspaceGeneric < Vector > Struct Template Reference

A generic workspace for storing temporary.

```
#include <workspace.hpp>
```

### **Public Types**

• typedef Vector State

#### **Public Member Functions**

template<HasLatticeParams Param>
 WorkspaceGeneric (const Param &param, auto &initializer)

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### **Public Attributes**

- long long int N
- double L
- · double m
- StaticEOSCosmology cosmology {}
- State state
- double lambda {0}
- double f\_a {1.0}
- · Vector Psi
- Vector dPsidt
- Vector Psi fft
- Vector dPsidt\_fft
- Vector R\_fft
- $std::vector < \frac{double}{t_list}$
- fftWrapperDispatcher< Vector >::Generic fft\_wrapper
- bool Psi\_approximation\_initialized {false}
- · long long int M
- std::unique\_ptr< typename fftWrapperDispatcher< Vector >::Generic > fft\_wrapper\_M\_ptr
- Vector cutoff\_R\_fft

### 8.26.1 Detailed Description

```
\begin{array}{l} \textbf{template} \!<\! \textbf{typename} \; \textbf{Vector} \!> \\ \textbf{struct} \; \textbf{WorkspaceGeneric} \!<\! \; \textbf{Vector} \;> \\ \end{array}
```

A generic workspace for storing temporary.

WorkspaceGeneric contains everything used during simulations, including the field state, gravitational potential, parameters, etc. It is initialized by a Param struct (containing just a few numbers) and an "initializer" (see initializer.hpp).

The lifetime of objects in the workspace are managed by us (this codebase), instead of external libraries (e.g. odeint).

The documentation for this struct was generated from the following file:

· src/workspace.hpp

# **Chapter 9**

# **File Documentation**

### 9.1 src/cuda\_wrapper.cuh File Reference

### Wrapper for CUDA Toolkit.

```
#include <iostream>
#include <Eigen/Dense>
#include <thrust/device_vector.h>
#include "cufft.h"
#include "cufftXt.h"
#include <cuda_runtime.h>
```

### Classes

- struct cufftWrapperD2Z
- struct cufftWrapperBatchedD2Z
- struct cufftWrapper

Wrapper for various cufft functions for a  $N^3$  grid. Similar to fftwWrapper.

struct cufftWrapperNoBatching

Wrapper for various cufft functions for a  $N^3$  grid. Similar to fftwWrapper.

### **Typedefs**

- typedef decltype(Eigen::VectorXd().begin()) eigen\_iterator
- typedef decltype(thrust::device vector< double >().begin()) thrust iterator
- typedef thrust::detail::normal\_iterator< thrust::device\_ptr< const double >> thrust\_const\_iterator
- typedef Eigen::internal::pointer\_based\_stl\_iterator< Eigen::Matrix< double, -1, 1 >> eigen\_iterator\_2

### **Functions**

- void copy\_vector (Eigen::VectorXd &out, const thrust::device\_vector< double > &in)
- void show\_gpu\_memory\_usage (void)

### 9.1.1 Detailed Description

Wrapper for CUDA Toolkit.

**Author** 

Siyang Ling

### 9.2 cuda wrapper.cuh

### Go to the documentation of this file.

```
00001
00006 #ifndef CUDA_WRAPPER_CUH
00007 #define CUDA_WRAPPER_CUH
00009 #include <iostream>
00010
00011 #include <Eigen/Dense>
00012
00013 #include <thrust/device_vector.h>
00014 // #include <thrust/host_vector.h>
00015 // #include <thrust/execution_policy.h>
00016 // #include <thrust/reduce.h>
00017 // #include <thrust/functional.h>
00018 // #include <thrust/fill.h>
00019 // #include <thrust/transform.h>
00021 #include "cufft.h"
00022 #include "cufftXt.h"
00023 #include <cuda_runtime.h>
00024
00025
00027 typedef decltype(Eigen::VectorXd().begin()) eigen_iterator;
00028 typedef decltype(thrust::device_vector<double>().begin()) thrust_iterator;
00029 typedef thrust::detail::normal_iterator<thrust::device_ptr<const double» thrust_const_iterator;
00030 typedef Eigen::internal::pointer_based_stl_iterator<Eigen::Matrix<double, -1, 1» eigen_iterator_2;
00031
00033 /*
00034 Explicit template instantiation declarations for the thrust library.
00035
       They are declared here so that they are instantiatiated in cuda_wrapper.cu (and compiled with nvcc),
        and don't get instantiated in other translation units.
00036
00037
       This is necessary since we want to call thrust functions in translation units compiled by other
      compilers (g++ / icpx).
00038 */
00039 extern template class thrust::device_vector<double>;
00040 extern template class thrust::device_ptr<double>;
00041 extern template thrust::device_ptr<double> thrust::for_each_n(const
      thrust::detail::execution_policy_base<thrust::cuda_cub::tag> &, thrust::device_ptr<double>, unsigned
      long, thrust::detail::device_generate_functor<thrust::detail::fill_functor<double»);</pre>
00042 extern template eigen_iterator thrust::copy(const
      thrust::detail::execution_policy_base<thrust::cuda_cub::cross_system<thrust::cuda_cub::tag,
      thrust::system::cpp::detail::tag> &, thrust_const_iterator, thrust_const_iterator, eigen_iterator);
00043
00044 extern template thrust_iterator thrust::copy(eigen_iterator, eigen_iterator, thrust_iterator);
00045 extern template eigen_iterator thrust::copy(thrust_iterator, thrust_iterator, eigen_iterator);
00047 //Eigen::VectorXd copy_vector(const thrust::device_vector<double> &in);
00048 void copy_vector(Eigen::VectorXd &out, const thrust::device_vector<double> &in);
00049 //void copy_vector(Eigen::VectorXd &out, const Eigen::VectorXd &in);
00050
00051
00052 void show_gpu_memory_usage(void);
00054 // Wrapper for 3D cufftPlan3d. Performs double to complex double FFT for a f N^3 f grid.
00055 struct cufftWrapperD2Z {
00056
       int N;
        cufftHandle plan;
00057
        explicit cufftWrapperD2Z(int N_);
00059
        ~cufftWrapperD2Z();
00060
        thrust::device_vector<double> execute(thrust::device_vector<double> &in);
00061
       cufftWrapperD2Z(const cufftWrapperD2Z &) = delete;
cufftWrapperD2Z &operator=(const cufftWrapperD2Z &) = delete;
cufftWrapperD2Z(cufftWrapperD2Z &&) = delete;
00062
00063
00064
        cufftWrapperD2Z &operator=(cufftWrapperD2Z &&) = delete;
```

```
00066 };
00067
00068
00069 // Wrapper for 3D cufftPlanMany. Performs two double to complex double FFT for a \f$ N^3 \f$ grid.
00070 struct cufftWrapperBatchedD2Z {
00071
        int N:
        cufftHandle plan;
00073
        explicit cufftWrapperBatchedD2Z(int N_);
00074
        ~cufftWrapperBatchedD2Z();
00075
       thrust::device_vector<double> execute(thrust::device_vector<double> &in);
00076
00077
        cufftWrapperBatchedD2Z(const cufftWrapperBatchedD2Z &) = delete;
        cufftWrapperBatchedD2Z &operator=(const cufftWrapperBatchedD2Z &) = delete;
00078
00079
        cufftWrapperBatchedD2Z(cufftWrapperBatchedD2Z &&) = delete;
08000
        cufftWrapperBatchedD2Z &operator=(cufftWrapperBatchedD2Z &&) = delete;
00081 };
00082
00088 struct cufftWrapper {
00089
       int N;
        cufftHandle plan_d2z;
        cufftHandle plan_batched_d2z;
00091
00092
        cufftHandle plan_z2d;
00093
       thrust::device_vector<double> work_area;
00094
        explicit cufftWrapper(int N_);
00095
        ~cufftWrapper();
00096
        thrust::device_vector<double> execute_d2z(thrust::device_vector<double> &in);
00097
00098
        thrust::device_vector<double> execute_batched_d2z(thrust::device_vector<double> &in);
00099
        thrust::device_vector<double> execute_z2d(thrust::device_vector<double> &in);
00100
00101
        cufftWrapper(const cufftWrapper &) = delete;
00102
        cufftWrapper &operator=(const cufftWrapper &) = delete;
00103
        cufftWrapper(cufftWrapper &&) = delete;
       cufftWrapper &operator=(cufftWrapper &&) = delete;
00104
00105 };
00106
00113 struct cufftWrapperNoBatching {
00114 int N;
00115
        cufftHandle plan_d2z;
00116
        cufftHandle plan_z2d;
00117
        thrust::device_vector<double> work_area;
00118
       explicit cufftWrapperNoBatching(int N_);
00119
        ~cufftWrapperNoBatching();
00120
00121
        thrust::device_vector<double> execute_d2z(thrust::device_vector<double> &in);
00122
        thrust::device_vector<double> execute_batched_d2z(thrust::device_vector<double> &in);
00123
        thrust::device_vector<double> execute_z2d(thrust::device_vector<double> &in);
00124
        void execute_inplace_z2d(thrust::device_vector<double> &inout);
00125
00126
        cufftWrapperNoBatching(const cufftWrapperNoBatching &) = delete;
        cufftWrapperNoBatching &operator=(const cufftWrapperNoBatching &) = delete;
00128
        cufftWrapperNoBatching(cufftWrapperNoBatching &&) = delete;
00129
        cufftWrapperNoBatching &operator=(cufftWrapperNoBatching &&) = delete;
00130 };
00131
00132
00133 #endif
```

### 9.3 src/dispatcher.hpp File Reference

Automatically dispatching between using FFTW and CUFFT libraries.

```
#include "fftw_wrapper.hpp"
#include <thrust/device_vector.h>
#include "cuda_wrapper.cuh"
```

#### Classes

- · struct empty
- struct fftWrapperDispatcher< Vector >
- struct fftWrapperDispatcher< thrust::device\_vector< double >>
- struct fftWrapperDispatcher< Eigen::VectorXd >

### Macros

#define ALGORITHM\_NAMESPACE thrust

### 9.3.1 Detailed Description

Automatically dispatching between using FFTW and CUFFT libraries.

**Author** 

Siyang Ling

### 9.4 dispatcher.hpp

### Go to the documentation of this file.

```
00006 #ifndef DISPATCHER_HPP
00007 #define DISPATCHER_HPP
80000
00009 #include "fftw_wrapper.hpp"
00010
00011 #ifndef DISABLE_CUDA
00012 #include <thrust/device_vector.h>
00013 #include "cuda_wrapper.cuh"
00014 #define ALGORITHM_NAMESPACE thrust
00015 #else
00016 #define ALGORITHM_NAMESPACE std
00017 #endif
00018
00019
00020 // An empty placeholder object
00021 struct empty {};
00022
00023 // Dispatcher for fftWrapper* types
00024 template<typename Vector>
00025 struct fftWrapperDispatcher {
00026 typedef empty D2Z;
00027 typedef empty BatchedD2Z;
00028 typedef empty Generic;
00029 };
00030
00031 #ifndef DISABLE CUDA
00032 template<>
00033 struct fftWrapperDispatcher<thrust::device_vector<double» {
00034 typedef cufftWrapperD2Z D2Z;
00035 typedef cufftWrapperBatchedD2
        typedef cufftWrapperBatchedD2Z BatchedD2Z;
00036
        //typedef cufftWrapper Generic;
00037
        typedef cufftWrapperNoBatching Generic;
00038 };
00039 #endif
00040
00041 template<>
00042 struct fftWrapperDispatcher<Eigen::VectorXd> {
00043 typedef empty D2Z;
00044 typedef empty BatchedD2Z;
00045 typedef fftwWrapper Generic;
00046 };
00047
00048
00049 #endif
```

## 9.5 src/eigen\_wrapper.hpp File Reference

Wrap some Eigen functionalites.

```
#include <Eigen/Dense>
```

### **Functions**

• void copy\_vector (Eigen::VectorXd &out, const Eigen::VectorXd &in)

### 9.5.1 Detailed Description

Wrap some Eigen functionalites.

