Low Power Coding with Balanced Ternary for Communications using spread spectrum

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2020/04/04

Abstract

Coding with balanced ternary for communications using spread spectrum is proposed in this article. The proposed code with large number of codewords can achieve lower power cost than binary code. The proposed code with long spreading code even can achieve larger line rate than binary code.

1 Power Cost of Binary and Balanced Ternary

Like as binary represents numbers with symbols $\{0, 1\}$ or $\{\bar{1}, 1\}$, balanced ternary represents numbers with symbols $\{\bar{1}, 0, 1\}$.

According to Hartley, if transmitter has amplitude range $[-A, ..., +A]$ volts and the precision of the receiver is ΔV volts, the maximum number of distinguishable pulse levels N has relation represented as equation1.

$$\begin{aligned} N &= 1 + \frac{A}{\Delta V} \\ A &= \Delta V(N-1) \end{aligned} \quad (1)$$

The amplitude of each symbol can be represented as $\{-\Delta V, +\Delta V\}$ for binary and $\{-2\Delta V, 0, +2\Delta V\}$ for balanced ternary. With power cost P_2 of a binary symbol, power cost of ternary symbol is represented as $\{4P_2, 0, 4P_2\}$. The power cost is different with the symbol in balanced ternary. Coding with less symbol 1 and $\bar{1}$, the power cost of the code can be decreased.

2 Low Power Coding

I propose variable-length code in this article. Padding with 0 will convert it to fixed-length code. Communication is assumed to use spread spectrum. A single spreading code is used, and it is binary because 0 does not contribute to voltage level.

Code with minimum number of symbol 1 and $\bar{1}$ can be represented as $\{1, \bar{1}, 01, 0\bar{1}, 001, 00\bar{1}, 0001, 000\bar{1}, ...\}$. With spreading code s, the code is $\{s, \bar{s}, 0s, 0\bar{s}, 00s, 00\bar{s}, 000s, 000\bar{s}, ...\}$. Each codeword can be send with the power cost of a single spreading code. Because 0 cannot be spreaded, time length of the symbol 0 can be shortened from the time length of the symbol s. In other words, the codeword consists of the sign of the spreading code and the period between the spreading code and the former spreading code.

3 Characteristics

In this section, I reveal the characteristics of the proposed code comparing to the binary code.

Each codeword appears in equal possibility. Equations uses T_0 as the time length of a symbol 0, T_s as the time length of the spreading code, N_c as the number of multiple access, N_w as the number of the codewords, N_0 as the number of the levels of the continuous time length sending 0, and H as the mean information content of the codewords. N_0 is represented as equation2. H is represented as equation3.

$$N_0 = \frac{N_w}{2} \quad (2)$$

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$$
\begin{aligned}
H &= \log_2(N_w) \\
&= \log_2(2N_0)
\end{aligned} \tag{3}
$$

For the proposed code, equations4 to 7 reapresent P_{a3} as the power cost of a codeword, T_{a3} as the average time length of codewords, P_{h3} as the mean power cost per information content, and R_3 as the line rate.

$$
P_{a3} = 4P_2 \tag{4}
$$

$$
T_{a3} = T_s + \frac{T_0(N_0 - 1)}{2} \tag{5}
$$

$$
\begin{aligned}
P_{h3} &= \frac{P_{a3}}{H} \\
&= \frac{4P_2}{\log_2(N_w)}
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
R_3 &= N_c \frac{H}{T_{a3}} \\
&= \frac{N_c \log_2(2N_0)}{T_s + \frac{T_0(N_0-1)}{2}}
\end{aligned} \tag{7}
$$

For the binary code, equations8 to 11 reapresent P_{a2} as the power cost of a codeword, T_{a2} as the average time length of codewords, P_{h2} as the mean power cost per information content, and R_2 as the line rate.

$$
P_{a2} = P_2 \log_2(N_w) \tag{8}
$$

$$
T_{a2} = T_s \log_2(N_w) \tag{9}
$$

$$
\begin{aligned}
P_{h2} &= \frac{P_{a2}}{H} \\
&= P_2
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
R_2 &= N_c \frac{H}{T_{a2}} \\
&= \frac{N_c}{T_s}
\end{aligned} \tag{11}
$$

P_{h3}/P_{h2}, the power cost ratio of the proposed code to the binary code, is represented as equation12. The condition to achieve $P_{h3}/P_{h2} < 1$ is represented as equation13. Figure1 shows equation12. You can see that the power cost ratio get smaller with the larger number of codewords.

