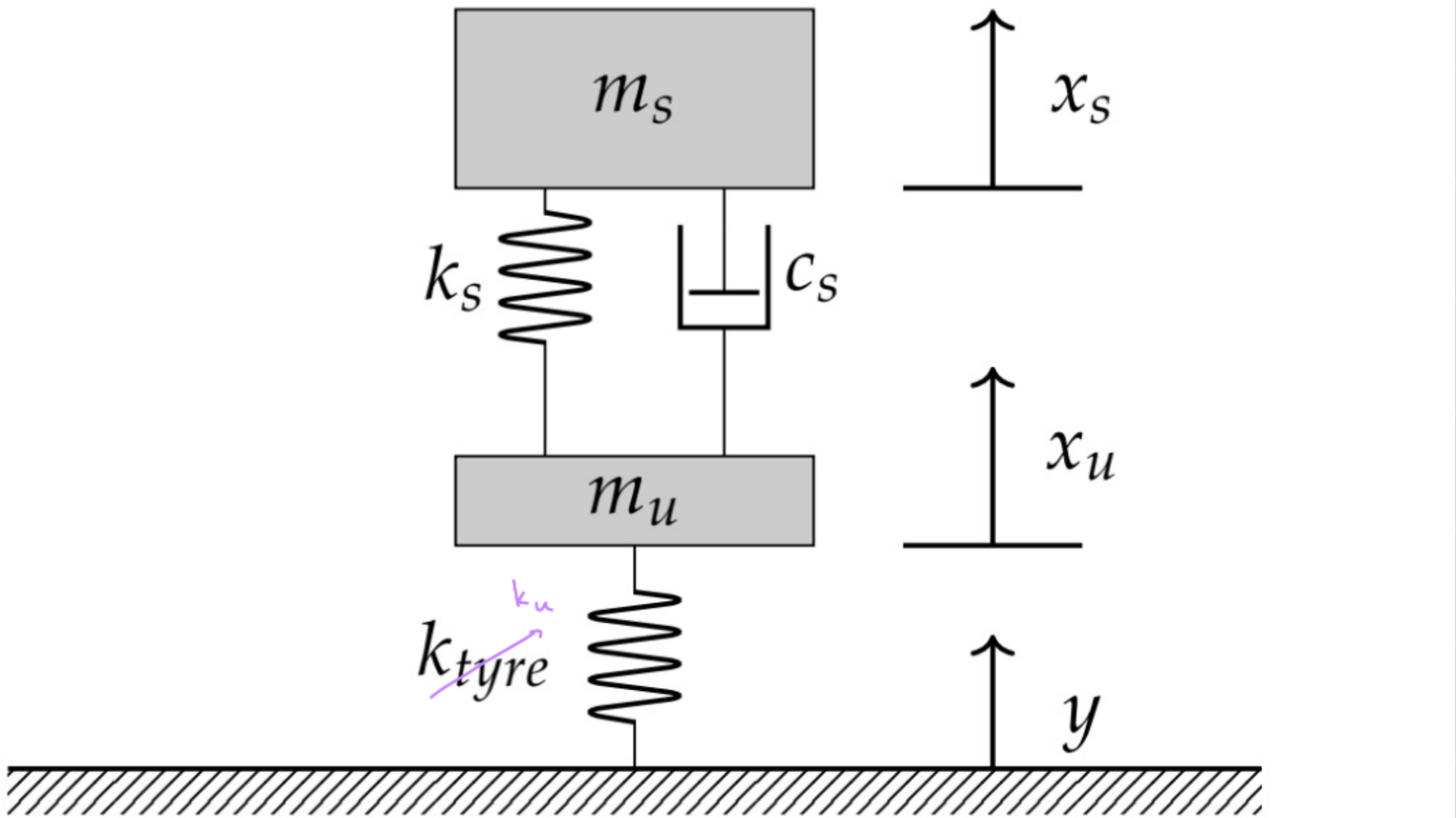
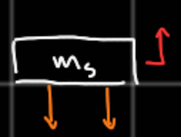
- Quarter Car

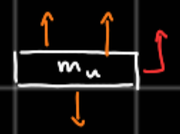


- Equations of motion



$$-k_s(x_s - x_u) - c_s(\dot{x}_s - \dot{x}_u) = m\ddot{x}_s$$

$$m\ddot{x}_s + c_s(\dot{x}_s - \dot{x}_u) + k_s(x_s - x_u) = 0$$



$$k_s(x_s - x_u) + c_s(\dot{x}_s - \dot{x}_u) - k_u(x_u - y) = m_u\ddot{x}_u$$

$$m_u\ddot{x}_u - c_s(\dot{x}_s - \dot{x}_u) - k_s(x_s - x_u) + k_u(x_u - y) = 0$$

$x_1 = x_u$

$x_2 = \dot{x}_1 = \dot{x}_u$

$x_3 = x_s$

$x_4 = \dot{x}_3 = \dot{x}_s$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$m_s \dot{x}_4 + c_s(x_4 - x_2) + k_s(x_3 - x_1) = 0$$

$$m_s \dot{x}_4 + c_s x_4 - c_s x_2 + k_s x_3 - k_s x_1 = 0$$

$$\dot{x}_4 = \frac{-c_s}{m_s} x_4 + \frac{c_s}{m_s} x_2 - \frac{k_s}{m_s} x_3 + \frac{k_s}{m_s} x_1 \quad , \quad \dot{x}_3 = x_4$$

$$m_u \dot{x}_2 - c_s(x_4 - x_2) - k_s(x_3 - x_1) + k_u(x_1 - y) = 0$$

$$m_u \dot{x}_2 - c_s x_4 + c_s x_2 - k_s x_3 + k_s x_1 + k_u x_1 - k_u y = 0$$

$$\dot{x}_2 = \frac{c_s}{m_u} x_4 - \frac{c_s}{m_u} x_2 + \frac{k_s}{m_u} x_3 - \frac{k_s}{m_u} x_1 - \frac{k_u}{m_u} x_1 + \frac{k_u}{m_u} y \quad , \quad \dot{x}_1 = x_2$$

