UofT-STA261-2015-Summer Note

hysw

Tuesday 14th July, 2015
at 15:44

Contents

1 TODO

section 7.2, 7.3

Example 8.1

$\mathcal{X}^2$ distribution, t distribution, F distribution

L^2 convergence

MGF, PDF

All exercises in section 8.2, read 8.3 and 8.4. Ex 8.36, 8.37. 8.38. Read section 9.3. try exercise 9.15 –9.36 and exercise in section 9.6.

2 Distributions

Notation	Name	pdf	cdf
$\mathcal{N}(\mu, \sigma^2)$	normal	-	-
$B(n, p)$	binomial		
	T-distribution		
	$\mathcal{X}^2$ distribution		

3 Abbreviations

r.v. random variable

i.i.d. independent and identically distributed

SD standard deviation

- $\overline{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$

4 Definitions

A **random variable** is a real-valued function for which the domain is a sample space.

A **statistic** is a function of the observable random variables in a sample and known constants.

Variance $\mathrm{Var}(X) = E\left[(X - \mu)^2\right]$

5 Basic Topics

6 Lecture Topics

6.1 Monday, June 29, 2015

6.1.1 Convergence

convergence in distribution Let $X_1, X_2, \ldots$ be a infinite sequence of random variables and let Y be another random variable. $X_n \xrightarrow{d} Y$ (the the sequence $\{X_n\}$ **converges** to Y **in distribution**) if

$$\forall x \in \mathbb{R} \left[\lim_{n\to\infty} \Pr(X_n \le x) = \Pr(Y \le x) \right]$$

note that $\Pr(X_n \le x) = F_{X_n}(x)$ and $\Pr(Y \le x) = F_Y(x)$

convergence in probobility Let $X_1, X_2, \ldots$ be a infinite sequence of random variables and let Y be another random variable. $X_n \xrightarrow{p} Y$ (the the sequence $\{X_n\}$ **converges** to Y **in probobility**) if

$$\forall \varepsilon > 0 \left[\lim_{n\to\infty} \Pr(|X_n - Y| > \varepsilon) = 0 \right]$$

almost surely convergence Let $X_1, X_2, \ldots$ be a infinite sequence of random variables with common $\mu < \infty$ and let Y be another random variable. $X_n \xrightarrow{a.s.} Y$ (the the sequence $\{X_n\}$ **converges** to Y **almost surely**) if

$$\Pr\left(\lim_{n\to\infty} X_n = Y \right) = 1$$

Weak Law of Large Number (WLLN) Let $X_1, X_2, \ldots, X_n$ be a sequence of independent random variables and each being $\mu < \infty$, $\delta < \infty$. Then $\overline{X}_n \xrightarrow{p} \mu$.

Strong Law of Large Number (SLLN) Let $X_1, X_2, \ldots, X_n$ be a infinite sequence of independent random variables with finite sequence mean($\mu < \infty$). Then $\overline{X}_n \xrightarrow{a.s.} \mu$.

6.1.2 Central Limit Theorem

Let $X_1, X_2, \ldots, X_n$ be iid rv with finite mean and variance. Let $Z_n = \frac{\overline{X}-\mu}{\sigma\sqrt{n}}$, then as $n \to \infty$, $Z_n \xrightarrow{d} Y$ where $Y \sim \mathcal{N}(0,1)$

6.1.3 Normal Distribution Theorem

Types of Convergences, WLLN and CLT

Convergence in distribution
$\forall x \in \mathbb{R} \left[\lim_{n\to\infty} F_n(x) = F(x) \right]$

Convergence in probability
$\lim_{n\to\infty} \Pr(|X_n - X| \ge \varepsilon) = 0$

Almost sure convergence
$\Pr(\lim_{n\to\infty} X_n = X) = 1$

Weak law of large number (WLLN)
$\overline{X}_n \xrightarrow{p} \mu$ when $n \to \infty$

Convergence on Wikipedia

6.1.4 Central Limit Theorem (CLT)

Let $Y_1, \ldots, Y_n$ be independent and identically distributed random variables with $\mathrm{E}(Y_i) = \mu$ and $\mathrm{Var}(Y_i) = \sigma^2 < \infty$.

Define $U_n = \frac{\overline{Y}-\mu}{\sigma/\sqrt{n}}$.

Then the distribution function of U_n converges to the standard normal distribution function as $n \to \infty$.

6.2 Monday, July 6, 2015

A **statistic** $t(X)$ is a function of data X or sample and not the parameter.

6.2.1 Statistical Model

A **statistical model** is a set of assertions that directly or indirectly specify the proboblity distribution of observed data.

6.2.2 Estimator

6.2.3 Statistical Inference

estimation($\hat{\theta}$, a estimator of θ)

hypothesis testing estimation($\hat{\theta}$)

estimator is a statistic $\hat{\theta}(X_1, \ldots, X_n)$ estimator $\hat{\theta}$ is a statistic of data sample $(X_1, \ldots, X_n)$

- Proboblity model
- statistic model
- statistic Inference
- principal of minimizaing MSE
- Method of moment estimator
- parameter space
- biased estimator / unbiased estimator
- consistant estimator
- MVUE - Minimum-variance unbiased estimator

An *estimator* is a rule, often expressed as a formula, that tells how to calculate the value of an estimate based on the measurements contained in a sample.

Let $\hat{\theta}$ be a point estimator for a parameter θ. Then $\hat{\theta}$ is an *unbiased estimator* if $\mathrm{E}(\hat{\theta}) = \theta$. If $E(\hat{\theta}) \neq \hat{\theta}$, $\hat{\theta}$ is said to be *biased*.

The *bias* of a point estimator $\hat{\theta}$ is given by $B(\hat{\theta}) = E(\hat{\theta}) - \theta$.

The *mean square error* of a point estimator $\hat{\theta}$ is

$$\mathrm{MSE}(\hat{\theta}) = \mathrm{E}[(\hat{\theta} - \theta)^2] = \mathrm{Var}(\hat{\theta}) + [\mathrm{B}(\hat{\theta})]^2$$

Define *standard error* $\sigma_{\hat{\theta}} = \sqrt{\sigma^2_{\hat{\theta}}}$.

The error of estimation ε is the distance between an estimator and its target parameter. That is, $\varepsilon = |\hat{\theta} - \theta|$.

Statistical Model and Inference Method of Moment Estimator

6.3 Wednesday, July 8, 2015

- Internal Estimator
- Confidence Interval (CI)
- Interpretation of CI
- Private Quantity
- Large Sample CI concerning μ
- Likelihood Inference
- Likelihood Principle
- Suffcient Statistics
- factorization theorem
- minimal sufficient statistic

Suffcient Statistics is not unique

6.4 Monday, July 13, 2015

Maximum Likelihood Estimator Rao-Blackwell Theorem Complete Statistic, Ancillary Statistic Lehmann-Scheffe Lemma

6.5 Wednesday, July 15, 2015

Exponential family model and its properties Score function and fisher information

6.6 Wednesday, July 22, 2015

Likelihood Asymptotic and Delta Method Cramer-Rao lower bound

6.7 Monday, July 27, 2015

Intro to Hypothesis Testing

6.8 Wednesday, July 29, 2015

Neyman-Pearson Lemma Likelihood Ratio Test

6.9 Monday, August 5, 2015

P-value Relationship between Confidence Interval and Hypothesis Testing