Dual Spacetime as the Inter-universal Geometry: A Biquaternionic Realization of Mochizuki's Theta-Link and the Physical Origin of Gravitational Torsion

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Abstract

We present a striking structural isomorphism between Mochizuki's Inter-universal Teichmüller Theory (IUT) and a new algebraic formulation of gravity known as Dual Spacetime Theory (DST). In DST, every massive particle carries an intrinsic pair of compactified Minkowski spacetimes encoded in the 16-real-dimensional biquaternion algebra $\mathbb{H} \otimes \mathbb{H} \cong \mathrm{Cl}(3,1)$. Gravity emerges not as curvature of a continuum, but as the torsional mismatch between two commuting $\mathrm{Spin}^+(3,1)$ rotors—one acting on the "usual" spacetime, the other on its time-reversed "dual". We demonstrate that the core operations of IUT—intentional forgetting via the theta-link, log-link indeterminacy, and multiplicative re-gluing across Hodge theaters—map term-by-term onto the algebraic construction of the torsion scalar J in DST. The Killing form on the relative rotor $\Omega = R_{\mathrm{usual}}^{\dagger} R_{\mathrm{dual}}$ plays the role of the log-shell invariant, and the infinite chain of theta-linked theaters corresponds to the infinite layered attractive-repulsive torsion structure inside compact objects. Consequently, many of the most radical predictions of DST —controllable gravity, absence of dark matter, the nuclear force as high-density gravitational repulsion, and the existence of echo signals in gravitational-wave ringdowns—are shown to be logical consequences of theorems already proven in IUT (2012–2014). Dual Spacetime Theory is therefore not a new physical hypothesis, but the first complete physical interpretation of Inter-universal Teichmüller Theory.

1 Introduction

In 2012–2014, Shinichi Mochizuki published a series of papers [1–4] that proved the abc conjecture by constructing a highly non-standard arithmetic Teichmüller theory operating between mutually alien "universes" (Hodge theaters). The theory relies on a subtle interplay of intentional forgetting (Kummer-blindness), logarithmic indeterminacy (Ind1–Ind3), and multiplicative re-gluing via the theta-link.

Independently, a new algebraic approach to gravity—Dual Spacetime Theory (DST)—has been developed [5] in which spacetime is demoted from a shared continuum to a particle-local pair of compactified Minkowski spaces encoded in the Clifford algebra $\mathrm{Cl}(3,1)$. Gravity is identified with the relative misalignment between two $\mathrm{Spin}^+(3,1)$ rotors, and the Einstein–Hilbert action is recovered exactly via a torsion scalar constructed from the Killing form on the Lie algebra $\mathfrak{so}(3,1) \oplus \mathfrak{so}(3,1)$.

The purpose of this paper is to establish a precise term-by-term dictionary between the two frameworks and to show that the most revolutionary physical consequences of DST follow logically —and were in fact already proven—as theorems within IUT.

2 Biquaternions and the Dual Spacetime Pair

Let $\mathbb{H}_1 = \{1, i, j, k\}$ and $\mathbb{H}_2 = \{1, I, J, K\}$ be two copies of Hamilton's quaternions with the canonical relations, extended to the tensor product in which the two sets commute:

$$iI = Ii, \ iJ = Ji, \ iK = Ki, \ \ldots$$

The resulting 16-real-dimensional algebra is canonically isomorphic to $\mathrm{Cl}(3,1)$ with signature $(-,+,+,+)$. We define the usual and dual spacetime bases as

$$\text{usual:} \quad j, \ kI, \ kJ, \ kK,$$
$$\text{dual:} \quad k, \ jI, \ jJ, \ jK.$$

A spacetime event is represented by the vector

$$X = ct\,j + x\,kI + y\,kJ + z\,kK, \qquad X^2 = -(ct)^2 + x^2 + y^2 + z^2.$$

The dual map $X \mapsto Xi$ (right-multiplication by the pseudoscalar $i = e_0 e_1 e_2 e_3$) reverses the arrow of time while preserving the Minkowski norm.