$$\dot{x} = Ax + Bu$$

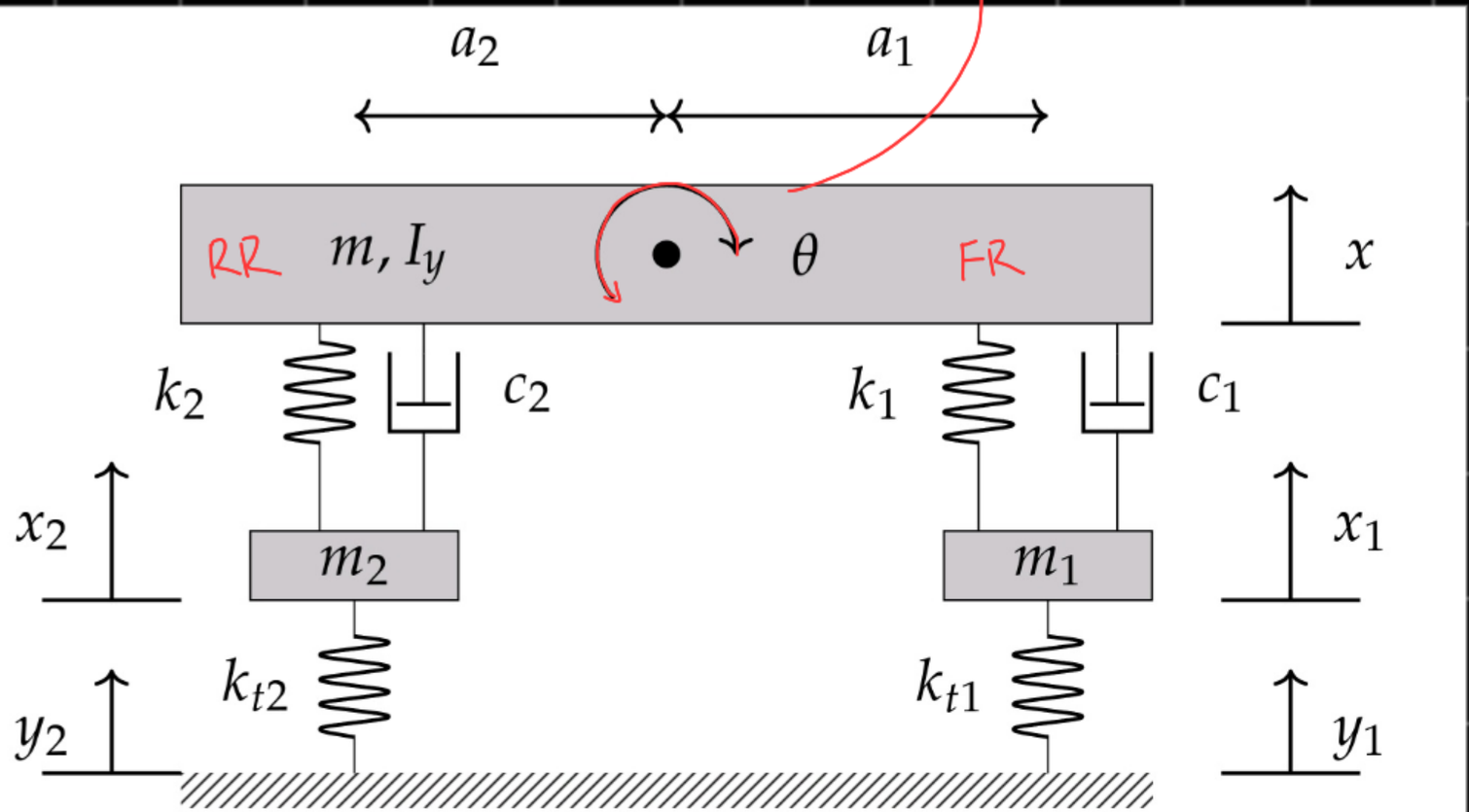
$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_s - k_u}{m_u} & \frac{-c_s}{m_u} & \frac{k_s}{m_u} & \frac{c_s}{m_u} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_s} & \frac{c_s}{m_s} & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k_u}{m_u} \\ 0 \\ 0 \end{bmatrix} y$$

$$y = Cx + Du$$

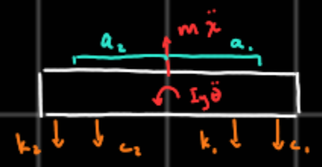
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

- Bicycle Car

Reverse pitch angle to match SAE



- Equations of motion



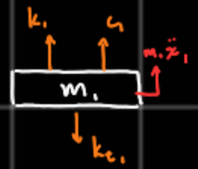
small angle approx.

$$-k_1(x - x_1 + a_1\theta) - k_2(x - x_2 - a_2\theta) - c_1(\dot{x} - \dot{x}_1 + a_1\dot{\theta}) - c_2(\dot{x} - \dot{x}_2 - a_2\dot{\theta}) = m\ddot{x}$$

$$\boxed{-\frac{k_1}{m}(x - x_1 + a_1\theta) - \frac{k_2}{m}(x - x_2 - a_2\theta) - \frac{c_1}{m}(\dot{x} - \dot{x}_1 + a_1\dot{\theta}) - \frac{c_2}{m}(\dot{x} - \dot{x}_2 - a_2\dot{\theta}) = \ddot{x}}$$

$$a_1[-k_1(x - x_1 + a_1\theta) - c_1(\dot{x} - \dot{x}_1 + a_1\dot{\theta})] - a_2[-k_2(x - x_2 - a_2\theta) - c_2(\dot{x} - \dot{x}_2 - a_2\dot{\theta}) = I_y\ddot{\theta}$$

$$\boxed{-\frac{a_1k_1}{I_y}(x - x_1 + a_1\theta) - \frac{a_1c_1}{I_y}(\dot{x} - \dot{x}_1 + a_1\dot{\theta}) + \frac{a_2k_2}{I_y}(x - x_2 - a_2\theta) + \frac{a_2c_2}{I_y}(\dot{x} - \dot{x}_2 - a_2\dot{\theta}) = \ddot{\theta}}$$



$$k_1(z - x_1 + a_1\theta) + c_1(\dot{z} - \dot{x}_1 + a_1\dot{\theta}) - k_{t1}(x_1 - y_1) = m_1\ddot{x}_1$$

$$\frac{k_1}{m_1}(z - x_1 + a_1\theta) + \frac{c_1}{m_1}(\dot{z} - \dot{x}_1 + a_1\dot{\theta}) - \frac{k_{t1}}{m_1}(x_1 - y_1) = \ddot{x}_1$$

$$\frac{k_2}{m_2}(z - x_2 - a_2\theta) + \frac{c_2}{m_2}(\dot{z} - \dot{x}_2 - a_2\dot{\theta}) - \frac{k_{t2}}{m_2}(x_2 - y_2) = \ddot{x}_2$$

$$\dot{x} = Ax + Bu \qquad x = \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \\ x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}$$

state x_1, not displacement x_1

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{k_1}{m}x_1 + \frac{k_1}{m}x_5 - \frac{k_1 a_1}{m}x_3 - \frac{k_2}{m}x_1 + \frac{k_2}{m}x_7 + \frac{k_2 a_2}{m}x_3 - \frac{c_1}{m}x_2 + \frac{c_1}{m}x_6 - \frac{c_1 a_1}{m}x_4 - \frac{c_2}{m}x_2 + \frac{c_2}{m}x_8 + \frac{c_2 a_2}{m}x_4$$

