Linear Theory of Hypersonic Shocks Interacting with Turbulence in Air

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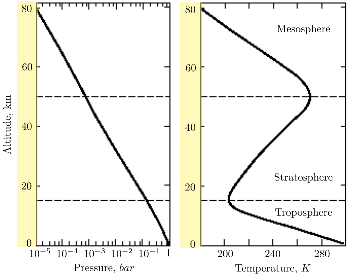
Motivation

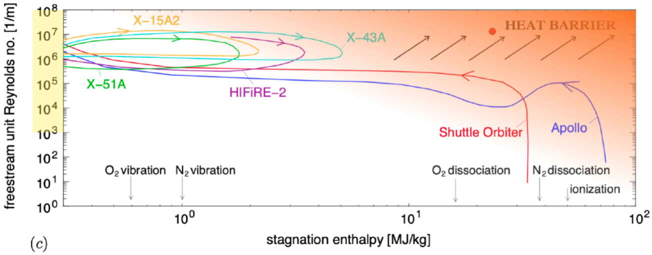


$$\frac{\Delta p}{p} \sim \frac{\Delta \rho}{\rho} = O(10^5), \quad \frac{\Delta T}{T} = O(1).$$

In hypersonic flight near the ground, the Reynolds number becomes large because of the comparatively larger densities

$$\frac{\Delta Re}{Re} = O(10^5).$$

Urzay, J., & Di Renzo, M. (2021). Annual Research Briefs, Center for Turbulence Research, 7-32.



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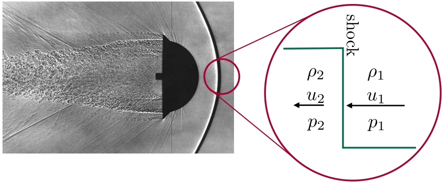
Hypersonic flight at low altitudes is characterized by:

- High free-stream Mach numbers $\mathcal{M} \geq 5$
- High free-stream and post-shock unit Reynolds numbers $Re \sim 10^7 - 10^9$ m^{-1}
- High stagnation enthalpies $h_0 \sim 5 - 30$ MJ/kg
- Small mean free paths $\lambda \sim 0.1\ \mu$m

- large normal Mach numbers
- turbulent boundary layers
- much higher than the vibrational specific energies of O_2 and N_2
- short vibrational relaxation distances

Base model for single diatomic species

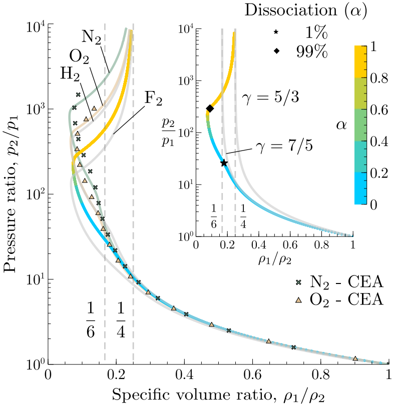


Integral conservation equations accross shock waves in dissociating gases

$$\rho_1 u_1 = \rho_2 u_2,$$
$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2,$$
$$e_1 + p_1/\rho_1 + u_1^2/2 = e_2 + p_2/\rho_2 + u_2^2/2 + q_d,$$

Upstream flow is sufficiently cold to be approximated as

$$e_1 = (5/2) R_{g,\mathrm{A}_2} T_1, \quad p_1 = \rho_1 R_{g,\mathrm{A}_2} T_1.$$



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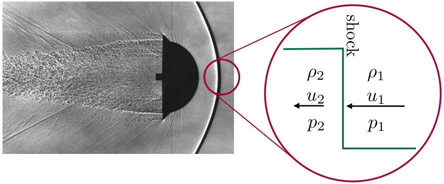

