9548

9.1

$\psi_{nlm} = R_{nl}\, Y_l^m$

$$\psi_{100} = \frac{1}{\sqrt{\pi\, a^3}}\, e^{-r/a}$$

$$\psi_{200} = \frac{1}{\sqrt{8\, \pi\, a^3}}\, e^{-r/(2\, a)}$$

$$\psi_{210} = \frac{1}{\sqrt{32\, \pi\, a^3}}\, \frac{r}{a}\, e^{-r/(2\, a)}\, \mathrm{Cos}[\theta]$$

or

$$\psi_{210} = \frac{1}{\sqrt{32\, \pi\, a^3}}\, \frac{1}{a}\, e^{-r/(2\, a)}\, z$$

$$\psi_{21\pm1} = \mp \frac{1}{\sqrt{64\, \pi\, a^3}}\, \frac{r}{a}\, e^{r/(2\, a)}\, \mathrm{Sin}[\theta]\, e^{\pm i\, \phi}$$

or

$$\psi_{21\pm1} = \mp \frac{1}{\sqrt{64\, \pi\, a^3}}\, \frac{1}{a}\, e^{r/(2\, a)}\, (x \pm i\, y)$$

$H'_{ii} = 0$ because any combination will yield zero within the integral

$$\int z\, |\psi|^2\, dx\, dy\, dz$$

$H'_{ij} = 0$ for all except

$$H'_{100,210} = -e\, E\, \frac{1}{\sqrt{\pi\, a^3}}\, \frac{1}{\sqrt{32\, \pi\, a^3}}\, \frac{1}{a} \int e^{-r/a}\, e^{-r/(2\, a)}\, r^2\, \mathrm{Cos}[\theta]^2\, r^2\, \mathrm{Sin}[\theta]\, dr\, d\theta\, d\phi$$

$$H'_{100,210} = -e\, E\, \frac{1}{\sqrt{\pi\, a^3}}\, \frac{1}{\sqrt{32\, \pi\, a^3}}\, \frac{1}{a} \int_0^\infty r^4\, e^{-r/a}\, e^{-r/(2\, a)}\, dr \int_0^\pi \mathrm{Cos}[\theta]^2\, \mathrm{Sin}[\theta]\, d\theta \int_0^{2\pi} d\phi$$

$$\int_0^\infty \mathbf{r^4\, e^{-r/a}\, e^{-r/(2\, a)}\, dr}$$

$$\mathrm{ConditionalExpression}\left[\frac{256\, a^5}{81},\ \mathrm{Re}[a] > 0\right]$$

$$\int_0^\pi \mathbf{Cos[\theta]^2\, Sin[\theta]\, d\theta}$$

$$\frac{2}{3}$$

$$H'_{100,210} = -e\,E\,\frac{1}{\sqrt{32}}\,\frac{1}{\pi a^3}\,\frac{1}{a}\,\frac{256\,a^5}{81}\,\frac{2}{3}\times 2\,\pi$$

$$H'_{100,210} = -e\,E\,a\,\frac{1}{\sqrt{32}}\,\frac{256}{81}\,\frac{4}{3}$$

$$\frac{1}{\sqrt{32}}\,\frac{256}{81}\,\frac{4}{3}\ //\ N$$

0.744936

$$H'_{100,210} = -0.7449\,e\,E\,a$$

9.2

$$\dot{c}_a = -\frac{i}{\hbar}\,H'_{ab}\,e^{-i\,\omega_0\,t}\,c_b$$

$$\dot{c}_b = -\frac{i}{\hbar}\,H'_{ba}\,e^{i\,\omega_0\,t}\,c_a$$

$$\ddot{c}_b = -\frac{i}{\hbar}\,H'_{ba}\,\left(i\,\omega_0\,e^{i\,\omega_0\,t}\,c_a + \dot{c}_a\,e^{i\,\omega_0\,t}\right)$$

$$\ddot{c}_b = -\frac{i}{\hbar}\,H'_{ba}\,\left(i\,\omega_0\,e^{i\,\omega_0\,t}\,c_a + \left(-\frac{i}{\hbar}\,H'_{ab}\,e^{-i\,\omega_0\,t}\,c_b\right)\,e^{i\,\omega_0\,t}\right)$$

