A Registered Protocol for Testing a Single-Scale Nonlinear Elliptic Gravity Law and a Biological "Sequention" Potential: Design, Blinding, Metrics, and Step-by-Step Draft-Guide

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Abstract

We present an *arXiv-ready preregistration* protocol spanning two domains that share the same mathematical architecture: (i) a single-scale, baryons-only nonlinear elliptic gravitational law intended to jointly explain galaxy rotation curves and strong lensing without dark halos; and (ii) a sparsity-regularized elliptic potential on genotype–phenotype–environment space ("Sequention") for predictive biology. The document codifies hypotheses, inclusion criteria, frozen model families, blinding, numerical solvers, uncertainty propagation, pass/fail thresholds, and audit procedures. We include a practical, step-by-step workflow (with file tree, configuration schemas, and shell scripts) to enable replication and registered analysis on OSF/AsPredicted and submission to journals. This manuscript is designed to be used as a living protocol: if the outcomes fail the predeclared metrics, the theory should be revised or rejected; if they pass, we expect increased interest despite philosophical disagreement.

Keywords: preregistration; blinded analysis; nonlinear elliptic PDE; galaxy rotation curves; strong lensing; modified gravity; MOND/AQUAL; PPN; multigrid; deep mutational scanning; canalization; group lasso; Gaussian process; reproducibility.

1 Statement of Contribution

This article does *not* present final fits or claims of empirical success. It contributes: (i) a full preregistration text suitable for OSF/AsPredicted; (ii) a frozen model family with *one* global acceleration scale a_* and a monotone constitutive law μ; (iii) a *blinded* evaluation plan for joint galaxy kinematics and lensing; (iv) a matched biological protocol using the same elliptic architecture; and (v) audit and reproducibility procedures stringent enough for skeptical review.

2 Background and Model Overview

2.1 Nonlinear elliptic gravity (physics)

We posit a scalar potential Φ governed by the nonlinear elliptic equation

$$\nabla\cdot\Big[\,\mu\big(\|\nabla\Phi\|/a_*\big)\,\nabla\Phi\,\Big] = 4\pi G\,\rho_b\,, \tag{1}$$

where ρ_b is the *baryonic* density, a_* is a global acceleration scale, and μ is monotone with high-acceleration limit $\mu\to 1$. We freeze the family

$$\mu(y) = \frac{y}{\big(1+y^n\big)^{1/n}}, \qquad n\in\{1,2,3\} \text{ (chosen once)}, \tag{2}$$

recovering the linear Poisson limit as $y\to\infty$. The model aims to fit galaxy rotation curves (RCs) and predict strong-lensing Einstein radii from baryons alone, using the *same* fixed a_* across samples, with post-Newtonian deviations bounded by solar-system tests.

2.2 Elliptic potential on genotype–phenotype–environment (biology)

We mirror (1) with a potential U on a discrete state space (genotype hypercube or GRN state lattice), solving

$$\nabla\cdot\Big[\,\mu_{\rm bio}\big(\|\nabla U\|/a^\dagger\big)\,\nabla U\,\Big] = \rho_{\rm var}\,, \qquad \mu_{\rm bio}(y) = \frac{y}{\big(1+y^m\big)^{1/m}},\ m\in\{1,2\}. \tag{3}$$

A sparsity prior (group lasso) constrains a parametric expansion of U. Benchmarks: (i) predictive lift on held-out variants in deep mutational scanning (DMS) or (ii) canalization assays with predeclared order-invariance metrics.

3 Registered Hypotheses and Inclusion Criteria

3.1 Physics

Hypotheses (frozen before analysis).

H1. (Kinematics) With a_* fixed from a preregistered calibration subset, median $\chi^2/\nu \leq 1.3$ on held-out RCs.

H2. (Lensing) With a_* and μ fixed, median fractional error ≤ 0.15 in Einstein radii $\hat{\theta}_E$ vs blinded ground truth.

H3. (Parsimony) Effective parameter count $p_{\rm eff} \leq$ that of a matched ΛCDM+NFW baseline with identical data rights.

H4. (Solar-system guardrail) Inferred $|\gamma-1| < 2\times 10^{-5}|$ at Cassini-like impact parameters.

