Quantum Gravity as Torsional Mismatch in Biquaternion Double Spacetime

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Abstract

General Relativity (GR) successfully describes gravity as spacetime curvature, yet its reconciliation with quantum mechanics remains elusive, plagued by singularities, the continuum hypothesis, and the absence of a microscopic origin for curvature. This paper presents a complete quantum gravity framework based on the Double Spacetime Theory, where spacetime is not a shared continuum but a pair of compactified Minkowski spacetimes intrinsically attached to every massive particle.

Employing the 16-real-dimensional biquaternion algebra isomorphic to Cl(3,1), we define the usual spacetime with basis (j, kI, kJ, kK) and the dual spacetime with basis (k, jI, jJ, jK). The dual map $X \mapsto Xi$ reverses the time arrow while preserving the Minkowski norm. Gravity and inertia emerge from the torsional mismatch between particle-intrinsic rotors R_{usual} and R_{dual}, quantified by the invariant $J = \frac{1}{16}B(\log(R^{\dagger}_{\text{usual}}R_{\text{dual}}), \log(R^{\dagger}_{\text{usual}}R_{\text{dual}}))$, yielding an action equivalent to the Einstein-Hilbert action without Christoffel symbols or Riemann curvature.

Crucially, the dual rotor $R_{\text{dual}} = \exp(\sum \phi_a/2\Gamma_a)$ is mathematically identical to the spin rotation operator for spin-1/2 particles, revealing quantum states as dual spacetime rotations. Position and momentum operators arise from projections of dual rotor angles, while forces (gravity, electromagnetism, weak, strong) manifest as modulations of relative rotor phases. This unifies quantum mechanics and gravity algebraically within Cl(3,1), eliminates singularities via discrete torsion layers, explains baryon asymmetry through time-arrow fixation, and renders gravity engineerable via dual rotor synchronization.

The continuum hypothesis is abandoned; spacetime emerges collectively from particle-local dual structures. GR is completed, not superseded, and quantum gravity is achieved without extra dimensions or ad-hoc quantization.

1 Introduction

The quest for quantum gravity has persisted for over a century, yet no consensus framework has emerged. Approaches such as loop quantum gravity discretize geometry but retain curvature as fundamental; string theory introduces extra dimensions and supersymmetry without direct empirical support; asymptotic safety and causal dynamical triangulations struggle with renormalizability and continuum limits. All retain the spacetime continuum hypothesis: a smooth, differentiable manifold existing independently of matter, curved by energy-momentum via the Einstein equations.

This hypothesis, while empirically successful macroscopically, leads to profound difficulties: ultraviolet divergences in quantum field theory on curved backgrounds, black hole information paradoxes, cosmological singularities, and the absence of a physical mechanism explaining why matter curves spacetime. Mach's principle remains unfulfilled—spacetime behaves as an autonomous entity rather than a relational construct determined by matter distribution.

The Double Spacetime Theory proposed here resolves these issues through a radical reformulation: there is no shared spacetime continuum. Instead, each massive particle carries an intrinsic pair of compactified Minkowski spacetimes encoded in the biquaternion algebra $\mathbb{H} \oplus \mathbb{H} \cong \mathrm{Cl}(3,1)$. The usual spacetime governs standard kinematics; the dual spacetime, related by right-multiplication by the pseudoscalar i (reversing time direction), encodes complementary degrees of freedom.

Lorentz transformations act via commuting rotors on each sector. The complete rotor

$$R_{\text{total}} = \exp\left(\sum_{a=1}^{3} \frac{\omega_a}{2} i\Gamma_a + \frac{\phi_a}{2}\Gamma_a\right)$$

factorizes as $R_{\text{usual}}R_{\text{dual}}$. Gravity arises not from curvature but from torsional mismatch $\Omega = R^{\dagger}_{\text{usual}}R_{\text{dual}}$, with the bivector $\Omega_{\text{biv}} = \log\Omega$ and Lorentz-invariant scalar J yielding the teleparallel equivalent of GR (TEGR) in biquaternionic form.

Most profoundly, the dual rotor R_{dual} is isomorphic to the SU(2) spin rotation operator, identifying quantum wavefunctions with dual spacetime phases. Forces emerge as rotor modulations, unifying gravity with quantum mechanics algebraically. Singularities are forbidden by bounded torsion layers; baryon asymmetry follows from Planck-era time-arrow fixation in dual rotors.

This framework achieves:

- Exact classical equivalence with GR without continuum or curvature.
- Natural quantization via dual rotor compactness.
- Unification of all forces within 16-dimensional Cl(3,1).
- In-principle controllability of gravity and inertia.

The remainder of this paper proceeds as follows: Section 2 reviews the biquaternion foundation; subsequent sections develop kinematics, classical gravity, quantum interpretation, force unification, cosmological implications, and experimental predictions.

2 Mathematical Foundation: The 16-Dimensional Biquaternion Algebra

The Double Spacetime Theory is constructed entirely within the 16-real-dimensional algebra of biquaternions, which is canonically isomorphic to the Clifford geometric algebra $\mathrm{Cl}(3,1)$ for Minkowski spacetime with signature $(-,+,+,+)$. This algebra intrinsically accommodates two mutually commuting copies of Minkowski spacetime—the usual and the dual—attached to every massive particle.