$$\dot{x}_2 = \left(\frac{-k_1-k_2}{m}\right)x_1 + \left(\frac{-c_1-c_2}{m}\right)x_2 + \left(\frac{-k_1 a_1 + k_2 a_2}{m}\right)x_3 + \left(\frac{-c_1 a_1 + c_2 a_2}{m}\right)x_4 + \left(\frac{k_1}{m}\right)x_5 + \left(\frac{c_1}{m}\right)x_6 + \left(\frac{k_2}{m}\right)x_7 + \left(\frac{c_2}{m}\right)x_8$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -\frac{a_1 k_1}{I_y}x_1 + \frac{a_1 k_1}{I_y}x_5 - \frac{a_1^2 k_1}{I_y}x_3 - \frac{a_1 c_1}{I_y}x_2 + \frac{a_1 c_1}{I_y}x_6 - \frac{a_1^2 c_1}{I_y}x_4 + \frac{a_2 k_2}{I_y}x_1 - \frac{a_2 k_2}{I_y}x_7 - \frac{a_2^2 k_2}{I_y}x_3 + \frac{a_2 c_2}{I_y}x_2 - \frac{a_2 c_2}{I_y}x_8 - \frac{a_2^2 c_2}{I_y}x_4$$

$$\dot{x}_4 = \left(\frac{-a_1 k_1 + a_2 k_2}{I_y}\right)x_1 + \left(\frac{-a_1 c_1 + a_2 c_2}{I_y}\right)x_2 + \left(\frac{-a_1^2 k_1 - a_2^2 k_2}{I_y}\right)x_3 + \left(\frac{-a_1^2 c_1 - a_2^2 c_2}{I_y}\right)x_4 + \left(\frac{a_1 k_1}{I_y}\right)x_5 + \left(\frac{a_1 c_1}{I_y}\right)x_6 + \left(\frac{-a_2 k_2}{I_y}\right)x_7 + \left(\frac{-a_2 c_2}{I_y}\right)x_8$$

$$\dot{x}_5 = x_6$$

$$\dot{x}_6 = \frac{k_1}{m_1}x_1 - \frac{k_1}{m_1}x_5 + \frac{a_1 k_1}{m_1}x_3 + \frac{c_1}{m_1}x_2 - \frac{c_1}{m_1}x_6 + \frac{a_1 c_1}{m_1}x_4 - \frac{k_{t1}}{m_1}x_5 + \frac{k_{t1}}{m_1}y_1$$

$$\dot{x}_6 = \left(\frac{k_1}{m_1}\right)x_1 + \left(\frac{c_1}{m_1}\right)x_2 + \left(\frac{a_1 k_1}{m_1}\right)x_3 + \left(\frac{a_1 c_1}{m_1}\right)x_4 + \left(\frac{-k_1 - k_{t1}}{m_1}\right)x_5 + \left(\frac{-c_1}{m_1}x_6\right) + \left(\frac{k_{t1}}{m_1}\right)y_1$$

$$\dot{x}_7 = x_8$$

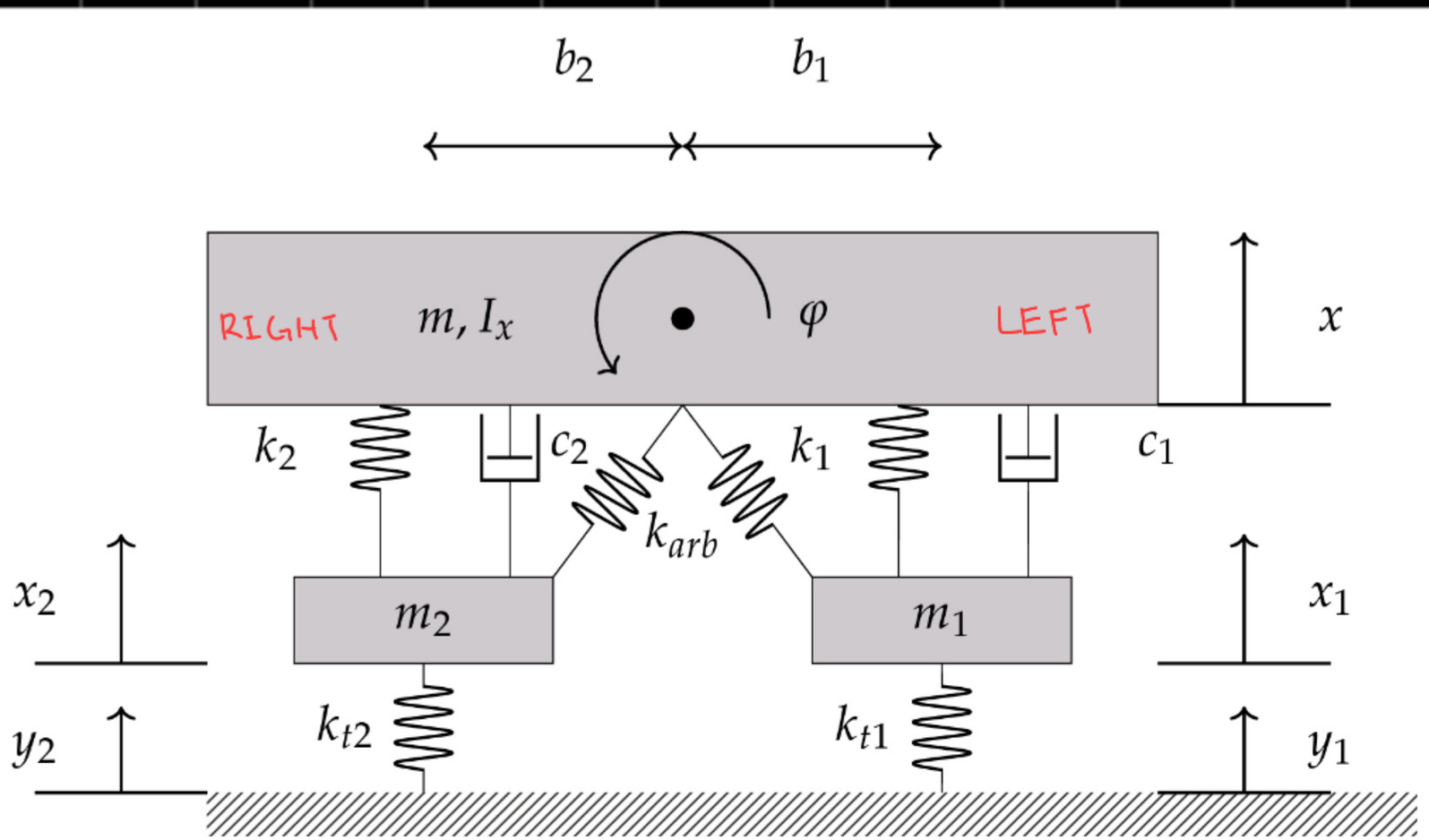
$$\dot{x}_8 = \frac{k_2}{m_2}x_1 - \frac{k_2}{m_2}x_7 - \frac{a_2k_2}{m_2}x_3 + \frac{c_2}{m_2}x_2 - \frac{c_2}{m_2}x_8 - \frac{a_2c_2}{m_2}x_4 - \frac{kt_2}{m_2}x_7 + \frac{kt_2}{m_2}y_2$$

$$\dot{x}_8 = \left(\frac{k_2}{m_2}\right)x_1 + \left(\frac{c_2}{m_2}\right)x_2 + \left(\frac{-a_2k_2}{m_2}\right)x_3 + \left(\frac{-a_2c_2}{m_2}\right)x_4 + \left(\frac{-k_2-kt_2}{m_2}\right)x_7 + \left(\frac{-c_2}{m_2}\right)x_8 + \left(\frac{kt_2}{m_2}\right)y_2$$

