Calibration of Ornstein-Uhlenbeck Process for FX Rate

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Abstract
Mean-reverting Ornstein-Uhlenbeck processes are widely used in model interest rates, FX rates and commodities. The current estimation approaches of the parameters have difficulties applied in financial practice. The paper presents a new approach of reversion speed and long-term mean calibration for FX rate. Our numerical results show that expectation value of simulation with new calibrated parameters is very close to the market forward FX rate. The new calibration approach generates more conservative FX derivatives values and credit risk exposure.

1 Introduction

Most common Ornstein-Uhlenbeck calibration approaches are least-square regression, maximum likelihood, and jackknife technique [3, 4, 5]. Both least-square minimization and maximum likelihood estimation techniques are known to be good at estimating volatility, but poor in estimating long-term mean and reversion speed. Taking a larger sample does not solve the problem, and nor do techniques such as the 'jackknife', which introduces more problems than it solves. Using least-square regression approach and ten-year FX history, we compare expectation of simulated FX rates with market FX forward rates, there are big discrepancy between them. Furthermore, incorrectly simulated FX rates causes FX derivatives like forward deal exposures counterintuitive decreasing.

In another way, if people can replicate same parameters from simulation data as from historical data, the calibration approach is stable and reliable. But least-square regression and maximum likelihood only can generate close parameters when calibrated with time series of more than 26,000 steps which equals to 100-year FX trading history. When calibrate data with 3,000 time steps equals to 11-year trading history, long-term mean and reversion speed with simulation data are far from parameters with history data 2003-2013. For most real FX rates with 10-20 year history, these calibrations have big standard deviations, so the parameters are not stable and correct. Please refer Table 1, Figure 1, 2 and 3 in appendix.

We propose a new approach to calibrate reversion speed and long-term mean. The calibration is under risk-neutral measure, find the optimal speed and mean of Ornstein-Uhlenbeck processes satisfying interest rate parity based on interest term structure. People think that FX forward rate is the best expectation of future spot rate. Exposure management generally focuses on real parameters [1]. One to five-year FX forward rate curves are available at the market. Our numerical results show the simulation based on risk-neutral calibration is very close to the true FX forward rate. Although FX classic risk-neutral Geometric Brownian Motion (GBM) simulation's expectation is also close to true FX forward rate, but GBM model usually get too big variance so that risk management considers it unreasonable and unrealistic. So risk management prefer mean-reversion model than GBM. Since risk-neutral Ornstein-Uhlenbeck calibration is only based on current interest rates, the method is more stable and reasonable than least-square regression and maximum likelihood etc.

The outline of this paper is as following: Section 2 briefly states FX simulation model and least-square regression calibration, Section 3 describes the calibration with risk-neutral method and compare ten major currencies calibration and simulation results with least-square, history quantiles, and risk-neutral GBM model, Section 4 presents the 97.5% exposure of FX forward, swap and option with ten-year maturity based on least-square, risk-neutral Ornstein-Uhlenbeck and GBM. Section 5 contains concluding remarks.

2 FX Simulation Model and Least-square Calibration [4]

2.1 FX Simulation Model- Ornstein-Uhlenbeck process

$$Z(t) = \ln(S(t)/S(0)) \quad (1)$$

$$dZ(t) = \alpha(\theta - Z(t))dt + \sigma dW(t) \quad (2)$$

where:

- S(t) is FX rate
- Z(t) is Ornstein-Uhlenbeck process, Z(0)=0
- σ is constant volatility
- α is constant mean reversion speed
- θ is constant long-term mean
- W(t) is a Brownian-Motion, so dW(t)~N(0, $\varepsilon\sqrt{dt}$)

Solution of SDE (2):

$$Z(t) = Z(0)e^{-\alpha t} + \theta(1-e^{-\alpha t}) + \int_0^t \sigma e^{\alpha(s-t)} dW_s \quad (3)$$

$$E(Z(t)) = Z(0)e^{-\alpha t} + \theta(1-e^{-\alpha t}) \quad (4)$$

$$Var(Z(t)) = E[(Z(t) - E(Z(t))^2] = \frac{\sigma^2}{2\alpha}(1-e^{-2\alpha t}) \quad (5)$$

$$E(S(t)) = S(0)e^{[\theta(1-e^{-\alpha t})+\frac{\sigma^2}{4\alpha}(1-e^{-2\alpha t})]} \xrightarrow{t->\infty} S(0)e^{[\theta+\frac{\sigma^2}{4\alpha}]} \quad (6)$$

Proof (6):

$$E(e^X) = E(e^{\mu_X + \frac{1}{2}\sigma_X{}^2}) \quad (7)$$

Based on (4), (5), (7):

$$E(S(t)) = S(0)E(e^{Z(t)}) = S(0)e^{\mu_{Z(t)} + \frac{1}{2}\sigma_{Z(t)}{}^2} = S(0)e^{[\theta(1-e^{-\alpha t})+\frac{\sigma^2}{4\alpha}(1-e^{-2\alpha t})]} \quad (8)$$

2.2 Calibration with Least-square

(2) rewritten as, refer the detail in Appendix: FX Least-square Calibration:

$$\Delta \ln S(t) = \alpha(\theta - \ln S(t)dt + \sigma dW(t) \quad (9)$$

$$\Delta \ln S(t) = \ln S(t) - \ln S(t-1)$$

$$\alpha = -\frac{Cov_{\Delta \ln S(t), \ln S(t)}}{\sigma_{\ln S(t)}{}^2} \quad (10)$$

$$\theta = \frac{mean_{\Delta \ln S(t)}}{\alpha} + mean_{\ln S(t)} \quad (11)$$

$$\sigma_{\Delta \ln S(t)}{}^2 = \alpha^2 \sigma_{\ln S(t)}{}^2 + \sigma^2 \quad (12)$$

As shown in Table 1, Figure 1, 2 and 3 of Appendix: Verification Least-square Regression and Maximum Likelihood Calibration, least-square calibration has big standard deviations for real historical data, so the parameters are not stable and correct.

