

Exercise 1

Barbara's mark in mathematics was one. Out of the five students that were marked there were three who scored the same mark in at least four out of the five subjects.

To figure out the total range of scores for each student we start by finding the lowest possible score any student could receive. Since the aggregate scores are in alphabetical order, Ellen has the lowest possible score out of the five.

Two of Ellen's scores are already given in the specification (5 and 3). This means the worst she could do in the other three subjects she took would be to place last in them. The sum of this would be 11 ($1 + 1 + 1 + 5 + 3$). This is the lower bound of aggregate scores and any sequence of marks which sum to a value lower than this are invalid sequences.

Updating the table...

	English	History	French	Math	Science
5				E	
4					
3					E
2					
1	E	E	E		

Alan has the highest aggregate score (supplied in the specification) of 24. The only possible combination of scores in each paper he could receive to get this number is to place 1st in four subjects and 2nd in the last (making his score range 5 5 5 5 4 or any variant). Since Ellen topped Mathematics, Alan must've placed 2nd here otherwise he wouldn't reach an aggregate score of 24. Now that the upper and lower bounds are in place we next figure out the middle score (Charles). Once we know this score finding Barbara and David's scores will be trivial as we know their boundaries.

Updating the table...

	English	History	French	Math	Science
5	A	A	A	E	A
4				A	
3					E
2					
1	E	E	E		

Charles has to receive the same score in at least four subjects. We can evaluate what score ranking he could possibly have by looking at each score line:

- (5 5 5 5 x)
- (4 4 4 4 x)
- (3 3 3 3 x)
- (2 2 2 2 x)
- (1 1 1 1 x)

X represents the subject Charles didn't receive a mark identical to the others. We can now begin a process of elimination which lines are feasible and which aren't.

- (5 5 5 5 x)

This line is **not possible** because Alan already occupies the top spot in five subjects

- (4 4 4 4 x)

This line is **not possible** because Barbara has a higher score than Charles (since the scores are in alphabetical order). If Charles scored in this line there is no possibility Barbara could beat his aggregate with the remaining marks available.

- (3 3 3 3 x)

This line is **possible** because it will beat David if he went for his next best option (2 2 2 2 x or any variant) while not beating Barbara on any iteration of her score.

- (2 2 2 2 x)

This line is **not possible**, as David will beat Charles score in whatever combination of available scores are left over. This cannot happen as David needs to score lower than Charles.

- (1 1 1 1 x)

This line is **not possible**, as Ellen already occupies three spots where she scored one. Since two people cannot place last in the same subject this is an impossible line.

Therefore Charles score should be 3 3 3 3 x. The next issue to solve is what the value of x is - the position where he didn't score the same as his other papers. Since Ellen scored third in Science, this paper has to be the one Charles placed differently in since he couldn't place differently in another paper and also maintain four identical marks in other papers.

- X = 5

Not possible because Alan has already been placed here

- X = 4

Not possible. If Charles placed 3 3 3 3 4 his aggregate score would be 16. Barbara would have to beat this score however the best line she could take to beat this would be 4 4 4 2 2, which is equal to 16.

- X = 3

Not possible because Ellen has already been placed here.

- X = 2

Not possible. If Charles scored 3 3 3 3 2 his aggregate score would be 14. Barbara could beat this with 4 4 4 2 4 giving her an aggregate score of 17. This would leave David with the remaining score line of 2 2 2 1 1 which equals 8. This is lower than Ellen, meaning this line in its entirety is not possible because it would place David's aggregate score out of place.

- X = 1

By process of elimination this is the only one left to pick, therefore it must be **possible**.

This makes Charles line of scores 3 3 3 3 1 giving him an aggregate score of 13. This means that David has to score exactly 12, as he cannot place higher than Charles at 13 and lower than Ellen at 11. Since there are also no identical aggregate scores this is the only score David can get.

With the number of scores left, we will look for David's score. This will leave the remaining scores to be Barbara's score so we can figure this out as well.

Updating the table...

	English	History	French	Math	Science
5	A	A	A	E	A
4				A	
3	C	C	C	C	E
2					
1	E	E	E		C

David needs 12, so we need to find all score combinations of 12 which will still give Barbara a higher score than Charles. Any score Barbara gets will be less than Alan's (because no score can beat 24) therefore we won't need to worry about the upper limit.

