Formally proving crypto properties of pseudorandom number generators

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advised by Andrew Appel and Matt Green


DILBERT By Scott Adams
TOUR OF ACCOUNTING
OVER HERE WE HAVE OUR RANDOM NUMBER GENERATOR.
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www.dilbert.com
NINE NINE NINE NINE NINE NINE
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ARE YOU SURE THAT'S RANDOM?
THAT'S THE PROBLEM WITH RANDOMNESS: YOU CAN NEVER BE SURE.

- Most modern cryptosystems rely on random numbers
- e.g. RSA generates random big primes that become a private key
- Reducing the entropy of a cryptosystem's pseudo-random number generator (PRG) is an easy way to break the entire cryptosystem

Random number generator

RNG

↓

1011100110101011000010110010000011111011110001111100110111010000000010

Pseudo-random number generator

1100101

PRG

111111011111101001010110011010001000111101111110101110001010101000110000

Pseudo-random number generator

1100101

$$\underbrace{\texttt{1100101}}$$

PRG

1111110111111010010101100110100010001111011111101011100010101010 0011000

≈

0010111001101010110000101100100000111111011110001111100110111010000000001

Pseudo-random number generator

1100101

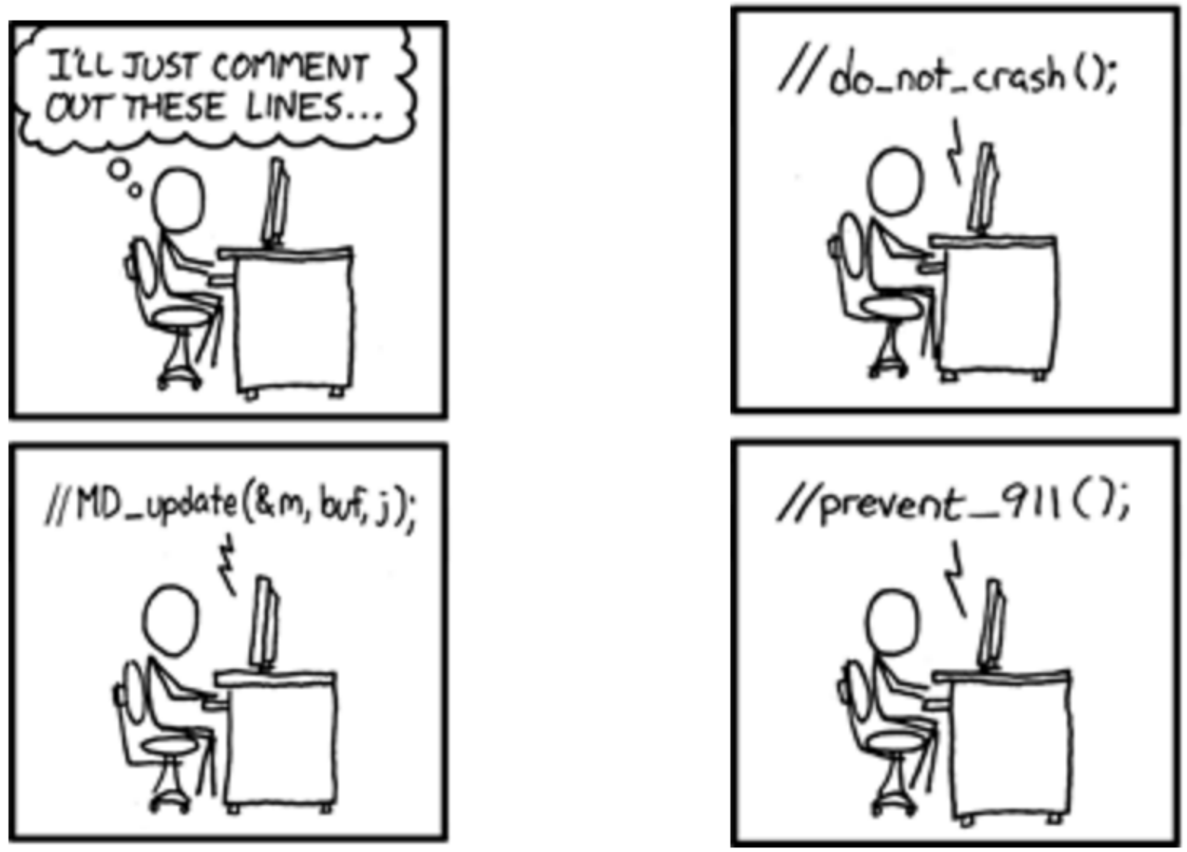
PRG

11111101111110100101011001101000100011110111111010111000101010100011000

! ≈

0010111001101010110000101100100000111111011110001111100110111010000000001

Debian OpenSSL PRG



https://www.xkcd.com/424/

- Removed sources of system entropy → only 32,767 choices
- Predictable SSL/SSH keys (Spotify, Yandex...)
- Can read encrypted traffic, log into remote servers, forge messages
- Have to patch servers AND replace weak keys

https://freedom-to-tinker.com/blog/kroll/software-transparency-debian-openssl-bug/

- **We need secure PRGs**

- But surprisingly little work exists on proving PRGs secure, either on paper or formally

Until now!