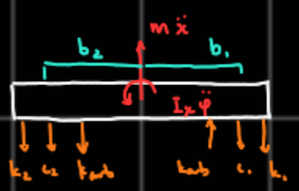
$$A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-k_1-k_2}{m} & \frac{-c_1-c_2}{m} & \frac{-k_1a_1+k_2a_2}{m} & \frac{-c_1a_1+c_2a_2}{m} & \frac{k_1}{m} & \frac{c_1}{m} & \frac{k_2}{m} & \frac{c_2}{m} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\frac{-a_1k_1+a_2k_2}{I_y} & \frac{-a_1c_1+a_2c_2}{I_y} & \frac{-a_1^2k_1-a_2^2k_2}{I_y} & \frac{-a_1^2c_1-a_2^2c_2}{I_y} & \frac{a_1k_1}{I_y} & \frac{a_1c_1}{I_y} & \frac{-a_2k_2}{I_y} & \frac{-a_2c_2}{I_y} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\frac{k_1}{m_1} & \frac{c_1}{m_1} & \frac{a_1k_1}{m_1} & \frac{a_1c_1}{m_1} & \frac{-k_1-kt_1}{m_1} & \frac{-c_1}{m_1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\frac{k_2}{m_2} & \frac{c_2}{m_2} & \frac{-a_2k_2}{m_2} & \frac{-a_2c_2}{m_2} & 0 & 0 & \frac{-k_2-kt_2}{m_2} & \frac{-c_2}{m_2}
\end{bmatrix}$$

$$B = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\frac{k_1}{m_1} & 0 \\
0 & 0 \\
0 & \frac{kt_2}{m_2}
\end{bmatrix}$$

- Roll Car



- Equations of motion

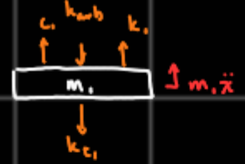


$$-k_1(x - x_1 + b_1\varphi) - c_1(\dot{x} - \dot{x}_1 + b_1\dot{\varphi}) - k_2(x - x_2 - b_2\varphi) - c_2(\dot{x} - \dot{x}_2 - b_2\dot{\varphi}) = m\ddot{x}$$

$$\boxed{\frac{-k_1}{m}(x - x_1 + b_1\varphi) - \frac{c_1}{m}(\dot{x} - \dot{x}_1 + b_1\dot{\varphi}) - \frac{k_2}{m}(x - x_2 - b_2\varphi) - \frac{c_2}{m}(\dot{x} - \dot{x}_2 - b_2\dot{\varphi}) = \ddot{x}}$$

$$-k_1b_1(x - x_1 + b_1\varphi) - c_1b_1(\dot{x} - \dot{x}_1 + b_1\dot{\varphi}) + k_2b_2(x - x_2 - b_2\varphi) + c_2b_2(\dot{x} - \dot{x}_2 - b_2\dot{\varphi}) - k_{arb}\varphi = I_x\ddot{\varphi}$$

$$\boxed{\frac{-k_1b_1}{I_x}(x - x_1 + b_1\varphi) - \frac{c_1b_1}{I_x}(\dot{x} - \dot{x}_1 + b_1\dot{\varphi}) + \frac{k_2b_2}{I_x}(x - x_2 - b_2\varphi) + \frac{c_2b_2}{I_x}(\dot{x} - \dot{x}_2 - b_2\dot{\varphi}) - \frac{k_{arb}}{I_x}\varphi = \ddot{\varphi}}$$



$$k_1(x - x_1 + b_1\varphi) + c_1(\dot{x} - \dot{x}_1 + b_1\dot{\varphi}) - k_{t_1}(x_1 - y_1) - \frac{k_{arb}}{b_1}\varphi = m_1\ddot{x}_1$$

$$\frac{k_1}{m_1}(x - x_1 + b_1\varphi) + \frac{c_1}{m_1}(\dot{x} - \dot{x}_1 + b_1\dot{\varphi}) - \frac{k_{t_1}}{m_1}(x_1 - y_1) - \frac{k_{arb}}{m_1 b_1}\varphi = \ddot{x}_1$$

$$\frac{k_2}{m_2}(x - x_2 - b_2\varphi) + \frac{c_2}{m_2}(\dot{x} - \dot{x}_2 - b_2\dot{\varphi}) - \frac{k_{t_2}}{m_2}(x_2 - y_2) + \frac{k_{arb}}{m_2 b_2}\varphi = \ddot{x}_2$$

$$+\frac{k_{arb}}{m_1 bl}\varphi - \frac{k_{arb}}{m_1 bl\, w}x_1 + \frac{k_{arb}}{m_1 bl\, w}x_2$$

$$-\varphi$$

$$-\frac{k_{arb}}{m_1}\left(-\varphi + \frac{x_1 - x_2}{w}\right)\left(\frac{1}{bl}\right)$$

Blade length

Full track width

$$+\frac{k_{arb}}{m_2 bl}\left(-\varphi + \frac{x_1 - x_2}{w}\right)$$

$$\dot{x} = Ax + Bu \qquad x = \begin{bmatrix} x \\ \dot{x} \\ x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \\ \varphi \\ \dot{\varphi} \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{-k_1}{m} x_1 + \frac{k_1}{m} x_3 - \frac{k_1 b_1}{m} x_7 - \frac{c_1}{m} x_2 + \frac{c_1}{m} x_4 - \frac{c_1 b_1}{m} x_8 - \frac{k_2}{m} x_1 + \frac{k_2}{m} x_5 + \frac{k_2 b_2}{m} x_7 - \frac{c_2}{m} x_2 + \frac{c_2}{m} x_6 + \frac{c_2 b_2}{m} x_8$$

$$\dot{x}_2 = \left(\frac{-k_1 - k_2}{m}\right) x_1 + \left(\frac{-c_1 - c_2}{m}\right) x_2 + \left(\frac{k_1}{m}\right) x_3 + \left(\frac{c_1}{m}\right) x_4 + \left(\frac{k_2}{m}\right) x_5 + \left(\frac{c_2}{m}\right) x_6 + \left(\frac{-k_1 b_1 + k_2 b_2}{m}\right) x_7 + \left(\frac{-c_1 b_1 + c_2 b_2}{m}\right) x_8$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{k_1}{m_1} x_1 - \frac{k_1}{m_1} x_3 + \frac{k_1 b_1}{m_1} x_7 + \frac{c_1}{m_1} x_2 - \frac{c_1}{m_1} x_4 + \frac{c_1 b_1}{m_1} x_8 - \frac{kt_1}{m_1} x_3 + \frac{kt_1}{m_1} y_1$$

$$\dot{x}_4 = \left(\frac{k_1}{m_1}\right) x_1 + \left(\frac{c_1}{m_1}\right) x_2 + \left(\frac{-k_1 - kt_1}{m_1}\right) x_3 + \left(\frac{-c_1}{m_1}\right) x_4 + \left(\frac{k_1 b_1}{m_1}\right) x_7 + \left(\frac{c_1 b_1}{m_1}\right) x_8 + \left(\frac{kt_1}{m_1}\right) y_1$$