David cannot place 2nd in more than two subjects as three scores of four would already place him above his necessary score of 12 while still needing to fill out 2 more scores.

The next highest score he could take is 4 4 2 2 1. This gives him 13 which is still too high.

The next he could take is 4 2 2 2 1. This would give him 11. Still wrong. If he chose 2 2 2 2 1 he would score 9. Too low. From this we realise that David cannot score any combination involving placing 2nd twice.

The next available sequence we try is 2 2 2 2 4. This gives him 12 exactly. This makes Barbara take the last remaining score line of 4 4 4 1 2 giving her an aggregate score of 15. This is the line David should take.

Barbara cannot swap the scores she placed 4th and 5th in (4 4 4 1 2 -> 4 4 4 2 1) as Charles already placed last in Science.

Inserting David's score (2 2 2 2 4):

	English	History	French	Math	Science
5	A	A	A	E	A
4				A	D
3	C	C	C	C	E
2	D	D	D	D	
1	E	E	E		C

Inserting Barbara's score (4 4 4 1 2):

	English	History	French	Math	Science
5	A	A	A	E	A
4	B	B	B	A	D
3	C	C	C	C	E
2	D	D	D	D	B
1	E	E	E	B	C

The table and score ranking is complete.

Exercise 2

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A						1	4
B	1						
C					5	0	6
D						4	
E	4		2	2		2	2

Team E can be filled out using only its row of information given, so I will do that first.

Team E has two points over 4 matches. This means that they couldn't have won any games (or they would have more points), making the other 2 games unaccounted for as draws. They could not have won only 1 game since we already know they lost two games. If they won once and drew once from the other matches they would have 3 points, which isn't the case.

Team E has 2 goals against them and have lost 2 matches. The lowest possible score they could've gotten to lose 2 matches would be to go 0 - 1 in both matches. This is how Team E has 2 goals against them. For Team E to draw a match they would've had to finish 0 - 0. They could finish their draw matches at 1 - 1, 2 - 2 etc. however in this case it is impossible since the goals against is already fixed in place. This means that Team E finished both their drawn matches at 0 - 0.

Updating the table...

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A						1	4
B	1						
C					5	0	6
D						4	
E	4	0	2	2	0	2	2

Next we can fill out the goals against Team B and get the total amount of goals in the tournament.

As a Round Robin tournament no two teams play each other more than once. From this we can confirm that Team E has played each team once (since they have 4 matches played and cannot play themselves for a 5th). Because of this the goals against Team B must be 0 as Team E have scored no goals in all four of their games. B has played one game in the tournament so far which must've been against Team E since they have played every team, and because Team E didn't score a goal in any match they played the goals against Team B must be 0.

Updating the table...

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A						1	4
B	1					0	
C					5	0	6
D						4	
E	4	0	2	2	0	2	2

Team C has a fixed point total of 6 from their games so far. They can get 6 points from either:

- 3W
- 2W 2D
- 1W 4D
- 0W 6D

The last 3 are all impossible cases since it would imply Team C has played 4 or more matches. One team is only able to play 4 matches in a Round Robin tournament and we know already that Team B has played their one match against Team E. This means Team C could've only played 3 matches otherwise they would play Team B as well (which couldn't happen if they've already played their single game against Team C). The first is the only score record where there are 3 games played so this must be the case.

Updating the table...

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A						1	4
B	1					0	
C	3	3	0	0	5	0	6
D						4	
E	4	0	2	2	0	2	2

Team A has played Team E once since E has played everyone. They have never played Team B since Team B has only played Team E so far in the competition. This means that Team A has played at least 1 match and no more than 4 since this isn't possible in a Round Robin tournament.

It isn't possible Team A could've played only 2 matches. If they only played 2 matches they would have to win them both to get 4 points, which is impossible since their opponent scored against them in one match. In order to win this match they would have to go 2 - 1 as well as handily win the other game of the two 1 - 0. This would suggest A has scored 3 goals, making the total amount of goals scored one higher than the total amount of goals scored against. These numbers need to be identical therefore Team A must've played more than 2 games.

