

1 Definitions and Explanation

Definition 1 (Constructive Function). Let $C: \mathbb{N}_{\text{odd}} \rightarrow \mathbb{N}$ be the constructive function defined by

$$C(x) = 3x + 1$$

for all $x \in \mathbb{N}_{\text{odd}}$.

Definition 2 (Destructive Function). Let $D: \mathbb{N} \rightarrow \mathbb{N}_{\text{odd}}$ be the destructive function defined by iteratively dividing a positive integer y by 2 until the result is odd. Formally, for $y = k \cdot 2^m$ where k is an odd positive integer and $m \in \mathbb{N}$, the destructive function is defined as

$$D(y) = k$$

The number of divisions, m , represents the magnitude of the destructive mode.

Definition 3 (Collatz Process). The Collatz process for $n \in \mathbb{N}$ is a sequence of applications of the constructive function C and the destructive function D , starting with $C(n)$ and alternating between C and D until reaching the value 1. The process is denoted as $\mathcal{P}(n)$.

Definition 4 (Magnitude of Constructive and Destructive Modes). Let $b: \mathbb{N} \rightarrow \mathbb{N}$ be a function that maps a positive integer to the number of bits in its binary representation. The magnitude of the constructive mode for $x \in \mathbb{N}_{\text{odd}}$, denoted by $G(x)$, is given by

$$G(x) = b(C(x)) - b(x)$$

The magnitude of the destructive mode for $y \in \mathbb{N}$ with $y = k \cdot 2^m$, denoted by $R(y)$, is given by

$$R(y) = m$$

Definition 5 (Mode Oscillation). The mode oscillation in the Collatz process refers to the alternation between the constructive mode, represented by the function C , and the destructive mode, represented by the function D . The oscillation between these modes serves as a clock for normalizing the x -axis, providing a uniform means for assessing the magnitudes of construction and destruction. The values of $G(x)$ and $R(y)$ represent the magnitudes of the constructive and destructive modes, respectively, for the given positive integers x and y . The Collatz process $\mathcal{P}(n)$ can be analyzed in terms of these magnitudes to study the behavior of the sequence.