**Author** 

Siyang Ling

### 9.6 eigen\_wrapper.hpp

### Go to the documentation of this file.

```
00001
00006 #ifndef EIGEN_WRAPPER_HPP
00007 #define EIGEN_WRAPPER_HPP
00008
00009 #include <Eigen/Dense>
00010
00011 void copy_vector(Eigen::VectorXd &out, const Eigen::VectorXd &in);
00012
00013 #endif
```

### 9.7 src/equations.hpp File Reference

Header for field equations that runs on the CPU.

```
#include "Eigen/Dense"
#include <boost/numeric/odeint.hpp>
#include <boost/numeric/odeint/external/eigen/eigen.hpp>
#include "odeint_eigen/eigen_operations.hpp"
#include "workspace.hpp"
```

### Classes

• struct KleinGordonEquation

```
The Klein Gordon equation, \ddot{\varphi} - \nabla^2 \varphi + m^2 \varphi = 0.
```

• struct KleinGordonEquationInFRW

```
The Klein Gordon in FRW equation, \ddot{\varphi}+3H\dot{\varphi}-\nabla^2\varphi/a^2+m^2\varphi=0.
```

• struct ComovingCurvatureEquationInFRW

Equation for free scalar field in FRW spacetime, including comoving metric perturbations (in radiation domination).

### Concepts

· concept LatticeEquationConcept

### 9.7.1 Detailed Description

Header for field equations that runs on the CPU.

**Author** 

Siyang Ling

This is the header for field equations that are supposed to run on CPU. Equations declared here will be used by the odeint library via operator(). See  $https://www.boost.org/doc/libs/1_85\_ \leftarrow 0/libs/numeric/odeint/doc/html/boost_numeric_odeint/getting_started/short_ \leftarrow example.html for an example of odeint equation. Typically, compute_energy_density is also implemented for saving energy density spectrum.$ 

### 9.8 equations.hpp

### Go to the documentation of this file.

```
00001
00011 #ifndef EQUATIONS HPP
00012 #define EQUATIONS_HPP
00013
00014
00015 #include "Eigen/Dense"
00016
00017 #include <boost/numeric/odeint.hpp>
00018 #include <boost/numeric/odeint/external/eigen/eigen.hpp>
00019
00020 #include "odeint_eigen/eigen_operations.hpp"
00022 #include "workspace.hpp"
00023
00024 template<typename Equation>
00025 concept LatticeEquationConcept = requires (Equation eqn)
00026
         //typename Equation::State;
00028
         eqn.workspace;
00029
         eqn.compute_energy_density(eqn.workspace, 0.0);
00030
00031
00032
00036 struct KleinGordonEquation {
        typedef Eigen::VectorXd Vector;
00038
        typedef Vector State;
00039
        typedef WorkspaceGeneric<State> Workspace;
00040
        Workspace &workspace;
00041
00042
        KleinGordonEquation(Workspace &workspace_) : workspace(workspace_) {}
00043
00050
        void operator()(const State &, State &, const double);
00051
00058
        static Vector compute_energy_density(const Workspace &workspace, const double t);
00059 };
00060
00061
00065 struct KleinGordonEquationInFRW
00066
        typedef Eigen::VectorXd Vector;
00067
        typedef Vector State;
00068
        typedef WorkspaceGeneric<State> Workspace;
00069
        Workspace &workspace;
00070
00071
        KleinGordonEquationInFRW(Workspace &workspace_) : workspace(workspace_) {}
00072
00073
        void operator()(const State &, State &, const double);
00074
00081
        static Vector compute_energy_density(const Workspace &workspace, const double t);
00082 };
00083
00084
00100 struct ComovingCurvatureEquationInFRW {
        typedef Eigen::VectorXd Vector;
00101
00102
        typedef Vector State;
00103
        typedef WorkspaceGeneric<State> Workspace;
00104
       Workspace &workspace;
```

### 9.9 src/equations\_cuda.cuh File Reference

Header for field equations that runs on the GPU.

```
#include "equations.hpp"
#include <thrust/device_vector.h>
#include "odeint_thrust/thrust.hpp"
```

#### Classes

- struct CudaKleinGordonEquationInFRW
- struct CudaLambdaEquationInFRW
- struct CudaSqrtPotentialEquationInFRW
- struct CudaFixedCurvatureEquationInFRW
- struct CudaComovingCurvatureEquationInFRW
- struct CudaApproximateComovingCurvatureEquationInFRW

### 9.9.1 Detailed Description

Header for field equations that runs on the GPU.

Author

Siyang Ling

This is the header for field equations that are supposed to run on GPU (via CUDA). Equations declared here will be used by the odeint library via <code>operator()</code>. See <code>https://www.boost.org/doc/libs/1\_85\_  $\leftarrow 0/libs/numeric/odeint/doc/html/boost_numeric_odeint/getting_started/short_ <math display="inline">\leftarrow example.html$  for an example of odeint equation. Typically, <code>compute\_energy\_density</code> is also implemented for saving energy density spectrum. Also see equations.hpp.</code>

### 9.10 equations cuda.cuh

### Go to the documentation of this file.

```
00001
00012 #ifndef EQUATIONS CUDA CUH
00013 #define EQUATIONS_CUDA_CUH
00014
00015 #include "equations.hpp"
00016
00017 #include <thrust/device_vector.h>
00018
00019 #include "odeint thrust/thrust.hpp"
00020
00021 struct CudaKleinGordonEquationInFRW {
00022
        typedef thrust::device_vector<double> Vector;
        typedef Vector State;
00023
        typedef WorkspaceGeneric<Vector> Workspace;
00024
00025
       Workspace &workspace;
00026
00027
        CudaKleinGordonEquationInFRW(Workspace &workspace_) : workspace(workspace_) {}
00028
00029
        void operator()(const State &, State &, const double);
00030
00031
        static Vector compute_energy_density(const Workspace &workspace, const double t);
00032
        static Vector compute_dot_energy_density(const Workspace &workspace, const double t);
00033 };
00034
00035
00036 struct CudaLambdaEquationInFRW {
        typedef thrust::device_vector<double> Vector;
00037
00038
        typedef Vector State;
        typedef WorkspaceGeneric<Vector> Workspace;
00039
00040
        Workspace &workspace;
00041
00042
        CudaLambdaEquationInFRW(Workspace &workspace_) : workspace(workspace_) {}
00043
00044
        void operator() (const State &, State &, const double);
00045
00046
        static Vector compute_energy_density(const Workspace &workspace, const double t);
00047 };
00048
00049
00050 struct CudaSqrtPotentialEquationInFRW {
00051
        typedef thrust::device_vector<double> Vector;
00052
        typedef Vector State;
00053
        typedef WorkspaceGeneric<Vector> Workspace;
00054
       Workspace &workspace;
00055
00056
        CudaSqrtPotentialEquationInFRW(Workspace &workspace_) : workspace(workspace_) {}
00057
00058
        void operator()(const State &, State &, const double);
00059
00060
        static Vector compute_energy_density(const Workspace &workspace, const double t);
00061 };
00062
00063
00064 struct CudaFixedCurvatureEquationInFRW {
00065
        typedef thrust::device_vector<double> Vector;
00066
        typedef Vector State;
00067
        typedef WorkspaceGeneric<Vector> Workspace;
00068
       Workspace &workspace:
00069
00070
        CudaFixedCurvatureEquationInFRW(Workspace &workspace_) : workspace(workspace_) {}
00071
00072
        void operator()(const State &, State &, const double);
00073
00074
       static Vector compute_energy_density(const Workspace &workspace, const double t);
00075 };
00076
00077
00078 struct CudaComovingCurvatureEquationInFRW {
00079
        typedef thrust::device_vector<double> Vector;
08000
        typedef Vector State;
        typedef WorkspaceGeneric<Vector> Workspace;
00081
00082
       Workspace &workspace:
00083
00084
        CudaComovingCurvatureEquationInFRW(Workspace &workspace_) : workspace(workspace_) {}
00085
00086
        void operator()(const State &, State &, const double);
00087
00088
        static Vector compute_energy_density(Workspace &workspace, const double t);
00089 };
00090
00092 struct CudaApproximateComovingCurvatureEquationInFRW {
```

```
typedef thrust::device_vector<double> Vector;
00094
        typedef Vector State;
00095
        typedef WorkspaceGeneric<Vector> Workspace;
00096
       Workspace &workspace;
00097
00098
        CudaApproximateComovingCurvatureEquationInFRW(Workspace &workspace) : workspace(workspace) {}
00099
00100
        void operator()(const State &, State &, const double);
00101
00102
        static Vector compute_energy_density(Workspace &workspace, const double t);
00103 };
00104
00105
00106 // Explicit template instantiation declaration for the thrust library.
00107 extern template double thrust::reduce(const
      thrust::detail::execution_policy_base<thrust::cuda_cub::tag> &, thrust_const_iterator,
      thrust_const_iterator, double, boost::numeric::odeint::detail::maximum<double>);
00108
00109 // Deprecated function for testing CUDA kernels.
00111 void compute_deriv_test(const Eigen::VectorXd &in, Eigen::VectorXd &out,
00112
                  const double m, const double lambda,
00113
                  const double a_t, const double H_t, const double inv_ah_sqr,
00114
                 const long long int N);
00115 */
00116 /*
00117 void kernel_test(const thrust::device_vector<double> &R_fft, thrust::device_vector<double> &Psi,
     thrust::device_vector<double> &dPsidt,
00118
              const long long int N, const double L, const double m,
00119
              const double a_t, const double H_t, const double eta_t, const double inv_ah_sqr,
00120
              const double t, fftWrapperDispatcher<thrust::device_vector<double»::Generic &fft_wrapper);</pre>
00121 */
00122 #endif
```

### 9.11 src/fdm3d.hpp File Reference

Common procedures for manipulating / summarizing field configuration on a 3D lattice.

```
#include "Eigen/Dense"
#include "dispatcher.hpp"
```

#### **Macros**

• #define IDX\_OF(N, i, j, k) ((N)\*(N)\*(i) + (N)\*(j) + (k))

Give the index of a lattice point, assuming row major ordering in (i,j,k).

#define PADDED\_IDX\_OF(N, i, j, k) ((N)\*2\*((N)/2+1)\*(i) + 2\*((N)/2+1)\*(j) + (k))

Give the index of a lattice point, assuming that the array is in FFTW padded format.

#### **Functions**

Eigen::VectorXd compute\_power\_spectrum (const long long int N, Eigen::VectorXd &f, fftWrapperDispatcher
 Eigen::VectorXd >::Generic &fft\_wrapper)

Sum Fourier mode power of a field over directions.

• Eigen::VectorXd compute\_mode\_power\_spectrum (const long long int N, const double L, const double m, const double a\_t, Eigen::VectorXd &state, fftWrapperDispatcher< Eigen::VectorXd >::Generic &fft\_wrapper)

Sum Fourier mode power of a field over directions, along with the power in time derivatives.

 Eigen::VectorXd compute\_inverse\_laplacian (const long long int N, const double L, Eigen::VectorXd &f, fftWrapperDispatcher< Eigen::VectorXd >::Generic &fft\_wrapper)

Compute the inverse Laplacian of a field. AKA solve the Poisson equation.

• Eigen::VectorXd compute\_field\_with\_scaled\_fourier\_modes (const long long int N, const double L, Eigen::VectorXd &f, std::function< double(const double)> kernel, fftWrapperDispatcher< Eigen::VectorXd >::Generic &fft\_wrapper)

Scale each Fourier mode of a field by a kernel, returning the new field.

• Eigen::VectorXd compute\_cutoff\_fouriers (const long long int N, const long long int M, Eigen::VectorXd &fft)

Downsample a Fourier transform on a  $N^3$  grid so that it looks like a Fourier transform on a  $M^3$  grid.

### 9.11.1 Detailed Description

Common procedures for manipulating / summarizing field configuration on a 3D lattice.

Author

Siyang Ling

### 9.11.2 Macro Definition Documentation

### 9.11.2.1 PADDED\_IDX\_OF

Give the index of a lattice point, assuming that the array is in FFTW padded format.