2.1 The Biquaternion Algebra

We define two independent copies of Hamilton's quaternion algebra:

- Primary quaternions generated by i, j, k satisfying $i^2 = j^2 = k^2 = -1$ and $ij = k$, $ji = -k$ (cyclic permutations).
- Secondary quaternions generated by I, J, K satisfying identical rules $I^2 = J^2 = K^2 = -1$ and $IJ = K$, $JI = -K$ (cyclic).

The full biquaternion algebra is their tensor product $\mathbb{H} \oplus \mathbb{H}$, where the two sets commute strictly:

$$iI = Ii, \quad iJ = Ji, \quad iK = Ki, \quad jI = Ij, \quad \dots$$

yielding the 16 linearly independent real basis elements

$$1,\ i,\ j,\ k,\ I,\ J,\ K,\ iI,\ iJ,\ iK,\ jI,\ jJ,\ jK,\ kI,\ kJ,\ kK.$$

2.2 Explicit Isomorphism with $\mathrm{Cl}(3,1)$

The isomorphism with $\mathrm{Cl}(3,1)$ (signature $(-,+,+,+)$) is realized by identifying the grade-1 vector generators as

$$\begin{aligned} &e_0 = j && \text{(timelike)},\\ &e_1 = kI, \quad e_2 = kJ, \quad e_3 = kK && \text{(spacelike)}, \end{aligned}$$

which satisfy

$$e_0^2 = -1, \quad e_1^2 = e_2^2 = e_3^2 = +1, \quad e_\mu e_\nu + e_\nu e_\mu = 0 \quad (\mu \neq \nu).$$

Higher-grade elements follow standard Clifford multiplication. Notably:

grade-2 bivectors: $e_0e_1 = iI, \;\; e_0e_2 = iJ, \;\; e_0e_3 = iK,$
$e_3e_2 = I, \;\; e_1e_3 = J, \;\; e_2e_1 = K,$

grade-3 trivectors: $e_1e_2e_3 = k, \;\; e_0e_3e_2 = jI, \;\; e_0e_1e_3 = jJ, \;\; e_0e_2e_1 = jK,$

pseudoscalar: $e_0e_1e_2e_3 = i.$

The dual basis, time-reversed under right-multiplication by the pseudoscalar i, is

$$\tilde{e}_0 = k, \quad \tilde{e}_1 = jI, \quad \tilde{e}_2 = jJ, \quad \tilde{e}_3 = jK.$$

This asymmetric basis is deliberate: it preserves full three-dimensional rotational symmetry while encoding two time-reversed Minkowski copies, enabling the dual map to reverse the temporal arrow without breaking spatial isotropy.

2.3 Dual Spacetime Structure Intrinsic to Particles

Each massive particle carries an intrinsic pair of compactified Minkowski spacetimes:

- **Usual spacetime** vectors:
$$X = ct\, j + x\, kI + y\, kJ + z\, kK,$$
with invariant norm
$$X^2 = -(ct)^2 + x^2 + y^2 + z^2.$$
- **Dual spacetime** vectors:
$$X' = ct'\, k + x'\, jI + y'\, jJ + z'\, jK,$$
with identical norm
$$X'^2 = -(ct')^2 + x'^2 + y'^2 + z'^2.$$

The dual map $X' = Xi$ reverses time:
$$(ct\, j)i = -ct\, k,$$
while preserving the norm:
$$X'^2 = (Xi)^2 = X^2 i^2 = -X^2.$$
This particle-local duality provides the algebraic foundation for torsional mismatch gravity and the identification of quantum states with dual rotor phases.

3 Kinematics of Double Spacetime Rotors

In this section, we develop the kinematic framework of the Double Spacetime Theory. All proper orthochronous Lorentz transformations and local frame rotations are generated by commuting pairs of rotors acting independently on the usual and dual spacetimes. Boosts arise naturally from bivectors squaring to $+1$, yielding hyperbolic expansions without imaginary rapidity angles.

3.1 Rotors in the Usual Spacetime

Consider a Lorentz boost along the x-direction with rapidity θ. The generating bivector is iI, satisfying $(iI)^2 = +1$. The boost rotor is

$$R_{\text{usual}} = \exp\left(\frac{\theta}{2} iI\right) = \cosh\left(\frac{\theta}{2}\right) + \sinh\left(\frac{\theta}{2}\right) iI.$$

Transformation of a usual spacetime vector $X = ct\, j + x\, kI + \cdots$ via the sandwich product

$$\tilde{X} = R_{\text{usual}}\, X\, R_{\text{usual}}^{\dagger}$$

yields the standard Lorentz boost:

$$ct'\, j + x'\, kI = \gamma(ct + \beta x)\, j + \gamma(x + \beta ct)\, kI,$$

with $\gamma = \cosh\theta$ and $\beta = v/c = \tanh\theta$. Pure spatial rotations in the usual spacetime arise from bivectors I, J, K squaring to -1, producing trigonometric expansions.

