9548

7.1

(a)

$V[x] = \alpha |x|$

$\psi[x] = A^2 e^{-b x^2}$

$\langle V\rangle = A^2 \int_0^\infty \alpha |x| \psi^* \psi \, dx$

$\langle V\rangle = 2 \alpha A^2 \int_0^\infty x e^{-2 b x^2} \, dx$

```
$Assumptions = {b > 0, b ∈ Reals};
ψ[x_] = A e^(-b x^2);
A = (2 b/π)^(1/4);
V = 2 α ∫_0^∞ x ψ[x]^2 dx // FullSimplify
```

$$\frac{\alpha}{\sqrt{b}\,\sqrt{2\pi}}$$

$\langle T\rangle = -\frac{\hbar^2}{2m} A^2 \int_{-\infty}^{\infty} \psi^* \frac{\partial^2}{\partial x^2} \psi \, dx$

```
T = -ℏ^2/(2 m) ∫_-∞^∞ ψ[x] D[ψ[x], {x, 2}] dx
```

$$\frac{b\,\hbar^2}{2\,m}$$

$\langle H\rangle = \langle T\rangle + \langle V\rangle$

```
H = T + V;
B = b /. (Solve[∂_b H == 0, b]) [[1]]
Hmin = H /. b → B // FullSimplify
```

$$\frac{1}{(2\pi)^{1/3}\left(\frac{\hbar^2}{m\alpha}\right)^{2/3}}$$

$$\frac{3\alpha\left(\frac{\hbar^2}{m\alpha}\right)^{1/3}}{2\,(2\pi)^{1/3}}$$

(b)

same process as previous but

$V[x] = \alpha x^4$

```
$Assumptions = {b > 0, b ∈ Reals};
```

$\psi[x\_] = A\, e^{-b x^2};$

$A = \left(\frac{2\,b}{\pi}\right)^{1/4};$

$V = 2 \int_0^\infty \alpha\, x^4\, \psi[x]^2\, dx$ // FullSimplify

$$\frac{3\,\alpha}{16\,b^2}$$

$T = -\frac{\hbar^2}{2\,m} \int_{-\infty}^{\infty} \psi[x]\, D[\psi[x], \{x, 2\}]\, dx$

$$\frac{b\,\hbar^2}{2\,m}$$

```
H = T + V;
```

$B = b$ /. $\left(\text{Solve}[\partial_b H == 0, b]\right)$ [[1]]

```
Hmin = H /. b → B
```

$$-\frac{(-3)^{1/3}\, m^{1/3}\, \alpha^{1/3}}{2^{2/3}\, \hbar^{2/3}}$$

$$-\frac{3\,(-3)^{1/3}\, \alpha^{1/3}\, \hbar^{4/3}}{4 \times 2^{2/3}\, m^{2/3}}$$

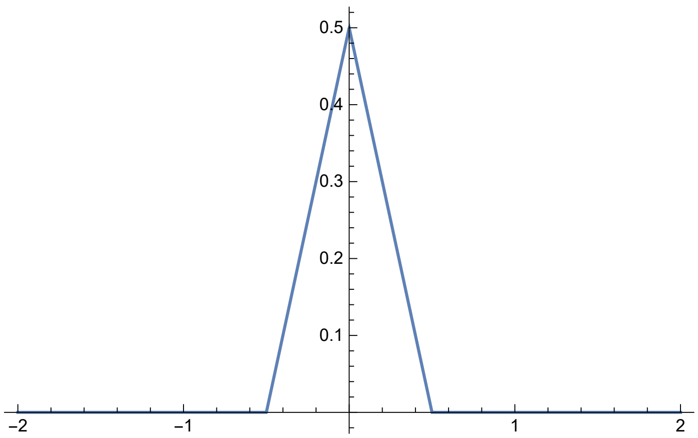
$$H_{min} = \frac{3}{4}\left(\frac{3\,\alpha\,\hbar^4}{4\,m^2}\right)$$

7.3

$$\psi = \begin{pmatrix} A\,(x + a/2) & -a/2 \le x \le 0 \\ A\,(a/2 - x) & 0 \le x \le a/2 \\ 0 & x < a/2,\ x > a/2 \end{pmatrix}$$

```
Print["ψ[x]=", ψ[x_] = Piecewise[{{0, x < -a/2 || x > a/2},
    {A (x + a/2), x > -a/2 && x < 0}, {A (a/2 - x), x > 0 && x < a/2}}]]
Plot[ψ[x] /. {A → 1, a → 1}, {x, -2, 2}, PlotRange → All]
```

$$\psi[x] = \begin{cases} 0 & x < -\frac{a}{2} \;||\; x > \frac{a}{2} \\ A\left(\frac{a}{2} + x\right) & x > -\frac{a}{2} \;\&\&\; x < 0 \\ A\left(\frac{a}{2} - x\right) & x > 0 \;\&\&\; x < \frac{a}{2} \\ 0 & \text{True} \end{cases}$$




```
$Assumptions = {a > 0, a ∈ Reals};
```