Goal: formally prove **functional correctness** and **cryptographic security** of a widely-used implementation of a PRG

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mbedTLS

HMAC-DRBG

Verified Software Toolchain

Foundational Crypto Framework

Goal: prove **functional correctness** and **cryptographic security** of a widely-used implementation of a PRG

mbedTLS

HMAC-DRBG

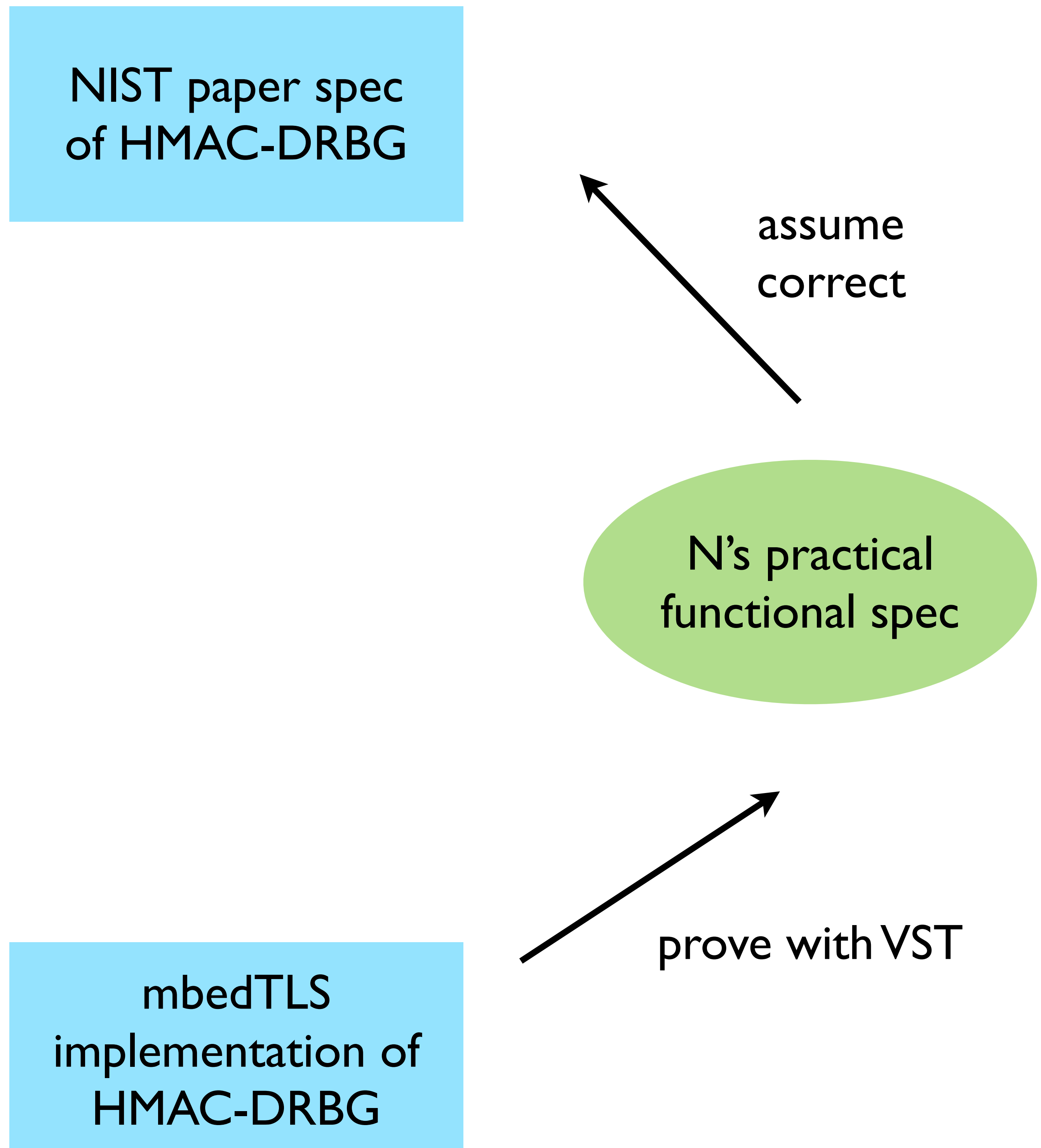
Our project

NIST paper spec
of HMAC-DRBG

mbedTLS
implementation of