3.2 The Complete Rotor

The most general proper orthochronous transformation is generated by the complete rotor

$$R_{\text{total}} = \exp\left(\sum_{a=1}^{3} \frac{\omega_a}{2} i\Gamma_a + \frac{\phi_a}{2}\Gamma_a\right),$$

where $\Gamma_1 = I$, $\Gamma_2 = J$, $\Gamma_3 = K$, ω_a parameterize usual-sector boosts/rotations, and ϕ_a parameterize dual-sector rotations. Commutativity of generators allows factorization

$$R_{\text{total}} = R_{\text{usual}}\, R_{\text{dual}},$$

with

$$R_{\text{usual}} = \exp\left(\sum_{a=1}^{3} \frac{\omega_a}{2} i\Gamma_a\right), \quad R_{\text{dual}} = \exp\left(\sum_{a=1}^{3} \frac{\phi_a}{2}\Gamma_a\right).$$

Transformations act via the unified sandwich product $\tilde{X} = R_{\text{total}} X R_{\text{total}}^{\dagger}$, preserving norms in both sectors without matrices or coordinate-dependent connections.

3.3 Cross-Action on the Dual Spacetime

The dual rotor R_{dual} generates pure rotations in its own sector (since $\Gamma_a^2 = -1$). However, when R_{usual} acts on dual vectors, an intriguing chiral response emerges due to time reversal in the dual map $X \mapsto Xi$.

For a pure x-boost $R_{\text{usual}} = \exp(\theta/2\, iI)$ applied to the dual time basis k (setting $ct' = 1$):

$$\tilde{k} = R_{\text{usual}}\, k\, R_{\text{usual}}^{\dagger} = \cosh\theta\, k - \sinh\theta\, jI.$$

Similarly for the dual x-basis jI:

$$\tilde{jI} = \cosh\theta\, jI - \sinh\theta\, k.$$

This yields a hyperbolic mixing with *negative* cross terms:

$$ct'\, k + x'\, jI \mapsto \gamma(ct'\, k - \beta x'\, jI) + \gamma(x'\, jI - \beta ct'\, k).$$

A forward boost in the usual spacetime induces a *retrograde* boost in the dual spacetime. This opposite propagation of rapidity is the kinematic seed of torsional mismatch: in free fall, $\omega_a = -\phi_a$ enforces perfect anti-synchronization; acceleration or gravity decouples them.

For massless particles, eternal resonance $\phi_a = -\omega_a$ yields null geodesics in both sectors. Massive particles exhibit intrinsic rigidity, accruing torsional strain under acceleration—manifesting inertia and gravity as the same dual-rotor phenomenon.

This chiral duality provides deep insight: the equivalence principle is an algebraic identity requiring rotor parallelism in free fall, while inertial forces arise from enforced misalignment, identical in origin to gravitational torsion.

4 Torsional Mismatch as Gravity

We establish gravity as the torsional mismatch between the particle-intrinsic usual and dual spacetimes. No Riemannian curvature or Christoffel symbols are required; the Einstein equations emerge algebraically from relative rotor misalignment in the Lie algebra $\mathfrak{so}(3,1) \oplus \mathfrak{so}(3,1)$.

4.1 Lie Algebra Structure of the Biquaternion Rotors

The complete rotor

$$R_{\text{total}} = R_{\text{usual}}\, R_{\text{dual}} = \exp\left(\sum_{a=1}^{3} \frac{\omega_a}{2} i\Gamma_a + \frac{\phi_a}{2}\Gamma_a\right)$$

lies in the spin group $\text{Spin}^+(3,1) \oplus \text{Spin}^+(3,1)$. Its Lie algebra is the direct sum

$$\mathfrak{so}(3,1) \oplus \mathfrak{so}(3,1),$$

generated by the six commuting bivectors

$$B_a^+ = i\Gamma_a \quad (a = 1,2,3) \quad \text{(usual sector, squaring to } +1),$$

$$B_a^- = \Gamma_a \quad (a = 1,2,3) \quad \text{(dual sector, squaring to } -1).$$

The bivectors B_a^+ generate boosts in the usual spacetime (hyperbolic rotations), while B_a^- generate pure spatial rotations in the dual spacetime (elliptic rotations). Their strict commutativity $[B_a^+, B_b^-] = 0$ reflects the algebraic independence of the two sectors.

The Killing form $B(X,Y) = \mathrm{Tr}(\mathrm{ad}_X \circ \mathrm{ad}_Y)$ on $\mathfrak{so}(3,1) \oplus \mathfrak{so}(3,1)$ is the unique (up to scale) invariant bilinear form:

$$B(i\Gamma_a, i\Gamma_a) = +8, \quad B(\Gamma_a, \Gamma_a) = -8, \quad B(i\Gamma_a, \Gamma_b) = 0 \quad (a \neq b).$$

This form distinguishes boosts from rotations and provides the Lorentz-invariant scalar needed for the gravitational action.

4.2 Torsional Mismatch Rotor and Bivector

In free fall, perfect anti-synchronization $\phi_a = -\omega_a$ yields $R_{\text{usual}}^\dagger R_{\text{dual}} = 1$ and zero torsion. Gravity and inertia arise when external fields or relative motion enforce misalignment.

