

Exercise 1

We will start by filling out the table from the supplementary information.

Alan had the highest aggregate score of 24. The only possible combination of scores in each paper he could receive to get this number is to place 1st in four subjects and 2nd in the last (making his score range 5 5 5 5 4 or any variant). Since Ellen placed 1st in Mathematics, Alan must've placed 2nd in this paper.

We can also add in Ellen's score in Science of 3rd.

Updating the table...

	English	History	French	Math	Science
5	A	A	A	E	A
4				A	
3					
2					
1					

We will now figure out Charles score.

The supplementary knowledge that Charles finished the same in four papers will help finding his score line. We can begin to deduce the sequences for him which doesn't disturb the ordering of the scores for other members.

It is impossible for the matched scores to be all 5s since Alan already occupies these spots.

If the four papers Charles matched scores with were all 4s the minimum possible score he could get is 17 (4 4 4 1 4). This line is not possible because Barbara would need to beat it with a worse line of numbers out of the available pool (3 3 3 3 2 = 14).

If the four papers Charles matched scores with were all 2s the maximum score he could achieve would be 12 (4 2 2 2 2). This is impossible since David wouldn't beat Ellen's score. The worst Ellen could do is to get 11 (1 1 1 5 3). Since there are no duplicate scores David cannot beat 11 while also getting 11, so this line for Charles is impossible.

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Missing Scores

It is impossible for the matched scores to be all 1s since we already know four scores of 2 don't work.

This leaves Charles getting four scores of 3. From here we can list the valid sequences for Charles given the state of the table so far:

- 3 3 3 3 1 (13)
- 3 3 3 3 2 (14)
- 3 3 3 3 4 (16)

From these we can see the minimum possible score for Charles is 13. This means Barbara has to score 14 or greater to maintain the ordinality of the score ranking. David must score less than Charles at 13 and greater than Ellen's minimum score of 11, so he will have to get 12 exactly.

Knowing this we can posit Barbara scored at least 4 in two subjects since she has to score higher than Charles.

For the other 3 unaccounted subjects we can deduce down to the valid cases from the following list:

- 4 2 4 = 10 (+ 8 from two scores of 4 = **18**)
- 4 1 4 = 9 (+ 8 from two scores of 4 = **17**)
- 4 2 2 = 8 (+ 8 from two scores of 4 = **16**)
- 4 1 2 = 7 (+ 8 from two scores of 4 = **15**)
- 2 2 2 = 6 (+ 8 from two scores of 4 = **14**)

(4 4) 4 2 4 or (4 4) 4 1 4

David wouldn't be able to beat Ellen's minimum score, as the best he could do is 10 (2 2 2 2 2). This makes these two scores for Barbara impossible.

(4 4) 2 2 2

If she went for David could beat Ellen and maintain ranking ordinality by going 2 2 4 1 4. This gives him a score of 13. Charles would need to beat 13 with what is left, which he cannot because the best he could do is 13 (3 3 3 3 1).

(4 4) 4 2 2 or (4 4) 4 1 2

These are the only two valid sequences for Barbara. Since we can lock the fact she would have three 4s and a 2 regardless of which sequence we choose we can update the table with this. We can also add in the locked sequence for Charles too (all his sequences involve him getting four straight 3s).

Missing Scores

	English	History	French	Math	Science
5	A	A	A	E	A
4	B	B	B	A	
3	C	C	C	C	
2					B
1				B	

Charles needs to beat David in order to maintain the ordinality of the ranking. The only remaining line David can take for this to happen is 2 2 2 2 4 giving him 12. This will also put Charles in place to get 13 (3 3 3 3 1) and confirms that Barbara will have to take line 4 4 4 1 2 since there are no open 2 slots after David's score for her to go 4 4 4 2 2.

Knowing all this Ellen will fill in what is left, completing the table.

	English	History	French	Math	Science
5	A	A	A	E	A
4	B	B	B	A	D
3	C	C	C	C	E
2	D	D	D	D	B
1	E	E	E	B	C

To answer the questions posed, Barbara scored last in Mathematics (mark of 1). Alan, Charles and David received the same mark in 4 of the 5 subjects.

Exercise 2

We will start by filling out Team E's scores.

Team E has 2 points from 2 draws, therefore they couldn't have won any games (they would have more points if they did). With a total of 4 games played these other two must've been losses. They have 2 goals against and have lost 2 games, therefore both of these matches they lost both ended 0 - 1. This is the lowest possible score they could have to lose both matches. This means that in the two games they drew there were no goals scored against them. This means their opponents finished their matches with scores of 0 in the games they drew, and for them to both be draws means Team E scored zero goals.

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A						1	4
B	1						
C					5	0	6
D						4	
E	4	0	2	2	0	2	2

Team B has played 1 match so far in the tournament, and Team E with 4 matches has played everyone else in the tournament since it is a Round Robin format. This means that for Team B the only game they have played so far is against Team E. Since Team E has scored no goals this also means Team B has 0 goals against them. This means we can fill out the *Goals Against* column and can determine the total amount of goals in the competition so far as 7. Since *Goals For* and *Goals Against* need to match in order for the table to make sense there are still two goals needed to be placed in the *Goals For* column by other teams.

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A						1	4
B	1					0	
C					5	0	6
D						4	

Missing Scores

E				2		2	2
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Knowing that Team E has only played Team B, we can now shift onto filling out Team C.

Team C has 6 points which can be reached by winning 3 matches. There is no chance they could draw any matches since they would need 2W 2D in order to get 6 points. This is not possible as they would have to have played Team B, who have only played Team E. This means Team C played a maximum of 3 matches which they all won. They also didn't lose any matches because of the reasoning above that they would have to play Team B at some point, which cannot happen.

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A						1	4
B	1						
C	3	3	0	0	5	0	6
D						4	
E	4	0	2	2	0	2	2

Team A has 4 points they could get over a maximum of 3 games (couldn't have played Team B and cannot play themselves). The only valid outcomes of these matches to score 4 points would be either 2W 1L 0D or 1W 2D. It is possible in the first case they only played 2 matches and won both. Both scenarios require Team A to score 2 goals, putting their *Goals For* count to 2 and accounting for the rest of the *Goals For* columns missing goals. This means the other 2 missing values in this column can be filled out too. Since we can now confirm Team A and Team C are the only goalscorers of the tournament both of them must've won against Team E, making Team B and Team D both draw with Team E in their matches.

Since Team B drew the only match they played, they could not have won or lost any matches.

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A					2	1	4
B	1	0	0	1	0	0	1
C	3	3	0	0	5	0	6
D					0	4	

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Missing Scores

E	4	0	2	2	0	2	2
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Team A must have lost once to Team C knowing that Team C is the only other scoring team so far in the tournament. This confirms the conjecture of Team A going 2W 1L rather than 1W 2D. While we already knew it was the case it also confirms Team A played 3 matches total.

Since Team C scored one goal against Team E and Team respectively, the other 3 goals they scored have been against Team D.

This confirms Team C has played against Team D. The score for the match was 3 - 0.

This still leaves Team D with one unaccounted *Goal Against*. Team A must've scored this goal against them since Team A has an unaccounted *Goal For*.

TEAM	PLAYED	WON	LOST	DRAWN	GOALS FOR	GOALS AGAINST	POINTS
A	3	2	1	0	2	1	4
B	1	0	0	1	0	0	1
C	3	3	0	0	5	0	6
D	3	0	2	1	0	4	1
E	4	0	2	2	0	2	2