3 Risk-Neutral Calibration of Ornstein-Uhlenbeck process

3.1 Ornstein-Uhlenbeck (OU) Calibration under Risk-Neutral Measure

Consider a bond $B(t,T)$ in local currency under the risk-neutral measure. $D(t)$ is local interest rate discount process. $B(t,T)D(t)$ is a martingale under probability space (Ω, F, Q) and filtration $F(t)$ [2]:

$$B(t,T)D(t) = \mathrm{E}[D(T) \mid F(t)], 0 \le t \le T \tag{13}$$

$$B(T,T) = 1 \text{ , in local currency}$$

$$D(t) = e^{-\int_0^t R(s)ds}$$

Another bond $B_f(t,T)$ is in foreign currency under the risk-neutral measure. $D_f(t)$ is foreign interest rate discount process. $B_f(t,T)D_f(t)$ is a martingale under probability space (Ω, F, Q) and filtration $F(t)$:

$$B_f(t,T)D_f(t) = \mathrm{E}[D_f(T) \mid F(t)], 0 \le t \le T \tag{14}$$

$$B_f(T,T) = 1 \text{ , in foreign currency}$$

$$D_f(t) = e^{-\int_0^t R_f(s)ds}$$

Transform (13) and (14), S(t) is exchange rate at time t:

$$S(t) = \frac{B(t,T)}{B_f(t,T)} = \frac{D_f(t)\mathrm{E}[D(T) \mid F(t)]}{D(t)\mathrm{E}[D_f(T) \mid F(t)]} \tag{15}$$

$$S(0) = \frac{B(0,T)}{B_f(0,T)} = \frac{D_f(0)\mathrm{E}[D(T) \mid F(t)]}{D(0)\mathrm{E}[D_f(T) \mid F(t)]} = \frac{\mathrm{E}[D(T) \mid F(t)]}{\mathrm{E}[D_f(T) \mid F(t)]}$$

$$D(0) = 1, D_f(0) = 1 \tag{16}$$

Input (16) into (15):

$$S(t) = S(0)\frac{D_f(t)}{D(t)} = S(0)e^{-\int_0^t (R_f(s) - R(s))ds} \tag{17}$$

(17) is Interest Parity under no-arbitrage condition.
Transform (17):

$$\ln S(t) = \ln S(0) - \int_0^t (R_f(s) - R(s))ds \tag{18}$$

$$d \ln S(t) = -\int_{t-dt}^t (R_f(s) - R(s))ds = (R(t) - R_f(t))dt \tag{19}$$

Input (19) into (2), the OU process under risk-neutral measure:

$$(R(t) - R_f(t))dt = \alpha(\theta - Z(t))dt + \sigma dW(t) \tag{20}$$

Based on equation (4) and (20):

$$E(\ln S(t)) = \theta(1 - e^{-\alpha t}) = (R(t) - R_f(t))t \tag{21}$$

$$\theta = \frac{(R(t) - R_f(t))t}{(1 - e^{-\alpha t})} \tag{22}$$

Observe 1 month to 5 year interest rates of local currency and foreign currency $R(t), R_f(t)$. Put into the formula (23). Change speed and mean until sum of difference on each terms (23) is minimized. In risk-neutral world, we use implied FX volatility as volatility value. Appendix shows simulation result based on risk-neutral OU process. 50% percentiles are very close to forward rates.

$$\arg\min\left(\sum_{t=1M}^{5Y}(\theta(1-e^{-\alpha t})-(R(t)-R_f(t))t)\right) \tag{23}$$

3.2 Upper and Lower Bounds of Reversion Speed

For currencies CHF, HKD, JPY, NZD, the calculated reversion speeds are close to zero or negative, that is, never converge. When reversion speed<0.01, the simulation shapes are similar. So we set 0.1 as lower bound of reversion speed in order to calculate a reasonable mean level.

For currency EUR, reversion speed is found very large, which makes simulation shape converges too steeply. When reversion speed >0.5, the simulation shapes are not realistic for FX rate. So we set 0.5 as upper bound for mean reversion to avoid unreasonable steep convergence.

3.3 Compare Risk-Neutral OU with Geometric Brownian Motion (GBM) Process

Front office prefers risk-neutral GBM process instead of risk-neutral OU process for FX rate. Because risk-neutral GBM process is classical FX evolution model and shows bigger volatility than mean reversion process, it means more trading opportunities. And historical data do not always show mean-reversion feature based on Augmented Dickey-Fuller and variance ratio test, some people tend to support GBM model. Risk-neutral GBM uses the differences between domestic interest rate and foreign interest rate as its drift, uses implied FX volatility as volatility value. We test ten currencies with GBM term structure model. Both 50% values of OU risk-neutral and GBM risk-neutral are close to forward rate. But GBM risk neutral's scope between 2.5% and 97.5% are larger than OU risk-neutral's scope. Formula (24) is risk-neutral GBM process. Please review the test result in Appendix.

$$\Delta \ln S(t) = (r_d - r_f)dt + \sigma dW(t) \tag{24}$$

where:

- r_d is domestic interest rate
- r_f is foreign interest rate
- σ is implied FX volatility

3.4 Compare Risk-Neutral OU Simulation with Historical Data Quantiles

Historical FX rate log return's quantiles of 97.5% is critical benchmark for backtesting criteria. An ideal model can produce a distribution expected to occur with realized or practical distribution [6]. Since most FX rate histories are less than 20 years, which are much less than OU process required time series, so historical cones usually should be within ideal model's simulation cones. That is, historical 97.5% value should be smaller than or equal to simulation's 97.5% value, and historical 2.5% value should be bigger than or equal to simulation's 2.5% value. According to formula (4) and (5), OU simulation's mean and volatility depend on reversion speed and long-term mean. By inverse cumulative function of the normal distribution, find implicit reversion speed and long-term mean matching with historical 97.5% and 2.5% according to formula (25). We call this calibration approach as inversion of historical quantiles. So we can compare OU simulation quantiles with history-implicit quantiles, validate OU parameters from backtesting view. Rewrite (4) and (5):

$$\mu_{Z(t)} = \theta(1-e^{-\alpha t}) \tag{4}$$

$$\sigma^2_{Z(t)} = \frac{\sigma^2}{2\alpha}(1-e^{-2\alpha t}) \tag{5}$$

$$\left\{\begin{array}{l} \arg\min(\Phi^{-1}(97.5\%, \mu_{Z(t)}, \sigma^2_{Z(t)}) - hist(97.5\%)) \\ \Phi^{-1}(97.5\%, \mu_{Z(t)}, \sigma^2_{Z(t)}) >= hist(97.5\%) \\ \\ \arg\min(hist(2.5\%) - \Phi^{-1}(2.5\%, \mu_{Z(t)}, \sigma^2_{Z(t)})) \\ \Phi^{-1}(2.5\%, \mu_{Z(t)}, \sigma^2_{Z(t)}) >= hist(2.5\%) \end{array}\right\} \tag{25}$$

where:

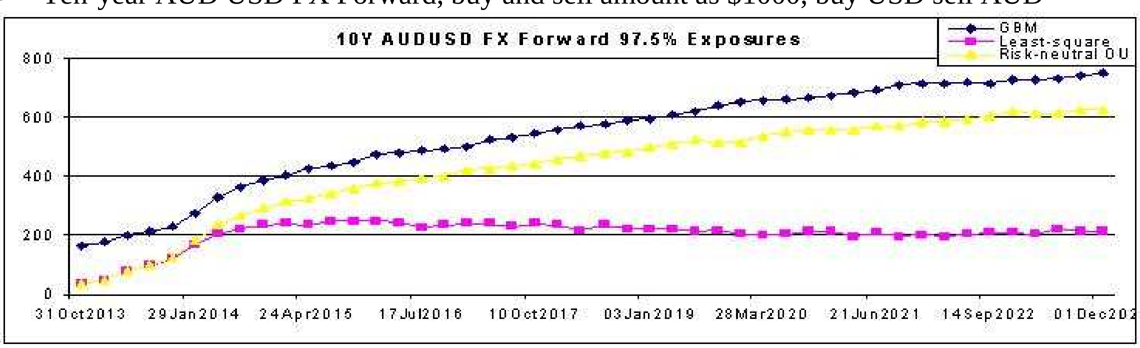
- $\Phi^{-1}(97.5\%, \mu_{Z(t)}, \sigma^2_{Z(t)})$ is inverse cumulative function of the normal distribution at 97.5%
- $\Phi^{-1}(2.5\%, \mu_{Z(t)}, \sigma^2_{Z(t)})$ is inverse cumulative function of the normal distribution at 2.5%

Based on test result of appendix, ten-currencies' historical values on 97.5% are all within the values of risk-neutral OU simulation 97.5% values, but least-square simulation values on 97.5% fail to cover most historical values on 97.5%. So risk-neutral OU calibration has good performance in backtesting.