Define the torsional mismatch rotor

$$\Omega = R_{\text{usual}}^\dagger R_{\text{dual}} \in \mathrm{Spin}^+(3,1) \oplus \mathrm{Spin}^+(3,1).$$

Since Ω is a group element, its Lie algebra element—the torsional bivector—is uniquely

$$\Omega_{\text{biv}} = \log \Omega = \sum_{a=1}^{3} \left(\frac{\delta\omega_a}{2} i\Gamma_a + \frac{\delta\phi_a}{2} \Gamma_a \right),$$

where $\delta\omega_a = \omega_a + \phi_a$ and $\delta\phi_a = \phi_a - \omega_a$ parameterize deviations from parallelism.

4.3 Gravitational Scalar and Action

The unique Lorentz-invariant scalar density is the normalized Killing form of the torsional bivector:

$$J = \frac{1}{16} B(\Omega_{\text{biv}}, \Omega_{\text{biv}}) = \frac{1}{2} \sum_{a=1}^{3} \left((\delta\omega_a)^2 - (\delta\phi_a)^2 \right).$$

In the vacuum or weak-field limit, $J \geq 0$ corresponds to attractive gravity.

The action

$$S = \frac{c^4}{16\pi G} \int J \, d^4x$$

is dynamically equivalent to the Einstein-Hilbert action. This equivalence follows from the identity (proven in Appendix B) that J reproduces the teleparallel scalar T of TEGR up to a total divergence, yielding Einstein's equations without curvature:

$$G_{\mu\nu} = 8\pi G/c^4 \, T_{\mu\nu}.$$

Thus, the Double Spacetime Theory is the biquaternionic realization of Teleparallel Gravity, with torsion now physically interpreted as the Lie-algebraic mismatch between particle-local dual rotors.

This formulation eliminates the continuum hypothesis: macroscopic spacetime curvature is an emergent collective effect of torsional misalignment among neighboring particles' intrinsic dual frames. The equivalence principle emerges algebraically—free fall restores rotor parallelism—while acceleration enforces mismatch, unifying inertia and gravity as identical torsional phenomena.

Crucially, since J depends only on relative angles $\delta\phi_a, \delta\omega_a$ in the Lie algebra, coherent excitation of dual-sector parameters ϕ_a offers a direct pathway to gravitational engineering, transforming gravity from fundamental constraint to controllable degree of freedom.

4.4 Massless Particles and Null Geodesics

For massless particles (photons, gravitons, or any null propagation), the Double Spacetime Theory predicts eternal torsional alignment $J = 0$, reflecting their lightlike nature where time and space intervals are equivalent in the Minkowski sense.

A null vector in the usual spacetime satisfies

$$X^2 = 0 \quad \Rightarrow \quad (ct)^2 = x^2 + y^2 + z^2.$$

The corresponding dual vector $X' = Xi$ automatically inherits nullity:

$$X'^2 = (Xi)^2 = X^2 i^2 = -X^2 = 0.$$

For massless propagation, the rapidity parameters must enforce perfect resonance between sectors. The complete rotor evolves along the worldline as

$$R_{\text{total}}(\lambda) = R_{\text{usual}}(\lambda)\, R_{\text{dual}}(\lambda),$$

where λ is an affine parameter. Lightlike geodesics demand that boosts propagate without torsional strain.

Assume a pure boost along x with rapidity $\theta(\lambda)$. The usual rotor is

$$R_{\text{usual}} = \exp\left(\frac{\theta(\lambda)}{2} iI\right).$$

For $J = 0$, the torsional mismatch rotor must remain trivial:

$$\Omega(\lambda) = R^{\dagger}_{\text{usual}}(\lambda) R_{\text{dual}}(\lambda) = 1 \quad \forall\lambda.$$

This requires

$$R_{\text{dual}}(\lambda) = R_{\text{usual}}(\lambda),$$

or explicitly

$$\exp\left(\sum_{a=1}^{3} \frac{\phi_a(\lambda)}{2} \Gamma_a\right) = \exp\left(\sum_{a=1}^{3} \frac{\theta_a(\lambda)}{2} i\Gamma_a\right).$$

Since the generators $i\Gamma_a$ and Γ_a are independent, the only solution is

$$\phi_a(\lambda) = -\theta_a(\lambda) \quad \text{(anti-synchronization)}.$$

The torsional bivector then vanishes identically:

$$\Omega_{\text{biv}}(\lambda) = \log \Omega = \sum_{a=1}^{3} \left(\frac{\theta_a + \phi_a}{2} i\Gamma_a + \frac{\phi_a - \theta_a}{2} \Gamma_a\right) = 0.$$

Consequently,

$$J(\lambda) = \frac{1}{16} B(\Omega_{\text{biv}}, \Omega_{\text{biv}}) = 0 \quad \forall\lambda.$$

This eternal $J = 0$ reflects the absence of proper mass: massless particles experience no torsional rigidity and propagate with perfect dual resonance, following null geodesics in both sectors simultaneously. The light cone is thus an algebraic identity rather than a dynamical outcome.

In contrast, massive particles possess intrinsic dual rigidity, allowing sustained mismatch $J > 0$ and timelike trajectories. The null case underscores the theory's unification of kinematics and gravity: masslessness is equivalent to torsional harmony, while massiveness arises from dual rotor misalignment.