$$
\frac{P_{h3}}{P_{h2}} = \frac{4}{\log_2(N_w)} \tag{12}
$$

$$
N_w > 16 \tag{13}
$$

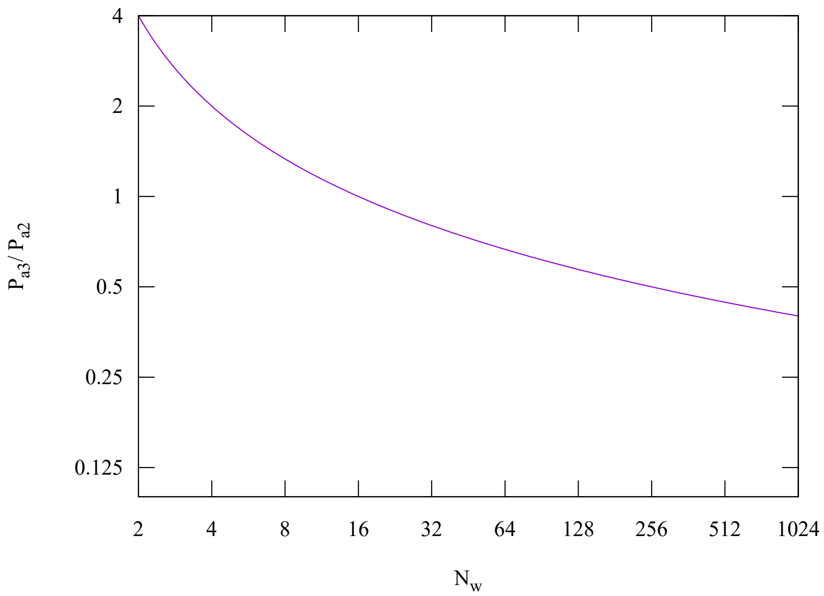

Figure 1: power cost ratio of the proposed code to the binary code

R_3/R_2, the line rate ratio of the proposed code to the binary code, is represented as equation14. The condition to achieve $R_3/R_2 > 1$ is represented as equation15. Figure2 shows equation14. In Figure2, horizontal axis shows N_w, vertical axis shows T_0/T_s, and color bar shows R_3/R_2. You can see that the line rate ratio get larger with the smaller T_0/T_s, or namely the larger length of the spreading code.

$$\begin{aligned} \frac{R_3}{R_2} &= \frac{T_s \log_2(2N_0)}{T_s + \frac{T_0(N_0-1)}{2}} \\ &= \frac{\log_2(N_w)}{1 + \frac{T_0}{T_s}\frac{N_w-2}{4}} \end{aligned} \tag{14}$$

$$4\frac{\log_2(N_w) - 1}{N_w - 2} > \frac{T_0}{T_s} \tag{15}$$

It is still unclear that the proposed code fits in Shannon-Hartley theorem. In Hartley's law, information content of a pulse is logarithm of the number of distinguishable pulse levels. But in proposed code, information content depends on the number of distinguishable time period levels between pulses rather than pulse levels.

4 Conclusion

Coding with balanced ternary for communications using spread spectrum is proposed in this article. The proposed code with large number of codewords can achieve lower power cost than binary code. The proposed code with long spreading code even can achieve larger line rate than binary code.

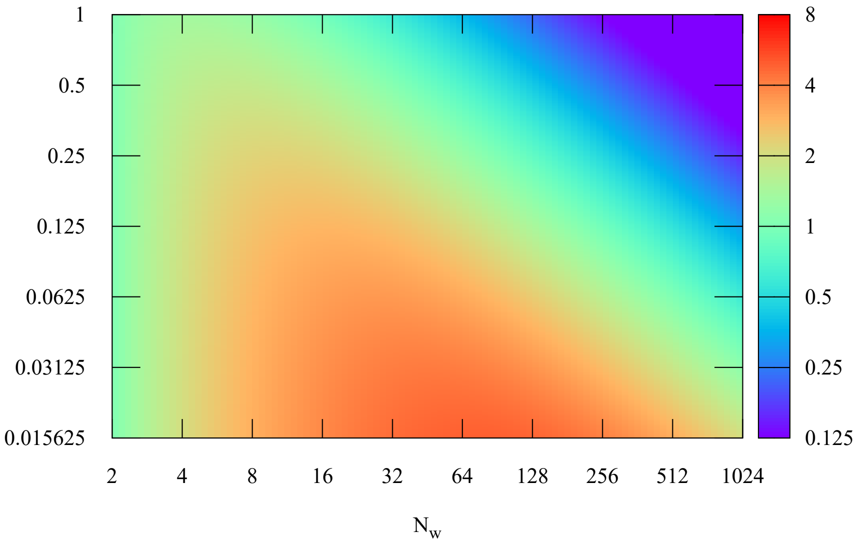

Figure 2: line rate ratio of the proposed code to the binary code