### 9.11.3 Function Documentation

### 9.11.3.1 compute\_cutoff\_fouriers()

Downsample a Fourier transform on a  ${\cal N}^3$  grid so that it looks like a Fourier transform on a  ${\cal M}^3$  grid.

### **Parameters**

Ν	Number of lattice points (of full grid).
М	Number of lattice points (of downsampled grid).
fft	The DFT of a real field. Should be a vector of size $2N^2(N/2+1)$ .

### Returns

The downsampled DFT the input DFT. Should be a vector of size  $2M^2(M/2+1)$ .

### 9.11.3.2 compute\_field\_with\_scaled\_fourier\_modes()

```
const double L,
Eigen::VectorXd & f,
std::function< double(const double) > kernel,
fftWrapperDispatcher< Eigen::VectorXd >::Generic & fft_wrapper )
```

Scale each Fourier mode of a field by a kernel, returning the new field.

#### **Parameters**

N	Number of lattice points.
L	Box size.
f	The field on a 3D lattice. Should be a vector of size ${\cal N}^3$ with row major ordering. See IDX_OF.
kernel	A function $K$ determining how the Fourier modes are scaled.
fft_wrapper	A fftwWrapper initialized to do Fourier transforms on grid size $N$ .

#### Returns

The field with  $f_{\mathbf{k}} \mapsto K(k) f_{\mathbf{k}}$ , where K is given by kernel.

### 9.11.3.3 compute\_inverse\_laplacian()

Compute the inverse Laplacian of a field. AKA solve the Poisson equation.

### **Parameters**

N	Number of lattice points.
L	Box size.
f	The field on a 3D lattice. Should be a vector of size $\mathbb{N}^3$ with row major ordering. See IDX_OF.
fft_wrapper	A fftwWrapper initialized to do Fourier transforms on grid size $N$ .

### Returns

The solution to the Poisson equation with RHS f, namely  $\nabla^{-2}f$ . The output have zero homogeneous mode regardless of whether f has one.

### 9.11.3.4 compute\_mode\_power\_spectrum()

```
Eigen::VectorXd & state,
fftWrapperDispatcher< Eigen::VectorXd >::Generic & fft_wrapper )
```

Sum Fourier mode power of a field over directions, along with the power in time derivatives.

#### **Parameters**

N	Number of lattice points.
L	Box size.
m	Mass $m$ of (free) scalar field.
a_t	Current scale factor.
state	The state $(\varphi,\dot{\varphi})$ of a scalar field on a 3D lattice. Should be a vector of size $2N^3$ , with the first half (first $N^3$ indices) containing $\varphi$ , and the second half containing $\dot{\varphi}$ .
fft_wrapper	A fftwWrapper initialized to do Fourier transforms on grid size $N$ .

### Returns

A vector of size  $3(N/2)^2 + 1$ , with its s index containing the power in Fourier modes with wavenumber  $\sqrt{s}k_{\rm IR}$ . Specifically:

output[s] = 
$$\sum_{i^2+j^2+k^2=s} |\tilde{\varphi}_{i,j,k}|^2 + \frac{|\dot{\tilde{\varphi}}_{i,j,k}|^2}{\omega_k^2}$$
$$\omega_k^2 = m^2 + sk_{\rm IR}^2/a^2(t)$$

Here,  $\tilde{\varphi}_{a,b,c}$  and  $\dot{\tilde{\varphi}}_{a,b,c}$  are the DFT's, (i,j,k) labels a site on the reciprocal lattice, and  $-N/2+1 \leq i,j,k \leq N/2$ ; see <a href="https://garrettgoon.com/gaussian-fields/">https://garrettgoon.com/gaussian-fields/</a> for details on this convention. Also see compute power spectrum.

### 9.11.3.5 compute\_power\_spectrum()

Sum Fourier mode power of a field over directions.

#### **Parameters**

N	Number of lattice points.
f	The field on a 3D lattice. Should be a vector of size $N^3$ with row major ordering. See IDX_OF.
fft_wrapper	A fftwWrapper initialized to do Fourier transforms on grid size $N$ .

#### Returns

A vector of size  $3(N/2)^2+1$ , with its s index containing the power in Fourier modes with wavenumber  $\sqrt{s}k_{\rm IR}$ . Specifically:

output[s] = 
$$\sum_{i^2+j^2+k^2=s} |\tilde{f}_{i,j,k}|^2$$
  
 $\tilde{f}_{i,j,k} = \sum_{a,b,c} e^{-2\pi i(a,b,c).(i,j,k)/N} f_{a,b,c}$ 

Here,  $\tilde{f}$  is the DFT of f, (i,j,k) labels a site on the reciprocal lattice, and  $-N/2+1 \le i,j,k \le N/2$ . See https://garrettgoon.com/gaussian-fields/ for details on this convention.

### 9.12 fdm3d.hpp

#### Go to the documentation of this file.

```
00006 #ifndef FDM3D_HPP
00007 #define FDM3D_HPP
00008
00009 #include "Eigen/Dense"
00010 #include "dispatcher.hpp"
00011
00016 #define IDX_OF(N, i, j, k) ((N)*(N)*(i) + (N)*(j) + (k))
00017
00018
00024 #define PADDED IDX OF(N, i, j, k) ((N) \star 2 \star ((N)/2+1) \star (i) + 2 \star ((N)/2+1) \star (j) + (k))
00025
00041 Eigen::VectorXd compute_power_spectrum(const long long int N,
00042
                              Eigen:: VectorXd &f,
00043
                              fftWrapperDispatcher<Eigen::VectorXd>::Generic &fft_wrapper);
00044
00065 Eigen:: VectorXd compute mode power spectrum(const long long int N, const double L, const double m,
     const double a_t,
00066
                               Eigen::VectorXd &state,
00067
                               fftWrapperDispatcher<Eigen::VectorXd>::Generic &fft_wrapper);
00068
00069 // Eigen::VectorXd compute_mode_power_spectrum(const long long int N, const double L, const double m,
00070 //
                              Eigen:: VectorXd &state,
00071 //
                               fftWrapperDispatcher<Eigen::VectorXd>::Generic &fft_wrapper);
00072
00082 Eigen::VectorXd compute_inverse_laplacian(const long long int N, const double L,
00083
                             Eigen::VectorXd &f,
00084
                             fftWrapperDispatcher<Eigen::VectorXd>::Generic &fft_wrapper);
00085
00086
00096 Eigen::VectorXd compute_field_with_scaled_fourier_modes(const long long int N, const double L,
00097
                                   Eigen:: VectorXd &f,
00098
                                   std::function<double(const double) > kernel,
00099
                                   fftWrapperDispatcher<Eigen::VectorXd>::Generic &fft_wrapper);
00100
00101
00109 Eigen::VectorXd compute_cutoff_fouriers(const long long int N, const long long int M,
00110
                          Eigen::VectorXd &fft);
00111
00112
00113 // Deprecated
00114 // Eigen::VectorXd compute_power_spectrum(const long long int N, Eigen::VectorXd &phi);
00115 // Eigen::VectorXd compute_fourier(const long long int N, const double L, Eigen::VectorXd &phi);
00116 // Eigen::VectorXd compute_laplacian(const long long int N, const double L, const Eigen::VectorXd &f);
00117
00118
00119
00120 #endif
```

### 9.13 src/fdm3d cuda.cuh File Reference

CUDA implementation for fdm3d.hpp. Common procedures for manipulating / summarizing field configuration on a 3D lattice.

```
#include <thrust/device_vector.h>
#include <thrust/reduce.h>
#include <thrust/functional.h>
#include "cuda_wrapper.cuh"
#include "dispatcher.hpp"
#include "fdm3d.hpp"
```

### **Functions**

 thrust::device\_vector< double > compute\_mode\_power\_spectrum (const long long int N, const double L, const double m, const double a\_t, thrust::device\_vector< double > &state, fftWrapperDispatcher
 thrust::device\_vector< double > >::Generic &fft\_wrapper) 9.14 fdm3d cuda.cuh 55

CUDA version of identically named function in fdm3d.hpp.

thrust::device\_vector< double > compute\_power\_spectrum (const long long int N, thrust::device\_vector< double > &f, fftWrapperDispatcher< thrust::device\_vector< double > >::Generic &fft\_wrapper)

CUDA version of identically named function in fdm3d.hpp.

- thrust::device\_vector< double > compute\_laplacian (const long long int N, const double L, thrust::device\_vector< double > &f)
- thrust::device\_vector< double > compute\_inverse\_laplacian (const long long int N, const double L, thrust::device\_vector< double > &f, fftWrapperDispatcher< thrust::device\_vector< double > >::Generic &fft wrapper)

CUDA version of identically named function in fdm3d.hpp.

thrust::device\_vector< double > compute\_cutoff\_fouriers (const long long int N, const long long int M, const thrust::device\_vector< double > &fft)

CUDA version of identically named function in fdm3d.hpp.

### 9.13.1 Detailed Description

CUDA implementation for fdm3d.hpp. Common procedures for manipulating / summarizing field configuration on a 3D lattice.

**Author** 

Siyang Ling

### 9.14 fdm3d cuda.cuh

### Go to the documentation of this file.

```
00006 #ifndef FDM3D_CUDA_CUH
00007 #define FDM3D_CUDA_CUH
80000
00009 #include <thrust/device vector.h>
00010 #include <thrust/reduce.h>
00011 #include <thrust/functional.h>
00012
00013 //#include "odeint_thrust/thrust.hpp"
00014 #include "cuda_wrapper.cuh"
00015 #include "dispatcher.hpp"
00016
00017 #include "fdm3d.hpp"
00018
00022 thrust::device_vector<double> compute_mode_power_spectrum(const long long int N, const double L, const
      double m, const double a_t,
00023
                                     thrust::device vector<double> &state,
00024
                                     fftWrapperDispatcher<thrust::device_vector<double>::Generic
      &fft_wrapper);
00025
00029 thrust::device_vector<double> compute_power_spectrum(const long long int N,
00030
                                    thrust::device_vector<double> &f,
00031
                                    fftWrapperDispatcher<thrust::device_vector<double>::Generic
      &fft wrapper):
00032
00033 thrust::device_vector<double> compute_laplacian(const long long int N, const double L,
00034
                               thrust::device_vector<double> &f);
00035
00039 thrust::device_vector<double> compute_inverse_laplacian(const long long int N, const double L,
00040
                                   thrust::device_vector<double> &f,
00041
                                   fftWrapperDispatcher<thrust::device_vector<double>::Generic &fft_wrapper);
00042
00046 thrust::device_vector<double> compute_cutoff_fouriers(const long long int N, const long long int M,
00047
                                     const thrust::device_vector<double> &fft);
00048
00049 // void compute_inverse_laplacian_test(const long long int N, const double L,
00050 //
                          thrust::device_vector<double> &fft);
00051 #endif
```

### 9.15 src/fftw\_wrapper.hpp File Reference

### Wrapper for FFTW library.

```
#include <iostream>
#include <Eigen/Dense>
#include <fftw3.h>
```

### Classes

struct fftwWrapper

Wrapper for various FFTW functions for a  $N^3$  grid.

### 9.15.1 Detailed Description

Wrapper for FFTW library.