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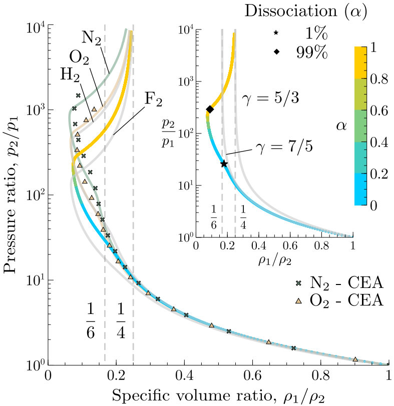
$$p_2 = \rho_2 R_{g,\mathrm{A}_2} T_2 (1+\alpha), \quad q_d = \alpha R_{g,\mathrm{A}_2} \Theta_d,$$

$$e_2 = R_{g,\mathrm{A}_2} T_2 \left[3\alpha + (1-\alpha)\left(\frac{5}{2} + \frac{\Theta_v/T_2}{\mathrm{e}^{\Theta_v/T_2} - 1} \right) \right],$$

$$\frac{\alpha^2}{1-\alpha} = G m \Theta_r \left(\frac{\pi m k_B}{\hbar^2} \right)^{3/2} \frac{\sqrt{T_2}}{\rho_2} \mathrm{e}^{-\frac{\Theta_d}{T_2}} \left(1 - \mathrm{e}^{-\frac{\Theta_v}{T_2}} \right),$$



where α is the degree of dissociation, defined as the mass fraction of A atoms in the reaction $\mathrm{A}_2 \rightleftharpoons \mathrm{A} + \mathrm{A}$, that must be solved with the aid of the chemical equilibrium condition.

A. Cuadra (presenter)
M. Vera, M. Di Renzo, C. Huete

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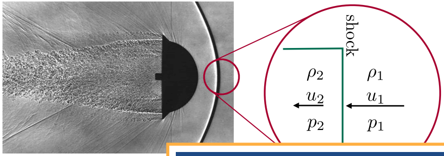

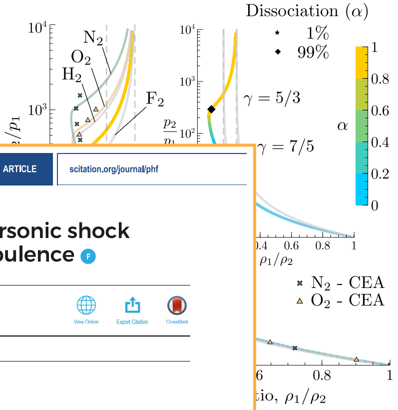

Integral conservatio
waves in dissociatin

$$\rho_1 u$$

$$p_1 + \rho_1 u$$

$$e_1 + p_1/\rho_1 + u_1^2/$$

$$p_2 = \rho_2 R_{g,A_2} T_2 (1$$

$$e_2 = R_{g,A_2} T_2 \left[3\alpha + (1-\alpha)\left(\frac{5}{2} + \frac{\Theta_v/T_2}{e^{\Theta_v/T_2} - 1}\right)\right],$$

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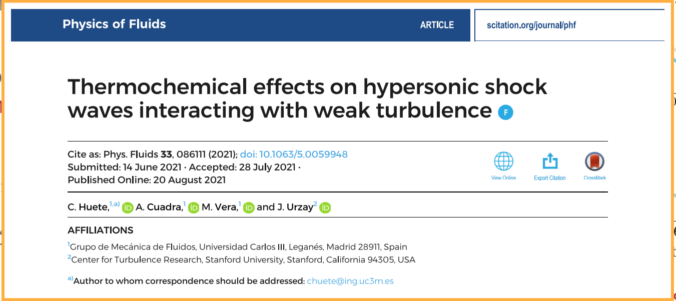

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Thermochemical effects on hypersonic shock waves interacting with weak turbulence

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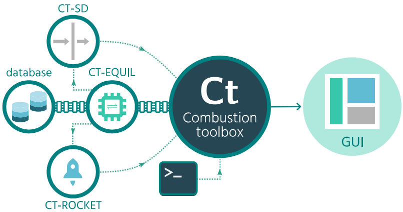
Extension to multi-species mixture and considering electronic excitation and ionization



