9548

8.13

$$\int_0^{r0} p[r]\, dr = \left(n - \frac{1}{4}\right)\pi\hbar$$

$$V[r] = V_0 \,\mathrm{Log}[r/a]$$

$$E = V_0 \,\mathrm{Log}[r0/a]$$

$$\left(n - \frac{1}{4}\right)\pi\hbar == \sqrt{2\, m\, V_0}\int_0^{r0}\sqrt{\mathrm{Log}[r0/a] - \mathrm{Log}[r/a]}\, dr$$

In[1]:= $\int_0^{r0}\sqrt{\mathrm{Log}[r0/a] - \mathrm{Log}[r/a]}\, dr$

Out[1]= $\frac{\sqrt{\pi}\; r0}{2}$

In[5]:= **ans = Solve**$\left[\left(n - \frac{1}{4}\right)\pi\hbar == \sqrt{2\, m\, V_0}\;\frac{\sqrt{\pi}\; r0}{2},\; r0\right]$

Out[5]= $\left\{\left\{r0 \to \frac{(-1+4\, n)\sqrt{\frac{\pi}{2}}\;\hbar}{2\sqrt{m\, V_0}}\right\}\right\}$

In[6]:= **En = V0 Log**$[r0/a]$ **/. ans[[1, 1]]**

Out[6]= $V0\;\mathrm{Log}\left[\frac{(-1+4\, n)\sqrt{\frac{\pi}{2}}\;\hbar}{2\, a\sqrt{m\, V_0}}\right]$

$$En = V_0\,\mathrm{Log}\left[n - \frac{1}{4}\right] + V_0\,\mathrm{Log}\left[\frac{\sqrt{2\pi}\;\hbar}{a\sqrt{m\, V_0}}\right]$$

$$E_{n+1} - E_n = V_0\,\mathrm{Log}\left[(n+1) - \frac{1}{4}\right] - V_0\,\mathrm{Log}\left[n - \frac{1}{4}\right]$$

$$E_{n+1} - E_n = V_0\,\mathrm{Log}\left[n + \frac{3}{4}\right] - V_0\,\mathrm{Log}\left[n - \frac{1}{4}\right] = V_0\,\mathrm{Log}\left[\frac{n+3/4}{n-1/4}\right]$$

8.16

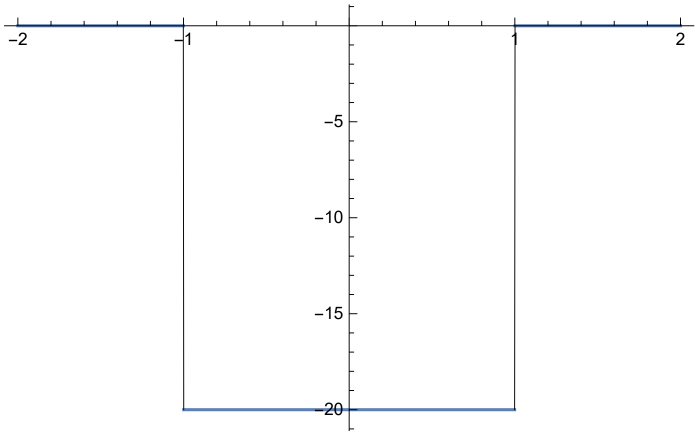
(a)

$$E_n = \frac{n^2\pi^2\hbar^2}{2\, m\,(2\, a)^2} = \frac{n^2\pi^2\hbar^2}{8\, m\, a^2}$$

(b)

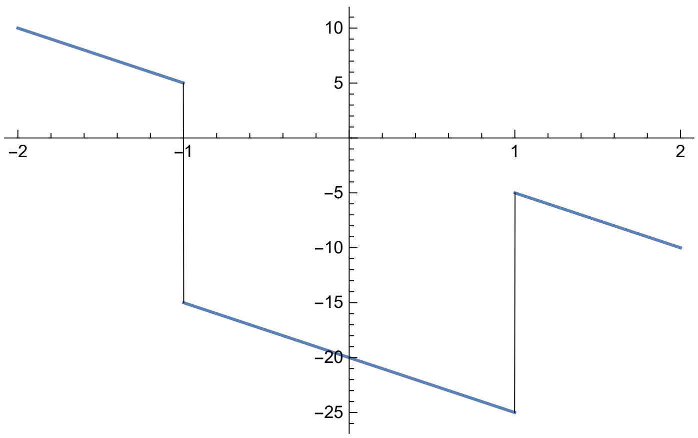
In[38]:=
```
V[x_] = 20 (-HeavisidePi[x/2]);
Plot[V[x], {x, -2, 2}, ExclusionsStyle → Opacity[1]]
```

Out[39]=



In[40]:=
```
H[x_] = V[x] - 5 x;
Plot[H[x], {x, -2, 2}, ExclusionsStyle → Opacity[1]]
```

Out[41]=



(c)

$$\gamma = \frac{1}{\hbar}\int_a^{x0} p[x]\, dx \text{ where } x_0 = \frac{V_0 - E_1}{\alpha}$$

$$p[x] = \sqrt{2\,m\,(E - V[x])} \text{ where } V[x] = -\alpha\, x$$

$$p[x] = \sqrt{2\,m\,(E_1 - V_0 + \alpha\, x)}$$

$$p[x] = \sqrt{2\,m\,\alpha}\,\sqrt{x_0 - x}$$

In[56]:= $\gamma = \frac{1}{\hbar}\int_a^{x0}\sqrt{2\,m\,\alpha}\,\sqrt{x0 - x}\, dx$

Out[56]= $\frac{2\sqrt{2}\,(-a + x0)^{3/2}\sqrt{m\,\alpha}}{3\,\hbar}$

In[57]:= $\gamma = \gamma$ /. x0 -> $\frac{V0 - E1}{\alpha}$ // FullSimplify

Out[57]= $\frac{2\sqrt{2}\,\sqrt{m}\,(-E1 + V0 - a\,\alpha)^{3/2}}{3\,\alpha\,\hbar}$

E1 term is small, and so is α a

In[58]:=
```
$Assumptions = {α ∈ Reals, α > 0};
γ = γ /. {E1 → 0, a → 0} // FullSimplify
```

Out[59]=
$$\frac{2\sqrt{2}\sqrt{m}\,V0^{3/2}}{3\,\alpha\,\hbar}$$

In[62]:=
$$v = \frac{\pi\hbar}{2\,m\,a};$$
$$\tau = \frac{4\,a}{v}\,e^{2\gamma}\ \text{// FullSimplify}$$

Out[63]=
$$\frac{8\,a^2\,e^{\frac{4\sqrt{2}\sqrt{m}\,V0^{3/2}}{3\alpha\hbar}}\,m}{\pi\hbar}$$

In[75]:=
```
V0 = 20 * 1.6*^-19;
a = 10^-10;
e = 1.6*^-19;
m = 9.11*^-31;
ℏ = 1.05*^-34;
α = e * 7*^6;
τ
```

This is an absurdly huge number. The age of the universe is about 10^{17} seconds, which is nothing compared to $10^{38\,000}$