$$\left(\frac{-k_1 - kt_1}{m_1} - \frac{karb}{m_1 bl\, w}\right) x_3 \qquad + \left(\frac{karb}{m_1 bl\, w}\right) x_5 \qquad \left(\frac{k_1 b_1}{m_1} + \frac{karb}{m_1 bl}\right) x_7$$

$$\dot{x}_5 = x_6$$

$$\dot{x}_6 = \frac{k_2}{m_2} x_1 - \frac{k_2}{m_2} x_5 - \frac{k_2 b_2}{m_2} x_7 + \frac{c_2}{m_2} x_2 - \frac{c_2}{m_2} x_6 - \frac{c_2 b_2}{m_2} x_8 - \frac{kt_2}{m_2} x_5 + \frac{kt_2}{m_2} y_2$$

$$\dot{x}_6 = \left(\frac{k_2}{m_2}\right) x_1 + \left(\frac{c_2}{m_2}\right) x_2 + \left(\frac{-k_2 - kt_2}{m_2}\right) x_5 + \left(\frac{-c_2}{m_2}\right) x_6 + \left(\frac{-k_2 b_2}{m_2}\right) x_7 + \left(\frac{-c_2 b_2}{m_2}\right) x_8 + \frac{kt_2}{m_2} y_2$$

$$+ \left(\frac{karb}{m_2 bl\, w}\right) x_3 \qquad + \left(\frac{-k_2 - kt_2}{m_2} - \frac{karb}{m_2 bl\, w}\right) x_5 \qquad \left(\frac{-k_2 b_2}{m_2} - \frac{karb}{m_2 bl}\right) x_7$$

$$\dot{x}_7 = x_8$$

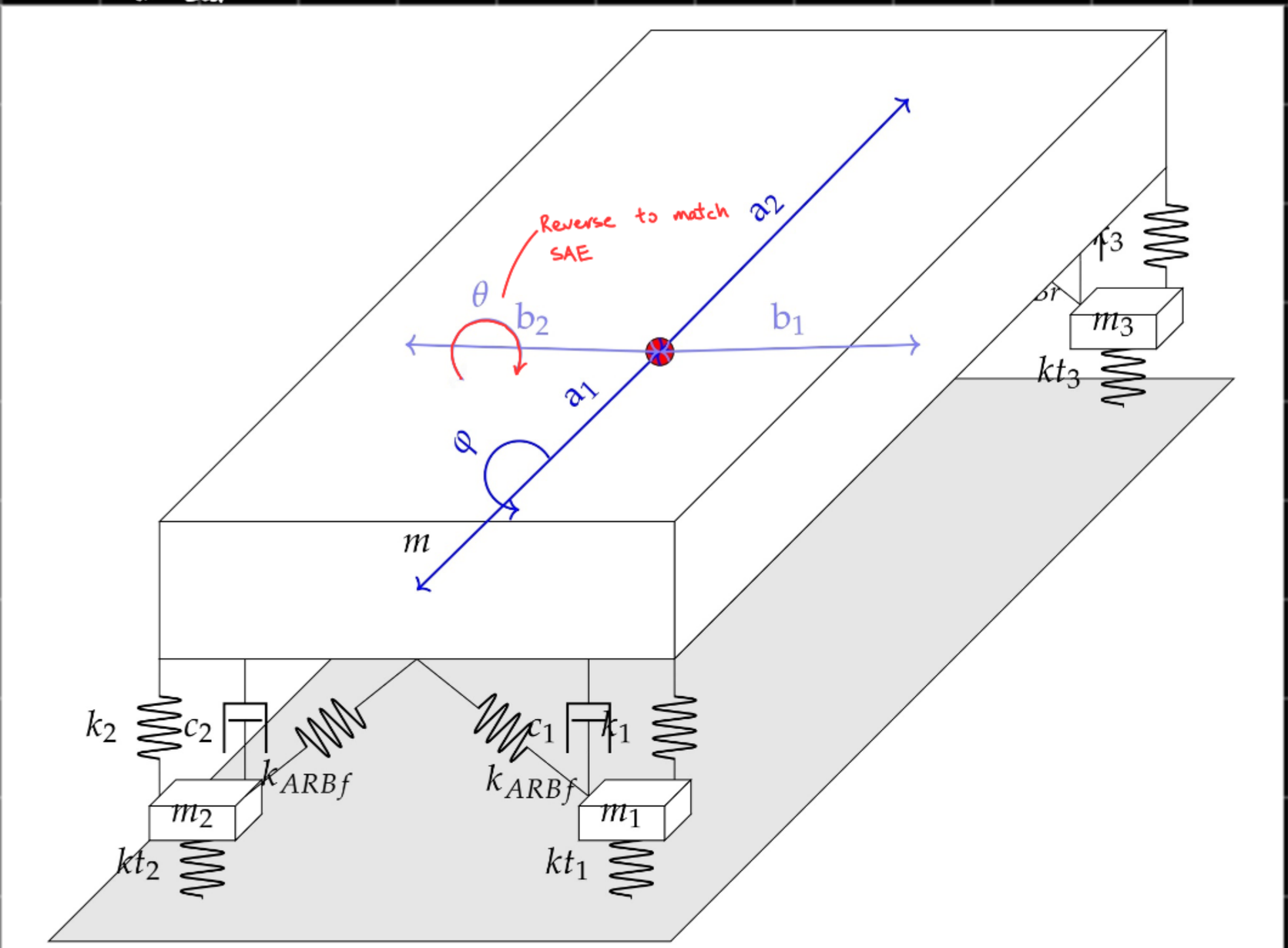
$$\dot{x}_8 = \frac{-k_1b_1}{I_x}x_1 + \frac{k_1b_1}{I_x}x_3 - \frac{k_1b_1^2}{I_x}x_7 - \frac{c_1b_1}{I_x}x_2 + \frac{c_1b_1}{I_x}x_4 - \frac{c_1b_1^2}{I_x}x_8 + \frac{k_2b_2}{I_x}x_1 - \frac{k_2b_2}{I_x}x_5 - \frac{k_2b_2^2}{I_x}x_3 + \frac{c_2b_2}{I_x}x_2 - \frac{c_2b_2}{I_x}x_6 - \frac{c_2b_2^2}{I_x}x_8 - \frac{k_{arb}}{I_x}x_7$$

$$\dot{x}_8 = \left(\frac{-k_1b_1+k_2b_2}{I_x}\right)x_1 + \left(\frac{-c_1b_1+c_2b_2}{I_x}\right)x_2 + \left(\frac{k_1b_1}{I_x}\right)x_3 + \left(\frac{c_1b_1}{I_x}\right)x_4 + \left(\frac{-k_2b_2}{I_x}\right)x_5 + \left(\frac{-c_2b_2}{I_x}\right)x_6 + \left(\frac{-k_1b_1^2-k_2b_2^2-k_{arb}}{I_x}\right)x_7 + \left(\frac{-c_1b_1^2-c_2b_2^2}{I_x}\right)x_8$$