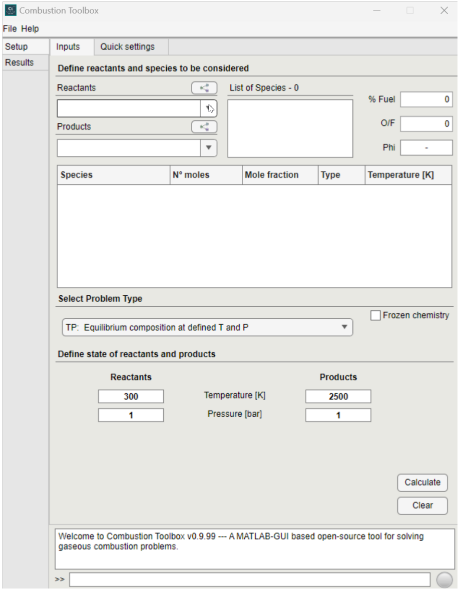
Combustion Toolbox is used to solve the Rankine-Hugoniot (RH) relations

$$p_2/p_1 = 1 - \rho_1 u_1^2/p_1 \left(\rho_1/\rho_2 - 1\right),$$
$$h_2 = h_1 + u_2^2/2 \left[1 - \left(\rho_1/\rho_2\right)^2\right].$$

These equations are supplemented with:

- the ideal-gas equation of state $p = \rho R_g T$,
- NASA-9 coefficient polynomials database to model the thermodynamic functions.

Combustion Toolbox is in excellent agreement with NASA's CEA and CANTERA within SD-Toolbox.



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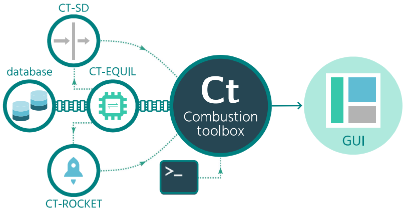

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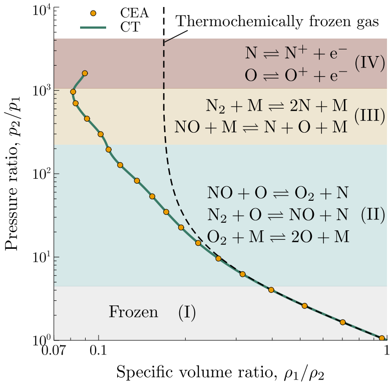
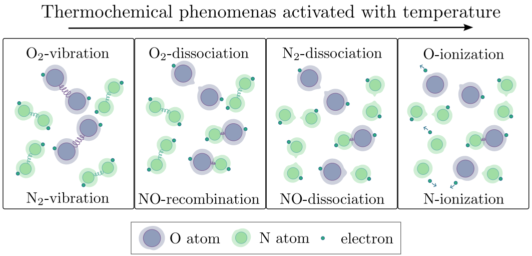


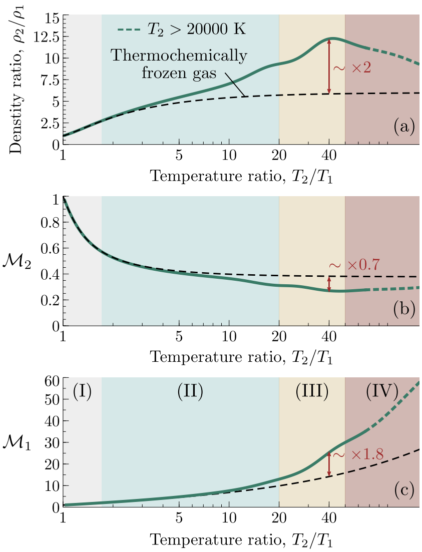
- Region (I): frozen chemistry.
- Region (II): mainly O_2 dissociation and recombination.
- Region (III): mainly N_2 dissociation and recombination.
- Region (IV): mainly electronic excitation and ionization.



Endothermicity due to dissociation, vibrational excitation, electronic excitation and ionization does the following:

- increases the mean post-shock density,
- decreases the mean post-shock velocity,
- decreases the mean post-shock temperature,
- which implies an increase in the mean pre-shock velocity.

Jump conditions



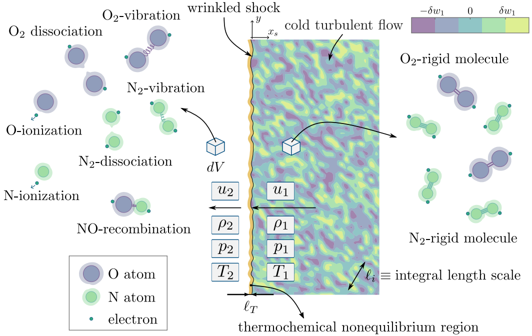
LIA of turbulence interacting with hypersonic shocks

Considering:

- turbulence is comprised of small fluctuations (weak),
- pre-shock turbulence is isotropic (no privileged direction),
- turbulent field composed of the superposition of vortical disturbances,

We can solve this problem analytically by using

- linearized Rankine-Hugoniot relations,
- linearized Euler equations in the post-shock gas.



