1 Bounded Destructive Magnitude: Formal Justification

Theorem 1 (Bounded Destructive Magnitude)**.** Let y be a positive integer with binary representation of the form $(d_1 d_2 \ldots d_p 0^q)_2$, where $d_1, d_2, \ldots, d_p$ are binary digits, $d_p = 1$, and 0^q represents a sequence of q trailing zeros. Then, the magnitude of the destructive mode $R(y)$ of the binary representation of y under the destructive function D is bounded above by q bits per even iteration.

Proof. Let y be a positive integer with binary representation $(d_1 d_2 \ldots d_p 0^q)_2$, where $d_p = 1$ and 0^q represents a sequence of q trailing zeros. The destructive function D is defined by iteratively dividing a positive integer y by 2 until the result is odd. Formally, for $y = k \cdot 2^m$ where k is an odd positive integer and $m \in \mathbb{N}$, the destructive function is defined as

$$D(y) = k$$

The number of divisions, m, represents the magnitude of the destructive mode.

Applying the destructive function D iteratively to y until the result is odd, we observe that each division by 2 removes one trailing zero from the binary representation of the number. After q divisions, we obtain an odd number $(d_1 d_2 \ldots d_p)_2$ with no trailing zeros.

The key observation is that the maximum reduction in the number of bits occurs when the number of trailing zeros q is maximized. In this case, the operation results in a reduction of q bits compared to the original number of bits in y.

Therefore, we can formally conclude that the magnitude of the destructive mode $R(y)$ is bounded above by q bits per even iteration, as the maximum reduction in the number of bits occurs when the number of trailing zeros q is maximized. This bound holds for all positive integers y in the sequence. $\square$