

Loop Currents

Text Reference: Section 1.10, p. 82

The purpose of this set of exercises is to provide more and larger examples of loop currents. Recall the two physical laws that are introduced in Section 1.10:

Ohm's Law: The voltage drop across a resistor is $V=RI$, where the voltage drop V is measured in volts, the resistance R is measured in ohms, and the current flow I is measured in amperes.

Kirchoff's Voltage Law: The algebraic sum of the RI voltage drops in one direction around a loop current equals the sum of the voltage sources in the same direction around the loop.

These laws may be used to find the current flow through any branch of a network of loop currents. For example, consider the simple circuit in Figure 1.

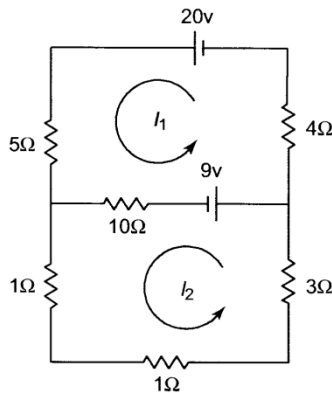


Figure 1

For the upper loop, the sum of the voltage drops for the current I_1 is $19I_1$ and the sum of the voltage drops for the current I_2 is $-10I_2$. Thus by Kirchoff's Voltage Law,

$$19I_1 - 10I_2 = 29$$

since the sum of the voltage sources on the upper loop is 29 volts. Likewise, the lower loop produces the equation

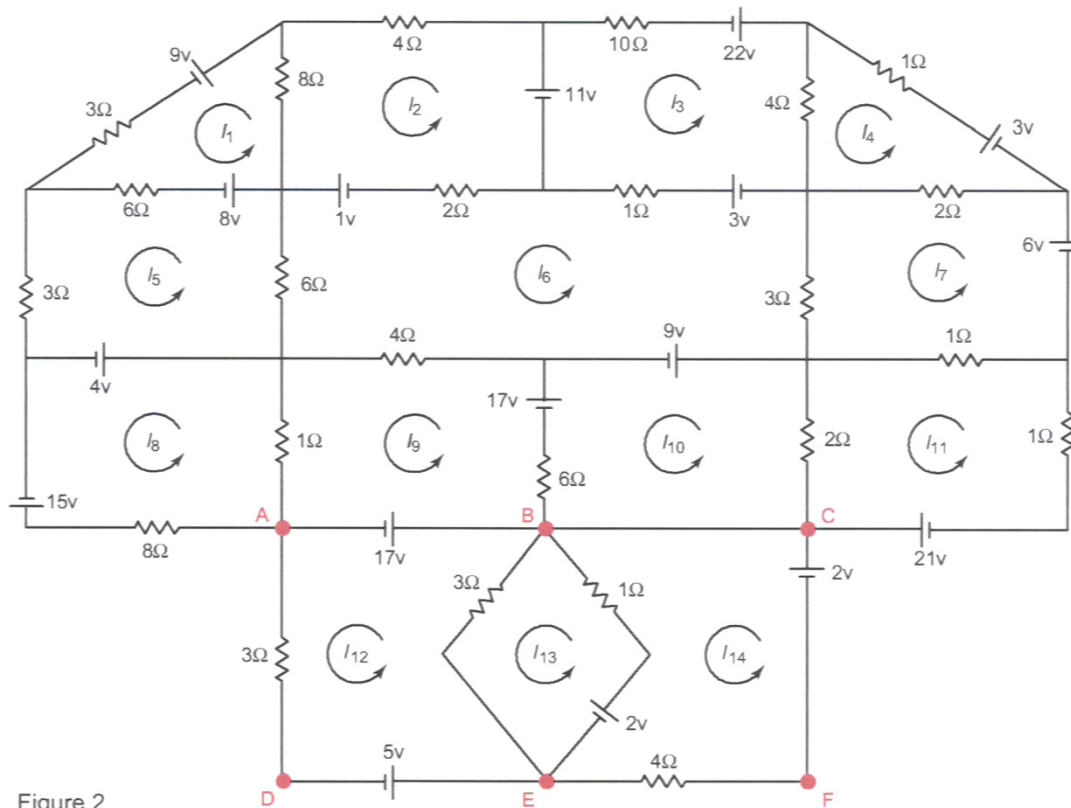
$$-10I_1 + 15I_2 = -9$$

Once the loop currents are known, the current in each branch of the network may be calculated. In the above example, the current in the branch connecting B and D is I_1 ; the current in the branch connecting C and D is $I_1 - I_2$ amperes in the direction of I_1 .

Questions:

*Note: The MATLAB M-file **currents.m** contains data for question 2. Save the accompanying file **currents.m** as a text file and with the .m extension. Set the path browser in MATLAB to the folder where the file **currents.m** is located. Then type **currents** in the MATLAB command window for the data.*

1. Confirm the above equations.
2. Solve the system of two equations given above to find the loop currents I_1 and I_2 .



Consider the network in Figure 2. Write down a system of equations which could be used to find the loop currents in this network. Check that the augmented matrix for this system is equivalent to matrix A which is given below.

$$A = \begin{pmatrix} 17 & -8 & 0 & 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -8 & 14 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 15 & -4 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -36 \\ 0 & 0 & -4 & 7 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ -6 & 0 & 0 & 0 & 15 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 \\ 0 & -2 & -1 & 0 & -6 & 16 & -3 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 13 \\ 0 & 0 & 0 & -2 & 0 & -3 & 6 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & -1 & 0 & 0 & 0 & 0 & 0 & 11 \\ 0 & 0 & 0 & 0 & 0 & -4 & 0 & -1 & 11 & -6 & 0 & 0 & 0 & 0 & 34 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 8 & -2 & 0 & 0 & 0 & -26 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -2 & 4 & 0 & 0 & 0 & -21 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & -3 & 0 & -12 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 4 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 5 & 0 \end{pmatrix}$$

Here is the matrix A in MATLAB format which can be found on the M-file **current.m**:

```
A = [17, -8, 0, 0, -6, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1;
     -8, 14, 0, 0, 0, -2, 0, 0, 0, 0, 0, 0, 0, 0, 10;
     0, 0, 15, -4, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, -36;
     0, 0, -4, 7, 0, 0, -2, 0, 0, 0, 0, 0, 0, 0, 3;
     -6, 0, 0, 0, 15, -6, 0, 0, 0, 0, 0, 0, 0, 0, -4;
     0, -2, -1, 0, -6, 16, -3, 0, -4, 0, 0, 0, 0, 0, 13;
     0, 0, 0, -2, 0, -3, 6, 0, 0, 0, -1, 0, 0, 0, 6;
     0, 0, 0, 0, 0, 0, 0, 9, -1, 0, 0, 0, 0, 0, 11;
     0, 0, 0, 0, 0, -4, 0, -1, 11, -6, 0, 0, 0, 0, 34;
     0, 0, 0, 0, 0, 0, 0, 0, -6, 8, -2, 0, 0, 0, -26;
     0, 0, 0, 0, 0, 0, -1, 0, 0, -2, 4, 0, 0, 0, -21;
     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 6, -3, 0, -12;
     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -3, 4, -1, 2;
     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 5, 0]
```

3. Find the loop currents for the network. What is the current in the branch between A and B? What is the current in the branches between B and E?
4. Suppose each power source in this circuit were replaced by a 9 volt battery. How do the loop currents change?