Homework #1

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Question1:

A.

1. 1*2^0+1*2^1+0*2^2+1*2^3+1*2^4+0*2^5+0*2^5+1*2^6 = 1+2+0+8+16+128 = **155**
2. 6*7^0+5*7^1+4*7^2 = 6+35+196 = **237**
3. 10*16^0+8*16^1+3*16^2 = 10+128+768 = **906**
4. 4*5^0+1*5^1+2*5^2+2*5^3 = 4+5+50+250 = **309**

B.

1.

Divided by		remainder
2	69	
2	34	1
2	17	0
2	8	1
2	4	0
2	2	0
2	1	0

	0	1

Answer: **(1000101)$_2$**

2.

Divided by		remainder
2	485	
2	242	1
2	121	0
2	60	1
2	30	0
2	15	0
2	7	1
2	3	1
2	1	1
2	0	1

Answer: **(111100101)$_2$**

3. 6 = 0110

   D = 1101

   1 = 0001

   A = 1010

Answer: $(\mathbf{0110110100011010})_2$

C.

1. 1011 = B

   0110 = 6

Answer: $(\mathbf{6B})_{16}$

2.

Divided by		remainder
16	895	
16	55	f
16	3	7
16	0	3

Answer: $(\mathbf{37F})_{16}$

Question 2:

1.

+1	7+1	5+1	6+1	6_8
+	4	5	1	5_8
1	4	3	0	3_8

Answer: **14303_8**

2.

	1	0+1	1+1	1+1	0+1	0+1	1+1	1_2
+					1	1	0	1_2
	1	1	0	0	0	0	0	0_2

Answer: **11000000_2**

3.

	7+1	A(10)+1	6	6_{16}
+	4	5	C(12)	5_{16}
	C	1	2	B_{16}

Answer: **$C12B_{16}$**

4.

	3-1	0-1	2-1	2_5
-	2	4	3	3_5
	0	0	3	4_5

Answer: **34_5**

Question3:

A.

1.

Divided by		Remainder
2	124	
2	62	0
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

Answer: $\mathbf{01111100}_{\mathbf{8\ bit\ 2's\ comp}}$

2.

		1+1	1+1	1+1	1+1	1	0	0
+	1	0	0	0	0	1	0	0
	1	0	0	0	0	0	0	0

Answer: $\mathbf{10000100}_{\mathbf{8\ bit\ 2's\ comp}}$

3.

Divided by		Remainder

2	109	
2	54	1
2	27	0
2	13	1
2	6	1
2	3	0
2	1	1
	0	1

Answer: **1101101**$_{\textbf{8 bit 2's comp}}$

4.

Convert the absolute value to binary:

Divided by		Remainder
2	79	
2	39	1
2	19	1
2	9	1
2	4	1
2	2	0
2	1	0
	0	1

1001111

Apply the two's complement:

		1+1	0+1	0+1	1+1	1+1	1+1	1

+	1	0	1	1	0	0	0	1
	1	0	0	0	0	0	0	0

Answer: **10110001**$_{\textbf{8 bit 2's comp}}$

B.

1. 0*2^0+1*2^1+1*2^2+1*2^3+1*2^4 = 0+2+4+8+16 = **30**

Answer: **30**

2. Apply the two's complement:

	1+1	1+1	1+1	0+1	0+1	1+1	1	0
+	0	0	0	1	1	0	1	0
1	0	0	0	0	0	0	0	0

11010 = 0*2^0+1*2^1+0*2^2+1*2^3+1*2^4 = 0+2+0+8+16 = 26

Add the negative sign

Answer: **-26**

3. 1*2^0+0*2^1+1*2^2+1*2^3+0*2^4+1*2^5 = 1+0+4+8+0+32 = **45**

Answer: **45**

4. Apply the two's complement:

	1+1	0+1	0+1	1+1	1+1	1+1	1	0
+	0	1	1	0	0	0	1	0
1	0	0	0	0	0	0	0	0

1100010 = 0*2^0+1*2^1+1*2^5+1*2^6 = 0+2+32+64 = 98

Add the negative sign

Answer: **-98**

Question4:

1. Exercise 1.2.4

b)

p	q	(pVq)	¬ (pVq)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

c)

r	p	q	¬ q	(p∧¬q)	rV(p∧¬q)
T	T	T	F	F	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

2. Exercise 1.3.4

b)

p	q	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

d)

p	q	$\neg q$	$(p \leftrightarrow q)$	$(p \leftrightarrow \neg q)$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Question 5:

1. Exercise 1.2.7

b) $(B \wedge D) \vee (B \wedge M) \vee (M \wedge D)$

c) $B \vee (D \wedge M)$

2. Exercise 1.3.7

b) $(s \vee y) \rightarrow p$

c) $p \rightarrow y$

d) $p \leftrightarrow (s \wedge y)$

e) $p \rightarrow (s \vee y)$

3. Exercise 1.3.9

c) $c \rightarrow p$

d) $p \rightarrow c$

Question 6:

1. Exercise 1.3.6

b) If Joe maintains a B average, then Joe is eligible for the honors program.

c) If Rajiv goes on the roller coaster, then he is at least four feet tall.

d) If Rajiv is at least four feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10

p = T, q = F, r = unknown

c) $(p \vee r) \leftrightarrow (q \wedge r)$

$= (T \vee r) \leftrightarrow (F \wedge r)$

$= T \leftrightarrow F$

$= F$

Answer: **False**

d) $(p \wedge r) \leftrightarrow (q \wedge r)$

$= (T \wedge r) \leftrightarrow (F \wedge r)$

When r is T

$= T \leftrightarrow F$

$= F$

When r is F

$= F \leftrightarrow F$

$= T$

Answer: **Unknown. When r is T, then the conclusion is F / when r is F, then the conclusion is T.**

e) $p \rightarrow (r \vee q)$

$= T \rightarrow (r \vee F)$

When r = T

$= T \rightarrow (T \vee F)$

$=T$

When r = F

$= T \rightarrow (F \vee F)$

$=F$

Answer: **Unknown. When r = T, the conclusion is T / when r = F, the conclusion is F.**

f) $(p \wedge q) \rightarrow r$

$= (T \wedge F) \rightarrow r$

When r is T

$= F \rightarrow T$

$= T$

When r is F

$= F \rightarrow F$

$= T$

Answer: **Unknown. When r equal either T or F, the conclusion is T.**

Question 7:

Exercise 1.4.5

b)

$\neg j \rightarrow (l \vee \neg r)$

$(r \wedge \neg l) \rightarrow j$

j	l	r	$\neg j \rightarrow l \vee \neg r$	$(r \wedge \neg l) \rightarrow j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

Answer: **The two sentences are logically equivalent**

c)

$j \rightarrow \neg l$

$\neg j \rightarrow l$

j	l	$j \rightarrow \neg l$	$\neg j \rightarrow l$
T	T	F	T

T	F	T	T
F	T	T	T
F	F	T	F

Answer: **The two sentences are NOT logically equivalent**

d)

(r∨¬l)→j

j→(r∧¬l)

j	l	r	(r∨¬l)→j	j→(r∧¬l)
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

Answer: **The two sentences are NOT logically equivalent**

Question 8:

1. Exercise 1.5.2

c)

$(p \rightarrow q) \land (p \rightarrow r)$	
$(\neg p \lor q) \land (\neg p \lor r)$	Conditional Identity Law
$\neg p \lor (q \land r)$	Distributive Law
$p \rightarrow (q \land r)$	Conditional Identity Law

f)

$\neg(p \lor (\neg p \land q))$	
$\neg p \land \neg(\neg p \land q)$	De Morgan's Law
$\neg p \land \neg\neg p \lor \neg q$	De Morgan's Law
$\neg(p \land p) \lor \neg q$	Double Negation Law
$\neg p \lor \neg q$	Idempotent Law

i)

$(p \land q) \rightarrow r$	
$\neg(p \land q) \lor r$	Conditional Identities Law
$\neg p \lor \neg q \lor r$	De Morgan's Law
$\neg(p \lor r) \lor \neg q$	Associative Law
$\neg p \land \neg r \lor \neg q$	De Morgan's Law
$\neg p \land \neg\neg r \lor \neg q$	Double Negation Law
$\neg(p \land \neg r) \lor \neg q$	De Morgan's Law
$(p \land \neg r) \rightarrow \neg q$	Conditional Identities Law

2. Exercise 1.5.3

c)

$\neg r \lor (\neg r \to p)$

$= \neg r \lor (\neg\neg r \lor p)$ (Conditional Identities Law)

$= \neg r \lor (r \lor p)$ (Double Negation Law)

$= (\neg r \lor r) \lor p$ (Associative Law)

$= T \lor p$ (Complement Law)

$= T$ (Domination Law)

d)

$\neg (p \to q) \to \neg q$

$= \neg(\neg p \lor q) \to \neg q$ (Conditional Identities Law)

$= \neg\neg p \land \neg q \to \neg q$ (De Morgan's Law)

$= p \land \neg q \to \neg q$ (Double Negation Law)

$= \neg(p \land \neg q) \lor \neg q$ (Conditional Identities Law)