Inclusion criteria. RCs: resolved discs with gas+stellar maps, inclinations 30–80°, distance error $<10\%$, and at least six radial bins beyond two scale lengths. Lenses: systems with measured θ_E, stellar light profiles, and baryonic mass maps with priors on M/L; external shear used if cataloged. Exclusions: dominant bars, strong warps, severe dust lanes.

3.2 Biology

Hypotheses.

B1. (Predictive lift) $\geq$10% RMSE reduction vs strong baselines (GLM up to pairwise interactions; GP with standard kernels) on held-out DMS variants or morphogenetic trajectories; paired bootstrap $p < 0.01$.

B2. (Order-invariance) Predeclared canalization cone-metrics within $\pm$5% tolerance.

Inclusion. DMS datasets with $\geq 10^5$ variants and replicates, minimal ceiling/floor. Canalization series with ≥ 20 time points.

4 Blinding and Freezing

- Choose n (physics) and m (biology) and calibrate a_* on a preregistered RC subset; serialize a final configuration JSON and publish its SHA256 hash in the registry.
- Keep lensing labels (θ_E, shears) *hidden* until predictions are committed (hash logged). For biology, hide labels for hold-out sets.
- Any deviation (e.g., solver change) triggers a new preregistration version.

5 Numerical Methods (frozen)

5.1 Physics PDE solver

Discretization. Finite-volume scheme on a 3D box enclosing baryons to $\geq$ 10 scale lengths; Neumann boundary (zero normal gradient). Adaptive mesh with 2–4 refinement levels; refine on $\|\nabla\Phi\|$ gradients. Face flux uses a monotone limiter to preserve ellipticity.

Multigrid. Full Approximation Scheme; Gauss–Seidel smoothing; line relaxation for anisotropy; V-cycles with residual ℓ_2 reduction $\geq 10^8$ and relative defect $< 10^{-10}$; maximum 200 cycles before Newton–Krylov fallback.

Validation. Manufactured-solution tests with second-order convergence; recovery of the linear Poisson limit as $\|\nabla\Phi\|/a_* \to \infty$.

Observables. Midplane circular velocity $v_c(r) = \sqrt{r\,\partial_r\Phi}$; lensing potential $\psi(\boldsymbol{\theta}) = \frac{2D_{ls}}{c^2 D_l D_s}\int \Phi(D_l\boldsymbol{\theta}, z)\,dz$; deflection $\boldsymbol{\alpha} = \nabla_\theta\psi$; Einstein radius from $\bar{\kappa}(<\theta_E) = 1$.

Uncertainty. Bootstrap $N = 1024$ over M/L, distance, inclination; report coverage of 68% CIs.

5.2 Biology solver

Discrete operator. On a genotype/GRN graph: for edge (i, j), flux $= \mu_{\text{bio}}(|U_i - U_j|/(a^\dagger\Delta))\,(U_i - U_j)/\Delta$; divergence is the signed sum over incident edges.

Optimization. Alternate nonlinear conjugate-gradient updates of U (Wolfe line search) with group-lasso proximal steps for weights (FISTA). Synthetic recovery tests verify identifiability.

6 Metrics and Pass/Fail

6.1 Physics

RCs: median χ^2/ν, CI coverage. Lensing: median $|\hat{\theta}_E - \theta_E|/\theta_E$ and MAE; CI coverage. Parsimony: WAIC and p_{eff} vs ΛCDM+NFW with matched data rights. Solar: $|\gamma - 1|$ bound from high-acceleration limit.

6.2 Biology

Held-out RMSE/MAE/Spearman-ρ; paired bootstrap for ΔRMSE; DOF within 1.2× of best baseline; canalization cone metrics.

7 Step-by-Step Guide (Practical Workflow)

1. **Create repository skeleton.** Suggested tree in Listing 1.
2. **Choose and freeze model family.** Select $n \in \{1, 2, 3\}$ (physics) and $m \in \{1, 2\}$ (biology); write `config.json`; compute and record its SHA256.
3. **Calibrate a_* on preregistered RC subset.** Do not touch lensing labels. Export a signed `config.hash` file.
4. **Register on OSF/AsPredicted.** Paste the prereg text (Appendix E), include hashes, and upload the frozen `config.json`.
5. **Run blinded predictions.** Produce RC and lensing predictions with the frozen config; store artifacts with hashes in `results/`.
6. **Unblind and evaluate.** Import ground truth; compute metrics; generate `report.md` and figures.
7. **Archive and release.** Push code and configs; upload artifacts and an audit report to OSF; submit this prereg manuscript with DOIs/links.