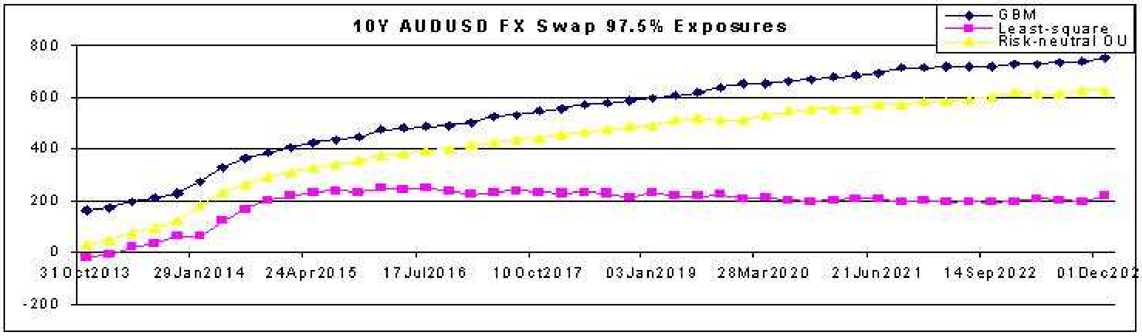
4 Test FX Derivatives with Risk-Neutral OU Process

Three deal profiles are similar at start date, but diverge with time. GBM exposure is more than risk-neutral OU, risk-neutral OU exposure is more conservative than least-square. Since least-square calibration is also unstable, it will cause big difference of deal profiles even if market conditions have no significant change. While risk-neutral OU calibration is only dependent on interest rate curve on the market, so the calibrated parameters are very stable and trusted, and deal profiles are also consistent with monthly calibration updating. It is valuable reference for risk managers and traders.

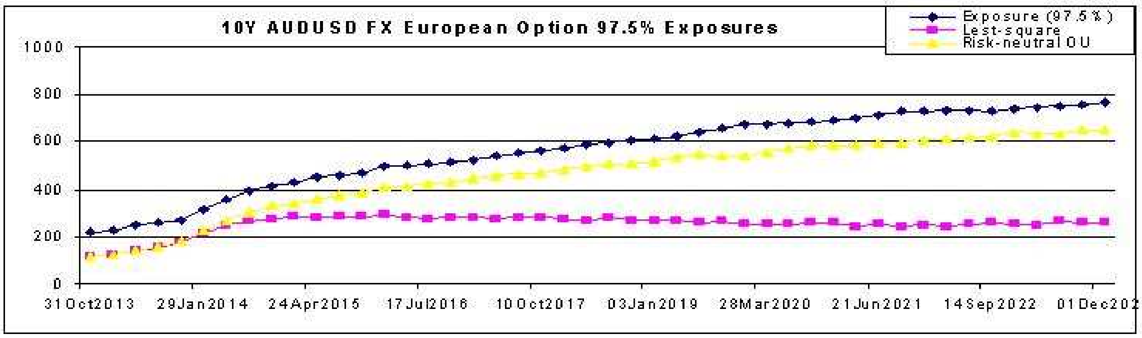
- Ten-year AUD USD FX Forward, buy and sell amount as $1000, buy USD sell AUD



- Ten-year AUD USD FX Swap, buy and sell amount as $1000, buy USD sell AUD



- Ten-year AUD USD FX European Option, strike is spot rate, amount as $1000



5 Conclusion

The calibration method outlined in this paper is very reasonable and feasible for FX mean-reversion model. There are three obvious advantages using our calibration approach: 1) It is stable and expectation consistent with FX forward rate; 2) Its conservative parameter estimation satisfies historical data backtesting; 3) It is a base to generate stable and correct FX derivative risk profile and exposure.

Appendix

1. Verification Least-square Regression and Maximum Likelihood Calibration

Calibration Verification		**Parameters from History (2003-2013)**			**Parameters from Simulation Data**					
3,000 steps 10,000 paths		**θ**	**σ**	**α**	μ_θ	σ_θ	μ_σ	σ_σ	μ_α	σ_α
Test case A	**= mean**	-0.288	0.143	0.426	-0.284	0.200	0.143	0.002	0.908	0.514
Test case B	**>mean**	-0.750	0.143	0.426	-0.733	0.192	0.143	0.002	0.690	0.369
Test case C	**<mean**	0.250	0.143	0.426	0.259	0.845	0.143	0.002	0.673	0.332
26,000 steps 10,000 paths		θ	σ	α	μ_θ	σ_θ	μ_σ	σ_σ	μ_α	σ_α
Test case A	**= mean**	-0.288	0.143	0.426	-0.287	0.011	0.143	0.000	0.430	0.030
Test case B	**>mean**	-0.750	0.143	0.426	-0.750	0.011	0.143	0.000	0.429	0.029
Test case C	**<mean**	0.250	0.143	0.426	0.250	0.011	0.143	0.000	0.430	0.030

Table 1 Verification Least-square Regression and Maximum Likelihood Calibration

- Test Case A/B/C: initial FX rate is at the mean level, above and below the mean level. Test with 3,000 and 26,000 time steps.
- Only when time steps are equal to or more than 26,000, the two calibrations can replicate mean level, reversion speed and volatility. The standard deviation of mean level is less than 3.9%. The standard deviation of reversion speed is less than 0.93%. The difference of real parameters and replicated parameters are less than 7% at the 95% confidence level.
- When time step decreases to normal 3,000 sample size, that is 11-year data, the reversion speed is more than 50% than its original value. Expectation of mean level is close to real value but with more than 50% standard deviation. It means calibration of mean level cannot be trusted. Please refer figure 1, 2, 3.
- Since there are usually 10-11 years FX historical data in practice, the two calibrations for mean level and reversion speed are not trusted.