HMAC-DRBG

Our project

x → y:
x implements y



Our project

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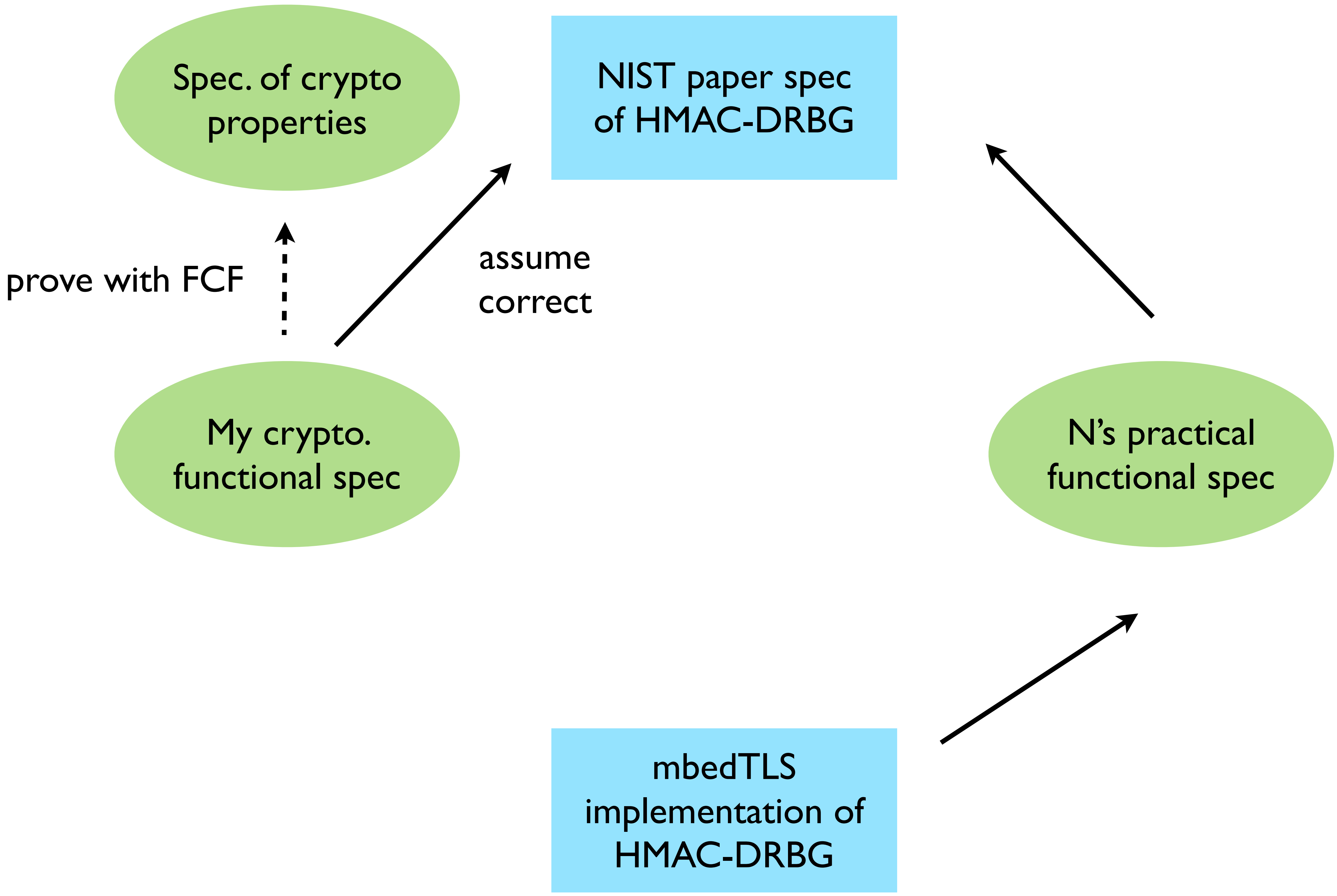
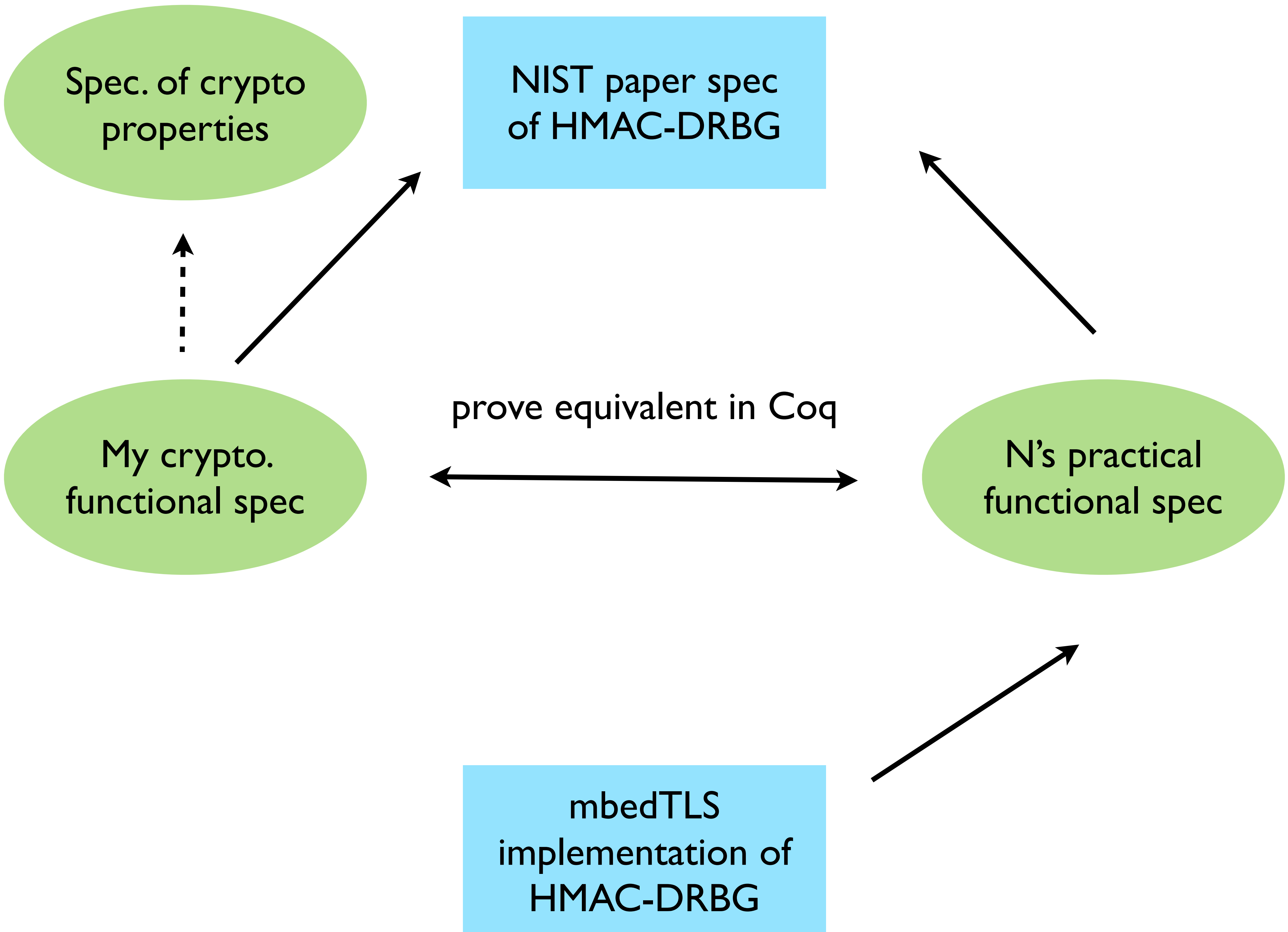
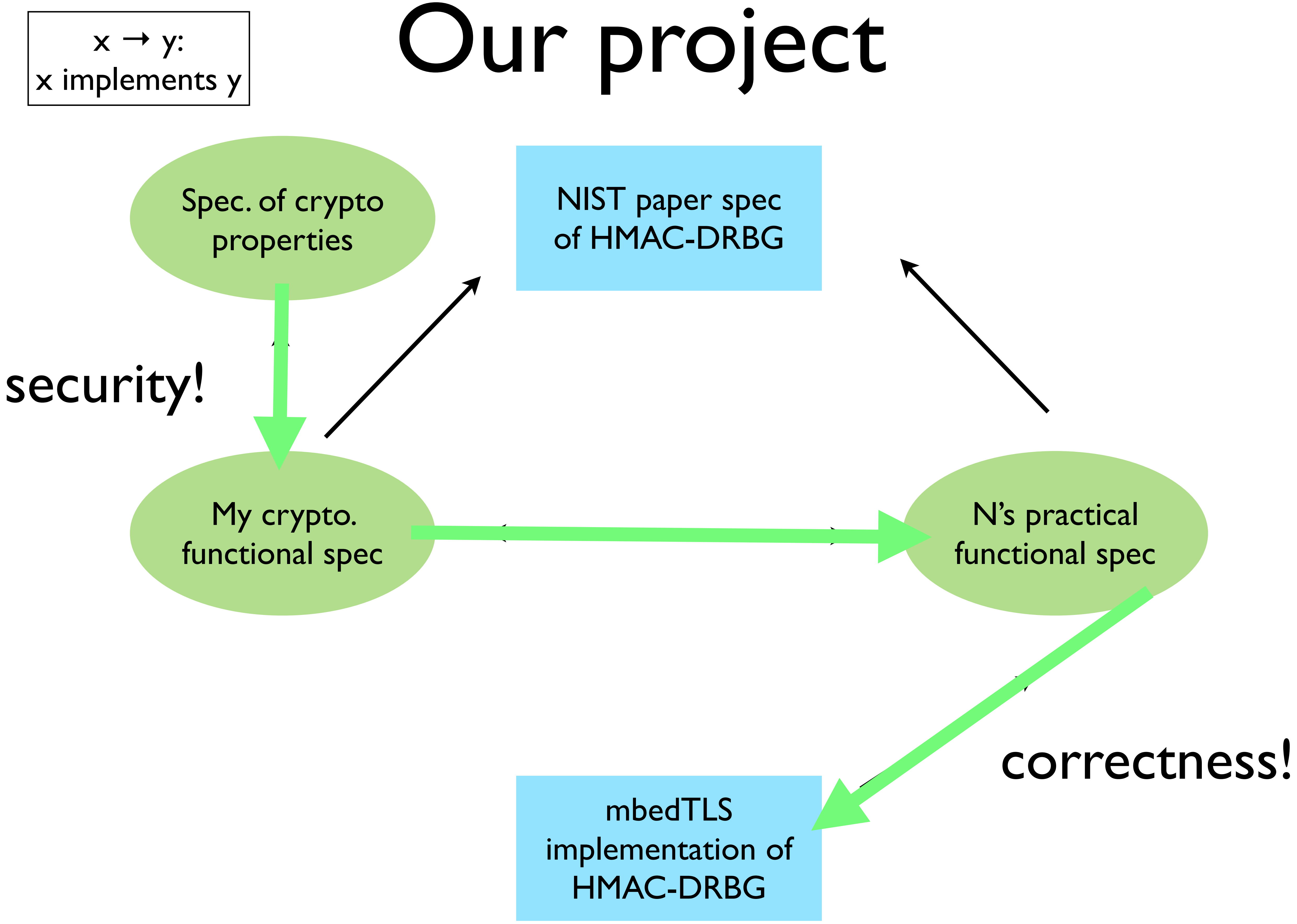