Listing 1: Suggested repository layout

```
repo/
  README.md
  env/                  # environment files (Dockerfile, requirements.txt)
  configs/
    config.json         # frozen configuration (hashed)
    config.hash         # SHA256 and git commit
  data/
    rc_calibration/     # rotation-curve calibration subset (public or NDA)
    rc_holdout/
    lens_blinded/       # lensing inputs without labels
  src/
    pde/                # solver modules (finite-volume, multigrid)
    lensing/
    kinematics/
    stats/
    bio/
```

```
  results/
    manifests/
    figures/
    metrics/
    reports/
  scripts/
    run_blinded.sh
    make_audit.sh
```

8 Statistical Details

WAIC and p_{eff}. We report WAIC and effective parameter count via the variance of the pointwise log-likelihood across bootstrap resamples [7]. For deterministic predictions with parametric uncertainty, we approximate likelihoods with Gaussian errors formed from bootstrap CIs.

Bootstrap. Paired bootstrap with $N = 10^4$ for ΔRMSE in biology and $N = 1024$ for RC/lensing metrics [6]. Bonferroni-corrected intervals are reported for multiple outcomes.

Solar-system bound. In the $\|\nabla\Phi\| \gg a_*$ regime, deviations scale with $\mu' - 1$; we translate this into a bound on $|\gamma - 1|$ using Cassini light-deflection results [3].

9 Ethics, Data Rights, and Reproducibility

Blinding. Lensing labels and biological hold-outs remain hidden until predictions are hashed and archived. Any modification triggers a new preregistration.

Data rights. Only public datasets or private data with written permission; an independent auditor may access under NDA.

Environment. Docker image digest is recorded; seeds are fixed. We archive hashes for every input/output artifact and provide a one-command audit script.

10 Audit Script and Config Examples

Listing 2 shows a minimal audit script; Listing 3 shows a frozen configuration example.

Listing 2: Minimal audit script

```
#!/usr/bin/env bash
set -euo pipefail
CONFIG="configs/config.json"
HASHFILE="configs/config.hash"
python -m src.solver.run --config $CONFIG --mode blinded
python -m src.metrics.eval --config $CONFIG --ground_truth data/unblinded
python -m src.reports.make --config $CONFIG --out results/reports/report.md
sha256sum $CONFIG > $HASHFILE
git rev-parse HEAD >> $HASHFILE
docker images --digests | grep YOUR_IMAGE >> $HASHFILE
```

Listing 3: Example config.json (physics)

```
{
  "model": {"mu_family": "y/(1+y^n)^(1/n)", "n": 2, "a_star_kms2_per_kpc": 3700.0},
  "grid": {"dx_pc": 100, "levels": 3, "bc": "neumann_zero_grad"},
  "solver": {"type": "FAS_multigrid", "max_cycles": 200, "residual_drop": 1e8, "
    defect_rel": 1e-10},
  "uncertainty": {"bootstrap": 1024, "seed": 424242},
  "priors": {"ML_band": {"r": {"mean": 0.6, "sigma": 0.1}, "i": {"mean": 0.5, "sigma":
    0.1}},
             "inclination_deg": {"sigma": 3.0}}
}
```

11 Limitations and Failure Modes

(1) Cosmology is not yet derived: distance–redshift, BAO, and CMB predictions are outside scope until a relativistic completion is delivered. (2) Cluster-scale lensing can be a stress test where many modified-gravity models fail; our preregistered metrics explicitly allow a decisive outcome. (3) In biology, identifiability without strong priors is challenging; we mitigate with sparsity and preregistered families.

12 Frequently Asked Questions (for Reviewers)

Is this just MOND? The constitutive law mirrors AQUAL deliberately; the novelty is a *single* global scale tested *jointly* on kinematics and lensing under blinding and parsimony metrics.

What would change minds? Matching or beating ΛCDM+NFW on joint RC+lensing with fewer effective parameters, or delivering a clearly negative result.

Why preregister? To avoid researcher degrees of freedom and make a null result valuable.

