Question5:

1. Exercise 1.12.2

b)

1	¬q	Hypothesis
2	$p \rightarrow (q \wedge r)$	Hypothesis
3	¬p	Modus, Tollens, 1,2

e)

1	pVq	Hypothesis
2	¬q	Hypothesis
3	p	Disjunctive Syllogism, 1,2
4	¬pVr	Hypothesis
5	r	Disjunctive Syllogism, 3,4

2. Exercise 1.12.3

c)

1	$(pVq) \wedge (\neg p) \rightarrow q$	
2	$(\neg p) \wedge (pVq)$	Commutative Law, 1

3	$(\neg p \wedge p) \vee (\neg p \wedge q)$	Distributive Law, 2
4	$F \vee (\neg p \wedge q)$	Complement Law, 3
5	$(\neg p \wedge q)$	Identity Law, 4
6	q	Simplification Rule, 5

3. Exercise 1.12.5

c)

p: I will buy a new car

q: I will buy a new house

r: I will get a job

$(p \wedge q) \rightarrow r$

$\neg r$

___

Therefore, $\neg p$

We can set up a truth table:

p	q	r	$(((p \wedge q) \rightarrow r) \wedge \neg r \rightarrow \neg p)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F

F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

The argument is not tautology, therefore the argument is invalid.

d)

p: I will buy a new car

q: I will buy a new house

r: I will get a job

$(p \wedge q) \rightarrow r$

$\neg r$

q

—-------------

Therefore $\neg p$

1	$(p \wedge q) \rightarrow r$	Hypothesis
2	$\neg r$	Hypothesis

3	$\neg(p \wedge q)$	Modus Tollens, 1,2
4	$\neg p \vee \neg q$	De Morgan's Law, 3
5	$\neg q \vee \neg p$	Commutative Law, 5
6	q	Hypothesis
7	$\neg p$	Disjunctive Syllogism, 5,6

<u>Question6:</u> (Proving algebraic statements with direct proofs.)

1. Exercise 2.4.1

d) The product of two odd integers is an odd integer.

Let two odd integers be 2a+1 and 2b+1

Therefore, two odd integers can be

2a+1+2b+1

= 2ab+1+b

=(2ab+b)+1

Since both a and b are integers, therefore (2ab+b)+1 must be an odd number.

2. Exercise 2.4.3

b) If x is a real number and $x \leq 3$, then $12 - 7x + x^2 \geq 0$.

We get $x \leq 3$

Subtract x from both sides:

$0 \leq 3\text{-}x$

$= (3\text{-}x) \geq 0$

As $(3\text{-}x) \geq 0$, (4-x) can be 1 more than (3-x)

Therefore, 4-x is also greater or equal to 0

$(3\text{-}x)*(4\text{-}x) \geq 0$

$= 12\text{-}7x+x^2 \geq 0$

Therefore, the statement is true.

Question7:

1. Exercise 2.5.1 (Proof by contrapositive of statements about odd and even integers.)

d) For every integer n, if n^2-2n+7 is even, then n is odd.

Let's assume n is an even number. Therefore, let's use 2a to represent n.

$n^2-2n+7 = (2a)^2-2(2a)+7$

$= 4a^2-4a+6+1$

$= 2(2a^2-2a+3)+1$

Let $b = (2a^2-2a+3)$, and we can get:

2b+1

a and b are both integers, therefore 2b+1 is an odd number, then n^2-2n+7 = odd

Therefore, if n is even, n^2-2n+7 is odd

Therefore, if n is odd, n^2-2n+7 is even.

2. Exercise 2.5.4

   a) For every pair of real numbers x and y, if $x^3+xy^2 \leq x^2y+y^3$, then $x \leq y$.

$x^3+xy^2 \leq x^2y+y^3$

$= x(x^2+y^2) \leq y(x^2+y^2)$

We cancel (x^2+y^2) on both sides

$= x \leq y$

Therefore, the statement is true.

   b) For every pair of real numbers x and y, if $x + y > 20$, then $x > 10$ or $y > 10$.

We have both $x > 10$ and $y > 10$

Therefore:

$x+y>10+10$

$x+y>20$

Therefore, the statement is true.

3. Exercise 2.5.5

c) For every non-zero real number x, if x is irrational, then 1/x is also irrational.

Let's prove by contrapositive

Suppose 1/x is rational, and x does not equal zero. Through the definition of a rational number, if a number is rational, then it can be expressed as the ratio of two integers (a/b), and b is not equal to 0.

Therefore, $1/x = a/b$ (a and b are both integers, and b is not equal to 0)

$x=b/a$

Since a and b are both integers and b is not equal to 0.

Therefore, x is a rational number.

Therefore, for a non-zero real number x, if x is irrational, then 1/x is also irrational, is true.

Question8:

1. Exercise 2.6.6 (Proofs by contradiction)

c) The average of three real numbers is greater than or equal to at least one of the numbers.

Let's set up three real numbers: a,b,c

The average of three real number: (a+b+c)/3

Case: $(a+b+c)/3 \geq a \vee (a+b+c)/3 \geq b \vee (a+b+c)/3 \geq c$

Contradiction: $(a+b+c)/3 < a \wedge (a+b+c)/3 < b \wedge (a+b+c)/3 < c$

Therefore:

$(a+b+c)/3 + (a+b+c)/3 + (a+b+c)/3 < a+b+c$

$= (a+b+c) < a+b+c$

However: a+b+c does not less than a+b+c

Therefore, the average of three real numbers is not greater than or equal to at least one of the numbers.

d) There is no smallest integer

Definition of integer: “An integer is a number that has no fractional part, and no digits after the decimal point. An integer can be positive, negative or zero. Zero is defined as neither negative nor positive.”

Therefore, x can be positive, negative, and zero

Negative < zero < positive

If there is a smallest integer, then it must be a negative number

Contradiction: We should assume that there is a smallest number: x (x is also a negative integer)

x/2 is always smaller than x

Therefore, there is no way to prove that “There is no smallest integer” is false.

Therefore, the argument “There is no smallest integer” is valid.

Question9:

1. 2.7.2

b)

There are two cases we need to prove: 1) when x and y are both even 2) when x and y are both odd

Case1: x and y are both even

How to prove an even number: x mod 2 = 0

Therefore, we set two variables for the integers: a and b

x=2a , y=2b

x+y=2(a+b)

Since (a+b) is multiplied by 2, therefore 2*(a+b) must be an even number. x+y is an even number.

Case 2: x and y are both odd

We set two variables for the integers: a and b

x=2a+1, y=2b+1

x+y=2(a+b+1)

Since (a+b+1) is multiplied by 2, therefore 2*(a+b+1) must be an even number. x+y is an even number.

Thus, when x+y is both even or both odd, the sum must be an even number.