Text reference page 49.

Solving Ax = b

Purpose

To use `gauss, swap, scale, replace, x == y, length,` and `lead` to study existence questions for the equation $Ax = b$ and Theorem 4 of Section 1.4 of the text.

MATLAB Functions

`swap, scale, replace, gauss, lead`

We can use the Laydata functions `swap, scale,` and `replace` to row reduce a matrix. We will use the following matrix to see how they work:

```
A=rand(3,4)
```

The function `swap` has the form `Y = swap(A,r,s)`, where the outcome `Y` is the result of interchanging rows `r` and `s` of `A`. So, even though swapping rows is not necessary to the row reduction of `A`, you can see how it works with

```
r = 1; s = 3;
A = swap(A,r,s)
```

To obtain a one in the (1,1) pivot position, use `scale` in the form `scale(A,r,c)`, which scales row `r` of matrix `A` by a nonzero scalar `c`.

```
r = 1; c = 1/A(1,1);
A = scale(A,r,c)
```

To zero out the entry in the (2,1) position use `replace` in the form `Y = replace(A,r,m,p)`, which replaces row `r` of matrix `A` by its sum with `m` times row `p`.

```
r = 2; m = -A(2,1); p = 1;
A = replace(A,r,m,p)
```

To finish the first column use

```
r = 3; m = -A(3,1); p = 1;
A = replace(A,r,m,p)
```

The Laydata function `Y = gauss(A,r)` can be used to zero out below a pivot entry in row `r`. Start over with

```
A = rand(3,4)
A = gauss(A,1)
A = gauss(A,2)
A = gauss(A,3)
```

The Laydata function `lead` in the form `L = lead(A)` returns a vector `L` containing the indices for the columns of the pivots (or leading ones) in the reduced row echelon form. With the form `[L,F] = lead(A)`, the vector `F` contains the indices for non-pivot columns. Together, the two vectors contain all numbers $1, 2, \ldots, n$ where n is the number of columns.

Suppose we want to know if $Ax = b$ is consistent. We look at `[L,F] = lead([A,b])` to see if there is a leading one in the augmented column. If `n` is the number of columns of `A`, we want to determine if `n + 1` is in the vector `F`. The numbers of `L` and `F` are arranged in order, and `n + 1` is the largest index for a column of the augmented matrix. The MATLAB function `length(F)` will find the number of entries in `F`. Thus, either `F(length(F))` is `n+1` or `L(length(L))` is `n+1`. So

```
F(length(F)) == n + 1
```

will return `1` if and only if there is not a leading one in the augmented column of `rref([A,b])`.

MATLAB Exercises

1. Use `scale`, `swap` and `replace` to determine which of the following systems $Ax = b$ has a solution.

a. `A = ones(5); b = (1:5)'`

b. `A = vander(1:5); b = ones(5,1)`

c. `A = [[rand(2,4); zeros(3,4)], [ones(2,3); rand(1,3); ones(2,3)]], b = ones(5,1)`

2. Use `gauss` to determine if the vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4$, and $\mathbf{a}_5$ span R^5 where

$$\mathbf{a}_1 = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \quad \mathbf{a}_2 = \begin{bmatrix} 7 \\ 9 \\ 11 \\ 13 \\ 15 \end{bmatrix} \quad \mathbf{a}_3 = \begin{bmatrix} 12 \\ 15 \\ 19 \\ 24 \\ 30 \end{bmatrix} \quad \mathbf{a}_4 = \begin{bmatrix} 17 \\ 21 \\ 28 \\ 39 \\ 55 \end{bmatrix} \quad \mathbf{a}_5 = \begin{bmatrix} 22 \\ 27 \\ 38 \\ 59 \\ 95 \end{bmatrix}$$

3. Use the function `lead` to determine which of the systems $Ax = b$ from Exercise 1 has a solution.

4. Use the function `lead` to solve Exercise 2.

5. Suppose `[L,F] = lead([A,b])` and `n = size(A,2)`. Which of the following can be used to determine if $Ax = b$ is consistent?

a. `L(length(L)) == n`

b. `1 - L(length(L)) == n + 1`

c. `max(L) < max(F)`

d. `find(F == n + 1)`

e. `min(L) > max(F)`

6. Suppose `[L,F] = lead([A,b])` and `[m,n] = size(A,2)` which of the following can be used to determine if $R^m = \text{Span}\{v_1, \ldots, v_n\}$?

a. `max(F) > max(L)`

b. `length(L) == m`

c. `max([L,F])`

d. `length(F) == n - length(L)`

e. `F == [ ]`