REVIEW SHEET FOR FIRST EXAM-A

The exam will cover the material we have discussed in class and studied in homework, from Section 1.1 to Section 2.3. The following list points out the most important definitions and theorems.

Definitions:

The definition of $A\mathbf{x}$ in both words and symbols.
Span$\{\mathbf{v}\}$, Span$\{\mathbf{u}, \mathbf{v}\}$ and geometric interpretation in $\mathbb{R}^2$ or $\mathbb{R}^3$.
Span$\{\mathbf{v}_1, \dots, \mathbf{v}_{\mathrm{p}}\}$.
Linearly independent and linearly dependent.
Linear transformation.
One-to-one and onto (for linear transformations).
The definition of a matrix product AB.

Theorems:

Chapter 1:
- Theorem 2 (Existence and Uniqueness Theorem).
- Theorem 3 (Matrix equation, vector equation, system of linear equations).
- Theorem 4 (When do the columns of A span $\mathbb{R}^{\mathrm{m}}$?).
- Theorem 5 (Properties of the Matrix-Vector Product $A\mathbf{x}$).
- Theorems 7, 8, 9 (Properties of linearly dependent sets).
  - Know the proofs of Theorems 8 and 9.
- Theorems 11 and 12 (one-to-one and onto linear transformations).

Chapter 2: Theorems 4, 5, 6, and 7.
- Know the proof of Theorem 5.
- The Invertible Matrix Theorem.

Important Skills:

Determine when a system is consistent. Write the general solution in parametric vector form.
Determine values of parameters that make a system consistent or make the solution unique.
Describe existence or uniqueness of solutions in terms of pivot positions.
Determine when a homogeneous system has a nontrivial solution.
Determine when a vector is in a subset spanned by specified vectors.
Exhibit a vector as a linear combination of specified vectors.
Determine whether the columns of an $m \times n$ matrix span $\mathbb{R}^{\mathrm{m}}$.
Determine whether the columns are linearly independent.
Write the parametric equation of a line through $\mathbf{p}$ in the direction parallel to $\mathbf{a}$.
Write the parametric equation of a line through points $\mathbf{p}$ and $\mathbf{q}$.
Write the equation of a line or a plane in parametric vector form.
Use linearity of matrix multiplication to compute $A(\mathbf{u} + \mathbf{v})$ or $A(c\mathbf{u})$.
Determine whether a set of vectors is linearly independent. Know several methods that can sometimes produce an answer "by inspection" (without much calculation).
Determine whether a specified vector is in the range of a linear transformation.
Find the standard matrix of a linear transformation.
Determine whether a linear transformation $T\colon \mathbb{R}^{\mathrm{n}} \to \mathbb{R}^{\mathrm{m}}$ is one-to-one or maps $\mathbb{R}^{\mathrm{n}}$ onto $\mathbb{R}^{\mathrm{m}}$.
Use an inverse matrix to solve a system of linear equations.
Use matrix algebra to solve equations involving matrices.

Applications:

Use linear combinations of vectors to describe various problems.
(See Example 7 and Exercises 27–28 in Section 1.3, Exercises 5-10 in Section 1.6, and Example 6 in Section 1.8.)
Set up a migration matrix and write the difference equation $\mathbf{x}_{k+1} = M\mathbf{x}_k$ $(k = 0, 1, \dots)$ that describes population movement in a region (assuming no births/deaths and no migration). Compute $\mathbf{x}_1$ or $\mathbf{x}_2$ when $\mathbf{x}_0$ is specified.