1 Decision Variables, Constraints, and Objective

In our model, we use a 2d array where the rows correspond to a facility, the columns a customer, and the values [0 1] the percentage of demand of a customer fulfilled by a facility.

We add three constraints in the model.

1. **The sum of the columns must be 1.** This constraint implies that the demand of all customers is met.
2. **The sum of the demand fulfilled by a facility does not exceed its capacity.** We ensure that the scalar product of any row with the corresponding customer demand vector is no greater than the capacity of a facility.
3. **The number of vehicles for a facility does not exceed the maximum value.** We calculate the total distance to fulfill demand by taking the scalar product of a row and the corresponding distance from facility to customer and then dividing this by the distance limit per truck, which gives the number of trucks needed to fulfill the demand.

1.1 Objective Function

Our objective function aims to minimize the total operational cost associated with facility operations, allocation costs, and truck usage. It consists of three main components:

1. **Facility Opening Cost**: This component calculates the opening cost for each facility proportional to how open it is (fraction of its capacity utilized). Specifically, it sums the product of each facility's opening cost and the proportion of demand fulfilled by the facility relative to its total capacity.
2. **Allocation Cost**: The allocation cost accounts for the expenses incurred from servicing each customer from the respective facilities. It is calculated as the scalar (dot) product of the allocation cost matrix (distance or other allocation-specific metrics between customers and facilities) and the decision variable matrix indicating the fraction of customer demand met by each facility.
3. **Truck Usage Cost**: The truck usage cost component incorporates the cost associated with the distance traveled by trucks in serving customer demand from each facility. It computes the product of the truck usage cost per unit distance and the total distance traveled, normalized by the maximum allowable distance per truck, to represent truck utilization efficiency.

2 Iterations

We had some initial struggles working with continuous variables in an inherently discrete problem, specifically in the facility opening cost. We first tried summing the percentage of demand fulfilled by a facility and then multiplied this value by the facility opening cost. This has the issue that the value is possibly greater than 1, meaning that the facility opening cost is greater than it will

be realistically. To account for this, we instead tried taking the maximum customer demand or clipping the first sum by 1. However, these introduced binary variables into the system, which violate linearity. Finally, we instead used the ratio of total demand fulfilled by a facility and capacity of the facility to gauge the opening cost. Although this is not exactly the definition of opening a facility, it does give a lower bound to the problem.

Time Spent: 4 hours