**Author** 

Siyang Ling

### 9.16 fftw\_wrapper.hpp

#### Go to the documentation of this file.

```
00001
00006 #ifndef FFTW_WRAPPER_HPP
00007 #define FFTW_WRAPPER_HPP
80000
00009 #include <iostream>
00010
00011 #include <Eigen/Dense>
00012 #include <fftw3.h>
00013
00014
00020 struct fftwWrapper {
00021 int N;
00022 fftw_plan plan_d2z;
        fftw_plan plan_z2d;
fftw_plan plan_inplace_z2d;
00023
00025
        explicit fftwWrapper(int N_);
00026
        ~fftwWrapper();
00027
        Eigen::VectorXd execute_d2z(Eigen::VectorXd &in);
00033
00034
00040
         Eigen::VectorXd execute_batched_d2z(Eigen::VectorXd &in);
00041
00049
         Eigen::VectorXd execute_z2d(Eigen::VectorXd &in);
00050
00056
        void execute_z2d(Eigen::VectorXd &in, Eigen::VectorXd &out);
00057
00064
        void execute_inplace_z2d(Eigen::VectorXd &inout);
00065
00066
        fftwWrapper(const fftwWrapper &) = delete;
        fftwWrapper &operator=(const fftwWrapper &) = delete;
fftwWrapper(fftwWrapper &&) = delete;
fftwWrapper &operator=(fftwWrapper &&) = delete;
00067
00068
00069
00070 };
00071
00072
00073 #endif
```

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### 9.17 field booster.hpp

```
00001 /*
       Tools related to boosting (adding velocity to) fields.
00002
00003 */
00004 #ifndef FIELD_BOOSTER_HPP
00005 #define FIELD_BOOSTER_HPP
00006
00007 #include "Eigen/Dense"
80000
00009 void add_phase_to_state(Eigen::VectorXd &state, const Eigen::VectorXd &phase);
00010
00011 void boost_klein_gordon_field(Eigen::VectorXd &varphi, Eigen::VectorXd &dt_varphi, const
     Eigen::VectorXd &theta,
00012
                        const long long int N, const double L, const double m);
00013
00014
00015 #endif
```

### 9.18 src/initializer.hpp File Reference

Snippets for initializing workpaces. (e.g. field initial conditions, etc)

```
#include "fdm3d.hpp"
#include "random_field.hpp"
#include "fftw_wrapper.hpp"
#include "special_function.hpp"
#include <thrust/device_vector.h>
#include "cuda_wrapper.cuh"
```

#### Macros

#define ALGORITHM\_NAMESPACE thrust

### **Variables**

auto unperturbed\_grf

Initialize a field and its derivative from a white noise power spectrum with cutoff k\_ast.

auto unperturbed\_grf\_with\_background

Initialize a field and its derivative from a white noise power spectrum with cutoff k\_ast, plus homogeneous background.

· auto perturbed grf

Setup a scalar field with inhomogeneous Gaussian random initial conditions.

auto perturbed\_grf\_without\_saving\_Psi

Same as perturbed\_grf, but does not store Psi in workspace to save memory.

auto unperturbed\_grf\_with\_Psi

Same as unperturbed\_grf, but with an extra scale-invariant Psi.

auto unperturbed\_grf\_and\_fixed\_curvature

Initialize a homogeneous Gaussian random field and some scale invariant curvature perturbation.

auto perturbed\_grf\_and\_comoving\_curvature\_fft

Initialize an inhomogeneous Gaussian random field and the fft of some scale invariant comoving curvature perturbation.

· auto homogeneous\_field

Initialize a homogeneous field with amplitude f and time derivative dt\_f. For testing the numerical code.

· auto homogeneous\_field\_with\_fluctuations

Initialize a homogeneous field with amplitude f, plus scale-invariant perturbations (resembling quantum fluctutations).

· auto plane\_wave

Plane wave initial condition. For testing the numerical code.

· auto wave packet

Wave packet initial condition.

### 9.18.1 Detailed Description

Snippets for initializing workpaces. (e.g. field initial conditions, etc)

**Author** 

Siyang Ling

Contains a collection of lambda functions used to initialize the workspace for simulation. Each lambda function initializer is meant to be passed to a workspace constructor Workspace (param, initializer). The use of lambda's makes it easy to switch between param types and workspace types. Moreover, in order for easy switching between CPU code and GPU code, the initializers compute everything on CPU first, and then decide whether the results should be copied to CPU or GPU memory.

#### 9.18.2 Variable Documentation

### 9.18.2.1 homogeneous\_field

```
Initial value:

[] (const auto param, auto &workspace) {
    const long long int N = param.N;
    Eigen::VectorXd varphi = Eigen::VectorXd::Constant(N*N*N, param.f);
    Eigen::VectorXd dt_varphi = Eigen::VectorXd::Constant(N*N*N, param.dt_f);

    auto &state = workspace.state;
    state.resize(varphi.size() + dt_varphi.size());
    ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
    ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
}
```

Initialize a homogeneous field with amplitude f and time derivative dt\_f. For testing the numerical code.

### 9.18.2.2 homogeneous\_field\_with\_fluctuations

```
auto homogeneous_field_with_fluctuations [inline]
```

### Initial value:

```
[](const auto param, auto &workspace) {
   const long long int N = param.N;
   Eigen::VectorXd varphi = Eigen::VectorXd::Constant(N*N*N, param.f);
   Eigen::VectorXd dt_varphi = Eigen::VectorXd::Constant(N*N*N, 0.0);

Spectrum P_delta_varphi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L,
        param.delta_varphi_std_dev, param.k_delta_varphi, -3);
   Eigen::VectorXd delta_varphi = generate_gaussian_random_field(param.N, param.L, P_delta_varphi);
   varphi += delta_varphi;

auto &state = workspace.state;
   state.resize(varphi.size() + dt_varphi.size());
   ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
   ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
}
```

Initialize a homogeneous field with amplitude f, plus scale-invariant perturbations (resembling quantum fluctutations).

### 9.18.2.3 perturbed\_grf

```
auto perturbed_grf [inline]
Initial value:
  [](const auto param, auto &workspace) {
    Spectrum P_Psi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.Psi_std_dev,
      param.k_Psi, -3);
    Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
      param.k_ast, 0);
    Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
    Eigen::VectorXd Psi = generate_gaussian_random_field(param.N, param.L, P_Psi);
    Eigen::VectorXd varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L, Psi, P_f);
    Eigen::VectorXd dt_varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L, Psi, P_dtf);
    auto &state = workspace.state;
    state.resize(varphi.size() + dt_varphi.size());
    workspace.Psi.resize(Psi.size());
    ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
    ALGORITHM_NAMESPACE::copy(Psi.begin(), Psi.end(), workspace.Psi.begin());
```

Setup a scalar field with inhomogeneous Gaussian random initial conditions.

Initialize a field and its derivative from a white noise power spectrum with cutoff k\_ast, but with a large scale perturbation specified by Psi. Psi is initialized from a scale-invariant power spectrum with cutoff k\_Psi.

#### 9.18.2.4 perturbed grf and comoving curvature fft

```
auto perturbed_grf_and_comoving_curvature_fft [inline]
```

Initialize an inhomogeneous Gaussian random field and the fft of some scale invariant comoving curvature perturbation.

This is the procedure used for section 4.2.2 of the paper.

### 9.18.2.5 perturbed\_grf\_without\_saving\_Psi

```
auto perturbed_grf_without_saving_Psi [inline]
```

#### Initial value:

Same as perturbed\_grf, but does not store Psi in workspace to save memory.

#### 9.18.2.6 plane\_wave

```
Initial value:

[] (const auto param, auto &workspace) {
    const long long int N = param.N;
    Eigen::VectorXd varphi(N*N*N);
    for(int a = 0; a < N; ++a) {
        for(int b = 0; b < N; ++b) {
        for(int c = 0; c < N; ++c) {
            varphi(IDX_OF(N, a, b, c)) = cos(2 * std::numbers::pi * c / N);
        }
        }
    }
    Eigen::VectorXd dt_varphi = Eigen::VectorXd::Constant(N*N*N, 0);
    auto &state = workspace.state;
    state.resize(varphi.size() + dt_varphi.size());
    ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
    ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
}</pre>
```

Plane wave initial condition. For testing the numerical code.

### 9.18.2.7 unperturbed grf

```
Initial value:

[] (const auto param, auto &workspace) {
    Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
    param.k_ast, 0);
    Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
    Eigen::VectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f);
    Eigen::VectorXd dt_varphi = generate_gaussian_random_field(param.N, param.L, P_dtf);
    auto &state = workspace.state;
    state.resize(varphi.size() + dt_varphi.size());
    ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin() + varphi.size());
}
```

Initialize a field and its derivative from a white noise power spectrum with cutoff k\_ast.

### 9.18.2.8 unperturbed\_grf\_and\_fixed\_curvature

```
auto unperturbed_grf_and_fixed_curvature [inline]
```

### Initial value:

Initialize a homogeneous Gaussian random field and some scale invariant curvature perturbation.

### 9.18.2.9 unperturbed\_grf\_with\_background

```
auto unperturbed_grf_with_background [inline]
```

#### Initial value:

```
[](const auto param, auto &workspace) {
   Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
        param.k_ast, 0);
   Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
   Eigen::VectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f);
   varphi.array() += param.varphi_mean;
   Eigen::VectorXd dt_varphi = generate_gaussian_random_field(param.N, param.L, P_dtf);

auto &state = workspace.state;
   state.resize(varphi.size() + dt_varphi.size());
   ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
   ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
}
```

Initialize a field and its derivative from a white noise power spectrum with cutoff k\_ast, plus homogeneous background.

### 9.18.2.10 unperturbed\_grf\_with\_Psi

```
auto unperturbed_grf_with_Psi [inline]
```

#### Initial value:

Same as unperturbed grf, but with an extra scale-invariant Psi.

### 9.18.2.11 wave\_packet

```
Initial value:

[] (const auto param, auto &workspace) {
    const long long int N = param.N;
    Eigen::VectorXd varphi(N*N*N);
    Eigen::VectorXd dt_varphi(N*N*N);
    Eigen::VectorXd Psi(N*N*N);

for (int a = 0; a < N; ++a) {
    for (int b = 0; b < N; ++b) {
    for (int c = 0; c < N; ++c) {
        double dist_to_center = sqrt(std::pow(std::min((double)a, (double)std::abs(N-a)), 2) + (b - N/3) * (b - N/3) + (c - N/3) * (c - N/3)) * (param.L / param.N);
    varphi(IDX_OF(N, a, b, c)) = exp(- dist_to_center * dist_to_center / 40.0);
    dt_varphi(IDX_OF(N, a, b, c)) = 0;

    Psi(IDX_OF(N, a, b, c)) = - param.Psi_std_dev * cos(2 * std::numbers::pi * c / N);
}</pre>
```

```
auto &state = workspace.state;
state.resize(varphi.size() + dt_varphi.size());
ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
workspace.Psi.resize(Psi.size());
ALGORITHM_NAMESPACE::copy(Psi.begin(), Psi.end(), workspace.Psi.begin());
}
```

Wave packet initial condition.

### 9.19 initializer.hpp

### Go to the documentation of this file.

```
00001
00012 #ifndef INITIALIZER HPP
00013 #define INITIALIZER_HPP
00015 #include "fdm3d.hpp'
00016 #include "random_field.hpp"
00017 #include "fftw_wrapper.hpp"
00018 #include "special_function.hpp"
00019 //#include "dispatcher.hpp"
00020 //#include "field_booster.hpp"
00021 //#include "param.hpp"
00022 //#include "physics.hpp"
00023
00024 #ifndef DISABLE CUDA
00025 #include <thrust/device_vector.h>
00026 #include "cuda_wrapper.cuh"
00027 #define ALGORITHM_NAMESPACE thrust
00028 #else
00029 #define ALGORITHM_NAMESPACE std
00030 #endif
00031
00033 inline auto unperturbed grf =
        [](const auto param, auto &workspace) {
           Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
      param.k_ast, 0);
00036
           Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
00037
           Eigen::VectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f); // Initial ULDM
      field
00038
           Eigen::VectorXd dt_varphi = generate_gaussian_random_field(param.N, param.L, P_dtf); // Initial
      ULDM field time derivative
00039
           auto &state = workspace.state;
00040
00041
           state.resize(varphi.size() + dt_varphi.size());
           // thrust::copy handles both copies between Eigen::VectorXd and copies from Eigen::VectorXd to
00042
      thrust::device vector<double>
00043
          ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00044
           ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00045
00046
00047
00049 inline auto unperturbed grf with background =
        [](const auto param, auto &workspace) {
           Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
      param.k_ast, 0);
00052
           Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
00053
           Eigen::VectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f);
           refer::vectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f);
varphi.array() += param.varphi_mean;
Eigen::VectorXd dt_varphi = generate_gaussian_random_field(param.N, param.L, P_dtf);
00054
00055
00056
00057
           auto &state = workspace.state;
00058
           state.resize(varphi.size() + dt_varphi.size());
           ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00059
00060
           ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00061
         };
00062
00063
00071 inline auto perturbed_grf =
00072
        [](const auto param, auto &workspace) {
           Spectrum P_Psi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.Psi_std_dev,
00073
      param.k_Psi, -3);
00074
          Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
      param.k_ast, 0);
00075
           Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
```

