COMP3821 Homework 4

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Question 4.

4.1 Proof of TSP in NP.

For all true results of TSP, check whether $m[C_n, C_i] + \sum_{1<i<n} M[C_i, C_{i+1}] \leq k$. Also check whether $M[C_i, C_{i+1}] \in M$ such that all vertices have been visited once. These checks can be done in polynomial time (linear). Thus there exists a verification algorithm to verify the results of TSP in polynomial time.

4.2 Construction of reduction.

Consider $E \subseteq V \times V$ for some specific set of edges in a graph G. Let E' be the set of edges of the complete graph G', where all nodes are connected exactly once to every other node. Then, for all pairs $e \in E$, let those edges weight 0 in E', and let all other edges be 1. Hence M is analoguos to E', i.e. a matrix containing 1 and 0's for the above representation.

Now consider $k = 0$ as the length of a possible cycle through all the vertices in G', and so our target construction is $k \leq 0$ for which whether the TSP is a YES instance.

4.3 Required properties.

The following are selected,

- The reduction takes polynomial time.
- The reduction is NP-Hard.
- The reduction maps YES instances to YES instances.
- The reduction maps NO instances to NO instances.

4.4 Proof that requirements are met.

It is known that the Hamiltonian Cycle is NP-Complete, so it is iin both NP and NP-Hard. Hence any valid reduction from the Hamiltonian Cycle to a problem Y in NP will mean Y is also NP-Hard. If Y is the TSP, then reducing Hamiltonian Cycle to it means TSP is NP-Hard.

If a hamiltonian cycle exists in G, then the reduction in 4.2 will return a YES instance. This is where G' has a cycle of length $l = 0$ through all its vertices. Hence we have a directly mapping to a YES instance in the TSP, since a hamiltonian cycle with length 0 means there is a solution in TSP for $k \leq 0$.

Similarly, if no hamiltonian cycle exists in G, then it will return a NO instance. This means there is no cycle of length $k = 0$ passing through all the vertices in G'. Hence there is no solution for the TSP for which $k \leq 0$, and we also get a NO instance.

It can be seen that the reduction takes no more than $O(n^2)$, i.e. polynomial time, and thus all the requirements are met.

Therefore, TSP is NP-Complete.