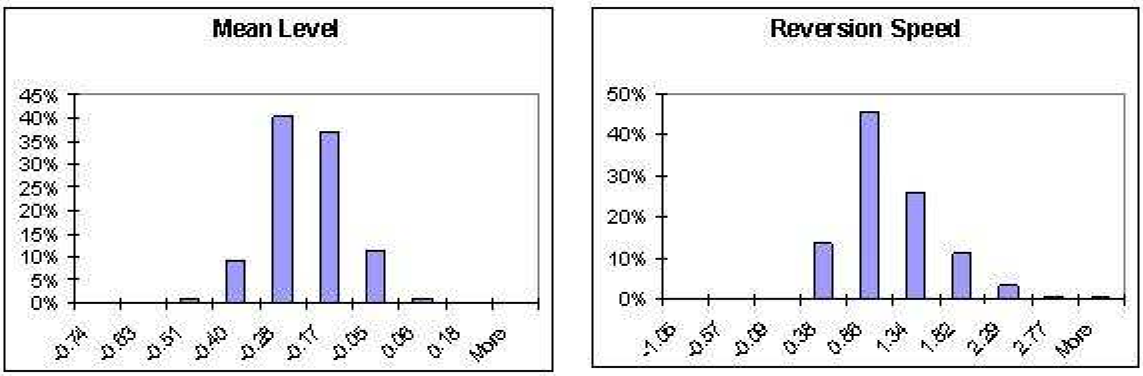


Figure 1 Histogram of Mean Level and Reversion Speed, Initial Value= Mean level

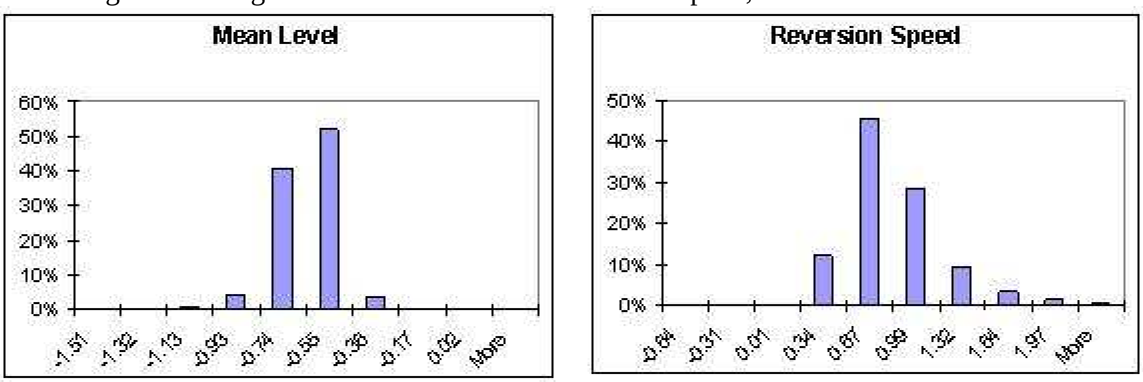


Figure 2 Histogram of Mean Level and Reversion Speed, Initial Value< Mean level

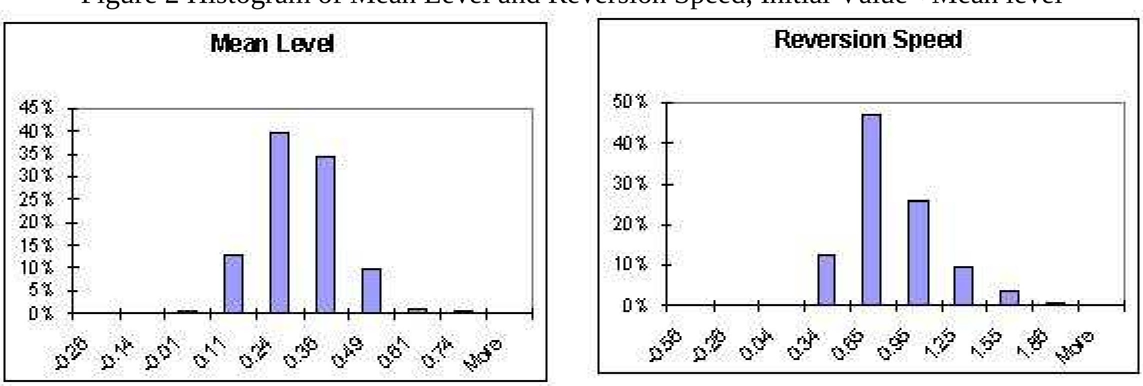


Figure 3 Histogram of Mean Level and Reversion Speed, Initial Value> Mean level

2. FX Least-square Calibration

$$\begin{aligned}&Based(1),(2)\\&d\ln S(t)=\alpha(\theta-\ln S(t))dt+\sigma dW(t)\end{aligned} \qquad (A1)$$

$$\begin{aligned}&Annualize(A1): dt=1\\&d\ln S(t)=\alpha(\theta-\ln S(t))+\sigma dW(t)\end{aligned} \qquad (A2)$$

$$\begin{aligned}&Apply: E(e)=0,\operatorname{var}(e)=1,\operatorname{cov}(e,\ln S(t))=0\\&E(e)=0\\&E(d\ln S(t))=E(\alpha(\theta-\ln S(t))+\sigma dW(t))\\&mean_{d\ln S(t)}=\alpha(\theta-mean_{\ln S(t)})\end{aligned} \qquad (A3)$$

$$\begin{aligned}&\operatorname{var}(e)=1\\&Var(d\ln S(t)-\alpha(\theta-\ln S(t)))=\sigma^2\\&\sigma_{d\ln S(t)}{}^2+\alpha^2\sigma_{\ln S(t)}{}^2+2\alpha Cov_{d\ln S(t),\ln S(t)}=\sigma^2\end{aligned} \qquad (A4)$$

$$\begin{aligned}&\operatorname{cov}(e,\ln S(t))=0\\&Transform(A2):\\&d\ln S(t)-\alpha\theta=-\alpha\ln S(t)+\sigma dW(t)\\&Var(d\ln S(t)-\alpha\theta)=Var(-\alpha\ln S(t)+\sigma dW(t))\\&\sigma_{d\ln S(t)}{}^2=\alpha^2\sigma_{\ln S(t)}{}^2+\sigma^2\end{aligned} \qquad (A5)$$

$$\begin{aligned}&Input(A4)to(A5)\\&\sigma_{d\ln S(t)}{}^2=\alpha^2\sigma_{\ln S(t)}{}^2+\sigma_{d\ln S(t)}{}^2+\alpha^2\sigma_{\ln S(t)}{}^2+2\alpha Cov_{d\ln S(t),\ln S(t)}\\&\alpha=-\frac{Cov_{d\ln S(t),\ln S(t)}}{\sigma_{\ln S(t)}{}^2}\end{aligned} \qquad (A6)$$

$$\begin{aligned}&Input(A6)in(A3):\\&\theta=\frac{mean_{d\ln S(t)}}{\alpha}+mean_{\ln S(t)}\end{aligned} \qquad (A7)$$

$$\begin{aligned}&Input(A6)in(A5):\\&\sigma^2=\sigma_{d\ln S(t)}{}^2-\alpha^2\sigma_{\ln S(t)}{}^2\end{aligned} \qquad (A8)$$

2. Compare Calibration and Simulation of Least-square, Risk-neutral OU, Risk-neutral GBM with Historical Quantiles and Forward FX Rate

- Use 2003-2013 FX Historical Data as Parameter Calibration and Historical Quantiles
- Use Forward Rate on: http://www.investing.com as Reference
- Risk-Neutral OU has bigger reversion speed than speed of least-square and inversion of historical quantiles.
- Risk-Neutral OU Simulation’s 50% values are close forward rates, while least-square and historical-inversion’s 50% values usually are very different from forward rate
- GBM divergence is more than risk-neutral OU divergence, risk-neutral OU divergence is more than least-square and inversion of historical quantiles.
- Ten-currencies 97.5% values are within risk-neutral OU and GBM 97.5% distribution, but out of least-square 97.5% distribution.