x → y:
x implements y

Our project

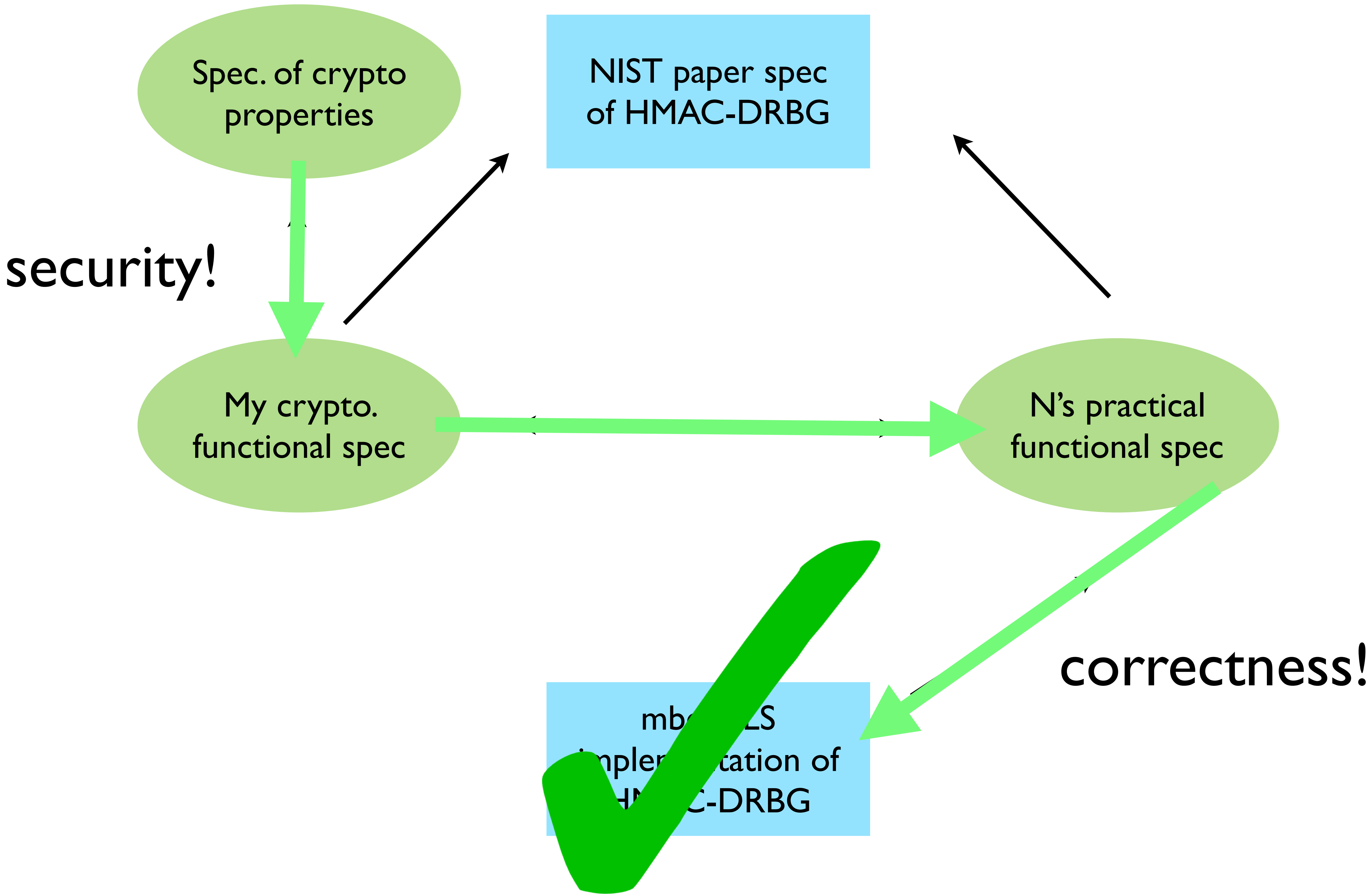


Our project


x → y:
x implements y
Spec. of crypto properties
NIST paper spec of HMAC-DRBG
security!
My crypto. functional spec
N's practical functional spec
correctness!
mbedTLS implementation of HMAC-DRBG