$$A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-k_1-k_2}{m} & \frac{-c_1-c_2}{m} & \frac{k_1}{m} & \frac{c_1}{m} & \frac{k_2}{m} & \frac{c_2}{m} & \frac{-k_1b_1+k_2b_2}{m} & \frac{-c_1b_1+c_2b_2}{m} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\frac{k_1}{m_1} & \frac{c_1}{m_1} & \frac{-k_1-kt_1}{m_1}-\frac{k_{arb}}{m_1bIw} & \frac{-c_1}{m_1} & \frac{k_{arb}}{m_1bIw} & 0 & \frac{k_1b_1}{m_1}+\frac{k_{arb}}{m_1bI} & \frac{c_1b_1}{m_1} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\frac{k_2}{m_2} & \frac{c_2}{m_2} & \frac{k_{arb}}{m_2bIw} & 0 & \frac{-k_2-kt_2}{m_2}-\frac{k_{arb}}{m_2bIw} & \frac{-c_2}{m_2} & \frac{-k_2b_2}{m_2}-\frac{k_{arb}}{m_2bI} & \frac{-c_2b_2}{m_2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\frac{-k_1b_1+k_2b_2}{I_x} & \frac{-c_1b_1+c_2b_2}{I_x} & \frac{k_1b_1}{I_x} & \frac{c_1b_1}{I_x} & \frac{-k_2b_2}{I_x} & \frac{-c_2b_2}{I_x} & \frac{-k_1b_1^2-k_2b_2^2-k_{arb}}{I_x} & \frac{-c_1b_1^2-c_2b_2^2}{I_x}
\end{bmatrix}$$

$$B = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
\frac{kt_1}{m_1} & 0 \\
0 & 0 \\
0 & \frac{kt_2}{m_2} \\
0 & 0 \\
0 & 0
\end{bmatrix}$$

- Full Car



- Equations of motion

$$-k_1(x - x_1 + b_1\varphi + a_1\theta) - k_2(x - x_2 - b_2\varphi + a_1\theta) - k_3(x - x_3 + b_1\varphi - a_2\theta) - k_4(x - x_4 - b_2\varphi - a_2\theta)$$

$$-c_1(\dot{x} - \dot{x}_1 + b_1\dot{\varphi} + a_1\dot{\theta}) - c_2(\dot{x} - \dot{x}_2 - b_2\dot{\varphi} + a_1\dot{\theta}) - k_3(\dot{x} - \dot{x}_3 + b_1\dot{\varphi} - a_2\dot{\theta}) - c_4(\dot{x} - \dot{x}_4 - b_2\dot{\varphi} - a_2\dot{\theta}) = m\ddot{x}$$

$$k_1(x - x_1 + b_1\varphi + a_1\theta) + c_1(\dot{x} - \dot{x}_1 + b_1\dot{\varphi} + a_1\dot{\theta}) - k_{t_1}(x_1 - y_1) - k_{arb_f}(-\varphi + \frac{x_1 - x_2}{w_f}) = m_1\ddot{x}_1$$

$$k_2(x - x_2 - b_2\varphi + a_1\theta) + c_2(\dot{x} - \dot{x}_2 - b_2\dot{\varphi} + a_1\dot{\theta}) - k_{t_2}(x_2 - y_2) + k_{arb_f}(-\varphi + \frac{x_1 - x_2}{w_f}) = m_2\ddot{x}_2$$

$k_{arbf}\varphi - \frac{k_{arbf}}{w_f}x_1 + \frac{k_{arbf}}{w_f}x_2$

* $k_{arb} = \frac{k_{arb}}{bl}$ bl = arb blade length

$$k_3(x - x_3 + b_1\varphi - a_2\theta) + c_3(\dot{x} - \dot{x}_3 + b_1\dot{\varphi} - a_2\dot{\theta}) - k_{t3}(x_3 - y_3) - k_{arbr}\left(-\varphi + \frac{x_3 - x_4}{w_r}\right) = m_3\ddot{x}_3$$

$$k_4(x - x_4 - b_2\varphi - a_2\theta) + c_4(\dot{x} - \dot{x}_4 - b_2\dot{\varphi} - a_2\dot{\theta}) - k_{t4}(x_4 - y_4) + k_{arbr}\left(-\varphi + \frac{x_3 - x_4}{w_r}\right) = m_4\ddot{x}_4$$

$$-b_1k_1(x - x_1 + b_1\varphi + a_1\theta) + b_2k_2(x - x_2 - b_2\varphi + a_1\theta) - b_1k_3(x - x_3 + b_1\varphi - a_2\theta) + b_2k_4(x - x_4 - b_2\varphi - a_2\theta)$$

$$-b_1c_1(\dot{x} - \dot{x}_1 + b_1\dot{\varphi} + a_1\dot{\theta}) + b_2c_2(\dot{x} - \dot{x}_2 - b_2\dot{\varphi} + a_1\dot{\theta}) - b_1c_3(\dot{x} - \dot{x}_3 + b_1\dot{\varphi} - a_2\dot{\theta}) + b_2c_4(\dot{x} - \dot{x}_4 - b_2\dot{\varphi} - a_2\dot{\theta})$$

$$+k_{arbf}\left(-\varphi + \frac{x_1 - x_2}{w_f}\right) + k_{arbr}\left(-\varphi + \frac{x_3 - x_4}{w_r}\right) = I_x\ddot{\varphi}$$

$$-a_1k_1(x - x_1 + b_1\varphi + a_1\theta) - a_1k_2(x - x_2 - b_2\varphi + a_1\theta) + a_2k_3(x - x_3 + b_1\varphi - a_2\theta) + a_2k_4(x - x_4 - b_2\varphi - a_2\theta)$$

$$-a_1c_1(\dot{x} - \dot{x}_1 + b_1\dot{\varphi} + a_1\dot{\theta}) - a_1c_2(\dot{x} - \dot{x}_2 - b_2\dot{\varphi} + a_1\dot{\theta}) + a_2c_3(\dot{x} - \dot{x}_3 + b_1\dot{\varphi} - a_2\dot{\theta}) + a_2c_4(\dot{x} - \dot{x}_4 - b_2\dot{\varphi} - a_2\dot{\theta}) = I_y\ddot{\theta}$$

$$\dot{x} = Ax + Bu \qquad x = \begin{bmatrix} x \\ \dot{x} \\ x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \\ x_3 \\ \dot{x}_3 \\ x_4 \\ \dot{x}_4 \\ \varphi \\ \dot{\varphi} \\ \theta \\ \dot{\theta} \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \end{bmatrix}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{k_1}{m}x_1 + \frac{k_1}{m}x_3 - \frac{k_1 b_1}{m}x_{11} - \frac{k_1 a_1}{m}x_{13} - \frac{k_2}{m}x_1 + \frac{k_2}{m}x_5 + \frac{k_2 b_2}{m}x_{11} - \frac{k_2 a_1}{m}x_{13} - \frac{k_3}{m}x_1 + \frac{k_3}{m}x_7 - \frac{k_3 b_1}{m}x_{11} + \frac{k_3 a_2}{m}x_{13} - \frac{k_4}{m}x_1 + \frac{k_4}{m}x_9 + \frac{k_4 b_2}{m}x_{11} + \frac{k_4 a_2}{m}x_{13}$$

