Text reference page 325.

Exploring Diagonalization

Purpose
To use MATLAB to find the eigenvalues and eigenvectors of a matrix, to determine the dimension of the eigenspace of an eigenvalue, and to determine if the matrix is diagonalizable.

MATLAB Functions
`eig, null, rank, poly, det`

MATLAB's `eig` function can be used to find the eigenvalues of a matrix. Using `eig(A)` gives us a list of all the eigenvectors of A. The MATLAB function `null` finds a basis for the null space, by invoking

```
B = null(A,'r')
```

We get the familiar parametric vector form from Section 1.5. We can get a basis for the eigenspace of `s` with

```
null(A - s*eye(n),'r')
```

Try this with MATLAB

```
A = [[2*eye(2); zeros(2)], ones(4,2)]
eig(A)
null(A - 2*eye(4))
```

We see that the multiplicity of the eigenvalue $\lambda = 2$ is 3. Is `A` diagonalizable? We can compute the dimension of the eigenspace easily with the MATLAB function `rank`:

```
4 - rank(A - 2*eye(4)
```

MATLAB Exercises

1. Use `eig` to find the eigenvalues and `null` to find the eigenvectors of these matrices.

a. `A = pascal(5)`
b. `A = zeros(5); A(:)= 1:25;`
c. `A = 3*eye(5) + diag(ones(4,1),1)`
d. `A = ones(5)`
e. `A = magic(6)`

2. Determine which of the matrices in Exercise 1 are diagonalizable.

3. Use `rank` to compute the dimension of each eigenspace for each of the matrices in Exercise 1.

4. Let `A = rand(5)`, `A = A'*A`, and `B = ref(A)`. What are the eigenvalues of `A` and `B`? What can you conclude about applying row operations when you are looking for eigenvalues?

5. The MATLAB function `prod(v)` computes the product of the entries of the vector `v`. For each of the matrices in Exercise 1, compute `prod(eig(A))` and `det(A)`. Describe what happens, and make a conjecture about this. Verify your conjecture, assuming that `A` is diagonalizable.

6. The MATLAB function `poly(A)` computes the coefficients of the characteristic polynomial of A. For each of the matrices in Exercise 1 compute `poly(A)` and `det(A)`. Describe what happens and make a conjecture about this. Verify your conjecture, assuming `A` that is diagonalizable.

7. Try the following:

```
A = pascal(10); ev = eig(A); r = ev(1),
det(A - r*eye(10))
```

Explain why or why not MATLAB's answer is reasonable.

Now look at

```
s = ev(10);
det(A - s*eye(10))
```

What would you expect to get here? Explain why or why not MATLAB's answer is reasonable.

Now replace `det` in both MATLAB examples with `rank`. Explain why or why not MATLAB's answers are reasonable. Which function, `det` or `rank`, should be used? Explain why.