x → y:
x implements y

Our project



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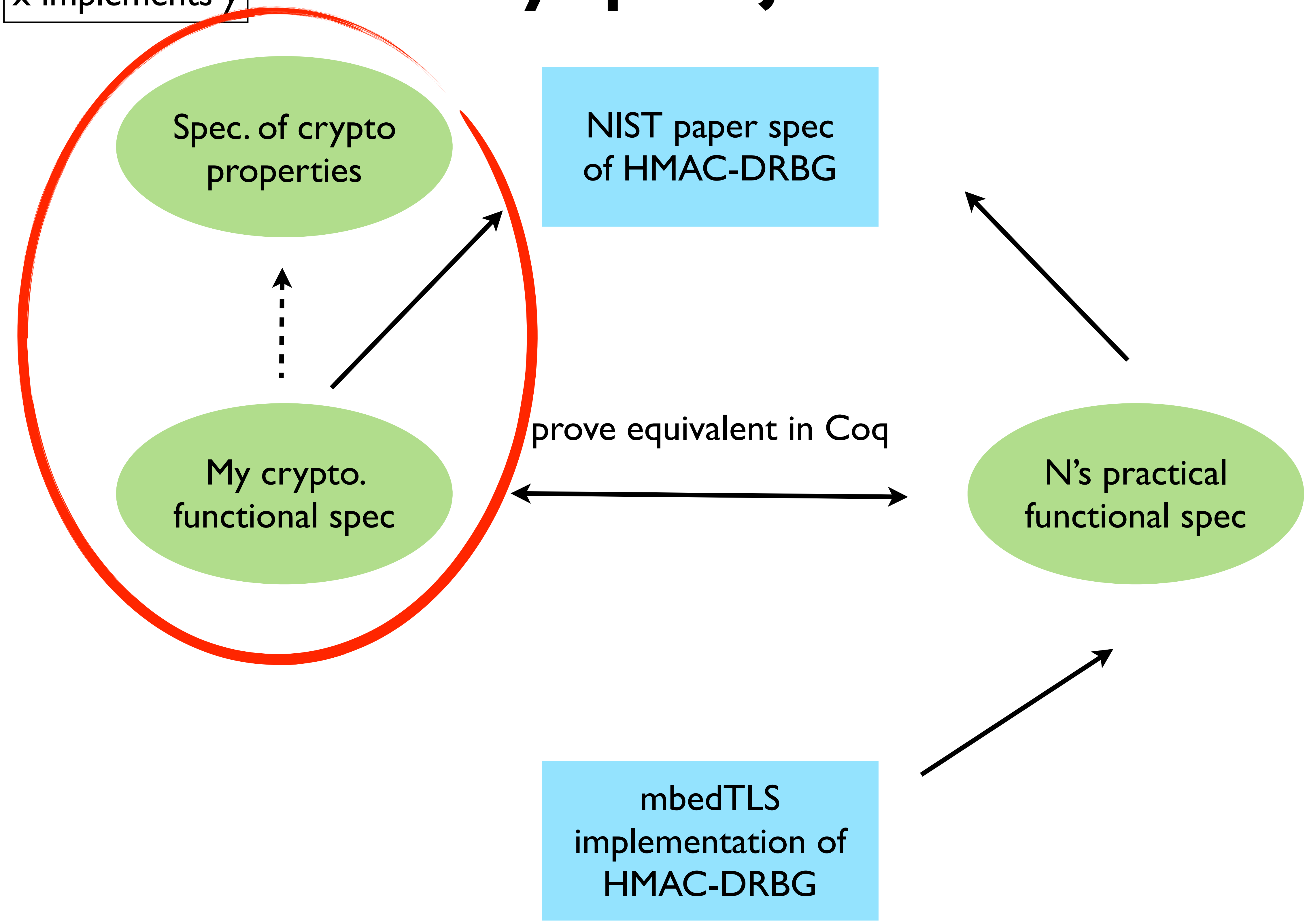

Security properties of PRGs

- Output indistinguishable from random to a computationally-bounded adversary

Security properties of PRGs

- Backtracking-resistant (compromise at time t does not compromise output from time < t)
- Eventually recovers from compromises of internal state

Related work

(there isn't much)

Our group

- Appel (2015) does the first "full formal machine-checked verification of a C program: the OpenSSL implementation of SHA-256."
- Petcher, Beringer, Ye, and Appel (2015) do the same for HMAC, adding a proof of crypto security depending on SHA

[Verification of a Cryptographic Primitive: SHA-256]

[Verified Correctness and Security of OpenSSL HMAC]

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hence HMAC-DRBG

[*Verification of a Cryptographic Primitive: SHA-256*]

[*Verified Correctness and Security of OpenSSL HMAC*]