		AUD	CAD	CHF	EUR	GBP	HKD	JPY	NOK	NZD	SEK
Least Square	**θ**	0.09	0.07	0.13	0.05	0.0005	0.0005	0.058	0.04	0.07	0.04
	α	0.62	0.76	0.43	1.35	0.45	2.26	0.32	1.5	0.95	1.48
Risk Neutral	**θ**	-0.49	-0.06	0.06	-0.003	-0.015	0.008	0.061	-0.21	-0.276	-0.075
	α	0.085	0.162	0.1	0.5	0.479	0.1	0.1	0.087	0.1	0.165
History Quantiles	**θ**	-0.106	-0.086	-0.145	0.01	0.062	0.013	0.03	0.026	-0.2	-0.057
	α	0.7188	0.7674	0.537	1.1	0.635	0.148	0.429	1.375	0.914	1.461

Table 2 Risk-Neutral OU, Least-square and Historical Parameters of FX Rate

Term	USD	AUD	CAD	CHF	EUR	GBP	HKD	JPY	NOK	NZD	SEK
1D	0.0042	0.041	0.014	0.0005	0.0091	0.0124	0.0002	0.0014	0.015	0.027	0.0218
1M	0.0044	0.044	0.013	0.0005	0.0091	0.012	0.003	0.0014	0.015	0.0288	0.0218
2M	0.0045	0.045	0.013	0.0005	0.0083	0.0116	0.0037	0.0014	0.0172	0.029	0.0218
3M	0.0047	0.045	0.013	0.0005	0.0081	0.0113	0.004	0.0014	0.0194	0.0292	0.0218
6M	0.0047	0.044	0.013	0.0006	0.0073	0.0108	0.0054	0.0015	0.0226	0.0289	0.0187
9M	0.0048	0.044	0.013	0.0007	0.0069	0.0105	0.0049	0.0015	0.0225	0.0292	0.0174
1Y	0.005	0.044	0.014	0.0008	0.0068	0.0104	0.0043	0.0015	0.0224	0.0295	0.0162
18M	0.0056	0.044	0.014	0.0007	0.007	0.0107	0.0051	0.0012	0.0229	0.0311	0.0164
2Y	0.0063	0.044	0.015	0.0006	0.0075	0.0112	0.0058	0.0008	0.0235	0.0326	0.0187
3Y	0.0081	0.045	0.017	0.0009	0.0092	0.0127	0.0076	0.0004	0.0254	0.0355	0.0198
4Y	0.0107	0.047	0.019	0.0016	0.0114	0.0142	0.01	0.005	0.0274	0.0383	0.021
5Y	0.0135	0.048	0.02	0.0028	0.0137	0.0162	0.0126	0.001	0.0292	0.0411	0.0223

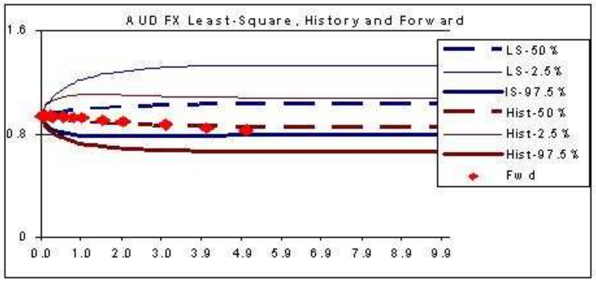
Table 3 Interest Rate

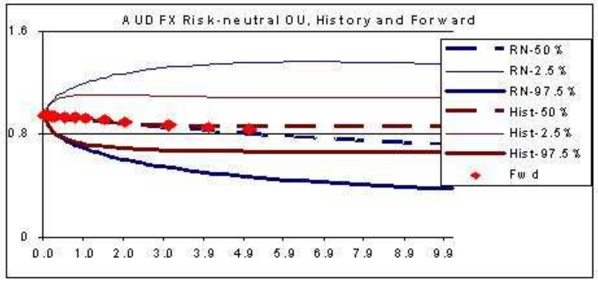
	AUD	CAD	CHF	EUR	GBP	HKD	JPY	NOK	NZD	SEK
σ	0.147	0.103	0.115	0.103	0.097	0.0054	0.108	0.133	0.146	0.135

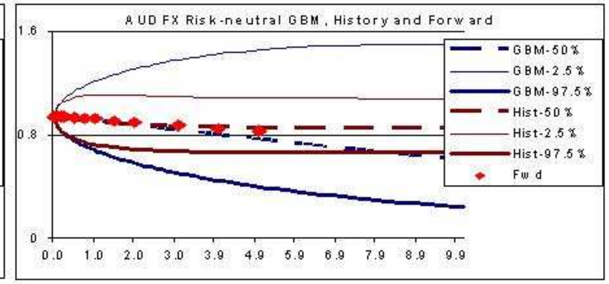
Table 4 FX Volatility

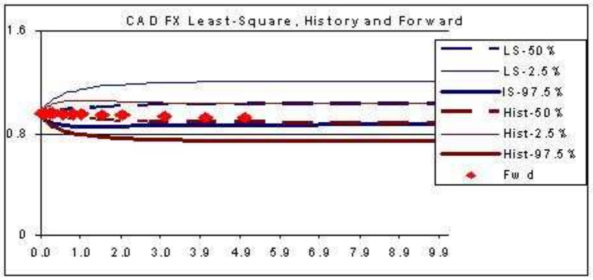
	AUD	CAD	CHF	EUR	GBP	HKD	JPY	NOK	NZD	SEK
ON	0.942437	0.9578296	1.0967402	1.352805	1.609787	0.1290058	0.009979	0.1642813	0.834945	0.1509716
1M	0.940583	0.9571118	1.0970355	1.35295	1.609415	0.1290135	0.0099791	0.1641031	0.833255	0.1508687
2M	0.93866	0.9564321	1.0973576	1.353086	1.609086	0.1290223	0.0099791	0.1639183	0.831512	0.1507653
3M	0.936832	0.9556484	1.097743	1.353181	1.60871	0.1290339	0.0099791	0.1637321	0.829792	0.1506662
6M	0.93134	0.9535729	1.0989615	1.353526	1.60779	0.1290589	0.0099792	0.163204	0.824508	0.1503714
9M	0.92575	0.9514069	1.1002916	1.353915	1.6068	0.1290764	0.0099793	0.1626902	0.818662	0.150052
1Y	0.91994	0.9491718	1.1019284	1.35435	1.60572	0.1290872	0.0099794	0.1621495	0.812304	0.1497156
18M	0.90719	0.9445107	1.1065988	1.3559	1.60366	0.1291089	0.0099799	0.1610347	0.798204	0.1489731
2Y	0.89444	0.9398496	1.1112692	1.35745	1.6016	0.1291306	0.0099803	0.1599199	0.784104	0.1482305
3Y	0.86886	0.9315758	1.1276195	1.3655	1.6001	0.1290972	0.0099822	0.1582704		0.146981
4Y	0.84912	0.9269559	1.15047	1.37955	1.6029	0.1291765	0.0099849	0.157397		0.1464697
5Y	0.83263	0.925626	1.1773709	1.3977	1.6117	0.1292557	0.0099882	0.15715		0.1465234