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```
00076
                Eigen::VectorXd Psi = generate_gaussian_random_field(param.N, param.L, P_Psi);
                Eigen::VectorXd varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L, Psi, P_f);
00077
00078
                Eigen::VectorXd dt_varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L, Psi,
         P_dtf);
00079
08000
                auto &state = workspace.state;
                state.resize(varphi.size() + dt_varphi.size());
00082
                workspace.Psi.resize(Psi.size());
00083
                ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
                ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size()); ALGORITHM_NAMESPACE::copy(Psi.begin(), Psi.end(), workspace.Psi.begin());
00084
00085
00086
00087
                //std::cout « boost::typeindex::type id runtime(workspace.Psi).pretty name() « '\n';
00088
00089
00090
00092 inline auto perturbed_grf_without_saving_Psi =
00093
             [](const auto param, auto &workspace) {
                Spectrum P_Psi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.Psi_std_dev,
00094
         param.k_Psi, -3);
00095
                \label{eq:spectrum} \textbf{Spectrum P\_f} = \textbf{power\_law\_with\_cutoff\_given\_amplitude\_3d(param.N, param.L, param.varphi\_std\_dev, param.Var
         param.k_ast, 0);
00096
                Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
                Eigen::VectorXd Psi = generate_gaussian_random_field(param.N, param.L, P_Psi);
Eigen::VectorXd varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L, Psi, P_f);
00097
00098
                Eigen::VectorXd dt_varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L, Psi,
00099
         P_dtf);
00100
00101
                auto &state = workspace.state;
00102
                state.resize(varphi.size() + dt_varphi.size());
00103
                ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00104
                ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00105
00106
00107
00109 inline auto unperturbed_grf_with_Psi =
             [](const auto param, auto &workspace)
00110
                Spectrum P_Psi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.Psi_std_dev,
00111
         param.k_Psi, -3);
00112
               Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
         param.k_ast, 0);
00113
                Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
                Eigen::VectorXd Psi = generate_gaussian_random_field(param.N, param.L, P_Psi);
Eigen::VectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f);
00114
00115
00116
                Eigen::VectorXd dt_varphi = generate_gaussian_random_field(param.N, param.L, P_dtf);
00117
00118
                auto &state = workspace.state;
00119
                state.resize(varphi.size() + dt_varphi.size());
00120
                workspace.Psi.resize(Psi.size());
00121
                ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00122
                ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
                ALGORITHM_NAMESPACE::copy(Psi.begin(), Psi.end(), workspace.Psi.begin());
00123
00124
00125
00126
00128 inline auto unperturbed grf and fixed curvature =
             [] (const auto param, auto &workspace)
00130
                Spectrum P_Psi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.Psi_std_dev,
         param.k_Psi, -3);
00131
               Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
         param.k_ast, 0);
00132
                Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
00133
                Eigen::VectorXd Psi = generate_gaussian_random_field(param.N, param.L, P_Psi);
                Eigen::VectorXd varphi = generate_gaussian_random_field(param.N, param.L, P_f);
00134
00135
                Eigen::VectorXd dt_varphi = generate_gaussian_random_field(param.N, param.L, P_dtf);
00136
00137
                auto &state = workspace.state;
00138
                state.resize(varphi.size() + dt_varphi.size());
00139
                workspace.Psi.resize(Psi.size());
00140
                ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00141
                ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00142
                ALGORITHM_NAMESPACE::copy(Psi.begin(), Psi.end(), workspace.Psi.begin());
00143
            };
00144
00150 inline auto perturbed_grf_and_comoving_curvature_fft =
00151
             [] (const auto param, auto &workspace) {
00152
                using namespace std::numbers;
00153
00154
                // Generate comoving curvature perturbation
00155
                double eta_i = workspace.cosmology.eta(param.t_start);
                double A_s = (-576 * pow(pi, 6) * pow(eta_i, 6) * pow(param.Psi_std_dev, 2)) / (-81 * pow(param.L, 4) * (pow(param.L, 2) + 2 * pow(pi, 2) * pow(eta_i, 2)) +
00156
00157
00158
00159
                     (81 * pow(param.L, 5) - 54 * pow(param.L, 3) * pow(pi, 2) * pow(eta_i, 2) +
                48 * param.L * pow(pi, 4) * pow(eta_i, 4)) * cos((4 * pi * eta_i) / (sqrt(3) * param.L)) + 256 * pow(pi, 6) * pow(eta_i, 6) * Ci_pade_approximant_12_12((4 * pi * eta_i) / (sqrt(3) *
00160
00161
00162
```

```
param.L)) +
           4 * sqrt(3) * param.L * pi * eta_i *
00163
             (27 * pow(param.L, 4) + 6 * pow(param.L, 2) * pow(pi, 2) * pow(eta_i, 2) -
00164
          16 * pow(pi, 4) * pow(eta_i, 4)) * sin((4 * pi * eta_i) / (sqrt(3) * param.L)));
00165
00166
00167
          Spectrum P R = scale invariant spectrum 3d(param.N, param.L, A s);
00168
00169
          // Manual cutoff for P_R at around horizon. The effect of imposing this cutoff is negligible.
00170
          // Spectrum P_R_with_cutoff = [P_R] (double k) { return k <= 0.5 ? P_R(k) : 0.0; };
00171
00172
          Eigen::VectorXd R = generate_gaussian_random_field(param.N, param.L, P_R);
          // std::cout « "A_s = " « A_s « '\n';
00173
00174
00175
          // Calculate initial gravitational potential Psi.
00176
          // Convention for potentials: \mathbb{R}_k = (3 / 2) \text{Psi}_k for superhorizon.
     auto kernel = [eta_i] (double k) {
    return k == 0.0 ? 0.0 : (6 * sqrt(3) * (-((k * eta_i * cos((k * eta_i) / sqrt(3))) /
sqrt(3)) + sin((k * eta_i) / sqrt(3)))) / (pow(k, 3) * pow(eta_i, 3));
00177
00178
00179
                };
00180
           auto fft_wrapper = fftwWrapper(param.N);
          Eigen::VectorXd Psi = compute_field_with_scaled_fourier_modes(param.N, param.L, R, kernel,
00181
      fft_wrapper);
00182
          // Calculate \ \pi^2, \det{\pi^2} perturbations as a multiple of Psi. // See Eqn (3.17) of paper.
00183
00184
          // There is an extra factor of 0.5 in front since "generate_inhomogeneous_gaussian_random_field"
00185
      use \exp(2\Psi) \sim 1 + 2 \Psi for variance perturbation convention.
          double v = param.k_ast / (param.al * param.m);
double alpha_varphi_sqr = 0.5 * (- 3 * pow(4*pow(v,2)+5, 2)) / (12*pow(v,4) + 50*pow(v,2) + 50);
00186
00187
          double alpha_dot_varphi_sqr = 0.5 * (25 - 20*pow(v,2)) / (12*pow(v,4) + 50*pow(v,2) + 50);
00188
00189
00190
          Spectrum P_f = power_law_with_cutoff_given_amplitude_3d(param.N, param.L, param.varphi_std_dev,
      param.k_ast, 0);
          Spectrum P_dtf = to_deriv_spectrum(param.m, P_f);
00191
00192
          Eigen::VectorXd varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L,
      alpha_varphi_sqr * Psi, P_f);
    Eigen::VectorXd dt_varphi = generate_inhomogeneous_gaussian_random_field(param.N, param.L,
alpha_dot_varphi_sqr * Psi, P_dtf);
00193
00194
00195
           auto &state = workspace.state;
00196
          state.resize(varphi.size() + dt_varphi.size());
          ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00197
00198
          ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00199
00200
           // Save the comoving curvature perturbation for reference
00201
00202
            decltype(workspace.state) R_dvec(R.size());
00203
            ALGORITHM_NAMESPACE::copy(R.begin(), R.end(), R_dvec.begin());
00204
             workspace.R_fft = workspace.fft_wrapper.execute_d2z(R_dvec);
00205
00206
        };
00207
00208
00209
00214 inline auto homogeneous_field =
00215
        [](const auto param, auto &workspace) {
          const long long int N = param.N;
00216
          Eigen::VectorXd varphi = Eigen::VectorXd::Constant(N*N*N, param.f);
00217
00218
          Eigen::VectorXd dt_varphi = Eigen::VectorXd::Constant(N*N*N, param.dt_f);
00219
00220
          auto &state = workspace.state;
00221
          state.resize(varphi.size() + dt_varphi.size());
00222
          ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00223
          ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00224
00225
00226
00230 inline auto homogeneous field with fluctuations =
00231
        [](const auto param, auto &workspace) {
00232
          const long long int N = param.N;
00233
          Eigen::VectorXd varphi = Eigen::VectorXd::Constant(N*N*N, param.f);
00234
          Eigen::VectorXd dt_varphi = Eigen::VectorXd::Constant(N*N*N, 0.0);
00235
          Spectrum P_delta_varphi = power_law_with_cutoff_given_amplitude_3d(param.N, param.L,
00236
Eigen::VectorXd delta_varphi = generate_gaussian_random_field(param.N, param.L, P_delta_varphi);
00238
          varphi += delta_varphi;
00239
          auto &state = workspace.state;
00240
00241
          state.resize(varphi.size() + dt varphi.size());
          ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00242
          ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00243
00244
00245
00246
00251 inline auto plane wave =
00252
        [] (const auto param, auto &workspace) {
```

```
const long long int N = param.N;
          Eigen::VectorXd varphi(N*N*N);
00255
          for (int a = 0; a < N; ++a) {
00256
           for (int b = 0; b < N; ++b) {
          for(int c = 0; c < N; ++c){</pre>
00257
00258
           varphi(IDX OF(N, a, b, c)) = cos(2 * std::numbers::pi * c / N);
00260
00261
         }
00262
00263
         Eigen::VectorXd dt_varphi = Eigen::VectorXd::Constant(N*N*N, 0);
00264
00265
         auto &state = workspace.state;
          state.resize(varphi.size() + dt_varphi.size());
00266
00267
          ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00268
          ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00269
00270
00275 inline auto wave_packet =
00276
      [](const auto param, auto &workspace) {
00277
          const long long int N = param.N;
00278
         Eigen::VectorXd varphi(N*N*N);
00279
         Eigen:: VectorXd dt varphi(N*N*N);
00280
         Eigen::VectorXd Psi(N*N*N);
          for (int a = 0; a < N; ++a) {
00282
00283
           for (int b = 0; b < N; ++b) {
00284
          for (int c = 0; c < N; ++c) {
     double dist_to_center = sqrt(std::pow(std::min((double)a, (double)std::abs(N-a)), 2) + (b - N/3)  
* (b - N/3) + (c - N/3) * (c - N/3)) * (param.L / param.N);
00285
            varphi(IDX_OF(N, a, b, c)) = exp(- dist_to_center * dist_to_center / 40.0);
00286
            dt_varphi(IDX_OF(N, a, b, c)) = 0;
00287
00288
            param.L / 3.0 / 3.0)));
00289
           Psi(IDX_OF(N, a, b, c)) = - param.Psi_std_dev * cos(2 * std::numbers::pi * c / N);
00290
           }
00292
         }
00293
00294
          auto &state = workspace.state;
00295
          state.resize(varphi.size() + dt_varphi.size());
          ALGORITHM_NAMESPACE::copy(varphi.begin(), varphi.end(), state.begin());
00296
00297
         ALGORITHM_NAMESPACE::copy(dt_varphi.begin(), dt_varphi.end(), state.begin() + varphi.size());
00298
00299
          workspace.Psi.resize(Psi.size());
00300
         ALGORITHM_NAMESPACE::copy(Psi.begin(), Psi.end(), workspace.Psi.begin());
00301
00302
00303
00304 #endif
```

# 9.20 src/io.hpp File Reference

#### Input/output utilities.