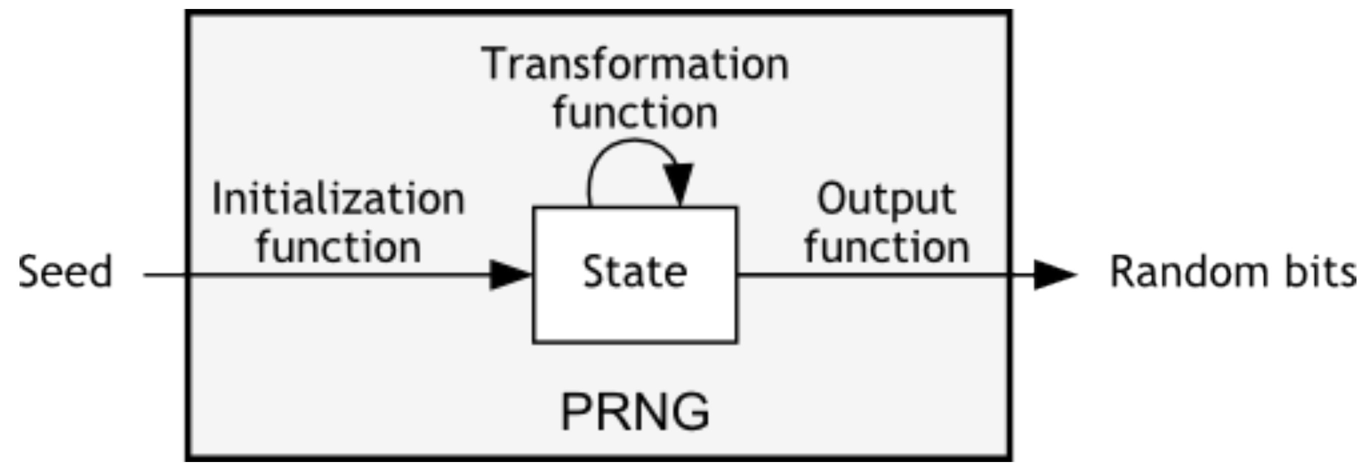
Paper proofs

- One proof by Hirose (2009) about HMAC-DRBG; not peer-reviewed
- Several crypto papers analyze the security of PRGs and propose new security properties, e.g. Dodis et al.

http://repo.flib.u-fukui.ac.jp/dspace/bitstream/10098/2126/1/art.pdf
https://eprint.iacr.org/2013/338.pdf

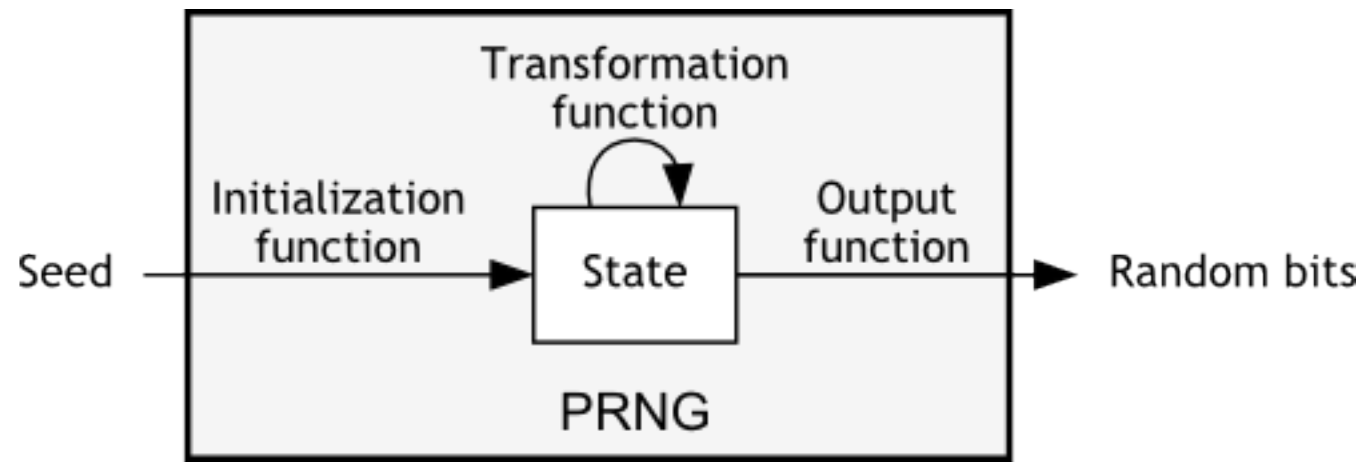
PRG internals

Pseudo-random number generator



http://pit-claudel.fr/clement/blog/how-random-is-pseudo-random-testing-pseudo-random-number-generators-and-measuring-randomness/

Pseudo-random number generator




```
Instantiate
Generate (bits)
Reseed (add entropy)
Update (internal state)
```

http://pit-claudel.fr/clement/blog/how-random-is-pseudo-random-testing-pseudo-random-number-generators-and-measuring-randomness/

Generate (simplified)

Chaining:
K = secret key; V = initialization vector;
H = hash function (e.g. HMAC); || = concatenate

```
rand_bits =
H(K, V)
|| H(K, H(K, V))
|| H(K, (H(K, H(K, V)))...
```

outputs used again as inputs

Generate (simplified)

n blocks of output: recursion

```
rec loop K V n =
  if n = 0 then ([], V)
  else
    let (result, V’) := loop K V (n-1) in
    let V’’ := HMAC K V’ in
    (result ++ V’’, V’’)

fun Generate K V n reseed_ctr =
  if reseed_ctr >= max then reseed_required
  else
    let (bits, V’) := loop K V n in
    let (K’, V’’) := Update K V’ in
    (K’, V’’, bits)
```

PRG run

```
User/Adversary:

Instantiate,
Generate 10 blocks,
Generate 20 blocks,
Generate 1 block,
Generate 10000000 blocks,
Generate 1 block,
...
```

Another loop

PRG run

```
User/Adversary:

Instantiate,
Generate 10 blocks,
  Update K and V
Generate 20 blocks,
  Update K and V
Generate 1 block,
  Update K and V,
Generate 10000000 blocks,
  Update K and V,
  RESEED,
Generate 1 block,
  Update K and V,
...
```