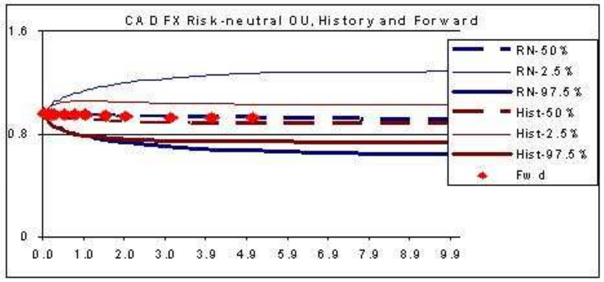
Table 5 FX Forward Rate

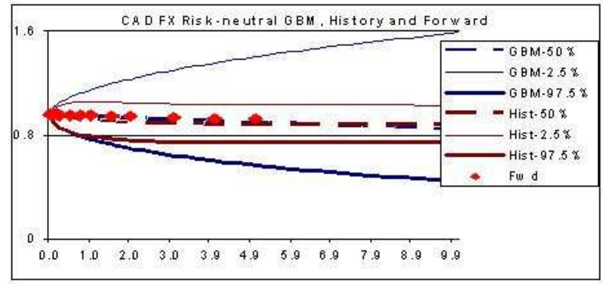

AUD FX Least-Square, History and Forward
LS-50%
LS-2.5%
IS-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fwd

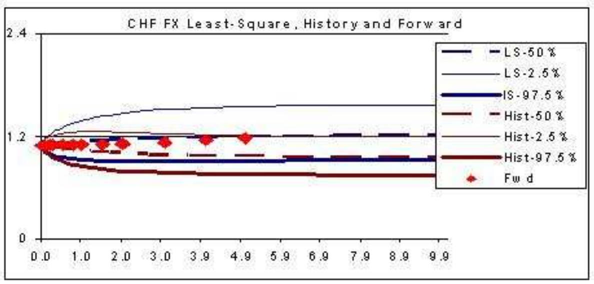

AUD FX Risk-neutral OU, History and Forward
RN-50%
RN-2.5%
RN-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fwd

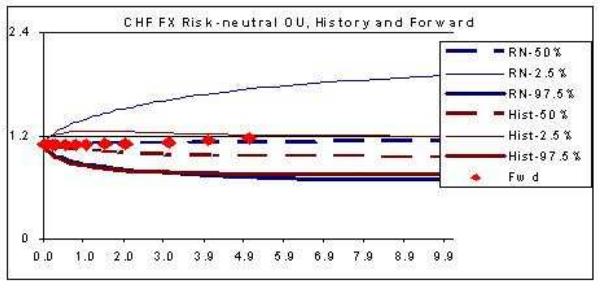

AUD FX Risk-neutral GBM, History and Forward
GBM-50%
GBM-2.5%
GBM-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fwd

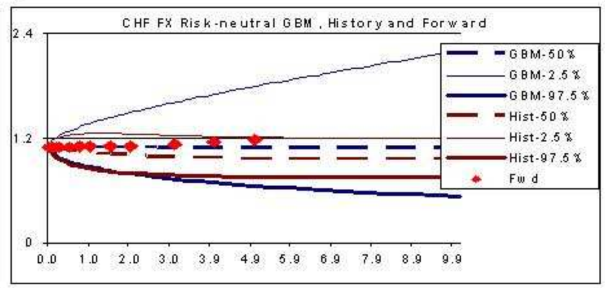

CAD FX Least-Square, History and Forward
LS-50%
LS-2.5%
IS-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fwd

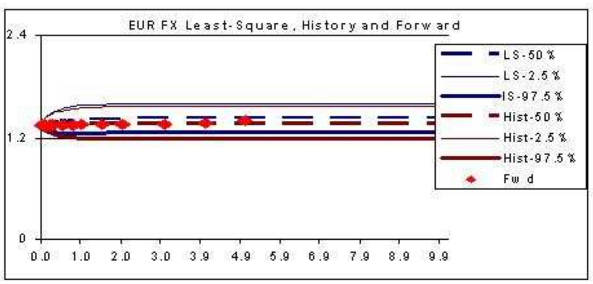

CAD FX Risk-neutral OU, History and Forward
RN-50%
RN-2.5%
RN-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fwd

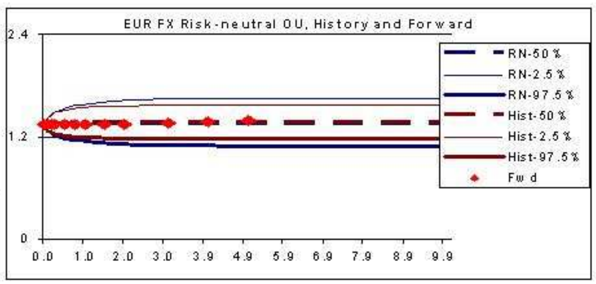

CAD FX Risk-neutral GBM, History and Forward
GBM-50%
GBM-2.5%
GBM-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fwd

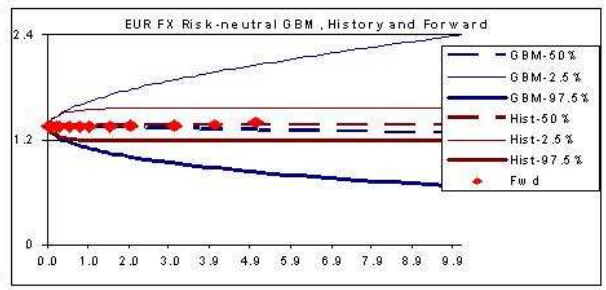

CHF FX Least-Square, History and Forward
LS-50%
LS-2.5%
IS-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fwd

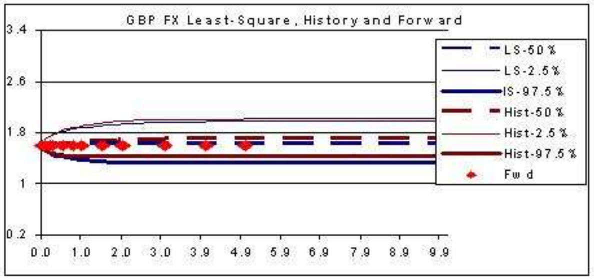

CHF FX Risk-neutral OU, History and Forward
RN-50%
RN-2.5%
RN-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fwd