```
#include <cstdlib>
#include <iostream>
#include <fstream>
#include <string>
#include <vector>
#include <iomanip>
#include <Eigen/Dense>
```

#### **Functions**

- std::vector< double > load vector from file (std::string filename)
- void write\_vector\_to\_file (std::vector< double > vector, std::string filename)
- void write\_data\_to\_file (const char \*buf, ssize\_t size, std::string filename)
- void write\_VectorXd\_to\_file (const Eigen::VectorXd &vector, std::string filename)
- void write\_VectorXd\_to\_filename\_template (const Eigen::VectorXd &vector, const std::string format\_string, const int idx)
- Eigen::VectorXd load\_VectorXd\_from\_file (const std::string &filename)

### 9.20.1 Detailed Description

Input/output utilities.

**Author** 

Siyang Ling

# 9.21 io.hpp

#### Go to the documentation of this file.

```
00006 #ifndef IO_HPP
00007 #define IO_HPP
00008 #include <cstdlib>
00009 #include <iostream>
00010 #include <fstream>
00011 #include <string>
00012 #include <vector>
00013 #include <iomanip>
00014
00015 #include <Eigen/Dense>
00016
00018 std::vector<double> load_vector_from_file(std::string filename);
00019 void write_vector_to_file(std::vector<double> vector, std::string filename);
00020 void write_data_to_file(const char *buf, ssize_t size, std::string filename);
00021
00022 void write_VectorXd_to_file(const Eigen::VectorXd &vector, std::string filename);
00023 void write_VectorXd_to_filename_template(const Eigen::VectorXd &vector, const std::string
      format_string, const int idx);
00024 Eigen::VectorXd load_VectorXd_from_file(const std::string &filename);
00025
00026
00027 #endif
```

# 9.22 src/midpoint.hpp File Reference

Midpoint method implementation for odeint stepper.

```
#include <boost/numeric/odeint/algebra/default_operations.hpp>
#include <boost/numeric/odeint/algebra/algebra_dispatcher.hpp>
#include <boost/numeric/odeint/algebra/operations_dispatcher.hpp>
#include <boost/numeric/odeint/util/state_wrapper.hpp>
#include <boost/numeric/odeint/util/is_resizeable.hpp>
#include <boost/numeric/odeint/util/resizer.hpp>
#include "cuda_wrapper.cuh"
```

#### Classes

class midpoint < State, Value, Deriv, Time, Algebra, Operations, Resizer >

### 9.22.1 Detailed Description

Midpoint method implementation for odeint stepper.

**Author** 

Siyang Ling

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# 9.23 midpoint.hpp

```
00001
00006 #ifndef MIDPOINT HPP
00007 #define MIDPOINT_HPP
00009
00010
00011 //#include <boost/numeric/odeint/stepper/base/explicit_stepper_base.hpp>
00012 //#include <boost/numeric/odeint/algebra/range_algebra.hpp>
00013 #include <boost/numeric/odeint/algebra/default_operations.hpp>
00014 #include <boost/numeric/odeint/algebra/algebra_dispatcher.hpp>
00015 #include <boost/numeric/odeint/algebra/operations_dispatcher.hpp>
00016
00017 #include <boost/numeric/odeint/util/state_wrapper.hpp>
00018 #include <boost/numeric/odeint/util/is_resizeable.hpp>
00019 #include <boost/numeric/odeint/util/resizer.hpp>
00020
00021 #ifndef DISABLE_CUDA
00022 #include "cuda_wrapper.cuh"
00023 #endif
00024
00025 template<
00026 class State,
       class Value = double,
        class Deriv = State,
00028
00029
        class Time = Value,
00030
        class Algebra = typename boost::numeric::odeint::algebra_dispatcher<State>::algebra_type,
00031
        class Operations = typename boost::numeric::odeint::operations_dispatcher<State>::operations_type,
00032
       class Resizer = boost::numeric::odeint::initially_resizer //boost::numeric::odeint::always_resizer
00034 class midpoint : public boost::numeric::odeint::algebra_stepper_base<Algebra, Operations>
00035 4
00036 public :
00037
        typedef State state_type;
00038
        typedef State deriv_type;
00039
        typedef Value value_type;
00040
        typedef Time time_type;
00041
        typedef unsigned short order_type;
00042
        typedef boost::numeric::odeint::stepper_tag stepper_category;
00043
00044
        typedef boost::numeric::odeint::algebra_stepper_base<Algebra, Operations> algebra_stepper_base_type;
00045
        typedef typename algebra_stepper_base_type::algebra_type algebra_type;
00046
        typedef typename algebra_stepper_base_type::operations_type operations_type;
00047
00048
        static order_type order(void) { return 2; }
00049
00050
        midpoint(){}
00051
00052
        template<class System>
00053
        void do_step(System system, State &in, Time t, Time dt)
00054
00055
          static const Value val1 = static_cast<Value>(1);
          const Time dh = dt / static_cast<Value>(2);
const Time th = t + dh;
00056
00057
00058
          //m_resizer.adjust_size(in, boost::numeric::odeint::detail::bind(&stepper_type::template
00059
     resize_impl<State>, boost::numeric::odeint::detail::ref(*this), boost::numeric::odeint::detail::_1));
00060
          m_resizer.adjust_size(in, [&](const auto &arg) { return resize_impl(arg); });
00061
00062
          typename boost::numeric::odeint::unwrap reference<System>::type &sys = system;
00063
00064
          sys(in, deriv_tmp.m_v, t);
00065
          algebra_stepper_base_type::m_algebra.for_each3(state_tmp.m_v, in, deriv_tmp.m_v,
00066
                                  typename operations_type::template scale_sum2<Value, Time>(val1, dh));
00067
          sys(state_tmp.m_v, deriv_tmp.m_v, th);
algebra_stepper_base_type::m_algebra.for_each3(state_tmp.m_v, in, deriv_tmp.m_v,
00068
00069
00070
                                  typename operations_type::template scale_sum2<Value, Time>(val1, dt));
00071
00072
          in.swap(state_tmp.m_v);
00073
00074
          // Release memory
00075
          //m_resizer.adjust_size(State(), [&](const auto &arg){ return resize_impl(arg); });
          // deriv_tmp.m_v.clear();
00077
          // State().swap(deriv_tmp.m_v);
00078
          // state_tmp.m_v.clear();
00079
          // State().swap(state_tmp.m_v);
08000
00081
        // template<class StateType>
00083
        // void adjust_size(const StateType &x)
00084
00085
             resize_impl(x);
```

```
00086
       // }
00087
00088
       bool resize_impl(const State &x)
00089
         bool resized = false;
00090
         resized |= boost::numeric::odeint::adjust_size_by_resizeability(deriv_tmp, x, typename
00091
     boost::numeric::odeint::is_resizeable<State>::type());
00092
         resized |= boost::numeric::odeint::adjust_size_by_resizeability(state_tmp, x, typename
     boost::numeric::odeint::is_resizeable<State>::type());
       return resized;
}
00093
00094
00095
00096 private:
00097
      Resizer m_resizer;
00098
00099
       boost::numeric::odeint::state_wrapper<State> deriv_tmp;
00100
       boost::numeric::odeint::state_wrapper<State> state_tmp;
00101 };
00103
00104
00105 #endif
```

# 9.24 src/observer.hpp File Reference

Implements "observers", which controls what gets saved during simulations.

```
#include <cstdlib>
#include <iostream>
#include <string>
#include "Eigen/Dense"
#include "Eigen/Dense"
#include <boost/numeric/odeint.hpp>
#include voost/numeric/odeint/external/eigen/eigen.hpp>
#include "odeint_eigen/eigen_operations.hpp"
#include "eigen_wrapper.hpp"
#include "fdm3d.hpp"
#include "io.hpp"
#include "io.hpp"
#include "physics.hpp"
#include "workspace.hpp"
#include "cuda_wrapper.cuh"
#include "fdm3d_cuda.cuh"
```

#### **Classes**

 $\bullet \ \, \text{struct ConstIntervalObserver} < \ \, \text{Equation, save\_field\_spectrum, save\_density\_spectrum, save\_density} > \\$ 

### 9.24.1 Detailed Description

Implements "observers", which controls what gets saved during simulations.

**Author** 

Siyang Ling

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# 9.25 observer.hpp

```
00007 #ifndef OBSERVER HPP
00008 #define OBSERVER_HPP
00009
00010 #include <cstdlib>
00011 #include <iostream>
00012 #include <string>
00013 #include <type_traits>
00014
00015 #include "Eigen/Dense"
00016
00017 #include <boost/numeric/odeint.hpp>
00018 #include <boost/numeric/odeint/external/eigen/eigen.hpp>
00020 #include "odeint_eigen/eigen_operations.hpp"
00021
00022 #include "eigen_wrapper.hpp"
00022 #include "fdm3d.hpp"
00024 #include "io.hpp"
00025 #include "physics.hpp'
00026 #include "workspace.hpp"
00027
00028 #ifndef DISABLE_CUDA
00029 #include "cuda_wrapper.cuh"
00030 #include "fdm3d_cuda.cuh"
00031 #endif
00032
00033
00034 template<typename Equation,
00035
           bool save_field_spectrum = true,
00036
           bool save_density_spectrum = true,
00037
           bool save density = false>
00038 struct ConstIntervalObserver
00039 typedef typename Equation::Workspace Workspace;
00040
        typedef typename Workspace::State State;
        typedef State Vector;
00041
00042
        Workspace &workspace;
00043
        int idx;
        std::string dir;
00044
00045
        double t_start;
00046
        double t_end;
00047
        double t_interval;
00048
        double t last:
00049
00050
        template<typename Param>
00051
        ConstIntervalObserver(const std::string &dir_, const Param &param, Equation &eqn) :
00052
          workspace(eqn.workspace), idx(0), dir(dir_),
00053
          t_start(param.t_start), t_end(param.t_end), t_interval(param.t_interval), t_last(param.t_start) { }
00054
00055
        ConstIntervalObserver(const ConstIntervalObserver &) = default;
00056
00057
        void operator()(const State &x, double t)
00058
00059
          if(t >= t_last + t_interval || t == t_end || t == t_start) {
            const long long int N = workspace.N;
const double L = workspace.L;
const double m = workspace.m;
00060
00061
00062
            const double a_t = workspace.cosmology.a(t);
00063
00064
00065
            if constexpr(save_field_spectrum) {
00066
            Vector varphi_plus_spectrum = compute_mode_power_spectrum(N, L, m, a_t, workspace.state,
      workspace.fft wrapper);
00067
            Eigen::VectorXd varphi_plus_spectrum_out(varphi_plus_spectrum.size());
            copy_vector(varphi_plus_spectrum_out, varphi_plus_spectrum);
write_VectorXd_to_filename_template(varphi_plus_spectrum_out, dir +
00068
00069
      "varphi_plus_spectrum_%d.dat", idx);
00070
00071
            if constexpr(save_density_spectrum) {
Vector rho = Equation::compute_energy_density(workspace, t);
00072
00074
            Vector rho_spectrum = compute_power_spectrum(N, rho, workspace.fft_wrapper);
00075
            Eigen::VectorXd rho_spectrum_out(rho_spectrum.size());
00076
            copy_vector(rho_spectrum_out, rho_spectrum);
00077
            write_VectorXd_to_filename_template(rho_spectrum_out, dir + "rho_spectrum_%d.dat", idx);
00078
00079
00080
            if constexpr(save_density) {
00081
            Vector rho = Equation::compute_energy_density(workspace, t);
00082
            Eigen::VectorXd rho_copy(rho.size());
            00083
00084
            Eigen::VectorXd rho_axis_average = rho_copy.reshaped(N*N, N).rowwise().mean(); // Save the
00085
      density overaged over a axis.
```

```
00087
            write_VectorXd_to_filename_template(rho_slice, dir + "rho_slice_%d.dat", idx);
00088
            write_VectorXd_to_filename_template(rho_axis_average, dir + "rho_axis_average_%d.dat", idx);
00089
00090
00091
            workspace.t_list.push_back(t);
00092
            t_{last} = t;
00093
            ++idx;
00094
00095
00096 };
00097
00098
00099
00100 #endif
```

# 9.26 src/param.hpp File Reference

Utilities for managing simulations parameters.