Limits of validity

Assumptions standard LIA:

(a) $\mathrm{rms}(u_\ell) \ll a_1$ and a_2,

(b) $\xi_s \ll \ell$,

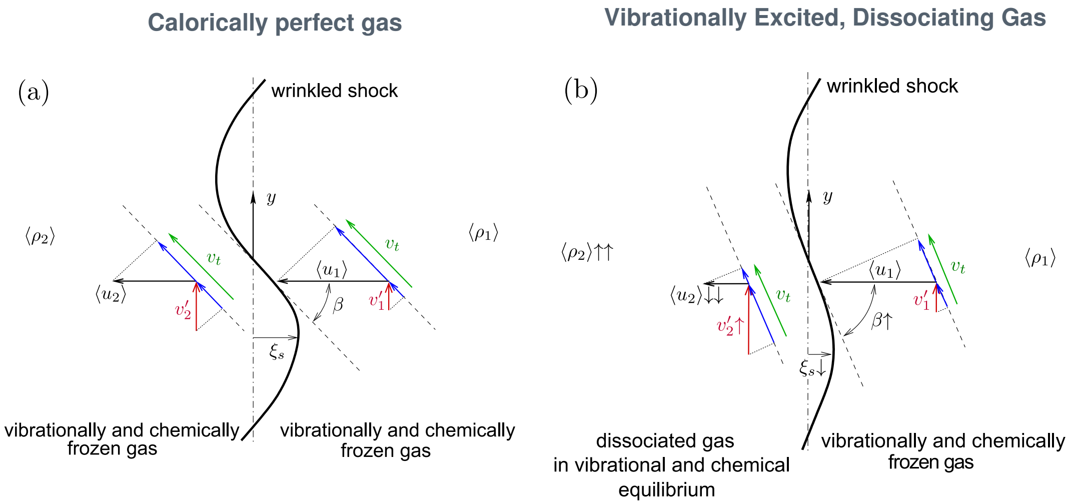
(c) $\ell/u_\ell \ll \ell^2/\nu$.

With thermochemical effects:

(d) $\ell_T \ll \ell$

For a given ℓ, this condition becomes increasingly more accurate as the altitude decreases.

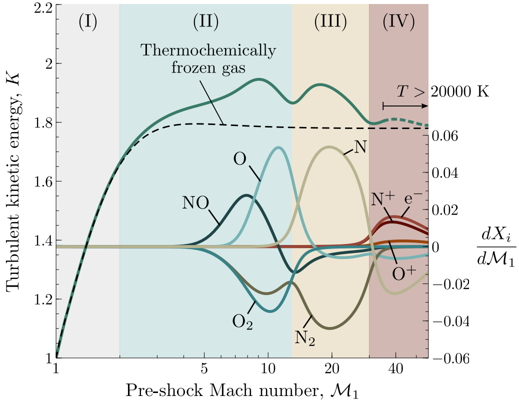
LIA of turbulence interacting with hypersonic shocks: amplification of TKE



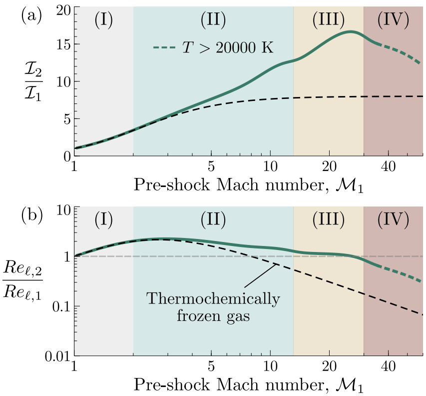
Conservation of tangential momentum dictates that the transverse velocity fluctuations should increase across the shock – these are larger at hypersonic velocities because of the associated larger post-shock densities induced by endothermic thermochemical effects.

LIA of turbulence interacting with hypersonic shocks: integral formulas

Turbulent Kinetic Energy



Turbulence intensity and Turbulent Reynolds number



In contrast, the incorporation of dissociation, vibrational excitation (regions II, III, IV), electronic excitation (regions III, IV) and ionization (region IV) predicts larger kinetic energy and turbulence intensity amplification rates, along with an increase in the turbulent Reynolds number accross the shock.