Complications with Updating key and Reseed

First steps

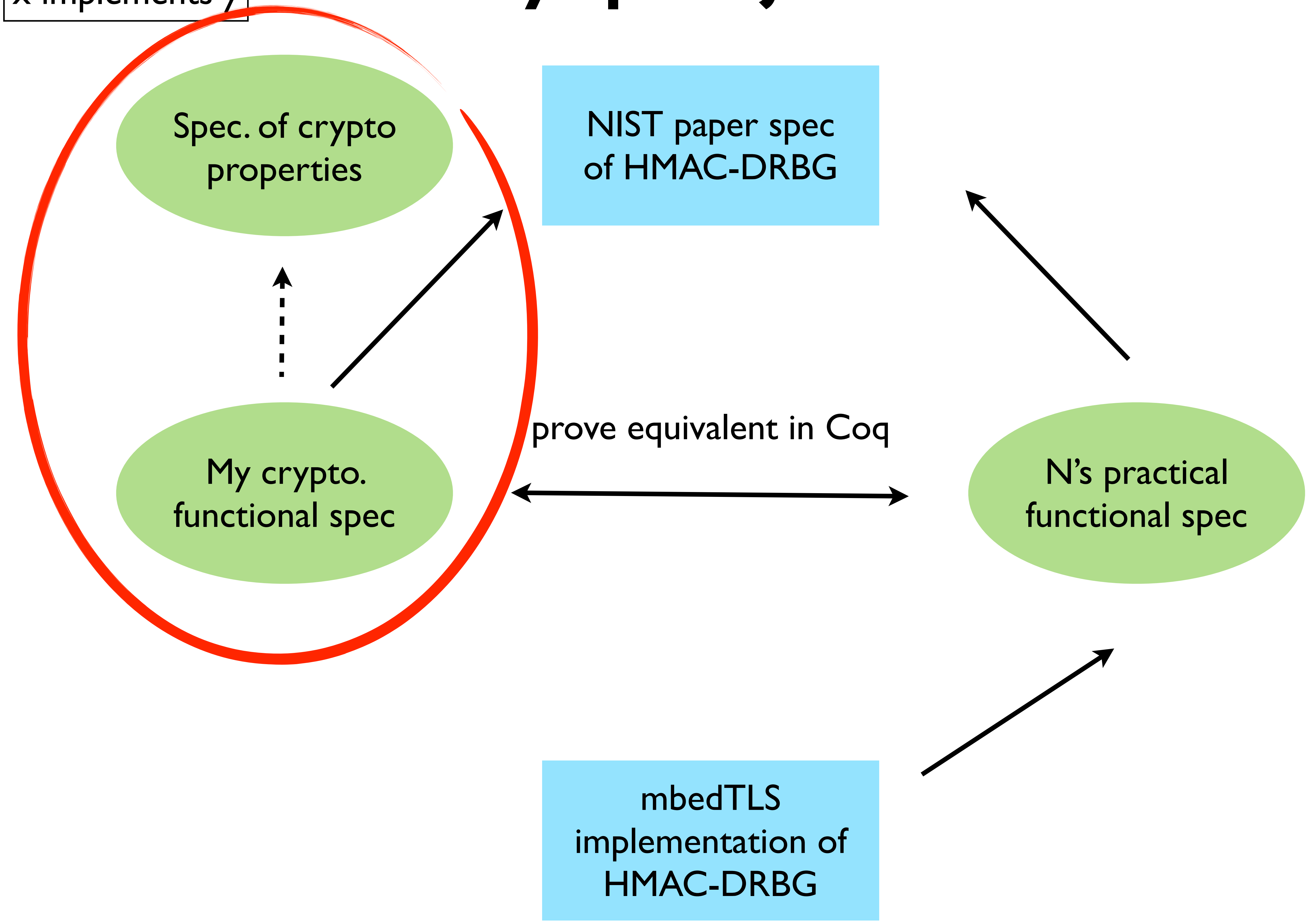
- Proof of indistinguishability for inner loop of PRG (Generate function): done by collaborator
- Extend to proof of indistinguishability for outer loop of PRG (multiple Generate calls with Update): working on

Method

- Proofs in the "sequence of games" style
- Bound probability of adversary distinguishing correctly by 1/2 + negligible amount
- Done in FCF in Coq, so correctness is verified

My project

x → y:
x implements y



Progress to date

Progress

- Matt and I have a proof outline for HMAC-DRBG indistinguishability written in text
- Started (separate) proof of entropy as a function of user number of calls

Progress

- Steep learning curve for FCF; also learning crypto as I go
- Collaborator wrote a bare-bones proof for the inner loop
- Collaborator gave us an outline for hybrid game strategy
- Studying the above; started formalizing our proof!

To do

- Formalize simplified HMAC-DRBG proof
- Add features to proof (e.g. additional input)
- Write full functional spec of HMAC-DRBG
- Connect it with concrete functional spec

Measure of success

Questions

- How automated?
- How much effort? (time, lines of code?)
- Did we contribute original math?
- Note: we hope to publish a paper on the whole system.

Measures of success

- How many properties were we able to verify, and how important are they?
- Is our verification actually right?

Measures of success

- What attacks can be definitively ruled out by our verification? What attacks are still possible?
- Are the security and formal verification communities excited about using or building on our work?

Thanks!