```
#include "utility.hpp"
#include "boost/pfr.hpp"
#include "boost/type_index.hpp"
#include <fstream>
#include <string>
```

#### Classes

struct SampleParam

• template<typename T >

A sample parameter type specifying a lambda-phi-4 theory in an FRW background.

#### **Functions**

```
    void print_param (const T &param)
        Pretty prints a parameter struct T.
    template<typename T >
        void save_param_names (const std::string &filename)
        Save the member names of parameter struct T to filename.
    template<typename T >
        void save_param_Mathematica_formats (const std::string &filename)
        Save the member types of parameter struct T to filename. Type names are in Mathematica convention.
    template<typename T >
        void save_param_for_Mathematica (const T &param, const std::string &dir)
        Save member names, types and values of param to directory dir.
    template<typename T >
        void save_param_types (const std::string &filename)
```

### 9.26.1 Detailed Description

Utilities for managing simulations parameters.

**Author** 

Siyang Ling

This header file contains utilities for pretty-printing and saving parameters of a simulation. By convention, we collect all parameters in a (trivial, standard layout) struct containing double's or long long int's. (e.g. SampleParam) The utilities here are generic for different parameter structs; you can define your own new type containing new parameters, and use the utilities here as usual. Typically, we use these utilities to export a struct along with some meta-information, so that external code (Mathematica / Python) can also use the parameters.

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# 9.27 param.hpp

```
00001
00014 #ifndef PARAM HPP
00015 #define PARAM_HPP
00016
00017 #include "utility.hpp"
00018 #include "boost/pfr.hpp"
00019 #include "boost/type_index.hpp"
00020 #include <fstream>
00021 #include <string>
00022
00026 struct SampleParam {
00027
       long long int N;
00028
        double L;
00029
        double m;
       double lambda;
00030
00031
        double k_ast;
00032
       double varphi_std_dev;
00033
        double a1;
00034
       double H1;
00035
       double t1;
00036 };
00037
00041 template<typename T>
00042 void print_param(const T &param) {
00043
       auto names = boost::pfr::names_as_array<T>();
       00044
00045
00046
00047
00048
       // std::cout « line_separator_with_description("The parameters for the simulation") « ' \n';
00049
       // boost::pfr::for_each_field(param, func);
       // std::cout « line_separator_with_description() « '\n';
00050
       auto c = [$](){ boost::pfr::for_each_field(param, func); };
run_and_print("The parameters for the simulation", c);
00051
00052
00053 }
00054
00058 template<typename T>
00059 void save_param_names(const std::string &filename) {
00060 std::ofstream outstream(filename);
00061
        auto names = boost::pfr::names_as_array<T>();
00062
       for(auto name : names) {
00063
         outstream « name « '\n';
00064
00065 }
00066
00067 /
00068 // Compiles with Intel icpx, but doesn't compile with gcc due to "Explicit template specialization
      cannot have a storage class"
00069 template<typename T> std::string_view Mathematica_format;
00070
00071 template<> constexpr static std::string_view Mathematica_format<double> = "Real64";
00072
00073 template<> constexpr static std::string_view Mathematica_format<long long int> = "Integer64";
00074 */
00075
00076 /
00077 // Compiles with gcc, fails at link stage with Intel icpx due to multiple definitions
00078 template<typename T> std::string_view Mathematica_format;
00079
00080 template<> constexpr std::string_view Mathematica_format<double> = "Real64";
00082 template<> constexpr std::string_view Mathematica_format<long long int> = "Integer64";
00083 */
00084
00085 namespace {
00086 template<typename T> std::string_view Mathematica_format;
00088 template<> constexpr std::string_view Mathematica_format<double> = "Real64";
00089
00090 template<> constexpr std::string_view Mathematica_format<long long int> = "Integer64";
00091 }
00092
00096 template<typename T>
00097 void save_param_Mathematica_formats(const std::string &filename) {
00098 std::ofstream outstream(filename);
00099
       auto func = [&](const auto &field) {
              typedef std::remove_const_t<std::remove_reference_t<decltype(field)» type_of_field;</pre>
00100
00101
              outstream « Mathematica_format<type_of_field> « '\n';
                };
00103
       boost::pfr::for_each_field(T(), func);
00104 }
00105
```

```
00109 template<typename T>
00110 static void save_param(const T &param, const std::string &filename){
00111 std::ofstream outstream(filename, std::ios::binary);
00112 if(outstream.is_open()){
00113
          outstream.write((const char *)&param, sizeof(T));
00114 }
00115 }
00116
00120 template<typename T>
00121 void save_param_for_Mathematica(const T &param, const std::string &dir) {
00122 save_param_names<T>(dir + "paramNames.txt");
00123 save_param_Mathematica formats<T>(dir + "paramNames.txt");
        save_param_Mathematica_formats<T>(dir + "paramTypes.txt");
        save_param<T>(param, dir + "param.dat");
00124
00125 }
00126
00127
00128 template<typename T>
00129 void save_param_types(const std::string &filename) {
00130 std::ofstream outstream(filename);
00131 auto func = [&] (const auto &field) {
00132 outstream « boost::typeindex::
               outstream « boost::typeindex::type_id_runtime(field) « '\n';
00133
00134 boost::pfr::for_each_field(T(), func);
00135 }
00136
00137
00138
00139
00140 #endif
```

# 9.28 src/physics.hpp File Reference

Collection of repeatedly used physics formulas. (e.g. FRW cosmology related formulas)

```
#include <cmath>
```

#### Classes

struct StaticEOSCosmology

A convenience class used to calculate FRW related quantities for constant EOS spacetimes.

### 9.28.1 Detailed Description

Collection of repeatedly used physics formulas. (e.g. FRW cosmology related formulas)

Author

Siyang Ling

# 9.29 physics.hpp

```
00001

00006 #ifndef PHYSICS_HPP

00007 #define PHYSICS_HPP

00008

00009 #include <cmath>

00010 //#include "param.hpp"

00011

00015 struct StaticEOSCosmology {

00016 double a1;
```

```
double H1;
00017
00018
00019
         double p;
00021
        StaticEOSCosmology(const double al_, const double H1_, const double t1_, const double p_)
00022
           : a1(a1_), H1(H1_), t1(t1_), p(p_) {}
00023
        template<typename T>
00026
        StaticEOSCosmology(const T &param)
00027
          : al(param.al), H1(param.H1), t1(param.t1), p(1.0) {}
00028
00030
        StaticEOSCosmology(void)
00031
           : a1(1.0), H1(0), t1(0), p(1.0) {}
00032
        double a(const double t) const {
00034
00035
           return al * pow(1 + (1 + 1 / p) * H1 * (t - t1), p / (1 + p));
00036
00037
00039
        double H(const double t) const {
  return H1 * pow(1 + (1 + 1 / p) * H1 * (t - t1), -1);
00040
00041
00042
00052
         double eta(const double t) const {
         //return etal + (p / (al * H1)) * (-1 + pow(1 + (1 + 1 / p) * H1 * (t - t1), 1 / (1 + p)));
return (p / (al * H1)) * pow(1 + (1 + 1 / p) * H1 * (t - t1), 1 / (1 + p));
00053
00054
00055
00056 };
00057
00058
00059 #endif
```

# 9.30 src/random field.hpp File Reference

Utilities for generating Gaussian random fields of given spectrum and inhomogeneity.

```
#include "Eigen/Dense"
#include <functional>
#include <random>
#include <vector>
```

#### **Typedefs**

typedef std::function < double(const double) > Spectrum

Typedef for spectrum P(k). Given momentum k, the spectrum should return P(k).

#### **Functions**

- void RandomNormal::set\_generator\_seed (std::mt19937::result\_type\_seed)
- std::mt19937 RandomNormal::get generator from device ()
- double RandomNormal::generate\_random\_normal()
- Spectrum power\_law\_with\_cutoff\_given\_amplitude\_3d (const long long int N, const double L, const double sigma, const double k ast, const double alpha)

 $k^{\alpha}$  power law spectrum with a sharp cutoff at  $k_*$ .

• Spectrum broken\_power\_law\_given\_amplitude\_3d (const long long int N, const double L, const double sigma, const double k ast, const double alpha, const double beta)

Broken power law spectrum with the break at  $k_*$ .

- Spectrum scale\_invariant\_spectrum\_3d (const long long int N, const double L, const double As)
  - $k^{\alpha}$  power law spectrum with a sharp cutoff at  $k_*$ .
- Spectrum to\_deriv\_spectrum (const double m, const Spectrum &P\_f)

Given spectrum  $P_{\varphi}$ , return a new spectrum given by  $P_{\dot{\varphi}}(k) = (k^2 + m^2)P_{\varphi}(k)$ .

• Spectrum to\_deriv\_spectrum (const double m, const double a, const Spectrum &P\_f)

Given spectrum  $P_{\varphi}$ , return a new spectrum given by  $P_{\dot{\varphi}}(k) = (k^2/a^2 + m^2)P_{\varphi}(k)$ .

Eigen::VectorXd generate\_gaussian\_random\_field (const long long int N, const double L, const Spectrum &P)

Special case of generate inhomogeneous gaussian random field.

Eigen::VectorXd generate\_inhomogeneous\_gaussian\_random\_field (const long long int N, const double L, const Eigen::VectorXd &Psi, const Spectrum &P)

Generate an inhomogeneous 3D real Gaussian random field from spectral data P(k).

### 9.30.1 Detailed Description

Utilities for generating Gaussian random fields of given spectrum and inhomogeneity.

**Author** 

Siyang Ling

This file contains utilities for generating Gaussian random fields (GRF), including some example spectra and a function for generating field realizations from a spectra. See function generate\_inhomogeneous\_gaussian\_random\_field for details.

#### 9.30.2 Function Documentation

#### 9.30.2.1 broken\_power\_law\_given\_amplitude\_3d()

Broken power law spectrum with the break at  $k_*$ .

### **Parameters**

N	Number of lattice points.	
L	Box size.	
sigma	Standard deviation $\sigma$ of generated function $f$ .	
k_ast	The break $k_*$ .	
alpha	Power law index $\alpha$ .	
beta	Power law index $\beta$ .	

#### Returns

The spectrum P, which can be called to get P(k).

The spectrum is given by

$$P(0) = 0 
P(k) = P(k_0)(k/k_0)^{\alpha} \text{ for } k < k_0 
P(k) = P(k_0)(k/k_0)^{\beta} \text{ for } k > k_0 
\overline{f^2} = \sigma^2$$

#### 9.30.2.2 generate\_inhomogeneous\_gaussian\_random\_field()

Generate an inhomogeneous 3D real Gaussian random field from spectral data P(k).

#### **Parameters**

Ν	Number of lattice points.
L	Box size.
Psi	The inhomogeneity function $\psi$ , given in terms of values on the lattice (of size $N^3$ ).
Р	The spectrum $P$ .

### Returns

The generated GRF, as values on the lattice (of size  $N^3$ ).

Generate an inhomogeneous Gaussian random field f, such that the spectrum of f is P, and the variance of the field has inhomogeneity like  $\langle f^2(x) \rangle \approx \overline{f^2} e^{2\psi(x)}$ . See section 3.2 of paper for details of this procedure.

### 9.30.2.3 power\_law\_with\_cutoff\_given\_amplitude\_3d()

 $k^{\alpha}$  power law spectrum with a sharp cutoff at  $k_{*}.$ 

### Parameters

N	Number of lattice points.
L	Box size.
<i>Sigma</i> Generated b	Standard deviation $\sigma$ of generated function $f$ .
k_ast	Cutoff $k_*$ .
alpha	Power law index $\alpha$ .

#### Returns

The spectrum P, which can be called to get P(k).

The spectrum is given by

$$P(0) = 0$$

$$P(k) = P(k_0)(k/k_0)^{\alpha} \text{ for } k < k_0$$

$$\overline{f^2} = \sigma^2$$

#### 9.30.2.4 scale\_invariant\_spectrum\_3d()

 $k^{\alpha}$  power law spectrum with a sharp cutoff at  $k_*$ .

#### **Parameters**

Ν	Number of lattice points.	
L	Box size.	
As	The height of the spectrum $A_s$ .	

#### Returns

The spectrum P, which can be called to get P(k).

The spectrum is given by

$$P(0) = 0$$

$$P(k) = A_s$$

# 9.31 random\_field.hpp