$= \neg p \lor \neg\neg q \lor \neg q$ (De Morgan's Law)

$= \neg p \lor q \lor \neg q$ (Double Negation Law)

$= \neg p \lor (q \lor \neg q)$ (Associative Law)

$= \neg p \lor T$ (Complement Law)

$= \neg T$ (Domination Law)

$= F$ (Complement Law)

Question 9:

1. Exercise 1.6.3

c) **Answer:** $\exists$ **x (x = x^2)**

d) **Answer:** $\forall$ **x (x ≤ x^2)**

2. Exercise 1.7.4

b) **Answer:** $\forall$ **x (¬S(x) $\wedge$ W(x))**

c) **Answer:** $\forall$ **x (S(x) → ¬W(x))**

d) **Answer:** $\exists$ **x (S(x) $\wedge$ W(x))**

Question 10:

1. Exercise 1.7.9

c) **True**

d) **True**

e) **True**

f) **True**

g) **False**

h) **True**

i) **True**

2. Exercise 1.9.2

b) **True**

c) **True**

d) **False**

e) **False**

f) **True**

g) **False**

h) **True**

i) **True**

Question 11:

1. Exercise 1.10.4

c) $\exists x \exists y\ (x+y = xy)$

d) $\forall x \forall y\ ((x > 0 \land y > 0) \rightarrow x/y > 0)$

e) $\forall x((x > 0 \land x < 1) \rightarrow 1/x > 1)$

f) $\neg \exists x \forall y\ (x <= y)$

g) $\forall x \exists y\ (x \neq 0 \rightarrow xy = 1)$

2. Exercise 1.10.7

c) $\exists x(N(x) \rightarrow D(x))$

d) $\exists x \forall y D(y) > P(Sam, y)$

e) $\exists x \forall y N(x) \rightarrow P(x,y)$

f) $\neg \exists x(S(x))$

3. Exercise 1.10.10

c) $\forall x \exists y((y \neq \text{Math } 101) \land T(x, y))$

f) $\exists x \forall y((y \neq \text{Math } 101) \rightarrow T(x, y))$

e) $\forall x \exists y \exists z((x \neq Sam) \rightarrow ((y \neq z) \land T(x, y) \land T(x, z)))$

f) $\exists y \exists z \forall w((z \neq y) \land T(Sam, y) \land T(Sam, z) \land ((w \neq y \land w \neq z) \rightarrow \neg T(Sam, w)))$

Question 12:

1. Exercise 1.8.2

b)

- $\forall x\ (D(x) \lor P(x))$
- Negation: $\neg \forall x\ (D(x) \lor P(x))$
- De Morgan’s Law: $\exists x\ (\neg D(x) \land \neg P(x))$
- There is one patient, who was not given the medication and was not given the placebo.

c)

- $\exists x\ (D(x) \land M(x))$
- Negation: $\neg \exists x\ (D(x) \land M(x))$
- De Morgan’s Law: $\forall x(\neg D(x) \lor M(x))$
- Every patient either or both did not take the medication or did not have migraines.

d)

- $\forall x\ (P(x) \rightarrow M(x))$
- Negation: $\neg(P(x) \rightarrow M(x))$
- De Morgan’s Law: $\exists x\ (P(x) \land \neg M(x))$
- Some patients took placebo and did not have migraines.

e)

- $\exists x\ (M(x) \land P(x))$
- Negation: $\neg \exists x\ (M(x) \land P(x))$
- De Morgan’s Law: $\forall x\ (\neg M(x) \lor \neg P(x))$
- Every patient either did not have migraine or was not given the placebo, or both.

2. Exercise 1.9.4

c)

$\exists x \forall y(P(x,y) \rightarrow Q(x,y))$

Negation: $\boldsymbol{\forall x \exists y(P(x, y) \wedge \neg Q(x, y))}$

d)

$\exists x \forall y(P(x,y) \leftrightarrow P(y,x))$

$= \neg (\exists x \ \forall y \ ((P(x,y) \rightarrow P(y,x)) \wedge (P(y,x) \rightarrow P(x,y))))$

$= \forall x \exists \ (\neg(\neg P(x,y) \vee P(y,x)) \vee \neg (\neg P(y,x) \vee P(x,y)) \ )$

Negation: $\boldsymbol{\forall x \exists y((P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y)))}$

e)

$\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y)$

$= \neg(\exists x \ \exists y P(x,y) \wedge \forall x \forall y Q(x,y))$

Negation: $\boldsymbol{\forall x \forall y \neg P(x, y) \vee \exists x \exists y \neg Q(x, y)}$