A. Cuadra (presenter)
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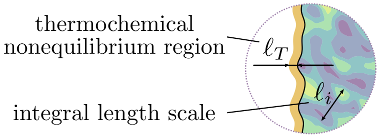
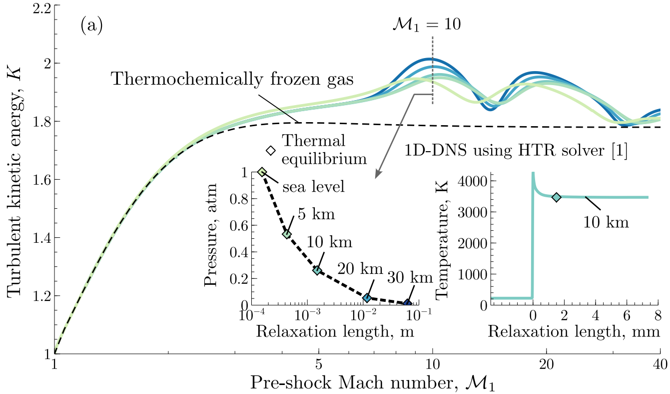
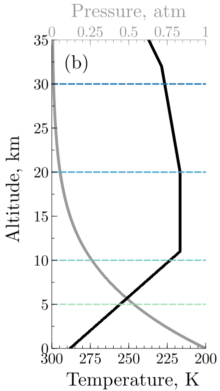
Flight altitude effects

- LIA with thermochemical effects demands $\ell_T/\ell_i \ll 1$
- ℓ_T varies with upstream conditions, i.e., with altitude
- An increase of pre-shock Mach number decreases ℓ_T
- An increase of altitude increases ℓ_T







[1] Di Renzo, M., Fu, L., and Urzay, J., "HTR solver: An open-source exascale-oriented task-based multi-GPU high-order code for hypersonic aerothermodynamics," Comput. Phys. Commun., Vol. 255, 2020, p. 107262.

Conclusions

Key takeaways

- The RH jump conditions have been computed using Combustion Toolbox*, an in-house thermochemical code capable of capturing high-temperature phenomena (dissociation, ionization, and recombination).
- Qualitative picture remains intact compared with the theoretical results obtained in [Physics of Fluids, 33(8), 086111 (2021)], which only accounted for vibrational and dissociation effects of single-species diatomic gases.
- Multi-species effects reshape the TKE curve by rendering two maxima that fit fairly well within the O_2 and N_2 dissociation processes.
- **Thermochemical effects arising at hypersonic velocities appear to enhance turbulent fluctuations in the post-shock gas.**

Get the code

* It is available under an open-source GPLv3 license via `https://combustion-toolbox-website.readthedocs.io`.

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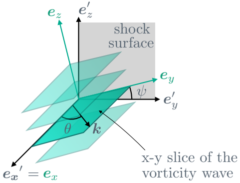
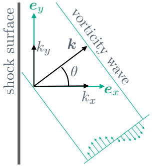
The amplitude of the vorticity fluctuations

$$\Omega = \begin{cases} \Omega_l & \text{if } \zeta \leq 1 \\ \Omega_s & \text{if } \zeta \geq 1 \end{cases}$$

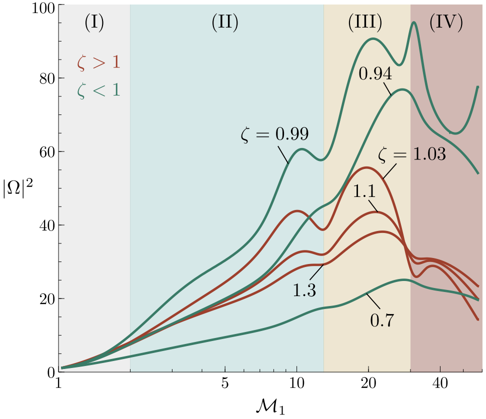
depends of the frequency parameter ζ:

- long-wavelength regime $\zeta < 1$ (vortical mode)
- short-wavelength regime $\zeta > 1$ (vortical and acoustic modes)





Square of the vorticity amplitude



Anticipating that the pre-shock turbulence is isotropic, there is no privileged direction of the wavenumber vector $\boldsymbol{k}$, which allows transforming the 3D problem into a 2D one with a simple rotation of the reference frame.