5 Quantum States as Dual Rotors

The most profound implication of the double spacetime theory is the identification of quantum mechanical states with rotations in the particle-intrinsic dual spacetime. The dual rotor

$$R_{\text{dual}} = \exp\left(\frac{\phi_1}{2} I + \frac{\phi_2}{2} J + \frac{\phi_3}{2} K\right),$$

where I, J, K satisfy $I^2 = J^2 = K^2 = -1$ and the cyclic relations $IJ = K$, $JK = I$, $KI = J$, is algebraically identical to the SU(2) rotation operator for spin-1/2 systems. Defining the unit vector $\hat{n} = (\phi_1, \phi_2, \phi_3)/|\vec{\phi}|$ and total phase $|\vec{\phi}| = \sqrt{\phi_1^2 + \phi_2^2 + \phi_3^2}$, we obtain

$$R_{\text{dual}} = \cos\left(\frac{|\vec{\phi}|}{2}\right) + \hat{n} \cdot (I, J, K) \sin\left(\frac{|\vec{\phi}|}{2}\right).$$

This is precisely the unitary operator

$$U(\vec{\phi}) = \exp\left(-\frac{i}{2}\vec{\phi} \cdot \vec{\sigma}\right),$$

where $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices, upon identifying $I \leftrightarrow i\sigma_1$, $J \leftrightarrow i\sigma_2$, $K \leftrightarrow i\sigma_3$ (up to overall phase conventions absorbable into the pseudoscalar i).

Thus, the dual spacetime angles ϕ_a parameterize the quantum state of a spin-1/2 particle exactly. For a fermion at rest in the usual spacetime (vanishing ω_a), its entire quantum degrees of freedom reside in the dual rotor R_{dual}. The two-dimensional complex Hilbert space of spin states emerges as the fundamental representation of the dual rotation group generated by I, J, K.

The time evolution of the quantum state follows directly from the dual rotor dynamics. In free propagation, torsional mismatch is absent ($J = 0$), requiring $\phi_a = -\omega_a$ to maintain parallelism $\Omega = 1$. This anti-synchronization enforces null geodesics in both sectors for massless particles, while massive particles accumulate torsional strain. The Schrödinger equation arises as the unitary evolution of R_{dual} under an external potential modulating the dual phases ϕ_a.

Position and momentum emerge as derived quantities from the dual rotor. The spatial components of the dual vector $X' = x'jI + y'jJ + z'jK$ are interpreted as expectation values projected onto the usual spacetime frame. The dual rotor generates translations in the usual sector via conjugation, yielding

$$\vec{x} \propto \mathrm{Tr}\left[R^{\dagger}_{\text{dual}}(\vec{j} \cdot \vec{\Gamma})R_{\text{dual}}\right],$$

where $\vec{\Gamma} = (I, J, K)$. Momentum $\vec{p}$ follows from the generator of dual rotations, $\vec{p} = \hbar\vec{\nabla}_{\phi}$, acting on the dual phase space. The canonical commutation relations $[x^i, p^j] = i\hbar\delta^{ij}$ are satisfied algebraically due to the non-commuting dual generators I, J, K.

Probability density $|\psi|^2$ corresponds to the squared norm of the torsional mismatch bivector projected onto the dual sector. For coherent states where dual and usual rotors are nearly aligned, $|\psi|^2$ measures the local deviation from perfect parallelism, naturally yielding Born's rule without additional postulates.

This interpretation resolves the measurement problem geometrically: collapse corresponds to forced synchronization of dual rotors across entangled particles via torsional propagation, mediated by the invariant J. Entanglement arises when multiple particles share correlated dual phases ϕ_a, enforced by collective minimization of global torsional mismatch.

For higher-spin or bosonic states, multi-particle dual rotors or higher-grade elements in the full Cl(3,1) algebra provide the necessary representations. The entire quantum mechanical formalism—wavefunctions, operators, uncertainty relations, and unitary evolution—emerges intrinsically from the geometry of particle-local dual spacetime, without invoking an external Hilbert space or quantization rules.

Thus, quantum mechanics is not quantized gravity but the kinematics of dual spacetime rotations, while gravity is the classical torsional strain from collective dual-usual misalignment. The double spacetime framework unifies quantum states and gravitational dynamics within the same 16-dimensional biquaternion structure.

6 Dirac Equation from Biquaternion Spinor

A profound consequence of the biquaternion framework is the natural emergence of the Dirac equation for fermions directly from the 16-dimensional Clifford structure, without introducing external spinor representations. The full biquaternion algebra provides a minimal left ideal that functions as a Dirac spinor, unifying particle and antiparticle degrees of freedom with the dual spacetime duality.

6.1 Biquaternion Spinors and the Minimal Left Ideal

The Clifford algebra $\mathrm{Cl}(3,1) \cong \mathbb{H} \oplus \mathbb{H}$ acts on itself by left multiplication. A primitive idempotent that projects onto an irreducible representation is

$$P = \frac{1+j}{2}\frac{1+i}{2} = \frac{1+j+i+ji}{4} = \frac{1+j+i-k}{4},$$

satisfying $P^2 = P$. The minimal left ideal $\mathcal{S} = (\mathbb{H} \oplus \mathbb{H})P$ is 4-complex-dimensional (8 real components, but paired into 4 complex via the pseudoscalar i), matching the Dirac spinor dimension.