$$- \frac{c_1}{m}x_2 + \frac{c_1}{m}x_4 - \frac{c_1 b_1}{m}x_{12} - \frac{c_1 a_1}{m}x_{14} - \frac{c_2}{m}x_2 + \frac{c_2}{m}x_6 + \frac{c_2 b_2}{m}x_{12} - \frac{c_2 a_1}{m}x_{14} - \frac{c_3}{m}x_2 + \frac{c_3}{m}x_8 - \frac{c_3 b_1}{m}x_{12} + \frac{c_3 a_2}{m}x_{14} - \frac{c_4}{m}x_2 + \frac{c_4}{m}x_{10} + \frac{c_4 b_2}{m}x_{12} + \frac{c_4 a_2}{m}x_{14}$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{k_1}{m_1}x_1 - \frac{k_1}{m_1}x_3 + \frac{k_1 b_1}{m_1}x_{11} + \frac{k_1 a_1}{m_1}x_{13} + \frac{c_1}{m_1}x_2 - \frac{c_1}{m_1}x_4 + \frac{c_1 b_1}{m_1}x_{12} + \frac{c_1 a_1}{m_1}x_{14} - \frac{k_{t1}}{m_1}x_3 \boxed{+ \frac{k_{t1}}{m_1}y_1} + \frac{k_{arbf}}{m_1}x_{11} - \frac{k_{arbf}}{w_f m_1}x_3 + \frac{k_{arbf}}{w_f m_1}x_5$$

$$\dot{x}_5 = x_6$$

$$\dot{x}_6 = \frac{k_2}{m_2}x_1 - \frac{k_2}{m_2}x_5 - \frac{k_2 b_2}{m_2}x_{11} + \frac{k_2 a_1}{m_2}x_{13} + \frac{c_2}{m_2}x_2 - \frac{c_2}{m_2}x_6 - \frac{c_2 b_2}{m_2}x_{12} + \frac{c_2 a_1}{m_2}x_{14} - \frac{k_{t2}}{m_2}x_5 \boxed{+ \frac{k_{t2}}{m_2}y_2} - \frac{k_{arbf}}{m_2}x_{11} + \frac{k_{arbf}}{w_f m_2}x_3 - \frac{k_{arbf}}{w_f m_2}x_5$$

$$\dot{x}_7 = x_8$$

$$\dot{x}_8 = \frac{k_3}{m_3} x_1 - \frac{k_3}{m_3} x_7 + \frac{k_3 b_1}{m_3} x_{11} - \frac{k_3 a_2}{m_3} x_{13} + \frac{c_3}{m_3} x_2 - \frac{c_3}{m_3} x_8 + \frac{c_3 b_1}{m_3} x_{12} - \frac{c_3 a_2}{m_3} x_{14} - \frac{kt_3}{m_3} x_7 + \boxed{\frac{kt_3}{m_3} y_3} + \frac{karbr}{m_3} x_{11} - \frac{karbr}{w_r m_3} x_7 + \frac{karbr}{w_r m_3} x_9$$

$$\dot{x}_9 = x_{10}$$

$$\dot{x}_{10} = \frac{k_4}{m_4} x_1 - \frac{k_4}{m_4} x_9 - \frac{k_4 b_2}{m_4} x_{11} - \frac{k_4 a_2}{m_4} x_{13} + \frac{c_4}{m_4} x_2 - \frac{c_4}{m_4} x_{10} - \frac{c_4 b_2}{m_4} x_{12} - \frac{c_4 a_2}{m_4} x_{14} - \frac{kt_4}{m_4} x_9 + \boxed{\frac{kt_4}{m_4} y_4} - \frac{karbr}{m_4} x_{11} + \frac{karbr}{w_r m_4} x_7 - \frac{karbr}{w_r m_4} x_9$$

$$\dot{x}_{11} = x_{12}$$

$$\dot{x}_{12} = \frac{-b_1 k_1}{I_x} x_1 + \frac{b_1 k_1}{I_x} x_3 - \frac{b_1^2 k_1}{I_x} x_{11} - \frac{b_1 a_1 k_1}{I_x} x_{13} + \frac{b_2 k_2}{I_x} x_1 - \frac{b_2 k_2}{I_x} x_5 - \frac{b_2^2 k_2}{I_x} x_{11} + \frac{b_2 a_1 k_2}{I_x} x_{13} - \frac{b_1 k_3}{I_x} x_1 + \frac{b_1 k_3}{I_x} x_7 - \frac{b_1^2 k_3}{I_x} x_{11} + \frac{b_1 a_2 k_3}{I_x} x_{13} + \frac{b_2 k_4}{I_x} x_1 - \frac{b_2 k_4}{I_x} x_9 - \frac{b_2^2 k_4}{I_x} x_{11} - \frac{b_2 a_2 k_4}{I_x} x_{13}$$

$$\frac{-b_1 c_1}{I_x} x_2 + \frac{b_1 c_1}{I_x} x_4 - \frac{b_1^2 c_1}{I_x} x_{12} - \frac{b_1 a_1 c_1}{I_x} x_{14} + \frac{b_2 c_2}{I_x} x_2 - \frac{b_2 c_2}{I_x} x_6 - \frac{b_2^2 c_2}{I_x} x_{12} + \frac{b_2 a_1 c_2}{I_x} x_{14} - \frac{b_1 c_3}{I_x} x_2 + \frac{b_1 c_3}{I_x} x_8 - \frac{b_1^2 c_3}{I_x} x_{12} + \frac{b_1 a_2 c_3}{I_x} x_{14} + \frac{b_2 c_4}{I_x} x_2 - \frac{b_2 c_4}{I_x} x_{10} - \frac{b_2^2 c_4}{I_x} x_{12} - \frac{b_2 a_2 c_4}{I_x} x_{14}$$

$$- \frac{karbj}{I_x} x_{11} + \frac{karbj}{w_j I_x} x_3 - \frac{karbj}{w_j I_x} x_5 - \frac{karbr}{I_x} x_{11} + \frac{karbr}{w_r I_x} x_7 - \frac{karbr}{w_r I_x} x_9$$

$$\dot{x}_{13} = x_{14}$$

$$\dot{x}_{14} = \frac{-a_1 k_1}{I_y} x_1 + \frac{a_1 k_1}{I_y} x_3 - \frac{a_1 b_1 k_1}{I_y} x_{11} - \frac{a_1^2 k_1}{I_y} x_{13} - \frac{a_1 k_2}{I_y} x_1 + \frac{a_1 k_2}{I_y} x_5 + \frac{a_1 b_2 k_2}{I_y} x_{11} - \frac{a_1^2 k_2}{I_y} x_{13} + \frac{a_2 k_3}{I_y} x_1 - \frac{a_2 k_3}{I_y} x_7 + \frac{a_2 b_1 k_3}{I_y} x_{11} - \frac{a_2^2 k_3}{I_y} x_{13} + \frac{a_2 k_4}{I_y} x_1 - \frac{a_2 k_4}{I_y} x_9 - \frac{a_2 b_2 k_4}{I_y} x_{11} - \frac{a_2^2 k_4}{I_y} x_{13}$$