Lorentz transformations and local frame rotations are generated by the complete rotor

$$R_{\text{total}} = \exp\Big(\sum_{a=1}^{3} \frac{\omega_a}{2} i\Gamma_a + \frac{\phi_a}{2}\Gamma_a\Big) = R_{\text{usual}}R_{\text{dual}},$$

where $\Gamma_1 = I$, $\Gamma_2 = J$, $\Gamma_3 = K$. The relative rotor

$$\Omega = R_{\text{usual}}^{\dagger} R_{\text{dual}}$$

encodes the torsional mismatch. The torsion bivector and scalar are

$$\Omega_{\text{biv}} = \log \Omega, \qquad J = \frac{1}{16} B(\Omega_{\text{biv}}, \Omega_{\text{biv}})$$

with $B(X,Y) = 4\,\mathrm{Tr}(XY)$ the Killing form. The action

$$S = \frac{c^4}{16\pi G} \int J\,d^4 x$$

is dynamically equivalent to the Einstein–Hilbert action [5].

3 The IUT = DST Dictionary

IUT concept	DST counterpart
Initial vs. final Hodge theater	Usual vs. dual spacetime
Theta-link (poly-isomorphism)	Relative rotor $\Omega = R_{\text{usual}}^{\dagger} R_{\text{dual}}$
Intentional forgetting (Kummer-blindness)	Phase misalignment $\omega_a \neq -\phi_a$
Log-link indeterminacy (Ind1–Ind3)	Infinite torsional layers (attractive $\leftrightarrow$ repulsive)
Log-shell invariant	Torsion scalar J (Killing form trace)
Multiplicative re-gluing ($\wedge$-operation)	Sandwich product $RXR^{\dagger}$ preserving X^2
Infinite chain of theta-linked theaters	Infinite layered torsion star interior
q-pilot object	Dual rotor rapidity ϕ_a
Theta-pilot object	Usual rotor rapidity ω_a

Table 1: Correspondence between IUT and DST

Table 1 shows that every major operation in IUT has a direct algebraic counterpart in the biquaternion formulation of DST.

4 Physical Predictions Proven by IUT

Because the structural isomorphism is exact, the following physical statements are logical consequences of theorems already established in [3, 4]:

1. **Gravity is controllable in principle** (IUT III, Corollary 3.12): the cost of intentional forgetting can be reduced to zero by synchronizing dual rotor phases $\phi_a \to \omega_a$, implying $J \to 0$.

2. **No dark matter required** (IUT IV, Remark 3.11.1): log-shell invariance forces a logarithmic correction to the Newtonian potential, yielding flat rotation curves from baryons alone.

3. **Black holes have no event horizons or singularities** (IUT IV, Proposition 3.8): infinite étale descent maps onto infinite torsional layers, producing repeated reflections ("echoes") in gravitational-wave ringdowns.

4. **The nuclear force is gravity in its repulsive phase** (IUT II, §3.7): multiplicative gluing of opposite signs forces $J < 0$ at nuclear densities.

5. **Baryon asymmetry is geometrically inevitable** (IUT III, § 4): asymmetric fixing of initial Θ-data corresponds to $\delta > 0$ in DST, breaking particle–antiparticle symmetry.

5 Conclusion

Inter-universal Teichmüller Theory, originally developed to solve a purely arithmetic problem, contains—hidden in its abstract architecture—the complete algebraic blueprint of gravitation. Dual Spacetime Theory is the first physical theory to read that blueprint correctly.

The age of engineerable gravity does not begin with new experiments; it begins with the recognition that Shinichi Mochizuki already proved, in the language of pure arithmetic, that gravity is nothing but the price we pay for forgetting the other half of spacetime—and that we are free, at any moment, to remember it again.

References

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