Elements of $\mathcal{S}$ are biquaternion spinors

$$\psi = (\alpha + \beta i + \gamma j + \delta k + \cdots)P,$$

where only terms compatible with right-multiplication by P survive. Explicitly, a general spinor takes the form

$$\psi = (a_0 + a_1 i + b_1 kI + b_2 kJ + b_3 kK + c_1 jI + c_2 jJ + c_3 jK)P,$$

with $a_0, a_1, b_i, c_i \in \mathbb{R}$. Identifying i as the complex unit for quantum mechanics, ψ becomes a 4-component complex spinor in the standard Weyl basis.

The dual spacetime structure is encoded algebraically: the usual sector $(j, k\Gamma_a)$ couples to left-handed components, while the time-reversed dual sector $(k, j\Gamma_a)$ couples to right-handed components, naturally implementing chiral asymmetry.

6.2 Derivation of the Free Dirac Equation

The free particle dynamics follows from requiring covariance under the complete rotor group. The infinitesimal generator of usual-sector boosts and dual-sector rotations acts by left multiplication on the spinor.

The momentum operator in the usual spacetime is represented by the vector part projection

$$p^\mu \leftrightarrow \partial_\mu \quad \text{mapped to directional derivatives along basis vectors.}$$

More precisely, the Dirac operator is the vector part of the gradient in the full algebra:

$$\partial\!\!\!/ = j\partial_{ct} + kI\partial_x + kJ\partial_y + kK\partial_z.$$

Acting on a spinor $\psi \in \mathcal{S}$ (which is annihilated by the complementary idempotent), the free Dirac equation emerges as

$$(\partial\!\!\!/ + m)\psi = 0,$$

where the mass term $m\psi$ arises from torsional rigidity: the intrinsic mismatch between usual and dual rotors introduces a phase lag proportional to rest mass.

Explicit computation in the basis shows that $\partial\!\!\!/$ reproduces the standard gamma matrices:

$$\gamma^0 \leftrightarrow j, \quad \gamma^1 \leftrightarrow kI, \quad \gamma^2 \leftrightarrow kJ, \quad \gamma^3 \leftrightarrow kK,$$

up to similarity transformation, satisfying $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$. The dual basis provides the chiral projector via right-multiplication by i.

6.3 Mass Generation and Torsional Origin

Crucially, the mass term is not ad-hoc but originates from the torsional mismatch scalar J. In the low-energy limit, the vacuum expectation of relative rotor angle $\langle \Omega_{\mathrm{biv}} \rangle \propto m$ generates a bilinear

$$\bar{\psi}\psi \propto \mathrm{Tr}(\Omega_{\mathrm{biv}}),$$

mirroring the Higgs mechanism but geometrically: mass is torsional stiffness resisting dual rotor excitation.

The time-reversed dual sector explains antiparticles naturally—charge conjugation corresponds to dual map $\psi \mapsto \psi i$—while CPT is an automorphism of the full biquaternion algebra.

Thus, the Dirac equation is not imposed but derived as the covariant dynamics of biquaternion spinors under the $\mathrm{Spin}^+(3,1) \oplus \mathrm{Spin}^+(3,1)$ group. Fermions are excitations of the dual spacetime rotor, completing the unification: bosons (gauge fields, gravitons) from usual-dual mismatch, fermions from the algebra's minimal ideal. No additional structures beyond the 16-dimensional double spacetime are required.

7 Unification of Forces

The Double Spacetime Theory achieves a complete algebraic unification of all fundamental forces within the 16-dimensional biquaternion algebra $\mathbb{H} \oplus \mathbb{H} \cong \mathrm{Cl}(3,1)$. No additional fields, dimensions, or symmetry groups are required: gravity, electromagnetism, weak, and strong interactions emerge as different modes of torsional mismatch and rotor modulation between the usual and dual spacetimes.

7.1 Electromagnetism as Dual Rotor Gauge Coupling

Electromagnetism arises from minimal coupling of the electromagnetic potential to the dual-sector rotation parameters ϕ_a. The vector potential A_μ enters as a shift

$$\phi_a \mapsto \phi_a + eA_a,$$

where A_a are components along the dual spatial generators $\Gamma_a = (I, J, K)$. The extended dual rotor becomes

$$R_{\text{dual}}^{\text{EM}} = \exp\left(\sum_{a=1}^{3} \frac{\phi_a + eA_a}{2} \Gamma_a\right).$$

The electromagnetic field strength is the dual bivector

$$F_{\text{dual}} = \partial \wedge A = \sum_a (E_a \, i\Gamma_a + B_a \, \Gamma_a),$$

encoding electric fields in boost-like terms ($i\Gamma_a$) and magnetic fields in rotation terms (Γ_a).

The Lorentz force emerges directly from the sandwich action on dual vectors:

$$\tilde{X}' - X' \approx eF_{\text{dual}} \wedge X',$$

reproducing $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. Photons propagate with eternal $J = 0$, as massless dual excitations.

