1   Scope of Document

This document describes an AMM for swapping between three tokens that are all pegged to the same currency, e.g USD. This doc only covers the simplest possible version, where the AMM does not charge fees.

2   Definitions: State Variables

State variables are a set of time-varying attributes that are sufficient to describe contract state.

Variable	Explanation
x_t	the contract's initial balance of Stablecoin X prior to transaction t
y_t	the contract's initial balance of Stablecoin Y ""
z_t	the contract's initial balance of Stablecoin Z ""
l_t	the contract's initial issuance of liquidity provider tokens ""
$\{l_{it}\}$	the initial balance of liquidity provider tokens held in address i ""

3   Definitions: Parameters

Parameters are constants that are set in stone when the contract is deployed.

Parameter	Explanation
ρ	constant between 0 and 1 that determines slippage

4   Definitions: Flow Variables (not sure what the Comp Sci term would be)

Flow variables describe the change in state prior to and after transaction t. They do not need to retained after state has been updated.

Variable	Explanation
Δx_t	$\Delta x_t = x_{t+1} - x_t$, we must have that $\Delta x_t \geq -x_t$
Δy_t	$\Delta y_t = y_{t+1} - y_t$, we must have that $\Delta y_t \geq -y_t$
Δz_t	$\Delta z_t = z_{t+1} - z_t$, we must have that $\Delta z_t \geq -z_t$
Δl_t	$\Delta l_t = l_{t+1} - l_t$, we must have that $\Delta l_t \geq -l_t$
Δl_{it}	$\Delta l_{it} = l_{it+1} - l_{it}$, we must have that $\Delta l_{it} \geq -l_{it}$

5   Transaction Type: Sell Tokens Δx_t for $|\Delta y_t|$ tokens

The processes of selling Δx_t for $|\Delta y_t|$, Δx_t for $|\Delta z_t|$, Δy_t for $|\Delta x_t|$, Δz_t for $|\Delta x_t|$, and Δz_t for $|\Delta x_t|$ all work the same, so I only provide one example.

5.1 Send Δx_t Tokens to the contract

The user sends Δx_t tokens of type X to the contract and indicates that he is buying tokens of type Y.

We must have $0 < \Delta x_t \le (y_t^\rho + x_t^\rho)^{\frac{1}{\rho}} - x_t$ or the transaction will revert.

5.2 The contract sends $|\Delta y_t|$ tokens to the user's address

Δy_t is computed as shown in Equation 1.

$$\Delta y_t = (x_t^\rho + y_t^\rho - (x_t + \Delta x_t)^\rho)^{\frac{1}{\rho}} - y_t \tag{1}$$

Note that $-y_t \le \Delta y_t < 0$.

5.3 Update Contract State

Variable
$x_{t+1} = x_t + \Delta x_t$
$y_{t+1} = y_t + \Delta y_t$
$z_{t+1} = z_t$
$l_{it+1} = l_{it}$
$l_{t+1} = l_t$

6 Transaction Type: Add/Remove Liquidity

6.1 Add Liquidity Transaction

6.1.1 Deposit Tokens

The user deposits the following token amounts in the contract: $(\Delta x_t, \Delta y_t, \Delta z_t)$. These amounts must satisfy $\frac{\Delta x_t}{x_t} = \frac{\Delta y_t}{y_t} = \frac{\Delta z_t}{z_t}$ AND $\Delta x_t > 0$ or the transaction will revert.

6.1.2 Update Contract State

Variable
$l_{it+1} = l_{it} + \frac{\Delta x_t}{x_t} l_t$
$l_{t+1} = l_t + \frac{\Delta x_t}{x_t} l_t$
$x_{t+1} = x_t + \Delta x_t$
$y_{t+1} = y_t + \Delta y_t$
$z_{t+1} = z_t + \Delta z_t$

6.2 Withdraw Liquidity Transaction

6.2.1 Destroy Liquidity Tokens

The user destroys $|\Delta l_{it}|$ liquidity tokens, where Δl_{it} must satisfy $-l_{it} \le \Delta l_{it} < 0$ or the transaction will revert.

6.2.2 Unlock User Funds

The contract sends the following tokens to the user's address: $\frac{|\Delta l_{it}|}{l_{it}} x_t$ tokens of type X, $\frac{|\Delta l_{it}|}{l_{it}} y_t$ tokens of type Y, and $\frac{|\Delta l_{it}|}{l_{it}} z_t$ tokens of type Z.

6.2.3 Update Contract State

Variable
$x_{t+1} = x_t + \frac{\Delta l_{it}}{l_t} x_t$
$y_{t+1} = y_t + \frac{\Delta l_{it}}{l_t} y_t$
$z_{t+1} = z_t + \frac{\Delta l_{it}}{l_t} z_t$
$l_{it+1} = l_{it} + \Delta l_{it}$
$l_{t+1} = l_t + \Delta l_{it}$