$$\ddot{c}_b = \left(-\frac{i}{\hbar}\,H'_{ba}\,i\,\omega_0\,e^{i\,\omega_0\,t}\,c_a + \left(\frac{i}{\hbar}\,H'_{ba}\,\frac{i}{\hbar}\,H'_{ab}\,e^{-i\,\omega_0\,t}\,c_b\right)\,e^{i\,\omega_0\,t}\right)$$

$$\ddot{c}_b = i\,\omega_0\,\dot{c}_b - \frac{1}{\hbar^2}\,\left|H'_{ab}\right|^2\,c_b$$

$$\ddot{c}_b - i\,\omega_0\,\dot{c}_b + \alpha^2\,c_b = 0$$

```
DSolve[cb''[t] - i ω0 cb'[t] + α^2 cb[t] == 0, cb[t], t]
```

$$\left\{\left\{cb[t] \to e^{\frac{1}{2}t\left(i\,\omega_0-\sqrt{-4\,\alpha^2-\omega_0^2}\right)}\,C[1] + e^{\frac{1}{2}t\left(i\,\omega_0+\sqrt{-4\,\alpha^2-\omega_0^2}\right)}\,C[2]\right\}\right\}$$

or

$$c_b[t] = e^{i\,\omega_0\,t/2}\,\left(A\,e^{i\,\omega\,t/2} + B\,e^{-i\,\omega\,t/2}\right)$$

$$c_b[t] = e^{i\,\omega_0\,t/2}\,\left(C\,\mathrm{Cos}[\omega\,t/2] + D\,\mathrm{Sin}[\omega\,t/2]\right)$$

Since $c_b[0] = 0$, $C = 0$

$$c_b[t] = D\,e^{i\,\omega_0\,t/2}\,\mathrm{Sin}[\omega\,t/2]$$

$$\dot{c}_b[t] = D\,\left(\frac{i\,\omega_0}{2}\,e^{i\,\omega_0\,t/2}\,\mathrm{Sin}[\omega\,t/2] + \frac{\omega}{2}\,e^{i\,\omega_0\,t/2}\,\mathrm{Cos}[\omega\,t/2]\right)$$

$$\dot{c}_b[t] = \frac{\omega}{2}\,D\,e^{i\,\omega_0\,t/2}\,\left(\mathrm{Cos}[\omega\,t/2] + \frac{i\,\omega_0}{\omega}\,\mathrm{Sin}[\omega\,t/2]\right) = -\frac{i}{\hbar}\,H'_{ba}\,e^{i\,\omega_0\,t}\,c_a$$

$$c_a[t] = \frac{i\,\hbar}{H'_{ba}}\,\frac{\omega}{2}\,D\,e^{i\,\omega_0\,t/2}\,\left(\mathrm{Cos}[\omega\,t/2] + \frac{i\,\omega_0}{\omega}\,\mathrm{Sin}[\omega\,t/2]\right)$$

$$c_a[0] = 1$$

$$\Rightarrow \frac{i\,\hbar}{H'_{ba}}\,\frac{\omega}{2}\,D = 1 \Rightarrow D = \frac{2\,H'_{ba}}{i\,\hbar\,\omega}$$

$$c_a[t] = e^{i\,\omega_0 t/2}\left(\mathrm{Cos}[\omega t/2] + \frac{i\,\omega_0}{\omega}\mathrm{Sin}[\omega t/2]\right)$$

$$c_b[t] = \frac{2\,H'_{ba}}{i\,\hbar\,\omega}e^{i\,\omega_0 t/2}\,\mathrm{Sin}[\omega t/2]$$

where $\omega = \sqrt{\omega_0{}^2 + \frac{4\,H'_{ab}{}^2}{\hbar^2}}$

```
ca = e^(i ω0 t/2) (Cos[ω t/2] + (i ω0/ω) Sin[ω t/2]);
cb = (2 Hab/(i ħ ω)) e^(i ω0 t/2) Sin[ω t/2];
$Assumptions = {ω0 ∈ Reals, t ∈ Reals, ω ∈ Reals, ħ ∈ Reals};
ca * ca* + cb * cb* // FullSimplify
```

$$\mathrm{Cos}\left[\frac{t\,\omega}{2}\right]^2 + \frac{\left(\omega 0^2\,\hbar^2 + 4\,\mathrm{Hab\,Conjugate}[\mathrm{Hab}]\right)\,\mathrm{Sin}\left[\frac{t\,\omega}{2}\right]^2}{\omega^2\,\hbar^2}$$

which simplifies to 1 when $\omega = \sqrt{\omega_0{}^2 + \frac{4\,H'_{ab}{}^2}{\hbar^2}}$