$$\text{Solve}\left[1 == \int_{-a/2}^{a/2} \psi[x]^2 \, dx, A\right];$$

```
AA = A /. %[[2]]
```

$$\frac{2\sqrt{3}}{a^{3/2}}$$

$$\frac{d\psi}{dx} = \begin{pmatrix} A & -a/2 < x < 0 \\ -A & 0 < x < a/2 \\ 0 & \text{elsewhere} \end{pmatrix}$$

$$\frac{d^2\psi}{dx^2} = A\,\delta[x + a/2] - 2A\,\delta[x] + A\,\delta[x - a/2]$$

$$\langle T \rangle = -\frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \psi \frac{d^2\psi}{dx^2} \, dx$$

$$\langle T \rangle = -\frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \psi \,(A\,\delta[x + a/2] - 2A\,\delta[x] + A\,\delta[x - a/2]) \, dx$$

$$\langle T \rangle = \frac{\hbar^2}{2m} \int_{-\infty}^{\infty} 2A\,\psi[0] \, dx$$

$$\langle T \rangle = \frac{\hbar^2}{m} A^2 \frac{a}{2}$$

$$T = AA^2 \frac{a\,\hbar^2}{2\,m}$$

$$\frac{6\,\hbar^2}{a^2\,m}$$

$$\langle V \rangle = -\alpha \int \psi^2 \, \delta[x] \, dx$$

$$\langle V \rangle = -\alpha \int \psi[0]^2 \, dx$$

$$\langle V \rangle = -\alpha \left(A \frac{a}{2}\right)^2$$

$$\langle V \rangle = -\alpha \left(A \frac{a}{2}\right)^2$$

$$\mathbf{V = -\alpha \left(AA \frac{a}{2}\right)^2}$$

$$-\frac{3\alpha}{a}$$

H = T + V

$$-\frac{3\alpha}{a} + \frac{6\hbar^2}{a^2 m}$$

```
ans = Solve[∂_a H == 0, a]
```

$$\left\{\left\{a \to \frac{4\hbar^2}{m\alpha}\right\}\right\}$$

```
Hmin = H /. ans[[1]]
```

$$-\frac{3 m \alpha^2}{8\hbar^2}$$

7.4

(a)

$$\langle \psi \mid \psi_{gs} \rangle = 0 \Rightarrow \langle \psi \mid \psi_1 \rangle = 0$$

$$\sum c_n \langle \psi \mid \psi_1 \rangle = c_1 = 0$$

$$\langle H \rangle = \sum E_n \, |c_n|^2 \geq E_{fe} \sum |c_n|^2 = E_{fe} \text{ for } n = 1$$

(b)

$$\psi[x] = x \, e^{-b x^2}$$

```
$Assumptions = {b > 0, b ∈ Reals};
ψ[x_] = A x e^(-b x^2);
AA = A /. (Solve[1 == ∫_{-∞}^{∞} ψ[x]^2 dx, A] // FullSimplify) [[2]]
```

$$2\, b^{3/4} \left(\frac{2}{\pi}\right)^{1/4}$$

```
T = - ħ^2/(2 m) ∫_{-∞}^{∞} (ψ[x] D[ψ[x], {x, 2}] /. A → AA) dx // FullSimplify
```

$$\frac{3 b \hbar^2}{2 m}$$

```
V = 1/2 m ω^2 ∫_{-∞}^{∞} (x^2 ψ[x]^2 /. A → AA) ⅆx // FullSimplify
```

$$\frac{3\,m\,\omega^2}{8\,b}$$

```
H = T + V
```

$$\frac{3\,m\,\omega^2}{8\,b}+\frac{3\,b\,\hbar^2}{2\,m}$$

```
ans = Solve[∂_b H == 0, b]
```

$$\left\{\left\{b\to -\frac{m\,\omega}{2\,\hbar}\right\},\left\{b\to \frac{m\,\omega}{2\,\hbar}\right\}\right\}$$

```
Hmin = H /. ans[[2]]
```

$$\frac{3\,\omega\,\hbar}{2}$$

7.5

(a)

ψ^0_{gs} is the trial wave function

$\langle \psi^0_{gs} \mid H \mid \psi^0_{gs} \rangle \geq E^0_{gs}$

$\langle \psi^0_{gs} \mid H^0 \mid \psi^0_{gs} \rangle + \langle \psi^0_{gs} \mid H' \mid \psi^0_{gs} \rangle \geq E^0_{gs}$

$E^0_{gs} + E^1_{gs} \geq E^0_{gs}$

This is always true

(b)

$$E^2_{gs} = \sum_{m\neq gs} \frac{\left|\langle \psi^0_{gs} \mid H' \mid \psi^0_{gs} \rangle\right|^2}{E^0_{gs} - E^0_m}$$

$\left|\langle \psi^0_{gs} \mid H' \mid \psi^0_{gs} \rangle\right|^2$ is always positive

gs is the lowest state, so gs < m for all m. Thus,

$E^0_{gs} - E^0_m < 0$ for all m

E^2_{gs} is a positive number divided by a negative, thus it is always negative.