FIRST EXAM-A

<u>Instructions:</u> Begin each of the eight numbered problems on a new page in your answer book. **Show your work, and mention theorems when appropriate**.

1. [15 Pts] Find the general solution of the following homogeneous system of equations. Express your answer using vector notation. Use the method developed in class. What is an appropriate (mental) geometric picture for this solution set?

$$\begin{cases} x_1 - 3x_2 - 9x_3 + 5x_4 = 0 \\ \quad\quad\; x_2 + 2x_3 - x_4 = 0 \end{cases}$$

2. [15] Determine which of the following sets of vectors are linearly independent. Give reasons for your answers.

a. $\begin{bmatrix} 10 \\ -6 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ -4 \end{bmatrix}$ **c.** $\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -3 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 9 \\ -4 \\ 4 \end{bmatrix}$

3. [15] Let $A = \begin{bmatrix} 1 & 1 \\ -2 & -1 \\ -1 & -3 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 2 \\ -7 \\ 4 \end{bmatrix}$, and define $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ by $T(\mathbf{x}) = A\mathbf{x}$.

a. Determine if $\mathbf{y}$ is in the range of T. (Check your arithmetic.)

b. Does T map $\mathbb{R}^2$ onto $\mathbb{R}^3$? Why or why not? (Explain.)

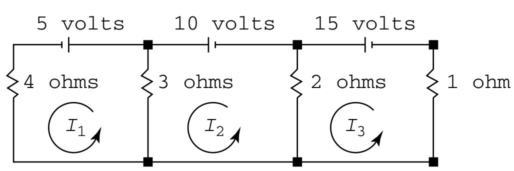
4. [18] **a.** Complete the definition in words: "If A is an $m \times n$ matrix and $\mathbf{x}$ is a vector in $\mathbb{R}^n$, then $A\mathbf{x}$ is ____________"

b. Suppose that A, B, C are $n \times n$ matrices with B invertible and $I - BAB^{-1} = C$.
Solve this equation for A. (Show your calculations.)

c. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. Suppose that T maps $\mathbf{u} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ into $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and T maps $\mathbf{v} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$ into $\begin{bmatrix} 3 \\ -7 \end{bmatrix}$. Use this information to compute $T(3\mathbf{u} + \mathbf{v})$.

5. [10] Prove the theorem: *If a set $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ in $\mathbb{R}^n$ contains more vectors than there are entries in each vector (i.e., if $k > n$), then the set is linearly dependent.*

6. [10] Use Ohm's law and Kirchoff's voltage law to *set up* a matrix equation that determines the loop currents in the system at the right. (Fill in the entries of the matrix and vectors.) Do <u>not</u> solve the equation.



7. [5] During each 10-year period, 8% of the people in a certain city move to the surrounding suburbs, and the rest remain in the city. Also, 3% of the people in the suburbs move to the city, and the rest remain in the suburbs. In 1990 there were 100,000 residents in the city and 200,000 in the suburbs. Set up the migration matrix and *use* it to compute the population vector for 2000 (one 10-year period).

8. [12] Write statements from the Invertible Matrix Theorem that are each equivalent to the statement that an $n \times n$ matrix A is invertible. Use the following concepts, one in each statement: (i) the equation $A\mathbf{x} = \mathbf{0}$, (ii) the transformation $\mathbf{x} \mapsto A\mathbf{x}$, (iii) span, and (iv) pivot position.