A PDE Well-Posedness Conditions (Physics)

Uniform ellipticity follows if μ is monotone with $\mu'(y) \geq \epsilon > 0$ in the calibration range and numerical fluxes respect a maximum principle; we enforce $\epsilon = 10^{-6}$ clipping in code. Manufactured-solution tests confirm second-order convergence.

B Manufactured Solution Recipe

Choose $\Phi_\star$ with analytic $\nabla\Phi_\star$; define $\rho_b = \frac{1}{4\pi G}\nabla\cdot\left[\mu(\|\nabla\Phi_\star\|/a_*)\nabla\Phi_\star\right]$. Solve numerically and verify $\|\Phi - \Phi_\star\|_{\ell_2} \propto \Delta^2$.

C PPN Guardrail Sketch

In the high-acceleration regime, linearize around $\mu \equiv 1$ and express deviations via $\mu' - 1$. Translate to a bound on $|\gamma - 1|$ using Cassini deflection measurements and typical impact parameters.

D Biology: Graph Operator and Optimization

On an undirected graph $G = (V, E)$ with edge lengths Δ_{ij}, define flux $F_{ij} = \mu_{\text{bio}}(|U_i - U_j|/(a^\dagger \Delta_{ij}))\,(U_i - U_j)/\Delta_{ij}$. The discrete divergence at i is $\sum_{j:(i,j)\in E} F_{ij}$. Alternate updates with proximal group-lasso on $U(x) = \sum_j w_j \phi_j(x)$.

E Preregistration Templates (OSF/AsPredicted)

Physics

Title: Baryons-only, single-scale fit to rotation curves and strong lensing using a non-linear elliptic gravitational law.
Hypotheses: H1–H4 as in main text.
Datasets: RC inclusion/exclusion; lensing inclusion; external shear if available.
Model family: $\mu(y) = y/(1 + y^n)^{1/n}$, $n \in \{1, 2, 3\}$; global a_* fixed from RC calibration subset.
Blinding: Lensing labels hidden; config SHA256 posted before unblinding.
Numerics: Finite-volume, multigrid (FAS), residual and defect thresholds as specified.
Metrics: RC χ^2/ν; lensing $|\hat{\theta}_E - \theta_E|/\theta_E$; WAIC/$p_{\text{eff}}$; PPN bound.
Pass/Fail: RC ≤ 1.3; lensing ≤ 0.15; PPN $< 2 \times 10^{-5}$; parsimony $\leq$ baseline.
Deviations: Convergence fallback and sensitivity runs logged as deviations.

Biology

Title: Predictive benchmarks for a sparsity-regularized elliptic potential on genotype–phenotype–environment space.
Hypotheses: B1–B2 as in main text.
Datasets: DMS with $\geq 10^5$ variants; canalization with ≥ 20 timepoints.
Model family: $\mu_{\text{bio}}(y) = y/(1 + y^m)^{1/m}$, $m \in \{1, 2\}$.
Blinding: Hold-out labels hidden; config hash posted before unblinding.
Metrics: RMSE/MAE/ρ; paired bootstrap; DOF constraint; cone metrics.
Pass/Fail: Lift $\geq 10\%$ and $p < 0.01$; DOF $\leq 1.2\times$ baseline; cone metrics within $\pm 5\%$.

F Config Schema and JSON Validation

A minimal JSON Schema for physics configs (submit alongside code):

```
{
  "$schema": "http://json-schema.org/draft-07/schema#",
  "type": "object",
  "properties": {
    "model": {"type": "object", "properties": {
      "mu_family": {"type": "string"},
      "n": {"type": "integer", "enum": [1,2,3]},
      "a_star_kms2_per_kpc": {"type": "number", "minimum": 0}
    }, "required": ["mu_family","n","a_star_kms2_per_kpc"]},
    "grid": {"type": "object", "properties": {
      "dx_pc": {"type": "number", "minimum": 1},
      "levels": {"type": "integer", "minimum": 1},
      "bc": {"type": "string"}
    }, "required": ["dx_pc","levels","bc"]}
```

```
  },
  "required": ["model","grid"]
}
```

G Data Availability and License

All preregistration text, configuration files, and evaluation notebooks will be released under a permissive open-source license (Creative Commons or BSD-3-Clause). Data usage complies with original licenses.

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