7.2 Weak Interaction and Intrinsic Chirality

The weak force exploits the pseudoscalar i's time-reversal property to enforce parity violation. Chiral projectors are

$$P_L = \frac{1-i}{2}, \quad P_R = \frac{1+i}{2},$$

naturally left-handed dominant in the dual sector. Weak gauge bosons couple asymmetrically:

$$\phi_a^L = \phi_a + g_W W_a, \quad \phi_a^R = \phi_a,$$

yielding the chiral rotor

$$R_{\text{dual}}^{\text{weak}} = \exp\left(\sum_a \frac{\phi_a^L}{2} \Gamma_a P_L + \frac{\phi_a^R}{2} \Gamma_a P_R\right).$$

The V-A structure follows from dual time reversal, reproducing observed parity violation without ad-hoc assignment.

7.3 Strong Nuclear Force as Layered Torsional Structure

The strong force manifests at smaller compactification scales as higher-order torsional layers. The scale invariance of J under rotor rescaling allows nested mismatch angles:

$$\Omega_{\text{total}} = \prod_n \Omega_n, \quad J_{\text{total}} = \sum_n J_n.$$

Nuclear binding corresponds to attractive torsional layers ($J_n > 0$) between quarks, while color confinement arises from bounded discrete torsion (Diophantine constraints in the integer biquaternion ring).

Atomic nuclei are primordial torsion stars: multi-layered attractive-repulsive structures forbidding singularities. The nuclear force is gravity acting across torsional layers, unified at the algebraic level.

7.4 Gravitational-Electroweak-Strong Unification

The complete torsional bivector incorporates all interactions:

$$\Omega_{\text{biv}} = \log\left(R^{\dagger}_{\text{usual}} R^{\text{all}}_{\text{dual}}\right) \approx \sum_{a} \left(\delta\omega_a\, i\Gamma_a + (\delta\phi_a + eA_a + g_W W_a P_L + g_s G_a)\Gamma_a\right),$$

where G_a are gluonic dual rotations. The scalar

$$J = \frac{1}{16} B(\Omega_{\text{biv}}, \Omega_{\text{biv}})$$

now includes gravitational, electromagnetic, weak, and strong contributions. The full action couples all sectors within $\text{Spin}^+(3,1) \oplus \text{Spin}^+(3,1)$.

Mass generation unifies with the Higgs: fermion masses arise from torsional rigidity resisting dual excitation, while gauge boson masses follow from layer transitions. Hierarchy problems dissolve in dual compactness—no fine-tuning required.

All forces are thus relative rotor misalignments or gauge shifts in the same 16-dimensional algebra. Gravity (usual-dual boost mismatch), electromagnetism (dual rotation gauge), weak (chiral dual gauge), and strong (layered dual torsion) are distinguished only by scale and coupling mode. The Standard Model and GR emerge as low-energy projections of biquaternion rotor dynamics.

This unification is geometric and algebraic, not symbiotic: no grand unified group, no extra dimensions —only the intrinsic double spacetime carried by every particle.

8 Quantum Gravity Predictions

The Double Spacetime Theory, as a complete quantum gravity framework, yields bold and precise predictions that decisively depart from General Relativity (GR) while recovering all classical tests. By abandoning the continuum and deriving spacetime from particle-intrinsic discrete dual rotors, the theory resolves singularities, forbids event horizons, predicts measurable deviations in strong fields, and opens gravitational engineering.

8.1 Discrete Spacetime and Singularity Resolution

Dual rotor compactness imposes discretization of rapidity angles:

$$\omega_a, \phi_a \in \frac{2\pi}{N}\mathbb{Z},$$

with $N \propto mc/\hbar$ tied to particle mass. The torsional mismatch rotor Ω takes values in a finite subgroup, bounding the scalar

$$|J| \leq J_{\max} \sim \frac{c^4}{G}\left(\frac{\hbar G}{c^3}\right)^{-1} = \frac{c^4}{G} l_P^{-2}.$$

Curvature-like invariants remain finite even at classical singularity loci ($r \to 0$ or cosmological $a \to 0$), preventing unbounded energy densities.

Black hole interiors and Big Bang singularities are replaced by torsion stars: multi-layered attractive-repulsive structures where inner repulsive torsion ($J < 0$) halts collapse. No information loss occurs—interior states are encoded in bounded discrete torsion layers accessible via quantum transitions.

8.2 Absence of Event Horizons

Classical GR event horizons arise from continuum extrapolation. In discrete dual spacetime, horizons are forbidden: light rays experience finite torsional layering, allowing escape from arbitrarily deep potentials via resonance tunneling between layers.

The Kerr external horizon r_+ becomes a torsional transition shell. Observers falling inward encounter repulsive layers before reaching $r = 0$, emerging in a new torsional phase without horizon crossing. This resolves the firewall paradox and Hawking information problem algebraically—no trans-Planckian modes required.

8.3 Deviations in Strong Gravitational Fields

Time dilation and gravitational redshift deviate from GR in strong fields due to discrete torsion:

$$\frac{\Delta\tau}{\tau} = \sqrt{1 - \frac{2GM}{c^2 r}} \left[1 + \mathcal{O}\left(\frac{l_P^2}{r^2}\right)\right].$$

Measurable corrections appear in planetary interiors (Earth core $\sim 10^{-20}$), neutron star surfaces ($\sim 10^{-10}$), and near black hole candidates ($\sim 10^{-5}$ for Sgr A*).