Team A couldn't have played 4 matches since that would mean they played every team once. Team B has played only Team E in their single game therefore they must've played a combination of 3 games.

Updating the table...

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A	3					1	4
B	1					0	
C	3	3	0	0	5	0	6
D						4	
E	4	0	2	2	0	2	2

The outcomes of their matches where Team A scored 4 points while only playing 3 games is:

- 2W 1L 0D
- 1W 0L 2D

Regardless of the outcome, Team A must've scored two goals. This is because if the first outcome were the case then they must've won 1 - 0 in both games and lost another in order to have been scored against. If they were scored against in a winning match then they would have to win another game going 0 - 0 which cannot happen.

In the next case of 1W 0L 2D, they could draw 1 - 1 and win 1 - 0 OR win 2 - 1 and draw both games 0 - 0. Either way accounts for the single goal against them. Therefore we can fill out the table that they must've scored two goals.

Knowing this we can fill out that Team B and Team D scored 0 goals since Team A has scored the other 2 goals available for the goals for and against to be the same number.

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A	3				2	1	4
B	1			1	0	0	
C	3	3	0	0	5	0	6
D					0	4	
E	4	0	2	2	0	2	2

We can solve the Team B line next.

Team B played one game against Team E. Since neither team scored in this match, evidenced by 0 goals for or against Team B, the game they played couldn't have ended at anything other than 0 - 0, meaning that the match was a draw. Since they only played one game that we know is a draw they also won 0 and lost 0, giving them a total point count of 1 (from the draw).

Updating the table...

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A	3				2	1	4
B	1	0	0	1	0	0	1
C	3	3	0	0	5	0	6
D					0	4	
E	4	0	2	2	0	2	2

Going back to the potential cases for Team A:

- 2W 1L 0D
- 1W 0L 2D

If we look at the second case we know that Team C won all of their matches and were the only team in the entire competition who scored goals other than Team A. This means for Team A the goals against must've come from their game with Team C. Since Team C has won all of their matches their game against Team A must've been a win for them, making the losses for Team A at least 1. Since Team A has lost a match it isn't possible for them to go 1W 0L 2D and must've gone 2W 1L 0D.

Updating the table...

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A	3	2	1	0	2	1	4
B	1	0	0	1	0	0	1
C	3	3	0	0	5	0	6
D					0	4	
E	4	0	2	2	0	2	2

We can now figure out the stats for Team D and complete the table.

Team C played every team but B (who only played Team E at this point) and won all of their games. This means Team D has at least 1 loss. Since Team D has additionally not scored any goals in their matches so they couldn't have won any (since they would obviously need to score to win). Because the total amount of wins in the competition needs to equal the total amount of losses, Team D has lost 2 games.

Updating the table...

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A	3	2	1	0	2	1	4
B	1	0	0	1	0	0	1
C	3	3	0	0	5	0	6
D		0	2		0	4	
E	4	0	2	2	0	2	2

Team E has drawn 2 games and lost 2. In order to lose a game there needs to be a goal difference of at least 1. In the entire tournament the only teams who scored goals were Team A and Team C. This means that the losses for Team E were to these two teams, and their two draws must've been against B and D. This means that D must have drawn a game.

This gives Team D a record of 0W 2L 1D, which gives them 1 point over 3 games.

Updating the table...

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A	3	2	1	0	2	1	4
B	1	0	0	1	0	0	1
C	3	3	0	0	5	0	6
D	3	0	2	1	0	4	1
E	4	0	2	2	0	2	2

The table is complete.

Team C has played every team but Team B (who has only played Team E), so Team C did end up playing Team D.

Team C won every game they played. In the games Team C played against Team E and Team A they finished both of their games with 0 - 1 losses. For Team A they lost their one match to Team C with 1 goal against. For Team E they lost 2 games where there were 2 goals against. This means they would need to lose both games 0 - 1. This means that in one of those games which was against Team C they went 0 - 1.

Since Team C scored 2 goals in these two games against Team A and Team E the remaining 3 goals they scored must've been against Team D. This means the result of Team C vs Team D was 3 - 0.