$$= \frac{-a_1 c_1}{I_y} x_2 + \frac{a_1 c_1}{I_y} x_4 - \frac{a_1 b_1 c_1}{I_y} x_{12} - \frac{a_1^2 c_1}{I_y} x_{14} - \frac{a_1 c_2}{I_y} x_2 + \frac{a_1 c_2}{I_y} x_6 + \frac{a_1 b_2 c_2}{I_y} x_{12} - \frac{a_1^2 c_2}{I_y} x_{14} + \frac{a_2 c_3}{I_y} x_2 - \frac{a_2 c_3}{I_y} x_8 + \frac{a_2 b_1 c_3}{I_y} x_{12} - \frac{a_2^2 c_3}{I_y} x_{14} + \frac{a_2 c_4}{I_y} x_2 - \frac{a_2 c_4}{I_y} x_{10} - \frac{a_2 b_2 c_4}{I_y} x_{12} - \frac{a_2^2 c_4}{I_y} x_{14}$$

$$
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-k_1-k_2-k_3-k_4}{m} & \frac{-c_1-c_2-c_3-c_4}{m} & \frac{k_1}{m} & \frac{c_1}{m} & \frac{k_2}{m} & \frac{c_2}{m} & \frac{k_3}{m} & \frac{c_3}{m} & \frac{k_4}{m} & \frac{c_4}{m} & \frac{-k_1b_1+k_2b_1-k_3b_1+k_4b_2}{m} & \frac{-c_1b_1+c_2b_1-c_3b_1+c_4b_2}{m} & \frac{-k_1a_1+k_2a_1+k_3a_2+k_4a_2}{m} & \frac{-c_1a_1+c_2a_1+c_3a_2+c_4a_2}{m} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{k_1}{m_1} & \frac{c_1}{m_1} & \frac{-k_1-kt_1}{m_1}-\frac{k_a b_1}{w_1 m_1} & -\frac{c_1}{m_1} & \frac{k_a b_1}{w_1 m_1} & 0 & 0 & 0 & 0 & 0 & \frac{k_1b_1+k_a b_1}{m_1} & \frac{c_1 b_1}{m_1} & \frac{k_1 a_1}{m_1} & \frac{c_1 a_1}{m_1} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{k_2}{m_2} & \frac{c_2}{m_2} & \frac{k_a b_1}{w_1 m_2} & 0 & \frac{-k_2-kt_2}{m_2}-\frac{k_a b_1}{w_1 m_2} & -\frac{c_2}{m_2} & 0 & 0 & 0 & 0 & \frac{-k_2b_2-k_a b_1}{m_2} & -\frac{c_2 b_2}{m_2} & \frac{k_2 a_1}{m_2} & \frac{c_2 a_1}{m_2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{k_3}{m_3} & \frac{c_3}{m_3} & 0 & 0 & 0 & 0 & \frac{-k_3-kt_3}{m_3}-\frac{k_a b_r}{w_r m_3} & -\frac{c_3}{m_3} & \frac{k_a b_r}{w_r m_3} & 0 & \frac{k_3b_1+k_a b_r}{m_3} & \frac{c_3 b_1}{m_3} & -\frac{k_3 a_2}{m_3} & -\frac{c_3 a_2}{m_3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
$$


$\frac{k_4}{m_4}$ $\frac{c_4}{m_4}$ 0 0 0 0 $\frac{k_4 b_r}{w_r m_4}$ 0 $\frac{-k_4 - k_{t4}}{m_4} - \frac{k_4 b_r}{w_r m_4}$ $-\frac{c_4}{m_4}$ $\frac{-k_4 b_2 - k_4 b_r}{m_4}$ $-\frac{c_4 b_2}{m_4}$ $-\frac{k_4 a_2}{m_4}$ $-\frac{c_4 a_2}{m_4}$
0 0 0 0 0 0 0 0 0 0 0 1 0 0
$\frac{-b_1 k_1 + b_2 k_2 - b_3 k_3 + b_4 k_4}{I_x}$ $\frac{-b_1 c_1 + b_2 c_2 - b_3 c_3 + b_4 c_4}{I_x}$ $\frac{b_1 k_1}{I_x} + \frac{k_{ab} f}{w_f I_x}$ $\frac{b_1 c_1}{I_x}$ $-\frac{b_2 k_2}{I_x} - \frac{k_{ab} f}{w_f I_x}$ $-\frac{b_2 c_2}{I_x}$ $\frac{b_3 k_3}{I_x} + \frac{k_{ab} r}{w_r I_x}$ $\frac{b_3 c_3}{I_x}$ $\frac{-b_2 k_4}{I_x} - \frac{k_{ab} r}{w_r I_x}$ $-\frac{b_2 c_4}{I_x}$ $\frac{-b_1^2 k_1 - b_2^2 k_2 - b_3^2 k_3 - b_4^2 k_4 - k_{ab} f - k_{ab} r}{I_x}$ $\frac{a_1 a_2 k_1 + b_1 a_2 k_2 + b_3 a_2 k_3 - b_4 a_2 k_4}{I_x}$ $\frac{-b_1^2 c_1 - b_2^2 c_2 - b_3^2 c_3 - b_4^2 c_4}{I_x}$ $\frac{-b_1 a_1 c_1 + b_2 a_1 c_2 + b_3 a_1 c_3 - b_4 a_1 c_4}{I_x}$
0 0 0 0 0 0 0 0 0 0 0 0 1 0
$\frac{-a_1 k_1 - a_2 k_2 + a_3 k_3 + a_4 k_4}{I_y}$ $\frac{-a_1 c_1 - a_2 c_2 + a_3 c_3 + a_4 c_4}{I_y}$ $\frac{a_1 k_1}{I_y}$ $\frac{a_1 c_1}{I_y}$ $\frac{a_1 k_2}{I_y}$ $\frac{a_1 c_2}{I_y}$ $-\frac{a_2 k_3}{I_y}$ $-\frac{a_2 c_3}{I_y}$ $-\frac{a_2 k_4}{I_y}$ $-\frac{a_2 c_4}{I_y}$ $\frac{-a_2 b_1 k_1 + a_2 b_2 k_2 + a_2 b_3 k_3 - a_2 b_4 k_4}{I_y}$ $\frac{-a_1^2 k_1 - a_2^2 k_2 - a_3^2 k_3 - a_4^2 k_4}{I_y}$ $\frac{-a_1 b_1 c_1 + a_1 b_2 c_2 + a_2 b_3 c_3 - a_2 b_4 c_4}{I_y}$ $\frac{-a_1^2 c_1 - a_2^2 c_2 - a_3^2 c_3 - a_4^2 c_4}{I_y}$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{k_{t1}}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{k_{t2}}{m_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{k_{t3}}{m_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{k_{t4}}{m_4} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$