Supplementary material: monochromatic waves

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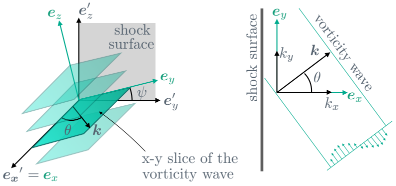

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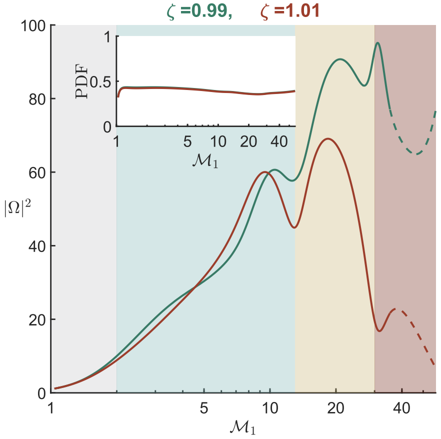

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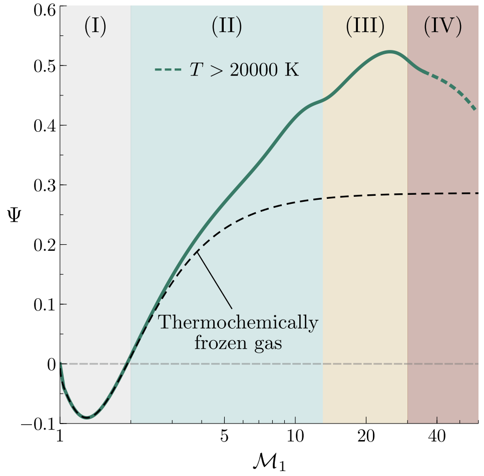
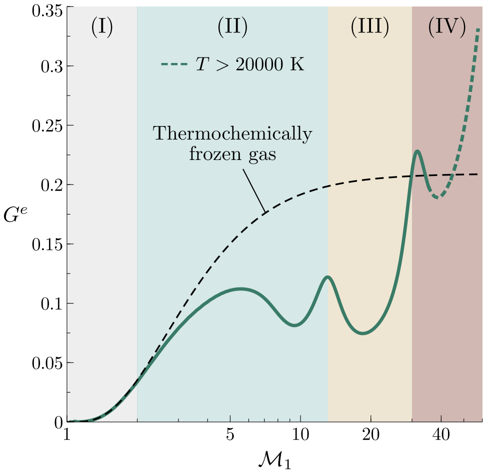
Anisotropy



Entropic prefactors of the post-shock density variance



Supplementary material: theoretical results of previous work (Huete 2021)

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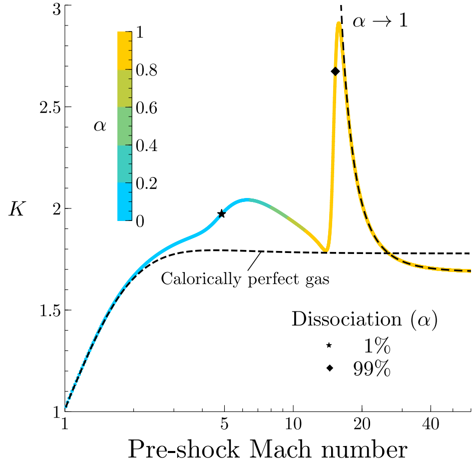

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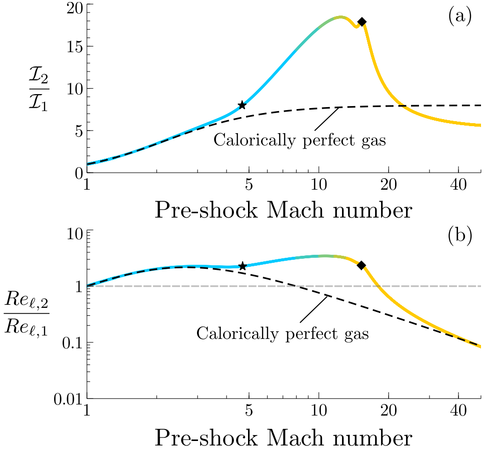

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