```
00001
00010 #ifndef RANDOM_FIELD_HPP
00011 #define RANDOM_FIELD_HPP
00012
00013 #include "Eigen/Dense"
00014
00015 #include <functional>
00016 #include <random>
00017 #include <vector>
00018
00019
00020 // A self-initializing random number generator for standard normal distribution
00021 namespace RandomNormal
00022 {
00023    void set_generator_seed(std::mt19937::result_type seed);
00024    std::mt19937 get_generator_from_device();
00025    double generate_random_normal();
00026 }
```

```
00031 typedef std::function<double(const double)> Spectrum;
00032
00033 // Typical spectra.
00034
00051 Spectrum power law with cutoff given amplitude 3d(const long long int N. const double L. const double
      sigma, const double k_ast, const double alpha);
00052
00071 Spectrum broken_power_law_given_amplitude_3d(const long long int N, const double L, const double
      sigma, const double k_ast, const double alpha, const double beta);
00072
00086 Spectrum scale invariant spectrum 3d(const long long int N, const double L, const double As);
00087
00091 Spectrum to_deriv_spectrum(const double m, const Spectrum &P_f);
00092
00096 Spectrum to_deriv_spectrum(const double m, const double a, const Spectrum &P_f);
00097
00101 Eigen::VectorXd generate_gaussian_random_field(const long long int N, const double L, const Spectrum
00102
00115 Eigen::VectorXd generate_inhomogeneous_gaussian_random_field(const long long int N, const double L,
      const Eigen::VectorXd &Psi, const Spectrum &P);
00116
00117
00118
00119
00120 #endif
```

# 9.32 special\_function.hpp

```
00001 #ifndef SPECIAL_FUNCTION_HPP
00002 #define SPECIAL_FUNCTION_HPP
00004 // Pade approximant for Si(x), with m=15, n=12
00005 inline double Si_pade_approximant_15_12(double x) {
00006
         using namespace std;
           return (x - 0.045439340981633 * pow(x, 3) + 0.0011545722575101668 * pow(x, 5) - 0.000014101853682133025 * pow(x, 7) + 9.432808094387131e-8 * pow(x, 9) - 3.5320197899716837e-10 * pow(x, 11) + 7.08240282274876e-13 * pow(x, 13) -
00007
00008
00009
                        6.053382120104225e-16 * pow(x, 15)) /
                        (1. + 0.010116214573922555 * pow(x, 2) + 0.000049917511616975513 * pow(x, 4) + 1.556549863087456e-7 * pow(x, 6) + 3.280675710557897e-10 * pow(x, 8) +
00011
00012
00013
                       4.5049097575386586e-13 * pow(x, 10) + 3.211070511937122e-16 * pow(x, 12));
00014 }
00015
00016 // Pade approximant for Ci(x), with m=12, n=12
00017 inline double Ci_pade_approximant_12_12(double x) {
00018 using namespace std;
           return log(x) + (0.5772156649015329 - 0.24231497614160186 * pow(x, 2) +
00019
                       0.007139183039136621 * pow(x, 4) - 0.00011466618094101764 * pow(x, 6) + 8.443734405201243e-7 * pow(x, 8) - 3.060472574705558e-9 * pow(x, 10) + 4.328624073851291e-12 * pow(x, 12)) /
00020
00021
00022
                       (1. + 0.013313955815300189 * pow(x, 2) + 0.00008836441800952094 * pow(x, 4) + 3.800404484365274e-7 * pow(x, 6) + 1.1376490214488613e-9 * pow(x, 8) + 2.297129602871981e-12 * pow(x, 10) + 2.510407760855278e-15 * pow(x, 12));
00023
00024
00025
00026 }
00027
00028 #endif
```

# 9.33 src/utility.hpp File Reference

Utilities for debugging / profiling / pretty printing.

```
#include <iostream>
#include <iomanip>
#include <chrono>
#include <filesystem>
```

#### **Functions**

template < typename Callable > void profile\_function (long long int repeat, Callable &&c)

### 9.33.1 Detailed Description

Utilities for debugging / profiling / pretty printing.

Author

Siyang Ling

# 9.34 utility.hpp

```
00001
00006 #ifndef UTILITY_HPP
00007 #define UTILITY_HPP
00008
00009 #include <iostream>
00010 #include <iomanip>
00011 #include <chrono>
00012 #include <filesystem>
00014 // Pretty print functions
00015 inline static std::string line_separator_with_description(const std::string &description) {
00016 std::string result(80, '=');
       const int length = description.length() + 2;
00017
       result.replace(80 / 2 - length / 2, length, " " + description + " ");
00018
00019
       return result;
00020 }
00021
00022
00025
       return result;
00026 }
00027
00028
00029 template<typename Callable>
00030 static void run_and_print(const std::string &description, const Callable &c) {
       std::cout « line_separator_with_description(description) « '\n';
00031
00033
       std::cout « line_separator_with_description() « '\n';
00034 }
00035
00036
00037 template<typename Callable>
00038 static void run_and_measure_time(const std::string &description, const Callable &c) {
       std::cout « line_separator_with_description(description) « '\n';
00040
       auto time_start = std::chrono::system_clock::now();
00041
00042
       auto time_end = std::chrono::system_clock::now();
00043
       std::chrono::duration<double> time diff = time end - time start;
00044
       std::cout « std::fixed « std::setprecision(9) « std::left;
00045
       std::cout « std::setw(9) « "time spent = " « time_diff.count() « " s" « '\n';
00046
       std::cout « line_separator_with_description() « '\n';
00047 }
00048
00049
00050 static void prepare_directory_for_output(const std::string &dir) {
       const std::filesystem::path dir_path(dir);
00052
       std::error_code ec;
00053
       std::cout « line_separator_with_description("Preparing directory for output") « '\n';
00054
       std::cout « "Saving results in directory: " « dir « '\n';
       std::filesystem::create_directories(dir_path, ec);
std::cout « "ErrorCode = " « ec.message() « '\n';
00055
00056
       std::cout « line_separator_with_description() « '\n';
00057
00058 }
00059
00060
00061 // Simple profiler for a big task, taking many cycles
00062 // Note that the function call incurs some time cost, so this is not totally accurate
00063 template<typename Callable>
00064 inline void profile_function(long long int repeat, Callable &&c) {
00065
       auto time_start = std::chrono::system_clock::now();
       for(long long int i = 0; i < repeat; ++i) {</pre>
00066
00067
         c();
00068
00069
       std::cout « line_separator_with_description("Profiling a callable") « '\n';
       auto time_end = std::chrono::system_clock::now();
```

```
00071    std::chrono::duration<double> time_diff = time_end - time_start;
00072    std::cout « std::fixed « std::setprecision(9) « std::left;
00073    std::cout « std::setw(9) « "total time spent = " « time_diff.count() « " s" « '\n';
00074    std::cout « std::setw(9) « "time spent per iteration = " « time_diff.count() / repeat « " s" « '\n';
00075    std::cout « line_separator_with_description() « '\n';
00076 }
00077
00078
00079
00080 #endif
```

# 9.35 src/wkb.hpp File Reference

Implementation of the WKB solution.

```
#include "Eigen/Dense"
#include "workspace.hpp"
```

#### Classes

struct WKBSolutionForKleinGordonEquationInFRW

### 9.35.1 Detailed Description

Implementation of the WKB solution.

Used to extend an existing field profile to a later time.

# 9.36 wkb.hpp

```
00001
00007 #ifndef WKB_HPP
00008 #define WKB_HPP
00009
00010 #include "Eigen/Dense"
00011 #include "workspace.hpp"
00012
00013 struct WKBSolutionForKleinGordonEquationInFRW {
00014
00015
        typedef Eigen::VectorXd Vector;
        typedef Vector State;
00016
00017
       typedef WorkspaceGeneric<State> Workspace;
00018
00019
        Workspace &workspace;
00020
        double t_i;
00021
        Vector phi_ffts;
00022
00023
       WKBSolutionForKleinGordonEquationInFRW(Workspace &workspace_, const double t_i_);
00024
00025
        Vector evaluate_at(const double t);
00026
00027 };
00028
00029 #endif
```

# 9.37 src/workspace.hpp File Reference

A generic "workspace" class, containing parameters / data / tools used during simulations.

```
#include <memory>
#include "param.hpp"
#include "physics.hpp"
#include "fftw_wrapper.hpp"
#include "dispatcher.hpp"
```

#### Classes

struct WorkspaceGeneric < Vector >
 A generic workspace for storing temporary.

#### Concepts

- concept HasLatticeParams
- concept HasMass
- concept HasLambda
- · concept HasFa
- concept HasFRWParameters
- concept HasPsiApproximationParameters

### Macros

• #define TYPE\_REQUIREMENT(value, type) {std::remove\_cvref\_t<decltype((value))>()} -> std::same\_as<type>;

### 9.37.1 Detailed Description

A generic "workspace" class, containing parameters / data / tools used during simulations.

Author

Siyang Ling

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### 9.38 workspace.hpp

```
00006 #ifndef WORKSPACE_HPP
00007 #define WORKSPACE_HPP
80000
00009 #include <memorv>
00010
00011 #include "param.hpp"
00012 #include "physics.hpp"
00013 #include "fftw_wrapper.hpp"
00014 #include "dispatcher.hpp"
00015
00016 #define TYPE_REQUIREMENT(value, type) {std::remove_cvref_t<decltype((value))>()} ->
std::same_as<type>;
00017
00018
00019
00020 template<typename Param>
00021 concept HasLatticeParams = requires (Param param)
       TYPE_REQUIREMENT(param.N, long long int)
TYPE_REQUIREMENT(param.L, double) };
00024
00025 template<typename Param>
00026 concept HasMass = requires (Param param) { TYPE_REQUIREMENT(param.m, double) };
00027
00028 template<typename Param>
00029 concept HasLambda = requires (Param param) { TYPE_REQUIREMENT(param.lambda, double) };
00030
00031 template<typename Param>
00032 concept HasFa = requires (Param param) { TYPE_REQUIREMENT(param.f_a, double) };
00033
00034 template<typename Param>
00035 concept HasFRWParameters = requires (Param param)
       { TYPE_REQUIREMENT(param.al, double)
00037
          TYPE_REQUIREMENT (param. H1, double)
00038
          TYPE_REQUIREMENT(param.t1, double) };
00039
00040 template<typename Param>
00041 concept HasPsiApproximationParameters = requires (Param param)
        { TYPE_REQUIREMENT(param.M, long long int) };
00043
00044
00053 template<typename Vector>
00054 struct WorkspaceGeneric {
00055 typedef Vector State;
        long long int N;
00057
        double L;
00058
        double m;
00059
        StaticEOSCosmology cosmology{};
00060
        State state;
00061
        double lambda{0};
        double f_a{1.0};
00062
00063
        Vector Psi;
00064
        Vector dPsidt;
00065
        Vector Psi_fft;
00066
        Vector dPsidt_fft;
00067
        Vector R fft;
00068
        std::vector<double> t_list;
00069
        typename fftWrapperDispatcher<Vector>::Generic fft_wrapper;
00070
00071
        bool Psi_approximation_initialized{false};
        long long int M;
std::unique_ptr<typename fftWrapperDispatcher<Vector>::Generic> fft_wrapper_M_ptr;
00072
00073
00074
        Vector cutoff R fft:
00075
00076
        template<HasLatticeParams Param>
00077
        WorkspaceGeneric(const Param &param, auto &initializer) :
00078
          N(param.N), L(param.L), fft_wrapper(param.N)
00079
          //static_assert(HasLatticeParams<Param>, "HasLatticeParams<Param> test failed.");
08000
          if constexpr(HasFRWParameters<Param>) { cosmology = StaticEOSCosmology(param); }
00081
00082
          if constexpr(HasMass<Param>) { m = param.m; }
00083
          if constexpr(HasLambda<Param>) { lambda = param.lambda; }
00084
          if constexpr(HasFa<Param>) { f_a = param.f_a; }
          if constexpr(HasPsiApproximationParameters<Param>) { M = param.M;
assert(N >= M); }
00085
00086
00087
          initializer(param, *this);
00088
00089 };
00090
00091
00092
00093
00094 #endif
```

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