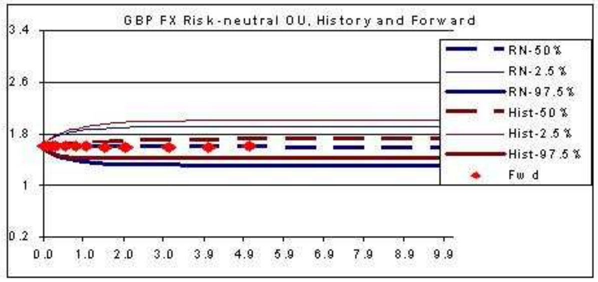

CHF FX Risk-neutral GBM, History and Forward
GBM-50%
GBM-2.5%
GBM-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fwd

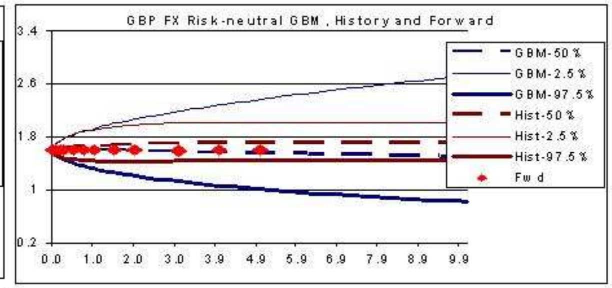

EUR FX Least-Square, History and Forward
LS-50%
LS-2.5%
IS-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fwd

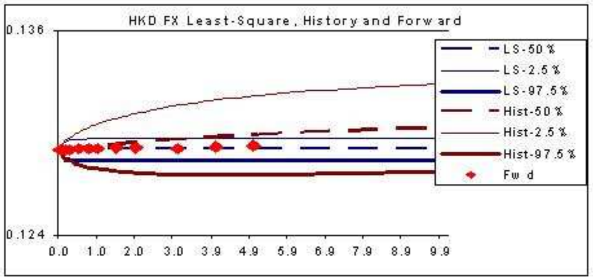

EUR FX Risk-neutral OU, History and Forward
RN-50%
RN-2.5%
RN-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fwd

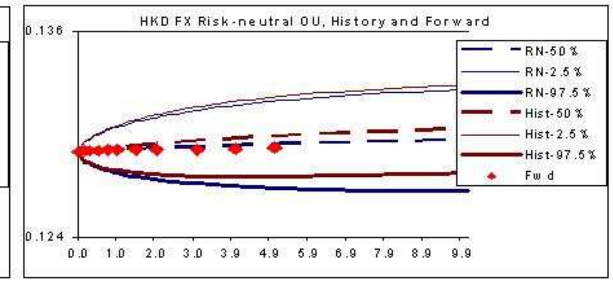

EUR FX Risk-neutral GBM, History and Forward
GBM-50%
GBM-2.5%
GBM-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fwd

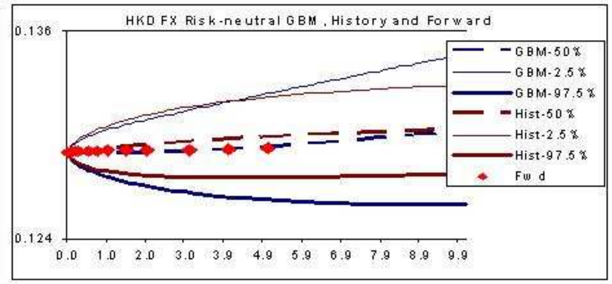

GBP FX Least-Square, History and Forward
3.4
2.6
1.8
1
0.2
0.0 1.0 2.0 3.0 3.9 4.9 5.9 6.9 7.9 8.9 9.9
LS-50%
LS-2.5%
IS-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fw d

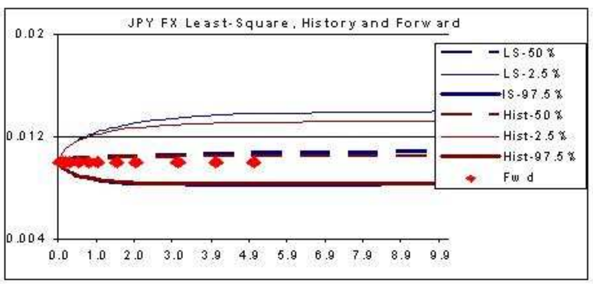

GBP FX Risk-neutral OU, History and Forward
3.4
2.6
1.8
1
0.2
0.0 1.0 2.0 3.0 3.9 4.9 5.9 6.9 7.9 8.9 9.9
RN-50%
RN-2.5%
RN-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fw d

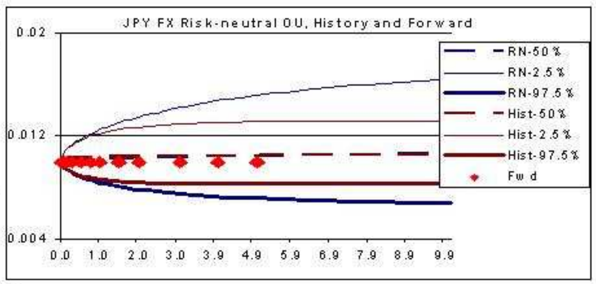

GBP FX Risk-neutral GBM, History and Forward
3.4
2.6
1.8
1
0.2
0.0 1.0 2.0 3.0 3.9 4.9 5.9 6.9 7.9 8.9 9.9
GBM-50%
GBM-2.5%
GBM-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fw d

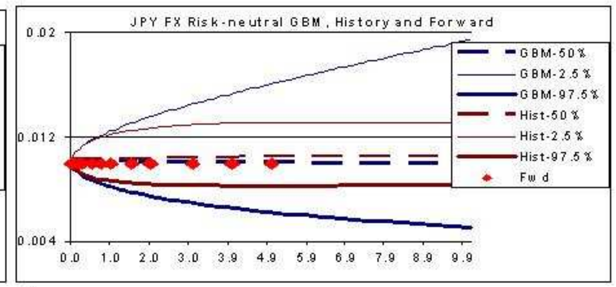

HKD FX Least-Square, History and Forward
0.136
0.124
0.0 1.0 2.0 3.0 3.9 4.9 5.9 6.9 7.9 8.9 9.9
LS-50%
LS-2.5%
LS-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fw d