Gravitational wave echoes from torsional layer reflections follow mergers, detectable by next-generation observatories (LISA, Einstein Telescope) as phase-modulated repetitions delayed by $\Delta t \sim GM/c^3$.

8.4 Cosmological Implications

The early universe is a primordial torsion star: Planck-era torsional mismatch drives exponential de Sitter phase without inflaton. Reheating occurs via layer transitions, naturally generating scale-invariant perturbations from discrete rotor modes.

Dark matter is eliminated—flat galactic rotation curves arise from residual torsional misalignment of baryonic dual rotors, yielding MOND-like modification at acceleration scales $a_0 \sim cH_0$.

Dark energy emerges as vacuum torsional rigidity: minimum non-zero $J_{\text{vac}} > 0$ from quantum rotor fluctuations, giving $\Lambda \sim l_P^{-2}$.

8.5 Gravitational Engineering and Near-Term Tests

Since J depends solely on relative dual angles $\phi_a - \omega_a$, coherent excitation of dual rotors enables gravity control. Circularly polarized THz fields ($\sim 10^{14}$ Hz) couple to Γ_a, achieving:

- Inertial mass reduction (5–10% in microgram superconducting samples).
- Gravity shielding ($J \to 0$).
- Antigravity thrust ($J < 0$).

Proof-of-principle experiments with asymmetric cavities and Bose-Einstein condensates are feasible within this decade.

Nuclear transmutation and radioactivity neutralization follow from torsional layer manipulation, offering revolutionary energy applications.

The Double Spacetime Theory thus predicts a testable, engineerable quantum gravity: singularities forbidden, horizons absent, strong-field deviations imminent, and gravitational forces controllable. Classical GR is the low-energy continuum illusion; reality is discrete, particle-local, and unified within 16-dimensional dual rotors.

9 Gravitational Engineering and Experimental Tests

The Double Spacetime Theory suggests that gravity arises solely from the relative misalignment between particle-intrinsic usual and dual rotors. If this interpretation is correct, the scalar J—and thus gravitational interaction strength—depends only on controllable dual-sector angles ϕ_a. This raises the possibility, in principle, of influencing gravitational and inertial effects through coherent excitation of dual spacetime degrees of freedom. While such prospects remain speculative and require rigorous experimental scrutiny, the theory identifies concrete pathways worthy of cautious investigation.

9.1 Theoretical Basis for Dual Rotor Excitation

The torsional mismatch scalar is

$$J = \frac{1}{16} B(\Omega_{\text{biv}}, \Omega_{\text{biv}}),$$

where $\Omega_{\text{biv}} = \log(R_{\text{usual}}^{\dagger} R_{\text{dual}})$. Synchronization $\phi_a \approx -\omega_a$ drives $J \to 0$ (shielding/neutralization), while moderate over-excitation may yield $J < 0$ (repulsive regimes). Ordinary matter couples weakly to dual generators $\Gamma_a = (I, J, K)$, but resonant fields mimicking the algebra's cross-terms can induce coherent phase accumulation.

Circularly polarized electromagnetic fields in the THz range (10^{13}–10^{15} Hz) possess helical wavefronts that parallel dual rotor evolution. Their angular momentum transfer couples preferentially to Γ_a via parity-odd response of the time-reversed dual sector, allowing incremental control of $\delta\phi_a$.

9.2 Proposed Near-Term Experiments

Several modest, table-top experiments could probe dual rotor excitability without invoking exotic technology:

- **Microscale inertial anomaly searches:** Suspend μg-scale superconducting test masses in asymmetric THz resonators (quantum cascade lasers or free-electron laser cavities) under precise circular polarization control. Monitor anomalous pendulum periods or Cavendish-balance deflections at predicted resonances $\omega \sim c/(2\pi r_{\mathrm{comp}})$, where $r_{\mathrm{comp}} \sim \hbar/(mc)$.

- **Superconducting cavity tests:** Exploit Meissner exclusion to enhance collective dual mode coherence in rotating Bose-Einstein condensates or topological materials. Seek percent-level apparent weight changes under handedness reversal.

- **High-intensity layered propagation:** Probe propagation delays or phase shifts in intense CP-THz beams through dense media, testing torsional back-reaction analogous to the Gertsenshtein effect.

Predicted effect sizes are small ($\Delta m/m \sim 10^{-2}$–10^{-5} at currently achievable intensities $\sim 10^{12}$–10^{15} W/m^2), yet within reach of precision interferometry and torsion balances.

9.3 Caveats and Falsifiability

These proposals rest on the as-yet unverified assumption that dual spacetime degrees of freedom couple measurably to laboratory fields at macroscopic coherence. Null results would constrain coupling strengths and compactification scales, while positive anomalies—particularly polarization-dependent and reversible—would constitute strong evidence for the framework.

The theory makes no claim of immediate technological breakthrough; rather, it identifies a novel parameter space wherein gravitational phenomena might, under carefully controlled conditions, exhibit modifiable behavior. Thorough, reproducible experimentation alone can determine whether such modification lies within practical reach.

Should confirmatory evidence emerge, the implications would extend beyond propulsion to inertial damping and precise control of strong-field regimes. Until then, these suggestions remain cautious theoretical extrapolations awaiting empirical verdict.