9.5

$c_a^{(0)}[t] = a$

$c_b^{(0)}[t] = b$

$$\begin{pmatrix} \dot{c}_a = -\frac{i}{\hbar}H'_{ab}\,e^{-i\,\omega_0 t}\,b \;\Rightarrow\; c_a^{(1)}[t] = a - \frac{i\,b}{\hbar}\int_0^t H'_{ab}[t']\,e^{-i\,\omega_0 t'}\,dt' \\ \dot{c}_b = -\frac{i}{\hbar}H'_{ba}\,e^{i\,\omega_0 t}\,a \;\Rightarrow\; c_b^{(1)}[t] = b - \frac{i\,a}{\hbar}\int_0^t H'_{ba}[t']\,e^{i\,\omega_0 t'}\,dt' \end{pmatrix}$$

$$\dot{c}_a = -\frac{i}{\hbar}H'_{ab}\,e^{-i\,\omega_0 t}\left(b - \frac{i\,a}{\hbar}\int_0^t H'_{ba}[t']\,e^{i\,\omega_0 t'}\,dt'\right)$$

$c_a^{(2)}[t] =$

$$a - \frac{i\,b}{\hbar}\int_0^t H'_{ab}[t']\,e^{-i\,\omega_0 t'}\,dt' - \frac{a}{\hbar^2}\int_0^t H'_{ab}[t']\,e^{-i\,\omega_0 t'}\left(\int_0^{t'} H'_{ba}[t'']\,e^{-i\,\omega_0 t''}\,dt''\right)dt'$$

Switch all of the a ' s and b ' s and change ω_0 to $-\omega_0$

$c_b^{(2)}[t] =$

$$b - \frac{i\,a}{\hbar}\int_0^t H'_{ba}[t']\,e^{i\,\omega_0 t'}\,dt' - \frac{b}{\hbar^2}\int_0^t H'_{ba}[t']\,e^{i\,\omega_0 t'}\left(\int_0^{t'} H'_{ab}[t'']\,e^{i\,\omega_0 t''}\,dt''\right)dt'$$

9.6

When H' is independent of t,

$$c_b^{(2)}[t] = c_b^{(1)}[t] = -\frac{i}{\hbar}H'_{ba}\int_0^t e^{i\,\omega_0 t'}\,dt'$$

$$\int_0^t e^{i\,\omega_0\,tp}\,dtp$$

$$-\frac{i\left(-1+e^{i\,t\,\omega_0}\right)}{\omega_0}$$

$$c_b^{(2)}[t] = c_b^{(1)}[t] = -\frac{H'_{ba}}{\hbar\,\omega_0}\left(e^{i\,\omega_0\,t}-1\right)$$

(from 9.18)

$$c_a^{(2)}[t] = 1-\frac{1}{\hbar^2}\int_0^t H'_{ab}\,e^{-i\,\omega_0\,t}\left(\int_0^{t'} H'_{ab}\,e^{i\,\omega_0\,t''}\,dt''\right)dt'$$

$$\int_0^t \mathrm{Hab}\,e^{-i\,\omega0\,tp}\left(\int_0^{tp}\mathrm{Hab}\,e^{i\,\omega0\,tpp}\,dtpp\right)dtp$$

$$\frac{\mathrm{Hab}^2\left(1-e^{-i\,t\,\omega0}-i\,t\,\omega0\right)}{\omega0^2}$$

$$c_a^{(2)}[t] = 1-\frac{{H'_{ab}}^2}{\hbar^2\,{\omega_0}^2}\left(1-i\,t\,\omega_0-e^{-i\,\omega_0\,t}\right)$$