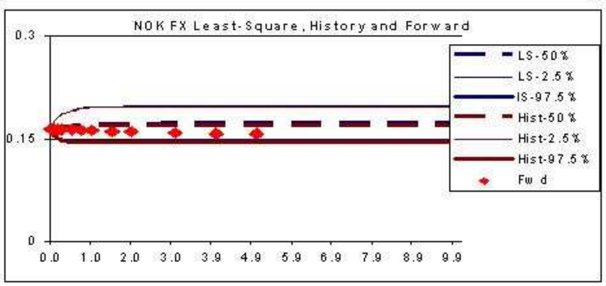

HKD FX Risk-neutral OU, History and Forward
0.136
0.124
0.0 1.0 2.0 3.0 3.9 4.9 5.9 6.9 7.9 8.9 9.9
RN-50%
RN-2.5%
RN-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fw d

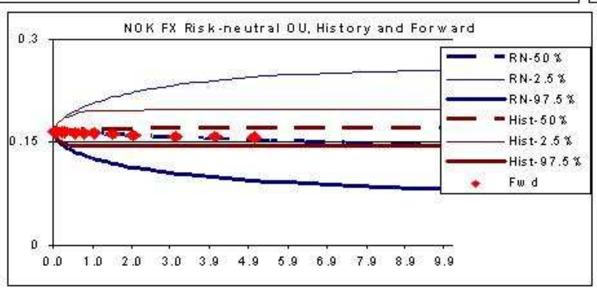

HKD FX Risk-neutral GBM, History and Forward
0.136
0.124
0.0 1.0 2.0 3.0 3.9 4.9 5.9 6.9 7.9 8.9 9.9
GBM-50%
GBM-2.5%
GBM-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fw d

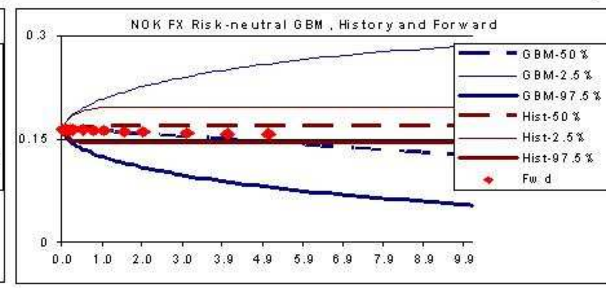

JPY FX Least-Square, History and Forward
0.02
0.012
0.004
0.0 1.0 2.0 3.0 3.9 4.9 5.9 6.9 7.9 8.9 9.9
LS-50%
LS-2.5%
IS-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fw d


JPY FX Risk-neutral OU, History and Forward
0.02
0.012
0.004
0.0 1.0 2.0 3.0 3.9 4.9 5.9 6.9 7.9 8.9 9.9
RN-50%
RN-2.5%
RN-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fw d


JPY FX Risk-neutral GBM, History and Forward
0.02
0.012
0.004
0.0 1.0 2.0 3.0 3.9 4.9 5.9 6.9 7.9 8.9 9.9
GBM-50%
GBM-2.5%
GBM-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fw d


NOK FX Least-Square, History and Forward
0.3
0.15
0
0.0 1.0 2.0 3.0 3.9 4.9 5.9 6.9 7.9 8.9 9.9
LS-50%
LS-2.5%
IS-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fw d


NOK FX Risk-neutral OU, History and Forward
0.3
0.15
0
0.0 1.0 2.0 3.0 3.9 4.9 5.9 6.9 7.9 8.9 9.9
RN-50%
RN-2.5%
RN-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fw d


NOK FX Risk-neutral GBM, History and Forward
0.3
0.15
0
0.0 1.0 2.0 3.0 3.9 4.9 5.9 6.9 7.9 8.9 9.9
GBM-50%
GBM-2.5%
GBM-97.5%
Hist-50%
Hist-2.5%
Hist-97.5%
Fw d

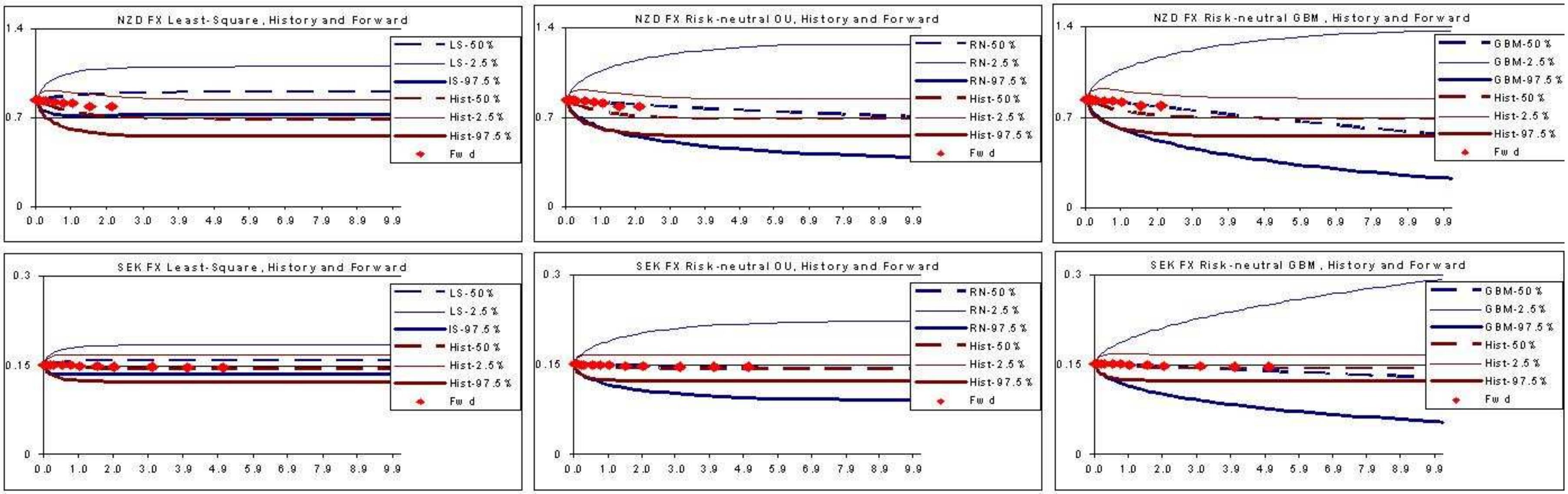


Figure 4 Compare Calibration and Simulation of Least-square, Risk-neutral OU, Risk-neutral GBM with Historical Quantiles and Forward Rate

References

[1] Counterparty Credit Risk and Credit Value Adjustment: A Continuing Challenge for Global Financial Markets, Hon Gregory, Oct 2012

[2] Stochastic Calculus for Finance II, Steven E Shreve, 2004

[3] Calibrating the Ornstein-Uhlenbeck (Vasicek) Model, Thijs van den Begr, http://www.sitmo.com/, May 2011

[4] On the Simulation and Estimation of the Mean-Reverting Ornstein-Uhlenbeck Process, William Smith, http://commoditymodels.files.wordpress.com, Feb 2010

[5] Jackknifing Bond Option Prices. Phillips, Peter C. B., and Jun Yu. The Review of Financial Studies 18, no. 2 (Summer): 707-742.

[6] Sound Practices for Backtesting Counterparty Credit Risk Models, Basel Committee on Banking Supervision, Dec 2010